ON DETERMINATION OF SOME CHARACTERISTICS OF SEMI-MARKOV PROCESS FOR DIFFERENT DISTRIBUTIONS OF TRANSIENT PROBABILITIES

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ABSTRACT

There is a model of transport system presented in the paper. The possible semi - Markov process definitions are included. The system is defined by semi –Markov processes, while functions distributions are assumed. There are attempts to assess factors for other than exponential functions distributions. The paper consist discussion on Weibull and Gamma distribution in semi – Markov calculations. It appears that some forms of distribution functions makes computations extremely difficult.

1 INTRODUCTION

The reliability model of intermodal transport was presented during ESREL'06 conference (Zajac 2006b). The model is described by semi – Markov processes. During the presentation assumed, that, probabilities of transition between states were exponential. Complex technical systems are usually assumed, that probabilities of transition between states or sojourn times' probabilities are exponential. Lack of information, too little number of samples or inaccurate assessment of data may cause that such assumption is abused. In some cases, when exponential distribution is assumed, there is also possibility to assess factors according to different distributions (Weibull, Gamma, etc.). Probabilities of transition between states are one of the fundamental reliability characteristic. The paper includes example of determination of above mention characteristic for one of the phases of combined transportation systems reliability model.

2 TRANSHIPMENT PHASE CHARACTERISTIC

There are three methods to define semi – Markov processes (Grabski 2002, Grabski&Jazwinski 2003):

- by pair ($\boldsymbol{p}$, $\mathbf{Q}(t)$),

when: $\boldsymbol{p}$ – vector of initial distribution, $\mathbf{Q}(t)$ – matrix of distribution functions of transition times between states;

- by threes ($\boldsymbol{p}$, $\boldsymbol{P}$, $\mathbf{F}(t)$),

where: $\boldsymbol{p}$ – vector of initial distribution, $\boldsymbol{P}$ – matrix of transition probabilities, $\mathbf{F}(t)$ – matrix of distribution functions of sojourn times in state *i-th*, when *j-th* state is next;

- by threes ($\boldsymbol{p}$, $\boldsymbol{e}(t)$, $\mathbf{G}(t)$), where: $\boldsymbol{p}$ – vector of initial distribution, $\boldsymbol{e}(t)$ – matrix of probabilities of transition between *i-th* and *j-th* states, when sojourn time in state *i-th* is x, $\mathbf{G}(t)$ – matrix of sojourn times distribution functions. For transshipment phase semi – Markov process is defined by ($\boldsymbol{p}$, $\boldsymbol{P}$, $\mathbf{F}(t)$). Phase of transshipment includes following states:

1. standby,
2. dislocation works,
3. transshipment,
4. preventive maintenance,
5. repair (after failure).

Activities which are involved into each of above states are described in papers (Zajac 2006a, Zajac 2007). The graph of state is presented in Figure 1.

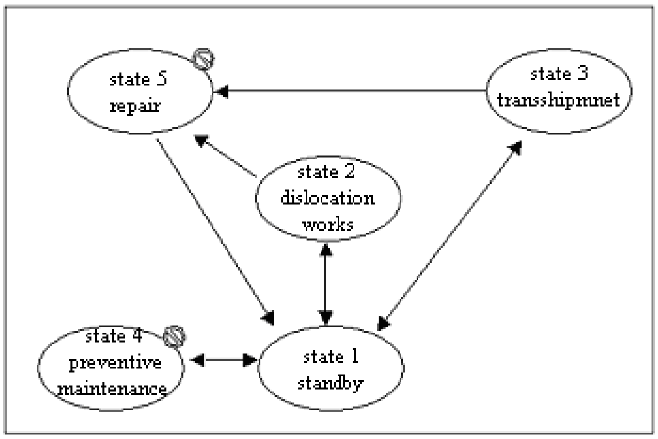


Figure 1. Graph of states in transshipment phase

3 CONDITIONS DETERMINATION FOR TRANSSHIPMENT PHASE RELIABILITY

Transshipment phase elements can stay in reliability states from the set S (0,1), where:
0 – unserviceability state,
1 – serviceability state.
Operation states takes values from the set T (1,2,3,4,5). Cartesian product of both states creates following pairs: (0,1), (1,1), (0,2), (1,2), (0,3), (1,3), (0,4), (1,4), (0,5), (1,5). The model allows for existence of following pairs, only: S_{1p} - (1,1), S_{2p} - (1,2), S_{3p} - (1,3), S_{4p} - (0,4), S_{5p} - (0,5).
Means of transport are in first operation state (standby) during time described by random variable ζ_{p1}. The distribution function of random variable is

