RELIABILITY AND CAPABILITY MODELING OF TECHNOLOGICAL SYSTEMS WITH BUFFER STORAGE

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ABSTRACT

The paper is devoted to reliability and capability investigation of technological systems, inclusive of development of dynamic reliability model for two-phase product line with buffer storages and multiphase line decomposition

Key words: Reliability analysis, markov process, multiphase systems, multi-flow structure decomposition

1 INTRODUCTION

Multiphase systems are the systems where technological process and supporting equipment are divided into sections referred as phases. One of the approaches to improving reliability and capability is to include into multiphase system time redundancy using buffer storages. When failure of input section equipment occurs buffer storage ensures uninterrupted technological process in output sections. Valid choice of placement location and capacity of buffer storages is impossible without reliability modeling and analysis of system projects alternatives. Common prediction models of multiphase systems describe only single-flow structures and suppose absolute reliability of buffer storage (*Cherkesov 1974*). In this paper we suggest analytical method for calculation reliability and capability of multiphase systems based on two-parameter markov process. The prediction model takes into account different ratio of input and output devices capability and unreliable buffers. The model decomposition technique is developed. This makes it possible to analyze multi-flow systems with tree-type structures. Procedure of construction state space and transition graph of the two-parameter markov process is created. The procedure is founded on selection of state subsets, corresponding to intermediate and marginal (maximum or minimum) level of resource (inventory) in buffer, and generation of boundary and limiting transition. Process of generation of difference equation and boundary condition are described.

2 TWO-PHASE SYSTEM DESCRIPTION

Schema of single-flow two-phase system with input (1) and output (2) processing devices and transient buffer (3) is shown in **Figure 1**.

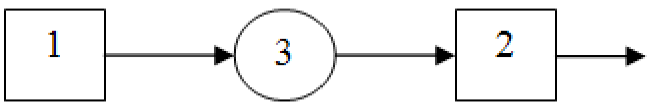

Figure 1. Single-flow two-phase system with buffer storage.

Each processing devices is characterized by capability q_i, failure rate λ_i, recovery rate μ_i; buffer is characterized by capacity z ($0 \le z \le z_M$), failure rate λ_H, recovery rate μ_H . Let us denote the state of markov graph for two-phase system by three-digit binary code. The first two digits indicate the states of the devices and the third digit indicates the buffer state. Digit 1 indicates that the state of device (buffer) is good, 0 – is failed.

Reliability behavior of the system depends on inventory level in the buffer:

- zero level ($z = 0$); we will designate zero level subset of markov reliability model state set as G
- maximum level ($z = z_M$); we will designate maximum level subset of markov reliability model state set as V
- intermediate level ($0 < z < z_M$); we will designate intermediate level subset of markov reliability model state set as W

3 METHODOLOGY OF TWO-PARAMETER MARKOV MODEL CONSTRUCTION

Let us define the markov model construction sequence:

1. Definition of all possible states for subsets G, V, W
2. Analysis of the states in compliance with characteristics of performance and failures, removing the states which can not stand in given subset and which have not transition from another states
3. Determination of the states which have marginal (limiting) transitions from another states (these are transitions from subset W into V and G, assignable with buffer inventory level maximization (minimization). Marginal transitions are indicated as dotted line.
4. Determination of boundary transitions from subsets V and G into subset W. These transitions exist for the states in subsets V and G, for which failure or recovery of the system devices result in buffer marginal inventory level decrease (increase). Boundary transitions are also indicated as dotted arc, waited with appropriate failure (recovery) rate.

After markov graph construction we can define mathematical model of the system. Let us denote state probability for subset W as $P(z,t)$ and for subsets V and G as $F(z_M,t)$ and $F(0,t)$ respectively. Now we can set up difference equation for characteristic states of the system. Characteristic states are the following:

1. The states which have input and output transitions in the range of one subset
2. The states which have input limiting transitions
3. The states which have output boundary transitions (equations for these states determine boundary conditions)

Figure 2 shows graphs with characteristic state α_i and input (output) transition. Graph I shows transitions in the range of one subset. Graph II shows boundary transition.

Difference equation for case I (transitions in the range of one subset) is of the form

$$P_{\alpha i}(z, t+\Delta t) - P_{\alpha i}(z \pm \Delta z_i, t) = -(\Delta t \sum_{i=1}^{m} \psi_i) \cdot P_{\alpha i}(z \pm \Delta z_i, t) + \Delta t \sum_{1}^{n} \cdot \varphi_i \cdot P_i(z \pm \Delta z_i, t) \tag{1}$$

Partial differential equation is:

$$q_{\alpha i} \cdot \frac{\partial P_{\alpha i}(z,t)}{\partial z} + \frac{\partial P_{\alpha i}(z,t)}{\partial t} = -(\sum_{i=1}^{m} \psi_i) \cdot P_{\alpha i}(z,t) + \sum_{i=1}^{n} \varphi_i \cdot P_i(z,t) \tag{2}$$

Let us consider stationary area and take into account the fact that $\frac{\partial P(z,t)}{\partial t} = 0$ when $t \to \infty$.

