DYNAMIC MODEL OF AIR APPARATUS PARK

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Abstract

The article is devoted to construction and research of dynamic stochastic model of park of aircrafts. A stochastic is enclosed in all of natural characteristic exploitations of this set of apparatuses: times of flight and landing, possibility of receipt of damage on flight, including the past recovery air apparatus; times of repair. The estimations of total possible flights are got for the any fixed interval of time.

Key Words Flight time, time on the ground, recoverable damage, loss of air apparatus, repair time, generating function, renewal equation.

1. INTRODUCTION

The important problem of management of the park of air apparatuses (PAA in short) maintenance, as stage of their life cycle, is an estimation of ability to provide the necessary amount of flights in given time interval of exploitation. The dynamics of exploitation of every apparatus consists of alternation of times of flight, times of repair and times of stand-down. These times are determined both external requests on flights and different damages during flight or loss of air apparatus (AA in short) on flight. Forecasting of the state of PAA is one of way of control of quality of management. This approach may be realized by modeling [1]. Analysis of literature in this direction shows that mainly authors develop of the models in a few lines. The authors of line [2-4] simulate of control of technical state of PAA with aim the optimization of preventive maintenance with respect to restoration of PAA parameters. The authors of next line [5-7] develop either methodological approach of operation adaptive control of technical state of PAA on basis of using of potential of corporative resources of unit information space (network-center environment) with purpose improving or support on the given level of reliable and durability indexes [5, 6] or task of definition of optimal type of PAA taking into account economical indexes. The authors of another line [8-10] build their own investigations on expert estimations. In this case, experience shows that decisions may be false. Therefore, it is urgency to develop models, which, first, consider of change of state of PAA by different manner.

Namely, which are grounded on the following probability indexes: probability of return of AA from flight without damages, probability of to receive certain damages of AA in flight, probability of to lose of AA in flight. Analysis of interaction of these indexes of random events is not simple process.

And so, it is actually, secondly, a development of such models that are based on analytical dependence with more complex mathematical filling.

In the article approaches are offered to the solution of the following task. We will designate through n_i the amount of AA able to fly up in some i-th moment of time. It is required to estimate of possibility to do given amount of flights Q in times of k successive time starts: j - th , $j+1$-th , $\cdots$, j-th starts. In other words, we must estimate possibility of implementation of relation $Q \le n_j + n_{j+1} + \cdots + n_{j+k}$ at any fixed integer j and k.

2. It is assumed that N units of AA, which are exploited from some initial moment of time. For definiteness we suppose that all (able to fly) AA fly up and land at the simultaneously.

Let us adopt the following notation.

We will denote by τ_k the flight time after k-th takeoff and by ξ_k the time on the ground after k-th landing. Thus moments of takeoffs $\{s_l\}$ are defined recurrently:

$$s_1 = 0, \quad s_2 = \tau_1 + \xi_1, \quad \dots \quad , s_l = \sum_{k=o}^{l} (\tau_k + \xi_k).$$

The moments of landing $\{t_l\}$ are defined analogy:

$$t_1 = \tau_1, \quad t_2 = \tau_1 + \xi_1 + \tau_2, \quad \dots \quad , t_l = \tau_l + \sum_{k=o}^{l-1} (\tau_k + \xi_k).$$

Further, we will consider the following probabilities as result of flight of every AA.

Let us denote by $p_i, i = 1,2$ the probabilities to obtain (in flight) eliminated damages;

by p_3 the probability of loss of AA in flight; by p_4 the probability to be safe and sound.

It is assume that $p_1 + p_2 + p_3 + p_4 = 1$.

We will use symbols β_l and α_l to denote the amount of AA at the $l-$ th takeoff ($l-$ th flight) and at the $l-$ th landing respectively.

The time of repair at the $i-$ th eliminated damage of the $k-th$ AA in the $l-th$ flight is equal to a random variables $d_i^{(k,l)}$, $i = 1,2;\ 1 \le k \le \beta_l;\ \ l \ge 1$ with the distribution functions

$$F_1(x) = P\left(d_1^{(k,l)} < x\right), \quad F_2(x) = P\left(d_2^{(k,l)} < x\right).$$

We will introduce sequences of independent events $A_i^{(k,l)}$ $i = 1,2,3,4;\ l \ge 1;\ 1 \le k \le \beta_l$.

These events are connected with aircraft events in flight so that the following equalities take place $P\left(A_i^{(k,l)}\right) = EI\left(A_i^{(k,l)}\right) = p_i$, here $I(\cdot)$ denotes the indicator of events.

In what follows, we shall be assuming that random variables form ensemble

$$\Pi := \left\{\tau_i,\, \xi_i,\, d_1^{(k,l)},\, d_2^{(k,l)},\, I\left(A_i^{(k,l)}\right),\, i,k,l \ge 1\right\}$$

are independent in common.

