SIMULATION OF FTA IN SIMULINK

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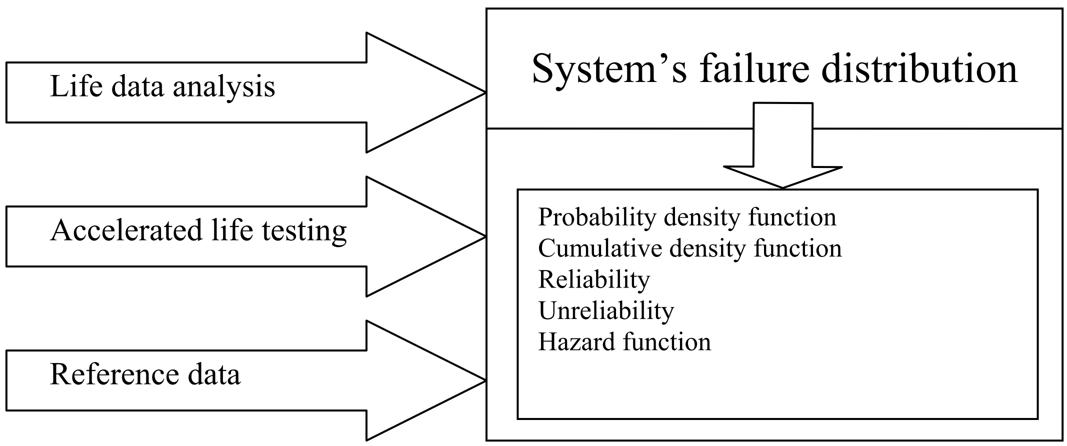
Abstract: This paper deals with possibility of simulation of reliability block diagrams, failure trees analysis as a time dependent analysis using Matlab/Simulnk.

Keywords: failure distribution, Simulink, reliability, probability density function, cumulative density function, reliability block diagrams, failure tree analysis

1. INTRODUCTION

In life data analysis and accelerated life testing data analysis, the objective is to obtain a life distribution that describes the times-to-failure of a component, subassembly, assembly or system. The analysis to determine the life distribution is based on the time of successful operation or time-to-failure data of the item, either under use conditions or from accelerated life tests.



The main objective of system reliability is the construction of a model (life distribution) that represents the times-to-failure of the entire system based on the life distributions of the system's elements. These elements can be components assemblies, sub-systems etc. There are many specific reasons for looking at component data to estimate the overall system reliability. One of the most important is that in many situations it is easier and less expensive to test single elements rather than entire systems, also properties of failure distributions of single elements can easily tuned and then changes of overall system distribution can be compared.

In general, most problems in reliability engineering deal with quantitative measures, such as the time-to-failure of a product, or qualitative measures, such as whether a product is defective or non-defective. We can then use a *random variable X* to denote these possible measures. In the case of times-to-failure, our random variable *X* is the time-to-failure of the product and can take on an infinite number of possible values in a range from 0 to infinity. Product can be found failed at any time after time 0, thus *X* can take on any value in this range. In this case, our random variable *X* is said to be a *continuous random variable*.

The probability density function (*pdf*) and cumulative distribution function (*cdf*) are two of the most important statistical functions in reliability and are very closely related. When these functions are known,, other reliability such as reliability *R(t)*, unreliability *Q(t)*, hazard function *h(t)* can be computed and obtained.

On the Figures 1.1-2 are depicted examples of *pdf* and *cdf* of the normal distribution.

