RESEARCHES IN IDENTIFICATION OF LOGICAL AND PROBABILISTIC RISK MODELS WITH GROUPS OF INCOMPATIBLE EVENTS

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Abstract: In this paper the results of the researches in identification of the logical and probabilistic (LP) risk models with groups of incompatible events are presented. The dependence of the criterion function on several parameters has been investigated. The parameters include: the total number of optimisations, the amplitude of parameters increments, the initial value of the criterion function (CF), the choice of identical or different amplitudes of increments for different parameters, objects risks distribution. An effective technology of defining the global extreme in the identification of LP-risk model for the calculation time, appreciable to practice has been suggested.

Key words: risk, logic, probability, model, identification, incompatible events

The logical and probabilistic risk models are almost twice as accurate and have seven times better robustness than other known classification methods [1,2]. However the task of multi-parameter and multi-criteria optimisation for training LP-models is characterised by exclusive difficulty [1-3]. In the process of identification of LP-risk models in business according to statistical data there arise a number of additional features and difficulties [1,2]:

- The criterion function F_{max} (CF) is a number of correctly recognised good and bad objects, i.e. it accepts the integer values and it is stepped;
- CF has some local extrema, and depends on the high number of real positive arguments;
- The derivatives of the criterion function with respect to probabilities $P1_{jr}$ cannot be computed.

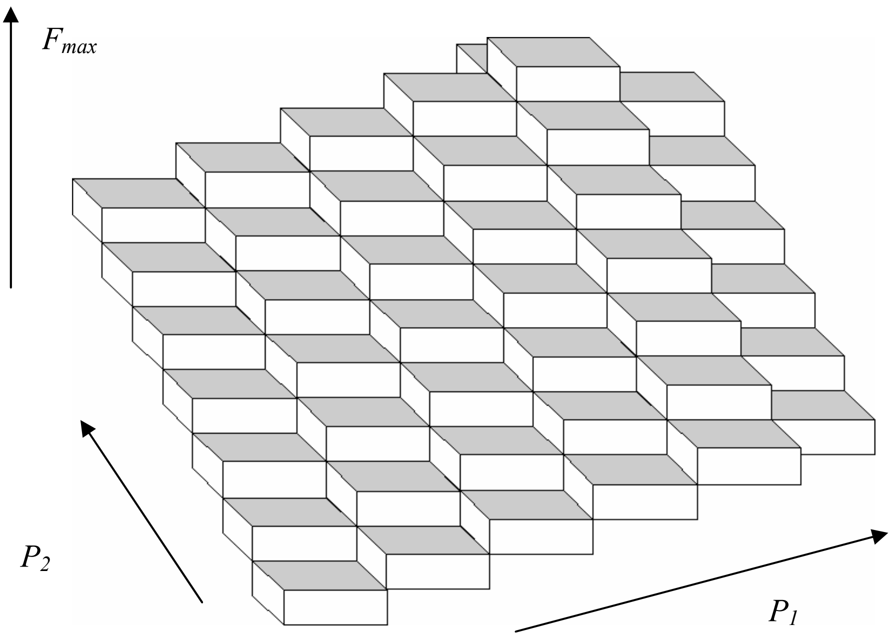

Fig.1. The stepped changing of the criterion function F_{max} from parameters P_1 and P_2

For each event-grade in GIE we consider three probabilities: W_{jr} is the relative frequency of the grade in the objects of the “object-signs” table, $P1_{jr}$ is the probability of the event-grade in GIE, P_{jr} is the probability of the event-grade to be substituted into the probability formula. The sums of the probabilities both W_{jr} and $P1_{jr}$ in GIE equal 1. Connection of these probabilities are considered in [1].

The criterion function F_{max}, presented in Fig.1, depends only on two arguments and changes with steps equal to 2. The platforms have different sizes. The arguments $P1_1$ and $P1_2$ belong to the interval [0,1], but their sizes can differ substantially. While approaching the extreme the platforms decrease in size.

The optimisation can get «stick» at any «platform», not reaching the maximum or crossing the maximum. The character of changing the criterion function in the multivariate space remains the same. Let us remind that the optimisation arguments space dimension for the credit risk LP-model equals 94 [1].

1. IDENTIFICATION OF LP-RISK MODELS

The risk object is described by a large number of signs, every sign has several grades. These signs and grades correspond to random events, which lead to a failure [1,2]. The events-signs ($j=1,n$) have logical connections and events-grades for each event-sign ($r=1,Nj$) form groups of incompatible events (GIE).

The identification of the P-risk model consists in the determination of optimal probabilities P_{jr}, $r=\overline{1,Nj}; j=\overline{1,n}$, *corresponding to* events-grades. Let us formulate the identification (training) problem for a B- risk model [1,2].

Available data: the 'object-signs' table with N_g good and N_b bad objects and the risk B-model;

Expected results: to determine the probabilities of P_{jr}, $r=\overline{1,Nj}; j=\overline{1,n}$ for events-grades and the acceptable risk P_{ad}, dividing the objects into good and bad according the amount of risk.