$$F_{\zeta p1}(t) = P\{\zeta_{p1} \le t\},\ t \ge 0\,.$$

The time of the second state (dislocation) is described by ζ_{p2}. The distribution function of random variable takes form

$$F_{\zeta p2}(t) = P\{\zeta_{p2} \le t\},\ t \ge 0$$

The time of third state (transshipment) is described by ζ_{p3}, where distribution function of random variable is given by formula:

$$F_{\zeta p3}(t) = P\{\zeta_{p3} \le t\},\ t \ge 0$$

If the time of realization of preventive maintenance is known (and lasts γ_p), than the distribution function of sojourn time in the fourth state (preventive maintenance) is

$$F_{\gamma_p}(t) = P\{\gamma_p \le t\},\ t \ge 0.$$

Some of activities can be interrupted by failures. It was assumed, that time of work without failure in states 2-nd and 3-rd is described by η_{pi}, $i = 2,3$. The distribution function is given by formula:

$$F_{\eta pi}(t) = P\{\eta_{pi} \le t\},\ t \ge 0,\ i = 2,3.$$

If there is known time, when the system is broken down, and that time is given by χ_p, then the distribution function of state 5-th (repair) is

$$F_{\chi_p}(t) = P\{\chi_p \le t\},\ t \ge 0.$$

States 4-th and 5-th are states of unserviceability, however only state 5-th requires repair after failure. We assume that random variables ζ_{pi}, η_{pi} and χ_p are independent.

3.1 Kernel determination and the definition of semi – Markov process in transshipment phase

The phase of transshipment can be described by semi – Markov process $\{X(t): t \ge 0\}$ with the finite set of states $S_p = \{1, 2, 3, 4, 5\}$. The kernel of the process is described by matrix

$$\mathbf{Q_p}(t) = \begin{bmatrix} 0 & Q_{p12} & Q_{p13} & Q_{p14} & 0 \\ Q_{p21} & 0 & 0 & 0 & Q_{p25} \\ Q_{p31} & 0 & 0 & 0 & Q_{p35} \\ Q_{p41} & 0 & 0 & 0 & 0 \\ Q_{p51} & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (1)$$

Transshipments from 1-st state to 2-nd, 3-rd and 4-th can be described by

$$Q_{p12}(t) = p_{12} F_{\zeta_{p1}}(t),$$

$$Q_{p13}(t) = p_{13} F_{\zeta_{p1}}(t),$$

$$Q_{p14}(t) = p_{14} F_{\zeta_{p1}}(t).$$

Transshipments from 2-nd state to 1-st and 5-th:

$$Q_{p21}(t) = p_{21} F_{\zeta_{p2}}(t),$$

$$Q_{p25}(t) = p_{25}F_{\zeta_{p2}}(t).$$

Transshipments from 3-rd state to 1-st and 5-th:

$$Q_{p31}(t) = p_{31}F_{\zeta_{p3}}(t),$$

$$Q_{p35}(t) = p_{35}F_{\zeta_{p3}}(t).$$

Transshipment from 4-th state to 1-st:

$$Q_{p41}(t) = P(\gamma_p < t) = F_{\gamma_p}(t).$$

Transshipment from 5-th state to 1-st:

$$Q_{p51}(t) = P(\chi_p < t) = F_{\chi_p}(t).$$

The vector $\boldsymbol{p} = [p_1, p_2, p_3, p_4, p_5]$ is initial distribution of the process. In this case vector takes values $\boldsymbol{p} = [1, 0, 0, 0, 0]$.
The matrix of transient probabilities is given by

$$\boldsymbol{P} = \begin{bmatrix} 0 & p_{12} & p_{13} & p_{14} & 0 \\ p_{21} & 0 & 0 & 0 & p_{25} \\ p_{31} & 0 & 0 & 0 & p_{35} \\ p_{41} & 0 & 0 & 0 & 0 \\ p_{51} & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{2}$$

