

PLANNING OF INSPECTION PROGRAM OF FATIGUE-PRONE AIRFRAME

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Abstract. To keep the fatigue ageing failure probability of an aircraft fleet on or below the certain level an inspection program is appointed to discover fatigue cracks before they decrease the residual strength of the airframe lower the level allowed by regulations. In this article the Minimax approach with the use one- and two-parametric Monte Carlo modelling for calculating failure probability in the interval between inspections is offered.

Keywords: failure probability, Minimax approach, inspection program, approval test

INTRODUCTION

Inspection program development should be made on the base of processing of approval lifetime test result, when we should make some redesign of the tested system if any requirement is not met. Here we consider some example of p-set function application to the problem of development and control of inspection program. We make assumption that some Structural Significant Item (SSI), the failure of which is the failure of the whole system, is characterized by a random vector (r.v.) (T_d, T_c) , where T_c is critical lifetime (up to failure), T_d is service time, when some damage (fatigue crack) can be detected. So we have some time interval, such that if in this interval some inspection will be fulfilled, then we can eliminate the failure of the SSI. We suppose also that a required operational life of the system is limited by so-called Specified Life (SL), t_{SL} , when system is discarded from service. P-set function for random vector is a special statistical decision function, which is defined in following way. Let Z and X are random vectors of m and n dimensions and we suppose that it is known the class $\{P_\theta, \theta \in \Omega\}$ to which the probability distribution of the random vector $W=(Z,X)$ is assumed to belong. The only thing we assume to be known about the parameter θ is that it lies in a certain set Ω , the parameter space. If $S_Z(x) = \bigcup_i S_{Z,i}(x)$ is such set of disjoint sets of z values as function of x that $\sup_{\theta} \sum_i P(Z \in S_{Z,i}(x)) \leq p$ then statistical decision function $S_Z(x)$ is p-set function for r.v. Z on the base of a sample $x=(x_1, \dots, x_n)$.

Later on the value x , observation of the vector X , would be interpreted as estimate $\hat{\theta}$ of parameter θ , Z would be interpreted as some random vector-characteristic of some SSI in service: $Z = (T_d, T_c)$.

FATIGUE CRACK GROWTH MODEL

The fatigue crack growth process flows in accordance with quite complicated rules, which depend on a big number of factors. An analytical approach describing that process could be considered as almost impossible. Nevertheless, it can be shown, that in general case crack growth process could be well enough approximated with the formula:

$$a(t) = \alpha \cdot e^{Qt}, \quad 1.$$

where $a(t)$ is a fatigue crack size at time t (the number of flight blocks); α is so called equivalent initial crack size (as if the airframe has been initially produced with the crack of such small size; α corresponds to the best fit of test data); and parameter Q defines the speed of growth of fatigue crack and depends on the loading mode (on the stress range in case of cycling loading).

For further needs, let us take a logarithm of both left and right sides of equation 1:

$$\ln a(t) = \ln \alpha + Qt. \quad 2.$$

Thus,

$$t = \frac{\ln a(t) - \ln \alpha}{Q}, \quad 3.$$

so the time when crack becomes detectable and the time when crack reaches its critical size can be calculated as

$$T_d = \frac{\ln a_{\det} - \ln \alpha}{Q} = C_d / Q, \quad T_c = \frac{\ln a_{\text{crit}} - \ln \alpha}{Q} = C_c / Q, \quad 4.$$

where a_{\det} is a crack size, at which chances to discover it tends to unit, a_{crit} is a crack size, which corresponds to the minimum residual strength of an aircraft component allowed by regulations, T_d is a time for crack to grow to its detectable size and T_c is a time for crack to grow to its critical size, C_c and C_d are appropriate constants.

Let us define *failure* as the situation, when we were unable to discover cracks with the size $a_{\det} \leq a < a_{\text{crit}}$, or, in other words, if there are no inspections performed in $[T_d; T_c]$ time interval.

It is clear, that varying the number of inspections $n_{\text{inspections}}$ in the service interval $[0, t_{SL}]$ we will discover a different number of cracks; therefore, the estimate of failure probability will vary as well. Unfortunately, we don't know the real values of parameters, so we are using their estimates from a small number (one, seldom two) of available observations (fatigue cracks during fatigue test) instead.