***We need:* to maximise the criterion function, which is the number of correctly classified objects:**

(1) $$F = N_{bs} + N_{gs} \Rightarrow MAX,$$

where N_{gs} *and* N_{bs} *are the* numbers of objects classified as good and bad using both by statistics and the P- risk model (both estimates should coincide). From (1) it follows, that the errors or accuracy indicators of the P-risk model in the classification of good E_g and bad E_b objects and in the classification of the whole set E_m are equal:

(2) $$E_g = (N_g - N_{gs}) / N_g; E_b = (N_b - N_{bs}) / N_b; E_m = (N - F) / N.$$

Assumed restrictions:

1) probabilities P_{jr} *and* $P1_{jr}$ *must satisfy the stipulation*:

(3) $$0 < P_{jr} < 1, j = \overline{1,n}; r = \overline{1,Nj}.$$

2) the average risks of objects P_m *based* on the P- risk model and on the table P_{av} must be equal; while training the P- risk model we must correct the P_{jr} probabilities on every step of iterative training under the formula

(4) $$P_{jr} = P_{jr} * (P_{av} / P_m); j = \overline{1,n}; r = \overline{1,Nj}.$$

3) the acceptable risk P_{ad} must be determined with the given ratio of incorrectly classified good and bad objects, because of non-equivalence losses at their wrong classification:

(5) $$E_{gb} = (N_g - N_{gs}) / (N_b - N_{bs}).$$

2. OPTIMISATION IN THE IDENTIFICATION TASK

Identification of the LP- risk model by the random search method is based on the ideas used in the training of neural networks [4]. With reference to the identification task of the LP- risk model, the following formula for the calculation of the changes of events-grades probabilities may be put down:

(6) $$dP1_{jr} = K_1 * (1 / N_t) * tg(K_3); j = \overline{1,n}; r = \overline{1,Nj},$$

where: K_1 *is a* coefficient; N_t is the current number of optimisation; K_3 *is a* random number from [- π /2, + π /2], ***n*** is a number of events-signs, N_j is a number of events-grades in each GIE, i.a. for every event–sign.

In the formula (6) the CF is a current error in training. The number of optimisations N_t, before the end of the training process, can be very big. The «tangent» operation is the consequence of the training error distribution recording to Cauchy. Theoretically, this error is distributed according to the normal law, but not spend a lot of time on tabulated values calculation, we use the distribution of the training error under the Cauchy's law. It allows to reduce in 100 times the calculation time, which otherwise, for real problems, would continue for days and weeks.

For failure risk LP-model training the following modification of the formula (6) is suggested [1]:

(7) $$dP1_{jr} = K_1 * (N_{opt} - N_t) * tg(K_3),\; j = \overline{1,n}; r = \overline{1,Nj},$$

where: N_{opt} is the given number of optimisations. The new values of $P1_{jr}$ and P_{jr}, obtained at $F > F_{max}$ on every step N_t of optimisation are considered optimal and saved.

In the LP-risk model identification task, the criterion function cannot exceed the total number of objects in the statistical data. The formula (7) is quite applicable, but the time of calculation is too big (about 10 hours for a session of optimisation).

To reduce the time of calculation, in the formula (7) the "tangent" operation is eliminated. As a result the following expression is obtained [3]:

(8) $$dP1_{jr} = K_1 * (N_{opt} - N_t) * K_3,\; j = \overline{1,n}; r = \overline{1,Nj}.$$

Using (7,8) the optimization happens so: if F>Fmax, then we remember the new $P1_{jr}$ and P_{jr}. If the criterion function does not strictly increase after the chosen number of trials N_{mc} in Monte-Karlo, then F_{max} is reduced by 2-4 units and optimisation continues.

In spite of the investigation in optimisation, carried out before, where the formulas (7) and (8) were used [1,2], the problem of optimisation in the identification task of LP-risk models is far from the final solution. The following fact proves it. In one of the research with the huge number of optimisations $N_{opt}=245\ 000$ and with the constant, almost optimal, value of the increment $dP1_{jr}$, we obtained $F_{max} = 824$ instead of $F_{max} = 810$ at the usual number of optimisations $N_{opt} \approx 245$. We had to carry out special investigations, the results of which are adduced below.

3. INVESTIGATIONS IN IDENTIFICATION / OPTIMISATION

If we generate a random number K_3 in the interval [-1, +1], then the absolute values of increments of probabilities $dP1_{jr}$, multiplied by 100, are transformed in percents (%). It is convenient, for practically it solves the problem of the evaluation of probabilities $P1_{jr}$ accuracy. For example, if the increment is $dP1_{jr}=0.0005$, it equals *0.05 %* . *W*e can say that the probability $P1_{jr}$ with the accuracy *0.05 %* is evaluated .

