MODELLING THE SHIP SAFETY ON WATERWAY ACCORDING TO NAVIGATIONAL SIGNS RELIABILITY

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ABSTRACT

An approach to safety analysis connected with consecutive "m out of n" systems is presented. Further, the consecutive "*m* out of *n*: G" system is defined and the recurrent formulae for its reliability function evaluation are proposed. Next the IALA buoys and leading lights system are introduced. Moreover, the safety states model for ship navigation are defined. Further, analysis of safety during manoeuvre in restricted area with curved draws is illustrated.

1 INTRODUCTION

The safety of passengers and cargo involved in the process of transport is one of the most important criteria for the evaluation of the process. In the maritime transport the most important factors making up the security include: the technical efficiency of the ship, the qualifications of the people in charge of the ship and the conditions under which the transport process takes a place. There are many hazard situations, in maritime transport, particularly in restricted waterways. In such situations it is useful to have methods to assess the safety of traffic. They allow the evaluation of the activities what lead to settle the hazard situation and allow the evaluation of quality control and assessment in terms of traffic safety (Pietrzykowski 2003, Purcz. 1998, Smolarek 2009). This assessment can help to develop the best control or the best manoeuvre for given hazard situation (Fuji 1977, Gucma 1998, Pietrzykowski 2003, Purcz. 1998, Smolarek 2009).

In the case of shipping on the restricted waters important aspects of safety are the technical characteristics of vessel, the type of waterway and its navigational infrastructure (Fuji 1977, IALA NAVGUIDE 2006, Kopacz et. al. 2001, Kopacz et. al. 2003).

In the case of shipping on the restricted waters the technical characteristics of vessel, the type of waterway and its navigational infrastructure are important aspects of its safety (Fuji 1977, Kopacz et. al. 2001, Kopacz et. al. 2003).

Navigational infrastructure is a set of basic navigation, stable and distributed objects and systems necessary to ensure adequate level of maritime safety (Kopacz et. al. 2003).

The paper is devoted to the combining the results on reliability of the two-state consecutive "*m* out of *n*: F" and consecutive "*m* out of *n*: G" systems (Antonopoulou et. al. 1987, Barlow et. al 1975, Guze 2007a,b, Hwang 1982, Kołowrocki 2004, Malinowski 2005) into the safety analysis of the ship on restricted waterway (Kopacz et. al. 2003).

2 TWO-STATE CONSECUTIVE "*M* OUT OF *N: F*" SYSTEMS

In the case of two-state reliability analysis of consecutive "*m* out of *n*" systems we assume that (Guze 2007a, Malinowski 2005):

- n is the number of system components,
- E_i, $i = 1,2,...,n,$ are components of a system,
- T_i are independent random variables representing the lifetimes of components E_i, $i = 1,2,...,n,$
- $R_i(t) = P(T_i > t), t \in <0,\infty),$ *is* a reliability function of a component E_i, $i = 1,2,...,n,$
- $F_i(t) = 1 - R_i(t) = P(T_i \le t), t \in <0,\infty),$ is the distribution function of the component E_i lifetime T_i, $i = 1,2,...,n,$ also called an unreliability function of a component E_i, $i = 1,2,...,n.$

Definition 1. A two-state system is called a two-state consecutive "*m* out of *n*: F" system if it is failed if and only if at least its *m* neighbouring components out of n its components arranged in a sequence of $E_1, E_2, \ldots, E_n$, are failed.

After assumption that:

- T is a random variable representing the lifetime of the consecutive "*m* out of *n*: F" system,
- $\boldsymbol{CR}_n^{(m)}(t) = P(T > t), t \in <0,\infty),$ is the reliability function of a non-homogeneous consecutive "*m* out of *n*: F" system,
- $\boldsymbol{CF}_n^{(m)}(t) = 1 - \boldsymbol{CR}_n^{(m)}(t) = P(T \le t), t \in <0,\infty),$ is the distribution function of a consecutive "*m* out of *n*: F" system lifetime T,

we can formulate the following auxiliary theorem [5].

