# MODEL OF A RELIABILITY FOR STRUCTURAL - COMPLICATED SYSTEMS, INCLUDING MULTYSTATE ELEMENTS 

Melnikov V.A.


#### Abstract

The problem of development of Boolean models of a reliability for systems, including elements with many states is considered on the basis of multivalued logic, algebra of trains, algebra of groups of incompatible events and classical logistic-probabilistic method (LPM). The inexpediency of development of Boolean models of a reliability on the basis of multivalued logic is displayed. The numerical examples demonstrating serviceability of LPM and their new possibilities are demonstrated. The perspective of development of methods of an evaluation of effectiveness of operation at different levels of operation rate by formulation of a set of different tasks, solved by the same LPM is underlined.


Key words: monotone Boolean model of a reliability, Boolean two-state algebra of logic, K-valued algebra of logic, multivalued logic, algebra of trains, algebra of logic of groups of incompatible events, LPM.

In § 3.1 of [1], written by Kurt Rainshke, is declared, that $s$ alternative "all or nothing " for investigations of a reliability of complicated engineering systems and devices appears too rough. Therefore engineers in the practice would like to use more fine division of levels of operation rate. In this fact the experts in a reliability theory in many countries are working in the field of creation of models of reliability in which monotone Boolean models of reliability are developed on a multivalued case. The reference to nine publications of the foreign authors using idea of multivalued logic is given.

In the foreword of the book [2] the authors, answering Kurt Rainshke on his criticism [3] of socalled "shortages" of LPM, considering only two states of elements of the system, underlined, that they are irrelevant with basic singularities of these methods, but reflected only rate of the development of the theory and practice of those years [4].

The classical Boolean algebra of logic envelops all binary - discrete world, in which the arguments $X_{i}$ of Boolean functions of algebra of logic (FAL) accept values from a two-element set $\{0,1\}$. L.Kroneker (1823-1891) wrote: " A Zero and unit from the god, remaining matter of human hands ".

So by the "hands" of Y.Lukasevich (1878-1956) was created the first continual algebra of logic multivalued logic, in which the limitation of $\mathrm{Xi}\{0,1\}$ is taken off. On a interval
[ $X_{\min }, X_{\max }$ ] of a numerical axes the minimax operations of a conjunction - min (X1, X2), disjunction-max (X1,X2) and refusing $-=\mathbf{K - X}$, where under K. K.Rainshke understands a maximum level of operation rate $\left[\frac{1}{X}\right.$, p.110], and Mc-Notan - [5] $2 \mathrm{X} 0=2\left[0,5\left[\mathrm{X}_{\min }+\mathrm{X}_{\max }\right]\right.$.

Without objecting the indicated generalizations and attempts of " magnifications of a potency " of logistic-mathematical methods in information processes of analog area [5], there is no need to introduce into use more complicated mathematical methods, using continual algebra of logic in tasks of a reliability (safety) of structural - complicated systems. Examples in [1], demonstrate the absence of actuality of presented situations and practical impossibility of substitution of logic variables by probabilities.

So in an example 3.3., called to show multivalued model of a reliability of a system, consisting of four elements, (each element can be in three states), the restricted operation rate of the first level is the same that refusals of not designated other elements:

X1 (Cpu) - refusal in additional devices of real time;
X2 (interface) - refusal of the output channel;
X3 (extended memory) - refusal of a HDD;
X4 (controller) - refusal of the device of a parallel printing.
On our opinion this system consists not from four elements but from eight, each of which is described by Boolean algebra of logic.

In the fact that in real life there are elements with three and more states, we shall overview methods and history of a research of a reliability of such systems without use of multivalued logic.

In the book [6, p. 165] there is a brief analysis of eleven publications since 1956 about a problem of three incompatible states in a reliability theory, without considering of 14 publications B.Dillon himself. In these publications the means of a calculus, alarm graphs, polynomial expansions for systems with elementary structures was used. For a case of systems of a bridge type B.Dillon offered the method of transformation of the "triangle" by the "star". As a result of such transformation the bridge structure was substituted by a system with sequential and parallel junction of elements.

In [7, p. 173] the reliability of systems, consisting of elements with three states, is considered by means of classical LPM. That function of serviceability of a system (FSS), is formed with the help $\boldsymbol{m}$ parallel SPSO (shortest paths of successful operation), and everyone SPSO- is a series connection of $\boldsymbol{n}$ arguments X 1 . In this case the fundamental regularities possible, thanks to polynomial expansion ( $\left.R_{i}+Q_{i 0}+Q_{i 3}\right)^{\mathrm{n}}$ to all possible hypotheses, allow to evaluate separately a refusal of a system as "cut off" $\left(Q_{c . o}\right)$ and as " short circuited" ( $Q_{c .3}$ ).

