FAILURE FREQUENCY CALCULATION TECHNIQUE IN LOGICAL-PROBABILISTIC MODELS

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The technique for calculation failure frequency measure of reliability in class of logical-probabilistic-models is proposed. The technique is applicable for models of redundant repairable systems which are not limited by serial-parallel structures. In conjunction with system decomposition the techniques makes it possible to analyze high dimensional systems very efficiently.

1. INTRODUCTION

Most of the papers regarding algorithmization of reliability&safety characterization in class of logical-probabilistic-models are devoted to truth probability estimation of some logical function defined on Boolean variables (elements of the analyzed system). It should be mentioned that in logical-probabilistic-models one can calculates only so-called differential measures, that is measures of some state or transition **in given moment of time, for example, system availability (unavailability).** In this paper we examine a system of repairable elements with given failure and repair time distribution and specify its availability and failure frequency (both stationary and not stationary). Using availability and failure frequency it is possible to calculate other reliability indices of the system.

Failure frequency is important reliability measure. It characterizes system transitions in the space of states, for example transition from good state to failed state. It is necessary to calculate this index when doing effectiveness, safety, risk analysis. Failure frequency is defined as time derivative of average number of failures. Therefore average number of failures (in the general case average number of transitions) one can calculate via integration failure frequency (transitions frequency) in given time interval. Failure frequency and number of failures are the main measures when calculating cost (loss) per unit time. Mean system cost on time interval $(0,t)$ (mean effectiveness $(0,t)$) in systems with multilevel performance is defined as:

$$E(0,t) = \sum_i \int_0^t \Pr_i(t)h_i dt + \sum_{i,j} \int_0^t \omega_{i,j}(t)h_{i,j} dt ,$$

where $\Pr_i(t)$ – system state i probability in time point t;

$\omega_{i,j}$ – frequency of transitions from the *ith* state to the *jth* state;

h_i – reward (gain or loss) per unit time associated with state i;

$h_{i,j}$ – nonrecurring gain or loss per transition from the *ith* state to the *jth* state.

The first integral presents average holding time in each system states on time interval $(0,t)$ multiplied by reward per unit. The second integral presents average number of transitions weighted by nonrecurring reward. So, if some faults bring to the damage of adjoining equipment or processed part, then with the help of failure frequency we can estimate total loss. Known, traditional estimation of average effectiveness $E(t)$ in time moment t gives too optimistic result:

$$E(t) = \sum_i \Pr_i(t)E_i(t) ,$$

$E_i(t)$ – effectiveness in state i (particularly, $E_i(t)$ can be equal to h_i).

It should be mentioned that using failure frequency one can calculate interval reliability index for repairable systems, which is not directly calculated in logical-probabilictic models.

Known method of failure frequency calculation, which is based on formula of joint event union, leads to considerable timetable and even to inability of obtaining accurate estimate at high dimensionality because of enumeration type of the algorithm.

In this paper we suggests less time-consuming method of calculation failure frequency for the high dimensional systems.

2. PROBLEM DESCRIPTION

Let elements of a system x_i, $i=\overline{1,n}$ and system $S(\mathbf{x})$, $\mathbf{x}=\{x_i\}$ can be in two state – good and failed

$$x_i = \begin{cases} 1, & \text{when element } i \text{ is good} \\ 0, & \text{when element } i \text{ is failed} \end{cases} \tag{1}$$

$$S(\mathbf{x}) = \begin{cases} 1, & \text{when system is good} \\ 0, & \text{when system is failed} \end{cases} \tag{2}$$

Let system state exhaustively defined by state of its elements in time point t. Denote minimal path sets of the system by $\mathbf{A}=\{A_j\}$, minimal cut sets by $\mathbf{C}=\{C_j\}$.

Then systems availability in time point t can be defined as

$$S(\mathbf{x},t) = \left\{ \bigvee_{j=1}^{r} A_j \right\} = 1, \tag{3}$$

and unavailability

$$\overline{S}(\mathbf{x},t) = \left\{ \bigvee_{j=1}^{l} C_j \right\} = 1. \tag{4}$$

Every minimal path (cut) corresponds to conjunction of some numbers of good (failure) elements $\mathbf{x}=\{x_i\}$.

Availability (unavailability) of a system is defined as:

$$P\{S(\mathbf{x},t) = 1\} = P\{\overline{S}(\mathbf{x},t) = 0\} = P\{\bigvee_{j=1}^{r} A_j = 1\} = 1 - P\{\bigvee_{j=1}^{l} C_j = 1\}, \tag{5}$$

$$P\{S(\mathbf{x},t)=0\} = P\{\overline{S}(\mathbf{x},t) = 1\} = P\{\bigvee_{j=1}^{l} C_j = 1\} = 1 - P\{\bigvee_{j=1}^{r} A_j = 1\}, \tag{6}$$

where $P\{.\}$ – occurrence probability of events in brackets in time point t.

Numerous methods and algorithms were designed for calculating availability (unavailability) indexes. The main purpose of these works was to increase efficiency of transformation of logical expressions (3) and /or (4) for obtaining probability (5) and /or (6). The problem lays in exponential growth of computational complexity in the system dimension increase (number of elements, number of minimal cut or path sets). Thus, in calculating unavailability by (6), using formula of joint event union, we obtain the following expression

$$Q(t) = \sum_{i_1=1}^{l} P\{C_{i_1}\} - \sum_{i_1=1}^{l-1} \sum_{i_2>i_1}^{l} P\{C_{i_1} \wedge C_{i_2}\} + \sum_{i_1=1}^{l-2} \sum_{i_2>i_1}^{l-1} \sum_{i_3>i_2}^{l} P\{C_{i_1} \wedge C_{i_2} \wedge C_{i_3}\} - \dots +$$

$$+ (-1)^{l-1} P\{C_1 \wedge C_2 \wedge \dots \wedge C_l\}. \tag{7}$$

Number of terms in right side of equation (7) will be $2^l - 1$. Besides generation software algorithm for crossing symbol subsets of paths (cuts) is complex task. For instance, famous test example of naval electrical power system, known as «I.A. Ryabinin task 35» [10], has 15 elements, 31 minimal cut sets and 92 minimal path sets ($2^{31} > 2 \cdot 10^9$). Software, implementing this

method for reliability indexes calculation (e.g., Risk Spectrum), in large dimension makes only approximate calculus, which gives rough estimate for system of elements with not a high reliability. It should be mentioned that in [1-3,5,6,8,9] were suggested quite effective calculation methods for availability (unavailability) in (5), (6) interpretation.