3.2 The transient probabilities

Transient probabilities are one of the most important characteristics of semi – Markov processes. They are defined as conditional probabilities

$$P_{ij}(t) = P\{X(t) = j \mid X(0) = i\},\ i, j \in S \tag{3}$$

Above probabilities obey Feller's equations (Grabski 2002, Grabski&Jazwinski 2003)

$$P_{ij}(t) = \delta_{ij}[1 - G_i(t)] + \sum_{k \in S} \int_0^t P_{kj}(t-x)dQ_{ik}(x) \tag{4}$$

Solution of that set of equations can be found by applying the Laplace – Stieltjes transformation. After that transformation the set takes form

$$\tilde{p}_{ij}(s) = \delta_{ij}[1 - \tilde{g}_i(s)] + \sum_{k \in S} \tilde{q}_{ik}(s)\tilde{p}_{kj}(s), \quad i, j \in S. \tag{5}$$

In matrix notation this set of equation has form

$$\tilde{\mathbf{p}}(s) = [I - \tilde{\mathbf{g}}(s)] + \tilde{\mathbf{q}}(s)\,\tilde{\mathbf{p}}(s)\,, \tag{6}$$

hence

$$\tilde{\mathbf{p}}(s) = [I - \tilde{\mathbf{q}}(s)]^{-1}[I - \tilde{\mathbf{g}}(s)]\ . \tag{7}$$

Determination of transient probabilities requires finding of the reverse Laplace – Stieltjes transformation of the elements of matrix $\tilde{\mathbf{p}}(s)$.

4 DATA AND ASSUMPTIONS FOR CALCULATIONS

Data were collected in 2006 in one of the Polish containers terminals. The data includes information about numbers of transient between states during 50 succeeded days. Selected data are presented in Table 1.

Table 1. Selected data about time of states [h]

	state 1 stand by	state 2 dis-location	state 3 trans-shipment	state 4 preventive maintenance	state 5 repair
average	3.610	3.766	3.805	0.538	0.28
variance	0.387	0.022	0.020	0.046	0.287
min. value	1.862	3.482	3.482	0.3	0
max. value	4.411	4.042	4.020	1.1	2
dispersion	2.548	0.560	0.538	0.8	2

Collected data didn't allow for verifying probabilities distribution. The information gave possibility to estimate necessary parameters to assess factors for exponential, Weibull and Gamma distribution functions. Factors are presented in Table 2.

Table 2. Distribution parameters for different distribution function

	state 1	state 2	state 3	state 4	state 5
Parameter of exponential distribution					
λ	0.28	0.27	0.26	1.86	0.86
Parameters of gamma distribution					
α	9.260	152.059	181.992	13.968	4.120
λ	33.208	531.455	665.314	9.129	4.878
Parameters of Weibull distribution					
α	1.020	1.01	1.014	0.985	1.021
λ	0.269	0.26	0.266	1.538	0.833

At first calculation has been done with assumption, that transient probabilities are exponential. The distribution function of sojourn times and their Laplace – Stieltjes transformation respectively, take form

$$F_{w1}(t) = 1 - e^{-0,28t}\ ,\ f_{w1}*(t) = \frac{0,28}{s+0,28}\,,$$

$$F_{w2}(t) = 1 - e^{-0,27t},\ f_{w2}*(t) = \frac{0,27}{s+0,27},$$

$$F_{w3}(t) = 1 - e^{-0,26t},\ f_{w3}*(t) = \frac{0,26}{s+0,26},$$

$$F_{w4}(t) = 1 - e^{-1,86t},\ f_{w4}*(t) = \frac{1,86}{s+1,86},$$

$$F_{w5}(t) = 1 - e^{-0,86t},\ f_{w5}*(t) = \frac{0,86}{s+0,86}.$$

Then, kernel of the process is given by matrix

$$\boldsymbol{Q_p}(t) = \begin{bmatrix} 0 & 0,81(1-e^{-0,28t}) & 0,16(1-e^{-0,28t}) & 0,04(1-e^{-0,28t}) & 0 \\ 0,98(1-e^{-0,27t}) & 0 & 0 & 0 & 0,02(1-e^{-0,27t}) \\ 0,99(1-e^{-0,26t}) & 0 & 0 & 0 & 0,01(1-e^{-0,26t}) \\ (1-e^{-1,86t}) & 0 & 0 & 0 & 0 \\ (1-e^{-0,86t}) & 0 & 0 & 0 & 0 \end{bmatrix} \quad (8)$$