Then

$$q_{\alpha i} \cdot \frac{\partial P_{\alpha i}(z)}{\partial z} = -(\sum_{i=1}^{m} \psi_i) \cdot P_{\alpha i}(z) + \sum_{i=1}^{n} \varphi_i \cdot P_i(z) \tag{3}$$

Under (3) we can formulate the following rule for setting up differential equation for any state with transitions in the range of one subset.

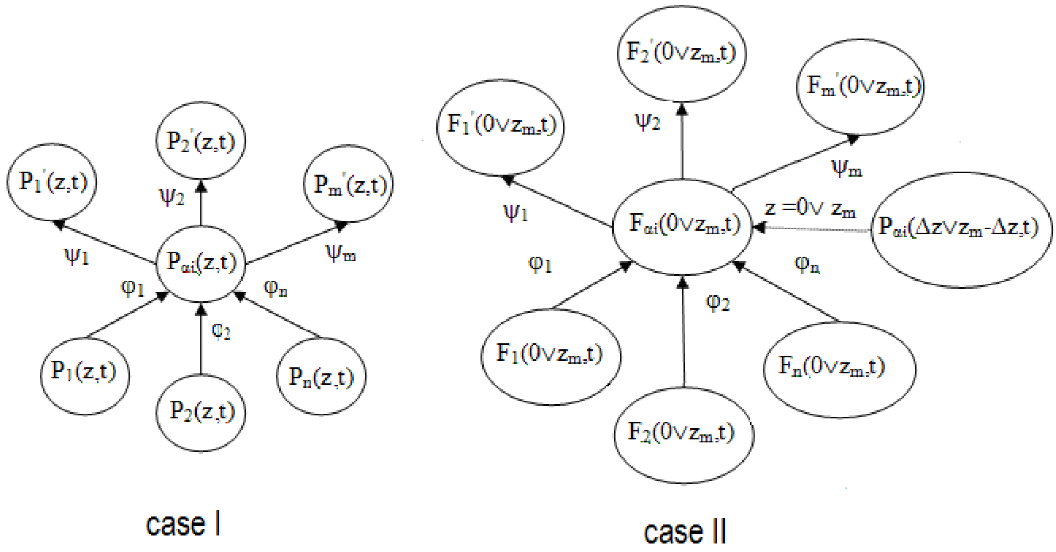

Figure 2. Graphs for case I (transitions in area of one state subset) and case II (limiting transition).

Rule 1. Derivative of state probability with respect to buffer inventory level (z) multiplied by rate of level change ($q_{\alpha i}$) is equal to product of state probability by sum of output transition rates, signed with minus, plus sum of product of input transition rate by probability of state from which transition is done.

Similarly we get differential equation for the case II. Here state α_i in the range of one subset has input transitions with rate φ_i, output transitions with rate ψ_I and limiting transition from subset W (z=0 or z=z_m).

$$\frac{\partial F_{\alpha i}(0 \vee z_m, t)}{\partial t} = \sum_{i=1}^{n} \varphi_i F_i(0 \vee z_m, t) - (\sum_{i=1}^{m} \psi_i) \cdot F_{\alpha i}(0 \vee z_m, t) + |q_{\alpha i}| \cdot P_{\alpha i}(0 \vee z_m, t) \tag{4}$$

For stationary area ($t \to \infty$) we have algebraic equation

$$(\sum_{i=1}^{m} \psi_i) \cdot F_{\alpha i}(0 \vee z_m) = \sum_{i=1}^{n} \varphi_i F_i(0 \vee z_m) + |q_{\alpha i}| \cdot P_{\alpha i}(0 \vee z_m) \tag{5}$$

Then it is possible to formulate the rule for states with input limiting transition.

Rule 2. Probability of considering state multiplied by sum of output transition rate is equal to sum of transition probabilities from other states to given state and probability of limiting transition. Probability of limiting transition is probability of state from which transition is done multiplied by absolute value of rate of level change.

Boundary condition occurs when transition exists from states of subsets V and G into states of subset W:

$$\begin{aligned} \psi \cdot F_{\alpha i}(z_m, t) &= P_{\alpha i}(z_m, t) \cdot |q_{\alpha i}| \\ \varphi \cdot F_{\alpha i}(0, t) &= P_{\alpha i}(0, t) \cdot |q_{\alpha i}| \end{aligned} \tag{6}$$

Stationary boundary condition is:

$$\begin{aligned} \psi \cdot F_{\alpha i}(z_m) &= P_{\alpha i}(z_m) \cdot |q_{\alpha i}| \\ \varphi \cdot F_{\alpha i}(0) &= P_{\alpha i}(0) \cdot |q_{\alpha i}| \end{aligned} \tag{7}$$

4 MARKOV RELIABILITY MODEL FOR TWO-PHASE SYSTEM

Proceeding from rules and equations of previous section one can construct reliability models for two-phase single-flow system. Models were constructed for three alternatives of relationship of processing devices capability ($q_1=q_2=q$; $q_1 > q_2$; $q_1 < q_2$).