Failure frequency is expected number of failures in given moment of time t (i. . in $(t, t+ t)$ when t 0). This implies appearance of at least one cut set in time moment $t+ t$. Let e_i – is occurrence event of ith cut set in $(t, t+ t)$, where $e_i(t+ t)$ – conjunction of n_i variables (elements), forming $_i$ cut set. At ordinary failure flow assumption appearance e_i in t means, that in moment t $(n_i – 1)$ elements of $_i$ were failing and then in t one good element failed. Denote this event as $e_i^{'}$. Using formula of total probability we can define occurrence probability of event e_i in t:

$$P\{e_i\} = {}_i^*(t)\Delta t = \sum_{j_i=1}^{n_i}[{}_{j_i}(t)\prod_{g_i \neq j_i}^{n_i} Q_{g_i}(t)]\Delta t , \qquad (8)$$

where ${}_{j_i}(t), Q_{g_i}(t)$ - failure frequency and unavailability of element j_i, g_i in time moment t;

${}_i^*(t)$ – failure frequency, conditioned of appearance C_i cut set.

Well known method [7] of calculation failure frequency ${}_S$ also is based on formula (7):

$$_S\Delta t = P\{(S(\mathbf{x},t) = 1) \wedge (\bigcup_{i=1}^{l} e_i)\} = P\{\bigcup_{i=1}^{l} e_i\} - P\{(S(\mathbf{x},t) = 0) \wedge (\bigcup_{i=1}^{l} e_i)\} = ({}_{S1} - {}_{S2})\Delta t \qquad (9)$$

$$_{S1}\Delta t = \sum_{i_1=1}^{l} P\{e_{i_1}\} - \sum_{i_1=1}^{l-1}\sum_{i_2>i_1}^{l} P\{e_{i_1}\bigcap e_{i_2}\} + \sum_{i_1=1}^{l-2}\sum_{i_2>i_1}^{l-1}\sum_{i_3>i_2}^{l} P\{e_{i_1}\bigcap e_{i_2}\bigcap e_{i_3}\} - \ldots$$

$$+ (-1)^{l-1} P\{e_1\bigcap e_2\bigcap\ldots\bigcap e_l\},$$

(10)

$$_{S2}\Delta t = \sum_{i=1}^{l}\Big[\sum_{j\neq i}^{l} P\{C_j \wedge e_i\} - \sum_{\substack{j_1=1 \\ j_1 \neq i}}^{l-1}\sum_{j_2=j_1+`1}^{l} P\{C_{j_1} \wedge C_{j_2} \wedge e_i\} + \ldots +$$

$$(-1)^{l-2} P\{C_1 \wedge\ldots\wedge C_{i-1} \wedge C_{i+1} \wedge\ldots\wedge C_l \wedge e_i\}\Big] -$$

$$- \sum_{i_1=1}^{l-1}\sum_{i_2=i_1+1}^{l}\Big[\sum_{j\neq i_1,i_2}^{l} P\{C_j \wedge (e_{i_1}\bigcap e_{i_2})\} - \sum_{\substack{j_1=1 \\ j_1,j_2\neq i_1,i_2}}^{l-1}\sum_{j_2=j_1+1}^{l} P\{C_{j_1} \wedge C_{j_2} \wedge (e_{i_1}\bigcap e_{i_2})\} + \ldots + \qquad (11)$$

$$(-1)^{l-3} P\{C_1 \wedge C_2 \wedge\ldots\wedge C_{i_1-1} \wedge C_{i_1+1} \wedge\ldots\wedge C_{i_2-1} \wedge C_{i_2+1} \wedge\ldots\wedge C_l \wedge (e_{i_1}\bigcap e_{i_2})\}\Big] +$$

$$\ldots + \sum_{j=1}^{l}(-1)^{l-1} P\{C_j \wedge (e_1\bigcap\ldots\bigcap e_{j-1}\bigcap e_{j+1}\bigcap\ldots\bigcap e_l)\}.$$

Event $\{C_{j_1} \wedge C_{j_2} \wedge (e_{i_1}\bigcap e_{i_2})\}$ implies, that in time point t the system was failed because of realization of two cut sets C_{j_1}, C_{j_2} and during t general elements for cut sets C_{i_1}, C_{i_2} have failed (i. . in $(t, t+ t)$ cut sets i_1 and i_2 have occurred). If there is no such element, then probability of occurrence just two or more cut sets during t is equal to zero. General term is

$$P\{C_{j_1} \wedge C_{j_2} \wedge\ldots\wedge\ldots\wedge C_{j_U} \wedge (e_{i_1}\bigcap e_{i_2}\bigcap\ldots\bigcap e_{i_G})\} = {}_{GU}(t)\Delta t\prod_{1\ldots U}^{1\ldots G} Q(t), \qquad (12)$$

where ${}_{GU}(t)$ - frequency of general elements group entering cut set G and not entering U from the others $(l – G)$; $\prod_{1\ldots U}^{1\ldots G} Q(t)$ - product of unavailability of all elements entering G and U cut sets from other $(l – G)$ except those elements, which are used in calculation of ${}_{GU}(t)$ (${}_{GU}(t)$ are calculated similarly to (5), but with regard to group of general elements entering G cut sets). Every

element in the product is included only once. Failure frequency calculation by (9) - (11) – is more laborious task (about three times), than availability (unavailability) calculation by (7).

Advantageous process of the high dimension problem solving is decomposition of structure or logical representation of the system. At structural decomposition one can appropriate:

1.   singly connected decomposition, when appropriated subsystems (assemblies, modules, …) connect each other only through two nodes, and at that one node is input, the other node is output, i. . this is series connection of the subsystems. Each of the subsystem can correspond to redundancy structure with some logical function (generally *k* out of *m*) in output node (element). In this process we avoid complicated calculation of reliability, safety, technical effectiveness indexes;

2.   multiply connected decomposition, when separating subsystems can involve any numbers of inputs/outputs. Only restriction on connection acyclicity exists:

a.   all input nodes of subsystem L^k are either heading nodes or they are connected with other elements (not entering into L^k) through the input edges of L^k;

b.   all output nodes are either terminal nodes or they are connected with other elements through the outgoing edges of L^k.

This process has disadvantages relating to complexity of the subsystem separation and indexes aggregation (e.g. failure frequency). But it is very efficien at solving high dimension problem and analyzing features of «reliability behaviour »;

3.   decomposition by divisible event group of element's states

4.   logical decomposition. In this process we do not make any transformation of the system structure. In this case the task of reliability modeling is simplified by dividing logical criteria of the system performance. For aggregation of indexes we can suggest method using theorem of probability of joint events sum, making easy to calculate bilateral estimation of the reliability indexes.