The refusal of a system as "abruption" is evaluated by LPM by substitution of logic variables Xi by the appropriate probabilities by rules:

$$
\mathrm{Xi}=\left\{\begin{array}{l}
1, \text { if i-element is efficient; }  \tag{1}\\
0, \text { if i-element has given up as " cut off ". }
\end{array}\right.
$$

The refusal of a system as " closure " is evaluated by same FSS, but the rules of substitution of a truth Xi will be inverted:

$$
\mathrm{Xi}=\left\{\begin{array}{l}
1, \text { if i-element is "short circuited"; }  \tag{2}\\
0, \text { if i-element is efficient. }
\end{array}\right.
$$

The authors of [8] came to a conclusion, that the offered LPM based mode of account of a reliability of complicated systems, with elements, which can be in three incompatible states, allows to apply logisticprobability methods without using of the formulas:

$$
\begin{align*}
R_{\text {посл. }}= & \prod_{i=1}^{n}\left(1-Q_{i o}\right)-\prod_{i=1}^{n} \mathcal{Z}_{i 3}  \tag{3}\\
& R_{\text {nap }}=\prod_{i=1}^{m}\left(1-Q_{i 3}\right)-\prod_{i=1}^{m} Q_{i o} \tag{4}
\end{align*}
$$

and transformations "star - triangle", and also complicated methods such as algebra of trains [9].

The author of algebra of trains (AT) B.Kulik. wrote [9, p. 19]: " Now, apparently, it is difficult to estimate the effect of AT in a solution of problems, connected with the reducing of complicated work while using of some LPM algorithms, but with the help AK it is possible to simplify statement and solution of such tasks, when the system consists of elements, for which the set of possible events is not restricted only by two states (for example, " the operation - refusal ") and supposes any additional set of intermediate states or events ". In the paragraph 4.4 [9] " Logistic-probability models at any number of states of elements " the numerical example for a bridge circuit from five elements is given. Four elements are submitted by three probabilities. They form a complete group of incompatible events. Taking into account difficulties, connected with small printing [9], and high value of a numerical example with the answer, we shall give FSS (5) and table of input data:

$$
\begin{equation*}
\mathrm{Y}\left(\mathrm{X}_{1}, \ldots, \mathrm{X}_{5}\right)=\mathrm{X}_{1} \mathrm{X}_{3} \vee \mathrm{X}_{2} \mathrm{X}_{4} \vee \mathrm{X}_{1} \mathrm{X}_{5} \mathrm{X}_{4} \vee \mathbf{X}_{\mathbf{2}} \mathbf{X}_{\mathbf{5}} \mathrm{X}_{\mathbf{3}} \tag{5}
\end{equation*}
$$

| $\mathrm{X}_{1}$ |  | $\mathrm{X}_{2}$ |  |  |  | $\mathrm{X}_{3}$ |  |  | $\mathrm{X}_{4}$ |  |  | $\mathrm{X}_{5}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ | $\mathrm{~B}_{1}$ | $\mathrm{~B}_{2}$ | $\mathrm{~B}_{3}$ | $\mathrm{c}_{1}$ | $\mathrm{c}_{2}$ | $\mathrm{c}_{3}$ | $\mathrm{~d}_{1}$ | $\mathrm{~d}_{2}$ | $\mathrm{~d}_{3}$ | $\mathrm{l}_{1}$ | $\mathrm{l}_{2}$ | $\mathrm{l}_{3}$ |  |
| 0,6 | 0,4 | 0,5 | 0,2 | 0,3 | 0,7 | 0,2 | 0,1 | 0,4 | 0,2 | 0,4 | 0,4 | 0,3 | 0,3 |  |

The answer will be $\mathrm{Rc}=\mathrm{P}\left\{\mathrm{y}\left(\mathrm{X}_{1}, \ldots, \mathrm{X}_{5}\right)=1\right\}=0,86988$.
We think, what not any expert in a reliability theory will manage to get the answer of this task, in spite of the fact that it seems very simple. The professor A.Mojaev has received this solution on the basis of general LPM (GLPM), combining classical algebra of logic and algebra of logic of groups of incompatible events (GIE) [10, p. 136].

In [10] in a conclusion the characteristics of some directions of the further development GLPM is given, where literally is told: " The methods of the account of GIE for the first time allowed to take off one of the most heavy limitation all LPM - requirement of dual representation of states of elements of investigated systems. Now, with the help GLPM, it is possible to get monotone and not monotone models of such systems, in which the elements can be not only in two, but also three, four and generally in any final set of incompatible states ... ".