Using the formula (8), in the beginning of optimisation we have the following maximum amplitude of probabilities increments :

(9) $$AP1_{max} = K_1 * N_{opt}\,.$$

In the end of optimisation the maximum amplitude of probabilities increments equals 0. Let us designate the current amplitude of probabilities increments as *AP1*. There is an optimal interval *OPT* of the amplitudes increments *AP1*, which position and width are unknown (Fig. 2). For the big values of *AP1* there is a small probability of increasing F_{max}, and for small values of *AP1* there is a high probability to stop at the local extreme of the reached value F_{max} (see Fig.1).

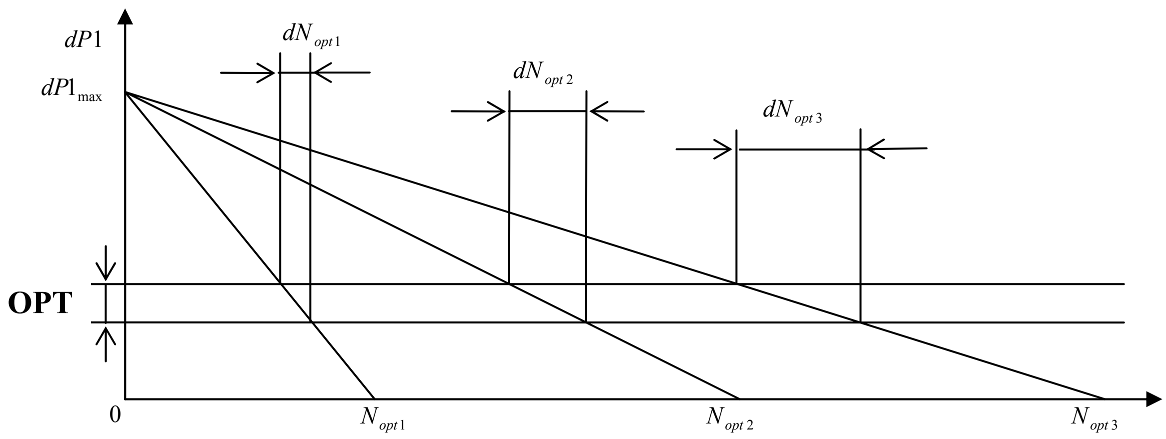

Fig.2. Graphs of relation between the number of optimisations N_{opt} and increments amplitudes AP1

The optimisation process (of training the LP-risk model) should be long enough in the optimal OPT interval . The value of dN_{opt} duration in the optimal *OPT* interval is equal

$$dN_{opt} = (OPT * N_{opt}) / AP1_{max} . \tag{10}$$

It also depends on the number of optimisations N_{opt} and the maximum amplitude of the increment $dP1_{max}$. The more N_{opt} is and the less $AP1_{max}$ is , the longer is the duration of dN_{opt}. The purpose of this work is the investigation of the dependence of the criterion function (accuracy of LP- risk model) on the following parameters in the training formula (8):

1. The number of optimisations N_{opt};
2. The increment minimum amplitude $AP1_{min}$, at which the optimisation is still possible;
3. The initial value of the criterion function F_{beg} ;
4. The choice of identical or different amplitudes *AP1* for different grades;
5. The increment maximum amplitude $AP1_{max}$;
6. Objects risk distribution in the statistical data.

Let us illustrate it. A question arises, whether to choose the identical or different values of increments amplitudes *AP1* for all events-grades ? In other words, whether the amplitudes $AP1_{jr}$ should depend on the values of probabilities $P1_{jr}$? In the training formulas of the LP-risk model (7) and (8) the increments amplitudes $AP1_{jr}$ are identical for all events-grades and do not depend on the values of their probabilities $P1_{jr}$. The increments $dP1_{jr}$ differ only because of the random simulation of the K_3 coefficient.

The model investigations for the LP-model of the credit risk were made on the PC. The credit risk structural LP-model has 20 events-signs (correspondingly GIE) and 94 events-grades. The credit risk L-function is [1,2] :

$$Y = X_1 \cup X_2 \cup \ldots \cup X_{20} \tag{11}$$

Verbally it can be formulated as follows: a failure occurs, if any one, or any two, … or all initiating events happen. After the orthogonalization of the L-function (11) the following P-risk model for the evaluation of the credit risk has been obtained:

$$P = P_1 + P_2 Q_1 + P_3 Q_1 Q_2 + \ldots . \tag{12}$$

The investigations were carried out in a set of 1000 credits, 700 of which were good and 300 - bad [5]. For calculation investigations we used the Software , designed in the object-oriented languages Visual C+++ and Java.

3.1 The choice of parameters N_{opt}, $AP1_{min}$, F_{beg}

In comparison with the optimal variant F_{max} = 824, the initial variant had the probabilities $P1_{jr}$ without the last four signs. So the optimisation starts at F_{beg} = 690-760. Such solution allowed to reduce calculation time.

