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RELIABILITY ASSESSMENT OF SYSTEMS WITH PERIODIC MAINTENANCE UNDER RARE FAILURES OF ITS ELEMENTS

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Abstract

There is investigated a model of a system with the highly reliable elements, where the periods of functioning are changed by the periods of maintenance. The system must be operational only in the periods of functioning although the restoration in these periods is not provided. The system is completely restored in the nearest period of maintenance. Since the elements of system are highly reliable, the reserve of system is rarely exhausted during each period of functioning. Therefore it is possible to use the results, obtained for the systems with fast restoration, for the reliability assessment of system, which is not restorable in the periods of the functioning. The estimations of indices of failure-free performance and maintainability of such systems are obtained.

1. Introduction and motivation

There is examined a system with periodic maintenance. The periods of functioning of the system, are changed by the periods, when the system is turned off and there is produced its maintenance and the restoration of all failed elements. The elements of the system fail only in the period of functioning, and their restoration is produced in the period of maintenance. User is interested in the high reliability of the system in the periods of functioning.

Examples of such systems are aircrafts, reusable spacecrafts, etc. So aircrafts in a flight are carrying out their flight mission. After landing their complete maintenance is carried out. After this, the aircrafts are ready to the fulfillment of a new flight mission. Another example is a nuclear power plant where users as a rule have no access or a limited access to its reactor compartment between periods of planned maintenance. At the time of the maintenance nuclear reactor can be adjusted and renewed to continue its mission.

For these systems the periods of functioning, where the system works and fulfills the assigned functions, are changed by the periods of maintenance, when the system is turned off and the maintenance is produced. The restoration of all failed elements as well as the elimination of all faultinesses, accumulated during the functioning period, is produced only in the second period and it manages to end up to the next functioning period. In the second period the reliability of operational elements is not changed. After gluing together the functioning periods of the system, we will obtain the model of the system with the periodic instantaneous maintenance.

In the work there is given assessment of the indices of failure-free performance and maintainability under the condition that elements of system fail rarely in the functioning period.

2. Model description

The duration of the periods of the time from initial moment of the functioning of the system till the nearest maintenance period or between the end of the period of maintenance and the beginning of the following period of maintenance is a random variable ξ with distribution function (DF) $P\{\xi < x\} = \Phi(x)$ and mean value $M\xi = T$.

The elements of the system can fail only in the functioning periods of the system, and their restoration is produced only in the nearest interval of maintenance. The failure of the system occurs when between the first failure of some element of the system and the nearest interval of maintenance there is completely exhausted the entire reserve of the system, including a time reserve, if it exists. The failures of elements and the failure of the system are instantly detected.

If the reserve of the system is spent not completely in the period of functioning, then the failure of the system is not observed, the reserve of the system is restored in the nearest interval of maintenance and the system is derived for the new period of functioning.

If the failures of elements do not occur in a certain period of the functioning, then the reliability of the system in the nearest period of maintenance is not changed.

The intervals of functioning can have different duration in sense of their mean value. Since in the intervals of functioning the system is not restored, the failure of the system is more probable in the more prolonged interval of functioning. After taking the worst (most prolonged in average) interval of functioning we can obtain the estimation of the worst reliability of the system. Therefore without limiting the generality we will consider that all intervals of functioning are distributed equally and correspond to the most prolonged by its mean value interval of functioning.

We will examine the highly reliable systems, in which the failures of elements in each interval of functioning occur rarely. This in particular means that the product of the maximum of summary failure rate of the system elements by the maximum duration of the functioning interval is considerably less than one. Actually this case corresponds to practice.

We are interested to study the behavior of the system only in the summary interval of functioning, obtained as a result of ignoring all intervals of maintenance. This summary interval can be considered as a new time axis with the point flow *T*. Each point of the flow corresponds to the interval of the maintenance, where the system is restored instantly and completely.

On this time axis it is possible to indicate the moments of the failures of the elements of the system. After the first failure of an element in any functioning interval an interval of malfunction (*IM*) is begun. *IM* ends at the moment of the beginning of the nearest interval of maintenance. Let us name an *IM* as a failure *IM*, if the failure of system occurs in it. Let us name a point of flow *T* as a failure point if it is contiguous with a failure *IM*. After such a failure point the system is fully restored (!).

We have a point flow *T* with rarefaction where some points may be failure points with probability converged to zero. But this means that there is possible to use an asymptotic approach for reliability assessment of the system.