Agreeing with a conclusion in [8] about difficulties in understanding of algebra of trains by the engineers, it is necessary nevertheless to admire scientific value of this algebra [9]. For other case illustration of real difficulties in understanding and practical use of multivalued logic not only by engineers, but also by scientists, we shall notice publication [11], in which task from [8] was solved with the help of three-value logic. The comparison goes not for the benefit of three-value logic.

Standing up for the models, in which instead of one " of a level of operation rate $"$ it would be possible to speak about "degrees of realization of the task", about "the effectiveness of operation " etc., in publication [1] the perspective declared only on the basis of multivalued logic. Rejecting this perspective in the beginning of paper as doubtful and more complicated, when the arisen problems solved on basis of LPM more simply and also more precisely, we shall illustrate now idea about an evaluation of the «effectivenesses of operation» at different « levels of operation rate».

If we have a real possibilities of formalization of these levels of operation rate, there is no reason to reject "«black-and-white" (" all or nothing ») model, and the set of such estimations will allow to see all gamma multicolor. In 1977 the professor I.A.Ryabinin wrote [12]: "Criticizing the "black-and-white" variant of a research (works - not works; yes - no; is true - is false) for it simplification, some experts stand up for multicolor model, in which ostensibly it is possible to take into account losses of even a part of percent of a system effectiveness. Frequently it appears, that such multicolored model cannot be checked up, it is not sufficiently determined and defined, and it seems that apparent multicolore is the same that irresponsibility ».

In [7] the example of a ship electric power system, consisting of 17 elements is given. 16 real tasks are formulated. FSS are placed in tables 31 and 32, all solutions at identical input data are given in the table 37.

Thus, the alternative «all or nothing », at its competent use, is not rough, and it is flexible enough, clear and responsible.

Most difficult and responsible in LPM is not an evaluation of probability function
$\mathrm{P}\left\{\mathrm{Y}\left(\mathrm{X}_{1}, \ldots, \mathrm{X}_{5}\right)=1\right\}$ for structural-complicated systems, but formalization of serviceability with the help of the shortest paths of successful (dangerous) operation -SPSO, (SPDO), which can be made by the highly experienced experts of the given subject field. "Automation" of the process of formalization with the help of raising in a degree of a matrix of nodal connections [13, p. 93], eliminations of intermediate knots [13, p. 96], solutions of a system of the logic equations [13, p. 100], is yet not less easy, than direct compiling of the list of SPSO, (SPDO) on the basis of common sense, which in the given context looks like engineering logic.

Thus, the Boolean logic helps engineering logic to formalize ideas about truth or false of our understanding of serviceability (danger) of structural-complicated systems .

## The literature

1. K.Rainshke, I.A.Ushakov. A reliability estimation of systems with use of the graphs. M.: «Radio and communications », 1988, 208 p .
2. I.A.Ryabinin, G.NCherkesov. Logistic-probability methods of a research of a reliability of structural - complicated systems. M.: " «Radio and communications», 1981, 264 p.
3. K.Rainshke. Models of a reliability and responsivity of systems. M.: Mir, 1979, 452 p.
4. I.A.Ryabinin. Reliabitity of engineering systems: Principles and analysis. M: Mir, 1976, 532 p.
5. L.I.Volgin. Continual logistic-algebraic calculations as a basis of information process engineerings in analog area. // A radio electronics Engineering, Computer science, Control. Zaporozhye, 2000/2, p. 34-38.
6. B.Dillon, C. Singh. Engineering methods of security of a reliability. M.: Mir, 1984, 318 p.
7. I.A.Ryabinin. A reliability and safety of structural - complicated systems. Spb.: "Polytechnics", 2000, 248 p.
8. A.P.Kovalev, A.V. Spivakovsky. Application of logistic-probability methods for a reliability estimation of structural - complicated systems . // "Electricity", № 9/2000, p. 66-70.
9. B.A.Kulik. Logistic-probability methods and algebra of trains. // the Theory and information process engineering of simulation of safety of complicated systems .Spb.: IPMASH RAS. Issue 5, Preprint 123, 1995, p. 18-43.
10. A.S.Mojaev, V.N. Gromov. Theoretical basis of a general logistic-probability method of the automized simulation systems. // СПб. ВИТУ, 2000, 144 p.
11.A.S.Smirnov, D.O.Gaydamovich. The analysis of a reliability of the structural - complicated electrical circuits in view of two types of refusals. // "Electricity", № 2/2001, p.50-56.
11. I.A.Ryabinin. A reliability of a ship electric power supplying. // "The Sea collection ". №1. 1977, p. 79-82.
12. I.A.Ryabinin, Y.M.Parfenov. A reliability, survivance and safety of ship electric power systems. Spb., MNA, textbook, 1997, 430 p.
