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On multi-state safety analysis in shipping

Keywords

safety, multi-state system, operation process, complex system, shipping

Abstract

A multi-state approach to defining basic notions of the system safety analysis is proposed. A system safety function and a system risk function are defined. A basic safety structure of a multi-state series system of components with degrading safety states is defined. For this system the multi-state safety function is determined. The proposed approach is applied to the evaluation of a safety function, a risk function and other safety characteristics of a ship system composed of a number of subsystems having an essential influence on the ship safety. Further, a semi-markov process for the considered system operation modelling is applied. The paper also offers a general approach to the solution of a practically important problem of linking the multi-state system safety model and its operation process model. Finally, the proposed general approach is applied to the preliminary evaluation of a safety function, a risk function and other safety characteristics of a ship system with varying in time its structure and safety characteristics of the subsystems it is composed of.

1. Introduction

Taking into account the importance of the safety and operating process effectiveness of technical systems it seems reasonable to expand the two-state approach to multi-state approach in their safety analysis [2]. The assumption that the systems are composed of multi-state components with safety states degrading in time gives the possibility for more precise analysis and diagnosis of their safety and operational processes' effectiveness. This assumption allows us to distinguish a system safety critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operational process effectiveness. Then, an important system safety characteristic is the time to the moment of exceeding the system safety critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state safety function that is a basic characteristic of the multi-state system. Determining the multi-state safety function and the risk function of systems on the base of their components' safety functions is then the main research problem. Modelling of complicated systems operations' processes is difficult mainly because of large number of operations states and impossibility of precise describing of changes between these states. One of the useful approaches in modelling of these complicated processes is applying the semi-markov model [3]. Modelling of multi-state systems' safety and linking it with semi-markov model of these systems' operation processes is the main and practically important research problem of this paper. The paper is devoted to this research problem with reference to basic safety structures of technical systems [9], [10] and particularly to safety analysis of a ship series system [5] in variable operation conditions. This new approach to system safety investigation is based on the multi-state system reliability analysis considered for instance in [1], [4], [6], [7], [8], [11] and on semi-markov processes modelling discussed for instance in [3].

2. Basic notions

In the multi-state safety analysis to define systems with degrading components we assume that:
- n is the number of system's components,
- E_i, $i = 1,2,...,n$, are components of a system,
- all components and a system under consideration have the safety state set $\{0,1,...,z\}$, $z \ge 1$,
- the safety state indexes are ordered, the state 0 is the worst and the state z is the best,
- $T_i(u)$, $i = 1,2,...,n$, are independent random variables representing the lifetimes of components E_i in the safety state subset $\{u,u+1,...,z\}$, while they were in the state z at the moment $t = 0$,
- $T(u)$ is a random variable representing the lifetime of a system in the safety state subset $\{u,u+1,...,z\}$ while it was in the state z at the moment $t = 0$,
- the system and its components safety states degrade with time t,
- $E_i(t)$ is a component E_i safety state at the moment t, $t \in < 0,\infty)$.
- $S(t)$ is a system safety state at the moment t, $t \in < 0,\infty)$.

The above assumptions mean that the safety states of the system with degrading components may be changed in time only from better to worse. The way in which the components and the system safety states change is illustrated in *Figure 1*.

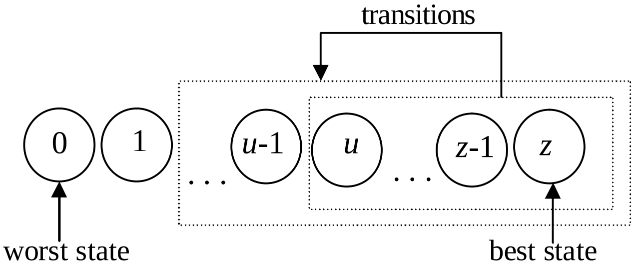

Figure 1. Illustration of a system and components safety states changing

The basis of our further considerations is a system component safety function defined as follows.

Definition 1. A vector

$$s_i(t,\cdot) = [s_i(t,0), s_i(t,1),..., s_i(t,z)],\ t \in < 0,\infty),\quad i = 1,2,\dots,n, \tag{1}$$

where

$$s_i(t,u) = P(E_i(t) \ge u \mid E_i(0) = z) = P(T_i(u) > t) \tag{2}$$

for $t \in < 0,\infty)$, $u = 0,1,...,z$, $i = 1,2,\dots,n$, is the probability that the component E_i is in the state subset $\{u, u+1,..., z\}$ at the moment t, $t \in < 0,\infty)$, while it was in the state z at the moment $t = 0$, is called the multi-state safety function of a component E_i.

Similarly, we can define a multi-state system safety function.

Definition 2. A vector

$$\boldsymbol{s}_n(t,\cdot) = [\boldsymbol{s}_n(t,0), \boldsymbol{s}_n(t,1),..., \boldsymbol{s}_n(t,z)],\ t \in < 0,\infty), \tag{3}$$

where

$$\boldsymbol{s}_n(t,u) = P(S(t) \ge u \mid S(0) = z) = P(T(u) > t) \tag{4}$$

for $t \in < 0,\infty)$, $u = 0,1,...,z$, is the probability that the system is in the state subset $\{u, u+1,..., z\}$ at the moment t, $t \in < 0,\infty)$, while it was in the state z at the moment $t = 0$, is called the multi-state safety function of a system.

Definition 3. A probability

$$\boldsymbol{r}(t) = P(S(t) < r \mid S(0) = z) = P(T(r) \le t),\quad t \in < 0,\infty), \tag{5}$$

that the system is in the subset of states worse than the critical state r, $r \in \{1,...,z\}$ while it was in the state z at the moment $t = 0$ is called a risk function of the multi-state system.

Under this definition, considering (4) and (5), we have

$$\boldsymbol{r}(t) = 1 - P(S(t) \ge r \mid S(0) = z) = 1 - \boldsymbol{s}_n(t,r),\quad t \in < 0,\infty), \tag{6}$$

and, if τ is the moment when the risk exceeds a permitted level δ, $\delta \in < 0,1 >$, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \tag{7}$$

where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$ given by (6).

3. Basic system safety structures

The proposition of a multi-state approach to definition of basic notions, analysis and diagnosing of systems' safety allowed us to define the system safety function and the system risk function. It also allows us to define basic structures of the multi-state systems of components with degrading safety states. For these basic systems it is possible to determine their safety functions. Further, as an example, we will consider a series system.

Definition 4. A multi-state system is called a series system if it is in the safety state subset $\{u, u+1, ..., z\}$ if and only if all its components are in this subset of safety states.

Corollary 1. The lifetime $T(u)$ of a multi-state series system in the state subset $\{u, u+1, ..., z\}$ is given by

$$T(u) = \min_{1 \le i \le n}\{T_i(u)\},\ u = 1,2,...,z.$$

The scheme of a series system is given in *Figure 2*.

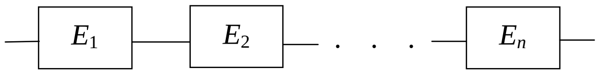

Figure 2. The scheme of a series system

It is easy to work out the following result.