The calculations were made for two values of increments maximum amplitudes: 1) $AP1_{max}$ *=0.05 (5 %)* , 2) $AP1_{max}$ *= 0.1 (10 %)* . We used the following numbers of optimisations N_{opt}*: 150, 300, 500, 750, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000.*

The results of investigations presented in Table 1 (Var.2-21) and Fig.3 , allow to make the following conclusions:

1) The criterion function F_{max} (column 6 in Table 1 and Fig.3) asymptotically increases with the growth of the number of N_{opt} optimisation ;
2) The minimum amplitude $AP1_{min}$ (column 9) equals approximately *0.0025 (0.25 %)*; at the smaller values of $AP1_{min}$ the optimisation does not happen and the number of the last optimisation N_{end} (column 10) is less, than the given number of N_{opt} optimisations. It is necessary to modify the law of the change of *AP1* during the training process *,* adding the constant line $AP1_{min}$ (Fig.4). It increases the chance to get the greater value of F_{max};
3) The big value of N_{opt} can lead to the disappearance of the B-C line (Fig. 4), which undoubtedly will deteriorate the process of optimisation.
4) The initial value of F_{beg} (column 5) should not be lowered, as it often leads to low final values of F_{max} (Fig. 5) because of the unsuccessful trajectory of optimisation process; in the considered case it is possible to accept F_{beg} *=750-760*.

Taking into consideration the just made conclusions , instead of the formula (8) the following formula for training the LP-risk model is suggested:

$$(13) \qquad \text{If } AP1 < AP1_{min}, \text{ then } dP1_{jr} = AP1_{min} * K_3,$$
$$\text{If } AP1 > AP1_{min}, \text{ then } dP1_{jr} = K_1 * (N_{opt} - N_t) * K_3.$$

The optimisation results using the formula (13) under $AP1_{min}$ *= 0.0025 (0.25 %),* different $AP1_{max}$ *= 0.098, 0.09, 0.03 (9.8 %, 9 %, 3 %), a* rather large number of optimisations N_{opt}*=5000-12000* and the high F_{beg} *=745* in Table 1 (var. 22-24) are shown. In all variants high values of F_{max} *=812-822* have been obtained.

Table 1. The investigations results in the choice of optimisation parameters

N	Nopt	K_1	AP1ma	Fbeg	Fmax	dPc	AP1min	Nend	Notes
1	2	3	4	5	6	7	8	9	10
1	2000	0.0001	0.2	776	786	0.204	0.1987	20	
2	300	0.000165	0.05	756	794	0.1969	0.00198	289	(3)
3	300	0.00033	0.1	712	790	0.221	0.00429	288	(3)
4	750	0.0000665	0.05	756	802	0.1641	0.00545	669	(3)
5	750	0.000133	0.1	692	790	0.2052	0.01316	652	(3)
6	1000	0.00005	0.05	750	802	0.1867	0.00350	931	(3)
7	1000	0.0001	0.1	708	792	0.2174	0.01580	843	(3)
8	2000	0.000025	0.05	776	808	0.1595	0.00747	1702	(3)
9	2000	0.00005	0.1	724	798	0.1802	0.01405	1720	(3)
10	3000	0.0000166	0.05	748	806	0.1867	0.00699	2581	(3)
11	3000	0.000033	0.1	708	806	0.1867	0.00501	2849	(3)
12	4000	0.0000125	0.05	744	812	0.1945	0.00791	3368	(3)
13	4000	0.000025	0.1	740	802	0.2121	0.00862	3656	(3)
14	5000	0.00001	0.05	754	806	0.1663	0.00556	4445	(3)
15	5000	0.00002	0.1	738	803	0.1586	0.00400	4801	(3)
16	6000	0.000016	0.1	710	810	0.1598	0.00625	5610	(3)
17	6000	0.0000183	0.109	736	810	0.1618	0.00495	5730	(3)
18	7000	0.0000071	0.05	764	810	0.2096	0.00407	6430	(3)
19	7000	0.0000142	0.1	734	810	0.1692	0.00745	6479	(3)
20	8000	0.0000062	0.05	764	810	0.1755	0.00985	6425	(3)

N	*Nopt*	*K_1*	*AP1ma*	*Fbeg*	*Fmax*	*dPc*	*AP1min*	*Nend*	*Notes*
21	*8000*	*0.0000125*	*0.1*	*718*	*814*	*0.1802*	*0.00286*	*7772*	*(3)*
22	*12000*	*0.0000075*	*0.09*	*772*	*812*	*0.1737*	*0.0025*	*11754*	*(10)*
23	*8000*	*0.00000375*	*0.03*	*780*	*820*	*0.1526*	*0.0025*	*7662*	*(10)*
24	*8000*	*0.00000875*	*0.07*	*744*	*814*	*0.1733*	*0.0025*	*7801*	*(10)*
25	*5000*	*0.0000043*	*0.0215*	*812*	*820*	*0.1462*	*0.0025*	*23*	*(13)*
26	*5000*	*0.00000043*	*0.0025*	*810*	*824*	*0.1511*	*0.0025*	*34*	*(13)*
27	*8000*	*0.00000002*	*0.0025*	*810*	*826*	*0.1538*	*0.0025*	*678*	*(13)*
28	*8000*	*0.0000025*	*0.00458*	*806*	*822*	*0.1604*	*0.00609*	*507*	*(13)*
29	*8000*	*0.00000312*	*0.00572*	*806*	*822*	*0.1677*	*0.00452*	*1757*	*(13)*

