RELIABILITY AND RISK ASSESSMENT OF SYSTEMS OF PROTECTION AND BLOCKING WITH FAST RESTORATION

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Abstract

There is examined a system with fast restoration which should be operational beginning from some moments of time. If beginning from these moments of time the system is defective during the time more than the assigned random time interval it is considered failed. Such system includes the models of systems with the protection and blocking and systems with the discrete periodic functions. The estimations of indices of failure-free performance and maintainability of these systems and the estimation of indices of risk and losses, connected with the failure (accident) of the system with protection are obtained. This material was presented in the Mathematical Methods in Reliability 2007 Conference in Glasgow, UK.

1. Introduction and Motivation

August 2003. The largest in the history of the USA a de-energizing of eastern regions of the USA and Canada for several days has left extensive territories and huge quantity of the population without the electric energy. Losses from this blackout were incredible. What did cause this catastrophic failure? The system of protection and blockings decided that power supply became too high and not to harm power plants they were cut off. In just three minutes the system of protection and blockings produced a cascade avalanche-like cut-off of 21 power plants.

September 2003. The "power outage that affected all of Italy—except the island of Sardinia—for 9 hours and part of Switzerland near Geneva for 3 hours on 28 September 2003. It was the largest blackout in the series of blackouts in 2003, affecting a total of 56 million people" (*http://en.wikipedia.org/wiki/2003_Italy_blackout*).

May 2005. The blackout in Moscow and in the adjacent regions of Russia.

November 2006. "Two high voltage power lines in Germany failed. This triggered a cascade of cuts as automatic safety devices cut millions of customers in order to prevent a total blackout of the continent. Parts of Germany, Belgium, France (including parts of Paris), Spain, and Italy were affected. High speed railways were also impacted. Power was restored within two hours. Later reports said that Austria and Croatia were also affected"
(*http://en.wikinews.org/wiki/Europe_suffers_widespread_power_cuts*).

These events show the importance of the reliable and correct functioning of systems with protection and blockings. The same events tightly connect the concepts of reliability and engineering risk (risk, appearing as a result of human activity), under which follow to Henley and Kumamoto [10] we understand consequences (on resources, on environment, victims and so on) arising as results of unreliable work (failures) of technical systems and/or intentional terrorists activity.

There are examined systems, for which it is required, that the system would be serviceable only beginning from specific moments of time. If, beginning from specific moments of time, the system is malfunction during the time not less than η, P{η < x} = H(x), the system is considered failed. Such systems include the models of systems with discretely periodical functions and the models of systems with the protection and blockings.

The system with discrete periodic function is considered failed, if it is faulty during the time not less than η after a demand for the function's service arrival. At other moments of time the system can be faulty, but this does not affect the reliability of the system with discrete periodic function.

The system with protection and blockings (SPB) contains a certain object, which periodically falls into a before accident situation (BAS), and a unit of protection and blockings (UPB) that should prevent an accident. Such an object can be, for example, a system of power plants together with power lines and end users. Power stations can fail; short circuits or breaks are possible on power lines and at end users. Many of such events represent a BAS which signals should be sent to UPB. Then during a "short" interval of time UPB should make a reconfiguration of system in purpose to disconnect or the failed plant, or/and failed power lines and the end user, and to redistribute the power supply that was delivered by the failed plant (if any) between other plants and/or power lines. Otherwise there can be an accident, for example not authorized redistribution of power supply which indeed can lead to cascade switching-off of plants.

For given above examples with de-energizing the BAS events could be failures of some power plants (a deficiency of electric power), or some short circuits on the power lines (as it was in the USA), or a short circuit on transformer substations (as it was in Russia), or a result of an external intentional terrorist activity directed toward the destruction of systems of electric power's production and delivery. Power plants are united into the power grid. Some power plants can temporarily stop the electric energy generation. But the electric energy generation by remained power plants is sufficient to ensure the needs of all basic users.

UPB receiving a BAS signal should prevent the accident. For our examples it means that during the assigned time interval, which is less then η, some actions must be executed in order or to block the failed power plants, temporarily excluding them from the power grid, or to block some sections of the power transmission line, where the short circuit or the break occurred, or to block users who had a short circuit that led to the big power consumption and to reconfigure the layout of the power delivery. UPB being also a technical system itself is subjected to failures and is continuously monitoring so that failures arising in it are eliminating.

A failure of SPB (the accident) occurs when BAS signal arrives at the failed UPB who therefore cannot prevent accident. In this case the UPB failure leads to the SPB failure (accident). But the event "UPB failed but had time to be restored before arrival of the BAS signal" will not affect on the SPB reliability. So, not every UPB failure leads to the SPB failure. There can be "dangerous" and "safe" failures of UPB that should be considered in the criterion of the SPB failure (accident).

In this article we will use terms, relating to the systems with protection and blockings. For evaluating the reliability of SPB we will use ideas of the reliability assessment of systems with fast restoration presented in our article [1].

2. Model Description

In the general case UPB is a space-distributed repairable system, and contains n elements and k repair units (RU). UPB come to the models of section 2 of [1]. Each element of system can be only in the operational or the failed state. Each operational element can be located in the loaded or unloaded regime. Let $F_i(x)$ and $f_i(x)$ are accordingly the distribution function (DF) and the distribution density (DD) of the time of failure-free operation of the i-th element in the system, $i = \overline{1,n}$, and m_i is the mean value of this time, $m_i < \infty$.

We will examine only the systems of the 1st and the 3d types [1], working in the steady-state operation. The systems of the 2nd type [1], that don't have the steady-state operation section of work, are not examined within the framework of this article. But taking into account [1] and the ideas given below in the sections 4 and 5 it is also possible to carry out the estimation of the reliability of the systems of the 2nd type.