Corollary 2. The safety function of the multi-state series system is given by

$$\overline{\boldsymbol{s}}_n(t, \cdot) = [1, \overline{\boldsymbol{s}}_n(t,1), ..., \overline{\boldsymbol{s}}_n(t,z)],\ t \in <0, \infty), \tag{8}$$

where

$$\overline{\boldsymbol{s}}_n(t,u) = \prod_{i=1}^{n} s_i(t,u),\ t \in <0, \infty),\ u = 1,2,...,z. \tag{9}$$

Corollary 3. If components of the multi-state series system have exponential safety functions, i.e., if

$$s_i(t, \cdot) = [1, s_i(t,1), ..., s_i(t,z)],\ t \in <0, \infty),$$

where

$$s_i(t,u) = \exp[-\lambda_i(u)t] \text{ for } t \in <0, \infty),\ \lambda_i(u) > 0,$$
$$u = 1,2,...,z,\ i = 1,2,...,n,$$

then its safety function is given by

$$\overline{\boldsymbol{s}}_n(t, \cdot) = [1, \overline{\boldsymbol{s}}_n(t,1), ..., \overline{\boldsymbol{s}}_n(t,z)], \tag{10}$$

where

$$\overline{\boldsymbol{s}}_n(t,u) = \exp[-\sum_{i=1}^{n} \lambda_i(u)t] \text{ for } t \in <0, \infty), \tag{11}$$
$$u = 1,2,...,z.$$

4. Basic system safety structures in variable operation conditions

We assume that the system during its operation process has v different operation states. Thus we can define $Z(t),\ t \in <0, +\infty>$, as the process with discrete operation states from the set

$$Z = \{z_1, z_2, \ldots, z_v\},$$

In practice a convenient assumption is that $Z(t)$ is a semi-markov process [3] with its conditional lifetimes θ_{bl} at the operation state z_b when its next operation state is z_l, $b, l = 1,2,..., v$, $b \ne l$. In this case the process $Z(t)$ may be described by:

- the vector of probabilities of the process initial operation states $[p_b(0)]_{1xv}$,
- the matrix of the probabilities of the process transitions between the operation states $[p_{bl}]_{vxv}$, where $p_{bb}(t) = 0$ for $b = 1,2,..., v$.
- the matrix of the conditional distribution functions $[H_{bl}(t)]_{vxv}$ of the process lifetimes θ_{bl}, $b \ne l$, in the operation state z_b when the next operation state is z_l, where $H_{bl}(t) = P(\theta_{bl} < t)$ for $b, l = 1,2,..., v$, $b \ne l$, and $H_{bb}(t) = 0$ for $b = 1,2,..., v$.

Under these assumptions, the lifetimes θ_{bl} mean values are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t),\ b, l = 1,2,..., v,\ b \ne l. \tag{12}$$

The unconditional distribution functions of the lifetimes θ_b of the process $Z(t)$ at the operation states z_b, $b = 1,2,..., v$, are given by

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t),\ b = 1,2,..., v.$$

The mean values $E[\theta_b]$ of the unconditional lifetimes θ_b are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl},\ b = 1,2,..., v,$$

where M_{bl} are defined by (12).

Limit values of the transient probabilities at the operation states

$$p_b(t) = P(Z(t) = z_b),\ t \in <0, +\infty),\ b = 1,2,..., v,$$

are given by

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \quad b = 1,2,...,v, \tag{13}$$

where the probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases}$$

We assume that the system is composed of n components E_i, $i = 1,2,...,n,$ the changes of the process $Z(t)$ operation states have an influence on the system components E_i safety and on the system safety structure as well. Thus, we denote the conditional safety function of the system component E_i while the system is at the operational state z_b, $b = 1,2,...,v,$ by

$$s_i^{(b)}(t,\cdot) = [1,\ s_i^{(b)}(t,1),\ s_i^{(b)}(t,2),\ ...,\ s_i^{(b)}(t,z)],$$

where

$$s_i^{(b)}(t,u) = P(T_i^{(b)}(u) > t \,|\, Z(t) = z_b)$$

for $t \in <0,\infty)$, $b = 1,2,...,v,$ $u = 1,2,..., z,$ and the conditional safety function of the system while the system is at the operational state z_b, $b = 1,2,...,v,$ by

$$\boldsymbol{s}_{n_b}^{(b)}(t,\cdot) = [1,\ \boldsymbol{s}_{n_b}^{(b)}(t,1),\ \boldsymbol{s}_{n_b}^{(b)}(t,2),\ ...,\ \boldsymbol{s}_{n_b}^{(b)}(t,z)],$$
$$n_b \in \{1,2,...,n\},$$

where n_b are numbers of components in the operation states z_b and

$$\boldsymbol{s}_{n_b}^{(b)}(t,u), = P(T^{(b)}(u) > t \,|\, Z(t) = z_b)$$

for $t \in <0,\infty)$, $n_b \in \{1,2,...,n\}$, $b = 1,2,...,\nu$, $u = 1,2,..., z.$

The safety function $s_i^{(b)}(t,u)$ is the conditional probability that the component E_i lifetime $T_i^{(b)}(u)$ in the state subset $\{u, u+1,...,z\}$ is not less than t, while the process $Z(t)$ is at the operation state z_b. Similarly, the safety function $\boldsymbol{s}_{n_b}^{(b)}(t,u)$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the state subset $\{u, u+1,...,z\}$ is not less than t, while the process $Z(t)$ is at the operation state z_b.

In the case when the system operation time is large enough, the unconditional safety function of the system is given by

$$\boldsymbol{s}_n(t,\cdot) = [1,\ \boldsymbol{s}_n(t,1),\ \boldsymbol{s}_n(t,2),\ ...,\ \boldsymbol{s}_n(t,z)],\ t \ge 0,$$

where

$$\boldsymbol{s}_n(t,u) = P(T(u) > t) \cong \sum_{b=1}^{\nu} p_b \boldsymbol{s}_{n_b}^{(b)}(t,u) \tag{14}$$

for $t \ge 0$, $n_b \in \{1,2,...,n\}$, $u = 1,2,...,z,$ and $T(u)$ is the unconditional lifetime of the system in the safety state subset $\{u, u+1,...,z\}$.

The mean values and variances of the system lifetimes in the safety state subset $\{u, u+1,...,z\}$ are

$$m(u) = E[T(u)] \cong \sum_{b=1}^{\nu} p_b m^{(b)}(u),\ u = 1,2,\ldots,z, \tag{15}$$

where [2]

$$m^{(b)}(u) = \int_0^\infty \boldsymbol{s}_{\boldsymbol{n_b}}^{(b)}(t,u)dt,\ n_b \in \{1,2,...,n\}, \tag{16}$$
$$u = 1,2,\ldots,z,$$

and

$$[\sigma^{(b)}(u)]^2 = 2\int_0^\infty t\boldsymbol{s}_{n_b}(t,u)dt - [m^{(b)}(u)]^2, \tag{17}$$
$$u = 1,2,\ldots,z,$$

for $b = 1,2,...,\nu$, and

$$[\sigma(u)]^2 = 2\int_0^\infty t\boldsymbol{s}_n(t,u)dt - [m(u)]^2, \quad u = 1,2,..., z. \tag{16}$$

The mean values of the system lifetimes in the particular safety states u, are [2]

$$\overline{m}(u) = m(u) - m(u+1),\ u = 1,2,\ldots,z-1,$$
$$\overline{m}(z) = m(z). \tag{19}$$

5. Ship safety Model in constant operation conditions

We preliminarily assume that the ship is composed of a number of main technical subsystems having an essential influence on its safety. There are distinguished her following technical subsystems:

S_1 - a navigational subsystem,

S_2 - a propulsion and controlling subsystem,

S_3 - a loading and unloading subsystem,

S_4 - a hull subsystem,

S_5 - a protection and rescue subsystem,

S_6 - an anchoring and mooring subsystem.

