SENSITIVITY ANALYSIS OF OPTIMAL COUNTER-TERRORISM RESOURCES ALLOCATION UNDER SUBJECTIVE EXPERT ESTIMATES

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1. Preliminary

The problem of optimal resources allocation for counter-terrorism measures is naturally based on subjective estimates made by experts in this area. In the future we will continue the word “optimal allocation” though in this case it would be more correct to say about “rational allocation” due to not accurate mathematical model and unreliable input data. Nevertheless, dealing with a mathematical model we will continue to say about “optimal allocation”.

Unreliability of the data is inevitable: there is no possibility of “experiments” to get, for instance, such parameters like expected damage of a terrorist act or level of protection of an object due to using some defending measures.

So, making decision for planning counter-terrorism operations we are forced to rely on experts’ opinions that could supply us with extremely uncertain input data.

The objective of this presentation is analysis of sensitivity of solutions of the optimal allocation problem under uncertainty of experts’ estimates.

2. Analysis of solution sensitivity: Expenses estimates variation

Let us analyze first how variation of expenses estimates influence on the solution on the level of a single object. For simplicity, we avoid to consider the influence of defense on the Federal and State levels. Assume that we have three variants of estimates: lower, middle and upper as it presented in the table below. Here the lower estimates are 20% lower of the corresponding middle estimates, and the upper ones are 20% higher.

Case of lower estimates

OBJECT-1		Vuln.	ΔE
Act-1	Measure-11	0,25	0,8
	Measure-12	0,2	2
	Measure-13	0,1	4
	Measure-14	0,01	3,2
Act-2	Measure-21	0,2	1,6
	Measure-22	0,16	0,8
	Measure-23	0,07	3,2
	Measure-24	0,02	5,6
Act-3	Measure-31	0,11	4
	Measure-32	0,1	2
	Measure-33	0,05	2,4
	Measure-34	0,04	1,6
	Measure-35	0,01	5,6

Case of middle estimates

OBJECT-1		Vuln.	ΔE
Act-1	Measure-11	0,25	1
	Measure-12	0,2	2.5
	Measure-13	0,1	5
	Measure-14	0,01	4
Act-2	Measure-21	0,2	2
	Measure-22	0,16	1
	Measure-23	0,07	4
	Measure-24	0,02	7
Act-3	Measure-31	0,11	5
	Measure-32	0,1	2.5
	Measure-33	0,05	3
	Measure-34	0,04	2
	Measure-35	0,01	7

Case of upper estimates

OBJECT-1		Vuln.	ΔE
Act-1	Measure-11	0,25	1,2
	Measure-12	0,2	3
	Measure-13	0,1	6
	Measure-14	0,01	4,8
Act-2	Measure-21	0,2	2,4
	Measure-22	0,16	1,2
	Measure-23	0,07	4,8
	Measure-24	0,02	8,4
Act-3	Measure-31	0,11	6
	Measure-32	0,1	3
	Measure-33	0,05	3,6
	Measure-34	0,04	2,4
	Measure-35	0,01	8,4

In these tables the last column contains expenses increments for applying the current protection measure.

Using the method of equal defense levels against every type of terrorist attack, suggested in the previous presentation, we build the "trajectory" of object protection improvement. For the sake of brevity, we consider only the case of middle estimates.

Case of lower estimates

Object 1			
Step Number	**Undertaken measures**	**Resulting Vulnerability**	**Total Expenses**
1	M-11, M-21, M-31	max {**0.25**, 0.2, 0.11}=0.25	0.8+1.6+4=6.4
2	**M-12**, M-21, M-31	max {**0.2**, **0.2**, 0.11}=0.2	6.4+2=8.4
3	**M-13**, **M-22**, M-31	max {0.1, **0.16**, 0.11}=0.16	8.4+4+0.8=13.2
4	M-13, **M-23**, M-31	max {0.1, 0.07, **0.11**}=0.11	13.2+3.2=16.4
5	M-13, M-23, **M-32**	max {**0.1**, 0.07, **0.1**}=0.1	16.4+2=18.4
6	**M-14**, M-23, **M-33**	max {0.01, **0.07**, 0.05}=0.07	18.4+3.2+2.4=24
7	M-14, **M-24**, M-33	max {0.01, 0.02, **0.05**}=0.05	24+5.6=29.6
8	M-14, M-24, **M-34**	max {0.01, 0.02, **0.04**}=0.04	29.6+1.6=31.2
9	M-14, M-24, **M-35**	max {0.01, **0.02**, 0.01}=0.02	31.2+5.6=36.8