Lemma 1. The reliability function of the two-state consecutive "*m* out of *n*: F" system is given by the following recurrent formula

$$\boldsymbol{CR}_n^{(m)}(t) = \begin{cases} 1 & \text{for } n < m, \\ 1 - \prod_{i=1}^{n}\left(1 - R_i(t)\right) & \text{for } n = m, \\ R_n(t)\boldsymbol{CR}_{n-1}^{(m)}(t) & \\ + \sum_{j=1}^{m-1} R_{n-j}(t)\boldsymbol{CR}_{n\text{-}j\text{-}1}^{(m)}(t) & \\ \cdot \prod_{i=n-j+1}^{n}\left(1 - R_i(t)\right) & \text{for } n > m, \end{cases} \tag{1}$$

for $t \in <0,\infty)$.

Definition 2. The consecutive "*m* out of *n*: F" system is called homogeneous if its components lifetimes T_i have an identical distribution function

$F(t) = P(T_i \leq t),\ i = 1,2,\dots,n,\ t \in <0,\infty),$

i.e. if its components E_i have the same reliability function

$R(t) = 1 - F(t),\ t \in <0,\infty).$

Lemma 1 simplified form for homogeneous systems takes the following form.

Lemma 2. The reliability function of the homogeneous two-state consecutive "*m* out of *n*: F" system is given by the following recurrent formula

$$\boldsymbol{CR}_n^{(m)}(t) = \begin{cases} 1 & \text{for } n < m, \\ 1-(1-R(t))^n & \text{for } n = m, \\ R(t)\boldsymbol{CR}_{n-1}^{(m)}(t) & \\ + R(t)\sum_{j=1}^{m-1}(1-R^j(t)) & \\ \cdot \boldsymbol{CR}_{n-j-1}^{(m)}(t) & \text{for } n > m, \end{cases} \tag{2}$$

for $t \in <0,\infty)$.

3 TWO-STATE CONSECUTIVE "*M* OUT OF *N*: G" SYSTEMS

Definition 3. A two-state system is called a two-state consecutive "*m* out of *n*: G" system if it is good if and only if at least its *m* neighbouring components out of n its components arranged in a sequence of $E_1, E_2, \dots, E_n$, are good.

In further analysis we assume, that:

- T is a random variable representing the lifetime of the consecutive "*m* out of *n*: G" system,
- $\boldsymbol{CRG}_n^{(m)}(t) = P(T > t),\ t \in <0,\infty),$ is the reliability function of a non-homogeneous consecutive "*m* out of *n*: G" system,
- $\boldsymbol{CFG}_n^{(m)}(t) = 1 - \boldsymbol{CRG}_n^{(m)}(t) = P(T \leq t),\ t \in <0,\infty),$ is the distribution function of a consecutive "*m* out of *n*: G" system lifetime T.

Thus, we can formulate the following auxiliary theorem (Malinowski 2005).

Lemma 3. The reliability function of the two-state consecutive "*m* out of *n*: G" system is given by the following recurrent formula

$$\boldsymbol{CRG}_n^{(m)}(t) = \begin{cases} 0 & \text{for } n < m, \\ \prod_{i=1}^{n} R_i(t) & \text{for } n = m, \\ \left(1 - R_n(t)\right)\boldsymbol{CRG}_{n-1}^{(m)}(t) & \\ + \sum_{j=1}^{m-1} \left(1 - R_{n-j}(t)\right)\left(1 - \boldsymbol{CRG}_{n\text{-}j\text{-}1}^{(m)}(t)\right) \cdot & \\ \cdot \prod_{i=n-j+1}^{n} R_i(t) & \text{for } n > m, \end{cases} \tag{3}$$

for $t \in <0, \infty)$.

From the above theorem, as a particular case for the homogeneous system, i.e. system composed of components with identical reliability, we immediately get the following corollary.