According to *Definition 1*, we mark the safety functions of these subsystems respectively by vectors

$$s_i(t,\cdot) = [s_i(t,0), s_i(t,1),..., s_i(t,z)],\ t \in < 0,\infty),\ i = 1,2,...,6, \qquad (20)$$

with co-ordinates

$$s_i(t,u) = P(S_i(t) \geq u \mid S_i(0) = z) = P(T_i(u) > t) \qquad (21)$$

for $t \in < 0,\infty)$, $u = 0,1,...,z$, $i = 1,2,...,6$, where $T_i(u)$, $i = 1,2,...,6$, are independent random variables representing the lifetimes of subsystems S_i in the safety state subset $\{u,u+1,...,z\}$, while they were in the state z at the moment $t = 0$ and $S_i(t)$ is a subsystem S_i safety state at the moment t, $t \in < 0,\infty)$.

Further, assuming that the ship is in the safety state subset $\{u,u+1,...,z\}$ if all its subsystems are in this subset of safety states and considering *Definition 4*, we conclude that the ship is a series system of subsystems S_1, S_2, S_3, S_4, S_5, S_6 with a scheme presented in *Figure 3*.

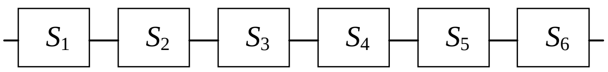

Figure 3. The scheme of a structure of ship subsystems

Therefore, the ship safety is defined by the vector

$$\bar{\mathbf{s}}_6(t,\cdot) = [\bar{\mathbf{s}}_6(t,0),\ \bar{\mathbf{s}}_6(t,1),...,\ \bar{\mathbf{s}}_6(t,z)],\ t \in < 0,\infty), \qquad (22)$$

with co-ordinates

$$\bar{\mathbf{s}}_6(t,u) = P(S(t) \geq u \mid S(0) = z) = P(T(u) > t) \qquad (23)$$

for $t \in < 0,\infty)$, $u = 0,1,...,z$, where $T(u)$ is a random variable representing the lifetime of the ship in the safety state subset $\{u,u+1,...,z\}$ while it was in the state z at the moment $t = 0$ and $S(t)$ is the ship safety state at the moment t, $t \in < 0,\infty)$, according to *Corollary 2*, is given by the formula

$$\bar{\mathbf{s}}_6(t,\cdot) = [1,\ \bar{\mathbf{s}}_6(t,1),...,\ \bar{\mathbf{s}}_6(t,z)],\ t \in < 0,\infty), \qquad (24)$$

where

$$\bar{\mathbf{s}}_6(t,u) = \prod_{i=1}^{6} s_i(t,u),\ t \in < 0,\infty),\ u = 1,2,...,z. \qquad (25)$$

6. Ship operation process

Technical subsystems S_1, S_2, S_3, S_4, S_5, S_6 are forming a general ship safety structure presented in *Figure 3*. However, the ship safety structure and the ship subsystems safety depend on her changing in time operation states.

Considering basic sea transportation processes the following operation ship states have been specified:

z_1 - loading of cargo,

z_2 - unloading of cargo,

z_3 - leaving the port,

z_4 - entering the port,

z_5 - navigation at restricted water areas,

z_6 - navigation at open sea waters.

In this case the process $Z(t)$ may be described by:

- the vector of probabilities of the initial operation states $[p_b(0)]_{1x6}$,
- the matrix of the probabilities of its transitions between the operation states $[p_{bl}]_{6x6}$, where $p_{bb}(t) = 0$ for $b = 1,2,...,6$,
- the matrix of the conditional distribution functions $[H_{bl}(t)]_{6x6}$ of the lifetimes θ_{bl}, $b \neq l$, where $H_{bl}(t) = P(\theta_{bl} < t)$ for $b, l = 1,2,...,6$, $b \neq l$, and $H_{bb}(t) = 0$ for $b = 1,2,...,6$.

Under these assumptions, the lifetimes θ_{bl} mean values are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t),\ b, l = 1,2,...,6,\ b \neq l. \qquad (26)$$

The unconditional distribution functions of the lifetimes θ_b of the process $Z(t)$ at the operation states z_b, $b = 1,2,...,6$, are given by

$$H_b(t) = \sum_{l=1}^{6} p_{bl} H_{bl}(t),\ b = 1,2,...,6.$$

The mean values $E[\theta_b]$ of the unconditional lifetimes θ_b are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{6} p_{bl} M_{bl},\ b = 1,2,...,6, \qquad (27)$$

where M_{bl} are defined by (26).

Limit values of the transient probabilities at the operation states

$$p_b(t) = P(Z(t) = z_b),\ t \in <0,+\infty),\ b = 1,2,...,6,$$

are given by

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{6}\pi_l M_l},\ b = 1,2,...,6, \qquad (28)$$

where the probabilities π_b of the vector $[\pi_b]_{1x6}$ satisfy the system of equations

$$\begin{cases}[\pi_b] = [\pi_b][p_{bl}]\\ \sum_{l=1}^{6}\pi_l = 1.\end{cases} \qquad (29)$$

7. Safety model of ship in variable operation conditions

We assume as earlier that the ship is composed of $n = 6$ subsystems S_i, $i = 1,2,...,6$, and that the changes of the process $Z(t)$ of ship operation states have an influence on the system subsystems S_i safety and on the ship safety structure as well. Thus, we denote the conditional safety function of the ship subsystem S_i while the ship is at the operational state z_b, $b = 1,2,...,6$, by

$$s_i^{(b)}(t,\cdot) = [1,\ s_i^{(b)}(t,1),\ s_i^{(b)}(t,2),\ ...,\ s_i^{(b)}(t,z)],$$

where

$$s_i^{(b)}(t,u) = P(T_i^{(b)}(u) > t | Z(t) = z_b)$$

for $t \in <0,\infty)$, $b = 1,2,...,6$, $u = 1,2,...,z$, and the conditional safety function of the ship while the ship is at the operational state z_b, $b = 1,2,...,6$, by

$$\boldsymbol{s}_{n_b}^{(b)}(t,\cdot) = [1,\ \boldsymbol{s}_{n_b}^{(b)}(t,1),\ \boldsymbol{s}_{n_b}^{(b)}(t,2),\ ...,\ \boldsymbol{s}_{n_b}^{(b)}(t,z)],$$

where

$$\boldsymbol{s}_{n_b}^{(b)}(t,u), = P(T^{(b)}(u) > t | Z(t) = z_b)$$

for $t \in <0,\infty)$, $b = 1,2,...,6$, $n_b \in \{1,2,3,4,5,6\}$, $u = 1,2,...,z$.

The safety function $s_i^{(b)}(t,u)$ is the conditional probability that the subsystem S_i lifetime $T_i^{(b)}(u)$ in the state subset $\{u,u+1,...,z\}$ is not less than t, while the process $Z(t)$ is at the ship operation state z_b. Similarly, the safety function $\boldsymbol{s}_{n_b}^{(b)}(t,u)$ is the conditional probability that the ship lifetime $T^{(b)}(u)$ in the state subset $\{u,u+1,...,z\}$ is not less than t, while the process $Z(t)$ is at the ship operation state z_b.