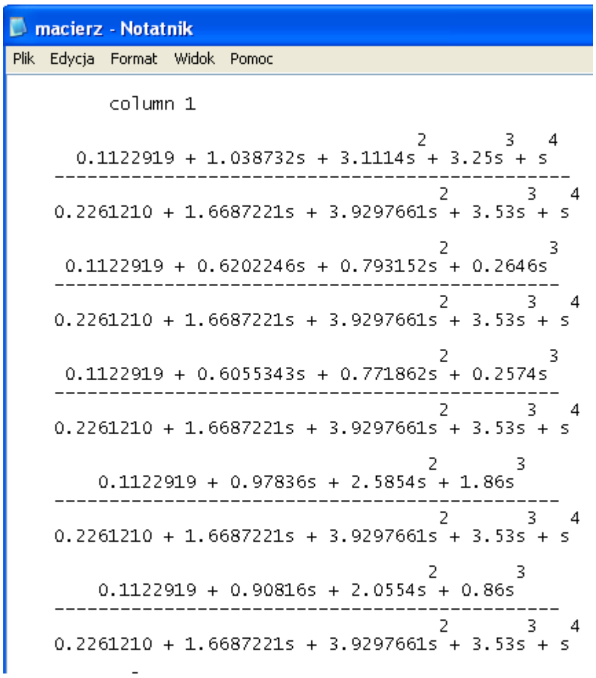
Matrices $\tilde{\mathbf{q}}(s)$ and $\tilde{\mathbf{g}}(s)$ have been determined according to equations (5) – (7). In considered example we obtain

$$\tilde{\mathbf{q}}(s) = \begin{bmatrix} 1 & 0,80\frac{0,28}{s+0,28} & 0,16\frac{0,28}{s+0,28} & 0,04\frac{0,28}{s+0,28} & 0 \\ 0,98\frac{0,27}{s+0,27} & 1 & 0 & 0 & 0,02\frac{0,27}{s+0,27} \\ 0,99\frac{0,26}{s+0,26} & 0 & 1 & 0 & 0,01\frac{0,26}{s+0,26} \\ \frac{1,86}{s+1,86} & 0 & 0 & 1 & 0 \\ \frac{0,86}{s+0,86} & 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

and

$$\tilde{\mathbf{g}}(s) = \begin{bmatrix} \frac{0,28}{s+0,28} & 0 & 0 & 0 & 0 \\ 0 & \frac{0,27}{s+0,27} & 0 & 0 & 0 \\ 0 & 0 & \frac{0,26}{s+0,26} & 0 & 0 \\ 0 & 0 & 0 & \frac{1,86}{s+1,86} & 0 \\ 0 & 0 & 0 & 0 & \frac{0,86}{s+0,86} \end{bmatrix} \tag{10}$$

According to (7), matrix $\tilde{\mathbf{p}}(s)$ is a result of multiplying of two matrices. Elements from first column of obtained matrix $\tilde{\mathbf{p}}(s)$ are shown on Figure 2.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column 1

$$\frac{0.1122919 + 1.038732s + 3.1114s^2 + 3.25s^3 + s^4}{0.2261210 + 1.6687221s + 3.9297661s^2 + 3.53s^3 + s^4}$$

$$\frac{0.1122919 + 0.6202246s + 0.793152s^2 + 0.2646s^3}{0.2261210 + 1.6687221s + 3.9297661s^2 + 3.53s^3 + s^4}$$

$$\frac{0.1122919 + 0.6055343s + 0.771862s^2 + 0.2574s^3}{0.2261210 + 1.6687221s + 3.9297661s^2 + 3.53s^3 + s^4}$$

$$\frac{0.1122919 + 0.97836s + 2.5854s^2 + 1.86s^3}{0.2261210 + 1.6687221s + 3.9297661s^2 + 3.53s^3 + s^4}$$

$$\frac{0.1122919 + 0.90816s + 2.0554s^2 + 0.86s^3}{0.2261210 + 1.6687221s + 3.9297661s^2 + 3.53s^3 + s^4}$$

