SAFETY AND RISK EVALUATION OF STENA BALTICA FERRY IN VARIABLE OPERATION CONDITIONS

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ABSTRACT

Basic safety structures of multi-state systems of components with degrading safety states related to their variable operation conditions are defined. For these systems the conditional and unconditional multi-state safety functions are determined. A semi-markov process for the considered systems operation modelling is applied. Further, the paper offers an approach to the solution of a practically important problem of linking the multi-state systems safety models and the systems operation processes models.

Theoretical definitions and results are illustrated by the example of their application in the safety and risk evaluation of the Stena Baltica ferry operating at the Baltic Sea. The ferry transportation system has been considered in varying in time operation conditions. The system safety structure and its components safety functions were changing in variable operation conditions.

1 INTRODUCTION

Taking into account the importance of the safety and operating process effectiveness of technical systems it seems reasonable to expand the two-state approach to multi-state approach in their safety analysis. The assumption that the systems are composed of multi-state components with safety states degrading in time gives the possibility for more precise analysis and diagnosis of their safety and operational processes' effectiveness. This assumption allows us to distinguish a system safety critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operational process effectiveness. Then, an important system safety characteristic is the time to the moment of exceeding the system safety critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state safety function that is a basic characteristic of the multi-state system. Determining the multi-state safety function and the risk function of systems on the base of their components' safety functions is then the main research problem. Modelling of complicated systems operations' processes is difficult mainly because of large number of operations states and impossibility of precise describing of changes between these states. One of the useful approaches in modelling of these complicated processes is applying the semi-markov model (Grabski 2002). Modelling of multi-state systems' safety and linking it with semi-markov model of these systems' operation processes is the main and practically important problem of this paper. This new approach to system safety investigation is based on the multi-state system reliability analysis considered for instance in (Aven 1985, Kolowrocki 2004) and on semi-markov processes modeling discussed for instance in (Soszynska 2006, Soszynska 2007). This paper using the results of the report (Soszynska et all. 2007), is devoted to optimizing the multi-state safety function, the risk function of the ship technical system on the base of its components' safety functions and its variable in time operation process.

2 SYSTEM SAFETY IN VARIABLE OPERATION CONDITIONS

We assume that the system during its operation process has v different operation states. Thus we can define the system operation process $Z(t)$, $t \in <0,+\infty>$, as the process with discrete operation states from the set

$$Z = \{z_1, z_2, \ldots, z_v\}.$$

In practice a convenient assumption is that $Z(t)$ is a semi-markov process (Grabski 2002) with its conditional lifetimes θ_{bl} at the operation state z_b when its next operation state is z_l, $b, l = 1,2,...,v$, $b \neq l.$ In this case the process $Z(t)$ may be described by:

- the vector of probabilities of the process initial operation states $[p_b(0)]_{1xv}$,
- the matrix of the probabilities of the process transitions between the operation states $[p_{bl}]_{vxv}$, where $p_{bb}(t) = 0$ for $b = 1,2,..., v.$
- the matrix of the conditional distribution functions $[H_{bl}(t)]_{vxv}$ of the process lifetimes θ_{bl}, $b \neq l,$ in the operation state z_b when the next operation state is z_l, where $H_{bl}(t) = P(\theta_{bl} < t)$ for $b, l = 1,2,...,v,$ $b \neq l,$ and $H_{bb}(t) = 0$ for $b = 1,2,..., v.$

Under these assumptions, the lifetimes θ_{bl} mean values are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t), \ \ b, l = 1,2,...,v, \ b \neq l. \tag{1}$$

The unconditional distribution functions of the lifetimes θ_b of the process $Z(t)$ at the operation states z_b, $b = 1,2,..., v,$ are given by

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t), \ \ b = 1,2,..., v.$$

The mean values $E[\theta_b]$ of the unconditional lifetimes θ_b are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}, \ \ b = 1,2,..., v, \tag{2}$$

where M_{bl} are defined by (1).

Limit values of the transient probabilities at the operation states

$$p_b(t) = P(Z(t) = z_b), \ \ t \in <0,+\infty), \ \ b = 1,2,..., v,$$

are given by

$$p_b = \lim_{t \to \infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \ \ b = 1,2,..., v, \tag{3}$$

where the probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \quad (4)$$

We assume that the system is composed of n components E_i, $i = 1,2,...,n,$ and that the changes of the operation process $Z(t)$ states have an influence on the system components E_i safety and on the system safety structure as well.

Consequently, we denote the component E_i lifetime by $T_i^{(b)}$ and by

$$s_i^{(b)}(t, \cdot) = [1,\ s_i^{(b)}(t,1),\ s_i^{(b)}(t,2),\ ...,\ s_i^{(b)}(t,z)\,],$$

where for $t \in <0, \infty)$, $b = 1,2,...,v$, $u = 1,2,..., z$,

$$s_i^{(b)}(t,u) = P(T_i^{(b)}(u) > t \,|\, Z(t) = z_b),$$

its conditional safety function while the system is at the operational state z_b, $b = 1,2,...,v$.