Case of middle estimates

Object 1			
Step Number	**Undertaken measures**	**Resulting Vulnerability**	**Total Expenses**
1	M-11, M-21, M-31	max {**0.25**, 0.2, 0.11}=0.25	1+2+5=8
2	**M-12**, M-21, M-31	max {**0.2**, **0.2**, 0.11}=0.2	8+2.5=10.5
3	**M-13**, **M-22**, M-31	max {0.1, **0.16**, 0.11}=0.16	10.5++1+5=16.5
4	M-13, **M-23**, M-31	max {0.1, 0.07, **0.11**}=0.11	16.5+4=20.5
5	M-13, M-23, **M-32**	max {**0.1**, 0.07, 0.1}=0.1	20.5+2.5=23
6	**M-14**, M-23, **M-33**	max {0.01, **0.07**, 0.05}=0.07	23+4+3=30
7	M-14, **M-24**, M-33	max {0.01, 0.02, **0.05**}=0.05	30+7=37
8	M-14, M-24, **M-34**	max {0.01, 0.02, **0.04**}=0.04	37+2=39
9	M-14, M-24, **M-35**	max {0.01, **0.02**, 0.01}=0.02	39+7=46

Case of upper estimates

Object 1			
Step Number	**Undertaken measures**	**Resulting Vulnerability**	**Total Expenses**
1	M-11, M-21, M-31	max {**0.25**, 0.2, 0.11}=0.25	1.2+2.4+6=9.6
2	**M-12**, M-21, M-31	max {**0.2**, **0.2**, 0.11}=0.2	9.6+3=12.6
3	**M-13**, **M-22**, M-31	max {0.1, **0.16**, 0.11}=0.16	12.6+6+1.2=19.8
4	M-13, **M-23**, M-31	max {0.1, 0.07, **0.11**}=0.11	19.8+4.8=24.6
5	M-13, M-23, **M-32**	max {**0.1**, 0.07, **0.1**}=0.1	24.6+3=27.6
6	**M-14**, M-23, **M-33**	max {0.01, **0.07**, 0.05}=0.07	27.6+4.8+3.6=36
7	M-14, **M-24**, M-33	max {0.01, 0.02, **0.05**}=0.05	36+8.4=44.4
8	M-14, M-24, **M-34**	max {0.01, 0.02, **0.04**}=0.04	44.4+2.4=46.8
9	M-14, M-24, **M-35**	max {0.01, **0.02**, 0.01}=0.02	46.8+8.4=55.2

Such a table gives a possibility to find what measures should be undertaken for each required level of protection (or admissible level of vulnerability) and given limited resources.

The final “trajectory” of the “Expenses vs. Vulnerability” dependency s presented below.

Case of lower estimates

Sum. Exp.	Vulner-ability
6,4	0,25
8,4	0,2
13,2	0,16
16,4	0,11
18,4	0,1
24	0,07
29,6	0,05
31,2	0,04
36,8	0,02