Decomposition methods, especially those, which have described in pt. 1 and 3, are known [7, 8, 11] and are used extensively for availability (unavailability) indexes calculation. In this paper we suggest method of failure frequency calculation based on decomposition technique by pt.3. For reduction calculation effort we also suggest decomposition by pt.1. Expression for failure frequency calculation, using decomposition by pt.1 and separation into series and parallel groups of elements and consequent convolution in one element with equivalent value of failure frequency, is the following:

− parallel schema *(1 out of m)*

$$_1\quad_m\{t\} = \sum_{j=1}^{m}[\quad_j(t)\prod_{g\neq j}^{m}Q_g(t)], \tag{13}$$

− series schema (*m out of m*)

$$_m\quad_m\{t\} = \sum_{j=1}^{m}[\quad_j(t)\prod_{g\neq j}^{m}R_g(t)], \tag{14}$$

− parallel schema (*k out of m*)

$$_k\quad_m\{t\} = \sum_{i_1<i_2<...<i_{m-k}}^{m}Q_{i_1}Q_{i_2}...Q_{i_{m-k}}\left[\sum_{j=1,j\neq i_1,i_2,...i_{m-k}}^{m}[\quad_j(t)\prod_{g=1,g\neq j,i_1,i_2,...i_{m-k}}^{m}R_g(t)]\right], \tag{15}$$

where $R_i(t)$, $Q_i(t)=1-R_i(t)$, $_i(t)$ - availability, unavailability, failure frequency of element *i*.

Expression (13) – (15) can be derived from (9) – (11) or drawn directly from

$$_S = P\{(S(\mathbf{x},t)=1)\bigwedge(\bigcup_{i=1}^{l}e_i^{'})\}. \tag{16}$$

Redundant structure *k out of m* very often consists of identical elements, in this case expression (15) takes on form

$$_k\ _m\{t\} = \mathrm{C}_m^{m-k} Q^{m-k}(t) k\ (t) R^{k-1}(t)\,, \tag{17}$$

where C_m^{m-k} - number of ($m - k$) out of m combinations of elements.

In [1] method of recursive variables increase was suggested for availability (unavailability) calculation. The kernel of problem is follows. Let

$$p^{(k)} = P\{S(x_1,...,x_k;t){=}1/x_{k+1} = 1, x_{k+2} = 1,...,x_n = 1\}$$
$$r^{(k)} = P\{S(x_1,...,x_k;t){=}1/x_{k+1} = 0, x_{k+2} = 1,...,x_n = 1\} \tag{18}$$

When calculating we use the formula

$$p^{(k+1)} = R_{k+1}(t) p^{(k)} + Q_{k+1}(t) r^{(k)}\,, \tag{19}$$

where $R_{k+1}(t) = 1 - Q_{k+1}(t) = P\{x_{k+1}(t) = 1\}, \quad p^{(0)} = 1$. Sequentially calculating $p^{(1)}, ..., p^{(n)}$, on the *nth* last step of recursion we'll get system availability.

3. TECHNIQUE FOR FAILURE FREQUENCY CALCULATION

Method of recursive variables increase (18), (19) is also applicable for failure frequency calculation. Let

$$^{(k)}(t)\Delta t = P\Big\{ (S(\mathbf{x},t) = 1) \wedge (\bigcup_{i=1}^{l} e_i') \big/ x_{k+1} = x_{k+2} = ... = x_n = 1 \Big\},$$

$$\mathrm{v}^{(k)}(t)\Delta t = P\Big\{ (S(\mathbf{x},t) = 1) \wedge (\bigcup_{i=1}^{l} e_i') \big/ x_{k+1} = 0, x_{k+2} = ... = x_n = 1 \Big\}. \tag{20}$$

Lemma.
System failure frequency can be recursively calculated like that:

$$^{(k+1)}(t) = R_{k+1}(t)\ ^{(k)}(t) + Q_{k+1}(t) \mathrm{v}^{(k)}(t) + P^{x_{k+1}}\ (t)\ _{k+1}(t),\quad ^{(0)}(t) = 0,\quad (t) = \ ^{(n)}(t), \tag{21}$$

$$P^{x_{k+1}}\ (t) = (p^{(k)} - r^{(k)})\,, k =(0,1,\ ...,\ n\text{-}1)$$

Lemma Proving. On the (k+1)th recursion step elements x_{k+2}, …, x_n of the system are completely reliable and we consider divisible group of disjoint events relative to the element x_{k+1}:

- element x_{k+1} is good in time point t. Probability of this event is $R_{k+1}(t)$, and failure frequency is equal to $^{(k)}$ in accordance with expression (20);
- element x_{k+1} in time point t is failed. Probability of this event is $Q_{k+1}(t)$, and failure frequency is equal to $^{(k)}$ in accordance with expression (20);
- element x_{k+1} failed on ($t,t+\ t$). Probability of this event is $_{k+1}(t)\ t$ (i. . failure frequency is equal to $_{k+1}(t)$). For the system to transfer to failed state at failure of element x_{k+1} it is necessary to be in the state that are previous to failure and the element x_{k+1} is good, but it's further failure results in failure of the system. Let us denote probability of such subsets as $P^{x_{k+1}}\ (t)$. It is proved in *appendix* that

$$P^{x_i}\ (t) = R\ (t)\big/\{x_i = 1\} - R\ (t)\big/\{x_i = 0\}\,, \tag{22}$$

where $R\ (t)\big/\{A\}$ - conditional system availability subject to .

Taking into account (22) we can come up on the (k+1)th step $P^{x_{k+1}}\ (t) = p^{(k)} - r^{(k)}$.

In accordance with formula of total probability we come up on (21). Note that (22) is Birnbaum reliability measure [7, 11].

Failure frequency calculation method (20), (21) (like availability calculation technique (18), (19)) can be used without system decomposition. But we advise to use system decomposition for overcoming dimensionality problem and rising performance of numerical algorithms. In case of system decomposition we suggest the following algorithm for failure frequency calculation.

    − All series, parallel, *k out of m* reliability schemes are enlarged in one element with failure frequency, calculated by (13) - (15) (for availability calculation one may use well-known formulas for series-parallel schemes).

    − As a result of several enlarging iterations one can get irreducible system part. In this case failure frequency calculation is implemented by (20), (21). And it is recommended to assign greater numbers to those elements, which incomes in different conjunctions several times. After that while these elements are treated as good (in accordance with expression (20)) it is possible to use simple formulas for series-parallel structures. At formalizing the step of minimal cut sets calculation one make use algorithm, proposed in [4, 5], which allows to pick out the elements, "making" reliability structure irreducible.

General way of $r^{(k)}$ and $\nu^{(k)}$ calculation includes the following. At each recursion step value x_i, stated in condition, are substituted in logical expressions (3), (4) and final expressions are transformed into probabilistic functions relative to availability index and failure frequency (in given step). At "hand-made" calculation one can figure the resulting structures, then all advantages of formulas (13) – (15), (17) together with decomposition and aggregation will be evident. Note, that in general case some steps of $r^{(k)}$ and $\nu^{(k)}$ calculation will include substeps, if resulting logical expressions will not fit series-parallel structures. In this case at given step one have to solve new task with resultant reduced logical description.