Similarly, we denote the system lifetime by $T^{(b)}(u)$ and by

$$\boldsymbol{s}_{n_b}^{(b)}(t,\ \cdot) = [1,\ \boldsymbol{s}_{n_b}^{(b)}(t,1),\ \boldsymbol{s}_{n_b}^{(b)}(t,2),\ ...,\ \boldsymbol{s}_{n_b}^{(b)}(t,z)\,]$$

for $n_b \in \{1,2,..., n\},$ where n_b are numbers of components in the operation states z_b and for $t \in <0, \infty)$, $n_b \in \{1,2,..., n\}$, $b = 1,2,..., v$, $u = 1,2,..., z$,

$$\boldsymbol{s}_{n_b}^{(b)}(t,u), = P(T^{(b)}(u) > t \,|\, Z(t) = z_b).$$

the conditional safety function of the system while the system is at the operational state z_b, $b = 1,2,...,v$.

Thus, the safety function $s_i^{(b)}(t,u)$ is the conditional probability that the component E_i lifetime $T_i^{(b)}(u)$ in the state subset $\{u, u+1,..., z\}$ is not less than *t*, while the process *Z*(*t*) is at the operation state z_b. Similarly, the safety function $\boldsymbol{s}_{n_b}^{(b)}(t,u)$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the state subset $\{u, u+1,..., z\}$ is not less than *t*, while the process *Z*(*t*) is at the operation state z_b.

In the case when the system operation time is large enough, the unconditional safety function of the system is given by

$$\boldsymbol{s}_n(t,\ \cdot) = [1,\ \boldsymbol{s}_n(t,1),\ \boldsymbol{s}_n(t,2),\ ...,\ \boldsymbol{s}_n(t,z)\,],\ t \geq 0,$$

where

$$\boldsymbol{s}_n(t,u) = P(T(u) > t) \cong \sum_{b=1}^{v} p_b \boldsymbol{s}_{n_b}^{(b)}(t,u) \quad (5)$$

for $t \geq 0$, $n_b \in \{1,2,..., n\}$, $u = 1,2,..., z,$ and $T(u)$ is the unconditional lifetime of the system in the safety state subset $\{u, u+1,..., z\}$.

The mean values of the system lifetimes in the safety state subset $\{u, u+1,..., z\}$ are

$$\mu(u) = E[T(u)] \cong \sum_{b=1}^{v} p_b \mu_b(u),\ u = 1,2,...,\ z, \quad (6)$$

where (Lisnianski, 2003, Soszynska 2006)

$$\mu_b(u) = \int_0^\infty \boldsymbol{s}_{\boldsymbol{n_b}}^{(b)}(t,u)dt,\ \ n_b \in \{1,2,...,n\},\ u = 1,2,...,\ z. \quad (7)$$

The mean values of the system lifetimes in the particular safety states u, are (Kolowrocki 2004)

$$\bar{\mu}(u) = \mu(u) - \mu(u+1),\ u = 1,2,...,\ z-1,\ \bar{\mu}(z) = \mu(z). \quad (8)$$

3 THE STENA BALTICA FERRY DESCRIPTION

The m/v Stena Baltica is a passenger Ro-Ro ship operating in Baltic Sea between Gdynia and Karlskrona ports on regular everyday line. Her owner is Stena Line Scandinavia AB. She was build in Gdańsk Shipyard in 2005.

Figure 1. Stena Baltica Ferry

She is characterized by the following parameters: the length of 164.41m, the breadth moulded of 27.60 m, the summer load draft of 6.313 m, DWT of 4456, the displacement of 16618 tons, the cargo capacity of 466 cars, the total numbers of passengers and crew capacity of 1200 + 96 = 1296. The number of cabins is 379 with the number of beds 949 and total number of seats on a board is 981. The main engines are 4 of the kind MAN 4840 kW, the propellers are 2 of the kind Ka Me Wa with diameter 4800 mm, the BOW thrusters are 2 of the kind 1275 kW and 735 kW and the aft thruster is 1 of the kind 735 kW. The navigation and communication equipments are according to SOLAS Convention. The ferry speed is 19.5 knots (calm water) (RPM – 178). The service restriction are: maximum of 350 NM from land and wave height of 3.1 m, according to the Stockholm Agreement.

4 STENA BALTICA FERRY IN VARIABLE OPERATION CONDITIONS

We preliminarily assume that the Stena Baltica ferry is composed of five subsystems S_1, S_2, S_3, S_4, S_5 having an essential influence on her safety (Soszynska et all 2007). These subsystems are:

S_1 - a navigational subsystem,

S_2 - a propulsion and controlling subsystem,
S_3 - a loading and unloading subsystem,
S_4 - a hull subsystem,
S_5 - an anchoring and mooring subsystem,
S_6 - a protection and rescue subsystem,
S_7 - a social subsystem.

In our further ship safety analysis we will omit the protection and rescue subsystem S_6 and the social subsystem S_7 and we will consider its strictly technical subsystems S_1, S_2, S_3, S_4 and S_5 only.