In the case when the ship operation time is large enough, the unconditional safety function of the system is given by

$$\boldsymbol{s}_6(t,\cdot) = [1,\ \boldsymbol{s}_6(t,1),\ \boldsymbol{s}_6(t,2),\ ...,\ \boldsymbol{s}_6(t,z)],\ t \ge 0,$$

where

$$\boldsymbol{s}_6(t,u) = P(T(u) > t) \cong \sum_{b=1}^{6} p_b \boldsymbol{s}_{n_b}^{(b)}(t,u) \qquad (30)$$

for $t \ge 0$, $n_b \in \{1,2,3,4,5,6\}$, $u = 1,2,...,z$, and $T(u)$ is the unconditional lifetime of the ship in the safety state subset $\{u,u+1,...,z\}$.

The mean values and variances of the ship lifetimes in the safety state subset $\{u,u+1,...,z\}$ are

$$m(u) = E[T(u)] \cong \sum_{b=1}^{6} p_b m^{(b)}(u),\ u = 1,2,...,z, \qquad (31)$$

where

$$m^{(b)}(u) = \int_0^\infty \boldsymbol{s}_{\boldsymbol{n_b}}^{(b)}(t,u)dt, \qquad (32)$$

for $b = 1,2,...,6$, $n_b \in \{1,2,3,4,5,6\}$, $u = 1,2,...,z$, and

$$[\sigma(u)]^2 = D[T(u)] = 2\int_0^\infty t\boldsymbol{s}_6(t,u)dt - [m(u)]^2, \qquad (33)$$
$$u = 1,2,...,z,$$

The mean values of the system lifetimes in the particular safety states u, are

$$\bar{m}(u) = m(u) - m(u+1),\ u = 1,2,...,z-1,$$

$$\bar{m}(z) = m(z). \qquad (34)$$

8. Preliminary application of general safety model of ship in variable operation conditions

According to expert opinions [5] in the ship operation process, $Z(t)$, $t \ge 0$, we distinguished seven operation states: z_1, z_2, z_3, z_4, z_5, z_6. On the basis of data coming from experts, the probabilities of transitions between the operation states are approximately given by

$$[p_{bl}]_{6x6} = \begin{bmatrix} 0.00 & 0.00 & 0.96 & 0.00 & 0.02 & 0.02 \\ 0.48 & 0.00 & 0.48 & 0.00 & 0.02 & 0.02 \\ 0.00 & 0.00 & 0.00 & 0.02 & 0.96 & 0.02 \\ 0.49 & 0.49 & 0.02 & 0.00 & 0.00 & 0.00 \\ 0.02 & 0.02 & 0.00 & 0.48 & 0.00 & 0.48 \\ 0.02 & 0.02 & 0.00 & 0.01 & 0.95 & 0.00 \end{bmatrix},$$

and the distributions of the ship conditional lifetimes in the operation states are exponential of the following forms:

$$H_{13}(t) = 1 - \exp[-0.5t],\ H_{15}(t) = 1 - \exp[-1.0t],$$

$$H_{16}(t) = 1 - \exp[-1.0t],\ H_{21}(t) = 1 - \exp[-0.5t],$$

$$H_{23}(t) = 1 - \exp[-0.5t],\ H_{25}(t) = 1 - \exp[-1.0t],$$

$$H_{26}(t) = 1 - \exp[-1.0t],\ H_{34}(t) = 1 - \exp[-25.0t],$$

$$H_{35}(t) = 1 - \exp[-25.0t],\ H_{36}(t) = 1 - \exp[-12.5t],$$

$$H_{51}(t) = 1 - \exp[-0.33t],\ H_{52}(t) = 1 - \exp[-0.33t],$$

$$H_{54}(t) = 1 - \exp[-0.5t],\ H_{56}(t) = 1 - \exp[-0.5t],$$

$$H_{61}(t) = 1 - \exp[-0.2t],\ H_{62}(t) = 1 - \exp[-0.2t],$$

$$H_{64}(t) = 1 - \exp[-0.25t],\ H_{65}(t) = 1 - \exp[-0.25t]$$

for $t \geq 0$.

Hence, by (26), the conditional mean values of lifetimes in the operation states are

$$M_{13} = 2,\ M_{15} = 1,\ M_{16} = 1,$$

$$M_{21} = 2,\ M_{23} = 2,\ M_{25} = 1,\ M_{26} = 1,$$

$$M_{34} = 0.04,\ M_{35} = 0.04,\ M_{36} = 0.08,$$

$$M_{41} = 0.08,\ M_{42} = 0.08,\ M_{43} = 0.04,$$
$$M_{51} = 3,\ M_{52} = 3,\ M_{54} = 2,\ M_{56} = 2,$$

$$M_{61} = 5,\ M_{62} = 5,\ M_{64} = 4,\ M_{65} = 4.$$

Whereas, by (27), the unconditional mean lifetimes in the operation states are

$$M_1 = E[\theta_1] = p_{13}M_{13} + p_{15}M_{15} + p_{16}M_{16}$$

$$= 0.96 \cdot 2 + 0.02 \cdot 1 + 0.02 \cdot 1 = 1.96,$$

$$M_2 = E[\theta_2]$$

$$= p_{21}M_{21} + p_{23}M_{23} + p_{25}M_{25} + p_{26}M_{26}$$

$$= 0.48 \cdot 2 + 0.48 \cdot 2 + 0.02 \cdot 1 + 0.02 \cdot 1 = 1.96,$$

$$M_3 = E[\theta_3] = p_{34}M_{34} + p_{35}M_{35} + p_{36}M_{36}$$

$$= 0.02 \cdot 0.04 + 0.96 \cdot 0.04 + 0.02 \cdot 0.08 = 0.0408,$$

$$M_4 = E[\theta_4] = p_{41}M_{41} + p_{42}M_{42} + p_{43}M_{43}$$

$$= 0.49 \cdot 0.08 + 0.49 \cdot 0.08 + 0.02 \cdot 0.04 = 0.0792,$$

$$M_5 = E[\theta_5]$$

$$= p_{51}M_{51} + p_{52}M_{52} + p_{54}M_{54} + p_{56}M_{56}$$

$$= 0.02 \cdot 3 + 0.02 \cdot 3 + 0.48 \cdot 2 + 0.48 \cdot 2 = 2.04,$$

$$M_6 = E[\theta_6]$$

$$= p_{61}M_{61} + p_{62}M_{62} + p_{64}M_{64} + p_{65}M_{65}$$

$$= 0.02 \cdot 5 + 0.02 \cdot 5 + 0.01 \cdot 4 + 0.95 \cdot 4 = 4.04.$$

Since from the system of equations

$$\begin{cases} [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6] \\ = [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6][p_{bl}]_{6x6} \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 = 1, \end{cases}$$

we get

$$\pi_1 = 0.126,\ \pi_2 = 0.085,\ \pi_3 = 0.165,$$

$$\pi_4 = 0.155,\ \pi_5 = 0.312,\ \pi_6 = 0.157,$$

then the limit values of the transient probabilities $p_b(t)$ at the operational states z_b, according to (28), are given by

$$p_1 = 0.145,\ \ p_2 = 0.098,\ \ p_3 = 0.004,\ \ p_4 = 0.007,$$

$$p_5 = 0.374, \quad p_6 = 0.372. \qquad (35)$$

We assume that the ship subsystems S_i, $i = 1,2,...,6,$ are its five-state components, i.e. z = 4, with the multi-state safety functions

$$s_i^{(b)}(t, \cdot) = [1,\ s_i^{(b)}(t, 1),\ s_i^{(b)}(t, 2),\ ...,\ s_i^{(b)}(t, z)],$$
$$b = 1,2,...,6,\ i = 1,2,...,6,$$

with exponential co-ordinates different in various ship operation states z_b, $b = 1,2,...,6.$

At the operation states z_1 and z_2, i.e. at the cargo loading and un-loading state the ship is built of $n_1 = n_2 = 4$ subsystems S_3, S_4, S_5 and S_6 forming a series structure shown in *Figure 4*.

