RISK ANALYSIS OF MILITARY OPERATIONS

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1. Introduction

The contemporary war-time military operations can be divided into the following two stages. The first and rather short stage includes very active actions with considerable losses. It is followed by a much longer stage, during which the events associated with losses occur at a much lower rate. The examples of such military operations are the current international military operations in Afghanistan, Iraq and the recent Russian military actions in Chechnya (whatever their political status is). Below, an approach to analysis of the military operations performance during the mentioned above second stage is suggested.

Because the daily losses during the second stage is much less compared to the first stage, the military operations during the second stage, from the reliability standpoint, can be considered as a functioning repairable system, which is rapidly restored after each failure to at least the same condition as it was just before the failure.

In this paper, an analysis of such military operations is developed in the framework of the so-called *repairable system analysis*. The approach can be applied not only to the system failures, but to successes (as the respective adversary's losses) as well.

2. Military Operations as Improving/Deteriorating Repairable Systems

This section begins with introducing some basic notions of repairable system analysis needed for the following discussion.

2.1. Point Processes as Models for Repairable Systems

Basic Definitions

A point process can be informally defined as a mathematical model for highly localized events distributed randomly in time. The major random variable of interest related to such processes is the number of failures (or generally speaking *events*) $N(t)$ observed in the time interval $[0, t]$, which is why such processes are also referred to as *counting processes*. Using the nondecreasing integer-valued function $N(t)$, the point process $\{N(t), t \geq 0\}$ is defined as the one satisfying the following conditions:

1. $N(t) \geq 0$
2. $N(0) = 0$
3. If $t_2 > t_1$, then $N(t_2) \geq N(t_1)$
4. If $t_2 > t_1$, then $[N(t_2) - N(t_1)]$ is the number of events (e.g., failures) occurred in the interval $(t_1, t_2]$

A *trajectory* (*sample path*) or *realization* of a point process is the successive failure times of an item: $T_1, T_2, \ldots, T_k \ldots$. It is expressed in terms of the integer-valued function $N(t)$ i.e., the number of events observed in the time interval $[0, t]$.

$$N(t) = \max(k | T_k \leq t) \tag{1}$$

It is clear that $N(t)$ is a random function. The mean value $E(N(t))$ of the number of failures $N(t)$ observed in the time interval $(0, t]$ is called *cumulative intensity function* (CIF) [Hoyland and Rausand, 1994, Moddares, et al., 1999], or *mean cumulative function* (MCF) [Nelson, 2003] In the following the term *cumulative intensity function* (CIF) is used. The CIF is usually denoted by W, i.e.,

$$W(t) = E(N(t))$$

Another important characteristic of point processes is the *rate of occurrence of failures* (ROCOF), which is defined as the derivative of CIF with respect to time, i.e.

$$w(t) = \frac{dW(t)}{dt}$$

Based on the above definition of ROCOF, the CIF is sometimes called *cumulative* ROCOF.

Most of the processes, which are discussed in the following have monotone ROCOF. The system modeled by a point process with an increasing ROCOF is called *deteriorating* (*aging*, *unhappy*, or *sad*) system. Analogously, the system modeled by a point process with a decreasing ROCOF is called *improving* (*happy* or *rejuvenating*) system.

The distribution of time to the first failure of a point process is called *the underlying distribution*. For some point processes this distribution coincides with the distribution of time between successive failures (which is also called *interarrival time*), for others it does not. The *underlying distribution* is included in definition of any particular point process used as a model for failure/repair process of reparable systems.

2.2 Nonhomogeneous Poisson Process

A point process, having independent increments, is called Nonhomogeneous Poisson Process (NHPP) with time dependent ROCOF $\lambda(t) > 0$, if the probability that exactly n events (failures) occur in any interval (a , b] has the Poisson distribution with the mean equal to $\int_a^b \lambda(t)dt$, i.e.,

$$P[N(b) - N(a) = n] = \frac{\left[\int_a^b \lambda(t)dt\right]^n e^{-\int_a^b \lambda(t)dt}}{n!} \tag{2}$$

for $n = 0, 1, 2, \ldots,$, and $N(0) = 0$.

