A FUZZY RELIABILITY MODEL FOR "SAFETY SYSTEM-PROTECTED OBJECT" COMPLEX

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ABSTRACT

The paper presents a new fuzzy reliability model for automated "safety system-protected object" complex. It is supposed that parameters of reliability model and reliability indices are fuzzy variables. Scheduled periodic inspections of safety system are also taken into account. Asymptotic estimates of mean time to accident membership function are proposed.

1 INTRODUCTION

Many researchers applied the concept of fuzzy reliability on various systems (Cai et al 1991, Cai et al. 1993, Cai 1996, Chen & Mon 1993, Onisava & Kacprzyk 1995, Utkin & Gurov 1995, Verma et al. 2007). Those researches are based on possibility instead of probability assumption or fuzzy state instead of binary state assumption. This paper presents a slightly different concept of fuzzy reliability. It is supposed that parameters of the reliability model are fuzzy variables. According to the random fuzzy variables theory presented by Liu (Liu 2002) reliability indices in this case are also fuzzy variables. In order to develop a complete practical methodology of the fuzzy reliability assessment we also consider some aspects of the fuzzy parameter estimation and numerical methods of the fuzzy arithmetic.

In the present study we set out to analyze the reliability of the automated "safety system-protected object" complex. Systems of such kind are quite common in the nuclear power engineering. We follow Pereguda (Pereguda 2001) in assuming that the operation of the complex can be described using a superposition of alternating renewal processes. We also utilize the concept of a random fuzzy renewal process (Shen et al. 2008, Zhao et al. 2006). Our objective is to provide an asymptotic estimation for the mean time to accident membership function.

2 MODEL DESCRIPTION

Let us consider an automated complex of safety system and protected object. The safety system and the protected object are repairable. They are restored to an as-good-as-new state. It is assumed that failures of the safety system can be detected only during periodic inspections of the safety system. All failures are supposed to be independent.

Let $\{F_\chi(t;\boldsymbol{\lambda}(\theta)),\theta\in\Theta\}$ be a family of probability distributions on the probability space (Ω, A, P) with a common fuzzy parameters vector $\boldsymbol{\lambda}$ on the credibility space (Θ, Π, Cr) which induces a joint membership function, $\mu_{\boldsymbol{\lambda}}(\mathbf{x})$, then χ is a random fuzzy variable (Guo et al. 2007). For example, χ is an exponentially distributed random fuzzy variable, χ~EXP(λ) if

$$F_\chi(t;\lambda(\theta))=\begin{cases}1-e^{-\lambda(\theta)t}, & if\ t\ge 0\\ 0, & otherwise.\end{cases}$$

If χ is a random fuzzy variable defined on the credibility space (Θ, Π, Cr) than the probability $\Pr(\chi \in A)$ is a fuzzy variable for any Borel set $A \subseteq R$ and the expected value $E[\chi]$ is a fuzzy variable provided that $E[\chi(\theta)]$ is finite for each $\theta \in \Theta$ (Liu 2002).

By χ_i, i = 1,2,… denote the time to the i-th protected object failure. Let χ_i, i = 1,2,… be independent and identically distributed (i.i.d) random fuzzy variables (Li & Liu 2006) with CDF $F_\chi(t; \boldsymbol{\lambda}_\chi(\theta))$. By γ_i, i = 1,2,… denote the time to the protected object repair after it's i-th failure. Let γ_i, i = 1,2,… be i.i.d. random fuzzy variables with CDF $F_\gamma(t; \boldsymbol{\lambda}_\gamma(\theta))$. Suppose that moments of the protected object repair are renewal points of the operation process of the complex. By ξ_i, i = 1,2,… denote the time to the i-th failure of the safety system. Let ξ_i, i = 1,2,… be i.i.d. random fuzzy variables with CDF $F_\xi(t; \boldsymbol{\lambda}_\xi(\theta))$. By η_i, i = 1,2,… denote the time to the safety system repair after it's i-th failure. Let η_i, i = 1,2,… be i.i.d. random fuzzy variables with CDF $F_\eta(t; \boldsymbol{\lambda}_\eta(\theta))$. Suppose that moments of the safety system repair are renewal points of the operation process of the safety system. By T denote the period of scheduled inspections of the safety system. By δ denote the duration of scheduled inspections of the safety system. The safety system is inactive during the inspection. By ν denote the number of renewal intervals before the accident. Let ν be an integer random fuzzy variable. By ω denote the time to accident. An accident takes place when the protected object fails during the period of the safety system inactivity. Our aim is to estimate the membership function $\mu_{M\omega}(y)$ of the mean time to accident.

