METHODS AND ALGORITHMS FOR EVALUATING UNKNOWN PARAMETERS OF COMPONENTS RELIABILITY OF COMPLEX TECHNICAL SYSTEMS

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ABSTRACT

The paper objectives are to present the methods and tools useful in the statistical identifying the unknown parameters of the components reliability and safety of complex industrial systems and to apply them in the maritime industry. There are presented statistical methods of estimating the unknown intensities of departure from the reliability state subsets of the exponential distribution of the component lifetimes of the multistate systems operating in various operation states. The goodness-of-fit method applied to testing the hypotheses concerned with the exponential form of the multistate reliability function of the particular components of the complex technical system in variable operations conditions is suggested. An application of these tools to reliability characteristics of a ferry operating at the Baltic Sea waters is presented as well.

1 INTRODUCTION

Many real transportation systems belong to the class of complex systems. It is concerned with the large numbers of components and subsystems they are built and with their operating complexity. Modeling the complicated system operation processes, first of all, is difficult because of the large number of the operation states, impossibility of their precise defining and because of the impossibility of the exact describing the transitions between these states. The changes of the operation states of the system operations processes cause the changes of these systems reliability structures and also the changes of their components reliability functions (Blokus-Roszkowska et all 2008a). The models of various multistate complex systems are considered in (Blokus-Roszkowska et all 2008b). The general joint models linking these system reliability models with the models of their operation processes, allowing us for the evaluation of the reliability and safety of the complex technical systems in variable operations conditions, are constructed in (Kolowrocki, Soszynska 2008). In these general joint reliability and safety models of the complex systems it was assumed that the conditional multistate reliability functions of the considered systems components in the particular operations states are exponential.

In order to be able to apply these general models practically in the evaluation and prediction the reliability of real complex technical it is necessary to elaborate the statistical methods concerned with determining the unknown parameters of the proposed models. Namely, the probabilities of the initials system operation states, the probabilities of transitions between the system operation states and the distributions of the sojourn times of the system operation process in the particular operation states and also the unknown parameters of the conditional multistate reliability functions of the system components in various operation states. It is also necessary the elaborating the methods of testing the hypotheses concerned with the conditional sojourn times of the system operations process in particular operations states and the hypotheses concerned with the conditional multistate reliability functions of the system components in the system various operation states. In this paper, the methods for evaluating unknown parameters of the exponential reliability functions in various experimental cases with a special stress on small samples and unfinished investigations are defined and formulae for estimating the intensities of departure from the reliability state subsets in all cases are proposed. The common principle to formulate and to verify the hypotheses about the exponential distribution functions of the lifetimes in the reliability state subsets of the multistate system components by chi-square test is also discussed. These tools on exemplary application to estimating unknown intensity of departure on Stena Baltica ferry component is shown.

2 IDENTYFICATION OF CONDITIONAL MULTISTATE RELIABILITY FUNCTIONS OF THE SYSTEM COMPONENTS

2.1. Estimation of intensities of departure from the reliability state subsets

We assume as in (Blokus-Roszkowska et all 2008b) that the changes of operations states of the multistate system operations process $Z(t)$ have an influence on the reliability functions of the system components and we mark the conditional multistate reliability function of the system component when the system is in the operation state z_b, $b = 1,2,...,\nu$, by

$$[R(t,\cdot)]^{(b)} = [1,\ [R(t,1)]^{(b)}, ...,\ [R(t,z)]^{(b)}\ ], \qquad (1)$$

where

$$[R(t,u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_b) \qquad (2)$$

for $t \in < 0,\infty)$, $u = 1,2,...,z$, $b = 1,2,...,\nu$,

is the conditional reliability function standing the probability that the conditional lifetime $T^{(b)}(u)$ of the system component in the reliability states subset $\{u,u+1,...,z\}$, $u = 1,2,...,z$, is greater than t, while the system operation process $Z(t)$ is in the operation state z_b, $b = 1,2,...,\nu$.