Further, assuming that the ship is in the safety state subset $\{u,u+1,...,4\}$ if all its subsystems are in this subset of safety states and considering *Definition 3.4* (Kolowrocki 2004), we conclude that the ship is a series system of subsystems S_1, S_2, S_3, S_4, S_5 with a general scheme and detailed scheme presented respectively in *Figure2*.

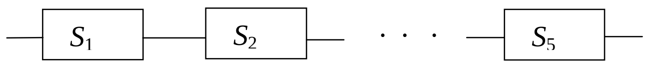

Figure 2. General scheme of ship safety structure

Taking into account the operation process of the considered ferry we distinguish the following as its eighteen operation states:

- an operation state z_1 – loading at Gdynia Port,
- an operation state z_2 – unmooring operations at Gdynia Port,
- an operation state z_3 – leaving Gdynia Port and navigation to “GD” buoy,
- an operation state z_4 – navigation at restricted waters from “GD” buoy to the end of Traffic Separation Scheme,
- an operation state z_5 – navigation at open waters from the end of Traffic Separation Scheme to “Angoring” buoy,
- an operation state z_6 – navigation at restricted waters from “Angoring” buoy to “Verko” Berth at Karlskrona,
- an operation state z_7 – mooring operations at Karlskrona Port,
- an operation state z_8 – unloading at Karlskrona Port,
- an operation state z_9 – loading at Karlskrona Port,
- an operation state z_{10} – unmooring operations at Karlskrona Port,
- an operation state z_{11} – ship turning at Karlskrona Port,
- an operation state z_{12} – leaving Karlskrona Port and navigation at restricted waters to “Angoring” buoy,
- an operation state z_{13} – navigation at open waters from “Angoring” buoy to the entering Traffic Separation Scheme,
- an operation state z_{14} – navigation at restricted waters from the entering Traffic Separation Scheme to “GD” buoy,
- an operation state z_{15} – navigation from “GD” buoy to turning area,
- an operation state z_{16} – ship turning at Gdynia Port,
- an operation state z_{17} – mooring operations at Gdynia Port,
- an operation state z_{18} – unloading at Gdynia Port.

On the basis of data coming from experts, the probabilities of transitions between the operation states are approximately given by

$$[p_{bl}] = \begin{bmatrix} 0\,1\,0 \ldots 0\,0 \\ 0\,0\,1 \ldots 0\,0 \\ \ldots \\ 0\,0\,0 \ldots 0\,1 \\ 1\,0\,0 \ldots 0\,0 \end{bmatrix},$$

and the conditional mean values of lifetimes in the operation states are:

$$M_{12} = 54.33,\ M_{23} = 2.57,\ M_{34} = 36.57,\ M_{45} = 52.5,\ M_{56} = 525.95,\ M_{67} = 37.16,$$

$$M_{78} = 7.02,\ M_{89} = 21.43,\ M_{910} = 53.69,\ M_{1011} = 2.93,\ M_{1112} = 4.38,\ M_{1213} = 23.86,$$

$$M_{1314} = 509.69,\ M_{1415} = 50.14,\ M_{1516} = 34.28,\ M_{1617} = 4.52,\ M_{1718} = 5.62,\ M_{181} = 18.74.$$

Hence, by (2), the unconditional mean lifetimes in the operation states are:

$$M_1 = 54.33,\ M_2 = 2.57,\ M_3 = 36.57,\ M_4 = 52.5,\ M_5 = 525.95,\ M_6 = 37.16,$$

$$M_7 = 7.02,\ M_8 = 21.43,\ M_9 = 53.69,\ M_{10} = 2.93,\ M_{11} = 4.38,\ M_{12} = 23.86,$$

$$M_{13} = 509.69,\ M_{14} = 50.14,\ M_{15} = 34.28,\ M_{16} = 4.52,\ M_{17} = 5.62,\ M_{18} = 18.74.$$

Since from the system of equations (4) we get

$$\pi_1 = \pi_2 = \pi_3 = \pi_4 = \pi_5 = \pi_6 = \pi_7 = \pi_8 = \pi_9 = \pi_{10} = \pi_{11} = \pi_{12} = \pi_{13} = \pi_{14} =$$

$$\pi_{15} = \pi_{16} = \pi_{17} = \pi_{18} = 0.056,$$

then the limit values of the transient probabilities $p_b(t)$ at the operational states z_b, according to (3), are given by

$$p_1 = 0.037,\ p_2 = 0.002,\ p_3 = 0.025,\ p_4 = 0.036,\ p_5 = 0.364,\ p_6 = 0.025,\ p_7 = 0.005,\ p_8 = 0.014,$$

$$p_9 = 0.037,\ p_{10} = 0.002,\ p_{11} = 0.003,\ p_{12} = 0.017,\ p_{13} = 0.354,\ p_{14} = 0.035,\ p_{15} = 0.024,\ p_{16} = 0.003,$$

$$p_{17} = 0.004,\ p_{18} = 0.013. \tag{9}$$

We assume as earlier that the ship is composed of $n = 5$ subsystems S_i, $i = 1,2,...,5$, and that the changes of the process of ship operation states have an influence on the system subsystems S_i safety and on the ship safety structure as well. The subsystems S_i, $i = 1,2,3,4,5$ are composed of five-state components, i.e. $z = 4$, with the multi-state safety functions

$$s_i^{(b)}(t,\cdot) = [1,\ s_i^{(b)}(t,1),\ s_i^{(b)}(t,2),\ s_i^{(b)}(t,3),\ s_i^{(b)}(t,4)\,],\ t\in<0,\infty),\ b=1,2,...,18,\ u=1,2,3,4,$$

with exponential co-ordinates different in various operation states z_b, $b=1,2,...,18$.