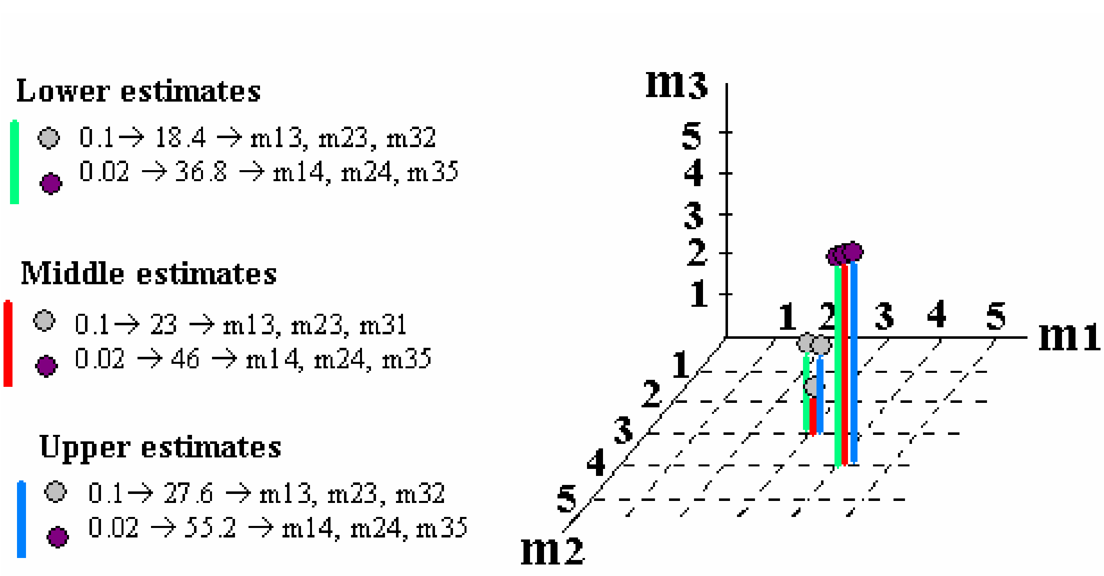
Case of middle estimates

Sum. Exp.	Vulner-ability
8	0,25
10,5	0,2
16,5	0,16
20,5	0,11
23	0,1
30	0,07
37	0,05
39	0,04
46	0,02

Case of upper estimates

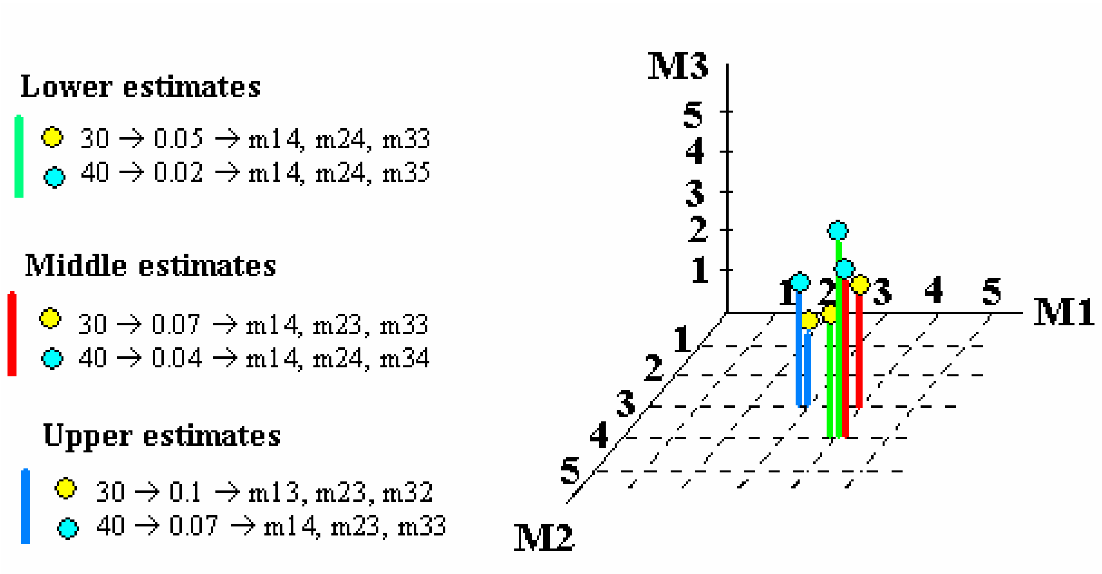
Sum. Exp.	Vulner-ability
9,6	0,25
12,6	0,2
19,8	0,16
24,6	0,11
27,6	0,1
36	0,07
44,4	0,05
46,8	0,04
55,2	0,02

Using the tables above, consider two solutions of the **Direct Problem** with required levels of vulnerability 0.1 and 0.02.



On the right, it is shown what measures correspond to these two solutions of the Direct Problem. One can see that with ± 20% variation of the values of expenses, vectors of solutions (that is, chosen measures) remain the same.

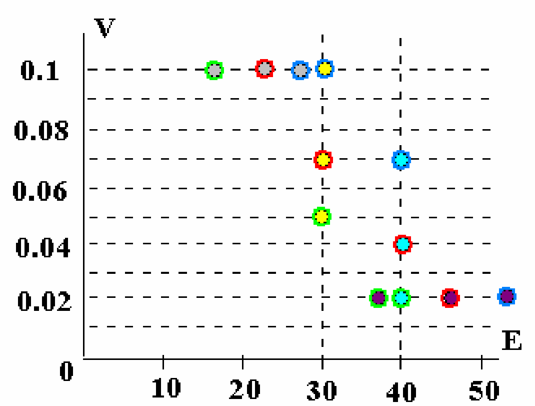
For the **Inverse Problem**, let us choose two fixed levels of admissible expenses: 30 c.c.u. and 40 c.c.u.



On the right, it is shown what measures correspond to these two solutions of the Inverse Problem. One can see that with the same variation of the values of expenses, vectors of solutions remain the same.