USING MONTE CARLO MODELLING TO ESTIMATE FAILURE PROBABILITY

We use the Monte Carlo method to generate a set of cracks to be processed in accordance with procedure, described in Section 0. The parameters for modelling can be derived from the full-scale fatigue tests or from other real crack observations.

We can never know how the certain fatigue crack curve will look like. Thus, performing approximation of that fatigue crack curve with a certain model, the fatigue crack growth model parameters (FCGMP) – we have two FCGM parameters $X = \ln Q$ and $Y = \ln C_c$ – will vary as well, so they are random values, and these random values have their own parameters of distribution. To perform Monte Carlo modelling of the fatigue crack growth process we have to know FCGMPs' distribution types and parameters, i.e. c.d.f. of each FCGMP. From the analysis of the fatigue test data it can be assumed, that the logarithm of time required the crack to grow to its critical size is distributed normally:

$$\ln T_c \sim N(\mu_{LT_c}, \sigma_{LT_c}^2). \quad 5.$$

From formulas 2 and 4 follows:

$$\ln T_c = \ln C_c - \ln Q. \quad 6.$$

From additive property of normal distribution comes that $\ln T_c$ could be normally distributed either if both $\ln C_c$ and $\ln Q$ are normally distributed:

$$X = \ln Q \sim N(\mu_X, \sigma_X^2), \quad Y = \ln C_c \sim N(\mu_Y, \sigma_Y^2), \quad 7.$$

or if one of them is normally distributed while another one is a constant. Thus, the value of logarithm of our FCGM parameters is distributed normally or, on other words, FCGM parameters have a log-normal distribution.

To get estimates of FCGMP distribution parameters ($\hat{\mu}$ and $\hat{\sigma}$) we consider statistics of several crack observations. For each of those cracks we calculate estimates of distribution parameters $\ln Q$ and $\ln C_c$, and then gather all data together into the table with two columns: $\ln Q$ and $\ln C_c$. From that table we then derive estimates of mean value and standard deviation for each column, as well as estimate of correlation between $\ln Q$ and $\ln C_c$.

The Monte Carlo modelling in fact means the process of getting a big number of pairs $[T_d; T_c]$ with upper mentioned specific distribution parameters. Having the array of $[T_d; T_c]$ pairs we apply an inspection

program looking for *failures* – situations, when both T_d and T_c are located between two consequent inspections. For each interval between inspections $[t_{i-1}; t_i]$ failure probability will be

$$P_{f_i} = P(t_{i-1} < T_d \leq T_c < t_i), \quad 8.$$

and for the entire inspection program

$$P_f = \sum_i P_{f_i}. \quad 9.$$

MINIMAX DECISION MAKING APPROACH

As it was stated above, the goal is to develop an inspection program, defined by the vector of inspection time moments

$$\vec{t} = (t_1, t_2, \dots, t_{n_{TIP}}), \quad 10.$$

i.e. to find a vector function $\vec{t}(\hat{\theta})$ ($\hat{\theta}$ is the estimate of FCGMP distribution parameters, n_{TIP} is the total number of inspections per inspection program, so $t_{n_{TIP}} = t_{SL}$) that limits aircraft failure probability at the required level $P_{f_{required}}$ with the minimum inspections n_{TIP} undertaken in service interval $[0, t_{SL}]$ ($t_{n_{TIP}} = t_{SL}$). In mathematical terms that can be presented as:

$$\sup_{\theta} (P_f(\theta, \vec{t})) \leq P_{f_{required}}, \quad 11.$$

where

$$P_f(\theta, \vec{t}) = \sum_{i=1}^{n_{TIP}} P(T_{i-1} \leq T_d \leq T_c < T_i). \quad 12.$$

In expression 12 $T_1, T_2, \dots, T_{n_{TIP}}$ are time moments of inspections: random value

$$T = (T_1, T_2, \dots, T_{n_{TIP}}) = \vec{t}(\hat{\theta}), \quad 13.$$

where $T_0 = 0$, $T_{n_{TIP}} = t_{SL}$, and $n_{TIP} = 0, 1, 2, \dots$. The expression $n_{TIP} = \infty$ symbolically means that the aircraft must be returned for redesign to the design office.

