
SOME COMMENTS ON STATISTICAL RISKS

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Abstract

In this work we make a detailed analysis of the concept of risk, the stress being focused then on various kinds of statistical risks: producer and consumer risks, technical risk, Taguchi's risk (making a connection with C_{pm} capability index) and a risk arising in SPC practice.

Key words: statistical risk, error, hazard rate, Taguchi's loss function, Taguchi's risk, SPC - Statistical Process Control.

1. Preliminaries: a discussion on the concept of risk

The notion of risk covers a broad area of interpretations. As in many cases, there is a man-in-the street approach and a scientific one which tries to offer quantitative measures of the underlying term.

Let us visit first some usual dictionaries. For instance, BBC English - Romanian Dictionary (Editura CORESI, București, 1998, page 966), risk is assimilated to a **danger**: if there is a risk of something, it might have unpleasant or even dangerous consequences (results).

We seize here the potentiality of such kind of results, which may or may not occur. Therefore, it is a suggestion that risk is associated with uncertainty: it might happen, but we do not know for sure if it will indeed happen.

Merriam - Webster's Collegiate Dictionary (Tenth Edition M. W. Incorporated, Springfield, Mass, U.S.A., 1996, page 1011) is more generous and specific: **possibility** of loss or injury, a peril but also **the degree of probability** of such loss (this is a new element in the usual definitions).

The very recent „Illustrated Oxford Dictionary of English Language” (2008, Dorling/Oxford Univ. Press, Litera International, București - Chișinău, page 709) defines it as a chance or possibility of danger, loss, injury etc.

The term „chance” is straightforwardly linked with that of uncertainty. Some authors consider that risk is characterized by possibility to be described by the aid of probability laws (see Bârsan-Pipu and Popescu, 2003 [2, page 2]). Uncertainty can be described also by quantitative measures - if we regard it from the metrological point of view (see Petrescu et al., 2006 [12]).

Webster's Unabridged Dictionary of English Language (edited 2002) advances the concept **risk management** (RM) and also that of risk manager. This RM is viewed as a technique of estimation, prevention and minimization the accidental losses which could appear in a business by taking some safety measures (insurances - for instance).

Risk appears therefore as an uncertain event which may take place if some **risk factors** actually act.

On the other hand, the risk is always associated with the anthropical element - that human factor which finally will suffer eventual losses of its „risky decisions”.

2. Kinds of risks

Generally speaking, there are several types of risk - depending on the domain we consider to be of interest. Isaac-Maniu et al (1999, page 492) [7] believe that the so-called economic risk is of great importance. This risk is defined as the incapacity (or just impossibility!) of a given organization to survive in a business environment: this means that its managers have no the skills (and knowledge) to adapt the economical policy of the company to variations (sometimes unexpected and unfriendly) of the social-economical reality at a specified moment.

This economic risk (quite general) has some components such as „bankruptcy subrisk” which seems to be essential: if an organization cannot pay its bills for current utilities, cannot reimburse its loans, cannot pay its subcontractors, suppliers etc. - all these are the signs that the above risk has already implemented its destructive effects.

Since the risk is regarded as a probability, therefore it is worth to investigate the nature of what is called the **statistical risk**. It plays an important role in the framework of statistical inference. One problem which has been not very deeply investigated is the following: how to manipulate (or to manage) this in order to minimize it, in the sense that the decision taken in an uncertain/risky situation, to be „the best” one?

3. Various types of statistical risks

Usually, in the theory of statistical hypotheses, founded mainly by the British School of Statistics (see Stoichițoiu - Vodă, 2002 [15]) we deal with the so-called **errors** we make as regards the decisions about the underlying hypotheses.

As it is well-known, a hypothesis (in general) is simply defined as a statement/assumption/supposition about a certain phenomenon, process, situation etc. This assumption may be true (that is in accordance with the real status of the entity considered) or may be not. For a **scientific** hypothesis it is sufficient to provide a counterexample, in order to reject as **false** the proposed hypothesis.