It means that relatively large dispersion of input data practically does not influence on the choice of the set of counter-terrorist measures.

Of course it does not mean that “Vulnerability-Expenses” dependence stays the same: it varies significantly.



It means that, for instance, when the upper estimates of expenses are used, the calculated level of protection will be low and a decision-maker could ask for more resources though it is, probably, unnecessary.

Naturally, for the direct problem solutions the difference between the final expenses remains ± 20%. At the same time the Inverse Problem solutions have significant dispersion due to a non-linear character of the “Vulnerability-Expenses” dependence. It can be seen from the figure above that corresponding “horizontal” dots lay closer than “vertical” ones. However, if one choose more natural scale for vulnerability, namely, logarithmical one, then “linear distance” between vertical dots on such a scale will be less significant.

The simple qualitative analysis of obtained data shows us that the solutions of both Direct and Inverse Problems in the case of expenses variation is stable enough.

3. Analysis of solution sensitivity: Vulnerability estimates variation

Let us conduct analogous reasoning for the case, when the estimations of vulnerability level of object are overstated (they are understated) by experts.

Case of lower estimates

OBJECT-1		Vuln.	Exp.
Act-1	Measure-11	0,200	1
	Measure-12	0,160	2.5
	Measure-13	0,080	5
	Measure-14	0,008	4
Act-2	Measure-21	0,160	2
	Measure-22	0,128	1
	Measure-23	0,056	4
	Measure-24	0,016	7
Act-3	Measure-31	0,088	5
	Measure-32	0,080	2.5
	Measure-33	0,040	3
	Measure-34	0,032	2
	Measure-35	0,008	7

Case of middle estimates

OBJECT-1		Vuln.	Exp.
Act-1	Measure-11	0,25	1
	Measure-12	0,2	2.5
	Measure-13	0,1	5
	Measure-14	0,01	4
Act-2	Measure-21	0,2	2
	Measure-22	0,16	1
	Measure-23	0,07	4
	Measure-24	0,02	7
Act-3	Measure-31	0,11	5
	Measure-32	0,1	2.5
	Measure-33	0,05	3
	Measure-34	0,04	2
	Measure-35	0,01	7

Case of upper estimates

OBJECT-1		Vuln.	Exp.
Act-1	Measure-11	0,3	1
	Measure-12	0,24	2.5
	Measure-13	0,12	5
	Measure-14	0,012	4
Act-2	Measure-21	0,24	2
	Measure-22	0,192	1
	Measure-23	0,084	4
	Measure-24	0,024	7
Act-3	Measure-31	0,132	5
	Measure-32	0,12	2.5
	Measure-33	0,06	3
	Measure-34	0,048	2
	Measure-35	0,012	7

Case of lower estimates

Object 1			
Step Number	**Undertaken measures**	**Resulting Vulnerability**	**Total Expenses**
1	M-11, M-21, M-31	max {**0.2**, 0.16, 0.088}=0.2	1+2+5=8
2	**M-12**, M-21, M-31	max {**0.16**, **0.16**, 0.088}=0.16	8+2.5=10.5
3	**M-13**, **M-22**, M-31	max {0.08, **0.128**, 0.088}=0.128	10.5+5+1=16.5
4	M-13, **M-23**, M-31	max {0.08, 0.056, **0.088**}=0.088	16.5+4=20.5
5	M-13, M-23, **M-32**	max {**0.08**, 0.056, **0.08**}=0.08	20.5+2.5=23
6	**M-14**, M-23, **M-33**	max {0.008, **0.056**, 0.04}=0.056	23+4+3=30
7	M-14, **M-24**, M-33	max {0.008, 0.016, **0.04**}=0.04	30+7=37
8	M-14, M-24, **M-34**	max {0.008, 0.016, **0.032**}=0.032	37+2=39
9	M-14, M-24, **M-35**	max {0.008, **0.016**, 0.008}=0.016	39+7=46