In (Soszynska et all 2007), on the basis of expert opinions concerned with the safety of the ship components the ship safety function in different operation conditions are determined.

At the operation state z_1, i.e. at the loading state the ship is built of $n_1=2$ subsystems S_3 and S_4 forming a series structure (Kolowrocki 2004) shown in *Figure 3*.

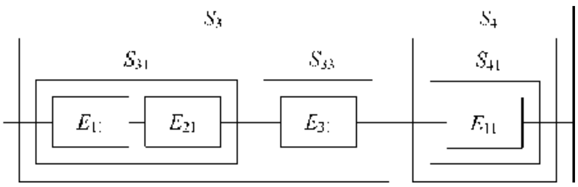

Figure 3. The scheme of the ship structure at the operation state z_1

Considering that the ship is in the safety state subsets $\{u,u+1,...,4\}$, $u=1,2,3,4,$ if all its subsystems are in this safety state subset, according to *Definition 3.4* (Kolowrocki 2004), the considered system is a five-state series system and the conditional safety function of the ship while the ship is at the operational state z_1 is given by

$$\bar{\boldsymbol{s}}_2^{(1)}(t,\cdot) = [1,\ \bar{\boldsymbol{s}}_2^{(1)}(t,1),\ \bar{\boldsymbol{s}}_2^{(1)}(t,2),\ \bar{\boldsymbol{s}}_2^{(1)}(t,3),\ \bar{\boldsymbol{s}}_2^{(1)}(t,4)\,], \quad (10)$$

where

$$\bar{\boldsymbol{s}}_2^{(1)}(t,u) = \boldsymbol{s}_{3;1,1,1}^{(1)}(t,u)\ \boldsymbol{s}_{1,1}^{(1)}(t,u) \ \text{ for } t\in<0,\infty),\ u=1,2,3,4, \quad (11)$$

i.e.

$$\bar{\boldsymbol{s}}_2^{(1)}(t,1) = \exp[-0.433t]\exp[-0.05t] = \exp[-0.483t], \quad (12)$$

$$\bar{\boldsymbol{s}}_2^{(1)}(t,2) = \exp[-0.59t]\exp[-0.06t] = \exp[-0.65t] \quad (13)$$

$$\bar{\boldsymbol{s}}_2^{(1)}(t,3) = \exp[-0.695t]\exp[-0.065t] = \exp[-0.76t], \quad (14)$$

$$\bar{\boldsymbol{s}}_2^{(1)}(t,4) = \exp[-0.85t]\exp[-0.07t] = \exp[-0.92t]. \quad (15)$$

The expected values and standard deviations of the ship conditional lifetimes in the safety state subsets calculated from the above result given by (10)-(15), according to (7), at the operational state z_1 are:

$$\mu_1(1)\cong 2.07,\ \mu_1(2)\cong 1.54,\ \mu_1(3)\cong 1.32,\ \mu_1(4)\cong 1.09 \text{ years}, \quad (16)$$

$$\sigma_1(1)\cong 2.07,\ \sigma_1(2)\cong 1.54,\ \sigma_1(3)\cong 1.32,\ \sigma_1(4)\cong 1.09 \text{ years}, \quad (17)$$

and further, using (8), it follows that the ship conditional lifetimes in the particular safety states at the operational state z_1 are:

$$\bar{\mu}_1(1) \cong 0.53,\ \bar{\mu}_1(2) \cong 0.22,\ \bar{\mu}_1(3) \cong 0.23,\ \bar{\mu}_1(4) \cong 1.09 \text{ years.} \quad (18)$$

At the operation states z_2, i.e. at the cargo loading and un-loading state the ship is built of $n_2 = 3$ subsystems S_1, S_2 and S_5 forming a series structure (Kolowrocki 2004) shown in Figure 4.