Further, we assume that the coordinates of the vector of the conditional multistate reliability function (1) are exponential reliability functions of the form

$$R^{(b)}(t,u) = R(t,\lambda^{(b)}(u)) = \exp[-\lambda^{(b)}(u)t] \text{ for } t \in < 0,\infty), \quad u = 1,2,...,z,\ b = 1,2,...,\nu. \qquad (3)$$

Te above assumptions mean that the density functions of the system component conditional life time $T^{(b)}(u)$ in the reliability states subset $\{u,u+1,...z\}$, $u = 1,2,...,z$, at the operations state z_b, $b = 1,2,...,\nu$, are exponential of the form

$$f^{(b)}(t,u) = f(t,\lambda^{(b)}(u)) = \lambda^{(b)}(u)\exp[-\lambda^{(b)}(u)t] \text{ for } t \in < 0,\infty), \qquad (4)$$

where $\lambda^{(b)}(u)$, $\lambda^{(b)}(u) \geq 0$, is an unknown intensity of departure from this subset of the reliability states.

We want to estimate the value of this unknown intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset $\{u,u+1,...z\}$, $u = 1,2,...,z$, on the basis empirical data. The estimators of the of the unknown intensity of departure $\lambda^{(b)}(u)$, i.e. the unknown failure rate $\lambda^{(b)}$, in the case of the two-state system reliability for various experimental conditions, are determined by maximum likelihood method in (Kolowrocki, Kwiatuszewska-Sarnecka 2009). The modified and transmitted to the multistate system reliability results obtained in (Soszynska et all 2009) are presented below.

Case 1.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Completed investigations, the same observation time on all experimental posts

We assume that during the time τ, $\tau > 0$, we are observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u,u+1,...z\}$, $u = 1,2,...,z$, at the operation state z_b, $b = 1,2,...,\nu$, on n identical experimental posts. Moreover, we assume that during the fixed observation time τ all components left the reliability states subset and we mark by $t_i^{(b)}(u)$, $i = 1,2,...,n$, the moment of departure from the reliability states subsets of the component on the $i-$th observational post, i.e. the realizations of the identical component lifetimes $T_i^{(b)}(u)$, $i = 1,2,...,n$, to the first departure from the reliability states subsets, that are the independent random variables with the exponential distribution defined by the density function (4).

In this case, the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset is

$$[\overline{\lambda}(u)]^{(b)} = \frac{n^{(b)}}{\sum_{i=1}^{n^{(b)}} t_i^{(b)}(u)}, \quad u = 1,2,...,z \tag{5}$$

Case 2.

The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, the same observation time on all experimental posts

We assume that during the time τ, $\tau > 0$, we are observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,...z\}$, $u = 1,2,...,z$, at the operation state z_b, $b = 1,2,...,\nu$, on n identical experimental posts. Moreover, we assume that during the fixed observation time τ not all components left the reliability states subset and we mark by m_1, $m_1 < n$, the number of components that left the reliability states subset and by $t_i^{(b)}(u)$, $i = 1,2,...,m_1$, the moments of their departures from the reliability states subsets, i.e. the realizations of the identical component lifetimes $T_i^{(b)}(u)$, $i = 1,2,...,m_1$, to the first departure from the reliability states subsets, that are the independent random variables with the exponential distribution defined by the density function (4).

In this case, the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset is

$$[\overline{\lambda}(u)]^{(b)} = \frac{m^{(b)}(u)}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \tau^{(b)}[n^{(b)} - m^{(b)}(u)]}, \ u = 1,2,...,z. \tag{6}$$

Assuming the observation time τ as the moment of departure from the reliability states subset of the components that have not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset of the form

$$[\overline{\lambda}(u)]^{(b)} = \frac{n^{(b)}}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \tau^{(b)}[n^{(b)} - m^{(b)}(u)]}, \ u = 1,2,...,z. \tag{6'}$$

Case 3.