In opposite to some other point processes, the times between successive events (e.g., failures) in the frame work of the NHPP model are neither independent nor identically distributed.

Based on the above definition, the CIF and ROCOF of NHPP obviously can be written as follows

$$W(t) = \int_0^t \lambda(\tau)d\tau \tag{3}$$

$$w(t) = \lambda(t) \tag{4}$$

The cumulative distribution function (CDF) of time to the *first* failure (i.e., the CDF of underlying distribution) for the NHPP can be found as

$$F(t) = 1 - \Pr[N(t) - N(0) = 0] = 1 - \exp(-W(t)) \tag{5}$$

where W(t) is given by (3).

Let's consider a series of failures occurring according to the NHPP with ROCOF $\lambda(t)$. Let t_k be time to kth failure, so, at this moment, the ROCOF is equal to $\lambda(t_k)$. The probability that no failure occurs in interval $(t_k, t]$, where $t > t_k$, can be written as

$$R(t_k, t) = e^{-\int_{t_k}^t \lambda(\tau)d\tau} = \frac{e^{-\int_0^t \lambda(\tau)d\tau}}{e^{-\int_0^{t_k} \lambda(\tau)d\tau}} = \frac{R(t)}{R(t_k)} \tag{6}$$

which is the conditional reliability function of a system having age t_k. In other words, we can consider NHPP as a process in which each failed system is instantaneously replaced by *identical one having the same age as the failed one*. This type of restoration model is referred to *same as old* (or *minimal repair*) condition.

Another very important property of NHPP under the given assumptions follows from Equation (6). If t_k is equal to zero, Equation (6) takes on the following form

$$R(t) = \exp(-\int_0^t \lambda(\tau))d\tau\ ), \tag{7}$$

which means that the ROCOF of NHPP coincides with the failure (hazard) rate function of the underlying distribution. In other words, all future behavior of repairable system is completely defined by this distribution. It also means that just after any repair/maintenance action carried out at time t, the

ROCOF is equal to the failure rate of the TTFF distribution $\lambda(t)$. So, we can also consider NHPP as a process, in which each failed system is instantaneously replaced by *identical one having the same failure rate as the failed one*.

Is "same as old" restoration a realistic assumption? The answer depends on a given application. Applied to a one-component system, it is definitely not a realistic assumption. For a complex system, composed of many components having close reliability functions, this assumption is rather realistic, because only a small fraction of the system components is repaired or replaced, which results in a small change of the system failure rate (Hoyland and Rausand, 1994). This can be definitely applied to the military operations considered in the following.

An important particular case of NHPP is the case when the CIF a power function of time, i.e,

$$W(t) = \left(\frac{t}{\alpha}\right)^{\beta} \quad t \geq 0, \quad \alpha, \beta > 0, \tag{8}$$

with the respective ROCOF given by

$$w(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} \quad t \geq 0, \alpha, \beta > 0, \tag{9}$$

which obviously results in the Weibull time to the first failure (underlying) distribution. Accordingly, the NHPP process with ROCOF given by (9) is sometimes referred to as the Weibull process, the power law NHPP process, or the "Crow-AMSAA model". It is clear that, if $\beta > 1$ the system is improving, and in the case of $\beta < 1$, the system is deteriorating.

Statistical procedures for this model were developed by Crow (1974, 1982), based on suggestions of Duane (1964). These procedures can also be found in MIL-HDBK-781, IEC Standard 1164 (1995). The main applications of the power law model are associated with reliability monitoring (which is optimistically called "reliability growth") of repairable products and as well as for non-reparable ones.