4.1 Model for equality of input and output capability

Markov graph for equality of capability of input and output processing devices ($q_1=q_2=q$) is shown on **Figure 3**.

System of partial differential equation is:

$$\begin{aligned}
& -q \cdot \frac{\partial P_{011}(z,t)}{\partial z} + \frac{\partial P_{011}(z,t)}{\partial t} = -(\mu_1 + \lambda_2 + \lambda_{н}) \cdot P_{011}(z,t) + \lambda_1 \cdot P_{111}(z,t) + \mu_2 \cdot P_{001}(z,t) + \mu_{н} \cdot P_{010}(z,t) \\
& q \cdot \frac{\partial P_{101}(z,t)}{\partial z} + \frac{\partial P_{101}(z,t)}{\partial t} = -(\mu_2 + \lambda_1 + \lambda_{н}) \cdot P_{101}(z,t) + \lambda_2 \cdot P_{111}(z,t) + \mu_1 \cdot P_{001}(z,t) + \mu_{н} \cdot P_{100}(z,t) \\
& \frac{\partial P_{111}(z,t)}{\partial t} = -(\lambda_1 + \lambda_2 + \lambda_{н}) \cdot P_{111}(z,t) + \mu_1 \cdot P_{011}(z,t) + \mu_2 \cdot P_{101}(z,t) + \mu_{н} \cdot P_{110}(z,t) \\
& \frac{\partial P_{001}(z,t)}{\partial t} = -(\mu_1 + \mu_2) \cdot P_{001}(z,t) + \lambda_2 \cdot P_{011}(z,t) + \lambda_1 \cdot P_{101}(z,t) \\
& \frac{P_{010}(z,t)}{\partial t} = -(\mu_1 + \mu_{н}) \cdot P_{010}(z,t) + \lambda_{н} \cdot P_{011}(z,t) \\
& \frac{\partial P_{100}(z,t)}{\partial t} = -(\mu_2 + \mu_{н}) \cdot P_{100}(z,t) + \lambda_{н} \cdot P_{101}(z,t) \\
& \frac{\partial P_{110}(z,t)}{\partial t} = -\mu_{н} \cdot P_{110}(z,t) + \mu_1 \cdot P_{010}(z,t) + \mu_2 \cdot P_{100}(z,t) + \lambda_{н} \cdot P_{111}(z,t) \\
& \frac{\partial F_{111}(0,t)}{\partial t} = -(\lambda_1 + \lambda_2 + \lambda_{н}) \cdot F_{111}(0,t) + \mu_1 \cdot F_{011}(0,t) + \mu_{н} \cdot F_{110}(0,t) \\
& \frac{\partial F_{011}(0,t)}{\partial t} = -\mu_1 \cdot F_{011}(0,t) + \lambda_1 \cdot F_{111}(0,t) + q \cdot P_{011}(0,t) \\
& \frac{\partial F_{110}(0,t)}{\partial t} = -\mu_{н} \cdot F_{110}(0,t) + \lambda_{н} \cdot F_{111}(0,t) \\
& \frac{\partial F_{111}(z_m,t)}{\partial t} = -(\lambda_1 + \lambda_2 + \lambda_{н}) \cdot F_{111}(z_m,t) + \mu_2 \cdot F_{101}(z_m,t) + \mu_{н} \cdot F_{110}(z_m,t) \\
& \frac{\partial F_{101}(z_m,t)}{\partial t} = -\mu_2 \cdot F_{101}(z_m,t) + \lambda_2 \cdot F_{111}(z_m,t) + q \cdot P_{101}(z_m,t) \\
& \frac{\partial F_{110}(z_m,t)}{\partial t} = -\mu_{н} \cdot F_{110}(z_m,t) + \lambda_{н} \cdot F_{111}(z_m,t)
\end{aligned} \tag{8}$$

Boundary condition:

$$\begin{aligned} q \cdot P_{011}(z_m,t) &= \lambda_1 \cdot F_{111}(z_m,t) \\ q \cdot P_{101}(0,t) &= \lambda_2 \cdot F_{111}(0,t) \end{aligned} \tag{9}$$