Since on the functioning intervals the restorations of elements are not produced the failure of the system in *IM* is developed only along the monotonic path. Monotonic path is such a trajectory of the failure of the system on which from the moment of the failure of the first element in *IM* (the beginning of *IM*) and to the moment of the failure of the system in this *IM* none restorations were finished.

Let us estimate the behavior of the system to the first failure of the system and between two adjacent failures of the system, when the elements of the system are highly reliable. This means that the probability of the failure of system in any period of functioning is small and more than zero, which corresponds to the conditions for fast restoration, introduced and studied in [1] and [2].

Thus the estimation of the reliability of system with the periodic maintenance, where there are no restorations on the intervals of functioning, can be and will be actually produced with using an apparatus, developed for the systems with the fast restoration. It sounds paradoxically, but this is actually so because the elements of the system are highly reliable, the periods of functioning have relatively small duration and the point interval of maintenance follows them.

3. Mathematical formulation of problem

Present article rest upon the results of the works of Genis [1] and [2]. In the work [1] there is given the general approach to the estimation of the reliability of systems with fast restoration. In the work [2] there is given the more detailed determination of the criterion of fast restoration.

For convenience of the reader let us give some details of the systems, already described in [1] and [2], which are used in this work. System consists of *n* elements. Each element of the system can be only in the operational or inoperative state. Each operational element can be located in the loaded or unloaded regime (lightened regime for the brevity is omitted). Let $F_i(x)$, $f_i(x)$, и m_i are accordingly the distribution function, the density of distribution (*DD*), and the mean value of the time of failure-free operation of the *i*-th element in the system, $m_i < \infty$, $i \in \overline{1,n}$.

Within the framework of this work we will examine the systems of the 1-st and the 3-d types, described in [1]. Systems of the 1-st type are systems with the exponentially distributed times of failure-free operation of elements. Systems of the 3-d type are the systems, whose elements have a limited *DD* of the failure-free operation time; there is required also, that these *DDs* in zero are not equal to zero, $f_i(0) = c_i \neq 0$, $i \in \overline{1,n}$.

There are no limitations to the structure of the system. There is assigned the criterion of the system's failure, which can include and the condition of time redundancy. The state of the elements of system at the moment *z* is described by the vector $\vec{v}(z) = \{v_1(z),...,v_n(z)\}$, where each component can take the values of $\{0, 1, \dots, n\}$. Number 0 corresponds to failed elements; numbers from 1 to *n* correspond to operational elements. These numbers make it possible to unambiguously assign the order of the replacement. Vector $\vec{v}(z)$ helps to estimate the reliability of a concrete system.

Let E is the set of the states of the system, $\{\vec{v}(z)\} = E = E_+ \cup E_-$, where E_+ is the area of the fully operational, and E_- is the area of the before failure states of the system. The system is considered as before failed at the moment *z* if $\vec{v}(z) \in E_-$ and failed if its malfunction lasts time not smaller then $\eta, P\{\eta < x\} = H(x)$. In the absence of the time reserve ($\eta \equiv 0$) the area of the before failed states of system is converted into the area of the failures of the system. All elements of the system were new and functioned properly at the initial moment of time.

Let $\vec{b}$ is a certain state vector of elements of the system directly before *IM*, and $\vec{b}^N$ is the state vector of the elements of the system on the same *IM* immediately after the moment of passing the state vector of system from the area E_+ into the area E_-. Let us name the way π leading from $\vec{b} \in E_+$ into the state $\vec{b}^N \in E_-$ on the *IM* the sequence of the state vectors of elements, beginning from the vector $\vec{b}$, which directly precede the beginning of the *IM*, and ending with the vector $\vec{b}^N$, which corresponds to the first before failed state of the system on this *IM*.

The path length is equal to the number of elements, which failed on this way. Let us name way monotonic, if on it there are no restorations of elements. Let us name monotonic way minimal for $\vec{b}$ if its length $l(\vec{b})$ equals to the minimum of the path lengths, which lead from $\vec{b}$ to E_-. Then the minimum number of elements, whose failure can cause the system's failure, equals $s = \min l(\vec{b})$ on $\vec{b} \in E_+$.

The value $s \geq 2$ corresponds to a fault-tolerant system with a structural reserve. If $s = 1$ a failure of some element may lead to a system failure unless there is not introduced a time reserve that will not be exhausted till the nearest moment of a maintenance period.

It is considered that the system works in the conditions of the fast restoration. Practically this means that the mean time of the interval of functioning is substantially less than the mean time between any two failures of elements in the system (see section 4).