2 MAIN RESULTS

Since the operation process of the complex is a superposition of alternating renewal processes, it follows that

$$\omega = \sum_{i=1}^{\nu-1} (\chi_i + \gamma_i) + \chi_\nu .$$

Taking into account the fact that $E[\omega(\theta)]$, $E[\chi_i(\theta)]$, $E[\gamma_i(\theta)]$, $E[\xi_i(\theta)]$, $E[\eta_i(\theta)]$ and $E[\nu(\theta)]$ are crisp variables for each fixed $\theta \in \Theta$ we obtain

$$E[\omega(\theta)] = E\left[\sum_{i=1}^{\nu(\theta)-1} (\chi_i(\theta) + \gamma_i(\theta)) + \chi_{\nu(\theta)}(\theta)\right]$$

for each fixed $\theta \in \Theta$. Since all random fuzzy variables of interest are independent it follows that

$$\Pr\left(\sum_{i=1}^{\nu(\theta)-1} (\chi_i(\theta) + \gamma_i(\theta)) + \chi_{\nu(\theta)}(\theta) \geq r\right)$$

$$= \sum_{k=1}^{\infty} \Pr(\nu(\theta) = k) \Pr\left(\sum_{i=1}^{k-1} (\chi_i(\theta) + \gamma_i(\theta)) + \chi_k(\theta) \geq r\right).$$

Note that

$$E[\omega(\theta)] = \int_0^\infty \Pr\left(\sum_{i=1}^{\nu(\theta)-1} (\chi_i(\theta) + \gamma_i(\theta)) + \chi_{\nu(\theta)}(\theta) \geq r\right) dr$$

$$= \sum_{k=1}^{\infty} \Pr(\nu(\theta) = k) E\left[\sum_{i=1}^{k-1} (\chi_i(\theta) + \gamma_i(\theta)) + \chi_k(\theta)\right].$$

Since all random fuzzy variables of interest are i.i.d. it follows that

$$E[\omega(\theta)] = \sum_{k=1}^{\infty} \Pr(\nu(\theta) = k) E[k\chi(\theta) + (k-1)\gamma(\theta)] = E[\nu(\theta)\chi(\theta) + (\nu(\theta) - 1)\gamma(\theta)].$$

Therefore

$$E[\omega] = E[\nu\chi + (\nu - 1)\gamma].$$

Taking into account the fact that all random fuzzy variables of interest are independent we obtain

$$E[\omega(\theta)] = \frac{1}{q(\theta)} E[\chi(\theta)] + \frac{1-q(\theta)}{q(\theta)} E[\gamma(\theta)]$$

$$= \frac{1}{q(\theta)} \int_0^\infty \left(1 - F_\chi\left(t; \boldsymbol{\lambda}_\chi(\theta)\right)\right) dt + \frac{1-q(\theta)}{q(\theta)} \int_0^\infty \left(1 - F_\gamma\left(t; \boldsymbol{\lambda}_\gamma(\theta)\right)\right) dt$$

for each fixed $\theta \in \Theta$, where $q(\theta)$ is the probability of the accident during a renewal interval.

Let $Q^+(\theta)$ be the set of intervals where the safety system is active and let $Q^-(\theta)$ be the set of intervals where safety system is inactive. We obviously have

$$q(\theta) = \int_0^\infty \Pr\left(t \in Q^-(\theta)\right) dF_\chi\left(t; \boldsymbol{\lambda}_\chi(\theta)\right)$$

for each fixed $\theta \in \Theta$. Note that

$$\Pr\left(t \in Q^-(\theta)\right) = 1 - \Pr\left(t \in Q^+(\theta)\right) = 1 - \Pr^+(t;\theta).$$

Applying the law of total probability we obtain

$$\Pr^+(t;\theta) = \int_0^\infty \int_0^\infty \Pr\left(t \in Q^+(\theta) \mid \xi(\theta) = x, \eta(\theta) = y\right) dF_\eta\left(y; \boldsymbol{\lambda}_\eta(\theta)\right) dF_\xi\left(x; \boldsymbol{\lambda}_\xi(\theta)\right).$$