Case of middle estimates

Object 1			
Step Number	**Undertaken measures**	**Resulting Vulnerability**	**Total Expenses**
1	M-11, M-21, M-31	max {**0.25**, 0.2, 0.11}=0.25	1+2+5=8
2	**M-12**, M-21, M-31	max {**0.2**, **0.2**, 0.11}=0.2	8+2.5=10.5
3	**M-13**, **M-22**, M-31	max {0.1, **0.16**, 0.11}=0.16	10.5++1+5=16.5
4	M-13, **M-23**, M-31	max {0.1, 0.07, **0.11**}=0.11	16.5+4=20.5
5	M-13, M-23, **M-32**	max {**0.1**, 0.07, **0.1**}=0.1	20.5+2.5=23
6	**M-14**, M-23, **M-33**	max {0.01, **0.07**, 0.05}=0.07	23+4+3=30
7	M-14, **M-24**, M-33	max {0.01, 0.02, **0.05**}=0.05	30+7=37
8	M-14, M-24, **M-34**	max {0.01, 0.02, **0.04**}=0.04	37+2=39
9	M-14, M-24, **M-35**	max {0.01, **0.02**, 0.01}=0.02	39+7=46

Case of upper estimates

Object 1			
Step Number	**Undertaken measures**	**Resulting Vulnerability**	**Total Expenses**
1	M-11, M-21, M-31	max {**0.3**, 0.24, 0.132}=0.3	1+2+5=8
2	**M-12**, M-21, M-31	max {**0.24**, **0.24**, 0.132}=0.24	8+2.5=10.5
3	**M-13**, **M-22**, M-31	max {0.12, **0.192**, 0.132}=0.192	10.5++1+5=16.5
4	M-13, **M-23**, M-31	max {0.12, 0.084, **0.132**}=0.132	16.5+4=20.5
5	M-13, M-23, **M-32**	max {0.12, 0.084, **0.12**}=0.12	20.5+2.5=23
6	**M-14**, M-23, **M-33**	max {0.012, **0.084**, 0.06}=0.084	23+4+3=30
7	M-14, **M-24**, M-33	max {0.012, 0.024, **0.06**}=0.06	30+7=37
8	M-14, M-24, **M-34**	max {0.012, 0.024, **0.048**}=0.048	37+2=39
9	M-14, M-24, **M-35**	max {0.012, **0.024**, 0.012}=0.024	39+7=46

Case of lower estimates

Sum. Exp.	Vulner-ability
8	0,200
10,5	0,160
16,5	0,128
20,5	0,088
23	0,080
30	0,056
37	0,040
39	0,032
46	0,016

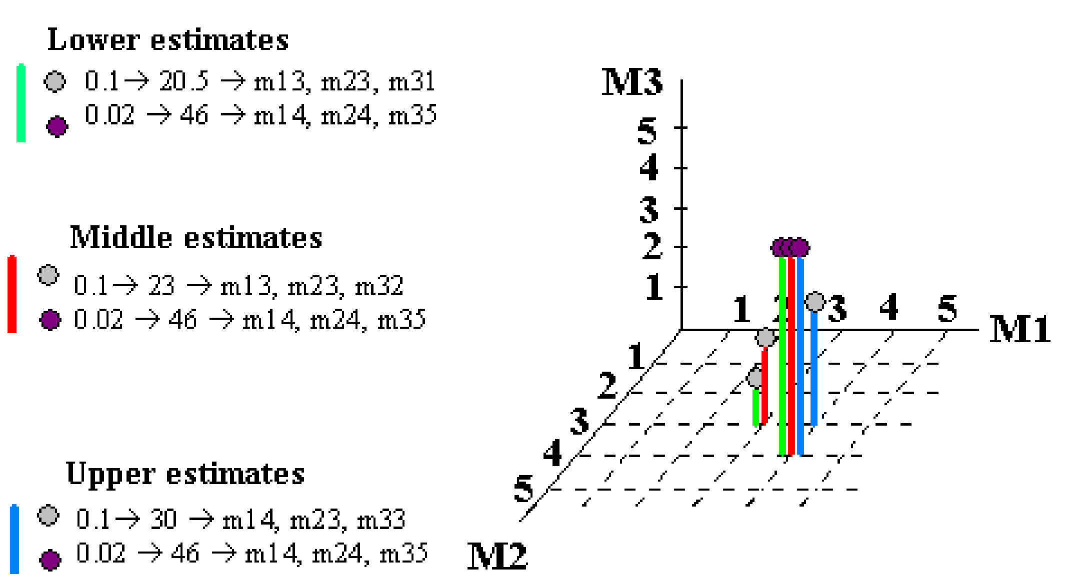
Case of middle estimates

Sum. Exp.	Vulner-ability
8	0,25
10,5	0,2
16,5	0,16
20,5	0,11
23	0,1
30	0,07
37	0,05
39	0,04
46	0,02

