SOME INFERENCES ON THE RATIO AVERAGE LIFETIME/TESTING TIME IN ACCEPTANCE SAMPLING PLANS FOR RELIABILITY INSPECTION

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Abstract

In this paper we construct effective single sampling plans for reliability inspection, when the distribution of failure times of underlying objects obey a Weibull law. To this purpose we use the index average lifetime (E (T)/testing time (T) for two values of E(T) - acceptable and non acceptable ones - and known shape parameter (K) of the Weibull cdf. We derive also a relationship between this index and reliability function R(t) of the assumed statistical law. A numerical illustrations is provided in the case of Rayleigh cdf - that is for a Weibull shape k = 2.

Key words: cdf - cumulative distribution function, two-parameter Weibull law, sampling plan, average lifetime, testing time, Rayleigh case.

1. Introduction

From the SQC (Statistical Quality Control) perspective, reliability is considered a dynamical quality characteristic since the performances of a given technical entity are put into light if the underlying element, component or system is in a functional / operational state, performing a prescribed mission for a specified period of time.

Static quality characteristics (such as hardness, length, pressure, volume a.s.o.) are observed and measured directly in units independent of time.

Metaphorically speaking, reliability is viewed as one of the special features of the general concept of quality. Vasiliu (1980, [7, page 26]) defines reliability as „the dimension in time of quality". Two decades later, Yang and Kapur (1997, [9, page 340]) state that „reliability is quality over time".

Anyway, no matter how good is a design, how performing is the production process, how careful is handled and exploited a technical system there is no way to stop its final decay. After a certain period of time - which may be short or quite long - every human made object sooner or later will fail. This event (failure) is due to natural causes (wear-out phenomenon) or to some „artificial" ones as for instance the use of the item in inappropriate conditions (aggressive environment, intensive operational tasks, lack of adequate maintenance actions, mishandling etc.).

A failure occurs in a random manner and usually after a certain period of time when the system was operating supposedly, satisfactorily.

Since we do not know the exact moment when a specified object will fail, we are forced to judge in terms of probabilities and averages involving the time elements as one of the main parameters. The failure behavior of that specific object has to be modeled and hence we are facing to the problem of choosing the most suitable class of life distributions describing this time-to-failure phenomenon.

Nevertheless, we may speak about the so-called „static reliability" where the time element is not instantly (or explicitly) involved (see Blischke and Murthy, 2000 [2, page 173 - 177]).

We refer here to the so-called „stress-strength models" where reliability is regarded as the capacity of item's strength (x) to resist to the action of stress (y). Actually a measure of reliability in this model is $R = \text{Prob}\ \{x > y\}$ where usually, both x and y are random variables. If this probability is greater than 50% we could expect a desirable reliability of the underlying entity.

In batch inspection procedures if the characteristic of interest is reliability (or durability) of underlying items we must take into account their failure behavior (where time element is the main parameter) in order to construct suitable sampling acceptance plans from economical point of view.

In this paper, we shall present some new results on the index average lifetime (durability)/testing time in the construction of acceptance sampling plans for reliability inspection, when time-to-failure distribution is a two parameter Weibull one.

2. Various approaches of reliability inspection

It is well-known that a very general approach for batch inspection - no matter the nature of quality characteristic investigated - is that called **attributive** one. All practical procedures have been already standardized - see the document MILSTD105 E „Sampling procedures and tables for inspection by attributes" (see Kirkpatrick, 1970 [5, page 354 - 415] where the variant D is entirely reproduced). The simplicity of attributive method lies in the fact that products are classified into categories: conforming and defective (nonconforming) ones regarding some specified criteria. In the case of reliability/durability inspection, this attributive approach ignores the very nature of failure behavior of inspected objects and this could lead to a larger sample (or samples) to be tested: if the items are quite expensive and since the specific test in this case is destructive, the procedure appears to be non-economic.

It is important to notice that the attributive approach ignores in the case of reliability/durability inspection, the following elements: a) what kind of samples we use for inspection: complete ones or censored ones; b) distributional assumption for time-to-failure; c) sampling is with replacement or non replacement; d) testing conditions are normal or accelerated ones; e) items are reparable or non-reparable (if they are non restoring, then E(T) is just the mean durability and $\overline{T}$ (sample mean) is computed with $(t_i)_{1 \le i \le n}$ values where t_i is the time to first - and last! - failure of the $i^{\underline{th}}$ item submitted to the test; it is senseless to speak in this case about MTBF - Mean Time Between Failures); f) what is the relationship between testing time (T_0) and the actual operating life of those items.

More useful are in such special case methods based on average operating time or on hazard rate associated to the failure time model specific for each peculiar instance.