Corollary 4. The reliability function of the homogeneous two-state consecutive “*m* out of *n*: G” system is given by the following recurrent formula

$$\boldsymbol{CRG}_n^{(m)}(t) = \begin{cases} 0 & \text{for n} < m, \\ [R(t)]^n & \text{for } n = m, \\ \left(1 - R(t)\right)\boldsymbol{CRG}_{n\text{-}1}^{(m)}(t) & \\ + \left(1 - R(t)\right)\sum_{j=1}^{m-1} R^j(t) \cdot & \\ \cdot \left(1 - \boldsymbol{CRG}_{n\text{-}j\text{-}1}^{(m)}(t)\right) & \text{for n} > m, \end{cases} \tag{4}$$

for $t \in <0, \infty)$.

4 THE MAIN KIND OF NAVIGATION INFRASTRUCTURE IN WATERWAYS DESIGN

The classification of navigation infrastructure is as follows (Kopacz et. al. 2001, 2003):

- signalling – warning and visual positioning infrastructure;
- radio-navigation positioning infrastructure;
- vessel traffic monitoring, information and navigation support infrastructure.

Every kind of the infrastructure has components in the form of an object or a system of navigation infrastructure.
An object is a simple element, for example a buoy or lighting tower. The objects create system of navigation infrastructure.
For safe navigation in restricted or limited areas IALA introduced the system of buoys and leading lights. It can be helpful to define a clearing line for the limits of safe navigation (IALA NAVIGUIDE 2006).
There are major parameters which are important for the optimum number and arrangement of buoys and leading lights. These parameters depend on the average channel width, the channel length, whether the section is straight or curved.
In the other hand the optimum separation distance between buoys and the numbers of buoys and leading lights are important. The distance is depended on the average width of the section concerned and its curvature. It is obvious that in the sections of waterway which have the greatest risk of groundings or collisions, the numbers of buoys and leading lights should be highest (IALA NAVIGUIDE 2006).

5 SAFETY ANALYSIS OF SHIP ON WATERWAY

Definition 4. The system is in safety state if the ship operator has full navigational information.

Definition 5. The system is in dangerous state if the ship operator has insufficient navigational information.

Under above definitions we define the set of safety states as

$$S = \{S_S, S_D\},$$

where:

S_S – state of safety,
S_D – state of dangerous.

Thus, after assumption that:

n_S – limit number for safety state;
n_D – limit number for dangerous state.

and considering formulae (1)-(4), we can define probabilities of states as follows:

$$- P(S_S) = \boldsymbol{CRG}_n^{(n_s)}(t), \text{ for } t \in <0, \infty).$$
$$- P(S_D) = 1\text{-}\boldsymbol{CR}_n^{(n_D)}(t), \text{ for } t \in <0, \infty).$$

It means that

- probability that the system is in safety state is equal to probability that at least n_S neighbouring components are good;

- probability that the system is in dangerous state is equal to probability that at least n_D neighbouring components are failed.

6 APPLICATION

Let us consider the vessel waterway given in Fig 1.

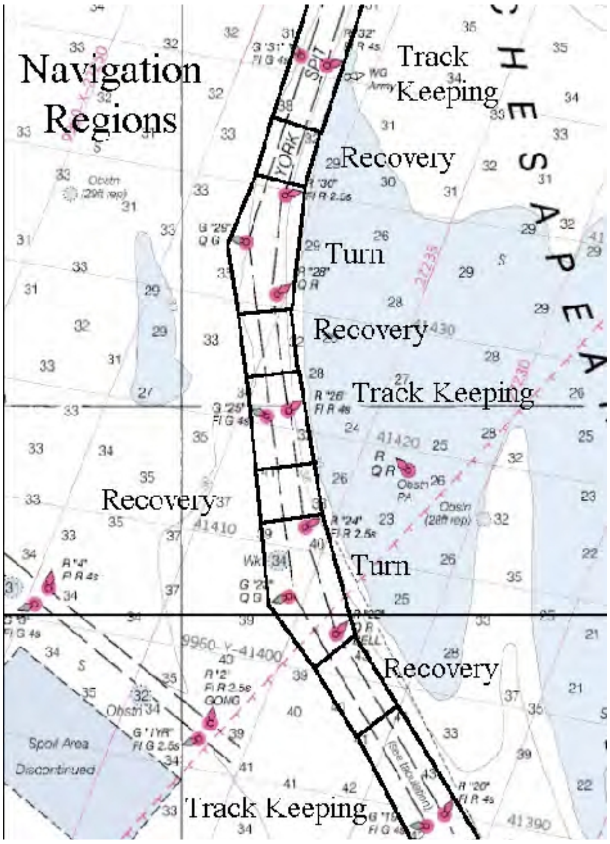

Figure 1. The vessel manoeuvring phases (IALA NAVIGUIDE 2006).

