MULTI-LINE MARKOV CLOSED QUEUING SYSTEM FOR TWO MAINTENANCE OPERATIONS

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ABSTRACT

In the given paper multi-component standby system with renewable elements is considered. For it multi-line closed Markov queuing model for two maintenance operations – replacements and renewals, is constructed and investigated. In this model the numbers of main elements as well as standby ones, also the numbers of replacement units as wall as renewal ones are arbitrary. An economic criterion for dependability planning (structural control) of considered system is introduced, the optimization problem is stated and partially investigated.

1. INTRODUCTION

The investigation of the problem of construction and investigation of maintenance models for multi-component systems is one of the topical directions in the modern reliability theory [10], [5]. This problem plays a key role in the dependability planning (structural control by economic criteria) of telecommunication and production systems [1], [4]. Let's consider the last statement in detail.

Since the beginning of the second half of the 80ies of the last century considerable changes have taken place in the field of telecommunication, affecting the interests of telecommunication service providers, as well as users and equipment manufacturers.

Here we mean de-monopolization in the field of telecommunication services in almost all developed countries of the world; creation of open market of traditional and new types of service and as a sequence, intensification of competition between telecommunication service providers; strengthening of requirements on the part of users; rapid development and inculcation of new technologies, architectures, etc.

The result of these changes is a considerable increase of interest to the problems of dependability of telecommunication networks and their components.

The essence of this is: low dependability threatens telecommunication service providers with not only possible loss of clients, but also directly affects the economical indices. The refuse of service to users, because of non-serviceability of telecommunication means, is expressed in lost income and frequently in direct losses caused by penal sanctions claimed by users.

Naturally, this problem was in the focus of ITU (International Telecommunications Union). Namely, ITU has issued recommendation E.862 "Dependability planning of telecommunication networks" (Geneva, 1992), which by ITU's assignment was prepared by Swedish experts.

Recommendation E.862 is concerned with models and methods for dependability planning, operation and maintenance of telecommunication networks and the application of these methods to various services in international network.

For all that, this Recommendation gives quite convincing preference to analytical methods for dependability planning, compared to other methods.

In particular, it is ascertained that the application of the analytical methods gives economically the best-balanced level of dependability, seen from the customer's point of view. This reduces the risk of customer's complaints and loss of business to competitors, as well as the risk of unnecessary investments. It is, therefore, considered to be the best general way of planning for administration, as well as for customers[5].

After this ITU has worked out a lot of documents on the problems of telecommunication service quality and dependability. According to ITU documents Quality of Service (QoS) is the degree of service a provider performs for a client, including the existing agreement relation between them(Service Level Agreement -SLA).

SLA between service providers and users regulates technical, organizational, legal and financial aspects and is becoming an important factor in competition in the developed countries.

A substantial part of SLA is the list of dependability indices and the values the guarantee of their provision being given by the provider. And this is not accidental, as dependability is one of the most important determining factor of quality.

Thus, service provider, i.e., company that leases telecommunication channels has to fundamentally study his possibilities, collect and process statistic data so that to have full idea about the quality of the provided service. Only in this way is it possible to rationally estimate the given guarantees, compensation forms and size he will have to pay in behalf of the client if SLA demands will be broken. In these conditions the optimization of dependability and structure of telecommunication networks and their components acquire particularly great importance.

Described situation, along with other new important developments (circumstances) yet again confirms necessity to construct and research complex systems' reliability and maintenance new, more adequate models.

Simultaneously, until now, models of standby multi-component systems with repairable elements, that would take into account a number of very essential factors, have been developed neither in the mathematical theory of reliability and maintenance nor in the queuing theory, which is the theoretical basis of the reliability models for complex repairable systems.

One of such factors is the length of time, required for the replacement of the failed element in a complex system. The necessity of consideration of this factor has long been emphasized by the leading reliability theory and practice experts [8], [9], [2]. However, only a few simple cases have so far been investigated in this direction. Simultaneously, modern methods of mathematical theory of reliability allow to construct and investigate such models [6], [7].

The matter is that in a majority of cases in which the reliability is investigated, the replacement of the failed element is not regarded as a separate operation.

For relatively simple systems there is no need for making such an assumption, since the replacement operation can be included in the complex maintenance operation, which is a sequence of two operations: repair and replacement.

However, after the failure of some element in renewable multi-element standby systems, the necessity of its replacement by a serviceable standby one comes to the foreground and thus the replacement is quite naturally distinguished as an independent maintenance operation.