4. EXAMPLE

Let us consider irreducible bridge structure (Figure 1) and make failure frequency calculation by two stated above methods.
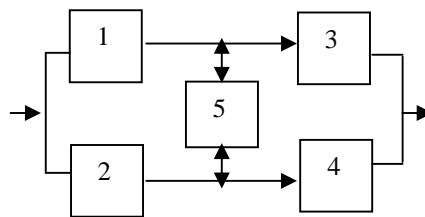

Figure 1. Bridge scheme.

$C_1 = \bar{1} \cdot \bar{2}$; $C_2 = \bar{3} \cdot \bar{4}$; $C_3 = \bar{1} \cdot \bar{5} \cdot \bar{4}$; $C_4 = \bar{2} \cdot \bar{5} \cdot \bar{3}$ (indexes are used instead of elements $_i$, conjunction symbol are substituted by product character).

1. Under (9) – (12).

$$\omega_{S1} = (\omega_1 Q_2 + \omega_2 Q_1) + (\omega_3 Q_4 + \omega_4 Q_3) + (\omega_1 Q_5 Q_4 + \omega_5 Q_1 Q_4 + \omega_4 Q_1 Q_5) + \quad (23)$$
$$(\omega_2 Q_5 Q_3 + \omega_5 Q_2 Q_3 + \omega_3 Q_2 Q_5) - \omega_1 Q_2 Q_5 Q_4 - \omega_2 Q_1 Q_5 Q_3 -$$
$$\omega_4 Q_1 Q_5 Q_3 - \omega_3 Q_2 Q_5 Q_4 - \omega_5 Q_1 Q_2 Q_3 Q_4.$$

All $_i$ and and cross product $_i$, $_j$ were included in ω_{S1} (cross product $_1$ $_2$ has not common elements, therefore for this event failure frequency is zero).

$$\omega_{S2} = [(\omega_1 Q_2 + \omega_2 Q_1) Q_3 Q_4 + \omega_2 Q_1 Q_5 Q_4 + \omega_1 Q_2 Q_5 Q_3]_1 - [\omega_2 Q_1 Q_3 Q_5 Q_4 +$$
$$\omega_1 Q_2 Q_5 Q_3 Q_4]_2 + [(\omega_3 Q_4 + \omega_4 Q_3) Q_1 Q_2 + \omega_3 Q_1 Q_5 Q_4 + \omega_4 Q_2 Q_5 Q_3]_3 -$$
$$[\omega_3 Q_1 Q_2 Q_5 Q_4 + \omega_4 Q_1 Q_2 Q_5 Q_3]_4 + [(\omega_4 Q_5 + \omega_5 Q_4) Q_1 Q_2 + (\omega_1 Q_5 + \omega_5 Q_1) Q_3 Q_4 +$$
$$(\omega_1 Q_4 + \omega_4 Q_1) Q_2 Q_5 Q_3]_5 - [\omega_5 Q_1 Q_2 Q_3 Q_4 + \omega_4 Q_1 Q_2 Q_5 Q_3 + \omega_1 Q_3 Q_4 Q_2 Q_5]_6 +$$
$$[(\omega_3 Q_5 + \omega_5 Q_3) Q_1 Q_2 + (\omega_2 Q_5 + \omega_5 Q_2) Q_3 Q_4 + (\omega_2 Q_3 + \omega_3 Q_2) Q_1 Q_5 Q_4]_7 - \quad (24)$$
$$[\omega_5 Q_1 Q_2 Q_3 Q_4 + \omega_3 Q_1 Q_2 Q_5 Q_4 + \omega_2 Q_1 Q_3 Q_5 Q_4]_8 - [\omega_1 Q_3 Q_4 Q_2 Q_5 + \omega_1 Q_3 Q_4 Q_2 Q_5 -$$
$$\omega_1 Q_3 Q_4 Q_2 Q_5]_9 - [\omega_2 Q_1 Q_3 Q_5 Q_4 + \omega_2 Q_1 Q_3 Q_5 Q_4 - \omega_2 Q_1 Q_3 Q_5 Q_4]_{10} -$$
$$[\omega_4 Q_1 Q_2 Q_5 Q_3 + \omega_4 Q_1 Q_2 Q_5 Q_3 - \omega_4 Q_1 Q_2 Q_5 Q_3]_{11} - \omega_3 Q_1 Q_2 Q_5 Q_4 + \omega_3 Q_1 Q_2 Q_5 Q_4 -$$

$\omega_3 Q_1 Q_2 Q_5 Q_4]_{12} - [\omega_5 Q_1 Q_2 Q_3 Q_4 + \omega_5 Q_1 Q_2 Q_3 Q_4 - \omega_5 Q_1 Q_2 Q_3 Q_4]_{13}$.

In ω_{S2} were included the following events: $\omega_1 e_i$ – the first square bracket (denoted as $[\ldots]_1$); $\omega_1 e_i e_j$ – the second square bracket (event $e_3 e_4$ is the failure of all system elements there fore failure frequency of this event is zero); $\omega_2 e_i$, $\omega_3 e_i$, $\omega_4 e_i$, - square brackets 3, 5, 7; $\omega_2 e_i e_j$, $\omega_3 e_i e_j$, $\omega_4 e_i e_j$ - square brackets 4, 6, 8; $\omega_1 e_3 e_2$, $\omega_1 e_3 e_4$, $\omega_1 e_3 e_2 e_4$ – square bracket 9; $\omega_1 e_4 e_2$, $\omega_1 e_4 e_3$, $\omega_1 e_4 e_2 e_3$ – square bracket 10; $\omega_2 e_3 e_1$, $\omega_2 e_3 e_4$, $\omega_2 e_3 e_1 e_4$ – square bracket 11; $\omega_2 e_4 e_1$, $\omega_2 e_4 e_3$, $\omega_2 e_4 e_1 e_3$ – square bracket 12; $\omega_3 e_4 e_1$, $\omega_3 e_4 e_2$, $\omega_3 e_4 e_1 e_2$ – square bracket 13.

Cross products ω_i of order 3 and 4 for ω_{S1} ω_{S2} have not common elements there fore this term of failure frequency is equal to zero. But all these cross products must be done (by men or by computer).