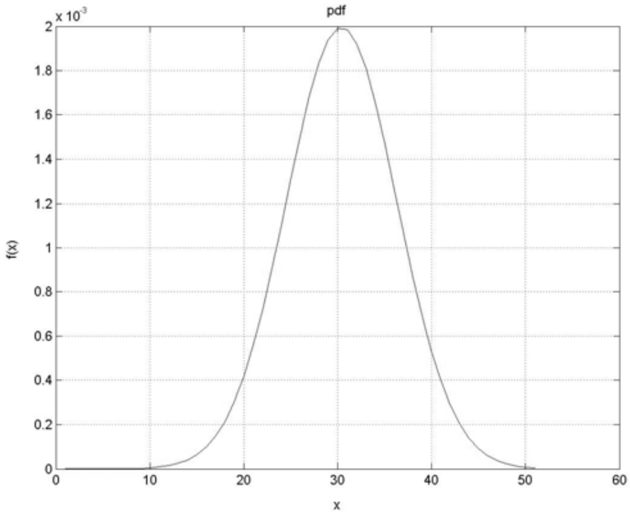


Fig. 1.1 Example of probability distribution function

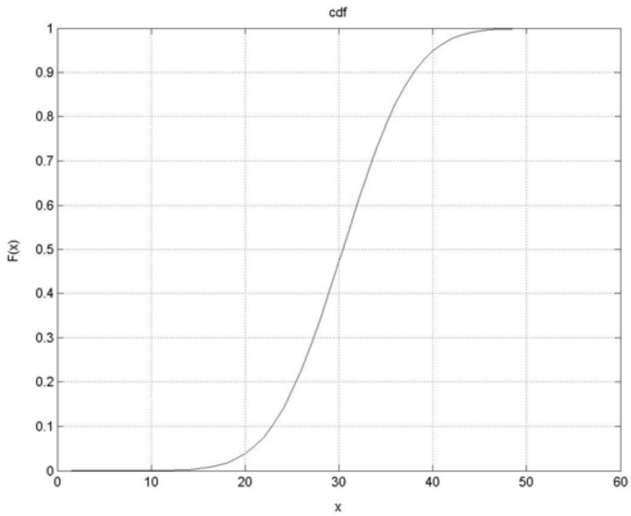


Fig. 1.1 Example of cumulative distribution function

The mathematical relationship between the *pdf* and *cdf* is given by eq. (1) and eq. (2)

$$F(x) = \int_{0}^{s} f(s)ds \qquad (1)$$

Conversely

$$f(x) = \frac{d(F(x))}{dx} \tag{2}$$

The *cdf* is the area under the probability density function up to a value of *x*. The total area under the *pdf* (Figure 1.1) is always equal to 1, or mathematically:

$$\int_0^{\infty} f(x)dx = 1 \tag{3}$$

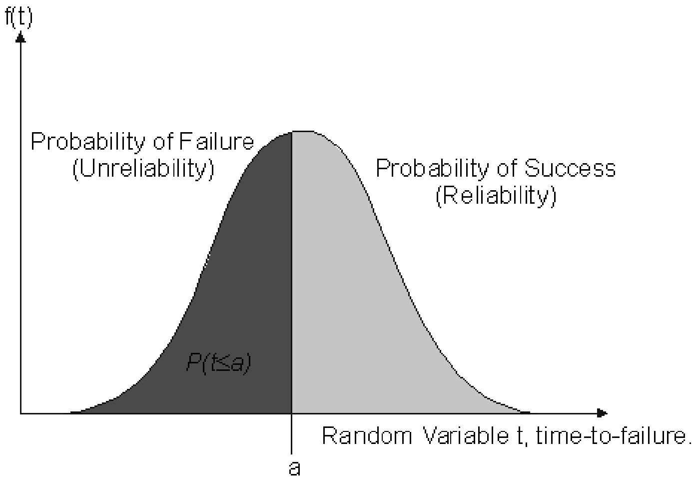


Fig. 1.3 Reliability and unreliability as areas under pdf

Other function as reliability *R(t)* and unreliability *Q(t)*can be computed according eq. (4-6)

$$Q(t) = F(t) = \int_0^t f(s)ds \tag{4}$$

$$Q(t) + R(t) = 1 \tag{5}$$

$$R(t) = \int_t^{\infty} f(s)ds \tag{6}$$

Other important function is failure rate also known as a hazard function. This function enables the determination of the number of failures per time. This function can be computed according eq. (7)

$$\lambda(t) = h(t) = \frac{f(t)}{R(t)} \tag{7}$$

2. APPROACHES OF SYSTEM RELIABILITY

In theory and in praxis exists two basic approaches (categories of approaches):

- Analytical calculations
    1. Static analytical calculations
    2. Time-dependent calculations
- Simulation calculations

Two types of analytical calculations can be performed using RBD or FTA: static reliability calculations and time-dependent reliability calculations. Systems can contain static blocks, time-dependent blocks or a mixture of the two.

Static analytical calculations are performed on RBD or failure trees that contain static blocks. A static block can be interpreted either as a block with a reliability value that is known only at a given time (but the block's entire distribution is unknown) or as a block with a probability of success that is constant with time. Static calculations can only be performed in the analytical mode and not in the simulation calculations.

Time-dependent analysis looks at reliability as a function of time. That is, a known failure distribution is assigned to each component. The time scale can be any quantifiable time measure, such as years, months, hours, minutes or seconds, and also units that are not directly related to time.

If one includes information on the repair and maintenance characteristics of the components and resources available in the system, other information can also be analyzed/obtained, such as i.e. system availabilty, maintability etc. This can be accomplished through discrete event simulation.

In simulation, random failure times from each component's failure distribution are generated. These failure times are then combined in accordance with the way the components are reliability-wise arranged within the system. The overall results are analyzed in order to determine the behavior of the entire system.