In particular case we have on the track 12 components of buoys system. We assume that for phase of track keeping ship operator need at least two navigational signs fo safety manoeuvring and in the phases of turn recovery the same operator need at least three signs. Thus, the number limits for safety states are give as

$n_S = 3,\ n_D = 2.$

Because the probabilities of buoys' visibility are the same, the probabilites of respective states are given as

$\boldsymbol{P(S_S)} = \boldsymbol{CRG}_{12}^{(3)}(t)$, where

- for $n < 3$

$$\boldsymbol{CRG}_1^{(3)}(t) = \boldsymbol{CRG}_2^{(3)}(t) = 0, \ \text{for } t \in < 0, \infty). \tag{5}$$

- for $n = 3$

$$\boldsymbol{CRG}_3^{(3)}(t) = [R(t)]^3, \text{ for } t \in <0, \infty). \tag{6}$$

- for $n > 3$

$$\boldsymbol{CRG}_n^{(3)}(t) = (1 - R(t))\big[\boldsymbol{CRG}_{n\text{-}1}^{(3)}(t)$$

$$- R(t)\boldsymbol{CRG}_{n\text{-}2}^{(m)}(t) - R^2(t)\boldsymbol{CRG}_{n\text{-}3}^{(m)}(t)$$

$$+ R(t) + R^2(t)\big] \text{ for } t \in <0, \infty). \tag{7}$$

And for

$$\boldsymbol{P(S_D)} = 1 - \boldsymbol{CR}_{12}^{(3)}(t), \text{ where}$$

- for $n < 2$

$$\boldsymbol{CR}_1^{(2)}(t) = 0, \ \text{ for } t \in <0, \infty). \tag{8}$$

- for $n = 2$

$$\boldsymbol{CR}_2^{(2)}(t) = 1 - 2R(t) + R^2(t), \text{ for } t \in <0, \infty). \tag{9}$$

- for $n > 3$

$$\boldsymbol{CR}_n^{(2)}(t) = 1 - R(t)\big[\boldsymbol{CR}_{n\text{-}1}^{(2)}(t) + (1 - R(t))\boldsymbol{CR}_{n\text{-}2}^{(2)}(t)\big], \tag{10}$$

for $t \in <0, \infty)$.

In particular case when the lifetimes of buoys have exponential distribution function of the form

$$F(t) = 1 - e^{-0.01t}, \text{ for } t \in <0, \infty),$$

i.e. if the reliability function of the particular buoys are given by

$$R(t) = e^{-0.01t}, \text{ for } t \in <0, \infty).$$

Considering (5)–(10), we get the following reccurent formula for the probabilities of safety states

a) safety state S_S

- for $n < 3$

$$\boldsymbol{CRG}_1^{(3)}(t) = \boldsymbol{CRG}_2^{(3)}(t) = 0, \ \text{ for } t \in <0, \infty). \tag{11}$$

- for $n = 3$

$$\boldsymbol{CRG}_3^{(3)}(t) = e^{-0.03t}, \text{ for } t \in <0, \infty). \tag{12}$$

- for $n > 3$

$$\boldsymbol{CRG}_n^{(3)}(t) = \left(1 - e^{-0.01t}\right)\left[\boldsymbol{CRG}_{n\text{-}1}^{(3)}(t)\right.$$
$$- e^{-0.01t}\boldsymbol{CRG}_{n\text{-}2}^{(m)}(t) - e^{-0.02t}\boldsymbol{CRG}_{n\text{-}3}^{(m)}(t)$$
$$\left. + e^{-0.01t} + e^{-0.02t}\right], \text{ for } t \in <0, \infty). \tag{13}$$

b) in the dangerous state S_D

- for $n < 2$

$$\boldsymbol{CR}_1^{(2)}(t) = 0, \quad \text{for } t \in <0, \infty). \tag{14}$$