In other cases it is supposed that the replacement time length is essentially smaller, than the repair time length, and therefore can be neglected (instant replacement).

But in many systems, especially in complex production ones, the time length of the replacement operation frequently has the same order as the repair time length and may even exceed it. Therefore the assumption about an instant replacement is quite rough. Such an assumption is especially inadmissible in the conditions of a generalized interpretation of the notions of the reliability theory given below.

Namely, the failure of some element is understood as the occurrence of an event when this element cannot execute the definite category of tasks with stated priority. This may be caused not only by the loss of element serviceability, but also by other factors, for example, when an element is switched over to the execution of higher priority tasks, readjustment, heating and so on. Within the framework of such an approach, under the repair time we mean the length of time during which an element is unserviceable in a generalized sense, i.e. is not able to fulfil the above-mentioned flow of tasks. From this generalized interpretation of the notions of failure and repair we easily come to the notion of standby and other notions in a generalized sense. For the background and usefulness of such an interpretation see [3].

2. SYSTEM DESCRIPTION

The basic model in the form of a closed queuing system with two types of service operations is described as follows.

The technical system consists of m main and n standby elements. All elements are identical. It is supposed that for the normal operation of the system, the serviceability of all m main elements is desired. However, if their number is less than m, then the system continues to function but with lower economic effectiveness. The main elements fail with intensity α and the standby ones - with intensity β. A failed main element is replaced by a serviceable standby one if there is such a possibility in the system. In the opposite case (all standby elements are non-serviceable, or the serviceable standby elements are already intended for the replacement of earlier failed main elements) the replacement will be carried out as soon as it becomes possible. The failed elements, both the main and the standby ones, are repaired and become identical to the new ones. There are k and replacement and l repair units in the system. The time lengths of replacement and repair operations have distribution functions $F(x)$ and $G(x)$, respectively(Figure 1).