Final expression for failure frequency in accordance with (9) will be:

$$\omega = \omega_{S1} - \omega_{S2} = [\omega_1(Q_2 + Q_4 Q_5 - Q_2 Q_4 Q_5 - Q_2 Q_3 Q_4 - Q_2 Q_3 Q_5 - Q_3 Q_4 Q_5 + 2 \cdot Q_2 Q_3 Q_4 Q_5)] + [\omega_2(Q_1 + Q_3 Q_5 - Q_1 Q_3 Q_5 - Q_1 Q_3 Q_4 - Q_1 Q_4 Q_5 - Q_3 Q_4 Q_5 + 2 Q_1 Q_3 Q_4 Q_5)] + [\omega_1(Q_2 + Q_4 Q_5 - Q_2 Q_4 Q_5 - Q_2 Q_3 Q_4 - Q_2 Q_3 Q_5 - Q_3 Q_4 Q_5 + 2 Q_2 Q_3 Q_4 Q_5)] + [\omega_3(Q_4 + Q_2 Q_5 - Q_2 Q_4 Q_5 - Q_1 Q_2 Q_4 - Q_1 Q_4 Q_5 - Q_1 Q_2 Q_5 + 2 Q_1 Q_2 Q_4 Q_5)] + [\omega_4(Q_3 + Q_1 Q_5 - Q_1 Q_3 Q_5 - Q_1 Q_2 Q_3 - Q_2 Q_3 Q_5 - Q_1 Q_2 Q_5 + 2 Q_1 Q_2 Q_3 Q_5)] + [\omega_5(Q_1 Q_4 + Q_2 Q_3 - Q_1 Q_2 Q_4 - Q_1 Q_3 Q_4 - Q_1 Q_2 Q_3 - Q_2 Q_3 Q_4 + 2 Q_1 Q_2 Q_3 Q_4)]. \quad (25)$$

2. After calculating conditional availability by (18), (19) we can calculate failure frequency. Let us define logical operability function throw minimal path sets

$$S(\mathbf{x}, t) = \left\{ \bigvee_{j=1}^{4} A_j \right\} = 1, \quad A_1 = 1 \cdot 3; \quad A_2 = 2 \cdot 4; \quad A_3 = 1 \cdot 5 \cdot 4; \quad A_4 = 2 \cdot 5 \cdot 3.$$

$$p^{(1)} = R_1 p^{(0)} + Q_1 r^{(0)} = R_1 \cdot 1 + Q_1 \cdot 1 = 1 \quad (r^{(0)} = P\{S(\mathbf{x}) = 1 / x_1 = 0, x_2 = \ldots = x_5 = 1\} = 1),$$

$$p^{(2)} = R_2 p^{(1)} + Q_2 r^{(1)} = R_2 \cdot 1 + Q_2 P\{S(\mathbf{x}) = S(x_1) = 1 / x_2 = 0, x_3 = x_4 = x_5 = 1\} = R_2 + Q_2 R_1,$$

$$p^{(3)} = R_3 p^{(2)} + Q_3 r^{(2)} = R_3(R_2 + Q_2 R_1) + Q_3 r^{(2)} = R_3(R_2 + Q_2 R_1) + Q_3 P\{S(\mathbf{x}) = S(x_1, x_2) = 1 / x_3 = 0, x_4 = x_5 = 1\} = R_3(R_2 + Q_2 R_1) + Q_3(R_2 + Q_2 R_1) = R_2 + Q_2 R_1,$$

$$p^{(4)} = R_4 p^{(3)} + Q_4 r^{(3)} = R_4(R_2 + Q_2 R_1) + Q_4 P\{S(\mathbf{x}) = S(x_1, x_2, x_3) = 1 / x_4 = 0, x_5 = 1\} = R_4(R_2 + Q_2 R_1) + Q_4 R_3(R_2 + Q_2 R_1) = (R_2 + Q_2 R_1)(R_4 + Q_4 R_3), \quad (26)$$

$$R = P\{S(\mathbf{x}, t) = 1\} = p^{(5)} = R_5 p^{(4)} + Q_5 r^{(4)} = R_5(R_2 + Q_2 R_1)(R_4 + Q_4 R_3) + Q_5 P\{S(\mathbf{x}) = S(x_1, x_2, x_3, x_4) = 1 / x_5 = 0\} = R_5(R_2 + Q_2 R_1)(R_4 + Q_4 R_3) + Q_5(1 - (1 - R_1 R_3)(1 - R_2 R_4)).$$

From (20), (21).failure frequency is

$$\omega^{(1)} = R_1 \omega^{(0)} + Q_1 v^{(0)} + \omega_1(p^{(0)} - r^{(0)}) = R_1 \cdot 0 + Q_1 P\{(S(\mathbf{x}, t) = 1) \wedge (\bigcup_{i=1}^{4} e_i') / x_1 = 0, x_2 = \ldots = x_5 = 1\} + \omega_1 \cdot (1 - 1) = 0;$$

$$\omega^{(2)} = R_2 \omega^{(1)} + Q_2 v^{(1)} + \omega_2(p^{(1)} - r^{(1)}) = R_2 \cdot 0 + Q_2 P\{(S(\mathbf{x}, t) = 1) \wedge (\bigcup_{i=1}^{4} e_i') / x_2 = 0, x_3 = x_4 = x_5 = 1\} + \omega_2(1 - R_1) = Q_2 \omega_1 + \omega_2(1 - R_1) = Q_2 \omega_1 + \omega_2 Q_1; \quad (27)$$

$$^{(3)} = R_3 \,^{(2)} + Q_3 \mathrm{v}^{(2)} + \,_3(p^{(2)} - r^{(2)}) = R_3(Q_2 \,_1 + \,_2 Q_1) +$$

$$Q_3 P\{(S(\mathbf{x},t) = 1) \wedge (\bigcup_{i=1}^{4} e_i^{'}) / x_3 = 0, x_4 = x_5 = 1\} + \,_3(R_2 + Q_2 R_1 - R_2 - Q_2 R_1) =$$

$$R_3(Q_2 \,_1 + \,_2 Q_1) + Q_3(Q_2 \,_1 + \,_2 Q_1) = Q_2 \,_1 + \,_2 Q_1;$$

$$^{(4)} = R_4 \,^{(3)} + Q_4 \mathrm{v}^{(3)} + \,_4(p^{(3)} - r^{(3)}) = R_4(Q_2 \,_1 + \,_2 Q_1) +$$

$$Q_4 P\{(S(\mathbf{x},t) = 1) \wedge (\bigcup_{i=1}^{4} e_i^{'}) / x_4 = 0, x_5 = 1\} + \,_4(R_2 + Q_2 R_1 - R_3(R_2 + Q_2 R_1))$$

$$= R_4(Q_2 \,_1 + \,_2 Q_1) + Q_4(\,_3 \cdot (1 - Q_1 Q_2) + R_3(\,_1 Q_2 + \,_2 Q_1)) + \,_4 Q_3(R_2 + Q_2 R_1);$$

$$= \,^{(5)} = R_5 \,^{(4)} + Q_5 \mathrm{v}^{(4)} + \,_5(p^{(4)} - r^{(4)}) = R_5[R_4(Q_2 \,_1 + \,_2 Q_1) +$$

$$Q_4[\,_3(1 - Q_1 Q_2) + R_3(\,_1 Q_2 + \,_2 Q_1)] + \,_4 Q_3(R_2 + Q_2 R_1)] +$$

$$Q_5[(\,_3 R_1 + \,_1 R_3)(1 - R_2 R_4) + (\,_2 R_4 + \,_4 R_2)(1 - R_1 R_3)] +$$

$$_5[(R_2 + Q_2 R_1)(R_4 + Q_4 R_3) - (1 - (1 - R_1 R_3)(1 - R_2 R_4))].$$

Let us make a comments to some calculation of $r^{(k)}$ and $\mathrm{v}^{(k)}$.