Figure 2. First column of matrix $\tilde{\mathbf{p}}(s)$

Determination of transient probabilities requires finding of reverse Laplace – Stieltjes transformation of each element of the $\tilde{\mathbf{p}}(s)$ matrix. For elements of the first column of matrix $\tilde{\mathbf{p}}(s)$ reverse transformations are as follow:

$$P_{11} = 0,4966 - 0,0087 \cdot e^{-0,856t} + 0,0002 \cdot e^{-0,262t} + 0,5033 \cdot e^{-0,539t} + 0,0086 \cdot e^{-1,873t} \tag{11}$$

$$P_{21} = 0,4966 - 0,0209 \cdot e^{-0,856t} + 0,006 \cdot e^{-0,262t} - 0,5221 \cdot e^{-0,5390t} - 0,0014 \cdot e^{-1,8734t} \tag{12}$$

$$P_{31} = 0,4966 - 0,0118 \cdot e^{-0,856t} - 0,0301 \cdot e^{-0,262t} - 0,4769 \cdot e^{-0,539t} - 0,0014 \cdot e^{-1,873t} \tag{13}$$

$$P_{41} = 0,4966 - 0,0162 \cdot e^{-0,856t} + 0,0002 \cdot e^{-0,262t} + 0,7087 \cdot e^{-0,539t} - 1,1894 \cdot e^{-1,873t} \tag{14}$$

$$P_{51} = 0,4966 - 1,1838 \cdot e^{-0,856t} + 0,0003 \cdot e^{-0,262t} + 1,3486 \cdot e^{-0,539t} + 0,0073 \cdot e^{-1,873t} \quad (15)$$

All other transient probabilities (for columns 2 – 5) have been calculated similar way. On the basis of above results, characteristics of transient probabilities from state 1-st (standby) to other, both serviceability and unserviceability, states were calculated. The values of those probabilities stabilize after few days of work of the system. The transient probabilities functions to serviceability states are shown on Figure 3, to unserviceability states on Figure 4.

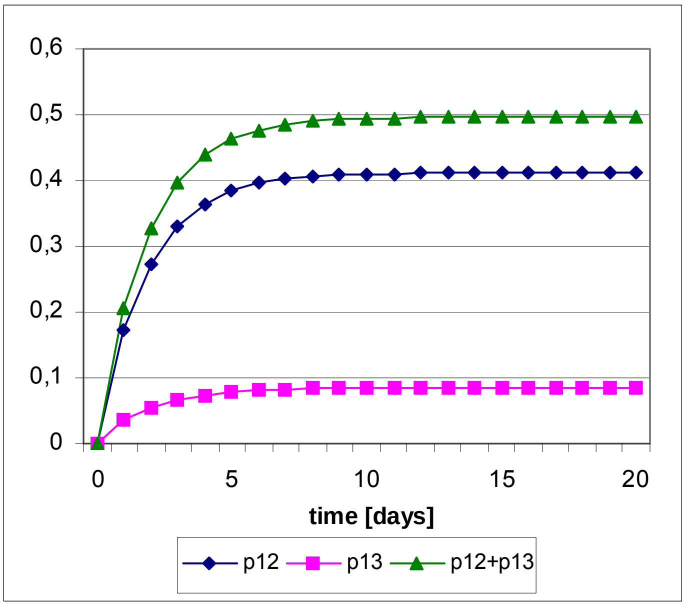

Figure 3. Graph of transient probabilities to serviceability states

For assumed conditions of phase of the system and distribution parameters, transient probability to serviceability states is: $p_{12} + p_{13} = 0.498$. Transient probabilities to unserviceability states achieve stable value for t = 4 days and don't change until t = 300. The calculation hasn't been done for greater values of t.