3. Case Study: US Military Operations in Iraq after Fall of Baghdad

3.1 Data Source

The data used for this case study are available from the website http://icasualties.org/oif/Details.aspx, which is called *Iraq Coalition Casualties*. According to the website, the data are being accumulated using the information from the following U.S. government sources:

1. CENTCOM (the United States Central Command in Tampa, FL), which news releases are published regularly on the Internet at http://www.centcom.mil/
2. News releases from the U.S. Department of Defense, that can be found on the Internet at http://www.defenselink.mil/releases/

3.2 Data Analysis

The above suggested approach to risk analysis of military operations is illustrated by an analysis of the US military operations in Iraq (the system). The performance of this system is considered with respect of two failure modes – *fatalities* (the weekly number of US soldiers killed) and *accidents*. In the first part of this case study, the system is analyzed with respect of the fatalities.

In the given context, the data related to fatalities include the following four causes of death:

1. Hostile fire
2. Accidents
3. Friendly fire
4. Other

In the second part of our case study, the system performance with respect to the failure mode called *accidents* is analyzed. Using the respective data analysis, it will be shown that in spite of slowly improving system performance with respect of the second failure mode (accidents), the system is slowly deteriorating with respect to the first failure mode (fatalities), which definition includes accidents as well.

3.2.1 Fatalities after Fall of Baghdad

Figure 1 below depicts the cumulative number (ECIF) of fatalities during first 10 weeks of military operations.

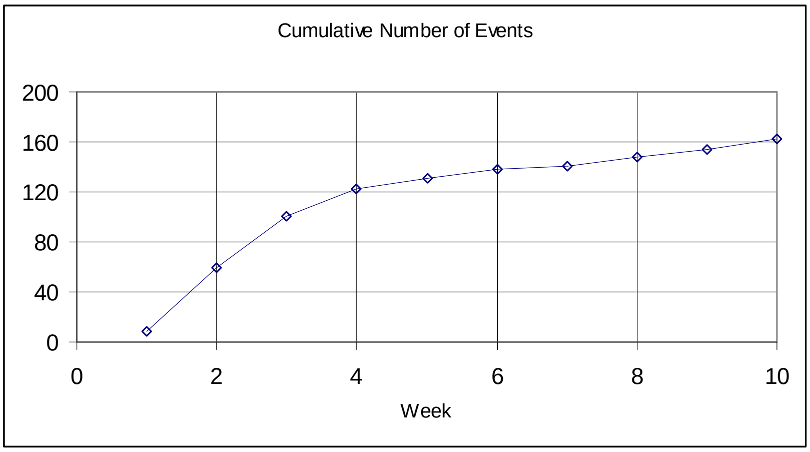

Figure 1. Cumulative Number of Fatalities during First 10 Weeks of Military Operations

The ECIF increases rapidly during first 4 weeks, and afterwards it slows down. The 4th week (04/06/03 – 04/12/03) includes the fall of Baghdad, so the first four weeks is the period of very active military operations with inevitably high losses (i.e., it is the first phase discussed in Introduction). The following case study is going to cover the military operations after this 4 week interval.

The plot of time dependence of cumulative number of fatalities during the time interval between the 5th and 151st week (04/13/2003 -- 02/04/2006) is presented in Figure 2. The data were fitted using the NHPP power low model (Equation (8). The fitted model for CIF is given by the following equation:

$$N(t) = 4.622(t-4)^{1.224}, \quad t \geq 5 \qquad (10)$$

The fitted cumulative number of fatalities (CIF) is shown as a continuous curve depicted in Figure 2 below.

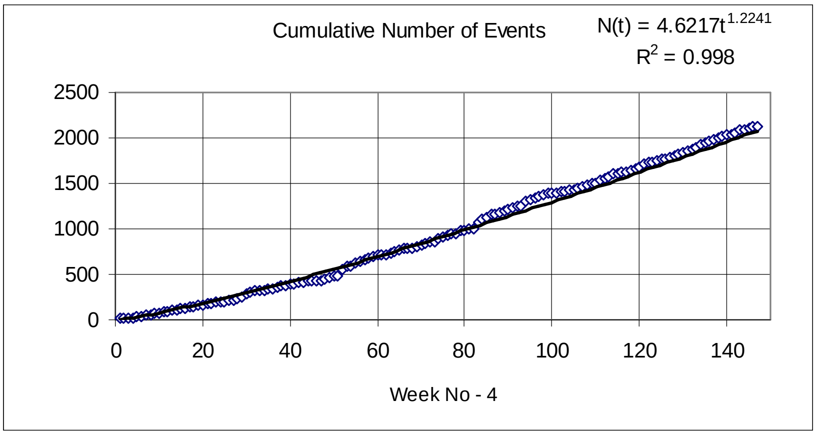

Figure 2. Cumulative number of fatalities during the time interval between the fifth and 151st week (04/13/2003 - 02/04/2006)