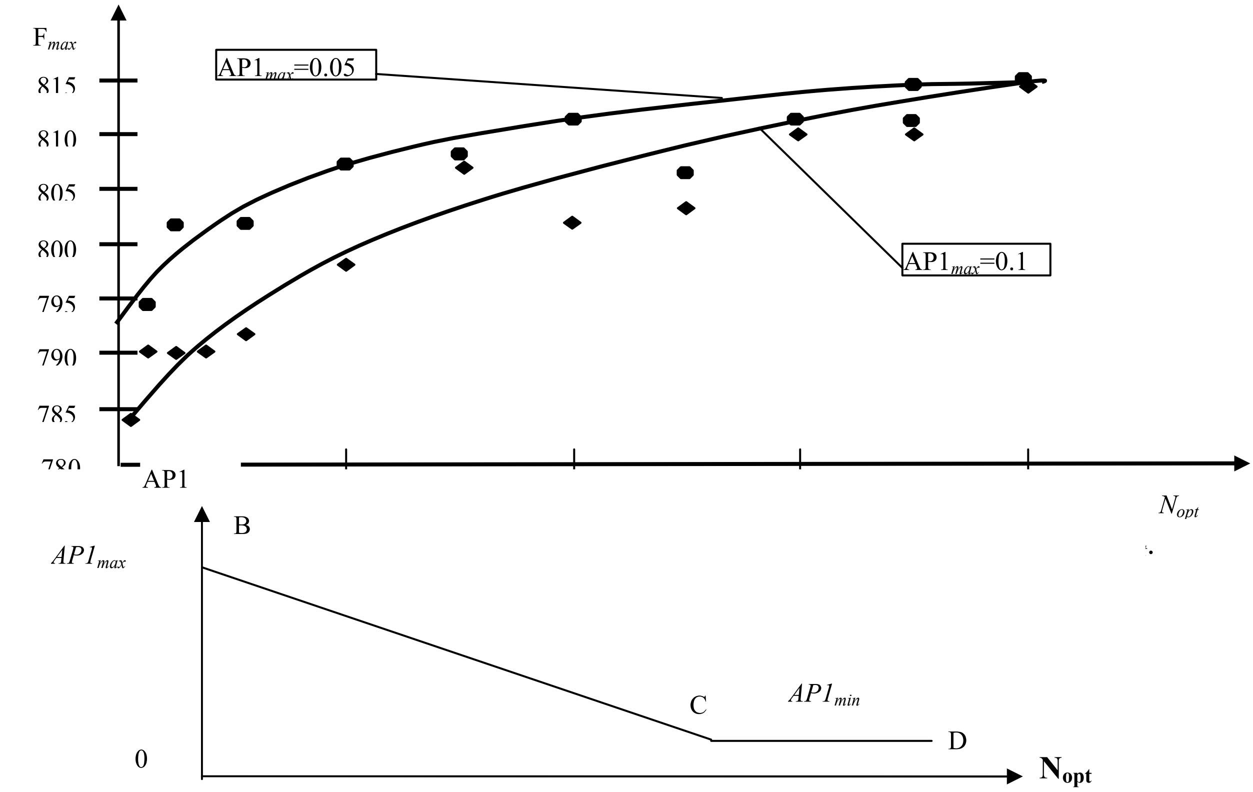

Fig.4. The graph of the current amplitude of increment AP1 modification

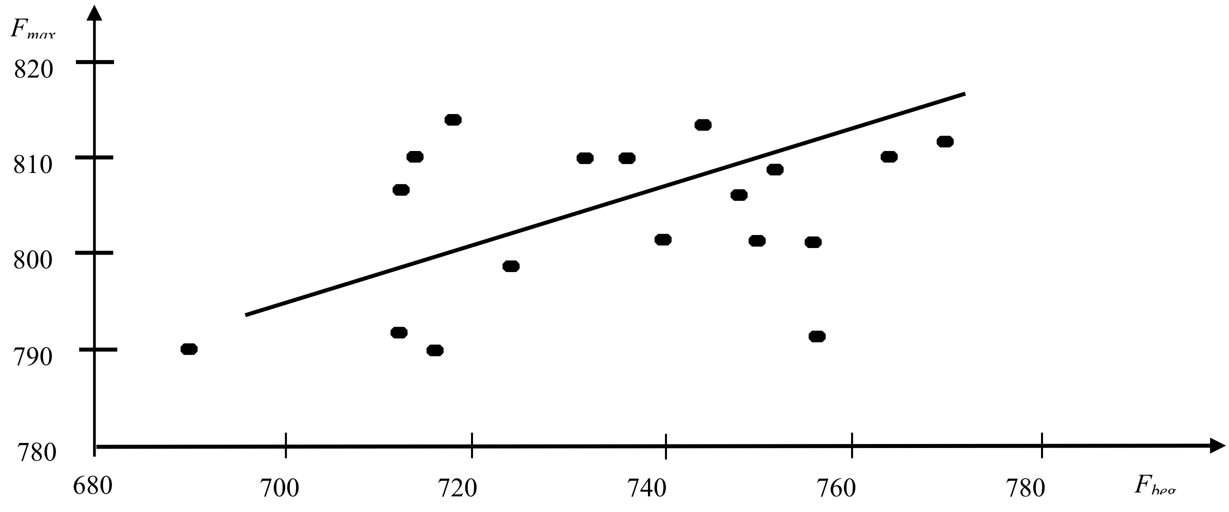

Fig. 5. Dependence of the criterion function F_{max} on its initial value

3.2 Different amplitudes $AP1_{jr}$ *of* increments for different grades

It should be noted, that the probabilities $P1_{jr}$ depend on: a number of grades in GIE, the frequencies of W_{jr} grades in objects and the grades contributions in the classification errors of objects. In the formula of training the LP-risk model (8) the increments amplitudes $AP1_{jr}$ are identical for all events-grades and do not depend on the magnitude of their probabilities $P1_{jr}$.

Let us change the formula of the training LP-risk model so that it takes into account the value of probability for each grade

(14) $$dP1_{jr} = K_1 * (N_{opt} - N_t) * K_3 * P1_{jr}..$$

Here the amplitudes for every event grade are equal

(15) $$AP1_{jr} = K_1*(N_{opt} - N_t)*P1_{jr}$$

and the formula (14) can be the following:

(16) $$dP1_{jr} = AP1jr * K_3.$$

Let us also put down the formula (14) with the following modification:

(17) $$dP1_{jr} = K_1 * (N_{opt} - N_t) * ((1-a) + a * P1_{jr}) * K_3,$$

where a is a coefficient from the interval $[0 < a < 1]$. It determines the formula (8) at $a=0$, the formula (14) at $a=1$ and all the modifications at other values of a.

In the formula (13) let us take into account the limitations, introduced earlier in the formula (8), and we shall get the following expression for training the LP-risk model:

(18) *If* $AP1_{jr} < AP1_{min}$ *, then* $dP1_{jr} = AP1_{min}$ *,*

If $AP1_{jr} > AP1_{min}$ *, then* $dP1_{jr} = K_1 * (N_{opt} - N_t) * ((1-a) + a * P1_{jr}) * K_3$,