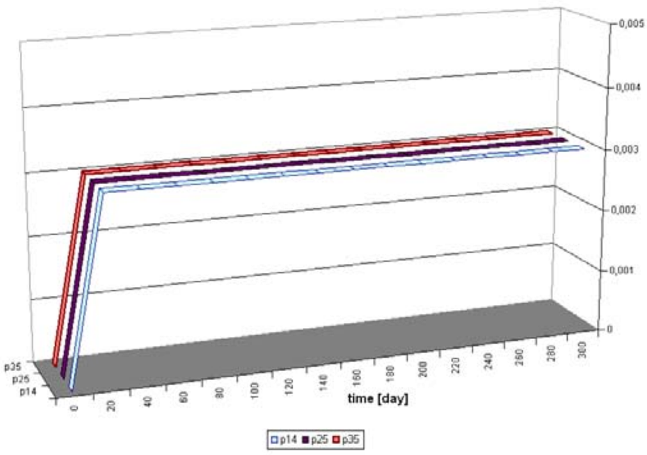

Figure 4. Graph of transient probabilities to unserviceability states

5 METHODS OF DETERMINING OF TRANSIENT PROBABILITIES FOR OTHER DISTRIBUTION FUNCTIONS

Gamma distribution is appropriate for describing age – hardening processes of technical object. There exists an assumption that sum of n independent random variables (with exponential distributions), with parameter λ, has two parameters gamma distribution (where α is shape parameter, and λ is scale parameter) (Jazwinski&Fiok 1990). Weibull distribution very often is used to object's durability modeling.

According to Table 2, collected data can be described by Weibull or gamma distributions. In the paper, for those distributions, only Laplace - Stieltjes transformation are presented. Sojourn times for Weibull distribution functions take form

$$F_{b1}(x) = 1 - e^{-0,269t^{1,02}},$$

$$F_{b2}(x) = 1 - e^{-0,26t^{1,01}},$$

$$F_{b3}(x) = 1 - e^{-2,66t^{1,014}},$$

$$F_{b4}(x) = 1 - e^{-1,538t^{0,985}},$$

$$F_{b5}(x) = 1 - e^{-0,833t^{1,021}}.$$

Derivative of Weibull distribution function (i.e. density function) is presented by

$$F'(t) = \lambda\alpha \cdot e^{-\lambda t^{\alpha}} \cdot t^{\alpha-1}$$

(16)

Laplace – Stieltjes transformations of Weibull distribution function can be obtained by using formula

$$f^*(t) = \int_0^\infty e^{-st} \cdot F'(t)\,dt = \int_0^\infty e^{-st}(1-e^{-\lambda t^{\alpha}})'\,dt = s\int_0^\infty e^{-st}(1-e^{-\lambda t^{\alpha}})\,dt - (1-e^{-\lambda \cdot 0^{\alpha}}) \tag{17}$$

Hence

$$f^*(t) = s\int_0^\infty e^{-st}(1-e^{-\lambda t^{\alpha}})\,dt = s\int_0^\infty e^{-st}dt - s\int_0^\infty e^{-st} \cdot e^{-\lambda t^{\alpha}}\,dt = 1 - s\int_0^\infty e^{-st} \cdot e^{-\lambda t^{\alpha}}\,dt \tag{18}$$

Using Maclaurin series for element "exp(-λt$^{\alpha}$)" we obtain Laplace – Stieltjes transformation of the Weibull distribution function

$$f^*(t) = \lambda\frac{\alpha \cdot \Gamma(\alpha)}{s^{\alpha}} - \frac{\lambda^2}{2!}\frac{2\alpha \cdot \Gamma(2\alpha)}{s^{2\alpha}} + \frac{\lambda^3}{3!}\frac{3\alpha \cdot \Gamma(3\alpha)}{s^{3\alpha}} - \ldots = \sum_{n=1}^{\infty}\frac{\lambda^n}{n!}\frac{n\alpha \cdot \Gamma(n\alpha)}{S^{n\alpha}}$$

(19)

For considered example, Weibull distribution Laplace – Stieltjes transformations take form, respectively

$$f_{b1}*(t)=\frac{0{,}2713}{s^{1,02}}-\frac{0{,}0774}{s^{2,04}}+\frac{0{,}0210}{s^{3,06}}-\frac{0{,}0059}{s^{4,08}}+\ldots$$

$$f_{b2}*(t)=\frac{0{,}2611}{s^{1,01}}-\frac{0{,}0689}{s^{2,02}}+\frac{0{,}0183}{s^{3,03}}-\frac{0{,}0049}{s^{4,04}}+\ldots$$