Put

$$s_2^l = \sum_{k=2}^{l} (\tau_k + \xi_k) \quad , \quad r_1 = E\sum_{i=1}^{2} I\left(A_i^{(k,1)}\right) I\left(d_i^{(k,1)} \in [0\,,\, \xi_1)\right)$$

$$r_l = E\sum_{i=1}^{2} I\left(A_i^{(k,l)}\right) I\left(d_i^{(k,l)} \in [\xi_1 + s_2^{l-1}\,,\, \xi_1 + s_2^l)\right), \quad l \ge 2.$$

here and in the sequel, we assume that $s_2^l = 0$, if $l < 2$.

By hypothesis on independence

$$r_1 = \sum_{i=1}^{2} p_i P\left(d_i^{(1,1)} < \xi_1\right);\ r_l = \sum_{i=1}^{2} p_i r_{li} \quad , \quad l \ge 2\,,$$

where $r_{1i} = P\left(d_i^{(1,1)} < \xi_1\right)$, $r_{li} = P\left(d_i^{(1,1)} \in [\xi_1 + s_2^{l-1}\,,\, \xi_1 + s_2^l)\right)$, $l \ge 2$.

We introduce the generating functions

$$B(s) = \sum_{m=1}^{\infty} s^m b_m \text{ , where } \quad b_m = E\beta_m . \quad R(s) = \sum_{m=1}^{\infty} s^m r_m \,, \quad s \in [0,1]\,.$$

Theorem 1. *The following formulas take place*

$$B(s) = \frac{sN}{1 - sp_4 - sR(s)}\,. \tag{1}$$

Proof. We shall establish the stochastic relations for sequences β_m, α_m $m \ge 1$. The designation $\omega \overset{w}{=} \zeta$ means that random variables ω and ζ have the same distribution function.
We will denote by $\overline{A}$ the complement of a set A.

$$\beta_1 = N, \qquad \alpha_1 \overset{w}{=} \sum_{k=1}^{N} I\left(\overline{A_3^{(k,1)}}\right).$$

$$\beta_2 \overset{w}{=} \sum_{k=1}^{\beta 1} I\left(A_4^{(k,1)}\right) + \sum_{k=1}^{\beta 1}\sum_{i=1}^{2} I\left(A_i^{(k,1)}\right) I\left(d_i^{(k,1)} < \xi_1\right), \quad \alpha_2 \overset{w}{=} \sum_{k=1}^{\beta 2} I\left(\overline{A_3^{(k,2)}}\right).$$

$$\vdots$$

$$\beta_m \overset{w}{=} \sum_{k=1}^{\beta m-1} I\left(A_4^{(k,m-1)}\right) + \gamma_m, \quad \alpha_m \overset{w}{=} \sum_{k=1}^{\beta m} I\left(\overline{A_3^{(k,m)}}\right),$$

where $$\gamma_m \overset{w}{=} \sum_{l=1}^{m-1}\sum_{k=1}^{\beta l}\sum_{i=1}^{2} I\left(A_i^{(k,l)}\right) I\left(d_i^{(k,l)} \in [s_{m-1} - t_l \vee 0, s_m - t_l)\right).$$

The random value γ_m is equal to amount of AA, which finished the repairs in the interval of time between $m-1-th$ and $m-th$ takeoffs.

By the construction of β_m, we have the following relations

$$b_1 = N, \quad b_m = p_4 b_{m-1} + \sum_{l=1}^{m-1} b_l\, r_{m-l}, \quad m \geq 2. \tag{2}$$

We introduce the functions

$$B(s) = \sum_{m=1}^{\infty} s^m b_m, \qquad R(s) = \sum_{m=1}^{\infty} s^m r_m, \quad s \in [0,1].$$

From the (1) we obtain

$$s^m b_m = s^m p_4 b_{m-1} + s^m \sum_{l=1}^{m-1} b_l\, r_{m-l}, \quad m \geq 2. \tag{3}$$

Summarizing left and right parts of (3) yields

$$B(s) - sN = s\, p_4\, B(s) + sB(s)R(s).$$

From the latter one we get

$$B(s) = \frac{sN}{1 - s p_4 - sR(s)}.$$

Proof is completed.

Corollary . *Assume that the sequences from* Π *satisfy the conditions*

$\lim_{n\to\infty} P\left(d_i^{(1,1)} < \xi_1 + \sum_{k=2}^{n} (\tau_k + \xi_k) \right) = 1,\ i = 1,2$. *Then the following equality is valid*

$$\sum_{m\ge 1} b_m = \frac{N}{p_3} \qquad (4).$$

Proof . Since, random variables from Π are independent, we have that

$$R_n := \sum_{l=1}^{n} r_l = \sum_{i=1}^{2} p_i P\left(d_i^{(1,1)} < \xi_1 + s_2^n\right). \qquad (5)$$

Combining (1), (5) and condition from the Corollary, we get

$$R(1) = \lim_{n\to\infty} R_n = \sum_{i=1}^{2} p_i\,, \quad B(1) = \frac{N}{1 - p_4 - p_1 - p_2} = \frac{N}{p_3}\,.$$

The proof is completed.

2.

We shall formulate the problems of estimations of b_m in terms of theory of renewal processes.