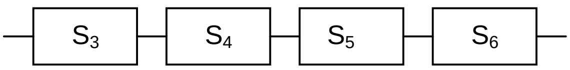

Figure 4. The scheme of the ship structure at the operation states z_1 and z_2

We assume that the ship subsystems S_i, $i = 3,4,5,6,$ are its five-state components, i.e. z = 4, having the multi-state safety functions

$$s_i^{(b)}(t,\cdot) = [1, s_i^{(b)}(t,1),\ s_i^{(b)}(t,2),\ s_i^{(b)}(t,3),\ s_i^{(b)}(t,4)\ ],$$
$$i = 3,4,5,6,\ \ b = 1,2,$$

with exponential co-ordinates, for $b = 1,2,$ respectively given by:

- for the loading subsystem S_3

$$s_3^{(b)}(t,1) = \exp[-0.06t],\ s_3^{(b)}(t,2) = \exp[-0.07t],$$

$$s_3^{(b)}(t,3) = \exp[-0.08t],\ s_3^{(b)}(t,4) = \exp[-0.09t],$$

- for the hull subsystem S_4

$$s_4^{(b)}(t,1) = \exp[-0.03t],\ s_4^{(b)}(t,2) = \exp[-0.04t],$$

$$s_4^{(b)}(t,3) = \exp[-0.06t],\ s_4^{(b)}(t,4) = \exp[-0.07t],$$

- for the protection and rescue subsystem S_5

$$s_5^{(b)}(t,1) = \exp[-0.10t],\ s_5^{(b)}(t,2) = \exp[-0.12t],$$

$$s_5^{(b)}(t,3) = \exp[-0.15t],\ s_5^{(b)}(t,4) = \exp[-0.16t],$$

- for the anchor and mooring subsystem S_6

$$s_6^{(b)}(t,1) = \exp[-0.06t],\ s_6^{(b)}(t,2) = \exp[-0.08t],$$

$$s_6^{(b)}(t,3) = \exp[-0.10t],\ s_6^{(b)}(t,4) = \exp[-0.12t].$$

Assuming that the ship is in the safety state subsets $\{u, u+1,..., z\}$, $u = 1,2,3,4,$ if all its subsystems are in this safety state subset, according to *Definition 1* and *Definition 4*, the considered system is a five-state series system. Thus, by *Corollary 3*, after applying (10)–(11), we have its conditional safety functions in the operation states z_1 and z_2 respectively for $b = 1,2,$ given by

$$\bar{\boldsymbol{s}}_4^{(b)}(t,\ \cdot)$$

$$= [1,\ \bar{\boldsymbol{s}}_4^{(b)}(t,1),\ \bar{\boldsymbol{s}}_4^{(b)}(t,2),\ \bar{\boldsymbol{s}}_4^{(b)}(t,3),\ \bar{\boldsymbol{s}}_4^{(b)}(t,4)\ ],$$
$$t \geq 0,\ b = 1,2,$$

where

$$\bar{\boldsymbol{s}}_4^{(b)}(t,1) = \exp[-(0.06 + 0.03 + 0.10 + 0.06)t]$$

$$= \exp[-0.25t],$$

$$\bar{\boldsymbol{s}}_4^{(b)}(t,2) = \exp[-(0.07 + 0.04 + 0.12 + 0.08)t]$$

$$= \exp[-0.31t],$$

$$\bar{\boldsymbol{s}}_4^{(b)}(t,3) = \exp[-(0.08 + 0.06 + 0.15 + 0.10)t]$$

$$= \exp[-0.39t],$$

$$\bar{\boldsymbol{s}}_4^{(b)}(t,4) = \exp[-(0.09 + 0.07 + 0.16 + 0.12)t]$$

$$= \exp[-0.44t] \text{ for } t \geq 0,\ b = 1,2\,.$$

The expected values and standard deviations of the ship conditional lifetimes in the safety state subsets calculated from the above result, according to (16)-(17), for $b = 1,2,$ are:

$$m^{(b)}(1) \cong 4.00,\ m^{(b)}(2) \cong 3.26,\ m^{(b)}(3) \cong 2.56,$$

$$m^{(b)}(4) \cong 2.27 \text{ years},$$

$$\sigma^{(b)}(1) \cong 4.00,\ \sigma^{(b)}(2) \cong 3.26,\ \sigma^{(b)}(3) \cong 2.56,$$

$\sigma^{(b)}(4) \cong 2.27$ years,

and further, from (10), the ship conditional lifetimes in the particular safety states, for $b = 1,2$, are:

$\overline{m}^{(b)}(1) \cong 0.74$, $\overline{m}^{(b)}(2) \cong 0.70$, $\overline{m}^{(b)}(3) \cong 0.29$,

$\overline{m}^{(b)}(4) \cong 2.27$ years.

At the operation states z_3 and z_4, i.e. at the leaving and entering state the ship is built of $n_3 = n_4 = 5$ subsystems S_1, S_2, S_4, S_5 and S_6 forming a series structure shown in *Figure 5*.

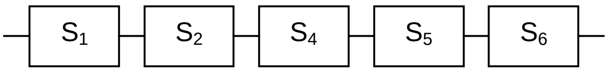

Figure 5. The scheme of the ship structure at the operation states z_3 and z_4

We assume that the ship subsystems S_i, $i = 1,2,4,5,6$, are its five-state components, i.e. $z = 4$, having the multi-state safety functions

$$s_i^{(b)}(t,\cdot) = [1,\ s_i^{(b)}(t,1), s_i^{(b)}(t,2), s_i^{(b)}(t,3), s_i^{(b)}(t,4)],$$
$$i = 1,2,4,5,6,\quad b = 3,4,$$

with exponential co-ordinates, for $b = 3,4$, respectively given by:

- for the navigational subsystem S_1

$$s_1^{(b)}(t,1) = \exp[-0.15t],\ s_1^{(b)}(t,2) = \exp[-0.20t],$$

$$s_1^{(b)}(t,3) = \exp[-0.22t],\ s_1^{(b)}(t,4) = \exp[-0.25t],$$

- for the propulsion and controlling subsystem S_2

$$s_2^{(b)}(t,1) = \exp[-0.05t],\ s_2^{(b)}(t,2) = \exp[-0.06t],$$

$$s_2^{(b)}(t,3) = \exp[-0.07t],\ s_2^{(b)}(t,4) = \exp[-0.08t],$$

- for the hull subsystem S_4

$$s_4^{(b)}(t,1) = \exp[-0.04t],\ s_4^{(b)}(t,2) = \exp[-0.05t],$$

$$s_4^{(b)}(t,3) = \exp[-0.07t],\ s_4^{(b)}(t,4) = \exp[-0.08t],$$

- for the protection and rescue subsystem S_5

$$s_5^{(b)}(t,1) = \exp[-0.12t],\ s_5^{(b)}(t,2) = \exp[-0.14t],$$

$$s_5^{(b)}(t,3) = \exp[-0.16t],\ s_5^{(b)}(t,4) = \exp[-0.18t],$$

- for the anchor and mooring subsystem S_6

$$s_6^{(b)}(t,1) = \exp[-0.02t],\ s_6^{(b)}(t,2) = \exp[-0.04t],$$

$$s_6^{(b)}(t,3) = \exp[-0.06t],\ s_6^{(b)}(t,4) = \exp[-0.08t].$$

Assuming that the ship is in the safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2,3,4$, if all its subsystems are in this safety state subset, according to *Definition 1* and *Definition 4*, the considered system is a five-state series system. Thus, by *Corollary 3*, after applying (10)–(11), we have its conditional safety functions in the operation states z_3 and z_4 respectively for $b = 3,4$, given by