The failed elements are restored. Different interruptions of restoration are permitted, but DF of summary recovery time of the i-th element by the j-th RU is equal to $G_{ij}(x)$, independent of the number and the duration of the interruptions Genis [3]. Class D of restoration disciplines [1] includes, in particular, the discipline FIFO d_1 with the straight order of maintenance, where the priority for the restoration have the elements, failed the first, discipline LIFO d_2 with the reverse order of maintenance, where the priority for the restoration have the elements, failed the last, discipline with time sharing d_3, where all failed elements are restored with the same speed, and discipline d_4, where the priority for the restoration have the elements with the shortest residual recovery time. The indices of reliability of the same systems for various restoration disciplines are essentially different. Therefore the reliability assessment for various restoration disciplines allows choosing the most effective discipline.

It is set the criterion of SPB failure that can include and a condition of time reservation.

UPB works in conditions of fast restoration (FR). Practically it means that the average time of restoration of a system's element is essentially less than the average time between any two failures of elements in the system [3].

The problem consists in estimating of SPB indices of non-failure operation and maintainability in conditions of fast restoration of UPB.

3. Mathematical Formulation of Problem

Behavior of UPB is described by the alternating process, in which the intervals where all elements are operational are changed by intervals, when in UPB there are any failures of elements, which possibly are not leading to the failure or the malfunction of UPB [1]. Let us call the last intervals as intervals of the malfunction (IM). Let us call the interval of malfunction, which begun in the interval (z, z+dz), as IM *z*.

The state of the elements of system at the moment z is assigned by the vector $\vec{v}(z) = \{v_1(z),...,v_n(z)\}$, where each component can take the values of $\{0, 1,..., n\}$. Number 0 corresponds to failed elements; numbers from 1 to n correspond to operational elements. Vector $\vec{v}(z)$ helps to estimate the reliability of concrete systems.

Let E is the set of the states of the system, $\{\vec{v}(z)\} = E = E_+ \cup E_-$, where E_+ is the area of the operational, and E_- is the area of the defective states of the system. The system is considered as defective at the moment z if $\vec{v}(z) \in E_-$ and failed if its malfunction lasts time not smaller then $\eta, P\{\eta < x\} = H(x)$ [1].

Let $\vec{b}$ is a certain state vector of elements of the system directly before IM, and $\vec{b}^N$ is the state vector of the elements of the system on the same IM immediately after the moment of passing the state vector of system from the region E_+ into the area E_-. Let π is the way leading from $\vec{b} \in E_+$ into the state $\vec{b}^N \in E_-$ on the IM. Then π is the sequence of the state vectors of elements, beginning from the vector $\vec{b}$, which directly precede the beginning of the IM, and ending with the vector $\vec{b}^N$, which corresponds to the first onset of malfunction of the system on this IM; the passage from one state vector to the following occurs only due to that, that exactly one element of system fails or ends to be restored [1].

The path length is equal to the number of state vectors, being contained on this path, not counting the initial state $\vec{b}$. Let us call the way monotonic if on it there are no restorations of elements. Let us call the monotonic way minimal for $\vec{b}$ if its length $l(\vec{b})$ is equal to the minimum of path lengths, leading from $\vec{b}$ into E_- [1].

All introduced notations help to understand the obtained results and are used to prove them.

Let us determine the concept of fast restoration. Let $G(x) = min\ G_{ij}(x)$, $G_*(x) = max\ G_{ij}(x)$, where the minimum and the maximum are taken according to the numbers j of RU, accessible to i-th element, and on $i = \overline{1,n}$ (here $G(x)$ and $G_*(x)$ are DF of the correspondingly greatest and shortest recovery time of elements); s is the minimum number of elements, failure of which can cause the malfunction of the system; $\overline{\Gamma}(\ ) = 1 - \Gamma(\ )$ for whichever DF $\Gamma(\ )$;

$m_R^{(j)} = j\int_0^\infty x^{j-1}\overline{G}(x)dx, \qquad m_R = m_R^{(1)}, \qquad m_{R^*}(\eta) = \int_0^\infty \int_0^\infty \overline{G}_*(x+u)\,dx\,dH(u);$

$\overline{\lambda}$ and $\underline{\lambda}$ are the maximum and the minimum failure rates of elements in the operational system [1].

Let us say, that in the system is satisfied the condition of FR if $\underline{\lambda} > 0$ and

$$\alpha = [\overline{\lambda}^s\, m_R^{(s)} / (m_R)^{s-1}] \to 0 \qquad (3.1)$$

and in this case for all DF $F_i(x)$, $i = \overline{1,n}$, there exist limited DDs.

In practice it is necessary to evaluate the reliability of a concrete system with fixed DF $F_k(x)$ and $G_{ij}(x)$. Therefore it is possible to count without the damage for the generality, that DF $F_k(x)$ are fixed, and DF $G_{ij}(x)$ are the element of a certain infinite sequence in the diagram of series. More precise we will assume, that are satisfied the following conditions introduced by A.D. Solovyev in [7]:

1) DF $F_k(x)$, $k = \overline{1,n}$, are fixed and have a limited and continuous in zero DD;
2) DF $G_{ij}(x)$ have the form

$$G_{ij}(x) = G_{ij}^{(0)}(x/\xi),$$

where $G_{ij}^{(0)}(x)$ are fixed, and

$$\xi \to 0; \qquad (3.2)$$

in this case $G(x) = G^{(0)}(x/\xi)$, $G_*(x) = G_*^{(0)}(x/\xi)$;

3) There is a finite moment $a^{(s)}$, where

$$a^{(j)} = j\int_0^\infty x^{j-1}\overline{G}^{(0)}(x)dx\,,\ a = a^{(1)}.$$

Let us note that the conditions (3.1) and (3.2) are equivalent under assumptions 1) – 3). Actually, under assumption 1) the value $\hat{\lambda}$ is limited and under assumptions 2) and 3) when $\xi \to 0$

$$\left(m_R^{(s)}/(m_R)^{s-1}\right) = \xi\left(a^{(s)}/a^{s-1}\right) \to 0.$$

Conversely, if we assume $\xi = \left(m_R^{(s)}/(m_R)^{s-1}\right)/\left(a^{(s)}/a^{s-1}\right)$, than from condition (3.1) and 3) it follows $\xi \to 0$.