Since in statistical analysis we work with **samples** (assumed to be obtained randomly), the conclusions will depend entirely on the sample (or samples) we have at hand. The sample could support the advanced hypothesis (called null-hypothesis, H_0) or it could sustain the alternative one (H_1). Therefore, we say that the couple (H_0, H_1) is accompanied by **two kinds of errors**, namely

$$\alpha = \text{Prob}\{\text{reject } H_0 \mid \text{if } H_0 \text{ is true}\} \quad (1)$$

and

$$\beta = \text{Prob}\{\text{accept } H_0 \mid \text{if } H_1 \text{ is true}\} \quad (2)$$

They are called respectively: error of the first Type (α) and error of the second Type (β) - see for details Blischke and Murthy, 2000 [3] page 157 - 162.

These authors draw the attention that Type I and Type II **errors rates** are the probabilities of making these kind of „mistakes” - namely „do reject H_0 ” (when H_0 is true) and „do not reject H_0 ” (when H_1 - the alternative is true).

In fact, $\alpha = \alpha(n; \theta_1)$ and $\beta = \beta(n; \theta_1)$ - that is they depend on the size of the sample we employ and on the **true value** of the parameter (θ) on which the hypothesis is made.

In SQC - Statistical Quality Control - especially in sampling inspection of batches, where practical procedures have been standardized (see American Standards MIL STD 105 D and MIL STD 414 - or their ISO equivalents, ISO 2859 and ISO 3951, α and β are called „producer risk” (α) and „consumer risk” (β) - respectively. In the above documents α and β are taken at fixed levels ($\alpha = 5\%$ and $\beta = 10\%$) and hence there is no possibility to modify these values if in practice we use these standards.

What we can do is diminish **the risk of non-acceptance** of a given lot/batch. This risk is expressed as $1 - P_a(p)$, where $P_a(p)$ is the probability of acceptance of the lot which depends on its defective (or nonconforming) fraction (p). If p is larger than the accepted value (AQL - Acceptable Quality Level), then the risk of non-acceptance is higher.

Other important element of the above mentioned documents is the so-called LQ - Limiting Quality - that is that value of p which we are ready to accept with a small probability (in 10% of the case at most).

If $p = p_0 > AQL$, the risk of non-acceptance increases as long as (p) approaches the LQ value.

It follows that the management of this risk has to be directed to those measures which can lead to a decline of the fraction defective (see Isaic-Maniu and Vodă, 1997, [6]).

3.1. Error of the Third Type?

In [14] has been discussed an argumentation of Malița and Zidăroiu (1980, [10]) in favor of a Raiffa's idea (1970, [13]) regarding the existence of a Type III error. This last author claims that if an experimenter (or an analyst) tries to solve a **false problem**, then he commits an error of the third kind! Raiffa did not establish clearly what he understands by a „false” problem: is it an ill - posed problem (improperly/wrongly formulated) or the falsity refers to the goal/purpose stated by the responsible authority?

Malița and Zidăroiu tried to justify Raiffa's proposal by linking it to the Type I and Type II errors, claiming that this Type III error „is expected to weight in a specific manner, the previous two classical type errors”. They say also that the main source of Type III error is the lack of communication between the analyst and the decisional factor. This communication must act in both directions: from the decision unit to the experimenter/analyst and conversely, in order to check/verify that indeed we detected the right problem!

Such an argumentation seems to be at most at a metaphoric level: nobody will ask himself or someone else if the problem he solves is false ...

We shall mention the Cambridge Dictionary of Statistics, Cambridge University Press, 1998 (author B. S. Everitt) where he draws the attention to not confound this risk with Type III error – term used for identifying the poorer of two treatments as the better (pages 116 and 338).

3.2. Technical risk

Irina Isaic-Maniu (see [8], page 51 - 65], 2003) gives a „risk interpretations” for the main indicators used in reliability theory; in fact, the distribution function $F(t)$ of a continuous and positive random variable (T) which describes the failure behavior of a given entity may be viewed as a „technical risk” - that is the complement of the reliability function:

$$\text{Prob}\{T < t_0\} = F(t_0) = 1 - R(t_0) = 1 - \text{Prob}\{T \geq t_0\} \quad (3)$$

Here $F(t_0)$ is hence the probability that the system operates less than a desired time t_0 . If the reliability $R(t_0)$ is low, consequently this technical risk is high.