$$\bar{\boldsymbol{s}}_5^{(b)}(t,\cdot)$$

$$= [1,\ \bar{\boldsymbol{s}}_5^{(b)}(t,1),\ \bar{\boldsymbol{s}}_5^{(b)}(t,2),\ \bar{\boldsymbol{s}}_5^{(b)}(t,3),\ \bar{\boldsymbol{s}}_5^{(b)}(t,4)],$$
$$t \ge 0,\ b = 3,4,$$

where

$$\bar{\boldsymbol{s}}_5^{(b)}(t,1) = \exp[-(0.15 + 0.05 + 0.04 + 0.12 + 0.02)t]$$

$$= \exp[-0.38t],$$

$$\bar{\boldsymbol{s}}_5^{(b)}(t,2) = \exp[-(0.20 + 0.06 + 0.05 + 0.14 + 0.04)t]$$

$$= \exp[-0.49t],$$

$$\bar{\boldsymbol{s}}_5^{(b)}(t,3) = \exp[-(0.22 + 0.07 + 0.07 + 0.16 + 0.06)t]$$

$$= \exp[-0.58t],$$

$$\bar{\boldsymbol{s}}_5^{(b)}(t,4) = \exp[-(0.25 + 0.08 + 0.08 + 0.18 + 0.08)t]$$

$$= \exp[-0.67t] \text{ for } t \ge 0,\ b = 3,4.$$

The expected values and standard deviations of the ship conditional lifetimes in the safety state subsets calculated from the above result, according to (16)-(17), for $b = 3,4$, are:

$m^{(b)}(1) \cong 2.63$, $m^{(b)}(2) \cong 2.04$, $m^{(b)}(3) \cong 1.72$,

$m^{(b)}(4) \cong 1.49$ years,

$\sigma^{(b)}(1) \cong 2.63$, $\sigma^{(b)}(2) \cong 2.04$, $\sigma^{(b)}(3) \cong 1.72$,

$\sigma^{(b)}(4) \cong 1.49$ years,

and further, from (10), the ship conditional lifetimes in the particular safety states, for $b = 1,2,$ are:

$\overline{m}^{(b)}(1) \cong 0.59$, $\overline{m}^{(b)}(2) \cong 0.32$, $\overline{m}^{(b)}(3) \cong 0.23$,

$\overline{m}^{(b)}(4) \cong 1.49$ years.

At the operation state z_5, i.e. at the navigation at restricted areas state the ship is built of $n_5 = 5$ subsystems S_1, S_2, S_4, S_5 and S_6 forming a series structure shown in *Figure 6*.

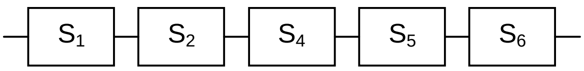

Figure 6. The scheme of the ship structure at the operation state z_5

We assume that the ship subsystems S_i, $i = 1,2,4,5,6,$ are its five-state components, i.e. $z = 4$, having the multi-state safety functions

$$s_i^{(5)}(t,\cdot) = [1, s_i^{(5)}(t,1), s_i^{(5)}(t,2), s_i^{(5)}(t,3), s_i^{(5)}(t,4)],$$
$i = 1,2,4,5,6,$

with exponential co-ordinates respectively given by:

- for the navigational subsystem S_1

$$s_1^{(5)}(t,1) = \exp[-0.18t],\ s_1^{(5)}(t,2) = \exp[-0.22t],$$

$$s_1^{(5)}(t,3) = \exp[-0.24t],\ s_1^{(5)}(t,4) = \exp[-0.26t],$$

- for the propulsion and controlling subsystem S_2

$$s_2^{(5)}(t,1) = \exp[-0.06t],\ s_2^{(5)}(t,2) = \exp[-0.07t],$$

$$s_2^{(5)}(t,3) = \exp[-0.08t],\ s_2^{(5)}(t,4) = \exp[-0.09t],$$

- for the hull subsystem S_4

$$s_4^{(5)}(t,1) = \exp[-0.06t],\ s_4^{(5)}(t,2) = \exp[-0.08t],$$

$$s_4^{(5)}(t,3) = \exp[-0.09t],\ s_4^{(5)}(t,4) = \exp[-0.10t],$$

- for the protection and rescue subsystem S_5

$$s_5^{(5)}(t,1) = \exp[-0.14t],\ s_5^{(5)}(t,2) = \exp[-0.15t],$$

$$s_5^{(5)}(t,3) = \exp[-0.17t],\ s_5^{(5)}(t,4) = \exp[-0.20t],$$

- for the anchor and mooring subsystem S_6

$$s_6^{(5)}(t,1) = \exp[-0.02t],\ s_6^{(5)}(t,2) = \exp[-0.03t],$$

$$s_6^{(5)}(t,3) = \exp[-0.04t],\ s_6^{(5)}(t,4) = \exp[-0.05t].$$

Assuming that the ship is in the safety state subsets $\{u, u+1, ..., z\}$, $u = 1,2,3,4,$ if all its subsystems are in this safety state subset, according to *Definition 1* and *Definition 4*, the considered system is a five-state series system. Thus, by *Corollary 3*, after applying (10)–(11), we have its safety function given by

$$\bar{\boldsymbol{s}}_5^{(5)}(t,\cdot)$$

$$= [1, \bar{\boldsymbol{s}}_5^{(5)}(t,1), \bar{\boldsymbol{s}}_5^{(5)}(t,2), \bar{\boldsymbol{s}}_5^{(5)}(t,3), \bar{\boldsymbol{s}}_5^{(5)}(t,4)],\quad t \geq 0,$$

where

$$\bar{\boldsymbol{s}}_5^{(5)}(t,1) = \exp[-(0.18 + 0.06 + 0.06 + 0.14 + 0.02)t]$$

$$= \exp[-0.46t],$$

$$\bar{\boldsymbol{s}}_5^{(5)}(t,2) = \exp[-(0.22 + 0.07 + 0.08 + 0.15 + 0.03)t]$$

$$= \exp[-0.55t],$$

$$\bar{\boldsymbol{s}}_5^{(5)}(t,3) = \exp[-(0.24 + 0.08 + 0.09 + 0.17 + 0.04)t]$$

$$= \exp[-0.62t],$$

$$\bar{\boldsymbol{s}}_5^{(5)}(t,4) = \exp[-(0.26 + 0.09 + 0.10 + 0.20 + 0.05)t]$$
$$= \exp[-0.70t] \text{ for } t \geq 0.$$

The expected values and standard deviations of the ship lifetimes in the safety state subsets calculated from the above result, according to (16)-(17), are:

$m^{(6)}(1) \cong 2.17$, $m^{(6)}(2) \cong 1.82$, $m^{(6)}(3) \cong 1.61$,

$m^{(6)}(4) \cong 1.43$ years,

$\sigma^{(6)}(1) \cong 2.17$, $\sigma^{(6)}(2) \cong 1.82$, $\sigma^{(6)}(3) \cong 1.61$,

$\sigma^{(6)}(4) \cong 1.43$ years,

and further, from (10), the ship lifetimes in the particular safety states are:

$\bar{m}^{(6)}(1) \cong 0.35$, $\bar{m}^{(6)}(2) \cong 0.21$, $\bar{m}^{(6)}(3) \cong 0.18$,

$\bar{m}^{(6)}(4) \cong 1.43$ years.