$r^{(0)} = P\{S(\mathbf{x}) = 1 / x_1 = 0, x_2 = x_3 = x_4 = x_5 = 1\} = 1$ - on conditions that in time point t $_1$ failed and all other elements are good, $S(\mathbf{x}) = 1$ is persistent event and required probability is 1.

$$^{(0)} \quad t = P\{(S(\mathbf{x},t) = 1) \wedge (\bigcup_{i=1}^{4} e_i^{'}) / x_1 = 0, x_2 = \ldots = x_5 = 1\}$$ - if elements 2 – 5 are good in time

point t system failure is impossible (i. . $\bigcup_{i=1}^{4} e_i$ - null event), so $\mathrm{v}^{(0)} = 0$.

$r^{(1)} = P\{S(\mathbf{x},t) = S(x_1,t) = 1 / x_2 = 0, x_3 = x_4 = x_5 = 1\} = R_1$ - at element $_2$ failure and elements $_3 - _5$ in good state and after substitution into $S(\mathbf{x})$ we get

$S(\mathbf{x}) = \,_1$, and $r^{(1)} = R_1$.

$$\mathrm{v}^{(1)} \quad t = P\{(S(\mathbf{x},t) = 1) \wedge (\bigcup_{i=1}^{4} e_i^{'}) / x_2 = 0, x_3 = x_4 = x_5 = 1\} = \,_1 \quad t$$ - substituting $_2 = 0$, $_3 = \,_4$

$= \,_5 = 1$ into paths $_i(t)$ and cuts $C_i(t + t)$, we have $S(\mathbf{x}) = \,_1$, $e_1(t+ t) = \bar{x}_1 \cdot 1 = \bar{x}_1$, other $e_i = 0$. In order to occur failure of $_1$ in $t+ t$, it is necessary that $_1$ was good in time point t:

$e_1^{'}(t) = x_1$. And, $\mathrm{v}^{(1)} = \,_1$.

$r^{(2)} = P\{S(\mathbf{x}) = S(x_1, x_2) = 1 / x_3 = 0, x_4 = x_5 = 1\} = R_2 + Q_2 R_1$ - substitute variables values into condition $S(\mathbf{x},t)$, expressed throw paths. As a result $S(\mathbf{x},t) = S(x_1,x_2,t) = x_1 + x_2$, and it is easy to define $r^{(2)} = R_2 + Q_2 R_1$ (parallel connection of $_1$ and $_2$).

$$\mathrm{v}^{(2)} \Delta t = P\{(S(\mathbf{x},t) = 1) \wedge (\bigcup_{i=1}^{4} e_i) / x_3 = 0, x_4 = x_5 = 1\} = Q_2 \,_1 + \,_2 Q_1$$ - we already got

expression for $S(\mathbf{x},t)$ subject to condition, at condition substitution into cuts, which must occur in point $(t, t+ t)$, we have $e_1(t, t+ t) = C_1(t, t+ t) = \bar{x}_1 \cdot \bar{x}_2$. For this cut to occur in $(t, t+ t)$ it is necessary to implement event $e_1^{'}(t) = x_1 \cdot \bar{x}_2 + x_2 \cdot \bar{x}_1$ in time point t. $(S(\mathbf{x},t) = x_1 + x_2)$ (
$e_1^{'}(t) = x_1 \cdot \bar{x}_2 + x_2 \cdot \bar{x}_1) = \,x_1 \cdot \bar{x}_2 + x_2 \cdot \bar{x}_1$, so $\mathrm{v}^{(2)} = Q_2 \,_1 + \,_2 Q_1$ (this can be write immediately in terms of $S(\mathbf{x},t)$ for parallel connection (13)).

If in (27) all R_i will be replaced by $(1-Q_i)$ we shall get (25).

<u>Resume from example</u>. When calculating by known method by (9) – (12) it was done L steps of logical expression transformation (logical expression includes 4 cut sets):

   – For $_{S1}$ calculation:

$$L(\omega_{S1}) = (C_4^1 = 4) + (C_4^2 = 6) + (C_4^3 = 4) + (C_4^4 = 1) = 15. \ (28) \tag{28}$$

In (28) the first bracket is equal to the number of terms of first sum in expression for ω_{S1} (10) and associates with first four parentheses in (23). The second bracket in (28) is equal to the number of terms of second (double) sum in ω_{S1} expression (10) and associates with other five summands (with minus sign; the sixth summand is zero in (23)). The other two brackets in (28) associate with the number of terms of the third (triple) sum and last summand for ω_{S1} in (10). In (23) this terms are absent because they are equal to zero.

– For ω_{S2} calculation in accordance with (24):

$$L(\omega_{S2}) = [(C_4^1 = 4)(C_3^1 + C_3^2 + C_3^3 = 7)]_1 + [(C_4^2 = 6)\cdot(C_2^1 + C_2^2 = 3)]_2 + [(C_4^3 = 4)\cdot(C_1^1 = 1)]_3 = 50. \tag{29}$$

The first parentheses in each square bracket defines number of occurrence of one, two, three cuts e_i on Δt. The second parentheses defines number of combinations of possible cuts in time point t.

As a result of fulfilment of some steps it is possible to get empty sets. Terms in expression (10, 11) for these steps are zero. It is necessary to emphasize that all these steps must be executed including those steps, which results in zero.

Thereby, total amount of steps $L = 65$.

In suggested method ((20), (21) taking into account (18), (19)) there are no combinatorial steps. In suggested method the calculation steps are recursive iteration of variables increase. For the example under consideration it was done 5 steps of calculation $r^{(k)}$ ($r^{(0)},\dots,r^{(4)}$) and 5 steps of calculation $v^{(k)}$ ($v^{(0)}, \dots, v^{(4)}$).

The form of final result in suggested method is well-behaved viz approximation to final result are calculated by summation of recursive iteration terms.