Under the condition of fast restoration almost always the failure of system occurs along the monotonic path [1], if only the probability of this failure is not zero. However, the sufficient condition of that, that the probability of the failure of system on the monotonic path is different from zero, is the condition

$$a_*(\eta) = \int_0^\infty\int_0^\infty \overline{G}_*(x+u)dxdH(u) > 0 \tag{3.3}$$

Next index unites the conditions for fast restoration (3.1) and (3.3)

$$\varphi_1 = [\hat{\lambda}^s\ m_R^{(s)} / \underline{\lambda}^{s-1}[m_{R*}(\eta]^{s-1}] \to 0,\ \underline{\lambda} > 0 \tag{3.4}$$

The condition

$$\varphi_2 = \hat{\lambda} m_R \to 0 \tag{3.5}$$

ensures the convergence of DF of time to the first failure for the system of the 1st and 3d type to the exponential function, and for the 2nd type to $exp\{-\int_0^x \beta(u)du\}$, that is shown in [1].

In practically important cases $m_R^{(s)} \le C\,(m_R)^s$, where C - some constant. In these cases at small s ($s \approx 2 \div 4$), closely related among themselves G (x) and G * (x), that is reached due to unification of procedure of restoration, and a small time reserve ($m_R \approx m_R(\eta)$) condition (3.4) is possible to replace by condition (3.1) or condition (3.5).

In section 8 [1] it is shown, that under the conditions of FR the estimation of the indices of the reliability of complex system can be brought to the estimations of the indices of reliability of its series-connected in the sense of the reliability schemes of the form p out of m, calculated under the assumption, that these schemes operate autonomously. The scheme p out of m has m of elements. Its malfunction occurs with the failure of not less than p elements out of m, $p \le m$, and its failure begins then, when the malfunction of the scheme lasts not less than $\eta, P\{\eta < x\} = H(x)$. Therefore within the framework of this article we will count that UPB is the scheme p out of m.

In our estimations we will count, that all RU are identical and therefore $G_{ij}(x) = G_i(x)$. Let $m_{Ri}^{(j)}(\eta/a) = j\int_0^\infty\int_0^\infty x^{j-1}\overline{G}_i(x+u/a)dxdH(u)$, $m_{Ri}(\eta) = m_{Ri}^{(1)}(\eta)$, $m_{Ri}^{(j)}(u) = j\int_0^\infty x^{j-1}\overline{G}_i(x+u)dx$, $\varphi_i(u) = m_{Ri}^{(1)}(u)$, $m_{Ri}^{(j)} = m_{Ri}^{(j)}(0)$, $m_{Ri} = m_{Ri}^{(1)}$. Let in the steady-state operation section of work with $d \in D$ and k RU $\beta_p(d,k)$ is the estimation of the failure rate of the scheme p out of m with k RU and the restoration discipline $d \in D$ taking into account only monotonic ways of failure, $\tau''(d,k)$ is the random system recovery time after failure, $T_R(d,k)$ is the average value of this time, $K_A(d,k)$ is the availability function of the system.

4. Estimation of the indices of failure-free performance

Let $\tau_i(t)$ is the interval from the moment t to the first failure of system after moment t. The details of the determination of time to the first failure of the system $\tau_1(t)$ and time between (j -1)-th and j-th failures of the system $\tau_j(t), j \geq 2$, are given in [1].

Let $F_*(x)$ and m_* are the distribution function of time between the adjacent BAS signals and mean time between them, moreover $F_*(x)$ is an absolutely continuous distribution function with the limited distribution density. In the steady-state operating conditions of the system the DF of residual time before the appearance of a BAS signal is

$$E(x) = \int_0^x \overline{F_*}(x)dx / m_*.$$

The object, which sends BAS signals, is considered as one of the elements of the system containing $(n+1)$ elements. We will investigate two cases:

1) the condition for the fast restoration is satisfied also relatively to the time between the appearances of the BAS signals (time between the adjacent BAS signals is considered as the object operating time between failures; the restoration of UPB leads to the restoration of SPB);
2) this condition is not satisfied, but in UPB the restoration is fast (restoration of UPB leads to the restoration of SPB);

In both cases is valid theorem 5.3 and estimation (5.8) from [1], and in steady-state operation DF $\tau_i(t)$, $i \geq 1$, converge to exponential. It remains to estimate the parameter of these distributions.

In the first case the SPB model come to the model of section 2 of [1], but containing (n+1) elements. In this model is fixed the element, failing the last before the failure of the system (if the time reserve is absent and $\eta \equiv 0$) or before the beginning of the malfunction of the system (when $\eta \neq 0$). This element is object. On IM this element is not restored, and the time reserve of system is equal to η. In this case all estimations (6.2) and (6.4) - (6.7) from [1] are carried out, if in them in all expressions, besides *dH(u)*, to replace "u" by "v+u" and $\int_0^\infty ...dH(u)$ by

$\int_0^\infty\int_0^\infty ...\overline{F_*}(v)dvdH(u)/m_*$ or $\int_0^\infty\int_0^\infty ..dvdH(u)/m_*$. There "$v$" represents the residual time to the arrival of BAS signal and "u" is the time reserve, during which must be executed actions on averting the accident. Replacement $\int_0^\infty ...dH(u)$ by $\int_0^\infty\int_0^\infty ..dvdH(u)/m_*$ is carried out on the basis of the condition for fast restoration that is satisfied also relatively to the time between the appearances of BAS signals, when $\overline{F_*}(v)dv/m_* \approx dv/m_*$.