The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, different observation times on particular experimental posts

We assume that we are observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,...z\}$, $u = 1,2,...,z$, at the operation state z_b, $b = 1,2,...,\nu$, on n identical experimental posts. We assume that the observation times on particular experimental posts are different and we mark by $\tau^{(i)}$, $\tau^{(i)} > 0$, $i = 1,2,...,n$, the observation time respectively on the i-th experimental post. Moreover, we assume that during the fixed observation times $\tau^{(i)}$ not all components left the reliability states subset and we mark by m_1, $m_1 < n$, the number of components

that left the reliability states subset and by $t_i^{(b)}(u)$, $i = 1,2,...,m_1$, the moments of their departures from the reliability states subsets, i.e. the realizations of the identical component lifetimes $T_i^{(b)}(u)$, $i = 1,2,...,m_1$, to the first departure from the reliability states subsets, that are the independent random variables with the exponential distribution defined by the density function (4).
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset is

$$[\overline{\lambda}(u)]^{(b)} = \frac{m^{(b)}(u)}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \sum_{i=m^{(b)}(u)+1}^{n^{(b)}} \tau_i^{(b)}}, \ u = 1,2,..., z. \tag{7}$$

Assuming the observation times $\tau^{(i)}$, $i = m_1$, $m_1 + 1,...n,$ as the moment of departure from the reliability states subset of the components that have not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset of the form

$$[\overline{\lambda}(u)]^{(b)} = \frac{n^{(b)}}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \sum_{i=m^{(b)}(u)+1}^{n^{(b)}} \tau_i^{(b)}}, \ u = 1,2,..., z. \tag{7'}$$

Case 4.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flow (stream) on one experimental post

We assume that we are observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z,$ at the operation state z_b, $b = 1,2,...,\nu$, on one experimental post. We assume that at the moment when the component is leaving the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z,$ it is replaced at once by the same new component staying at the best reliability state z. Moreover, we assume that the renewal process of the components is continuing during the observation time $\tau^{(b)}$, $\tau^{(b)} > 0,$ and that during this time $m_1^{(b)}(u) = m_1$, $m_1^{(b)}(u) < n^{(b)}$, components have left the reliability states subset $\{u, u+1,..., z\}$ and we mark by $t_i^{(b)}(u) = t_i$, $i = 1,2,..., m_1^{(b)}(u),$ the moments of their departures from the reliability states subsets, i.e. the realizations of the identical component lifetimes $T_i^{(b)}(u)$, $i = 1,2,..., m_1^{(b)}(u),$ to the first departure from the reliability states subset $\{u, u+1,..., z\}$, that are the independent random variables with the exponential distribution defined by the density function (4).
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset is

$$[\overline{\lambda}(u)]^{(b)} = \frac{m^{(b)}(u)}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + d^{(b)}(u)}, \ u = 1,2,..., z, \tag{8}$$

where

$$d^{(b)}(u) = \begin{cases} \tau^{(b)} - \sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(1) \ if \ m^{(b)}(u) = m^{(b)} \\ 0 \qquad if \ m^{(b)}(u) = m^{(b)} + 1, \ u = 1,2,..., z. \end{cases}$$

In the case if $m^{(b)}(u) = m^{(b)}$, $u = 1,2,..., z,$ after assuming the observation time $\tau^{(b)}$ as the moment of departure from the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z,$ of the last component that has not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the form

$$[\bar{\lambda}(u)]^{(b)} = \frac{m^{(b)}+1}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + d^{(b)}(u)}, \ u=1,2,...,z. \tag{8'}$$

Case 5.