3. FAULT TREE ANALYSIS, RELIABILITY BLOCK DIAGRAMS

Block diagrams are widely used in engineering in many different forms. Fault trees and reliability block diagrams are both symbolic analytical logic techniques that can be applied to analyze system reliability and related characteristics. They can also be used to describe the interrelation between the components and to define the system.

When blocks are connected with direction lines, that represent the reliability relationship between these blocks, it's referred as reliability block diagram (RBD). Example of RBD is depicted on fig. 3.1.

A fault tree diagram follows a top-down structure and represents a graphical model of the pathways within a system that can lead to a foreseeable, undesirable loss event (or a failure). The pathways interconnect contributory events and conditions using standard logic symbols (AND, OR, etc.). Fault tree diagrams consist of gates and events connected with lines. Example of RBD is depicted on fig. 3.2.

The most fundamental difference between fault tree diagrams and reliability block diagrams is that you work in the "success space" in an RBD while you work in the "failure space" in a fault tree. In other words, the RBD looks at success combinations while the fault tree looks at failure combinations. In addition, fault trees have traditionally been used to analyze fixed probabilities (*i.e.* each event that comprises the tree has a fixed probability of occurring) while RBDs may include time-varying distributions for the success (reliability equation) and other properties, such as repair/restoration distributions. In general (and with some specific exceptions), a fault tree can be easily converted to an RBD. However, it is generally more difficult to convert an RBD into a fault tree, especially if one allows for highly complex configurations.On fig. 3.2 is converted RBD from fig. 3.1

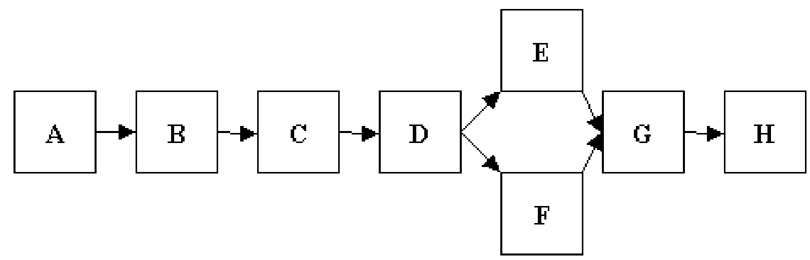


Fig. 3.1 Example of Reliability block diagram

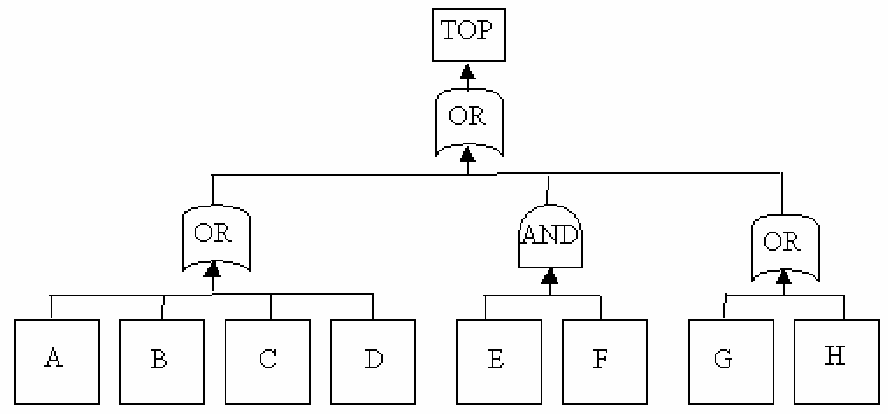


Fig. 3.2 Example of Failure tree

4. SIMULINK MODEL OF FAILURE TREE

In computer programming language Malab/Simulink, were constructed blocks for distributions (WEIBULL, NORMAL, EXPONENTIAL) and also blocks for AND gate and OR gate. Distributions contains all important outputs for computation of system reliability according failure tree or RBD diagram. Distributions outputs are: *f(t)* (*PDF*), *F(t) CDF, R(t), Q(t)* and *h(t).*

AND gate was made according fig.4.1, OR gate was made according fig. 4.2.