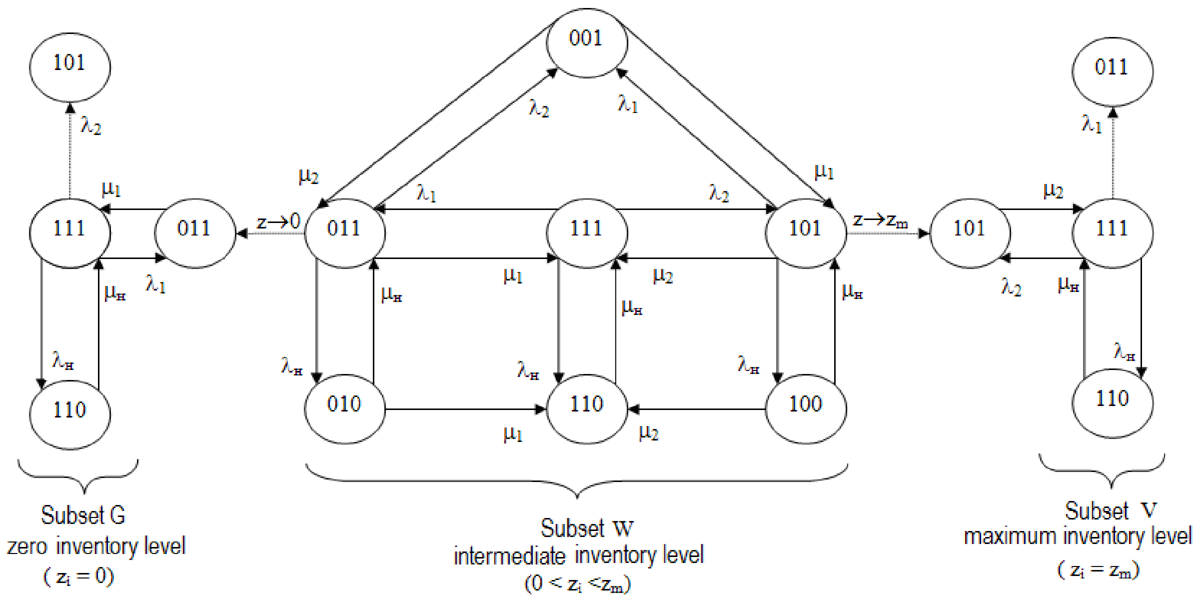

Figure 3. Markov graph for two-phase system ($q_1=q_2=q$).

At stationary area ($t \to \infty$) system (8) turns into the system of differential-algebraic equation:

$$\begin{aligned}
-q \cdot P'_{011}(z) &= -(\mu_1 + \lambda_2 + \lambda_H) \cdot P_{011}(z) + \lambda_1 \cdot P_{111}(z) + \mu_2 \cdot P_{001}(z) + \mu_H \cdot P_{010}(z) \\
q \cdot P'_{101}(z) &= -(\mu_2 + \lambda_1 + \lambda_H) \cdot P_{101}(z) + \lambda_2 \cdot P_{111}(z) + \mu_1 \cdot P_{001}(z) + \mu_H \cdot P_{100}(z) \\
0 &= -(\lambda_1 + \lambda_2 + \lambda_H) \cdot P_{111}(z) + \mu_1 \cdot P_{011}(z) + \mu_2 \cdot P_{101}(z) + \mu_H \cdot P_{110}(z) \\
0 &= -(\mu_1 + \mu_2) \cdot P_{001}(z) + \lambda_2 \cdot P_{011}(z) + \lambda_1 \cdot P_{101}(z) \\
0 &= -(\mu_1 + \mu_H) \cdot P_{010}(z) + \lambda_H \cdot P_{011}(z) \\
0 &= -(\mu_2 + \mu_H) \cdot P_{100}(z) + \lambda_H \cdot P_{101}(z) \\
0 &= -\mu_H \cdot P_{110}(z) + \mu_1 \cdot P_{010}(z) + \mu_2 \cdot P_{100}(z) + \lambda_H \cdot P_{111}(z) \\
0 &= -(\lambda_1 + \lambda_2 + \lambda_H) \cdot F_{111}(0) + \mu_1 \cdot F_{011}(0) + \mu_H \cdot F_{110}(0) \\
0 &= -\mu_1 \cdot F_{011}(0) + \lambda_1 \cdot F_{111}(0) + q \cdot P_{011}(0) \\
0 &= -\mu_H \cdot F_{110}(0) + \lambda_H \cdot F_{111}(0) \\
0 &= -(\lambda_1 + \lambda_2 + \lambda_H) \cdot F_{111}(z_m) + \mu_2 \cdot F_{101}(z_m) + \mu_H \cdot F_{110}(z_m) \\
0 &= -\mu_2 \cdot F_{101}(z_m) + \lambda_2 \cdot F_{111}(z_m) + q \cdot P_{101}(z_m) \\
0 &= -\mu_H \cdot F_{110}(z_m) + \lambda_H \cdot F_{111}(z_m)
\end{aligned} \tag{10}$$

We use the following boundary and normalizing condition when solving system (10):

$$q \cdot P_{011}(z_m) = \lambda_1 \cdot F_{111}(z_m)$$
$$q \cdot P_{101}(0) = \lambda_2 \cdot F_{111}(0) \qquad (11)$$
$$\sum_{ijk} \int_0^{z_m} P_{ijk}(z)\partial z + \sum_{ijk} F_{ijk}(z_m) + \sum_{ijk} F_{ijk}(0) = 1$$

4.2 Model for unequal input and output capability ($q_1 > q_2$)

Markov graph for unequal capability of input and output processing devices ($q_1 > q_2$) is shown on **Figure 4**.