In the same case with $n=1$ (structural reserve in UPB it is absent) because of the fast restoration and relative to the time between the appearances of the BAS signals $(\hat{\lambda}\, m_R^{(2)}/m_R) \to 0$, DF $\tau_i(t)$, $i \geq 1$, converge to exponential and with k = 1 in accordance with the criterion of the failure of the system

$$\beta_1(d,1) \approx \frac{1}{m_1 m_*}\int_0^\infty\int_0^\infty \overline{G_1}(u+v)dvdH(u) \approx \frac{m_{r1}(\eta)}{m_1 m_*} \tag{4.1}$$

Result (4.1) was obtained by Turbin with co-authors [5] for $\eta \equiv 0$, $d = d_1$ under more stronger assumptions. In particular, there was required the absolute continuity of d.f. $G_1(x)$.

In the second case the system with protection and blockings is reduced to the model of section 2 [1], in which in accordance with the criterion of the failure of the system this failure begins when malfunction of UPB lasts not less than the time $(\gamma+\eta)$, $P\{\eta < x\} = H(x)$, and in the steady-state operating conditions of the system $P\{\gamma < x\} = E(x)$. Therefore and in this case with the presence of structural reserve in SPB are carried out all previous estimations for the indices of failure-free performance, if in them in all expressions, besides *dH(u),* to replace "*u*" by "*v+u*" and $\int_0^\infty ...dH(u)$ by $\int_0^\infty\int_0^\infty ...\overline{F_*}(v)dvdH(u)/m_*$.

Thus it is proven

Theorem 4.1. For examined cases of systems with protection and blockings there are carried out the estimations $\beta_p(d,k)$, undertaken for the scheme *p* from *n*, if in these estimations in all expressions, besides *dH(u)*, to replace "*u*" by "*v+u*" and $\int_0^\infty ...dH(u)$ by $\int_0^\infty\int_0^\infty \overline{F_*}(v)dvdH(u)/m_*$. In the case if the fast restoration is satisfied also relatively to the time between the appearances of signals of before accident situations it is allowed to substitute $\int_0^\infty ...dH(u)$ by $\int_0^\infty\int_0^\infty ...dvdH(u)/m_*$.

Corollary 4.1. If UPB represents $n>1$ parallel-connected in the sense of the reliability elements used in the loaded regime and $k=1$ than for the system with protection and blockings under the conditions for fast restoration that are satisfied also relatively to the time between the appearances of BAS signals

$$\beta_n(d_1,1) \approx \frac{n-1}{m_1 ... m_n m_*}\sum_{j=1}^{n}\int_0^\infty\int_0^\infty\int_0^\infty x^{n-2}\overline{G_j}(x+v+u)dxdvdH(u) =$$

$$= \sum_{j=1}^{n} m_{Rj}^{(n)}(\eta)/(m_1 ... m_n m_* n);$$

(4.2)

$$\beta_n(d_2,1) \approx \frac{(n-1)!}{m_1 ... m_n m_*} \sum_{j=1}^{n} \int_0^\infty \int_0^\infty \overline{G_j}(v+u) \prod_{k \neq j} m_{Rk} dv dH(u) =$$

$$= (n-1)! \sum_{j=1}^{n} m_{Rj}(\eta) \prod_{k \neq j} m_{Rk} /(m_1 ... m_n m_*); \tag{4.3}$$

$$\beta_n(d_3,1) \approx \frac{(n-1)!}{m_1 ... m_n m_*} \sum_{j=1}^{n} \int_0^\infty \int_0^\infty \overline{G_j}\left(\frac{v+u}{n}\right) \prod_{k \neq j} \varphi_k\left(\frac{v+u}{n}\right) dv dH(u) =$$

$$= \frac{n!}{m_1 ... m_n m_*} \int_0^\infty \left(\prod_{j=1}^{n} \int_0^\infty \overline{G}_j(y + \frac{u}{n}) dy \right) dH(u);$$

(4.4)

$$\beta_n(d_4,1) \approx \sum_{j=1}^{n} \int_0^\infty \int_0^\infty \overline{G_j}(v+u) \prod_{k \neq j} \varphi_k(v+u) dv dH(u) =$$

$$= \frac{1}{m_1 ... m_n m_*} \int_0^\infty \left(\prod_{j=1}^{n} \int_0^\infty \overline{G}_j(y+u) dy \right) dH(u). \tag{4.5}$$

For the proof of Corollary 4.1 the probabilities of system failures along all minimal monotonic paths are summarized. At that for estimation of indices of failure-free performance for various disciplines of restoration the results of Corollary 6.1 of [1] were used.

All necessary proofs in the article are given in the Appendix.

For the system with the discrete periodic function the probability $Q_D(d,k)$ of the failure of system to the requirement on the fulfillment of the function is determined from the formula

$$Q_D(d,k) \approx m_* \beta(d,k).$$

(4.6)

5. Indices of maintainability and availability function

It is examined a system, described in paragraph 2, with the condition $\eta = h = const$. This condition is typical. Let us define recovery time of SPB as time, passed from the moment of the failure of the system to the moment of the restoration of SPB capacity for work. For evaluating the indices of the maintainability and availability function we will need the concept of *x*-failure of SPB [1]. Let us say, what the system *x*-failed, if its failure lasts not less than the time *x*. Through $\beta_x(d,r)$ we will designate the intensity of SPB *x*-failures. Let us recall, that in the SPB model the BAS signals enter the last before the UPB failure.

Since with $\eta = h = const$ $H(u) = 0$ with $u<h$ and $H(u) = 1$ with $u \geq h$ than taking into account theorem 4.1 we will obtain

Corollary 5.1. For the system with protection and blockings and the time reserve, equal to the constant, $\eta = h = const$, the estimation $\beta_x(d,r)$ is obtained

- or from the estimations $\beta_p(d,r)$ for the scheme *p* out of *m* from [1], if in them in all expressions, besides *dH(u)*, to replace the arguments "*u*" by "*h+x+v*", and replace $\int_0^\infty ...dH(u)$ by $\int_0^\infty ...\overline{F}_*(v)dv / m_*$; in the case of the fast restoration and relatively to the time between the appearances of the signals of before accident situations it is allowed to substitute $\int_0^\infty ...dH(u)$ by $\int_0^\infty ...dv$;
- or from the estimations $\beta_p(d,r)$ of section 4 of this work, if in them in all expressions, besides *dH(u)*, to replace the arguments "u" by "h+x" and to remove the external integral on *dH(u)*.