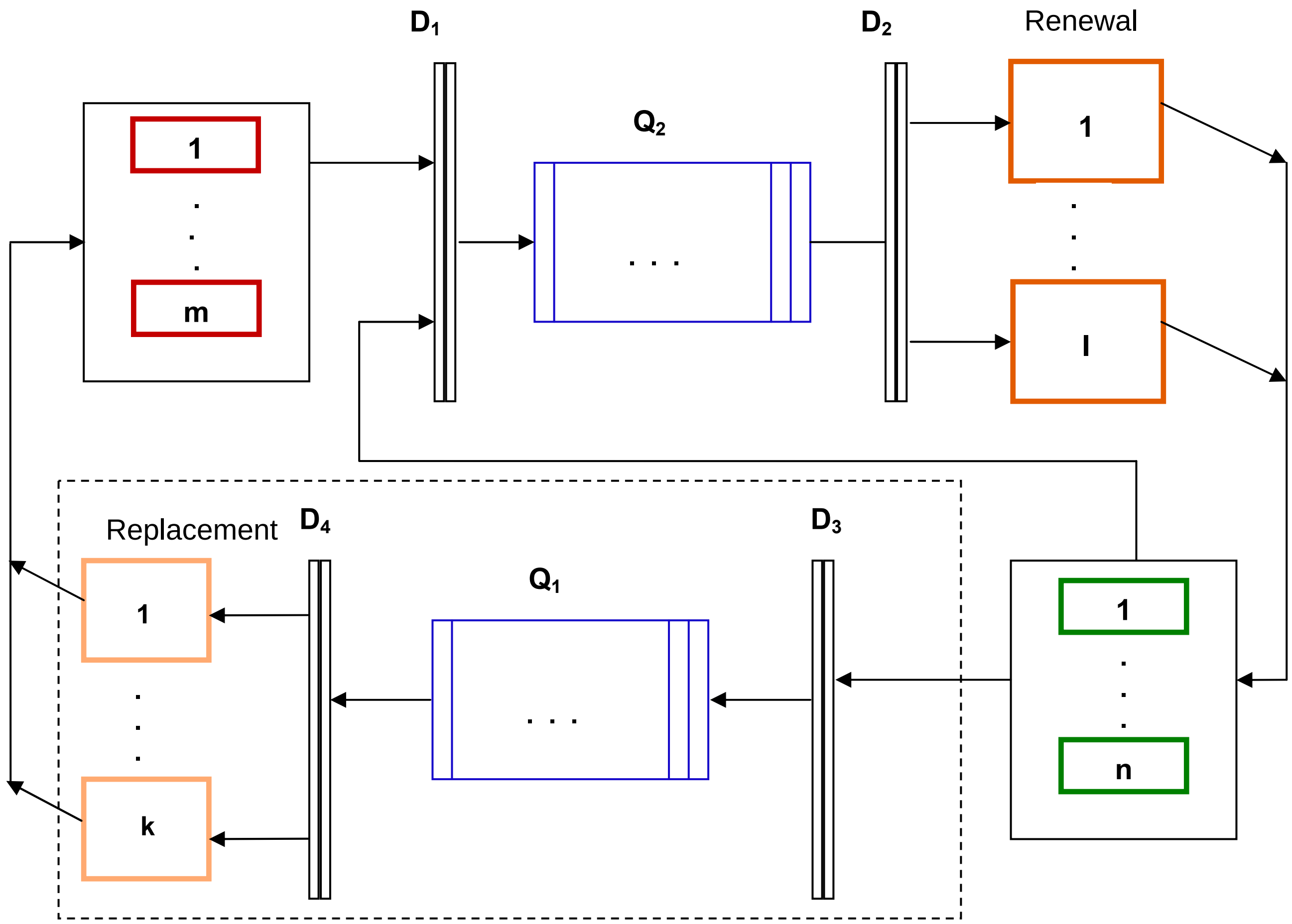

Figure 1.. General Scheme of Closed Queuing System for Replacements and Renewals

D_1, D_2, D_3, D_4 - disciplines of distribution to resources for requests; Q_1 – queue to replacement units; Q_2 – queue to repair units.

Only a few particular cases of the described system have so far been investigated in the reliability and queuing theories, namely:

1. $\boldsymbol{m}=1$, $\boldsymbol{n}=1$;
2. $\boldsymbol{m}=1$, $\boldsymbol{n}=2$;
3. $\boldsymbol{m}=2$, $\boldsymbol{n}=1$;
4. M/M/N, i.e. the repair time length has an exponential distribution, while the replacement time length equals zero (instant replacement);
5. some similar cases have also been investigated.

In the last 6-7 years the specialists of Georgian Technical University have succeeded in making considerable progress in the investigation along these lines. In particular, the models have been constructed and partly investigated for the following cases:

1. ***m***, ***n***, ***k***, ***l*** are arbitrary; the functions $F(x)$ and $G(x)$ are exponential;
2. ***m***, ***n*** and the function $F(x)$ are arbitrary; $\boldsymbol{k}=\boldsymbol{l}=1$ and the function $G(x)$ is exponential;
3. ***m***, ***n*** and the function $G(x)$ are arbitrary; $\boldsymbol{k}=\boldsymbol{l}=1$ and function $F(x)$ is exponential;
4. some similar statements have also been considered.

3. THE MATHEMATICAL MODEL

In this section we construct and investigate the mathematical model for the case where ***m, n, k*** and ***l*** are arbitrary. The replacement and repair time lengths have exponential distribution functions with parameters λ and μ respectively.

For describing of considered system we introduce the random processes, which determine the states of considered system at the moment t.

i(t) – the number of elements missed in main group of elements;

j(t) – the number of nonserviceable (failed) elements in the system;

Denote, $p(i,j,t)=P\{i(t)=i;\ j(t)=j\}$, $i=\overline{1,m};\quad j=\overline{0,n+i}$.

We suppose that there exists $\lim\limits_{t\to\infty} p(i,j,t)=p(i,j)$;

Regarding to the functions introduced above we construct a system of usual linear differential first order equations (Kolmogorov equations), which in steady mode transforms into the system of linear algebraic equations. They form four groups of equations describing the following merged states of the considered system:

1. A free state (in the system there are no requests neither for replacement nor for renewal);
2. Replacement (only the replacement operation is carried out in the system);
3. Repair (only the repair operation is carried out in the system);
4. Replacement and repair (both the replacement and the repair operation are carried out in the system).

Hence, in the case $k\le l\le n\le m$ we obtain the mathematical model in steady state mode in the form of the system of linear algebraic equations regarding $p(i,j)$, $i=\overline{1,m};\quad j=\overline{0,n+i}$.