More adequate to define this technical risk seems to be the hazard rate (or failure rate) function which may be called also „the danger of failure”:

$$h(t) = \frac{f(t)}{1 - F(t)} = \frac{d F(t)}{R(t)} = - \frac{d R(t)}{R(t)} \quad (4)$$

A high value of $h(t)$ means a low level of reliability ($h(t)$ is expressed usually in failures/hour).

4. Taguchi's risk

Genichi Taguchi (see Alexis, 1999, [1]) revitalized Gauss' quadratic function $f_{(x)} = a(x - x_0)^2$, $a > 0$, $x \geq x_0 \geq 0$ and associated it to the so-called quality loss

$$L(x_0; T) = k(x_0 - T)^2, \quad k > 0, x_0, T \in \mathbb{R} \quad (5)$$

where x_0 is the measured value of the quality characteristic (X) and (T) is its target value (k is a constant depending on the specific case at hand).

If $f(x; \theta)$ is the density of X ($x \in D$, D being a part of \mathbb{R} , $\theta \in \mathbb{R}$) then the average value

$$E[L(x; T)] = \int_D L(x; T) f(x; \theta) dx \quad (6)$$

is called Taguchi type risk (see Kackar, 1986, [9]).

Taking into account (5), we may write (6) as

$$E[L(x; T)] = k[\text{Var}(x) + (E(x) - T)^2] \quad (7)$$

and if X is normally distributed $N(\mu; \delta^2)$, we have.

$$E[L(x; T)] = k[\delta^2 + (\mu - T)^2] \quad (8)$$

The empirical risk (denoted $\widehat{R}_T(x)$) is therefore

$$\widehat{R}_T(x) = k[s^2 + (\bar{x} - T)^2] \quad (9)$$

where \bar{x} and s are the well-known sample statistics.

There is a straightforward link between Taguchi's risk and his own process capability index \widehat{C}_{pm} (see Chan et al, 1988, [4]):

$$\widehat{C}_{pm} = \frac{USL - LSL}{6 \cdot \sqrt{s^2 + (\bar{x} - T)^2}} \quad (10)$$

where $USL =$ Upper Specified Limit and $LSL =$ Lower Specified Limit of the given quality characteristic $X \sim N(\mu, \delta^2)$ with T as its target value.

We may write hence immediately

$$\widehat{R}_T(x) = k \left(\frac{USL - LSL}{6} \cdot \frac{1}{\widehat{C}_{pm}} \right)^2 \quad (11)$$

If $USL - LSL = 6s$ - that is the minimal level for admissible process capability, we get:

$$\widehat{R}_T(x; T) = k \left(\frac{s}{\widehat{C}_{pm}} \right)^2 \quad (12)$$

and we draw the conclusion that the Taguchi's risk can be regarded as a function of the length of the specified interval USL - LSL measured in standard deviations units.

The theoretical Taguchi risk corresponding to (12) is

$$R_T(x) = k \left(\frac{\delta}{C_{pm}} \right)^2 = \frac{k\delta^2}{C_{pm}^2}$$

Denoting $k\delta^2 = M$ and $C_{pm} = X$, we shall have a hyperbolic dependence of the type $R = M/X^2$. If in C_{pm} , the true mean-value μ is just the target T , then C_{pm} becomes the classical potential index of a process namely $C_{pm} = (USL - LSL)/6\delta$ (see figure 1).