Corollary 5.2. If UPB is a system *n* out of *n* with the loaded reserve (parallel in the sense of reliability connection of elements), $k = 1$ and the condition of fast restoration is satisfied and relatively to the time between the appearances of BAS signals, than

for $d = d_1$

$$P\{\tau''(d_1,1) \geq x\} \approx \sum_{i=1}^{n} m_{Ri}^{(n)}(h+x) / \sum_{j=1}^{n} m_{Ri}^{(n)}(h), \tag{5.1}$$

$$T_R(d_1,1) \approx \sum_{i=1}^{n} m_{Ri}^{(n+1)}(h) / \left((n+1) \sum_{j=1}^{n} m_{Rj}^{(n)}(h) \right), \tag{5.2}$$

$$K_A(d_1,1) \approx 1 - \sum_{i=1}^{n} m_{ri}^{(n+1)}(h) / \left(n(n+1) m_* \prod_{j=1}^{n} m_j \right); \tag{5.3}$$

for $d = d_2$

$$P\{\tau''(d_2,1\} \geq x\} \approx \left[\sum_{j=1}^{n} m_{Rj}(h+x) / m_{Rj} \right] / \left[\sum_{i=1}^{n} m_{Ri}(h) / m_{Ri} \right], \tag{5.4}$$

$$T_R(d_2,1) \approx \left[\sum_{j=1}^{n} m_{Rj}^{(2)}(h) / m_{Rj} \right] / \left[2 \sum_{i=1}^{n} m_{Ri}(h) / m_{Ri} \right], \tag{5.5}$$

$$K_A(d_2,1) \approx 1 - (n-1)! \sum_{j=1}^{n} m_{Rj}^{(2)}(h) \prod_{i \neq j} m_{Ri} / \left(2 m_* \prod_{k=1}^{n} m_k \right); \tag{5.6}$$

for $d = d_3$

$$P\{\tau''(d_3,1) \geq x\} \approx \prod_{i=1}^{n} m_{Ri}\left(\frac{h+x}{n}\right) / m_{Ri}\left(\frac{h}{n}\right), \tag{5.7}$$

$$T_R(d_3,1) \approx \int_0^\infty \prod_{i=1}^{n} \left[m_{Ri}\left(\frac{h+x}{n}\right) / m_{Ri}\left(\frac{h}{n}\right)\right] dx, \tag{5.8}$$

$$K_A(d_3,1) \approx 1 - n! \int_0^\infty \prod_{i=1}^{n} \left[m_{Ri}\left(\frac{h+x}{n}\right) /(m_* m_i)\right] dx; \tag{5.9}$$

for $d = d_4$

$$P\{\tau''(d_4,1) \geq x\} \approx \prod_{i=1}^{n} m_{Ri}(h+x) / m_{Ri}(h), \tag{5.10}$$

$$T_R(d_4,1) \approx \int_0^\infty \prod_{i=1}^{n} \left[m_{Ri}(h+x) / m_{Ri}(h)\right] dx, \tag{5.11}$$

$$K_A(d_4,1) \approx 1 - \int_0^\infty \prod_{i=1}^{n} \left[m_{Ri}(h+x) /(m_* m_i)\right] dx. \tag{5.12}$$

For the proof of Corollary 5.2 were used estimations of indices of maintainability and availability function for various disciplines of restoration that are given in Corollary 6.2 of [1].

For *k*=1 and *n*=1 (UPB consists of one element) and with any discipline $d \in D$

$$P\{\tau''(d,1) \geq x\} \approx m_R(h+x) / m_R(h),$$

$$T_R(d,1) \approx m_R^{(2)}(h) /(2m_R(h),$$

$$K_A(d,1) \approx 1 - m_R^{(2)}(h) /(2mm_*).$$

In much the same way it is possible to obtain the estimations of the indices of maintainability and availability function also for other types of UPB.

Let us note that it is possible to remove the requirement of absolute continuity of d.f . $F_*(x)$. Thus, the period between two adjacent entries of BAS signals (or of requirements for the fulfillment of a function in the system with the discretely carried out functions) can be constant. In this case it is possible to obtain the same estimations of the indices of reliability, using an apparatus, connected with the rare events in the regenerating process [6].

6. Estimation of risk and losses

It is natural to ask the question how to estimate the risk and losses connected with the failure of the system of protection and blockings? One of the possible approaches is the following.

When SPB fails the local systems of protection and blockings as a rule soften the losses from the accident. Let there are possible N different ways of the development of the accident when SPB fails. The probability of the *i*-th way of the development of the accident is p_i, $\sum p_i = 1$ on $i = \overline{1, N}$, and the losses on this way are L_i. Then risk and losses from the SPB failure [9] can be estimated as

$$R_1 = \sum_{i=1}^{N} p_i L_i \, . \tag{6.1}$$

In practice [9] the value L_i is substituted by the *loss function or the function of the usefulness* $U(L_i)$ (when some losses are disregarded, and some losses are exaggerated), and the probabilities p_i are substituted by the *subjective probabilities* $f(p_i)$ (when some small probabilities are disregarded). In this case risk and losses from the SPB failure can be estimated as

$$R_2 = \sum_{i=1}^{N} f(p_i) U(L_i) \tag{6.2}$$

With this approach the estimation of risk and losses can be conducted only for the concrete system in the stages of its design and operation.

Appendix

The following three lemmas will make it possible to simplify calculation formulas.

Let $M_J^{(l)}$ is the set of all permutations from the collection of the numbers $J = (j_1, ..., j_l)$ and $i^{(l)} = (i_1, ..., i_l)$ is a certain permutation from the set $M_J^{(l)}$.