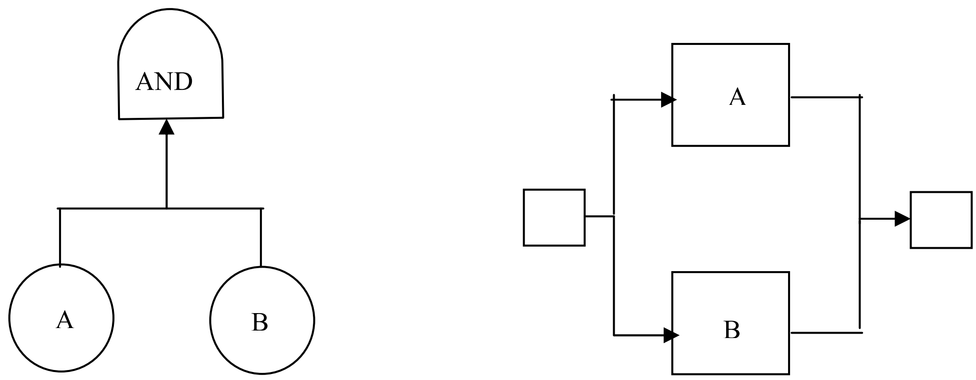


Fig. 4.1 FTA and RBD representation of parallel connection

The reliability equation for either configuration depicted on fig. 4.1

$$R_S = R_A + R_B - R_A R_B \tag{8}$$

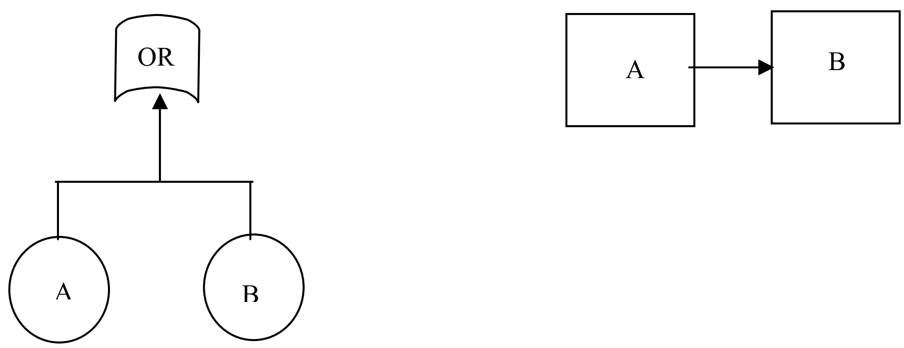


Fig. 4.2 FTA and RBD representation of serial connection

The reliability equation for either configuration depicted on fig. 4.2

$$R_S = R_A R_B \tag{9}$$

When more input events is needed in fig. 4.1 or fig. 4.2, the eq. (8) or (9) is automatically changed for correct input events.

As a example RBD, FTA depicted on fig. 3.1 and fig. 3.2 was simulated in Simulink. Table of distributions is shown in fig. 4.3

	Block							
	A	B	C	D	E	F	G	H
Failure distrib.	Weibull	Exponential	Normal	Weibull	Weibull	Weibull	Exponential	Normal
Param.	β=1,5	m=10000	σ=200	β=1,5	β=3	β=1,5	m=100000	σ=50
	η=1000		μ=1000	η=10000	η=1000	η=5000		μ=5000

Fig. 4.3 Table of failure distributions used in example depicted on fig.3.1 or fig. 3.2