The document MILSTD 781 Reliability test: exponential distribution (U.S. Dept. of Defense, Washington D. C., 1984) use the ratio E (T) / T_0 where E (T) is the average lifetime (durability) of underlying objects and T_0 is the testing time. In the exponential case $F(t;\theta) = 1 - \exp(-t/\theta), \quad t \geq 0, \quad \theta > 0$, F being the cdf (cumulative distribution function) of the representative variable (T), the mean-value of T is $E(T) = \theta$ and therefore the inference is done straight forwardly on the distributional parameter (details are given in Cătuneanu-Mihalache, 1989 [3], Vodă-Isaic Maniu [8]).

We shall examine now this ratio E (T) / T_0 in the case of a Weibull distribution.

3. The ratio E (T) / T_0 in the case of a Weibull distribution

Let T_w be a two-parameter Weibull distribution with the following cdf

$$T_w : F(t;\theta,k) = 1 - \exp(-t^k/\theta), \quad t \geq 0, \quad \theta, k > 0 \tag{1}$$

The corresponding reliability function is $R_t = \exp(-t^k/\theta)$ and the theoretical mean-value is

$$E(T) = \theta^{1/k} \cdot \Gamma(1 + 1/k) \text{ where } \Gamma(x) = \int_0^\infty u^{x-1} e^{-u} du \tag{2}$$

is the well-known Gamma function (see Isaic-Maniu, 1983 [6, page 21]). We have hence

$$\theta = \left[\frac{E(T)}{\Gamma(1+1/k)} \right]^k \tag{3}$$

and consequently we get

$$R(t) = \exp\left\{ - \left[\frac{t \cdot \Gamma(1+1/k)}{E(T)} \right]^k \right\} \tag{4}$$

By taking natural logarithms, we have

$$\ln R(t) = -\left[\frac{t \cdot \Gamma(1+1/k)}{E(T)} \right]^k \tag{5}$$

and finally

$$\frac{E(T)}{t} = [-\ln R(t)]^{-1/k} \cdot \Gamma(1+1/k) \tag{6}$$

Therefore, the ratio E(T)/t depends on the shape parameter (k) of Weibull's cdf and on its reliability function. If we fix t = T_0 and considering k to be known, we have either to estimate R (T_0) or to fix lower acceptable bound for it.

From (5) we can deduce

$$\ln E(T) = -\frac{1}{k} \cdot \ln[-\ln R(T_0)] + \ln\Gamma(1+1/k) + \ln T_0 \quad (7)$$

and taking into account a formula given in Abramowitz-Stegun (1979, [1, page 82]), namely

$$\ln\Gamma(1+1/k) \approx -\ln(1+1/k) + \frac{1-C}{k} \quad (8)$$

where C is the Euler-Mascheroni's constant (≈ 0.57721).

If we approximate now ln (1+1/k) as 1/k (let us recall the inequalities $x(1+x)^{-1} \le \ln(1+x) \le x, \quad 0 \le x < 1$) then, if k > 1 the relationship (7) becomes a very simple estimation equation for the shape parameter if it is not known.

4. Construction of acceptance sampling plans

We shall start with the following assumptions:

(1) the items subjected to inspection are non-reparable;

(2) the failure time distribution is a two-parameter Weibull one with known shape parameter;

(3) we use only one sample with no replacement, its size has to be determined;

(4) there is fixed an acceptable average lifetime $[E(T)]_1$ corresponding to a given risk α (usually $\alpha = 0.05$ or 5%), that is we wish to accept a lot with such average value with $1-\alpha = 0.95$ probability;

(5) there is fixed a non-acceptable average lifetime $[E(T)]_2$ corresponding to a given risk β (usually $\beta = 0.10$ or 10%), that is we with to reject a lot with such average value with $1-\beta = 0.90$ probability;

(6) there is fixed a testing time T_0 smaller than the actual operating life of the underling items.

Therefore, the sampling plan will be the system of objects {(n, A) | T_0} where n and A are respectively the sample size and acceptance number which has to be determined and T_0 is the previously fixed testing time.

The decision on the lot is taken as follows: submit to the specific reliability/durability test a sample of size n drawn randomly from a lot of size N (n < N), during a period of units of T_0 (usually, T_0 is given in hours); record then the number (d) of failed elements in the interval [0, T_0]; if $d \le A$, then the lot is accepted - otherwise, that is if $d \ge R = A+1$, the lot is rejected (here, R = A + 1 is the so-called „rejection number").

The elements n and A are determined via the OC - function (Operative Characteristic) of the plan which has the expression

$$L(p) = \sum_{d=0}^{A} \frac{1}{d!}(np)^d e^{-np} \quad (9)$$

where $d! = 1 \cdot 2 \cdot \ldots \cdot A$ and p is the defective fraction of the lot given by

$$p = 1 - \exp\left(-t^k/\theta\right), \quad t \ge 0, \quad \theta, k > 0 \quad (10)$$

and d is the number of failed elements during the testing period T_0 (see for other details US-MIL-HDBK-781 „Reliability Test Methods, Plans and Environments for Engineering Development. Qualification and Production” and Grant and Leavenworth, 1988 [4]).