Lemma A.1. For $i \geq 1$ and any fixed $x \geq 0$ and $u \geq 0$ the next inequality is correct

$$\sum_{i^{(l)} \in M_J^{(l)}} \int \ldots \int_{x < y_1 < \ldots < y_l} \prod_{k=1}^{l} \overline{G}_{i_k}(y_k + u) dy_1 \ldots dy_l = \prod_{k=1}^{l} \int_x^{\infty} \overline{G}_{i_k}(y + u) dy. \tag{A.1}$$

Proof of lemma A.1. When $l = 1$ (A.1) is obviously. Let (A.1) is true for $l = w$. Then we will show that (A.1) is true for $l = w + 1$.

$$\sum_{i^{(w+1)} \in M_J^{(w+1)}} \int \ldots \int_{x < y_1 < \ldots < y_{w+1}} \prod_{k=1}^{w+1} \overline{G}_{i_k}(y_k + u) dy_1 \ldots dy_{w+1} = \sum_{i^{(w)} \in M_J^{(w)}} \int \ldots \int_{x < z_1 < \ldots < z_w} \prod_{k=1}^{w} \overline{G}_{i_k}(z_k + u) dz_1 \ldots dz_w \, *$$

$$* \left[\int_x^{z_1} \overline{G}_{i_{w+1}}(y + u) dy + \int_{z_1}^{z_2} \overline{G}_{i_{w+1}}(y + u) dy + \ldots + \int_{z_w}^{\infty} \overline{G}_{i_{w+1}}(y + u) dy \right] = \prod_{k=1}^{w+1} \int_x^{\infty} \overline{G}_{i_k}(y + u) dy \, .$$

Lemma A.1 is proved.

Lemma A.2. For any integers $k \geq 1$ and $h = const$

$$\int_0^\infty m_{Ri}^{(k-1)}(h+x)dx = m_{Ri}^{(k)}(h)/k\,. \tag{A.2}$$

Proof of lemma A.2. After using the replacement of variable and a change in the order of integration, we will obtain:

$$\int_0^\infty m_{Ri}^{(k-1)}(h+x)dx = (k-1)\int_0^\infty\int_0^\infty u^{k-2}\overline{G}_i(u+h+x)dudx = (k-1)\int_0^\infty\int_x^\infty (u-x)^{k-2}\overline{G}_i(u+h)dudx =$$

$$= (k-1)\int_0^\infty \overline{G}_i(u+h)\int_0^u (u-x)^{k-2}dxdu = \int_0^\infty \overline{G}_i(u+h)\int_u^0 d_x(u-x)^{k-1}du =$$

$$= \int_0^\infty u^{k-1}\overline{G}_i(u+h)du = m_{Ri}^{(k)}(h)/k$$

and lemma A.2 is proved.

Lemma A.3. For any integers $N \geq 1$, any $u < \infty$, $0 < a < \infty$ and any functions $f_i(x)$ such, that $\int_0^\infty f_i(x)dx < \infty$, next identity is carried out

$$\sum_{i=1}^N \int_0^\infty f_i\left(\frac{u+y}{a}\right)dy \prod_{\substack{j=1,\\ j\neq i}}^N \int_{\frac{u+y}{a}}^\infty f_j(v)dv = a\prod_{j=1}^N \int_{\frac{u}{a}}^\infty f_j(y)dy\,. \tag{A.3}$$

Proof of lemma A.3. Since

$$\sum_{i=1}^N \int_0^\infty f_i\left(\frac{u+y}{a}\right)dy \prod_{\substack{j=1,\\ j\neq i}}^N \int_{\frac{u+y}{a}}^\infty f_j(v)dv = a\sum_{i=1}^N \int_{\frac{u}{a}}^\infty f_i(y)dy \prod_{\substack{j=1,\\ j\neq i}}^N \int_y^\infty f_j(v)dv,$$

then for the proof of (A.3) it is sufficient to show, that

$$\sum_{i=1}^N \int_u^\infty f_i(y)dy \prod_{\substack{j=1,\\ j\neq i}}^N \int_y^\infty f_j(v)dv = \prod_{k=1}^N \int_u^\infty f_k(y)dy\,. \tag{A.4}$$

Since

$$\prod_{k=1}^N \int_u^\infty f_k(y)dy = -\left(\prod_{k=1}^N \int_y^\infty f_k(v)dv\right)\Big|_{y=u}^{y=\infty} = -\int_u^\infty d_y\left(\prod_{k=1}^N \int_y^\infty f_k(v)dv\right) =$$

$$= - \sum_{i=1}^{N} \int_{u}^{\infty} f_i(v) \,|_{v=y}^{v=\infty} \, dy \prod_{\substack{j=1,\\ j\neq i}}^{N} \int_{y}^{\infty} f_j(v)dv \; = \; \sum_{i=1}^{N} \int_{u}^{\infty} f_i(y)dy \prod_{\substack{j=1,\\ j\neq i}}^{N} \int_{y}^{\infty} f_j(v)dv \,,$$

And lemma (A.3) is proved.

Proof of corollary 4.1. For the scheme *n* out of *n* all monotonic ways leave from $\vec{b} \equiv \vec{1}$, where all *n* of elements are operational, and they fall into $\vec{b}^{\,j} \equiv \vec{0}$, where all *n* of elements failed [1]. The length of the monotonic way leading from $\vec{b} \equiv \vec{1}$ into $\vec{b}^{\,j} \equiv \vec{0}$ equals (*n* + 1). The BAS signal on the monotonic way always enters the last.