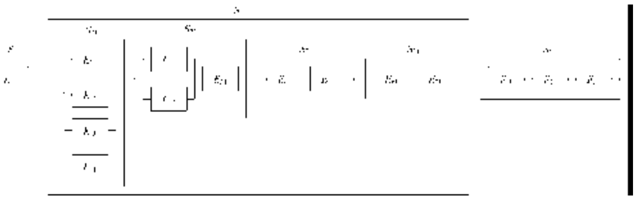

Figure 4. The scheme of the ship structure at the operation state z_2

Considering that the ship is in the safety state subsets $\{u, u+1,...,4\}$, $u = 1,2,3,4,$ if all its subsystems are in this safety state subset, according to *Definition 3.4* (Kolowrocki 2004), the considered system is a five-state series system and the conditional safety function of the ship while the ship is at the operational state z_2 is given by

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ \cdot) = [1, \bar{\boldsymbol{s}}_3^{(2)}(t,\ 1), \bar{\boldsymbol{s}}_3^{(2)}(t,\ 2), \bar{\boldsymbol{s}}_3^{(2)}(t,\ 3), \bar{\boldsymbol{s}}_3^{(2)}(t,\ 4)], (19)$$

where

$$\bar{\boldsymbol{s}}_3^{(2)}(t,u) = \boldsymbol{s}_{1,1}^{(2)}(t,\ u)\ \boldsymbol{s}_{7;4,2,1,1,1,1,1}^{(2)}(t,\ u)\ \boldsymbol{s}_{3;1,1,1}^{(2)}(t,\ u) \text{ for } t \in <0,\infty),\ u = 1,2,3,4, \quad (20)$$

i.e.

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 1) = \exp[-0.033t][\ 12\exp[-0.33t] + 8\exp[-0.429t]-16\exp[-0.363t]$$

$$- 3\exp[-0.462t]]\exp[-0.099t]= 12\exp[-0.462t] + 8\exp[-0.561t]$$

$$-16\exp[-0.495t]- 3\exp[-0.594t] \quad (21)$$

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 2) = \exp[-0.04t][\ 12\exp[-0.38t] + 8\exp[-0.49t] + 6\exp[-0.46t]$$

$$- 16\exp[-0.42t] - 6\exp[-0.45t] - 3\exp[-0.53t]]\exp[-0.12t]= 12\exp[-0.54t] + 8\exp[-0.65t]$$

$$+ 6\exp[-0.62t] - 16\exp[-0.58t] - 6\exp[-0.61t] - 3\exp[-0.69t], \quad (22)$$

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 3) = \exp[-0.045t][\ 12\exp[-0.43t] + 8\exp[-0.555t] + 6\exp[-0.53t]$$

$$- 16\exp[-0.48t] - 6\exp[-0.505t] - 3\exp[-0.605t]]\exp[-0.145t]= 12\exp[-0.62t] + 8\exp[-0.745t]$$

$$+ 6\exp[-0.72t] - 16\exp[-0.67t] - 6\exp[-0.695t] - 3\exp[-0.795t], \quad (23)$$

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 4) = \exp[-0.05t][\ 12\exp[-0.47t] + 8\exp[-0.605t] + 6\exp[-0.58t]$$

$$- 16\exp[-0.525t] - 6\exp[-0.55t] - 3\exp[-0.66t]]\exp[-0.165t]= 12\exp[-0.685t] + 8\exp[-0.82t]$$

$$+ 6 \exp[-0.795t] - 16 \exp[-0.74t] - 6 \exp[-0.765t] - 3 \exp[-0.875t]. \quad (24)$$

The expected values and standard deviations of the ship conditional lifetimes in the safety state subsets calculated from the above result given by (19)-(24), according to (7), at the operational state z_2 are:

$$\mu_2(1) \cong 2.86, \ \mu_2(2) \cong 0.43, \mu_2(3) \cong 2.14, \ \mu_2(4) \cong 1.93 \text{ years}, \quad (25)$$

$$\sigma_2(2) \cong 2.74, \ \sigma_2(2) \cong 2.35, \ \sigma_2(3) \cong 2.05, \ \sigma_2(4) \cong 1.85 \text{ years}, \quad (26)$$

and further, using (8), it follows that the ship conditional lifetimes in the particular safety states at the operational state z_2 are:

$$\bar{\mu}_2(1) \cong 0.43, \ \bar{\mu}_2(2) \cong 0.29, \ \bar{\mu}_2(3) \cong 0.21, \ \bar{\mu}_2(4) \cong 1.93 \text{ years}. \quad (27)$$

At the remaining operation states z_3, z_4, z_5, z_6, $z_7, z_8, z_9, z_{10}, z_{11}, z_{12}, z_{13}, z_{14}, z_{15}, z_{16}, z_{17}$ and z_{18} we proceed in an analogous way. We determined the system conditional safety functions in particular operation states and the expected values and standard deviations of the ship conditional lifetimes.