The inspection program definition vector $\vec{t}(\hat{\theta})$ is a function, where both number of inspections during service interval n_{TIP} and disposition of inspection time moments $T_1, T_2, \dots, T_{n_{TIP}}$ during $[0, t_{SL}]$ are to be chosen as a function of $\hat{\theta}$ and some limitations. It is clear, that there might be many ways how to position inspection time moments on $[0, t_{SL}]$ for a particular n_{TIP} . Let us apply the following inspection time moment disposition rule R_D : the time of the first inspection T_1 (T_1 is a random value because it is a function of $\hat{\theta}$) will be defined by procedure similar to the safe life approach (probability of failure without inspections is less than some small value P_{f1}), while all remaining inspections are distributed evenly in the interval $[T_1, t_{SL}]$. Of course, this rule R_D in general case does not minimise the total required number of inspections n_{TIP} ; there are other rules that are more optimal, but our choice of rule R_D is caused by its simplicity for further applications; inspection programs created by this rule are currently used in practice for commercial jet aircrafts.

To apply a particular rule R_D we have to find the total required number of inspections n_{TIP} , which depends on the limiting value of the failure probability $P_{f_{required}} = 1 - R_{required}$, where required reliability $R_{required}$ is mandated, for example, by JAR regulations.

As it was shown above, the failure probability is a function of the number of inspections n and parameter θ ; let us denote it as $P_f(\theta, n)$. We also suppose that $P_f(\theta, n)$ monotonically decreases when the

number of inspections n increases (at least when n is large enough) and $\lim_{n \rightarrow \infty} P_f(\theta, n) = 0$ for all θ . Let

n_{TIP} is a solution of the equation

$$P_f(\theta, n) = P_{f_{required}} \tag{14}$$

Then let us denote

$$n_{TIP} = P_f^{-1}(\theta, P_{f_{required}}) = n(\theta, P_{f_{required}}) \tag{15}$$

as the minimal inspection number at which failure probability $P_f(\theta, n_{TIP}) \leq P_{f_{required}}$. But the true value of the θ is unknown, so $\hat{n}_{TIP} = n(\hat{\theta}, P_{f_{required}})$ and $\hat{P}_f = P_f(\theta, \hat{n}_{TIP})$ are random values. We suppose that we begin the commercial production and operation of aircrafts only if some specific requirements to reliability are met. For the simplest case there is a limitation for the maximum allowed number of inspections n_{max} : we will return airframe project for redesign as unprofitable in case, when the required number of inspections in the inspection program n_{TIP} exceeds n_{max} (we need to inspect aircraft too often to ensure required reliability). It can be assumed, that the probability of failure for the returned projects is equal to zero, i.e.

$$\hat{P}_{f_{corrected}} = \begin{cases} P_f(\theta, \hat{n}_{TIP}) & , \hat{n}_{TIP} \leq n_{max} \\ 0 & , \hat{n}_{TIP} > n_{max} \end{cases} \tag{16}$$

In the more complex case there is a set of limitations. For example, in addition to limitation on the expected number of inspections $n_{calculated} = \hat{n}_{TIP}$ we will return airframe project for redesign if estimate of expectation value of T_c ($T_{c_{calculated}}$) is too small in comparison with t_{SL} (breaking minimum threshold $T_{c_{min}}$); if estimate of time between two consequent inspections $\Delta t_{calculated}$ is smaller than a threshold Δt_{min} ; if estimate of initial equivalent crack size $\alpha_{calculated}$ exceeds crack detectable size α_{det} and so on. Let us denote the vector of calculated values of limiting values $\vec{d}_L = \vec{d}_L(\hat{\theta})$ as

$$\vec{d}_L = \begin{cases} n_{calculated} \\ \Delta t_{calculated} \\ T_{c_{calculated}} \\ \alpha_{calculated} \end{cases}, \tag{17}$$

and the set of its allowed values D_L as

$$D_L = \begin{cases} (0, n_{max}] \\ [\Delta t_{min}, \infty) \\ [T_{c_{min}}, \infty) \\ [0, \alpha_{det}] \end{cases} \tag{18}$$

Actually, the number of elements in \vec{d}_L and, therefore, the number of dimensions in the set of the allowed values D_L may vary depending on modelling situation and specific requirements. For example, for inspection programs with the equal time between inspections in the whole service interval $[0, t_{SL}]$ the time between two consequent inspections $\Delta t = t_{SL} / n$, so it can be excluded from the set of limitations, but it is important in programs when the time between inspections may vary.