Let us denote by $\{\kappa_i \in \{1,2,\ldots\}, i \ge 1\}$ the sequence of independent discrete random values with common distribution law $\delta_1 = P(\kappa_1 = 1) = p_4 + r_1$, $\delta_l = P(\kappa_1 = l) = r_l$, $l \ge 2$.

It is well known that if $S_k = \sum_{i=1}^{k} \kappa_i$ and $\eta(m) = \min\{k : S_k \ge m\}$, then $E\eta(m), m \ge 1$ is unique solution of the renewal equation $E\eta(m) = \sum_{l=1}^{m-1} \delta_l E\eta(m-l)$.

Comparing latter one and (2), we conclude that $b_m = E\eta(m)$.

Let $G(m) = P(\kappa_1 \le m)$ and $h(m,M) = \sum_{i=m}^{m+M} b_i$.

Now, we obtain the following upper estimation

$$h(m,M) = \sum_{n=1}^{\infty} \sum_{i=0}^{M} P(S_n \le m+i) = \sum_{i=0}^{M} \sum_{n=1}^{m+i} P(S_n \le m+i) \le \sum_{i=0}^{M} \sum_{n=1}^{m+i} G^n(m+i) \qquad (6)$$

Since $G(m) = p_4 + \sum_{i=1}^{2} p_i P\left(d_i^{(1,1)} < \xi_1 + s_2^m\right),\ m \geq 1$, the estimation (6) is well calculated.

3.

Now we will consider the construction of $B(s)$ more detail for special case. We make the following additional assumptions:

– τ_k, $k \geq 1$ have the same distribution function with Laplace transformation $\psi(s) = E\exp\{-s\tau_1\}$, $s > 0$.

– ξ_k, $k \geq 1$ have the same distribution function with Laplace transformation $\varphi(s) = E\exp\{-s\xi_1\}$, $s > 0$.

– $F_i(x) = 1 - \exp(-\lambda_i x)$, $i = 1,2$.

For convenience we put $f(s) = \psi(s)\varphi(s)$.

Now we shall obtain more exact expression for $R(s)$.

By induction, we shall calculate the r_{li} for $l = 1,2,\ldots$

$$r_{li} = \int_0^\infty P(d_i^{(1,1)} < x)P(\xi_1 \in dx) = \int_0^\infty (1 - \exp(-\lambda_i x))P(\xi_1 \in dx) = 1 - \varphi(\lambda_i);$$

$$r_{2i} = \int_0^\infty \int_0^\infty (P(d_i^{(1,1)} < x + y) - P(d_i^{(1,1)} < x))P(\xi_1 \in dx)P(\tau_2 + \xi_2 \in dy) = \varphi(\lambda_i) - \varphi(\lambda_i)f(\lambda_i);$$

$$r_{3i} = \int_0^\infty \int_0^\infty \int_0^\infty (P(d_i^{(1,1)} < x + y + z) - P(d_i^{(1,1)} < x + y))P(\xi_1 \in dx)P(s_2^{l-1} \in dy)P(\tau_l + \xi_l \in dz) =$$

$$= \varphi(\lambda_i)f(\lambda_i) - \varphi(\lambda_i)f^2(\lambda_i) = \varphi(\lambda_i)f(\lambda_i)(1 - f(\lambda_i));$$

$$\vdots$$

$$r_{li} = \varphi(\lambda_i)f^{l-2}(\lambda_i)(1 - f(\lambda_i)), \qquad l \geq 4.$$

After routine calculations we get

$$R(s)=\sum_{i=1}^{2}p_i\sum_{m=1}^{\infty}s^m\, r_{mi}=\sum_{i=1}^{2}p_i\left\{s(1-\varphi(\lambda_i))+\frac{s^2\varphi(\lambda_i)(1-f(\lambda_i))}{1-s\,f(\lambda_i)}\right\}.$$

Thus, we have the following expression for this case

$$B(s)=N\frac{s-(f_1+f_2)s^2+f_1f_2s^3}{1-a_1s+a_2s^2-a_3s^3}\ , \tag{7}$$

where for convenience, we introduced notation $f_i := f(\lambda_i),\quad \varphi_i := \varphi(\lambda_i)$,

$$a_1 := p_4(1+f_1+f_2)+\sum_{i=1}^{2}p_i(1-\varphi_i),$$

$$a_2 := f_1f_2+p_4(f_1+f_2)-\sum_{i=1}^{2}p_i(\varphi_i-f_i-f_{3-i}(1-\varphi_i)),$$

$$a_3 := p_4f_1f_2-\sum_{i=1}^{2}p_i(\varphi_i-f_i)f_{3-i}\ .$$

Thus, in this case the term b_m poses no problem because expression (7) can be expanded into the convergent power series about s.

Further, it is easy to check that under such special assumptions the function $G(m)$ from Section 2 has the following form

$$G(m)=p_4+p_1+p_2-\sum_{i=1}^{2}p_i\varphi^m(\lambda_i)\psi^{m-1}(\lambda_i)\ .$$

Remark. It is clear, that restriction on number of different types of eliminated damages (only two) is not essentially. The proved formulas are transformed for more number of types easy.

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