In the case when the system operation time is large enough, the unconditional safety function of the ship is given by the vector

$$\boldsymbol{s}_5(t, \cdot) = [1, \ \boldsymbol{s}_5(t,1), \ \boldsymbol{s}_5(t,2), \ \boldsymbol{s}_5(t,3), \ \boldsymbol{s}_5(t,4)], \ t \geq 0, \quad (28)$$

where, according to (5) and after considering the values of p_b, $b = 1,2,...,18$, given by (9), its co-ordinates are as follows:

$$\boldsymbol{s}_5(t,1) = 0.037 \cdot \bar{\boldsymbol{s}}_2^{(1)}(t,1) + 0.002 \cdot \bar{\boldsymbol{s}}_3^{(2)}(t,1) + 0.025 \cdot \bar{\boldsymbol{s}}_2^{(3)}(t,1) + 0.036 \cdot \bar{\boldsymbol{s}}_3^{(4)}(t,1) + 0.364 \cdot \bar{\boldsymbol{s}}_3^{(5)}(t,1)$$

$$+ 0.025 \cdot \bar{\boldsymbol{s}}_3^{(6)}(t,1) + 0.005 \cdot \bar{\boldsymbol{s}}_3^{(7)}(t,1) + 0.014 \cdot \bar{\boldsymbol{s}}_2^{(8)}(t,1) + 0.037 \cdot \bar{\boldsymbol{s}}_2^{(9)}(t,1) + 0.002 \cdot \bar{\boldsymbol{s}}_3^{(10)}(t,1)$$

$$+ 0.003 \cdot \bar{\boldsymbol{s}}_2^{(11)}(t,1) + 0.017 \cdot \bar{\boldsymbol{s}}_3^{(12)}(t,1) + 0.354 \cdot \bar{\boldsymbol{s}}_3^{(13)}(t,1) + 0.035 \cdot \bar{\boldsymbol{s}}_3^{(14)}(t,1)$$

$$+ 0.024 \cdot \bar{\boldsymbol{s}}_2^{(15)}(t,1) + 0.003 \cdot \bar{\boldsymbol{s}}_2^{(16)}(t,1) + 0.004 \cdot \bar{\boldsymbol{s}}_3^{(17)}(t,1) + 0.013 \cdot \bar{\boldsymbol{s}}_2^{(18)}(t,1), \quad (29)$$

$$\boldsymbol{s}_5(t,2) = 0.037 \cdot \bar{\boldsymbol{s}}_2^{(1)}(t,2) + 0.002 \cdot \bar{\boldsymbol{s}}_3^{(2)}(t,2) + 0.025 \cdot \bar{\boldsymbol{s}}_2^{(3)}(t,2) + 0.036 \cdot \bar{\boldsymbol{s}}_3^{(4)}(t,2)$$

$$+ 0.364 \cdot \bar{\boldsymbol{s}}_3^{(5)}(t,2) + 0.025 \cdot \bar{\boldsymbol{s}}_3^{(6)}(t,2) + 0.005 \cdot \bar{\boldsymbol{s}}_3^{(7)}(t,2) + 0.014 \cdot \bar{\boldsymbol{s}}_2^{(8)}(t,2) + 0.037 \cdot \bar{\boldsymbol{s}}_2^{(9)}(t,2)$$

$$+ 0.002 \cdot \bar{\boldsymbol{s}}_3^{(10)}(t,2) + 0.003 \cdot \bar{\boldsymbol{s}}_2^{(11)}(t,2) + 0.017 \cdot \bar{\boldsymbol{s}}_3^{(12)}(t,2) + 0.354 \cdot \bar{\boldsymbol{s}}_3^{(13)}(t,2)$$

$$+ 0.035 \cdot \bar{\boldsymbol{s}}_3^{(14)}(t,2) + 0.024 \cdot \bar{\boldsymbol{s}}_2^{(15)}(t,2) + 0.003 \cdot \bar{\boldsymbol{s}}_2^{(16)}(t,2) + 0.004 \cdot \bar{\boldsymbol{s}}_3^{(17)}(t,2) + 0.013 \cdot \bar{\boldsymbol{s}}_2^{(18)}(t,2), \quad (30)$$

$$\boldsymbol{s}_5(t,3) = 0.037 \cdot \bar{\boldsymbol{s}}_2^{(1)}(t,3) + 0.002 \cdot \bar{\boldsymbol{s}}_3^{(2)}(t,3) + 0.025 \cdot \bar{\boldsymbol{s}}_2^{(3)}(t,3) + 0.364 \cdot \bar{\boldsymbol{s}}_3^{(5)}(t,3) + 0.025 \cdot \bar{\boldsymbol{s}}_3^{(6)}(t,3)$$

$$+0.005 \cdot \bar{s}_3^{(7)}(t,3) + 0.014 \cdot \bar{s}_2^{(8)}(t,3) + 0.037 \cdot \bar{s}_2^{(9)}(t,3) + 0.002 \cdot \bar{s}_3^{(10)}(t,3)$$

$$+0.003 \cdot \bar{s}_2^{(11)}(t,3) + 0.017 \cdot \bar{s}_3^{(12)}(t,3) + 0.354 \cdot \bar{s}_3^{(13)}(t,3) + 0.035 \cdot \bar{s}_3^{(14)}(t,3)$$

$$+0.024 \cdot \bar{s}_2^{(15)}(t,3) + 0.003 \cdot \bar{s}_2^{(16)}(t,3) \quad + 0.004 \cdot \bar{s}_3^{(17)}(t,3) + 0.013 \cdot \bar{s}_2^{(18)}(t,3), \quad (31)$$