The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flows (streams) on several experimental posts – The same observation time on all experimental posts

We assume that we are observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u,u+1,...z\}$, $u=1,2,...,z,$ at the operation state z_b, $b=1,2,...,\nu$, on n experimental posts. We assume that at the moment when the component is leaving the reliability states subset $\{u,u+1,...z\}$, $u=1,2,...,z,$ it is replaced at once by the same new component staying at the best reliability state z. Moreover, we assume that the renewal process of the components is continuing at all experimental posts during the same observation time τ, $\tau>0,$ and that during this time m_k, $k=1,2,...,n,$ components at the k-th experimental post left the reliability states subset $\{u,u+1,...z\}$ and we mark by $[t_i^{(b)}(u)]^{(k)}$, $i=1,2,...,m_k$, the moments of their departures from the reliability states subsets, i.e. the realizations of the identical component lifetimes $[T_i^{(b)}(u)]^{(k)}$, $i=1,2,...,m_k$, to the first departure from the reliability states subset $\{u,u+1,...z\}$, that are the independent random variables with the exponential distribution defined by the density function (4).
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset is

$$[\bar{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)}(u)}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} d_j^{(b)}(u)}, \ u=1,2,...,z, \tag{9}$$

where for $j=1,2,...,n^{(b)}$

$$d_j^{(b)}(u) = \begin{cases} \tau^{(b)} - \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(1)]^{(j)} & if \ m_j^{(b)}(u) = m_j^{(b)} \\ 0 & if \ m_j^{(b)}(u) = m_j^{(b)}+1, \ u=1,2,...,z. \end{cases}$$

In the case if there exist j, $j \in \{1,2,...,n^{(b)}\}$, such that $m_j^{(b)}(u) = m_j^{(b)}$, $u=1,2,...,z,$ assuming the observation time $\tau^{(b)}$ as the moment of departures from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the last components on all experimental posts that have not left this reliability states subset we get so called pessimistic evaluation of the intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the form

$$[\bar{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)} + n^{(b)}}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} d_j^{(b)}(u)}, \ u=1,2,...,z. \tag{9'}$$

Case 6.

The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flows (streams) on several experimental posts – Different observation times on experimental posts

We assume that we are observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u,u+1,...z\}$, $u=1,2,...,z,$ at the operation state z_b, $b=1,2,...,\nu$, on n experimental posts. We assume that at the moment when the component is leaving the reliability states subset $\{u,u+1,...z\}$, $u=1,2,...,z,$ it is replaced at once by the same new component staying at the best reliability state z. Moreover, we assume that the renewal process of the components is continuing at the *k*-th experimental post during the observation time $\tau^{(k)}$, $\tau^{(k)}>0,$ and that during this time m_k, $k=1,2,...,n,$ components at this experimental post left the reliability states subset $\{u,u+1,...z\}$ and we mark by $[t_i^{(b)}(u)]^{(k)}$, $i=1,2,...,m_k$, the moments of their departures from the reliability states subsets, i.e. the realizations of the identical component lifetimes $[T_i^{(b)}(u)]^{(k)}$, $i=1,2,...,m_k$, to the first departure from the reliability states subset $\{u,u+1,...z\}$, that are the independent random variables with the exponential distribution defined by the density function (4). In this case, the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset is

$$[\overline{\lambda}(u)]^{(b)}=\frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)}(u)}{\sum_{j=1}^{n^{(b)}}\sum_{i=1}^{m_j^{(b)}(u)}[t_i^{(b)}(u)]^j+\sum_{j=1}^{n^{(b)}}\overline{d}_j^{(b)}(u)},\ u=1,2,...,z, \tag{10}$$

where for $j=1,2,...,n^{(b)}$

$$\overline{d}_j^{(b)}(u)=\begin{cases}\tau_j^{(b)}-\sum_{i=1}^{m_j^{(b)}(u)}[t_i^{(b)}(1)]^{(j)} & \text{if } m_j^{(b)}(u)=m_j^{(b)}\\ 0 & \text{if } m_j^{(b)}(u)=m_j^{(b)}+1,\ u=1,2,...,z.\end{cases}$$