$$f_{b3}*(t)=\frac{2{,}6760}{s^{1,014}}-\frac{7{,}2619}{s^{2,028}}+\frac{19{,}845}{s^{3,042}}-\frac{54{,}489}{s^{4,056}}+\ldots$$

$$f_{b4}*(t)=\frac{1{,}5284}{s^{0,985}}-\frac{2{,}3013}{s^{1,97}}+\frac{3{,}4391}{s^{2,96}}-\frac{5{,}1139}{s^{3,94}}+\ldots$$

$$f_{b5}*(t)=\frac{0{,}8405}{s^{1,021}}-\frac{0{,}7216}{s^{2,042}}+\frac{0{,}6260}{s^{3,063}}-\frac{0{,}5468}{s^{4,084}}+\ldots$$

According to equation (7), after determining of reverse Laplace - Stieltjes transformation of elements of the matrix $\tilde{\mathbf{p}}(s)$, transient probabilities can be calculated.
Gamma distribution is given by formula

$$F(t)=\frac{\Gamma_{\lambda t}(\alpha)}{\Gamma(\alpha)}. \tag{20}$$

Density function of gamma distribution has form

$$F'(t)=\frac{\lambda^{\alpha}}{\Gamma(\alpha)}t^{\alpha-1}e^{-\lambda t}.$$

(21)
Laplace – Stieltjes transformation can be obtain by using formula

$$f*(t)=\int_0^\infty e^{-st}\cdot F'(t)\,dt=\int_0^\infty e^{-st}\frac{\lambda^{\alpha}}{\Gamma(\alpha)}t^{\alpha-1}e^{-\lambda t}dt=\frac{\lambda^{\alpha}}{\Gamma(\alpha)}\int_0^\infty e^{-(s+\lambda)t}t^{\alpha-1}dt \tag{22}$$

Taking into account equation

$$\int_0^\infty e^{-st}t^{a}dt=\frac{\Gamma(a+1)}{s^{a+1}}, \tag{23}$$

Laplace – Stieltjes transformation takes form

$$f*(t)=\frac{\lambda^{\alpha}}{\Gamma(\alpha)}\frac{\Gamma(\alpha)}{(s+\lambda)^{\alpha}}=\frac{\lambda^{\alpha}}{(s+\lambda)^{\alpha}}.$$

(24)

In this case sojourn times distribution functions and respective Laplace – Stieltjes transformations are as follows

$$F(t) = \frac{\Gamma_{33,21t}(9,26)}{\Gamma(9,26)},\ f^*(t) = \frac{33,21^{9,26}}{(s+33,21)^{9,26}},$$

$$F(t) = \frac{\Gamma_{531,46t}(152,06)}{\Gamma(152,06)},\ f^*(t) = \frac{531,46^{152,06}}{(s+531,46)^{152,06}},$$

$$F(t) = \frac{\Gamma_{665,31t}(181,99)}{\Gamma(181,99)},\ f^*(t) = \frac{665,31^{181,99}}{(s+665,31)^{181,99}},$$

$$F(t) = \frac{\Gamma_{9,13t}(13,97)}{\Gamma(13,97)},\ f^*(t) = \frac{9,13^{13,97}}{(s+9,13)^{13,97}},$$

$$F(t) = \frac{\Gamma_{4,88t}(4,12)}{\Gamma(4,12)},\ f^*(t) = \frac{4,88^{4,12}}{(s+4,88)^{4,12}}.$$

Using of equation (7) and calculating reverse Laplace - Stieltjes transformations transient probabilities can be obtained. In the case of gamma distribution, there are numerical problems with calculating of incomplete gamma functions values. Moreover, even values of gamma function for arguments larger than 50 cannot be easy obtained. Used software tools (SciLab 4.1.1 and Derive 6.1) don't allow for calculating such great values.

6 CONCLUSIONS

1. Semi - Markov processes allow for estimate basic reliability characteristics like availability or transient probabilities for systems, where distributions functions are discretional.
2. Usages of distribution functions other than exponential in case of semi Markov processes causes that further calculations are very complicated.
3. There is no easy available software which allow for calculations connected with semi – Markov processes. Because of that, profits from usage of semi – Markov processes are limited.
4. Lack of information about type of distribution and routine assessment of exponential distribution can bring not accurate assumptions and consequently false results.

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