$$\boldsymbol{s}_5(t,4) = 0.037 \cdot \bar{s}_2^{(1)}(t,4) + 0.002 \cdot \bar{s}_3^{(2)}(t,4) + 0.025 \cdot \bar{s}_2^{(3)}(t,4) + 0.036 \cdot \bar{s}_3^{(4)}(t,4) + 0.364 \cdot \bar{s}_3^{(5)}(t,4)$$

$$+0.025 \cdot \bar{s}_3^{(6)}(t,4) + 0.005 \cdot \bar{s}_3^{(7)}(t,4) + 0.014 \cdot \bar{s}_2^{(8)}(t,4) + 0.037 \cdot \bar{s}_2^{(9)}(t,4) + 0.002 \cdot \bar{s}_3^{(10)}(t,4)$$

$$+0.003 \cdot \bar{s}_2^{(11)}(t,4) + 0.017 \cdot \bar{s}_3^{(12)}(t,4) + 0.354 \cdot \bar{s}_3^{(13)}(t,4) + 0.035 \cdot \bar{s}_3^{(14)}(t,4) + 0.024 \cdot \bar{s}_2^{(15)}(t,4)$$

$$+0.003 \cdot \bar{s}_2^{(16)}(t,4) + 0.004 \cdot \bar{s}_3^{(17)}(t,4) + 0.013 \cdot \bar{s}_2^{(18)}(t,4) \text{ for } t \geq 0, \quad (32)$$

where

$\bar{s}_{n_b}^{(b)}(t,u)$, $u = 1,2,3,4$, $b = 1,2,\ldots,18$, are given in (Soszynska et all 2007)

The mean values and standard deviations of the system unconditional lifetimes in the safety state subsets, according to (6)-(7) respectively are:

$$\mu(1) \cong 0.037 \cdot 2.07 + 0.002 \cdot 2.86 + 0.025 \cdot 4.94 + 0.036 \cdot 4.2 + 0.364 \cdot 4.2 + 0.025 \cdot 4.01$$

$$+0.005 \cdot 2.86 + 0.014 \cdot 3.53 + 0.037 \cdot 3.53 + 0.002 \cdot 2.86 + 0.003 \cdot 3.91 + 0.017 \cdot 4.2$$

$$+0.354 \cdot 4.2 + 0.035 \cdot 4.2 + 0.024 \cdot 4.94 + 0.003 \cdot 3.91 + 0.004 \cdot 2.86 + 0.013 \cdot 2.07$$

$$\cong 4.07, \quad (33)$$

$$\sigma(1) \cong 4.1,$$

$$\mu(2) \cong 0.037 \cdot 1.54 + 0.002 \cdot 2.43 + 0.025 \cdot 3.9 + 0.036 \cdot 3.80 + 0.364 \cdot 3.80 + 0.025 \cdot 3.24$$

$$+0.005 \cdot 2.43 + 0.014 \cdot 2.50 + 0.037 \cdot 2.50 + 0.002 \cdot 2.43 + 0.003 \cdot 3.37 + 0.017 \cdot 3.80$$

$$+0.354 \cdot 3.80 + 0.035 \cdot 3.80 + 0.024 \cdot 3.90 + 0.003 \cdot 3.37 + 0.004 \cdot 2.43$$

$$\cong 3.59, \quad (34)$$

$$\sigma(2) \cong 3.34,$$

$$\mu(3) \cong 0.037 \cdot 1.32 + 0.002 \cdot 2.14 + 0.025 \cdot 3.44 + 0.036 \cdot 3.38 + 0.364 \cdot 3.38 + 0.025 \cdot 2.88$$

$$+0.005 \cdot 2.14 + 0.014 \cdot 2.17 + 0.037 \cdot 2.17 + 0.002 \cdot 2.14 + 0.003 \cdot 3.07 + 0.017 \cdot 3.38$$

$$+0.354 \cdot 3.38 + 0.035 \cdot 3.38 + 0.024 \cdot 3.44 + 0.003 \cdot 3.07 + 0.004 \cdot 2.14 + 0.013 \cdot 1.32$$

$$\cong 3.19, \tag{35}$$

$$\sigma(3) \cong 3.65,$$

$$\mu(4) \cong 0.037 \cdot 1.09 + 0.002 \cdot 1.93 + 0.025 \cdot 3.1 + 0.036 \cdot 3.05 + 0.364 \cdot 3.05 + 0.025 \cdot 2.61$$

$$+ 0.005 \cdot 1.93 + 0.014 \cdot 1.92 + 0.037 \cdot 1.92 + 0.002 \cdot 1.93 + 0.003 \cdot 2.76 + 0.017 \cdot 3.05$$

$$+ 0.354 \cdot 3.05 + 0.035 \cdot 3.05 + 0.024 \cdot 3.10 + 0.003 \cdot 2.76 + 0.004 \cdot 1.93 + 0.013 \cdot 1.09$$

$$\cong 2.87, \tag{36}$$

$$\sigma(4) \cong 2.75.$$

The mean values of the system lifetimes in the particular safety states, by (8), are

$$\overline{\mu}(1) = \mu(1) - \mu(2) = 0.48, \ \overline{\mu}(2) = \mu(2) - \mu(3) = 0.4,$$

$$\overline{\mu}(3) = \mu(3) - \mu(4) = 0.32, \ \ \overline{\mu}(4) = \mu(4) = 2.87. \tag{37}$$