Let in the steady-state operation section of work with the restoration discipline d_1 with the straight order of maintenance and *one* repair unit, $\beta(d_1,1)$ is the estimation of the system's failure rate taking into account only monotonic paths of the failure (but in the system with periodic maintenance there are no other paths), $\tau''(d_1,1)$ is the random system's restoration time after failure, $T_R(d_1,1)$ is the mean value of this time, $K_A(d_1,1)$ is the availability function of the system. In all of these indices the first parameter is the type of restoration discipline and the second one is the number of restoration units in the system.

The problem consists in estimating of the indices of failure-free performance and maintainability of the described system under the conditions of the fast restoration.

A system with the periodic maintenance fails, if the reserve of the system is completely exhausted during the period between two maintenances. In this case the failure of the system occurs only along the monotonic paths.

Let DF $\Phi(x)$ is absolutely continuous. In the steady-state operating conditions of the system DF of time from the beginning of IM to the nearest moment of the system's maintenance is equal to

$$A(x) = \int_0^x \bar{\Phi}(u)du / T. \qquad (3.1)$$

For the failure of the system during an *IM* there must be spent the reserve of the system. Therefore the behavior of the described system is analogous to the behavior of the system of section 2 [1] with restoration discipline $d = d_1$ with the straight order of maintenance FIFO, one repair unit $k = 1$, and with identical for all elements distribution function of the restoration time $G_i(x) = G(x) = A(x)$,

$i=\overline{1,n}$, with the additional assumption that only monotonic paths of the system's failure are permitted. Indeed, for such repairable system the first failed element will not be restored during *IM* time, in the same time the reserve of system should be spent, and under the conditions of fast restoration the failure of system will be developed along the monotonic path.

In [1] and [2] for establishing the criteria of the system's behavior and formulating the results of the study there were used the indices of failure-free performance of the elements m_i and c_i, *DF* of restoration time *G(x)* (if it is identical for all elements) and "shifted" for the time η moments *DF G(x)*

$$m_R^{(k)}(\eta)=\int_0^\infty\int_0^\infty kx^{k-1}\overline{G}(x+u)dxdH(u),\ \ m_R^{(k)}=m_R^{(k)}(0),\ m_R=m_R^{(1)}, \qquad (3.2)$$

where for any *DF* $\Gamma(x)$ $\overline{\Gamma}(x)=1-\Gamma(x)$.

In the present work let us replace $G(x)$ by $A(x)$ from (3.1) and let us introduce the designations

$$\tilde{m}_R^{(k)}(\eta)=\int_0^\infty\int_0^\infty kx^{k-1}\overline{\Phi}(x+u)dxdH(u),\ \ \tilde{m}_R^{(k)}=\tilde{m}_R^{(k)}(0). \qquad (3.3)$$

The following lemma makes it possible to pass from (3.2) to (3.3).

Lemma 3.1. For any numbers $k\geq 1$

$$m_R^{(k)}(\eta)=\frac{1}{(k+1)T}\tilde{m}_R^{(k+1)}(\eta), \qquad (3.4)$$

if $\tilde{m}_R^{(k+1)}(\eta)$ exists.

The proof of Lemma 3.1 is given in the Appendix.

4. Asymptotic approximation. Criterion of the fast restoration

Let *s* is the minimum number of elements, whose failure can cause the malfunction of the system; $\overline{\lambda}$ and $\underline{\lambda}$ are the maximum and the minimum failure rates of elements in the operational system [1].

Let us say that the condition of the fast restoration is satisfied for the system if $\underline{\lambda}>0$,

$$\alpha=[\overline{\lambda}\ m_R^{(s)}\ /\ (m_R)^{s-1}]=[\overline{\lambda}(\tilde{m}_R^{(s+1)}/(s+1)T)/(\tilde{m}_R^{(2)}/2T)^{s-1}]\to 0, \qquad (4.1)$$

and for all *DF* $F_i(x)$, $i=\overline{1,n}$, the limited distribution densities must exist [1, 2].

In the practically important cases $m_R^{(s)}\ \leq C\ (m_R)^s$, where *C* is a certain constant. In these cases the condition for fast restoration (4.1) is reduced to

$$\alpha = \bar{\lambda} m_R = \bar{\lambda} \tilde{m}_R^{(2)} / 2T \to 0 . \qquad (4.2)$$

For the fast restoration it is actually required that the probability of the system's failure on the period of functioning would be more than zero (it is enough to demand $\underline{\lambda} > 0$), and the probability of the system's failure on *IM* converges to zero ($\alpha \to 0$).