Choosing two values for p (p_1 and p_2) for which $L(p_1) = 1 - \alpha = 0.95$ and $L(p_2) = 1 - \beta = 0.10$ and using the ratios $[E(T)]_1/T_0$ and $[E(T)]_2/T_0$ we obtain a system which provides the elements of the plan, n and A.

In table 1 we present some values for n and A in the Rayleigh case, that is if k = 2, the input data being (in order to ease the computations) the following quantities: $100T_0/[E(T)]_1$ for which $L(p_1) = 0.95$ and $100T_0/[E(T)]_2$ for which $L(p_2) = 0.10$ (the first figure is given in parentheses).

We do notice that in this approach it is avoided the knowledge of R (T_0) since the input elements are only T_0 and $[E(T)]_{1,2}$ which are fixed previously taking into account the specific case at hand.

Table 1

Elements of the single sampling plan {(n, A) | given T_0} for the input ratios $100T_0$ / $[E(T)]_{1,2}$

A	n Values of $100T_0/[E(T)]_2$ for which $L(p_2) = 0.10$			
	100	50	25	15
0	3 (15)	12 (7.5)	46 (3.8)	130 (2.2)
1	6 (30)	21 (15)	80 (7.5)	224 (4.5)
2	8 (40)	30 (19)	110 (9.9)	305 (5.9)
3	11 (42)	35 (22)	139 (11)	383 (6.8)

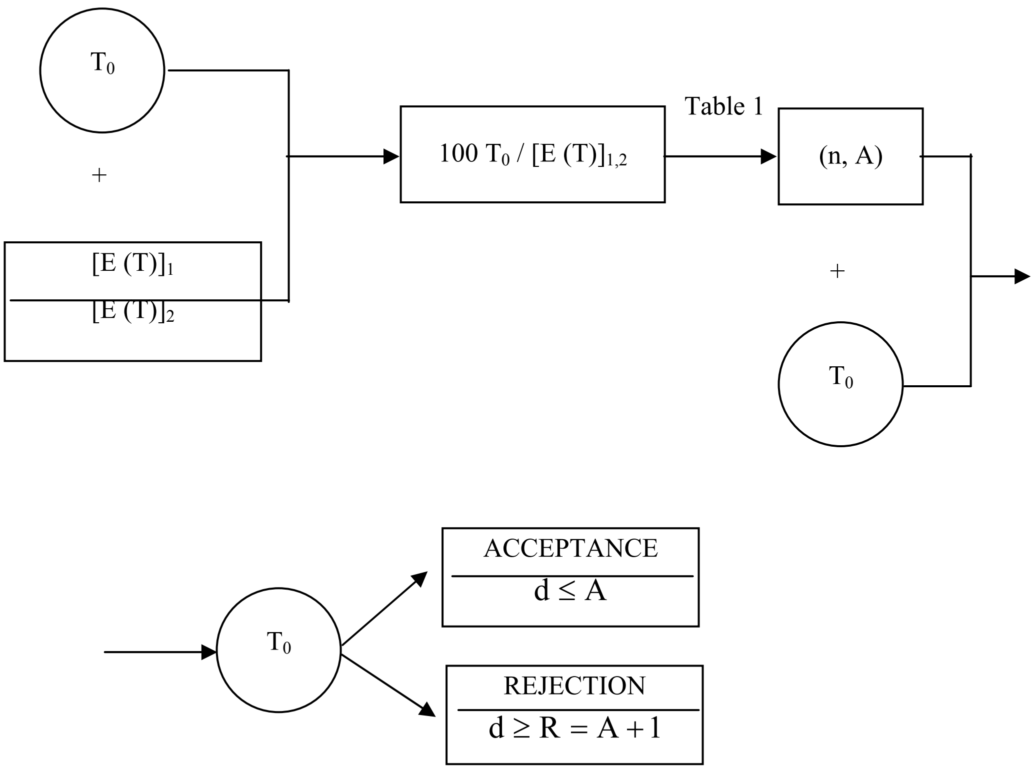
Example: Assume that we have an acceptable durability $[E(T)]_1$ = 5000 hours and a non-acceptable one as $[E(T)]_2$ = 1000 hours. Testing time was fixed at the value T_0 = 500 hours (the usual risks are $\alpha = 0.05$ and $\beta = 0.10$).

Therefore, to find the plan, we evaluate

$$\frac{100\,T_0}{[E(T)]_2} = \frac{100 \cdot 500}{1000} = 50 \text{ and } \frac{100\,T_0}{[E(T)]_1} = \frac{100 \cdot 500}{5000} = 10$$

In table 1, the nearest value of $100T_0 / [E(T)]_1$ for $100T_0 / [E(T)]_2 = 50$ is 15 and hence for the couple 50 (15) we read n = 21 (sample units) and A = 1 (the acceptance number). The plan is hence {(21, 1) | 500} and as a consequence we shall test n = 21 items on a period of 500 hours and record d - the number of failed elements. If d = 0 or 1, we shall accept the lot - otherwise (that is $d \geq 2$) we shall reject it.

The flow of operations is presented below.



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