Since the operation process of the safety system is an alternating renewal process, it follows that

$$\Pr^+(t;\theta) = \iint_{\tau_{SS}(x,y) \le t} \Pr\left(t \in Q^+(\theta) \mid \xi(\theta) = x, \eta(\theta) = y\right) dF_\eta\left(y; \boldsymbol{\lambda}_\eta(\theta)\right) dF_\xi\left(x; \boldsymbol{\lambda}_\xi(\theta)\right)$$

$$+ \iint_{\tau_{SS}(x,y) > t} \Pr\left(t \in Q^+(\theta) \mid \xi(\theta) = x, \eta(\theta) = y\right) dF_\eta\left(y; \boldsymbol{\lambda}_\eta(\theta)\right) dF_\xi\left(x; \boldsymbol{\lambda}_\xi(\theta)\right) = I_1 + I_2,$$

where $\tau_{SS}(\xi,\eta) = \left(\left\langle \frac{\xi}{T+\delta} \right\rangle + 1\right)(T+\delta) + \eta$ is the length of the renewal interval of the safety system operation process and $\langle x \rangle$ is an integer part of x. We see that

$$I_2 = \iint_{\tau_{SS}(x,y) > t} \left(\sum_{m=0}^{\left\langle \frac{x}{T+\delta} \right\rangle - 1} J_{t \in [m(T+\delta), m(T+\delta)+T)} + J_{t \in \left[\left\langle \frac{x}{T+\delta} \right\rangle (T+\delta), x\right)} \right) dF_\eta\left(y; \boldsymbol{\lambda}_\eta(\theta)\right) dF_\xi\left(x; \boldsymbol{\lambda}_\xi(\theta)\right),$$

where J_A is an indicator function of the event A. It now follows that

$$I_2 = \left(1 - F_\xi\left(t; \boldsymbol{\lambda}_\xi(\theta)\right)\right) - \sum_{m=1}^\infty \left(1 - F_\xi\left(m(T+\delta); \boldsymbol{\lambda}_\xi(\theta)\right)\right)\left(J_{(m-1)(T+\delta)+T \le t} - J_{m(T+\delta) \le t}\right)$$

$$= F_\zeta\left(t; \boldsymbol{\lambda}_\xi(\theta)\right) - F_\xi\left(t; \boldsymbol{\lambda}_\xi(\theta)\right),$$

where

$$F_\zeta\left(t; \boldsymbol{\lambda}_\xi(\theta)\right) = 1 - \sum_{m=1}^\infty \left(1 - F_\xi\left(m(T+\delta); \boldsymbol{\lambda}_\xi(\theta)\right)\right)\left(J_{(m-1)(T+\delta)+T \le t} - J_{m(T+\delta) \le t}\right).$$

Note that

$$I_1 = \iint_{\tau_{SS}(x,y) \le t} \Pr^+\left(t - \tau_{SS}(x,y); \theta\right) dF_\eta\left(y; \boldsymbol{\lambda}_\eta(\theta)\right) dF_\xi\left(x; \boldsymbol{\lambda}_\xi(\theta)\right) = \int_0^t \Pr^+(t-z;\theta) dF_{\tau_{SS}}\left(z; \boldsymbol{\lambda}_\eta(\theta), \boldsymbol{\lambda}_\xi(\theta)\right).$$

Finally,

$$\Pr^+(t;\theta) = f(t;\theta) + \int_0^t \Pr^+(t-z;\theta) dF_{\tau_{SS}}\left(z; \boldsymbol{\lambda}_\eta(\theta), \boldsymbol{\lambda}_\xi(\theta)\right),$$

where $f(t;\theta) = F_\zeta\left(t; \boldsymbol{\lambda}_\xi(\theta)\right) - F_\xi\left(t; \boldsymbol{\lambda}_\xi(\theta)\right)$ and $F_{\tau_{СБ}}\left(z; \boldsymbol{\lambda}_\eta(\theta), \boldsymbol{\lambda}_\xi(\theta)\right) = P\left(\left(\left\langle \frac{\xi(\theta)}{T+\delta} \right\rangle + 1\right)(T+\delta) + \eta(\theta) \le z\right).$