The squared correlation coefficient R^2 (proportion of the variance of $N(t)$ explained by t) is 0.998, which shows that the fitted model is adequate enough.

The parameter β of model (10) shows that the ROCOF of fatalities is not even constant, but slowly increasing ($\beta > 1$), which means that the system of military operation is slowly deteriorating with respect to the “fatalities” failure mode. Being on an optimistic side (in spite of the 2 sigma interval of (1.216, 1.232) on β), one can state with a very high confidence that the system performance with respect to the given failure mode (fatalities) is not improving.

The reader can notice a slight jump in ECIF at some time about 87th – 88th weeks. This increase might be explained by the emerging insurgent activity in September – October 2004. This could suggest breaking down the data into two subsets – before 87th week and after it, for a further analysis. The respective analysis is beyond the given paper scope.

3.2.2 Accidents from the Beginning of Military Operations

Now we are going to analyze the system performance with respect to the *accidents* only, which constitute one out four failure modes related to the *fatalities* considered in the previous section.

Figure 3 below depicts the cumulative number (ECIF) of accidents during the complete time interval between the first week of operations (03/16/2003 - 03/22/2003) and 151st week (01/29/2006 - 02/04/2006). The 98th week is definitely a “discontinuity point.” During this week, there were 43 accidental fatalities. Most of the respective causes were identified as "non-hostile – helicopter crash." There were two helicopter crashes during this week.

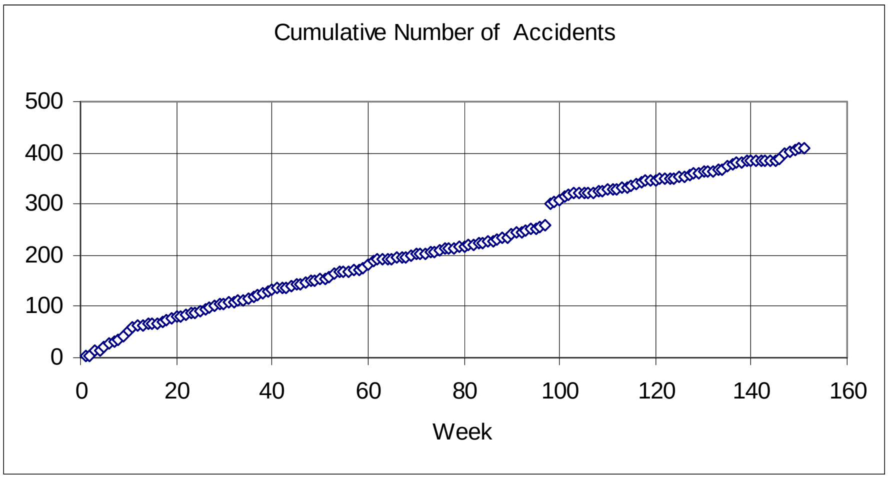

Figure 3. Cumulative number of fatalities during the time interval between the first week of operations (03/16/2003 - 03/22/2003) up to 151st week (01/29/2006 - 02/04/2006)

Due to the discontinuity point at 98th week, it is reasonable to break the data down into two subsets – before 98th week and after it. The plots of the respective ECIFs are shown in Figure 4 and in Figure 5 respectively. The fitted curves of the cumulative number of accidents (CIF) are shown as the continuous curves in the figures.

For the first time interval, the fitted model for CIF is given by the following equation:

$$N(t) = 4.917t^{0.884}, \tag{11}$$

The squared correlation coefficient R^2 (proportion of the variance of $N(t)$ explained by t) is 0.962, which shows that the fitted model is rather adequate. The parameter β = 0.884 indicates that the system is slowly improving with respect to the "accidents" failure mode.