With $d = d_1$ let us fix the first failed element, which will be restored by the single RU. With the fixed first element and with the fixed last element (object) remained (*n* – 1) elements will give (*n* - 1)! ways, leading from $\vec{b} \equiv \vec{1}$ into $\vec{b}^{\,j} \equiv \vec{0}$. Therefore taking into account (6.2) from [1] and theorem 4.1 it follows

$$\beta_n(d_1,1) \approx \frac{(n-1)!}{m_1 \dots m_n m_*} \sum_{j=1}^{n} \int_0^\infty \int_0^\infty \underset{0<x_2<\dots<x_n}{\int \dots \int} \overline{G}_j(x_n + v + u)dx_2 \dots dx_n dvdH(u) =$$

$$= \frac{(n-1)!}{m_1 \dots m_n m_*} \sum_{j=1}^{n} \int_0^\infty \int_0^\infty \int_0^\infty \overline{G}_j(x_n + v + u)dx_n \int_0^{x_n} dx_{n-1} \dots \int_0^{x_3} dx_2 dvdH(u) =$$

$$= \frac{(n-1)}{m_1 \dots m_n m_*} \sum_{j=1}^{n} \int_0^\infty \int_0^\infty \int_0^\infty x^{n-2} \overline{G}_j(x + v + u)dxdvdH(u) \,. \tag{A.5}$$

Passing in (A.5) from the internal double integral to the iterated and after making the change of variables (x + v) = y, dv = dy, 0<*x*<(*x* + *v*) = *y*, we will obtain

$$\beta_n(d_1,1) \approx \frac{(n-1)}{m_1 \dots m_n m_*} \sum_{j=1}^{n} \int_0^\infty \int_0^\infty \overline{G}_j(y+u) \int_0^y x^{n-2} dxdydH(u) =$$

$$= \frac{1}{m_1 \dots m_n m_* n} \sum_{j=1}^{n} n \int_0^\infty \int_0^\infty y^{n-1} \overline{G}_j(y+u)dydH(u)$$

and statement (4.2) is proven.

With $d = d_2$ let us fix the last failed element of UPB which precedes the BAS signal. Those remaining (*n* - 1) elements of UPB will give (*n* - 1)! ways leading from $\vec{b} \equiv \vec{1}$ into $\vec{b}^{\,j} \equiv \vec{0}$ with the last fixed element. Therefore taking into account (6.2) from [1] and theorems 4.1 it follows

$$\beta_n(d_2,1) \approx \frac{(n-1)!}{m_1 \dots m_n m_*} \sum_{j_n=1}^{n} \int_0^\infty \int_0^\infty \underset{0<x_2<\dots<x_n}{\int \dots \int} \overline{G}_{j_1}(x_2)\overline{G}_{j_2}(x_3 - x_2) \dots \overline{G}_{j_{n-1}}(x_n - x_{n-1})\overline{G}_{j_n}(v+u)dx_2 \dots dx_n dvdH(u) =$$

$$=\frac{(n-1)!}{m_1...m_n m_*}\sum_{j_n=1}^{n}\int_0^\infty\int_0^\infty\int_0^\infty \overline{G}_{j_1}(x_2)dx_2\int_{x_2}^{\infty}\overline{G}_{j_2}(x_3-x_2)dx_3...\int_{x_{n-1}}^{\infty}\overline{G}_{j_{n-1}}(x_n-x_{n-1})dx_n\overline{G}_{j_n}(v+u)dvdH(u).$$

After taking into consideration, that $x_1=0$, making the change of variables $x_j-x_{j-1}=y$, $dx_j=dy$, and changing limits of integration from $x_{j-1}<x_j<\infty$ то $0<y<\infty$, $j=\overline{2,n}$, we will obtain, that

$$\beta_n(d_2,1)\approx\frac{(n-1)!}{m_1...m_n m_*}\sum_{j=1}^{n}\int_0^\infty\int_0^\infty\overline{G}_j(v+u)\prod_{i\neq j}m_{Ri}dvdH(u) = \frac{(n-1)!}{m_1...m_n m_*}\sum_{j=1}^{n}m_{Rj}(\eta)\prod_{i\neq j}m_{Ri}.$$

Statement (4.3) is proven.

With $d=d_3$, let us fix the last failed element of UPB (i-th element), which precedes the BAS signal. Therefore taking into account (6.7) from [1] and theorem 4.1 we will obtain

$$\beta_n(d_3,1)\approx\frac{(n-1)!}{m_1...m_n m_*}\sum_{i=1}^{n}\sum_{j^{(n-1)}\in M_j^{(n-1)}}\int_0^\infty\int_0^\infty\int\limits_{0<y_1<...<y_{n-1}}\!\!...\!\!\int\overline{G}_{j_{n-1}}(y_1+\frac{v+u}{n})...\overline{G}_{j_1}(y_{n-1}+\frac{v+u}{n})dy_1...dy_{n-1}*$$

$$*\overline{G}_i(\frac{v+u}{n})dvdH(u). \tag{A.6}$$

Since integral expression, standing under the sign of sums in (A.6), is converged and it is equal to the probability of failure of SPB on one of the monotonic ways, leading from $\vec{b}\equiv\vec{1}$ into $\vec{b}^j\equiv\vec{0}$ than according [8] it is possible to interchange the positions of summing up and integration. Therefore

$$\beta_n(d_3,1)\approx\frac{(n-1)!}{m_1...m_n m_*}\sum_{i=1}^{n}\int_0^\infty\int_0^\infty\left(\sum_{j^{(n-1)}\in M_j^{(n-1)}}\int\limits_{0<y_1<...<y_{n-1}}\!\!...\!\!\int\overline{G}_{j_{n-1}}(y_1+\frac{v+u}{n})...\overline{G}_{j_1}(y_{n-1}+\frac{v+u}{n})dy_1...dy_{n-1}\right)*$$

$$*\overline{G}_i(\frac{v+u}{n})dvdH(u). \tag{A.7}$$

After using to the internal iterated integral in (A.7) lemma A.1, then after making the change of variables $[y+(v+u)/n]=y'$, and then using lemma A.3, we will obtain

$$\beta_n(d_3,1)\approx\frac{(n-1)!}{m_1...m_n m_*}\sum_{i=1}^{n}\int_0^\infty\int_0^\infty\overline{G}_i(\frac{v+u}{n})\prod_{\substack{j=1\\j\neq i}}^{n}\int_0^\infty\overline{G}_j(y+\frac{v+u}{n})dydvdH(u)=$$

$$=\frac{(n-1)!}{m_1...m_n m_*}\sum_{j=1}^{n}\int_0^\infty\int_0^\infty\overline{G}_j(\frac{v+u}{n})\prod_{i\neq j}\int_{\frac{v+u}{n}}^{\infty}\overline{G}_i(y)dydvdH(u)=$$

$$\frac{(n-1)!}{m_1...m_n m_*}\int_0^\infty\left(\sum_{i=1}^{n}\int_0^\infty\overline{G}_i(\frac{v+u}{n})\prod_{\substack{j=1\\j\neq i}}^{n}\int_{\frac{v+u}{n}}^{\infty}\overline{G}_j(y)dy\right)dvdH(u)=$$

$$= \frac{n!}{m_1 ... m_n m_*} \int_0^\infty \left(\prod_{j=1}^n \int_0^\infty \overline{G}_j (y + \frac{u}{n}) dy \right) dH(u),$$

and we obtained (4.4).