The application of Laplace-Stieltjes transform and tauberian theorems yields

$$\lim_{t\to\infty}\Pr^{+}(t;\theta)=\frac{E[\xi(\theta)]-E[\zeta(\theta)]}{E[\tau_{SS}(\theta)]},$$

where

$$E[\zeta(\theta)]=\delta E\left[\left\langle\frac{\xi(\theta)}{T+\delta}\right\rangle\right]=\delta\sum_{k=1}^{\infty}k\left(F_{\xi}\left((k+1)(T+\delta);\boldsymbol{\lambda}_{\xi}(\theta)\right)-F_{\xi}\left(k(T+\delta);\boldsymbol{\lambda}_{\xi}(\theta)\right)\right),$$

$$E[\xi(\theta)]=\int_{0}^{\infty}\left(1-F_{\xi}\left(t;\boldsymbol{\lambda}_{\xi}(\theta)\right)\right)dt,$$

$$E[\tau_{SS}(\theta)]=\int_{0}^{\infty}\left(1-F_{\eta}\left(t;\boldsymbol{\lambda}_{\eta}(\theta)\right)\right)dt+(T+\delta)\left(1+\sum_{k=1}^{\infty}k\left(F_{\xi}\left((k+1)(T+\delta);\boldsymbol{\lambda}_{\xi}(\theta)\right)-F_{\xi}\left(k(T+\delta);\boldsymbol{\lambda}_{\xi}(\theta)\right)\right)\right).$$

Therefore

$$q(\theta)\approx 1-\frac{E[\xi(\theta)]-E[\zeta(\theta)]}{E[\tau_{SS}(\theta)]}.$$

Taking into account the definition of a function of fuzzy variables and Zadeh Extension Principle we obtain

$$\mu_{E[\omega]}(y)=\sup_{y=f(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4)}\min\left(\mu_{\boldsymbol{\lambda}_\chi}(\mathbf{x}_1),\mu_{\boldsymbol{\lambda}_\gamma}(\mathbf{x}_2),\mu_{\boldsymbol{\lambda}_\xi}(\mathbf{x}_3),\mu_{\boldsymbol{\lambda}_\eta}(\mathbf{x}_4)\right),$$

where

$$f(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4)=\frac{1}{q(\mathbf{x}_3,\mathbf{x}_4)}\int_{0}^{\infty}\left(1-F_{\chi}(t;\mathbf{x}_1)\right)dt+\frac{1-q(\mathbf{x}_3,\mathbf{x}_4)}{q(\mathbf{x}_3,\mathbf{x}_4)}\int_{0}^{\infty}\left(1-F_{\gamma}(t;\mathbf{x}_2)\right)dt,$$

$$q(\mathbf{x}_3,\mathbf{x}_4)\approx 1-\frac{E[\xi(\mathbf{x}_3)]-E[\zeta(\mathbf{x}_3)]}{E[\tau_{SS}(\mathbf{x}_3,\mathbf{x}_4)]},$$

$$E[\zeta(\mathbf{x}_3)]=\delta\sum_{k=1}^{\infty}k\left(F_{\xi}\left((k+1)(T+\delta);\mathbf{x}_3\right)-F_{\xi}\left(k(T+\delta);\mathbf{x}_3\right)\right),$$

$$E[\xi(\mathbf{x}_3)]=\int_{0}^{\infty}\left(1-F_{\xi}(t;\mathbf{x}_3)\right)dt,$$

$$E[\tau_{SS}(\mathbf{x}_3,\mathbf{x}_4)]=\int_{0}^{\infty}\left(1-F_{\eta}(t;\mathbf{x}_4)\right)dt+(T+\delta)\left(1+\sum_{k=1}^{\infty}k\left(F_{\xi}\left((k+1)(T+\delta);\mathbf{x}_3\right)-F_{\xi}\left(k(T+\delta);\mathbf{x}_3\right)\right)\right).$$

Therefore it is now possible to estimate the fuzzy mean time to accident. In order to perform defuzzyfication we use expected value operator suggested by Liu (Liu & Liu 2003):

$$E[\omega]=\frac{1}{2}\int_{0}^{\infty}\left(\sup_{y\geq r}\mu_{M\omega}(y)+1-\sup_{y<r}\mu_{M\omega}(y)\right)dr.$$

It is clearly evident that the most difficult part in the proposed methodology is the evaluation of $\mu_{M\omega}(y)$ according to Zadeh Extension Principle. To overcome these difficulties one should use a suitable numerical method. We suggest to use General Transformation Method (Hanss 2005) for this task. This method consists of a decomposition of input fuzzy variables, a transformation of the input intervals, an evaluation of the model and a retransformation of the output array. Another important issue is the fuzzy parameter estimation. We suggest to use fuzzy estimators developed by Buckley (Buckley 2006). These estimators are based on confidence intervals and allow the estimation of the membership function of the distribution parameter from the sample data.