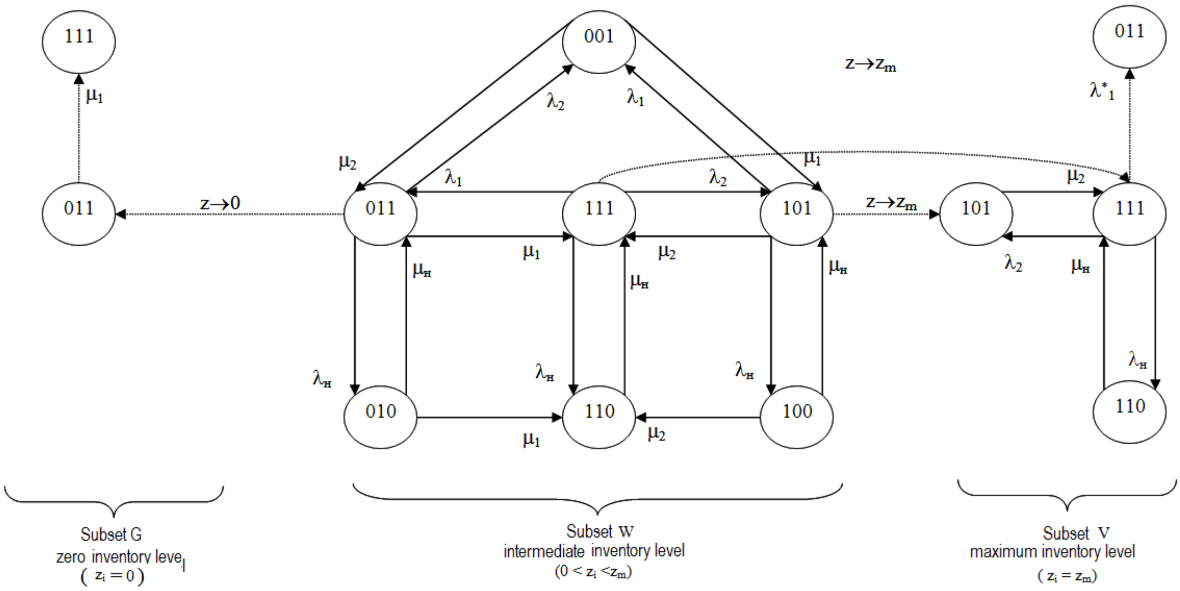

Figure 4. Markov graph for two-phase system ($q_1 > q_2$).

Let us directly consider stationary area ($t \to \infty$) and system of differential-algebraic equation:

$$-q_2 \cdot P'_{011}(z) = -(\mu_1 + \lambda_2 + \lambda_H) \cdot P_{011}(z) + \lambda_1 \cdot P_{111}(z) + \mu_2 \cdot P_{001}(z) + \mu_H \cdot P_{010}(z)$$
$$q_1 \cdot P'_{101}(z) = -(\mu_2 + \lambda_1 + \lambda_H) \cdot P_{101}(z) + \lambda_2 \cdot P_{111}(z) + \mu_1 \cdot P_{001}(z) + \mu_H \cdot P_{100}(z)$$
$$(q_1 - q_2) \cdot P'_{111}(z) = -(\lambda_1 + \lambda_2 + \lambda_H) \cdot P_{111}(z) + \mu_1 \cdot P_{011}(z) + \mu_2 \cdot P_{101}(z) + \mu_H \cdot P_{110}(z)$$
$$0 = -(\mu_1 + \mu_2) \cdot P_{001}(z) + \lambda_2 \cdot P_{011}(z) + \lambda_1 \cdot P_{101}(z)$$
$$0 = -(\mu_1 + \mu_H) \cdot P_{010}(z) + \lambda_H \cdot P_{011}(z)$$
$$0 = -(\mu_2 + \mu_H) \cdot P_{100}(z) + \lambda_H \cdot P_{101}(z) \qquad (12)$$
$$0 = -\mu_H \cdot P_{110}(z) + \mu_1 \cdot P_{010}(z) + \mu_2 \cdot P_{100}(z) + \lambda_H \cdot P_{111}(z)$$
$$0 = -\mu_1 \cdot F_{011}(0) + q_2 \cdot P_{011}(0)$$
$$0 = -(\lambda_1 + \lambda_2 + \lambda_H) \cdot F_{111}(z_m) + \mu_2 \cdot F_{101}(z_m) + \mu_H \cdot F_{110}(z_m) + (q_1 - q_2) \cdot P_{111}(z_m)$$
$$0 = -\mu_2 \cdot F_{101}(z_m) + \lambda_2 \cdot F_{111}(z_m) + q_1 \cdot P_{101}(z_m)$$
$$0 = -\mu_H \cdot F_{110}(z_m) + \lambda_H \cdot F_{111}(z_m)$$