1. Free state

$$(\mathrm{m}\alpha+\mathrm{n}\beta)\mathrm{P}(0,0)=\lambda\mathrm{P}(1,0)+\mu\mathrm{P}(0,1) \tag{1.1}$$

2. Replacement

$$((m-i)\alpha + n\beta + i\lambda)P(i,0) = (i+1)\lambda P(i+1,0) + \mu P(i,1), 0 < i < k\,; \qquad (2.1)$$
$$((m-i)\alpha + (n+1-k)\beta + k\lambda)P(i,0) = k\lambda P(i+1,0) + \mu P(i,1), \qquad k \le i < m\,; \qquad (2.2)$$
$$((m+n-k)\beta + k\lambda)P(m,0) = \mu P(m,1)\,; \qquad (2.3)$$

3. Renewal

$$(m\alpha + (n-j)\beta + j\mu)p(0,j) = \lambda p(1,j) + (j+1)\mu p(0,j+1) + (n-j+1)\beta.p(0,j-1)\,,$$
$$1 \le j < l; \qquad (3.1)$$
$$(m\alpha + (m-j)\beta + l\mu)p(0,j) = \lambda p(1,j) + l\mu p(0,j+1) + (n-j+1)\beta.p(0,j-1)\,,$$
$$l \le j < n; \qquad (3.2)$$

$$(m\alpha + l\mu)p(0,n) = \lambda p(1,n) + \beta.p(0,n-1)\,; \qquad (3.3)$$

4. Replacement and Renewal

$$((m-i)\alpha + (n-j)\beta + \mathrm{i}\lambda + j\mu)p(i,j) = (m-i+1)\alpha p(i-1,j-1) +$$
$$+ (n-j+1)\beta P(i,j-1) + (i+1)\lambda p(i+1,j) + (j+1)\mu p(i,j+1), 1 \le i < k, 1 \le j < l\,; \qquad (4.1)$$

$$((m-i)\alpha + (n-j)\beta + \mathrm{i}\lambda + l\mu)p(i,j) = (m-i+1)\alpha p(i-1,j-1) + (i+1)\lambda p(i+1,j) +$$
$$+ (n-j+1)\beta p(i,j-1) + l\mu p(i,j+1), 1 \le i < k, l \le j \le n\,; \qquad (4.2)$$

$$((m-j)\alpha + (n+i-j)\mu + l\mu)p(i,j) = p(i-1,j-1)(m-i+1)\alpha +$$
$$+ (n+i-j+1)\lambda p(i+1,j) + l\mu p(i,j+1), \qquad 1 \le i < k,\ n < j < n+i; \qquad (4.3)$$

$$((m-i)\alpha + l\mathrm{\mu})p(i,n+i) = (m-i+1)\alpha p(i-1,n+i-1) + \lambda p(i+1,n+i)$$
$$1 \le i < k; \qquad (4.4)$$
$$((m-i)\alpha + (n+i-j-k)\beta + l\mathrm{\mu} + \mathrm{k}\lambda)p(i,j) = (n+i-j+1-k)\beta p(i,j-1) +$$
$$+ \lambda kp(i+1,j) + (j+1)\mu p(i,j+1), k \le i < m, 1 \le j < l\,; \qquad (4.5)$$

$$((m-i)\alpha + (n+i-j-k)\beta + l\mathrm{\mu} + \mathrm{k}\lambda)p(i,j) = (m-i+1)\alpha p(i-1,j-1) +$$
$$(n+i-j+1)\beta P(i,j-1) + k\lambda P(i+1,j) + l\mu p(i,j+1), \qquad k \le i < m,\ l \le j \le n+i-k; \qquad (4.6)$$

$$((m-i)\alpha + (m-i-j)\lambda + l\mathrm{\mu})P(i,j) = (m-i+1)\alpha P(i-1,j-1) + (n+i+1-j)\lambda P(i+1,j) +$$
$$+ l\mu P(i,j+1), k \le i < m, \mathrm{n+i-k} < \mathrm{j} < \mathrm{n+i}\,; \qquad (4.7)$$

$$((m-i)\alpha + l\mathrm{\mu})P(i,n+i) = (m-i+1)\alpha p(i-1,n+i-1) + \lambda p(i+1,n+i) \quad k \le i < m; \qquad (4.8)$$

$$((m+n-j-k)\beta + \lambda k + j\mathrm{\mu})\mathrm{p}(m,j) = \alpha p(m-1,j-1) +$$
$$+ (m+n-j+1-k)\beta p(m,j-1) + (j+1)\mu p(m,j+1), 1 \le j < l\,; \qquad (4.9)$$

$$((m+n-j-k)\beta + \lambda k + l\mathrm{\mu})p(m,j) = \alpha p(m-1,j-1) +$$
$$+ (m+n-j+1-k)\beta P(m,j-1) + l\mu P(m,j+1), l \le j \le m+n-k\,; \qquad (4.10)$$

$((m+n-j)\lambda + l\mu)\mathrm{p}(m,j)p(m,j) = \alpha p(m-1,j-1) + l\mu p(m,j+1)$,

m+n-k<j<m+n; (4.11)

$l\mu\mathrm{p}(m,m+n) = \alpha p(m-1,m+n-1)$.