If vector of limiting values \vec{d}_L does not match the set of its allowed values D_L , then the project is considered as unprofitably and is returned back for redesign in the design office. As we stated above, the probability of failure for returned projects is equal to zero, thus

$$\hat{P}_{f_{corrected}} = \begin{cases} P_f(\theta, \vec{d}_L) & , \vec{d}_L \in D_L \\ 0 & , \vec{d}_L \notin D_L \end{cases} \tag{19}$$

The parameter θ , which defines the c.d.f. of vector (T_d, T_c) , is a vector parameter. For considered case in this work, if both crack model parameters are random and have normal distribution, it consists of five components:

$$\theta = [\theta_{0_{\ln C_c}}, \theta_{1_{\ln C_c}}, \theta_{0_{\ln Q}}, \theta_{1_{\ln Q}}, r], \tag{20}$$

where θ_0 stands for a location and θ_1 stands for a scale parameter of the appropriate crack growth model parameter $\ln C_c$ or $\ln Q$; r is a coefficient of correlation between $\ln C_c$ and $\ln Q$, and

$$\theta \in \Theta = \left\{ (-\infty, \infty); [0, \infty); (-\infty, \infty); [0, \infty); [0, 1] \right\}. \text{ As it was shown before, we don't know the real}$$

value of θ , thus we use its estimate $\hat{\theta}$. A part of elements of $\hat{\theta}$ may be assumed as known. For example, $\theta_{1_{\ln C_c}}$, $\theta_{1_{\ln Q}}$ and correlation coefficient r can be considered as constants, so processing fatigue crack growth data we should estimate only two remaining parameters $\theta_{0_{\ln C_c}}$ and $\theta_{0_{\ln Q}}$.

It can be shown that for considered decision making procedure random variable $\hat{P}_{f_{corrected}}$ has expectation value, which is a function of θ , and this function has a maximum value for $\theta \in \Theta$. To prove that let us fix one of two crack model parameters and look how the probability of failure depends on another one. Let us consider that the equivalent initial crack size is a constant: $\alpha = const$, i.e. $\theta_{0_\alpha} = \mu_\alpha = const$, $\theta_{1_\alpha} = \sigma_\alpha = 0$.

In accordance with upper defined rules the probability of failure tends to zero when the crack growth speed representing parameter $\theta_{0_{\ln Q}} = E\{\ln Q\}$ tends to zero: this is a case when the item is extremely reliable and cracks are growing so slowly, that have no chance to grow up to a_{crit} in interval $[0, t_{SL}]$, thus there are no inspections required. The failure probability without inspections is defined by formula:

$$P_{f_{wi}} = P(T_c \leq t_{SL}) = \Phi\left(\frac{\ln t_{SL} - \mu_{\ln T_c}}{\sigma_{\ln T_c}}\right), \tag{21}$$

or, in terms of reliability,

$$R_{wi} = P(T_c > t_{SL}) = 1 - P(T_c \leq t_{SL}) = 1 - \Phi\left(\frac{\ln t_{SL} - \mu_{\ln T_c}}{\sigma_{\ln T_c}}\right), \tag{22}$$

where $\ln T_c$ is distributed normally as $\ln T_c \sim N(\mu_{\ln T_c}; \sigma_{\ln T_c}^2)$.

From other side, if the $\theta_{0_{\ln Q}}$ is high, then the probability of failure tends to zero as well: with high probability we return for redesign all items due to the break of limiting rules, i.e. $\vec{d}_L \notin D_L$ (see formulas 17, 18 and 19). Between these zero values of $E\{P_{f_{corrected}}\}$ there can be non-zero values somewhere in between, when the fatigue cracks maybe can reach their critical size during the time between inspections, maybe not, but there are no sufficient reasons to return project for redesign so far. Let us call a value of failure probability used for calculations (at the choice of the number of inspections required, or choosing vector-function \vec{t}) as $P_{f_{calc}}$. The following conclusion can be made from the upper mentioned: the dependence of the probability of failure as a function of θ is a function which has a maximum, the value of that maximal value is unknown, but somehow depends on the value of failure probability $P_{f_{calc}}$ used for calculations.