Boundary and normalizing condition:

$$
\begin{aligned}
&q \cdot P_{011}(z_m) = \lambda_1^* \cdot F_{111}(z_m) \\
&(q_1 - q_2) \cdot P_{111}(0) = \mu_1 \cdot F_{011}(0) \qquad (13) \\
&\sum_{ijk} \int_0^{z_m} P_{ijk}(z)\partial z + \sum_{ijk} F_{ijk}(z_m) + \sum_{ijk} F_{ijk}(0) = 1
\end{aligned}
$$

4.3 Model for unequal input and output capability ($q_1 < q_2$)

Markov graph for unequal capability of input and output processing devices ($q_1 < q_2$) is shown on **Figure 5**.

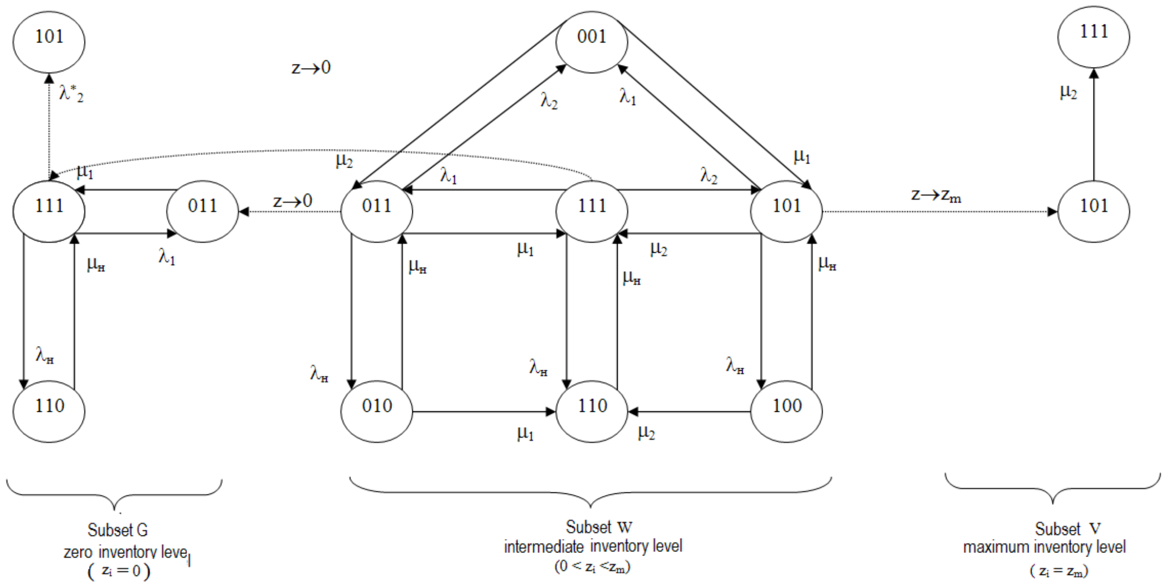

Figure 5. Markov graph for two-phase system ($q_1 < q_2$)

System of differential-algebraic equation $(t \to \infty)$:

$$
\begin{aligned}
&-q_2 \cdot P'_{011}(z) = -(\mu_1 + \lambda_2 + \lambda_H) \cdot P_{011}(z) + \lambda_1 \cdot P_{111}(z) + \mu_2 \cdot P_{001}(z) + \mu_H \cdot P_{010}(z) \\
&q_1 \cdot P'_{101}(z) = -(\mu_2 + \lambda_1 + \lambda_H) \cdot P_{101}(z) + \lambda_2 \cdot P_{111}(z) + \mu_1 \cdot P_{001}(z) + \mu_H \cdot P_{100}(z) \\
&-(q_2 - q_1) \cdot P'_{111}(z) = -(\lambda_1 + \lambda_2 + \lambda_H) \cdot P_{111}(z) + \mu_1 \cdot P_{011}(z) + \mu_2 \cdot P_{101}(z) + \mu_H \cdot P_{110}(z) \\
&0 = -(\mu_1 + \mu_2) \cdot P_{001}(z) + \lambda_2 \cdot P_{011}(z) + \lambda_1 \cdot P_{101}(z) \\
&0 = -(\mu_1 + \mu_H) \cdot P_{010}(z) + \lambda_H \cdot P_{011}(z) \\
&0 = -(\mu_2 + \mu_H) \cdot P_{100}(z) + \lambda_H \cdot P_{101}(z) \qquad (14) \\
&0 = -\mu_H \cdot P_{110}(z) + \mu_1 \cdot P_{010}(z) + \mu_2 \cdot P_{100}(z) + \lambda_H \cdot P_{111}(z) \\
&0 = -(\lambda_1 + \lambda_2^* + \lambda_H) \cdot F_{111}(0) + \mu_1 \cdot F_{011}(0) + \mu_H \cdot F_{110}(0) + (q_2 - q_1) \cdot P_{111}(0) \\
&0 = -\mu_1 \cdot F_{011}(0) + \lambda_1 \cdot F_{111}(0) + q_2 \cdot P_{011}(0) \\
&0 = -\mu_H \cdot F_{110}(0) + \lambda_H \cdot F_{111}(0) \\
&0 = -\mu_2 \cdot F_{101}(z_m) + q_1 \cdot P_{101}(z_m)
\end{aligned}
$$