5. Estimation of the indices of failure-free performance

Let $\tau_i(t)$ is the interval from the moment *t* to the nearest moment of the system's failure after moment *t*. Details of the determination of time to the first failure of the system $\tau_1(t)$ and time between (*j*-1)-th and *j*–th system's failures $\tau_j(t), j \ge 2$, are given in section 5 of [1].

Taking into account the theorem 5.3 of [1] we will obtain that for the dicussed system it is fullfilled

Corollary 5.1. In the steady-state operating conditions of the system with periodic maintenance and with structural reserve for $i \ge 1$ under the conditions of the fast restoration $\underline{\lambda} > 0$ and $\alpha \to 0$

$$P\{\tau_i(t) \ge x\} \approx \exp\{-\beta(d_1, 1)x\}. \qquad (5.1)$$

Let us name the intervals between the end of one *IM* and the beginning of the next *IM* in the new time axis as intervals of operability (*IO*). Let from $\vec{v}(z) = \vec{b} \in E_+$, $z \in$ *IO*, there are possible minimal paths $l = l(\vec{b})$, leading into the certain state $\vec{b}^j \in E_-$. Every of state $\vec{b}^j$ is characterized by the set l of numbers of failed elements, belonging to the set $J = J_+ \cup J_-$, where J_+ and J_- are accordingly the sets of the numbers of those elements, which in the state $\vec{b}$ were located in the loaded and unloaded regime. Let the set of minimum paths leading from $\vec{b}$ into $\vec{b}^j$ is Π^j. Let

$$\Lambda^j = \prod_{k \in J_-} c_k \prod_{i \in J_+} 1 / m_i . \qquad (5.2)$$

Let $R_j(\vec{b})$ is the number of minimal paths of the length $l = l(\vec{b})$, leading from $\vec{b}$ into $\vec{b}^j \in E_-$; *s* is the minimum number of elements, whose failure can cause the before failure system state; $\lambda(\vec{b})$ and $q^{(\vec{b})}$ are respectively the summary failure rate of elements and the probability of the occurrence of a failure *IM* in the steady-state operating conditions of the system in condition that the system directly before *IM* was in the state $\vec{b}$.

Then there is valid

Theorem 5.1. Under the conditions of the fast restoration with $\underline{\lambda} > 0$ and $\alpha \to 0$ in the steady-state operating conditions of the system with the periodic maintenance and structural reserve

$$\lambda(\vec{b}) q^{(\vec{b})} \approx \frac{1}{T} \sum_{\vec{b}^j \in E_-} R_j(\vec{b}) \Lambda^j \tilde{m}_R^{(l)}(\eta) / l! \qquad (5.3)$$

The prove of Theorem 5.1 is given in the Appendix.

Observation 5.1. In theorem 5.1 there is possible to remove the requirement of absolute continuity of DF $\Phi(x)$. Using an apparatus for a study of rare event in the regenerating process [3], it is possible to obtain estimation (5.3) after requiring existence of finite first and second moments for the period ξ. This makes it possible to extend the theorem 5.1 to the case, when $\xi \equiv T$, that has a great practical value.

In the section 8 of [1] it is shown that under the conditions of the fast restoration the estimation of the indices of the reliability of a complex system can be brought to the estimations of the indices of the reliability of its series-connected in the sense of the reliability schemes p out of m, calculated under the assumption of their autonomous operation. The scheme p out of m has m elements. Its malfunction occurs with the failure of not less then p elements from m, $p \le m$, and its failure begins when the malfunction of scheme lasts not less than time $\eta, P\{\eta < x\} = H(x)$. Therefore within the framework of this article we will consider that the system is a scheme p out of m, which failure rate is β_p.

Using (5.3), there is possible to obtain the estimations $\beta_p = \beta_p(d_1,1)$ for the system p out of n with the periodic maintenance for various types of structural reserve.

In particular let us examine schemes n out of n, corresponding to the parallel in the sense of reliability connection of elements. The scheme becomes defective, if all n elements fail.

Corollary 5.2. For the system n out of n with the periodic maintenance and the loaded reserve

$$\beta_n \approx \tilde{m}_R^{(n)}(\eta) / T \prod_{i=1}^{n} m_i. \qquad (5.4)$$

System n out of n with the unloaded reserve consists of the 1 basic element used in the loaded regime, and (n - 1) reserve elements, which are located in the unloaded regime. With the failure of the basic element its place takes the element from the unloaded reserve, which stayed in the reserve the greatest time. The malfunction of system begins when the basic element fails and there is absent a reserve element that is capable to replace the failed one.