Expected number of failures on time interval $(0,t)$ is

$$N_\omega(t) = \int_0^t \omega(t)dt. \tag{30}$$

Let assume exponential distribution of operating and recovery time ($\lambda\mu$) for all elements, and element 1 is identical to element 2 (i. . $\lambda_1 = \lambda_2 = \lambda_{1,2}$, $\mu_1 = \mu_2 = \mu_{1,2}$), element 3 is identical to element 4 ($\lambda_3 = \lambda_4 = \lambda_{3,4}$, $\mu_3 = \mu_4 = \mu_{3,4}$). Availability and failure frequency of the elements are

$$R_i(t) = \frac{\mu_i}{\lambda_i + \mu_i} + \frac{\lambda_i}{\lambda_i + \mu_i}\exp\{-(\lambda_i + \mu_i)t\}, \qquad \omega_i = \frac{\mu_i \cdot \lambda_i}{\lambda_i + \mu_i} + \frac{\lambda_i \cdot \lambda_i}{\lambda_i + \mu_i}\exp\{-(\lambda_i + \mu_i)t\}, i = (1\text{-}5).$$

Let $\lambda_{1,2} = 1\cdot10^{-3}$, $\mu_{1,2} = 1/20 = 0.05$, $\lambda_{3,4} = 2\cdot10^{-3}$, $\mu_{3,4} = 1/15 = 0.0667$, $\lambda_5 = 5\cdot10^{-3}$, $\mu_5 = 1/10 = 0.1$ (unit is hour^{-1}). Figures 2, 3 show failure frequency and expected number of failures dependence on time. One can see that interval of unstationarity is sufficiently small (~3 $\tau_{system\ recovery}$; this is known theoretical result at $\mu_i \gg \lambda_i$), that is why it is acceptable to calculate stationary measures for a systems with long operation life. In this case availability (R_i) and failure frequency (ω_i) will include only first terms, which is not dependant on time, and expected number of failures will be

$$N_\omega(t) = \omega_\cdot t. \tag{31}$$

5. CONCLUSION

In this paper we suggest the method of calculation failure frequency measure for system with complex structure and known availability and failure frequency of system's elements (in accordance with expressions (20), (21)). The method is well algorithmizied and together with considered techniques of decomposition and aggregation ((13) - (15), (17)) makes it possible to analyze the system with considerably high dimensions. The method is more effective in comparison with other known methods.
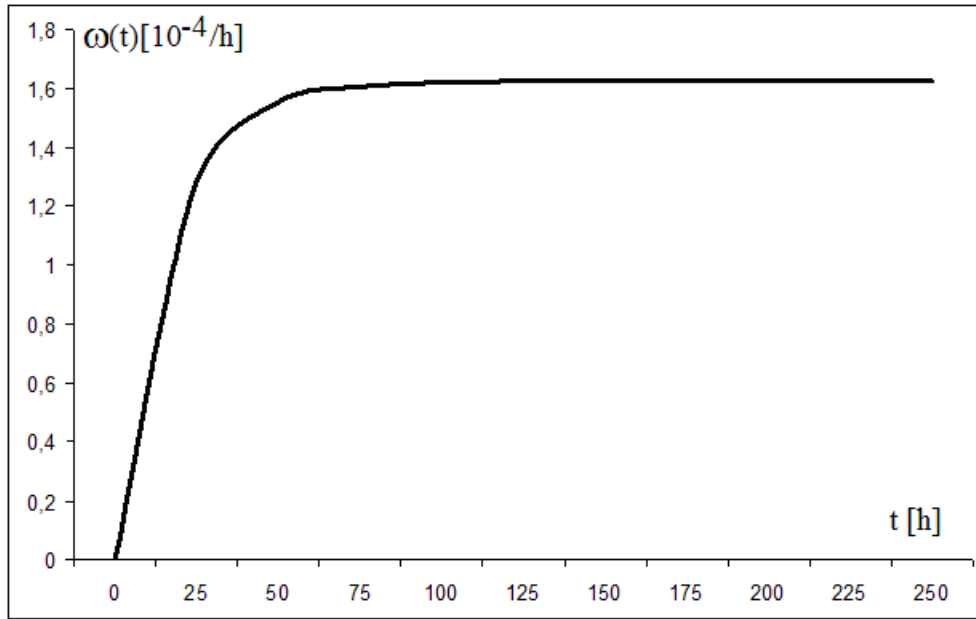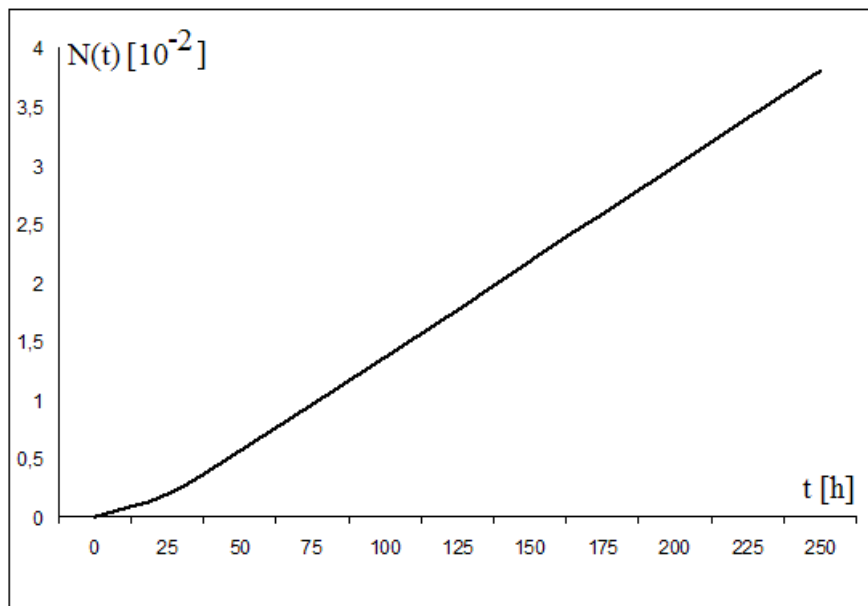
Figure 2. Failure frequency time dependence.

Figure 3. Expected number of failures time dependence.

6. APPENDIX

Proving the expression (22) for $P^{x_i}_{\text{prefailed}}(t)$ calculation.

Let logical function is defined through the cut sets $\{C_k\}$ in the form of (4) (we shall use product $\cdot$ and summation $+$ character instead of conjunction $\wedge$ and disjunction V symbols), i.e. unoperability function is $\overline{S(\mathbf{x},t)} = C_1 + C_2 + \ldots + C_l$, and operability function is $\overline{S(\mathbf{x},t)} = S(\mathbf{x},t) = \overline{C_1} \cdot \overline{C_2} \cdot \ldots \cdot \overline{C_l}$.

Let us divide cut set $\{C_k\}$ into two subset: $\{C_{ki}\} = \{C_{1i}, C_{2i}, \ldots, C_{ki}\}$ and $\{C_{kj}\} = \{C_{1j}, C_{2j}, \ldots, C_{kj}\} = \{C_k\} \setminus \{C_{ki}\}$; $\{C_{ki}\} \cup \{C_{kj}\} = \{C_k\}$, $\{C_{ki}\} \cap \{C_{kj}\} = \varnothing$; $\{C_{ki}\}$ – set of cuts, which

include element i, $\{C_{kj}\}$ - set of cuts, which do not include element i. Then system operability function is $S(\mathbf{x},t) = \overline{C_{1j}} \cdot \overline{C_{2j}} \cdot \ldots \cdot \overline{C_{kj}} \cdot \overline{C_{1i}} \cdot \overline{C_{2i}} \cdot \ldots \cdot \overline{C_{ki}}, \quad ki + kj = l$.