- for $n = 2$

$$\boldsymbol{CR}_2^{(2)}(t) = 1 - 2e^{-0.01t} + e^{-0.02t}, \text{ for } t \in <0, \infty). \tag{15}$$

- for $n > 3$

$$\boldsymbol{CR}_n^{(2)}(t) = 1 - e^{-0.01t}\left[\boldsymbol{CR}_{n\text{-}1}^{(2)}(t) + \left(1 - e^{-0.01t}\right)\boldsymbol{CR}_{n\text{-}2}^{(2)}(t)\right], \tag{16}$$

for $t \in <0, \infty)$.

Then the values of the particular probabilities of the safety states, calculated by the computer program based on the formulae (11)-(16), are presented in the Tables 1-2 and illustrated in Figure 2.

Table 1. The values of probabilities of the dangerous state of navigational signs

t	$\boldsymbol{P(S_D)} = 1 - \boldsymbol{CR}_{12}^{(2)}(t)$
0.0	0.0000
5.0	0.0248
10.0	0.0885
15.0	0.1762
20.0	0.2753
25.0	0.3766
30.0	0.4737
35.0	0.5626
40.0	0.6415
45.0	0.7095
50.0	0.7671
55.0	0.8149
60.0	0.8541
65.0	0.8857
70.0	0.9111
75.0	0.9312
80.0	0.9470
85.0	0.9594
90.0	0.9690

95.0	0.9764
100.0	0.9821
105.0	0.9865
110.0	0.9898
115.0	0.9923
120.0	0.9942
125.0	0.9957
130.0	0.9968
135.0	0.9976
140.0	0.9982
145.0	0.9987
150.0	0.9990
155.0	0.9993
160.0	0.9995

Table 2. The values of probabilities of the safety state of navigational signs

t	$\boldsymbol{P(S_S) = CRG^{(3)}_{12}}(t)$
0.0	0.0000
50.0	0.3990
100.0	0.4637
150.0	0.4871
200.0	0.4833
250.0	0.4156
300.0	0.3151
350.0	0.2194
400.0	0.1447
450.0	0.0924
500.0	0.0578
550.0	0.0357
600.0	0.0219
650.0	0.0134
700.0	0.0082
750.0	0.0050
800.0	0.0030
850.0	0.0018
900.0	0.0011
950.0	0.0007
1000.0	0.0004

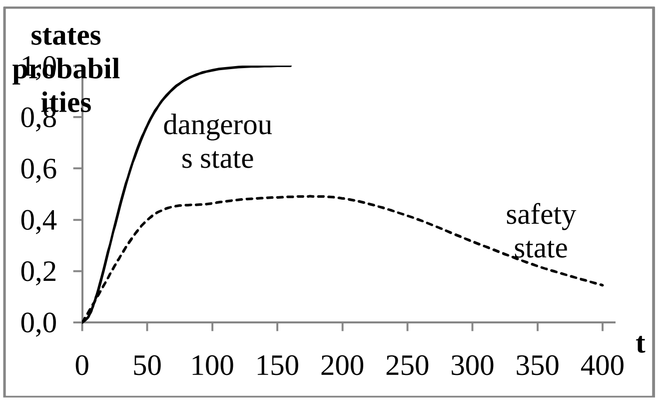

Figure 2. The graphs of particular states probabilities

7 CONCLUSIONS

The paper is devoted to an approach to safety analysis of ship in restricted waterways because of navigational infrastructure. The recurrent formulae for two-state reliability functions, a general one for non-homogeneous and its simplified form for homogeneous two-state consecutive "*m* out of *k*: G" systems have been proposed. The formulae for a homogeneous two-state consecutive "*m* out of *k*: F" and a homogeneous two-state consecutive "*m* out of *k*: G" has been applied to evaluation of ship safety in limited waterway.
Further, the safety model was used to the safety of ship on exemplary limited area with 12 navigational signs. The probabilities of respective states was evaluated and illustrated.
The transition probabilities between states depend of navigational signs technical reliability and waterway shape.
The calculated examples show us the possibilities of practical model usage.

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