Let us pass to discipline d_4. By definition of discipline d_4 $A(d_4, 1, \pi, u) = A(d_1, l, \pi, u)$. Furthermore, exactly so, as was proven lemma 1 in [4], it is possible to show, that with the condition for fast restoration and $k \geq 1$ $A(d_4, k, \pi, u) = A(d_1, l, \pi, u)$, and we obtain statement 4) of corollary 6.1 from [1]. Hence with $d = d_4$, $l = n, k = 1$ let us fix the last failed element of UPB and taking into account (6.5) from [1] and theorem 4.1 we will obtain

$$\beta_n(d_4,1) \approx \frac{1}{m_1 ... m_n m_*} \sum_{j_n=1}^n \int_0^\infty \int_0^\infty \left(\sum_{j^{(n-1)} \in M_J^{(n-1)}} \int_{0<y_1<...<y_{n-1}} \cdots \int \overline{G}_{j_{n-1}}(y_1 + v + u) ... \overline{G}_{j_1}(y_{n-1} + v + u) dy_1 ... dy_{n-1} \right) *$$

$$* \overline{G}_{j_n}(v + u) \ \ dvdH(u).$$

After using to the internal iterated integral lemma A.1, then after making the change of variables $[y + (v + u)] = y'$, and then using lemma A.3, we will obtain

$$\beta_n(d_4,1) \approx \frac{1}{m_1 ... m_n m_*} \sum_{j=1}^n \int_0^\infty \int_0^\infty \overline{G}_j(v + u) \prod_{i \neq j} \int_0^\infty \overline{G}_i(y + v + u) dydvdH(u) =$$

$$= \frac{1}{m_1 ... m_n m_*} \sum_{j=1}^n \int_0^\infty \int_0^\infty \overline{G}_j(v + u) \prod_{i \neq j} \int_{v+u}^\infty \overline{G}_i(y) dydvdH(u) =$$

$$= \frac{1}{m_1 ... m_n m_*} \int_0^\infty \left(\prod_{j=1}^n \int_0^\infty \overline{G}_j(y + u) dy \right) dH(u),$$

and we obtained (4.5). Corollary 4.1 is proven.

Proof of corollary 5.2. Let us take an advantage of corollary 6.2 of [1] and the second recommendation of corollary 5.1.

With $r = 1$, $d = d_1$, and $\eta = h = const$ from (4.2)

$$\beta_n(d_1,1) \approx \frac{1}{m_1 ... m_n m_* n} \sum_{j=1}^n n \int_0^\infty y^{n-1} \overline{G}_j(y + h) dy \ = \frac{1}{m_1 ... m_n m_* n} \sum_{j=1}^n m_{Rj}^{(n)}(h),$$

and

$$\beta_{xn}(d_1,1) \approx \frac{1}{m_1 ... m_n m_* n} \sum_{j=1}^n n \int_0^\infty y^{n-1} \overline{G}_j(y + h + x) dy \ = \frac{1}{m_1 ... m_n m_* n} \sum_{j=1}^n m_{Rj}^{(n)}(h + x).$$

Hence from (6.9) [1] it follows (5.1).

On the lemma A.2

$$\int_0^\infty m_{Rj}^{(n)}(h+x)dx = m_{Rj}^{(n+1)}(h)/(n+1)$$

and in accordance with (6.10) and (6.11) from [1] we obtain (5.2) and (5.3).

With $r=1$, $d=d_2$, and $\eta = h = const$ from (4.3) we will obtain

$$\beta_n(d_2,1) \approx \frac{(n-1)!}{m_1 \dots m_n m_*} \sum_{j=1}^n m_{Rj}(h) \prod_{i \neq j} m_{Ri} ,$$

$$\beta_{xn}(d_2,1) \approx \frac{(n-1)!}{m_1 \dots m_n m_*} \sum_{j=1}^n m_{Rj}(h+x) \prod_{i \neq j} m_{Ri} .$$

We obtained (5.4).

On lemma A.2

$$\int_0^\infty m_{Rj}(h+x)dx = m_{Rj}^{(2)}(h)/2 .$$

And we obtained (5.5) and (5.6).
With $r=1$, $d=d_3$, and $\eta = h = const$ from (4.4) we will obtain

$$\beta_n(d_3,1) \approx \frac{n!}{m_1 \dots m_n m_*} \prod_{j=1}^n m_{Rj}\left(\frac{h}{n}\right),$$

$$\beta_{xn}(d_3,1) \approx \frac{n!}{m_1 \dots m_n m_*} \prod_{j=1}^n m_{Rj}\left(\frac{h+x}{n}\right).$$

Hence taking into account corollary 6.2 from [1] we will obtain (5.7), (5.8) and (5.9).

With $r=1$, $d=d_4$, and $\eta = h = const$ from (4.4) we will obtain

$$\beta_n(d_4,1) \approx \frac{1}{m_1 \dots m_n m_*} \prod_{j=1}^n m_{Rj}(h) ,$$

$$\beta_{xn}(d_4,1) \approx \frac{1}{m_1 \dots m_n m_*} \prod_{j=1}^n m_{Rj}(h+x) ,$$

which proves (5.10), (5.11), and (5.12).

Corollary 5.2 is proven.

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