It is necessary to find probability of subset of pre failed states when the *ith* element is good. Failure of the *ith* element results in system failure. And this probability must be formed from original system description $S(\mathbf{x},t)$, but not from specially constructed logical function for this subset of pre failed states. Let define logical function of required pre failed states in the form of $y_{\text{prefailed}}(i) = y_1 \cdot y_2$. Cuts $\{C_{kj}\}$, in which the *ith* element is not included, must not exist in time point t (as these pre failed states are states of system operability), so $y_1 = \overline{C_{1j}} \cdot \overline{C_{2j}} \cdot \ldots \cdot \overline{C_{kj}}$. Let consider the cuts, which include the *ith* element. As the *ith* element failure on $(t, t+\Delta t)$ results in system failure, then at least one of events $\{C_{ki}\}$ must exist in time point t, provided the *ith* element is map out. So as to exist pre failed state in time t with the *ith* good element it is necessary to implement a function with $\{C_{ki}\}$ $x_i = 0$: $y_2 = (C_{1i}/\{x_i = 0\}) + (C_{2i}/\{x_i = 0\}) + \ldots + (C_{ki}/\{x_i = 0\})$.

Thereby

$$y_{\text{prefailed}}(i) = y_1 \cdot y_2 = \overline{C_{1j}} \cdot \overline{C_{2j}} \cdot \ldots \cdot \overline{C_{kj}} \cdot ((C_{1i}/\{x_i = 0\}) + (C_{2i}/\{x_i = 0\}) + \ldots + (C_{ki}/\{x_i = 0\})) =$$

$$\overline{C_{1j}} \cdot \overline{C_{2j}} \cdot \ldots \cdot \overline{C_{kj}} \cdot \overline{\overline{((C_{1i}/\{x_i = 0\}) + (C_{2i}/\{x_i = 0\}) + \ldots + (C_{ki}/\{x_i = 0\}))}} =$$

$$\overline{C_{1j}} \cdot \overline{C_{2j}} \cdot \ldots \cdot \overline{C_{kj}} \cdot (1 - (\overline{C_{1i}}/\{x_i = 0\}) \cdot (\overline{C_{2i}}/\{x_i = 0\}) \cdot \ldots \cdot (\overline{C_{ki}}/\{x_i = 0\})).$$

Let define this probability

$$P\{\overline{C_{1j}} \cdot \overline{C_{2j}} \cdot \ldots \cdot \overline{C_{kj}} \cdot (1 - (\overline{C_{1i}}/\{x_i = 0\}) \cdot (\overline{C_{2i}}/\{x_i = 0\}) \cdot \ldots \cdot (\overline{C_{ki}}/\{x_i = 0\})) = 1\} = P\{\overline{C_{1j}} \cdot \overline{C_{2j}} \cdot \ldots \cdot \overline{C_{kj}} = 1\} -$$

$$P\{\overline{C_{1j}} \cdot \overline{C_{2j}} \cdot \ldots \cdot \overline{C_{kj}} \cdot (\overline{C_{1i}}/\{x_i = 0\}) \cdot (\overline{C_{2i}}/\{x_i = 0\}) \cdot \ldots \cdot (\overline{C_{ki}}/\{x_i = 0\}) = 1\}.$$

Note, that

1. $P\{\overline{C_{1j}} \cdot \overline{C_{2j}} \cdot \ldots \cdot \overline{C_{kj}} = 1\} = P\{(S(\mathbf{x},t)/\{x_i = 1\}) = 1\}$ - system availability at $\lambda_i = 1$, because $(S(\mathbf{x},t)/x_i = 1) = (\overline{C_{1j}} \cdot \overline{C_{2j}} \cdot \ldots \cdot \overline{C_{kj}} \cdot (\overline{C_{1i}} \cdot \overline{C_{2i}} \cdot \ldots \cdot \overline{C_{ki}})/\{x_i = 1\}) = \overline{C_{1j}} \cdot \overline{C_{2j}} \cdot \ldots \cdot \overline{C_{kj}}$. Element x_i is a part of all $\{C_{ki}\}$ and his operability provides $((\overline{C_{1i}} \cdot \overline{C_{2i}} \cdot \ldots \cdot \overline{C_{ki}})/\{x_i = 1\}) = 1$ regardless of state of other elements.

2. $P\{(\overline{C_{1j}} \cdot \overline{C_{2j}} \cdot \ldots \cdot \overline{C_{kj}}) \cdot ((\overline{C_{1i}}/\{x_i = 0\}) \cdot (\overline{C_{2i}}/\{x_i = 0\}) \cdot \ldots \cdot (\overline{C_{ki}}/\{x_i = 0\})) = 1\} =$

$P\{(S(\mathbf{x},t)/\{x_i = 0\}) = 1\}$ - system availability at $\lambda_i = 0$, which are obvious.

Thus, $P^{x_i}(t) = R(t)\big/\{x_i = 1\} - R(t)\big/\{x_i = 0\}$.

REFERENCES

1. Akulova L.G. 1983. *About stochastic complexity of reliability calculation of Boolean systems.* Iaroslavl. ISU press. (in Russian).
2. Cherniak . . 1991. Combinatorial graph method of reliability analysis of complex systems with monotone Boolean functions. *Automatics and Remote Control.* 4. pp. 165 – 174. (in Russian).
3. Cherniak . ., Cherniak J. . 1998. Logical probabilistic method of analysis of high dimensional multivalued systems. *Automatics and Remote Control.* 1. pp. 171 – 180. (in Russian).
4. Filin B.P. 1982. About analytical techniques of approximate calculation of complex system reliability. *Automatics and Remote Control.* 11. .159 – 170. (in Russian).
5. Filin B.P. 1989. *Techniques of analysis structural reliability of communication network.* . Radio and Connection press. (in Russian).
6. Filin B.P. 1989. About duality principle in problems of analysis of complex systems structural reliability. *Automatics and Remote Control.* 6. pp. 158 – 172. (in Russian).
7. Henley E.G., Kumamoto H. 1984. *Reliability Engineering and Risk Assessment*. M. Industry press.
8. Mojaev A.S. 1988. *General logical-probabilistic method of complex system reliability analysis.* Leningrad. "VMA" Tutorial. (in Russian).
9. Mojaev A.S., Alekseev . . 1994. Automated structural-logical modeling and probabilistic analysis of complex system. *Theory and informational technology of complex systems safety modeling.* St. Petersburg. Issue 2. (in Russian).
10. *Ryabinin I. . 1971. Basis of theory and calculation of power systems. Leningrad. Shipbuilding press. (in Russian)*
11. *Ryabinin I. ., Parfenov U. . 1978. Reliability measures estimation. Proceedings of USSR Academy of Sciences. Energetics and Transport. 6. pp. 22 – 32. (in Russian).*