In the case if there exist $j,\ j\in\{1,2,...,n^{(b)}\},$ such that $m_j^{(b)}(u)=m_j^{(b)}$, $u=1,2,...,z,$ assuming the observation times $\tau_j^{(b)}$, $j=1,2,...,n^{(b)}$, as the moments of departures from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the last components on experimental posts that have not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the form

$$[\overline{\lambda}(u)]^{(b)}=\frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)}+n^{(b)}}{\sum_{j=1}^{n^{(b)}}\sum_{i=1}^{m_j^{(b)}(u)}[t_i^{(b)}(u)]^j+\sum_{j=1}^{n^{(b)}}\overline{d}_j^{(b)}(u)},\ u=1,2,...,z. \tag{10'}$$

2.2. Identification of distributions of conditional lifetimes of system components in reliability state subsets

To formulate and next to verify the non-parametric hypothesis concerning the exponential form of the coordinate

$$R^{(b)}(t,u)=R(t,\lambda^{(b)}(u))=\exp[-\lambda^{(b)}(u)t]\ \text{for}\ t\in<0,\infty),\quad u=1,2,...,z,\ b=1,2,...,\nu. \tag{11}$$

of the vector

$$[R(t,\cdot)]^{(b)}=[1,\ [R(t,1)]^{(b)},...,\ [R(t,z)]^{(b)}\ ], \tag{12}$$

of the conditional multistate reliability function of the system component when the system is at the operations state z_b, $b=1,2,...,\nu,$ it is necessary to ct according to the scheme below:

- to fix the realizations $t_1^{(b)}(u),\ t_2^{(b)}(u),\ \ldots,\ t_n^{(b)}(u),\ u=1,2,...,z,$ of the system component conditional lifetimes $T^{(b)}(u)$, $b=1,2,...,\nu$, in the reliability states subsets $\{u,u+1,...,z\},\ u=1,2,...,z,$
- to determine the number $\bar{r}$ of the disjoint intervals $I_j =< x_j, y_j)$, $j=1,2,...,\bar{r}$, that include the realizations $t_1^{(b)}(u),\ t_2^{(b)}(u),\ \ldots,\ t_n^{(b)}(u)$ of the system component conditional lifetimes $T^{(b)}(u)$ in the reliability states subset, according to the formula

$$\bar{r} \cong \sqrt{n},$$

- to determine the length d of the intervals $I_j =< x_j, y_j)$, $j=1,2,...,\bar{r}$, according to the formula

$$d = \frac{\bar{R}}{\bar{r}-1},$$

where

$$\bar{R} = \max_{1\le i\le n} t_i^{(b)} - \min_{1\le i\le n} t_i^{(b)},$$

- to determine the ends x_j, y_j, of the intervals $I_j =< x_j, y_j)$, $j=1,2,...,\bar{r}$, according to the formulae

$$x_1 = \min_{1\le i\le n} t_i^{(b)} - \frac{d}{2},\ y_j = x_1 + jd,\ \ j=1,2,...,\bar{r},\ x_j = y_{j-1},\ \ j=2,3,...,\bar{r},$$

in the way such that

$$I_1 \cup I_2 \cup ... \cup I_{\bar{r}} =< x_1, y_{\bar{r}}),$$

and

$$I_i \cap I_j = \emptyset \text{ for all } i \ne j,\ i,j \in \{1,2,...,\bar{r}\},$$

- to determine the numbers of realizations n_j in particular intervals I_j, $j=1,2,...,\bar{r}$, according to the formula

$$n_j = \#\{i : t_i^{(b)} \in I_j, i \in \{1,2,...,n\}\},\ \ j=1,2,...,\bar{r},$$

where

$$\sum_{j=1}^{\bar{r}} n_j = n,$$

whereas the symbols $\#$ means the number of elements of a set,
- to evaluate the value of the unknown intensity of the component departure $\lambda^{(b)}(u),$ from the reliability states subset, applying suitable formula from the section 3.1,
- to construct and to plot the realization of the histogram of the conditional system component lifetime $T^{(b)}(u),\ b=1,2,...,\nu,$ in the reliability states subset $\{u,u+1,...,z\},\ u=1,2,...,z,$ at the system operation state z_b, $b=1,2,...,\nu,$