The obtained system together with equilibrium condition: $\sum_{i=0}^{m}\sum_{j=0}^{n+i} p(i,j) = 1$ has, as a rule, unique solution. It's only computational difficulty to find that solution.

In the same way it can be written similar equations for the other arbitrary alignments of m,n,k,l parameters, but we'll not discuss this question here.

4. ECONOMIC ANALYSIS

The system elements as well as maintenance units give different profits depending on the their following states (positions).

For system elements: 1. main; 2. serviceability standby being under replacement; 3. serviceability standby not being under replacement; 4. failed standby being under repair; 5. failed standby not being under repair.

For maintenance units: 6. working replacement; 7. non-working replacement; 8. working repair; 9.non-working repair.

The profit amount per some fixed time interval (time unit) from one system element or one maintenance unit is c_i, $i=\overline{1,9}$ for above-mentioned states. Some of this c_i may be non-positive.

Denote by E_i, $i=\overline{1,9}$ the average number (mathematic expectation) of system elements or maintenance units at the arbitrary time moment in the steady mode of system functioning.

It is not difficult to obtain the expressions for E_i, $i=\overline{1,9}$.

Namely,

$$E_1 = \sum_{i=0}^{m}(m-i)\sum_{j=0}^{n+2} p(i,j);$$

$$E_2 = \sum_{r=1}^{k-1} r\left(\sum_{j=0}^{n} p(r,j) + \sum_{i=r+1}^{m} p(i,n+i-r)\right) + k\sum_{i=k}^{m}\sum_{j=0}^{n+i-k} p(i,j);$$

$$E_3 = \sum_{i=0}^{m}\sum_{j=0}^{n+i}(n+i-j)p(i,j) - E_2;$$

$$E_4 = \sum_{j=0}^{l-1} r\sum_{i=0}^{m} p(i,j) + l\sum_{i=0}^{m}\sum_{j=l}^{n+i} p(i,j);$$

$$E_5 = \sum_{i=0}^{m}\sum_{j=0}^{n+i} jp(i,j) - E_4;$$

$$E_6 = E_2;$$

$$E_7 = k - E_6 = k - E_2;$$

$$E_8 = E_4;$$

$$E_9 = l - E_8 = l - E_4;$$

We introduce the function for economic analysis of the system. This function expresses the profit of examined systems per time unit, taking into account the above-mentioned values.

Initial characteristics of considered systems – ***m***, ***n***, ***k***, ***l***, ***α***, ***β*** and ***λ*** enter into the expression of profit function through $p(i,j)$ - probabilistic characteristics of considered systems. Profit function also depends on $c_1, \dots, c_9$ in the following way $F = F(n,k,l) = \sum_{i=1}^{9} c_i E_i$.

Eventually, the problem of system optimization is stated as problem of mathematical programming (integer programming).

Namely, with those fixed ***m***, ***α***, ***β***, ***λ***, ***μ***, $c_1, \dots, c_9$ for the considered system to select such values of the parameters ***n***, ***k***, ***l*** (optimal numbers of standby elements, replacement units and renewal units) so that the profit function **F** would accept maximum value and to determine this value.

That means the solving of problem of analytical synthesis of multi-element recoverable standby system by economical criterion.

We believe, this result will be very useful for experts working in the field of design, control and management of complex systems.

5. CONCLUSIONS

Main goal of mathematical model, constructed by us, is structural optimization (structural management) of considered system by economic criteria. Since connections between system's elements is fixed, its structural optimization means selection of optimal quantity of standby elements, replacement units and repair units.

From this point of view, intermediate characteristic values p(i,j), introduced by us, are very fruitful: 1) using them it is not difficult to construct mathematical model; 2) system's effectiveness criterion F(n,k,l) is easily expressed by their means.

Finally, structural management problem is brought to integer programming problem, which's solution is only computational difficulty.

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