The investigations results in optimisation using the formula (18) at $a=1$ ($AP1_{max} = 2.15$ *%, 0.25 %, 0.45 %, 0.57%)* are represented in Table 1 (Var.25-29). They show that the high values of the $F_{max} = 822$-*826* can be obtained at the limited number of optimisation attempts N_{end} (column 10). Actually the first optimisation already gives the high value of CF ($F_{beg}=806$-*810)*. The optimisation process ends at N_{end} = 23-1750 instead of the given numbers of optimisations $N_{opt}=5000$-*8000* (column 6). It seems, that the number of optimisations N_{opt} can be essentially reduced. To verify this hypothesis some extra investigations have been carried out.

The investigations were carried out at small numbers of optimisations N_{opt} *= 600, 450, 300, 150, 100, 50 and K1=0.00033, 0.00025, 0.00015, 0.0001*. The increments maximum amplitude $AP1_{max}$ varied in an interval *0.5% - 20%* from $P1_{jr}$. In Table 2 the CF values and the difference between maximum and minimum risks of objects in the statistics F_{max} / APc are shown. The results of the investigations should be considered as good *(*F_{max} *=810-822)* and completely confirming the effectiveness of the formulas (14), (17) and (18).

Also the investigations of the influence of a parameter on the optimisation results have been carried out. It was done at the small numbers of optimisations $N_{opt}=150$ and $K_1=0.00015$. The maximum amplitude of an increment $AP1_{max}$ equals $0.0225 * P1_{jr}$.

Table 2. Values of F_{max} / APc at the small number of optimisations N_{opt} and $a=1$

Number of optimizations, N_{opt}	K_1=0.00033	K_1=0.00025	K_1=0.00015	K_1=0.0001
600	798 / 0.248	796 / 0.225	810 / 0.180	810 / 0.149
450	802 / 0.217	804 / 0.187	814 / 0.162	819 / 0.161
300	810 / 0.146	810 / 0.174	816 / 0.147	820 / 0.162
225	810 / 0.154	811 / 0.152	818 / 0.148	821 / 0.146
150	816 / 0.145	820 / 0.156	822 / 0.148	822 / 0.147
100	818 / 0.146	820 / 0.149	820 / 0.151	820 / 0.153
50	822 / 0.151	820 / 0.146	820 / 0.152	820 / 0.148

The investigations results, represented in Table 3, also confirm the effectiveness of the formulas (14),(17) and (18) at $a=1$. Really, at $a=1$ F_{max} equals *820*, and at $a=0$ F_{max} equals *802*.