$$\bar{f}_n^{(b)}(t,u) = \frac{n_j}{n} \text{ for } t \in I_j,$$

- to analyze the realization of the histogram, comparing it with the graph of the exponential density function

$$f^{(b)}(t,u) = f(t,\lambda^{(b)}(u)) = \lambda^{(b)}(u)\exp[-\lambda^{(b)}(u)t] \text{ for } t \in< 0,\infty),$$

of the system component lifetime $T^{(b)}(u)$ in the reliability states subset $\{u,u+1,...z\}$ at the operations state z_b, corresponding the reliability function coordinate

$$R^{(b)}(t,u) = R(t,\lambda^{(b)}(u)) = \exp[-\lambda^{(b)}(u)t] \text{ for } t \in< 0,\infty),$$

of the vector of the conditional multistate reliability function of the system component

$$[R(t,\cdot)]^{(b)} = [1,\ [R(t,1)]^{(b)}, ...,\ [R(t,z)]^{(b)}\ ],$$

and to formulate the null hypothesis H_0 and the alternative hypothesis H_A, concerned with the form of the component multistate reliability $[R(t,\cdot)]^{(b)}$ in the following form:

H_0 : The conditional multistate reliability function of the system component

$$[R(t,\cdot)]^{(b)} = [1,\ [R(t,1)]^{(b)}, ...,\ [R(t,z)]^{(b)}\ ],$$

has the exponential reliability functions coordinates of the form

$$R^{(b)}(t,u) = R(t,\lambda^{(b)}(u)) = \exp[-\lambda^{(b)}(u)t] \text{ for } t \in < 0,\infty),$$

H_A : The conditional multistate reliability function of the system component has different from the exponential reliability functions coordinates,

- to join each of the intervals I_j, that has the number n_j of realizations is less than 4 either with the neighbor interval I_{j+1} or with the neighbor interval I_{j-1}, this way that the numbers of realizations in all intervals are not less than 4,
- to fix a new number of intervals
$\bar{\bar{r}}$,
- to determine new intervals
$\bar{I}_j =< \bar{x}_j, \bar{y}_j),\quad j = 1,2,..,\bar{\bar{r}},$
- to fix the numbers $\bar{n}_j$ of realizations in new intervals $\bar{I}_j,\quad j = 1,2,..,\bar{\bar{r}},$
- to calculate the hypothetical probabilities that the variable $T^{(b)}(u)$ takes values from the interval $\bar{I}_j$, under the assumption that the hypothesis H_0 is true, i.e. the probabilities

$$p_j = P(T^{(b)}(u) \in \bar{I}_j) = P(\bar{x}_j \le T^{(b)}(u) < \bar{y}_j) = R^{(b)}(\bar{x}_j,u) - R^{(b)}(\bar{y}_j,u),\ j = 1,2,...,\bar{\bar{r}},$$

where $R^{(b)}(\bar{x}_j,u)$ and $R^{(b)}(\bar{y}_j,u)$ are the values of the coordinate reliability function $R^{(b)}(t,u)$ of the multistate reliability function defined in the null hypothesis H_0.

- to calculate the realization of the χ^2 (*chi-square*)-Pearson's statistics U_n, according to the formula

$$u_n = \sum_{j=1}^{\bar{\bar{r}}} \frac{(\bar{n}_j - np_j)^2}{np_j},$$

- to assume the significance level α ($\alpha = 0.01,\ \alpha = 0.02,\ \alpha = 0.05$ or $\alpha = 0.10$) of the test,

- to fix the number $\bar{\bar{r}} - l - 1$ of degrees of freedom, substituting $l = 1$,

- to read from the Tables of the χ^2 – Pearson's distribution the value u_α for the fixed values of the significance level α and the number of degrees of freedom $\bar{\bar{r}} - l - 1$ such that the following equality holds

$$P(U_n > u_\alpha) = 1 - \alpha,$$

and next to determine the critical domain in the form of the interval $(u_\alpha,+\infty)$ and the acceptance domain in the form of the interval $< 0, u_\alpha >$,

- to compare the obtained value u_n of the realization of the statistics U_n with the red from the Tables critical value u_α of the chi-square random variable and to verify previously formulated the null hypothesis H_0 in the following way: if the value u_n does not belong to the critical domain, i.e. when $u_n \le u_\alpha$, then we do not reject the hypothesis H_0, otherwise if the value u_n belongs to the critical domain, i.e. when $u_n > u_\alpha$, then we reject the hypothesis H_0 in favor of the hypothesis H_A.