Corollary 5.3. For the system n out of n with the periodic maintenance and the unloaded reserve

$$\beta_n \approx \tilde{m}_R^{(n)}(\eta) \sum_{k=1}^{n} \prod_{i \neq k} c_i / Tn! \sum_{j=1}^{n} m_j. \qquad (5.5)$$

6. Indices of maintainability and availability function

There is examined the system, described above, with $\eta \equiv h = const$. In this case

$$\tilde{m}_R^{(k)}(y) = \int_0^\infty kz^{k-1}\overline{\Phi}(z+y)dz .$$

For the system n out of n with the periodic maintenance and the loaded reserve
(the parallel in the sense of reliability connection of the elements)

$$P\{\tau'' \geq x\} \approx \tilde{m}_R^{(n)}(h+x) / \tilde{m}_R^{(n)}(h) , \tag{6.1}$$

$$T_R \approx \tilde{m}_R^{(n+1)}(h) / (n+1)\tilde{m}_R^{(n)}(h) , \tag{6.2}$$

$$K_A \approx 1 - \tilde{m}_R^{(n+1)}(h) / (n+1)T\prod_{i=1}^{n} m_i . \tag{6.3}$$

For the system n out of n with the periodic maintenance and the unloaded reserve the estimations of $P\{\tau'' \geq x\}$ and T_R are the same, as for the system with the loaded reserve, but the estimation of availability function takes the form

$$K_A \approx 1 - \tilde{m}_R^{(n+1)}(h)\sum_{i=1}^{n}\prod_{j \neq i} c_j / (n+1)!T\sum_{k=1}^{n} m_k . \tag{6.4}$$

Summary

The received results allow the developing of a practical methodology of reliability assessment of systems described in the first section of this article. Together with technologist who knows the system very good it should be established the criteria of the system failure. After that the system’s reliability assessment is provided. If results are not satisfactory there is introduced an additional reserve in the system and reliability assessment is repeated. The process is continued until satisfactory results are obtained.

Appendix

Proof of the Lemma 3.1. Taking into account (3.1) for any value $u \ge 0$

$$\bar{G}(x+u) = \frac{1}{T}\int_{x+u}^{\infty}\bar{\Phi}(y)dy = \frac{1}{T}\int_{x}^{\infty}\bar{\Phi}(y+u)dy .$$

In this case for any integer $k \ge 1$

$$\int_0^{\infty} kx^{k-1}\bar{G}(x+u)dx = \frac{1}{T}\int_0^{\infty} kx^{k-1}\int_x^{\infty}\bar{\Phi}(y+u)dydx .$$

After replacing the order of integration for 0< x< ∞ and x<y< ∞ to 0<y< ∞ and 0<x<y, and then replacing the variable of integration y to x we will obtain

$$\int_0^{\infty} kx^{k-1}\bar{G}(x+u)dx = \frac{1}{T}\int_0^{\infty}\bar{\Phi}(y+u)\int_0^{y} kx^{k-1}dxdy = \frac{1}{(k+1)T}\int_0^{\infty}(k+1)x^k\bar{\Phi}(x+u)dx . \quad \text{(A.1)}$$

After adding to (A.2) integration by value *u* with measure *H(u)* we will obtain the assertion (3.4) of Lemma 3.1.

Proof of the Theorem 5.1. We will use the Theorem 6.1 of [1] taking into account that the discipline of the restoration in the system is d_1, the number of repair units is *k* = 1, and for any element i_1 failed the first on *IM* the DF $G_{i_1}(x) = G(x) = A(x)$. Then we will obtain that from (6.4) of [1] follows

$$\lambda(\vec{b})q^{(\vec{b})} \approx \sum_{\vec{b}^j \in E_-}\Lambda^j \sum_{\pi\in\Pi^j}\int_0^{\infty}\int_0^{\infty}\frac{x^{l-2}}{(l-2)!}\bar{G}(x+u)dxdH(u) . \quad \text{(A.2)}$$

Since for any path $\pi \in \Pi^j$ the integral in (A.2) is the same and it is equal to $m_R^{(l-1)}(\eta)/(l-1)!$, and the number of minimal paths in Π^j equals $R_j(\vec{b})$, then

$$\lambda(\vec{b})q^{(\vec{b})} \approx \frac{1}{(l-1)!}\sum_{\vec{b}\in E_-} R_j(\vec{b})\Lambda^j m_R^{(l-1)}(\eta) \quad \text{(A.3)}$$

After applying assertion of lemma 3.1 to (A.3) we will obtain assertion (5.3) of theorem 5.1.

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