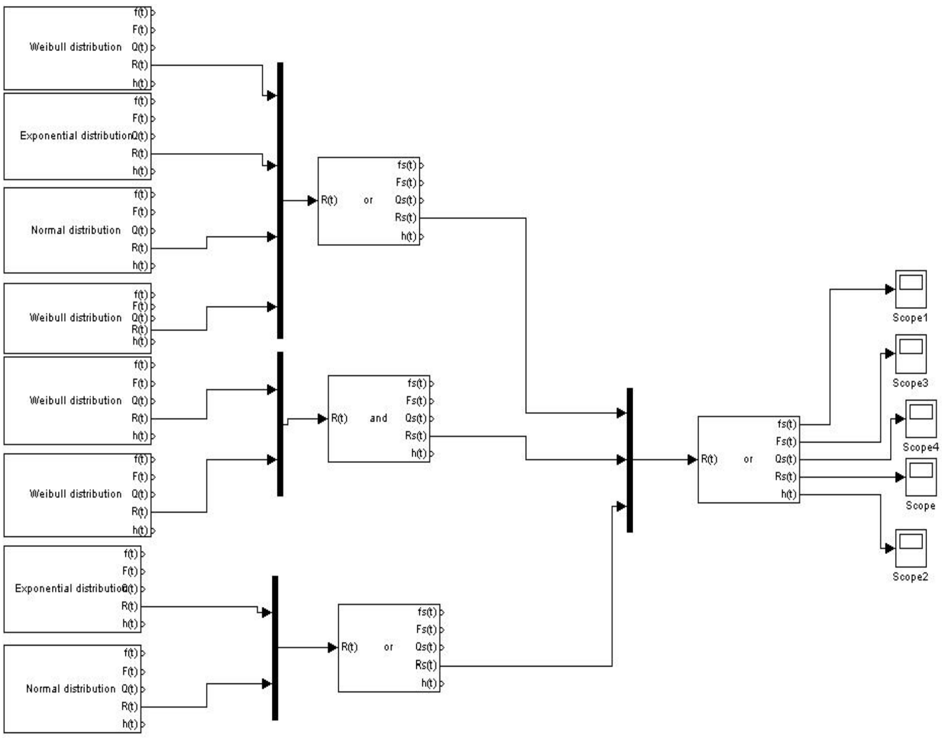


Fig. 4.4 Simulink model of FTA shown in fig. 3.2

5. SIMULATION RESULTS

On the fig. 5.1 up to fig. 5.4 are shown simulation results. Simulation was made in simulink with constant time step, using method ode5 (Dormand-Prince). The end of simulation was at time 1000 tu.

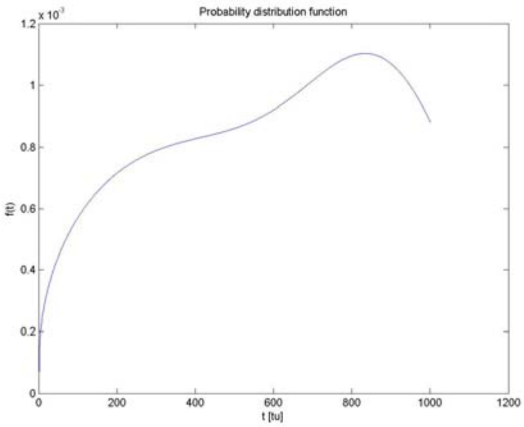


Fig. 5.1 PDF of the system

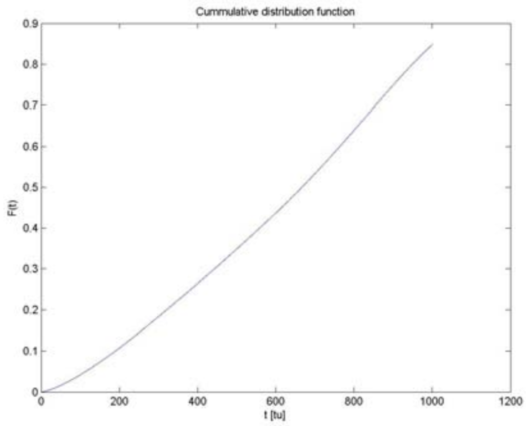


Fig. 5.2 CDF of the system

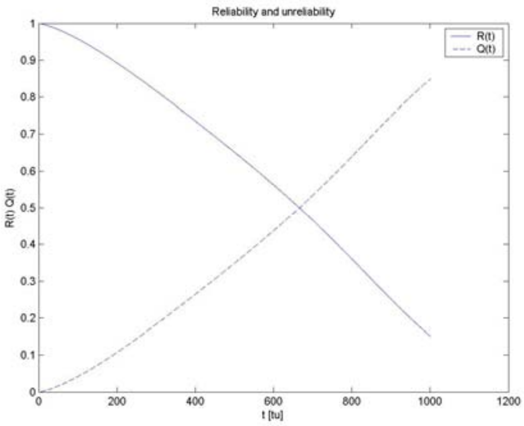


Fig. 5.3 R(t) and Q(t) of the system

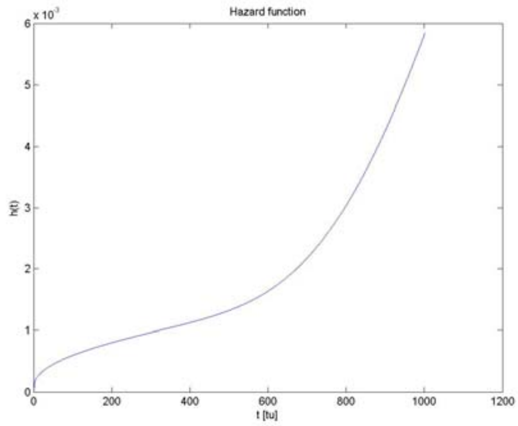


Fig. 5.4 Hazard function of the system

6. CONCLUSION

The purpose of this paper was to shown possibility of simulation reliability block diagrams or failure tree analyses in Matlab/Simulink. The main advantage of the programming in Simulink instead of Matlab is possibility to create appropriate blocks and then easily change FTA or RBD diagrams, Simulink also provide repeatable simulations with various input failure distributions and observing the changes of system failure distributions. The outputs from simulations can be easily processed in other computer programms.

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