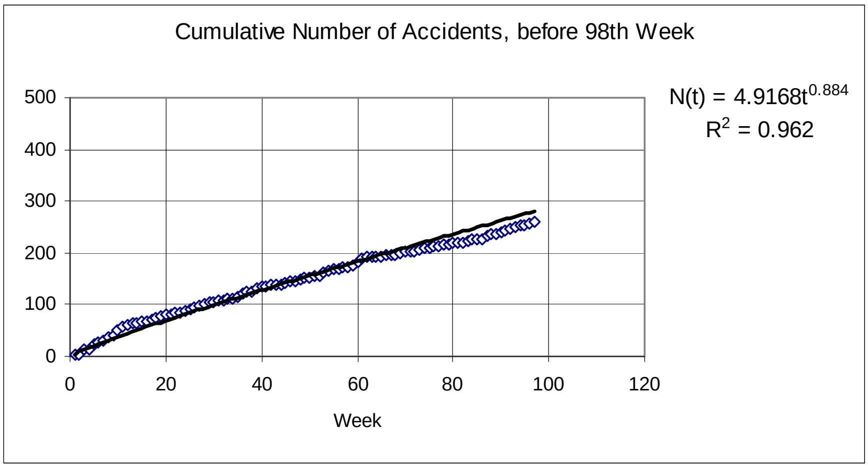

Figure 4. Cumulative number of accidents from the beginning the military operations up to 97th week (01/16/2005 - 01/22/2005)

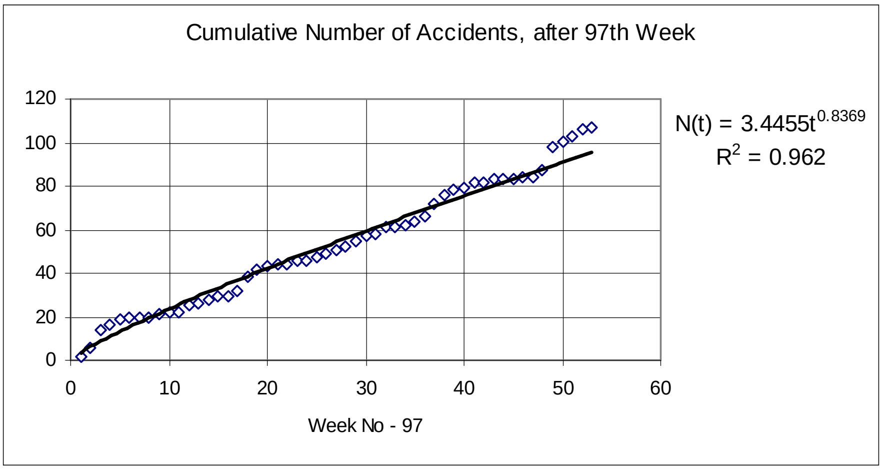

Figure 5. Cumulative number of accidents from the beginning the military operations after 97th week

For the second time interval (after the 97th week), the fitted model for CIF is given by the following equation:

$$N(t) = 3.445t^{0.834} \tag{12}$$

The equation reveals that the system continues to improve with a possible slight decrease in the accidence rate of occurrence, compared to the one related to the first interval, which might indicate that some safety improvements have been done.

The squared correlation coefficient R^2 for model (12) is the same as the one related to model (11), which shows that both models have approximately the same accuracy. Both models have the parameter $\beta < 1$.

Based on the above analysis one can come to the following conclusion. In spite of slowly improving system performance with respect to the "accidents", as a failure mode, the system is slowly deteriorating with respect to the "fatalities" failure mode, which definition includes accidents as a specific case.

4. Concluding Remarks

The considered case study shows that the risk analysis of military operations can be performed in the framework of traditional repairable system data analysis using the notions of improving and deteriorating systems. This approach can be suggested as a potentially useful tool among other decision making support tools and techniques.

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