Case of upper estimates

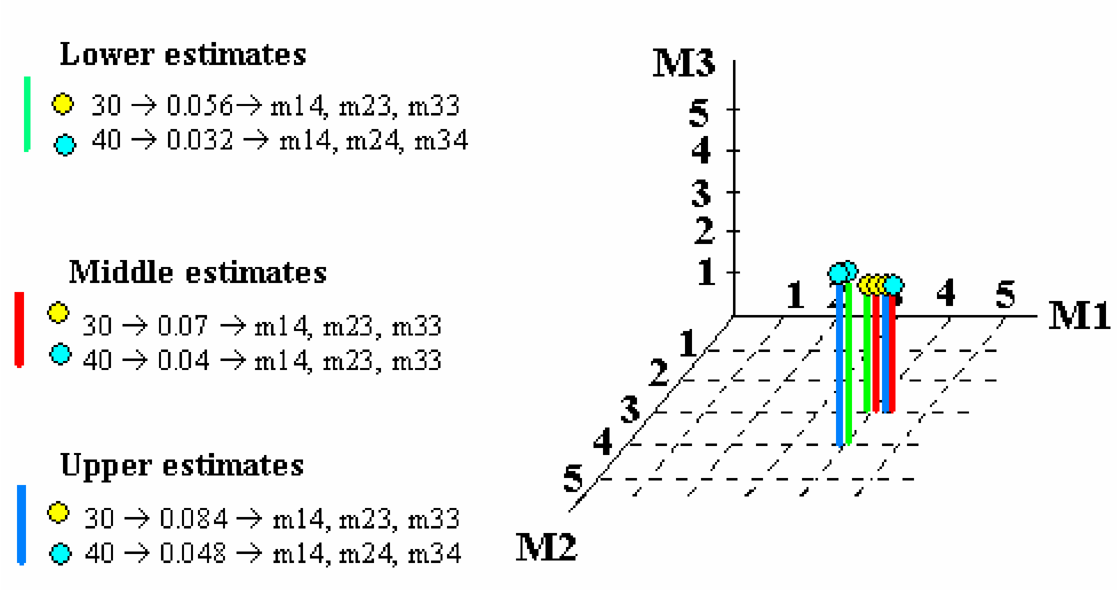
Sum. Exp.	Vulner-ability
8	0,3
10,5	0,24
16,5	0,192
20,5	0,132
23	0,12
30	0,084
37	0,06
39	0,048
46	0,024

Using the tables above, consider two solutions of the **Direct Problem** with the same required levels of vulnerability 0.1 and 0.02, as it was in the previous section.



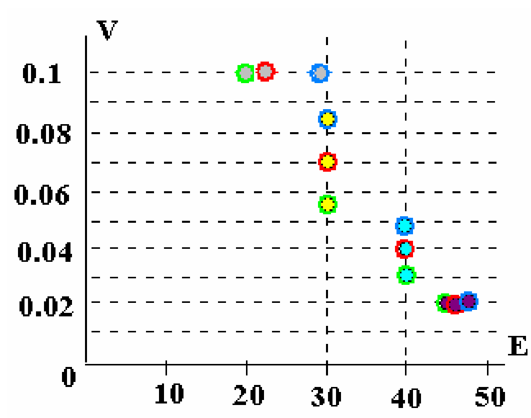
On the right, it is shown what measures correspond to these two solutions of the Direct Problem. One can see that with ±20% variation of the values of vulnerability, vectors of solutions remain almost the same.

For the **Inverse Problem**, let us choose again the same two fixed levels of admissible expenses: 30 c.c.u. and 40 c.c.u.



On the right, it is shown what measures correspond to these two solutions of the Inverse Problem. One can see that again, as in the precious section, ±20% variation of the values of vulnerability, vectors of solutions remain almost the same.

In this case, "Vulnerability-Expenses" dependence also varies significantly, especially for the solutions of the Inverse Problem.



For this case, one can make approximately the same conclusions, as in the previous case.

4. Conclusion

The presented analysis shows that presented model of optimal allocation of counter-terrorism resources, suggested in the previous report, is working stably enough.

Development of improved computer model will allow analyzing more realistic situations, including random insensitivity of input data. However, it seems that such "one-side biased" expert estimates should lead to more serious errors than random variations of the parameters.

References

1. **I.Ushakov.** Counter-terrorism: Protection Resources Allocation. Part I. Minimax Criterion. *Reliability: Theory and Applications*" (vol.1, No.2), 2006.
2. **I.Ushakov.** "Counter-terrorism: Protection Resources Allocation. Part II. Branching System. *Reliability: Theory and Applications* (vol.1, No.3), 2006.