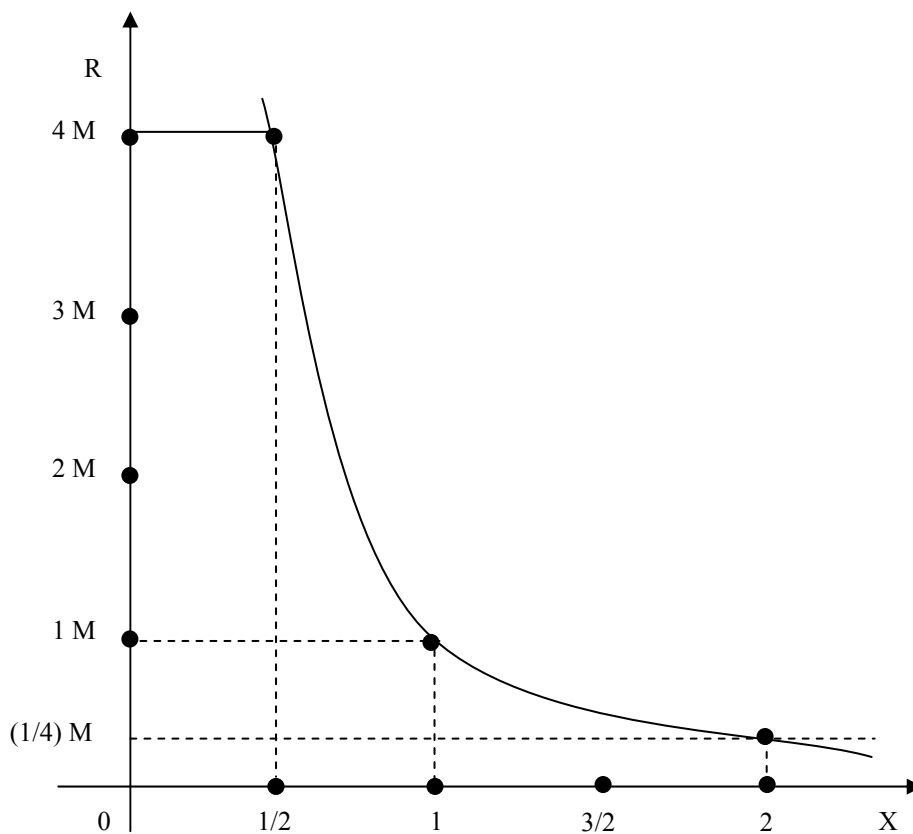


Fig. 1. The relationship between Taguchi's risk and C_{pm}

5. Risk in SPC practice

SPC - or Statistical Process Control is mainly based on the theory and practice of Shewhart control charts (see ISO document ISO 8258 „Shewhart control charts”, 1991 or Petrescu-Vodă, 2002 [11]).

From a statistical point of view, Shewhart control charts can be viewed as a continuously testing of by hypothesis $H_0 : \text{Mean} = \mu$ versus the alternative $H_1 : \text{Mean} \neq \mu$ at the significance level $\alpha = \text{Prob}\{|Z| > 3\} = 0.0027$ (see Derman and Ross, [5]).

From a practical perspective, this means that even when a certain process is in the state of statistical stability (remains in control) there is a chance - a risk (0.0027) - that a subgroup average will fall outside the control limits $UCL = \mu + 3\delta / \sqrt{n}$, $LCL = \mu - 3\delta / \sqrt{n}$ and the experimenter would incorrectly take the „risky decision” to correct the process that is to dig for an illusory cause of trouble.

Numerical example: Consider a measurable characteristic for which two specified limits are fixed, namely $LSL = 263.48$ c.u. (c. u. = conventional units) and $USL = 263.68$ c. u. The target value is $T = 263.58$ c. u. If we ask a performance level for C_p to be 2 and if from data we get the mean value $\bar{x} = 263.58$ c. u. and standard deviation $s = 0.011$ c. u., we shall get \hat{C}_p approximately 0.40 – that is a very weak potential index of the process. The estimated Taguchi risk is therefore $\hat{R}_T(x) \approx 0.007$ k and this risk is expressed in monetary units. This values shows that if the defective unit is cheap, then the risk is small. For such low production cost items it is not necessary to impose a performance at the level of SIX SIGMA (see the excellent monograph of Praveen Gupta “The Six Sigma Performance Handbook. A Statistical Guide to Optimizing Results”, McGraw-Hill Book Co., 2005, New York

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