At the operation state z_6, i.e. at the navigation at open sea state the ship is built of $n_6 = 4$ subsystems S_1, S_2, S_4, and S_5 forming a series structure shown in *Figure 7*.

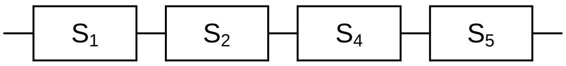

Figure 7. The scheme of the ship structure at the operation state z_6

We assume that the ship subsystems S_i, $i = 1,2,4,5$, are its five-state components, i.e. z = 4, having the multi-state safety functions

$$s_i^{(6)}(t,\cdot) = [1, s_i^{(6)}(t,1), s_i^{(6)}(t,2), s_i^{(6)}(t,3), s_i^{(6)}(t,4)],$$
$i = 1,2,4,5$,

with exponential co-ordinates respectively given by:

- for the navigational subsystem S_1

$$s_1^{(6)}(t,1) = \exp[-0.18t],\ s_1^{(6)}(t,2) = \exp[-0.22t],$$

$$s_1^{(6)}(t,3) = \exp[-0.24t],\ s_1^{(6)}(t,4) = \exp[-0.26t],$$

- for the propulsion and controlling subsystem S_2

$$s_2^{(6)}(t,1) = \exp[-0.06t],\ s_2^{(6)}(t,2) = \exp[-0.07t],$$
$$s_2^{(6)}(t,3) = \exp[-0.08t],\ s_2^{(6)}(t,4) = \exp[-0.09t],$$

- for the hull subsystem S_4

$$s_4^{(6)}(t,1) = \exp[-0.05t],\ s_4^{(6)}(t,2) = \exp[-0.06t],$$

$$s_4^{(6)}(t,3) = \exp[-0.07t],\ s_4^{(6)}(t,4) = \exp[-0.08t],$$

- for the protection and rescue subsystem S_5

$$s_5^{(6)}(t,1) = \exp[-0.15t],\ s_5^{(6)}(t,2) = \exp[-0.16t],$$

$$s_5^{(6)}(t,3) = \exp[-0.18t],\ s_5^{(6)}(t,4) = \exp[-0.22t].$$

Assuming that the ship is in the safety state subsets $\{u, u+1,..., z\}$, $u = 1,2,3,4$, if all its subsystems are in this safety state subset, according to *Definition 1* and *Definition 4*, the considered system is a five-state series system. Thus, by *Corollary 3*, after applying (10)–(11), we have its safety function given by

$$\bar{\boldsymbol{s}}_4^{(7)}(t,\cdot)$$

$$= [1,\ \bar{\boldsymbol{s}}_4^{(7)}(t,1),\ \bar{\boldsymbol{s}}_4^{(7)}(t,2),\ \bar{\boldsymbol{s}}_4^{(7)}(t,3),\ \bar{\boldsymbol{s}}_4^{(7)}(t,4)\ ], t \geq 0,$$

where

$$\bar{\boldsymbol{s}}_4^{(6)}(t,1) = \exp[-(0.18 + 0.06 + 0.05 + 0.15)t]$$

$$= \exp[-0.44t],$$

$$\bar{\boldsymbol{s}}_4^{(6)}(t,2) = \exp[-(0.22 + 0.07 + 0.06 + 0.16)t]$$

$$= \exp[-0.51t],$$

$$\bar{\boldsymbol{s}}_4^{(6)}(t,3) = \exp[-(0.24 + 0.08 + 0.07 + 0.18)t]$$

$$= \exp[-0.57t],$$

$$\bar{\boldsymbol{s}}_4^{(6)}(t,4) = \exp[-(0.26 + 0.09 + 0.08 + 0.22)t]$$

$$= \exp[-0.67t] \text{ for } t \geq 0.$$

The expected values and standard deviations of the ship lifetimes in the safety state subsets calculated from the above result, according to (16)-(17), are:

$m^{(6)}(1) \cong 2.27$, $m^{(6)}(2) \cong 1.96$, $m^{(6)}(3) \cong 1.75$,

$m^{(6)}(4) \cong 1.49$ years,

$\sigma^{(6)}(1) \cong 2.27$, $\sigma^{(6)}(2) \cong 1.96$, $\sigma^{(6)}(3) \cong 1.75$,

$\sigma^{(6)}(4) \cong 1.49$ years,

and further, from (18), the ship lifetimes in the particular safety states are:

$\overline{m}^{(6)}(1) \cong 0.31,\ \overline{m}^{(6)}(2) \cong 0.21,\ \overline{m}^{(6)}(3) \cong 0.26,$

$\overline{m}^{(6)}(4) \cong 1.49$ years.

In the case when the system operation time is large enough, the unconditional safety function of the ship is given by the vector

$$\boldsymbol{s}_6(t,\cdot)$$

$$= [1,\ \boldsymbol{s}_6(t,1),\ \boldsymbol{s}_6(t,2),\ \boldsymbol{s}_6(t,3),\ \boldsymbol{s}_6(t,4)],\ t \geq 0,$$

where, according to (14), the co-ordinates are

$$\boldsymbol{s}_6(t,1) = p_1 \bar{\boldsymbol{s}}_4^{(1)}(t,1) + p_2 \bar{\boldsymbol{s}}_4^{(2)}(t,1) + p_3 \bar{\boldsymbol{s}}_5^{(3)}(t,1)$$

$$+ p_4 \bar{\boldsymbol{s}}_5^{(4)}(t,1) + p_5 \bar{\boldsymbol{s}}_5^{(5)}(t,1) + p_6 \bar{\boldsymbol{s}}_4^{(6)}(t,1)$$

$$= 0.145 \cdot \exp[-0.25t] + 0.098 \cdot \exp[-0.25t]$$

$$+ 0.004 \cdot \exp[-0.38t] + 0.007 \cdot \exp[-0.38t]$$

$$+ 0.374 \cdot \exp[-0.46t] + 0.374 \cdot \exp[-0.44t],$$

$$\boldsymbol{s}_6(t,2) = p_1 \bar{\boldsymbol{s}}_4^{(1)}(t,2) + p_2 \bar{\boldsymbol{s}}_4^{(2)}(t,2) + p_3 \bar{\boldsymbol{s}}_5^{(3)}(t,2)$$

$$+ p_4 \bar{\boldsymbol{s}}_5^{(4)}(t,2) + p_5 \bar{\boldsymbol{s}}_5^{(5)}(t,2) + p_6 \bar{\boldsymbol{s}}_4^{(6)}(t,2)$$

$$= 0.145 \cdot \exp[-0.31t] + 0.098 \cdot \exp[-0.31t]$$

$$+ 0.004 \cdot \exp[-0.49t] + 0.007 \cdot \exp[-0.49t]$$

$$+ 0.3.74 \cdot \exp[-0.55t] + 0.372 \cdot \exp[-0.51t],$$

$$\boldsymbol{s}_6(t,3) = p_1 \bar{\boldsymbol{s}}_4^{(1)}(t,3) + p_2 \bar{\boldsymbol{s}}_4^{(2)}(t,3) + p_3 \bar{\boldsymbol{s}}_5^{(3)}(t,3)$$