Boundary and normalizing condition:

$$\begin{gathered} q_1 \cdot P_{101}(0) = \lambda_2^* \cdot F_{111}(0) \\ (q_2 - q_1) \cdot P_{111}(z_m) = \mu_2 \cdot F_{101}(z_m) \\ \sum_{ijk} \int_0^{z_m} P_{ijk}(z)\partial z + \sum_{ijk} F_{ijk}(z_m) + \sum_{ijk} F_{ijk}(0) = 1 \end{gathered} \tag{15}$$

Computer-oriented procedure was developed for analytical solving systems (10), (12), (14). In accordance with this procedure at first one have to obtain probability density function $P_{101}(z)$:

$$P_{101}(z) = P_{101}(0) \cdot e^{\frac{a}{q}z}, \tag{16}$$

where $a = -(\lambda_1 + \mu_2 + \lambda_н) + \lambda_2 b + \frac{\mu_1(\lambda_1 + \lambda_2)}{\mu_1 + \mu_2} + \frac{\mu_н \lambda_н}{\mu_2 + \mu_н}$

Further probability $F_{101}(z)$ is defined via density function

$$F_{101}(z) = \int_0^{z_m} P_{101}(z)dz = \frac{q}{a} \cdot P_{101}(0)\left[e^{\frac{a}{q}z_m} - 1\right] = C_1 \cdot H_1 . \tag{17}$$

Then each i^{th} unknown $F_{ijk}(z)$ is represented as product of invariable and variable actors $C_1 \cdot H_i$, where H_i recursively calculated from H_{i-1}, and C_1 is calculated from normalizing condition (11,13,15).

Stationary availability $K_r(z)$ and mathematical expectation of capability C(z) of two-phase system are

$$K_r(z)=F_{111}(z)+F_{011}(z)+F_{111}(0)+F_{111}(z_m); \quad C(z)=K_r(z) \cdot q. \tag{18}$$

5 RELIABILITY AND CAPABILITY ANALYSIS OF MULTIPHASE SYSTEM

Multiphase systems are aggregate of two-phase systems. Examples of multiphase multiflow systems, specified in graphical editor of software implemented described above models, are shown in **Figure 6**.

The procedure of calculation estimate of availability of multiphase system includes the following steps:

1. Pick out the triplet (buffer, input device, output device) with minimum buffer capacity
2. Calculate availability and average capability indexes (18) for evolved triplet via appropriate models (10, 12, 14)
3. Replace the triplet by one processing device with equivalent availability and capability calculated on previous step
4. Repeat steps 1-3 until all multiphase structure will be represented by one equivalent device

It was shown in (*Victorova 2009)* that above procedure ensures derivation of availability low estimate.