If the critical safety state is r = 2, then the system risk function, according to (12) (Kolowrocki, Soszynska 2008) , is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{s}_5(t,2) = 1 - [0.037 \cdot \overline{\boldsymbol{s}}_2^{(1)}(t,2) + 0.002 \cdot \overline{\boldsymbol{s}}_3^{(2)}(t,2) + 0.025 \cdot \overline{\boldsymbol{s}}_2^{(3)}(t,2) + 0.036 \cdot \overline{\boldsymbol{s}}_3^{(4)}(t,2)$$

$$+ 0.364 \cdot \overline{\boldsymbol{s}}_3^{(5)}(t,2) + 0.025 \cdot \overline{\boldsymbol{s}}_3^{(6)}(t,2) + 0.005 \cdot \overline{\boldsymbol{s}}_3^{(7)}(t,2) + 0.014 \cdot \overline{\boldsymbol{s}}_2^{(8)}(t,2)$$

$$+ 0.037 \cdot \overline{\boldsymbol{s}}_2^{(9)}(t,2) + 0.002 \cdot \overline{\boldsymbol{s}}_3^{(10)}(t,2) + 0.003 \cdot \overline{\boldsymbol{s}}_2^{(11)}(t,2) + 0.017 \cdot \overline{\boldsymbol{s}}_3^{(12)}(t,2)$$

$$+ 0.354 \cdot \overline{\boldsymbol{s}}_3^{(13)}(t,2) + 0.035 \cdot \overline{\boldsymbol{s}}_3^{(14)}(t,2) + 0.024 \cdot \overline{\boldsymbol{s}}_2^{(15)}(t,2) + 0.003 \cdot \overline{\boldsymbol{s}}_2^{(16)}(t,2)$$

$$+ 0.004 \cdot \overline{\boldsymbol{s}}_3^{(17)}(t,2) + 0.013 \cdot \overline{\boldsymbol{s}}_2^{(18)}(t,2)] \quad \text{for } t \geq 0. \tag{38}$$

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (13) (Kolowrocki, Soszynska 2008), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.19 \text{ years.} \tag{39}$$

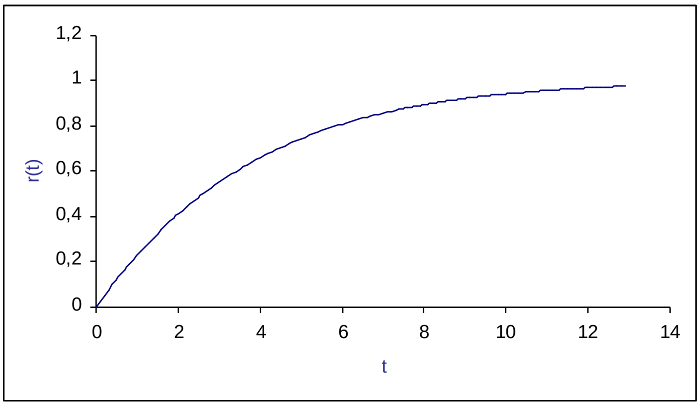

Figure 5. A graph of a risk function $r(t)$ of the ship

5 CONCLUSION

In the paper the multi-state approach (Kolowrocki 2004) to the safety analysis and evaluation of systems related to their variable operation processes has been considered. The ship safety structure and its safety subsystems characteristics are changing in different states what makes the analysis more complicated. A semi-markov model (Grabski 2002) of this ferry operation process is applied and its parameters statistical identification is performed. The Stena Baltca ferry operation process is analyzed and its operation states are defined. Preliminary collected statistical data is applied to the ferry operation process identification. Basic safety structures of multi-state systems of components with degrading safety states related to their variable operation conditions are applied to the considered ferry safety determination. For the ferry technical subsystems the conditional and unconditional multi-state safety functions are determined. The proposed approach to the solution of a practically important problem of linking the multi-state systems safety models and the systems operation processes models is applied to the preliminary evaluation of the safety function, the risk function and other safety characteristics of the Stena Baltica ferry operating with varying in time her structure and safety characteristics of the subsystems and components it is composed. The system safety structures are fixed generally with not high accuracy in details concerned with the subsystems structures because of their complexity and concerned with the components safety characteristics because of the luck of statistical data necessary for their estimation. Whereas, the input characteristics of the ferry operation process are of high quality because of the very good statistical data necessary for their estimation.

The results presented in the paper suggest that it seems reasonable to continue the investigations focusing on the methods of safety analysis for other more complex multi-state systems and the methods of safety evaluation related to the multi-state systems in variable operation processes and their applications to the ship transportation systems (Soszynska et all 2007).

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