Table 3. Values F_{max} at different values of a

Value a	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	1.0
Value F_{max}	802	800	798	804	808	810	808	810	818	820

3.3 Determination of the amplitude $AP1_{max}$ and the global extreme F_{max}

Let us consider again the choice of the increment maximum amplitude of probabilities $AP1_{max}$. The results of the change of F_{max} at the change of $AP1_{max}=K_1*N_{opt}$ in the interval *0.5-20 %* of $P1_{jr}$ are represented in Table 2. They demonstrate that the higher is $AP1_{max}$ the less is F_{max}. In Fig.6 the dynamics and the results of optimisation for five variants, having $N_{opt}=2000$, are shown:

- Variant 1: $AP1_{max}=0.05(5\%)$, $F_{max}=808$ (Var.8 in Table1), training under the formula (3);
- Variant 2: $AP1_{max}=0.1(10\%)$, $F_{max}=798$(Var.9 in Table1), training under the formula (3);
- Variant 3: $AP1_{max}=0.05$ *(5%)*, $F_{max}=820$, training under the formula (14) with a=1;
- Variant 4: $AP1_{max}=0.1$ *(10%)*, $F_{max}=804$, training under the formula (14) with a=1;
- Variant 5: $AP1_{max}=0.2$ *(20%)*,$F_{max}=786$(Var.1 in Table1),training under the formula (14).

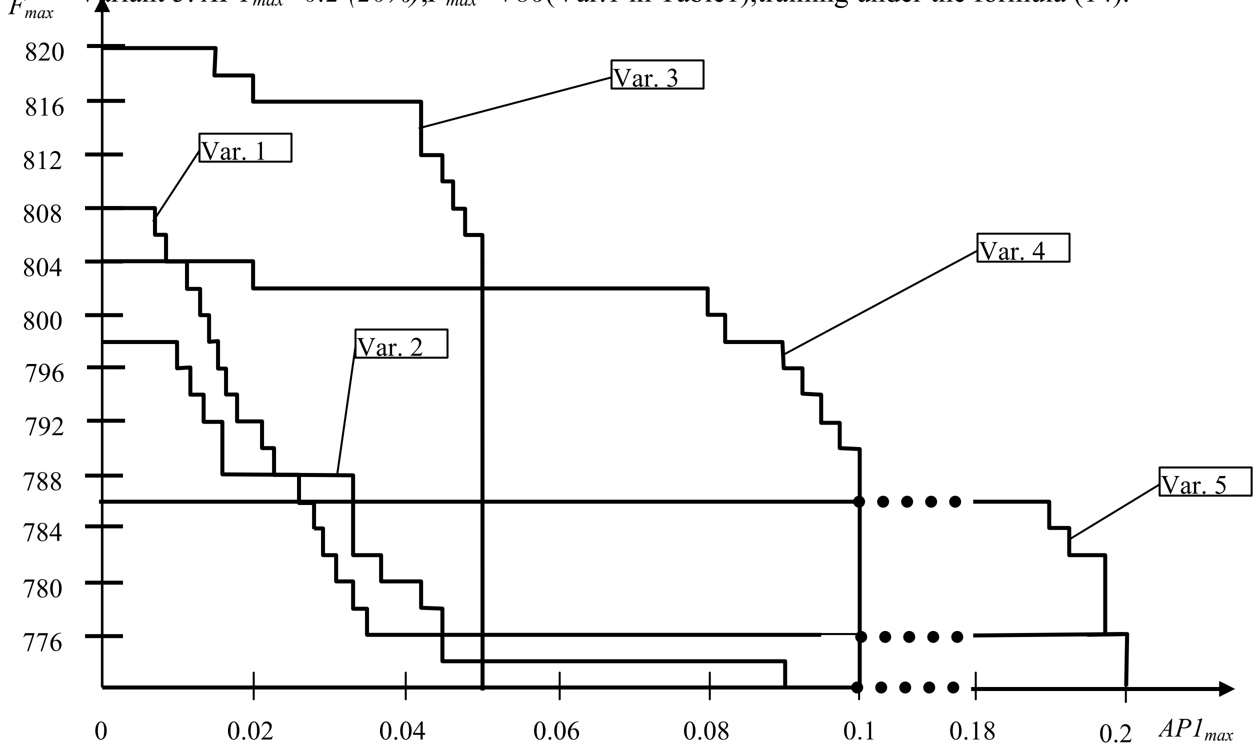

Fig.6. Dynamics and results of optimization depending on the $AP1_{max}$ parameter