Consider now the following example. Suppose χ~EXP(λ_χ), γ~EXP(λ_γ), ξ~EXP(λ_ξ), η~EXP(λ_η). Therefore

$$\mu_{E[\omega]}(y)=\sup_{y=f(x_1,x_2,x_3,x_4)}\min\left(\mu_{\lambda_\chi}(x_1),\mu_{\lambda_\gamma}(x_2),\mu_{\lambda_\xi}(x_3),\mu_{\lambda_\eta}(x_4)\right),$$

where

$$f(x_1, x_2, x_3, x_4) = \frac{1}{q(x_3, x_4)} \frac{1}{x_1} + \frac{1 - q(x_3, x_4)}{q(x_3, x_4)} \frac{1}{x_2},$$

$$q(x_3, x_4) \approx 1 - \frac{E[\xi(x_3)] - E[\zeta(x_3)]}{E[\tau_{SS}(x_3, x_4)]},$$

$$E[\zeta(x_3)] = \delta \frac{e^{-x_3(T+\delta)}}{1 - e^{-x_3(T+\delta)}}, \quad E[\xi(x_3)] = \frac{1}{x_3},$$

$$E[\tau_{SS}(x_3, x_4)] = \frac{1}{x_4} + (T + \delta)\left(1 + \frac{e^{-x_3(T+\delta)}}{1 - e^{-x_3(T+\delta)}}\right).$$

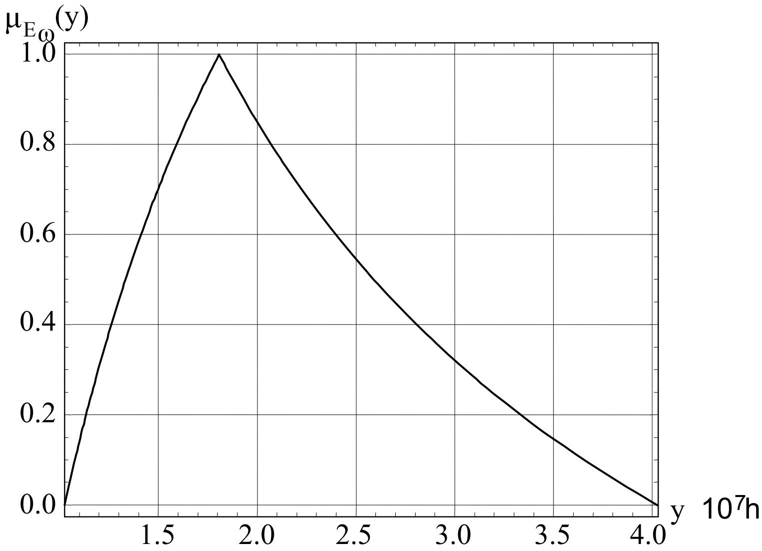

Figure 1. Membership function of the mean time to accident.

Let all fuzzy parameters be triangular fuzzy variables: $\mu_{\lambda_\chi}(x) = \Delta(1 \times 10^{-6} h^{-1}, 1.5 \times 10^{-6} h^{-1}, 2 \times 10^{-6} h^{-1})$, $\mu_{\lambda_\gamma}(x) = \Delta(1h^{-1}, 1.5h^{-1}, 2h^{-1})$, $\mu_{\lambda_\chi}(x) = \Delta(1 \times 10^{-4} h^{-1}, 1.5 \times 10^{-4} h^{-1}, 2 \times 10^{-4} h^{-1})$, $\mu_{\lambda_\eta}(x) = \Delta(1h^{-1}, 1.5h^{-1}, 2h^{-1})$, T = 500h, δ = 0.1h. Membership function of the mean time to accident is shown on Figure 1.

3 CONCLUSIONS

The proposed model permits to assess the reliability of one specific class of technological systems with fuzzy parameters. In particular the suggested approach allows to evaluate the membership function of the mean time to accident for the "safety system-protected object" complex. The proposed approach allows to take into account the uncertainty of reliability model parameters and reliability indices. The solution obtained is useful for reliability assessment of nuclear power plants and similar dangerous technological objects.

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