Let us call the value of expectation of failure probability for all allowable θ as $E\{\hat{P}_{f_{corrected}}\}$. We have named it as ‘‘corrected’’ to distinguish it from $P_{f_{calc}}$, because we take into consideration some limitations. The goal is to find such a maximum value of failure probability for calculations $P_{f_{calc}}^*$ that the corrected value of

failure probability $P_{f_{corrected}}$ does not exceed the required limiting value of failure probability

$$P_{f_{required}} = 1 - R_{required} : \tag{23}$$

$$P_{f_{calc}}^* : \tilde{P}(P_{f_{calc}}) \leq P_{f_{required}},$$

where

$$\tilde{P}(P_{f_{calc}}) = \max_{\theta} \left\{ E_{\theta} \left\{ \hat{P}_{f_{corrected}} \right\} \right\}. \tag{24}$$

Graphically this approach for a two-dimensional case (when either $\alpha = const$ or $Q = const$) is presented in Figure 3:

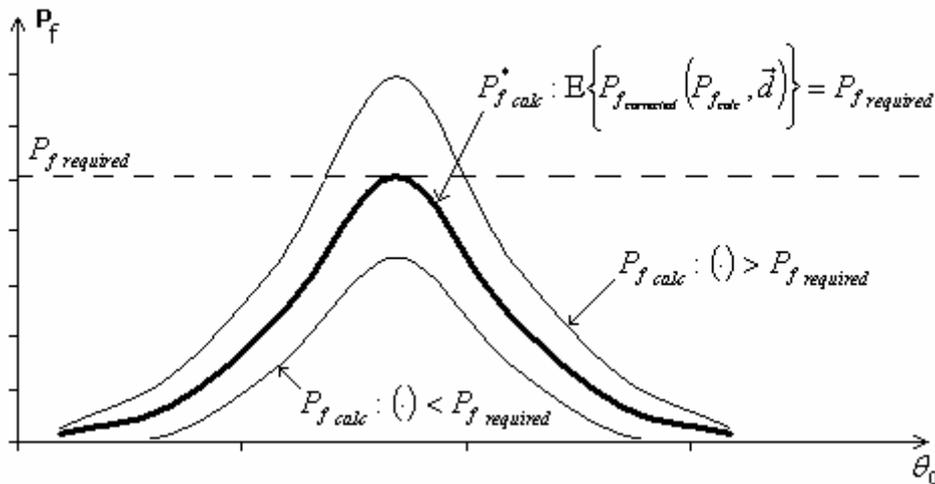


Figure 3. Minimax approach example ($\alpha = const$ or $\ln Q = const$)

For the more complex case we get a three-dimensional picture like in

Figure 4:

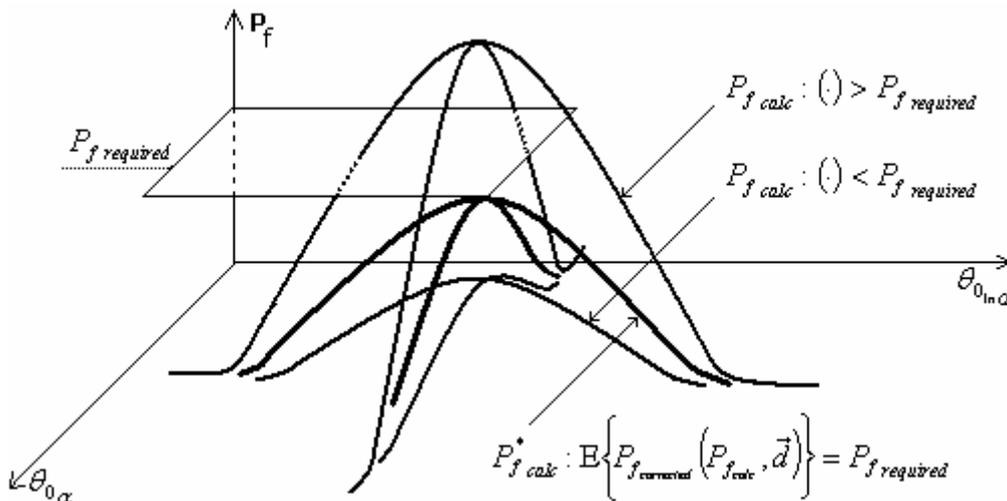


Figure 4. Minimax approach example (general case)

Depending on parameters the shapes of these two- or three-dimensional failure probability curves may vary, but this does not affect our conclusions.