$$+ p_4 \bar{\boldsymbol{s}}_5^{(4)}(t,3) + p_5 \bar{\boldsymbol{s}}_5^{(5)}(t,3) + p_6 \bar{\boldsymbol{s}}_4^{(6)}(t,3)$$

$$= 0.145 \cdot \exp[-0.39t] + 0.098 \cdot \exp[-0.39t]$$

$$+ 0.004 \cdot \exp[-0.58t] + 0.007 \cdot \exp[-0.58t]$$

$$+ 0.0374 \cdot \exp[-0.62t] + 0.372 \cdot \exp[-0.57t],$$

$$\boldsymbol{s}_6(t,4) = p_1 \bar{\boldsymbol{s}}_4^{(1)}(t,4) + p_2 \bar{\boldsymbol{s}}_4^{(2)}(t,4) + p_3 \bar{\boldsymbol{s}}_5^{(3)}(t,4)$$

$$+ p_4 \bar{\boldsymbol{s}}_5^{(4)}(t,4) + p_5 \bar{\boldsymbol{s}}_5^{(5)}(t,4) + p_6 \bar{\boldsymbol{s}}_4^{(6)}(t,4)$$

$$= 0.145 \cdot \exp[-0.44t] + 0.098 \cdot \exp[-0.44t]$$

$$+ 0.004 \cdot \exp[-0.67t] + 0.007 \cdot \exp[-0.67t]$$

$$+ 0.374 \cdot \exp[-0.70t] + 0.372 \cdot \exp[-0.67t] \text{ for } t \geq 0.$$

The mean values and variances of the system unconditional lifetimes in the safety state subsets, according to (31) and (33), respectively are

$$m(1) = p_1 m^{(1)}(1) + p_2 m^{(2)}(1) + p_3 m^{(3)}(1)$$

$$+ p_4 m^{(4)}(1) + p_5 m^{(5)}(1) + p_6 m^{(6)}(1),$$

$$\cong 0.145 \cdot 4.00 + 0.098 \cdot 4.00 + 0.004 \cdot 2.63$$

$$+ 0.007 \cdot 2.63 + 0.374 \cdot 2.17 + 0.372 \cdot 2.27 = 2.66.$$

$$[\sigma(1)]^2 \cong 2[0.145 \cdot [4.00]^2 + 0.098 \cdot [4.00]^2$$

$$+ 0.004 \cdot [2.63]^2 + 0.007 \cdot [2.63]^2 + 0.374 \cdot [2.17]^2$$

$$+ 0.372 \cdot [2.27]^2] - [2.66]^2 = [2.87]^2,\ \sigma(1) \cong 2.87,$$

$$m(2) = p_1 m^{(1)}(2) + p_2 m^{(2)}(2) + p_3 m^{(3)}(2)$$

$$+ p_4 m^{(4)}(2) + p_5 m^{(5)}(2) + p_6 m^{(6)}(2),$$

$$\cong 0.145 \cdot 3.26 + 0.098 \cdot 3.26 + 0.004 \cdot 2.04$$

$$+ 0.007 \cdot 2.04 + 0.374 \cdot 1.82 + 0.372 \cdot 1.96 = 2.22,$$

$$[\sigma(2)]^2 \cong 2[0.145 \cdot [3.26]^2 + 0.098 \cdot [3.26]^2$$

$$+ 0.004 \cdot [2.04]^2 + 0.007 \cdot [2.04]^2 + 0.374 \cdot [1.82]^2$$

$$+ 0.372 \cdot [1.96]^2] - [2.22]^2 = [2.38]^2,\ \sigma(2) \cong 2.38,$$

$$m(3) = p_1 m^{(1)}(3) + p_2 m^{(2)}(3) + p_3 m^{(3)}(3)$$

$$+ p_4 m^{(4)}(3) + p_5 m^{(5)}(3) + p_6 m^{(6)}(3),$$

$$\cong 0.145 \cdot 2.56 + 0.098 \cdot 2.56 + 0.004 \cdot 1.72$$

$$+ 0.007 \cdot 1.72 + 0.374 \cdot 1.61 + 0.372 \cdot 1.75 = 1.89,$$

$$[\sigma(3)]^2 \cong 2[0.145 \cdot [2.56]^2 + 0.098 \cdot [2.56]^2$$

$$+ 0.004 \cdot [1.72]^2 + 0.007 \cdot [1.72]^2 + 0.374 \cdot [1.61]^2$$

$$+0.372\cdot[1.75]^2\,]-[1.89]^2=[1.97]^2,\ \sigma(3)\cong 1.97,$$

$$m(3)=p_1 m^{(1)}(4)+p_2 m^{(2)}(4)+p_3 m^{(3)}(4)$$

$$+p_4 m^{(4)}(4)+p_5 m^{(5)}(4)+p_6 m^{(6)}(4),$$

$$\cong 0.145\cdot 2.27+0.098\cdot 2.27+0.004\cdot 1.49$$

$$+0.007\cdot 1.49+0.374\cdot 1.43+0.372\cdot 1.49=1.66,$$

$$[\sigma(4)]^2\cong 2[0.145\cdot[2.27]^2+0.098\cdot[2.27]^2$$

$$+0.004\cdot[1.49]^2+0.007\cdot[1.49]^2+0.374\cdot[1.43]^2$$

$$+0.372\cdot[1.49]^2\,]-[1.66]^2=[1.73]^2,\ \sigma(4)\cong 1.3.$$

The mean values of the system lifetimes in the particular safety states, by (34), are

$$\overline{m}(1)=m(1)-m(2)=0.44,$$

$$\overline{m}(2)=m(2)-m(3)=0.33,$$

$$\overline{m}(3)=m(3)-m(4)=0.23,$$

$$\overline{m}(4)=m(4)=1.66.$$

If the critical safety state is r = 2, then the system risk function, according to (6), is given by

$$\boldsymbol{R}(t)=1-\boldsymbol{s}_6(t,2)$$

$$=0.145\cdot\exp[-0.31t]+0.098\cdot\exp[-0.31t]$$

$$+0.004\cdot\exp[-0.49t]+0.007\cdot\exp[-0.49t]$$

$$+0.3.74\cdot\exp[-0.55t]+0.372\cdot\exp[-0.51t]\ \text{for}\ t\geq 0.$$

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (7), is

$$\tau=\boldsymbol{r}^{-1}(\delta)\cong 0.11\ \text{years}.$$

9. Conclusion

In the paper the multi-state approach to the safety analysis and evaluation of systems related to their variable operation processes has been considered. Theoretical definitions and preliminary results have been illustrated by the example of their application in the safety evaluation of a ship transportation system with changing in time its operation states. The ship safety structure and its safety subsystems characteristics are changing in different states what makes the analysis more complicated but also more precise than the analysis performed in [2]. However, the varying in time ship safety structure used in the application is very general and simplified and the subsystems safety data are either not precise or not real and therefore the results may only be considered as an illustration of the proposed methods possibilities of applications in ship safety analysis. Anyway, the obtained evaluation may be a very useful example in simple and quick ship system safety characteristics evaluation, especially during the design and when planning and improving her operation processes safety and effectiveness.

The results presented in the paper suggest that it seems reasonable to continue the investigations focusing on the methods of safety analysis for other more complex multi-state systems and the methods of safety evaluation related to the multi-state systems in variable operation processes [9], [10] and their applications to the ship transportation systems [5].

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