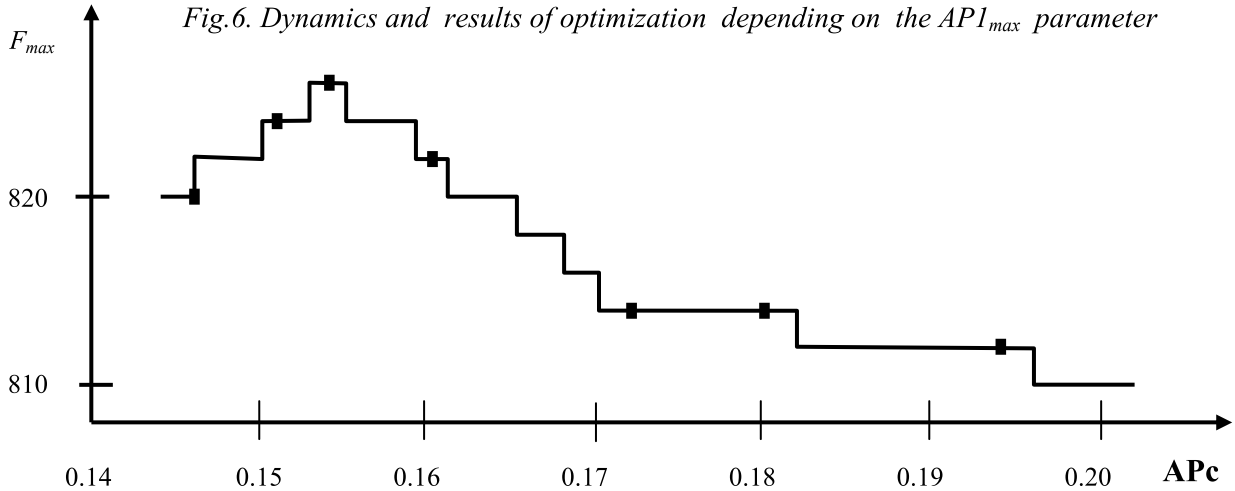

Fig.7.The connection of parameters F_{max} and APc

Variants 4 and 5 with high $AP1_{max}$, despite using the effective formula (18) and $a=1$, have bad training dynamics and results. In these variants CF are correspondingly *786* and *804*. The optimisation process finishes early, ($N_{end}=1608$ *and* $N_{end}=20$). Additional optimisation attempts $N_{opt} - N_{end}$ have not increased CF. This example confirms that the increment amplitude $AP1_{max}$ should not be more than *0.02 - 0.05 (2-5 %)*.

We check the calculation of the global extreme of the CF by the graph (Fig.7). The function F_{max} has an extreme at some value of the difference *APc* between the maximum risk and the minimum risk of objects in statistics [2]. This difference, constructed for variants of computational investigations , presented in Table 1 and 2, demonstrates the robustness (stability) of solutions at a small dispersion of *APc* in the area of the global extreme of CF.

4. CONCLUSION

In the investigations the following main results have been obtained:

1. The effective technology of the criterion function global extreme search in the tasks of identification of LP-risk models under statistical data has been offered .It permits to solve the task of multi-parameter multi-criteria optimisation with integer CF for the time, applicable to practice (less than before).
2. We suggest to generate in the training formula a random number K_3 in the interval [-1, +1]. It permits to consider the absolute values of increments $dP1_{jr}$, multiplied by 100, in percents (%)) and to estimate the accuracy of probabilities $P1_{jr}$.
3. In the technology of the CF global extreme search, the following regularities of changing the CF should be used:
   - The CF asymptotically increases with the growth of N_{opt} optimisation number ;
   - The minimum amplitude $AP1_{min}$ of probabilities $P1_{jr}$ increments is established by 2-3 test calculations; at smaller values of $AP1_{min}$ the optimisation does not happen (less than 0.25 %);
   - The initial CF F_{beg} should not be lowered , as low values more often result in low final values of F_{max} because of the unsuccessful trajectory of the optimisation process;
   - Maximum amplitude of increments of $AP1_{max}$ must not exceed *0.02 - 0.05 (2-5%),* as the training speed lows down and the value of the CF F_{max} becomes less.
4. For the criterion function global extreme search new , more effective formulas of training (14), (17), (18) have been suggested ; they use different amplitudes of increments for probabilities of different events-grades.
5. It has been confirmed that we can test the determination of the global extreme of CF F_{max} by the graph of change of F_{max} in the function of difference *APc* between maximum and minimum risks of objects in statistics. The function F_{max} has an extreme at a certain value of *APc* .

REFERENCES

1. Solojentsev E.D., Karasev V.V.,Solojentsev V.E. Logic and probabilistic models of risk in banks, business and quality / edited by E.D.Solojentsev. SPb.: Nauka, 1999.-120 p.
2. Solojentsev E.D., Karassev V.V. (2001) Risk logic and probabilistic models in business and identification of risk models. - Informatica 25 (2001) 49-55.
3. Taha X. Introduce in research of operations . v.1,2. Moscow, Mir , 1985.
4. Wasserman Philips D. Neural Computing Theory and Practice. ANSA Research , Inc. VAN NOSTRARD REINOLD, New York, 1990.
5. Seitz J., Stickel E. Consumer Loan Analysis Using Neural Network. Adaptive Intelligent Systems. Proceed. of the Bankai Workshop, Brussels, 14-19 October, 1992. P.177-189