NUMERICAL EXAMPLE AND CONCLUSION

The upper mentioned approach lets us to ensure reliability of the airframe on or above the required level by developing appropriate inspection program for the case of lack of the initial fatigue test data. There are examples of numerical modelling for one- and two-parametric models shown in Figure 5 and

Figure 6 below (please note: in pictures $LQ=LN(Q)=\theta_{0_{ln Q}}$, $LC=LN(Cc)=\theta_{0_{Cc}}$).

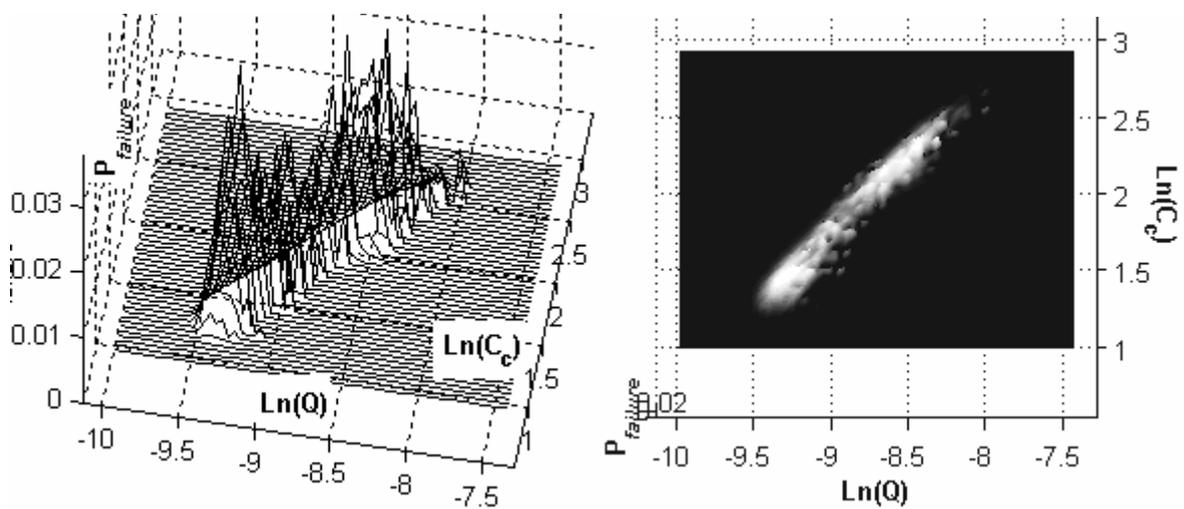
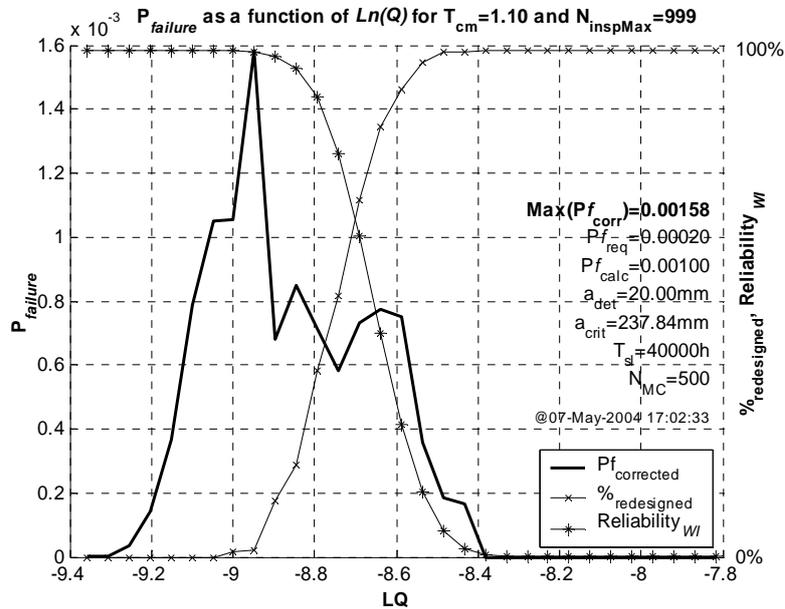


Figure 6. Two-parametric numerical example (3D and projection)

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