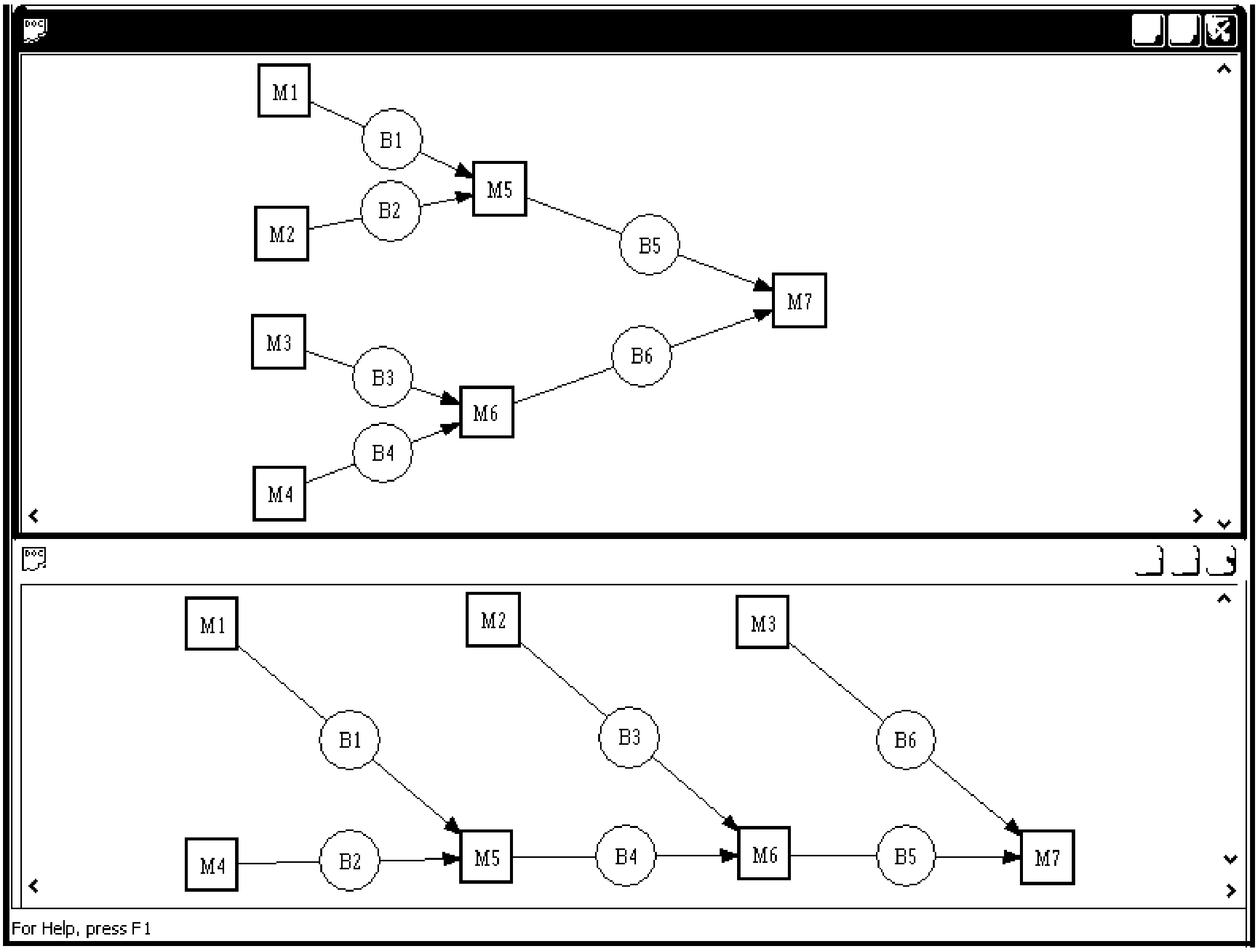

Figure.6. Screen shot of software for reliability and capability analysis of multiphase systems.

6 CONCLUSION

For adequate reliability and capability modeling of technological systems it is necessary to take into account unreliability of buffer storages. Statistical analysis of failure data of buffer storages shows failure rate growth with increasing capacity. On assumption of absolute buffer reliability one can make pitfall about continuous capability growth with increasing capacity (see upper curves on **Figure 7**). Analysis based on the models suggested in this paper shows that inflection point exist on the curve of capability as function of capacity. After this point one can observe decrease of reliability and capability of multiphase systems as it is shown on lower curves of **Figure 7**.

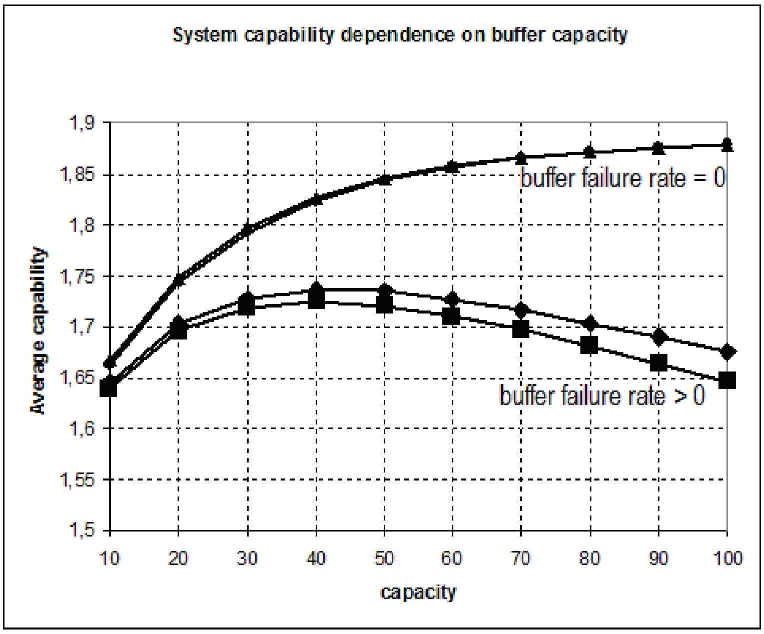

Figure 7. Multiphase system capability dependence on buffer storage capacity.

7 REFERENCES

Cherkesov, G.N. (1974). *Reliability of Technical Systems with Time Redundancy*, Sovetskoe Radio: Moscow.

Victorova V.C. (2009). *Aggregation of reliability and safety models of complex technical systems*, Abstract of Dr.Sci thesis, Institute of Control Sciences RAS: Moscow