3 APPLICATION IN MARITIME TRANSPORT

3.1. The Stena Baltica ferry reliability characteristic statistical identification

The exact evaluation of the Stena Baltica ferry is not possible at the moment because of the complete lack of statistical data about the changes the reliability state subsets by the ferry components and subsystems. Currently, we have only one information about the change from the reliability state subset $\{1.2\}$ into the worst reliability state $z = 0$ (a failure) one of two stern loading platforms operating at the ferry main deck. This departure happened after its good working for around 22 years. The remaining components and subsystems of the ferry under considerations are high reliable and none of them failed during the observation time $\tau = 22.5$ years.

The estimation of this failed component intensity of departure from the reliability states subset $\{1.2\}$ can be done by the formula (6) derived in *Case 2*. Substituting in this formula $\tau = 22.5$, $u = 1$, $n = 2$, $m_1 = 1$ and $t_1^{(b)}(1) = 22$, we get the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(1)$ from the reliability states subset $\{1.2\}$ is

$$\overline{\lambda}^{(b)}(1) = \frac{1}{22 + 22.5(2-1)} \cong 0.0225.$$

The estimation of this failed component intensity of departure from the reliability states subset $\{1.2\}$ can also be done by the formula (9) derived in *Case 5*. Substituting in this formula $\tau = 22.5$, $u = 1$, $n = 2$, $m_1 = 1$ and $t_1^{(b)}(1) = 22$, we get the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(1)$ from the reliability states subset $\{1.2\}$ is

$$\overline{\lambda}^{(b)}(1) = \frac{1+0}{2 \cdot 22.5} \cong 0.0222.$$

The unknown intensities of departures from the reliability state subsets for the components that have not failed during the observation time can be evaluated using so called pessimistic estimation (7')-(11'), derived in (Kolowrocki, Kwiatuszewska-Sarnecka 2009).

4 CONCLUSION

The statistical methods estimating the unknown intensities of the components' exponential reliability functions existing in the joint general model of complex technical systems reliability operating in variable operation conditions linking a semi-markov modeling of the system operation processes with a multi-state approach to system reliability and availability analysis are proposed. Next, these methods are applied to estimating the reliability characteristics of Stena Baltica ferry operating between Gdynia Port in Poland and Karsklone Port in Sweden. The proposed methods other very wide applications to port and shipyard transportation systems reliability and safety characteristics evaluation are obvious. The results are expected to be the basis to the reliability and safety of complex technical systems optimization and their operation processes effectiveness and cost analysis.

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5 REFERENCES

Blokus-Roszkowska, A., Guze, S., Kolowrocki, K., Kwiatuszewska-Sarnecka, B., Soszynska, J. 2008a. *Models of safety, reliability and availability evaluation of complex technical systems related to their* *operation processes*. WP 4 – Task 3.1 – 31.05.2008. Poland-Singapore Joint Research Project, 2007-2010.

Blokus-Roszkowska, A., Guze, S., Kolowrocki, K., Kwiatuszewska-Sarnecka, B., Soszynska, J. 2008b. *Models of safety, reliability and availability evaluation of complex technical systems related to their operation processes*. WP 4 – Task 4.1 – 31.05.2008. Poland-Singapore Joint Research Project, 2007-2010.

Blokus-Roszkowska, A. Guze, S. Kolowrocki, K., Kwiatuszewska-Sarnecka, B., Milczek B., Soszynska, J. 2009. *Data Mining for Identification and Prediction of Safety and Reliability Characteristics of Complex Industrial Systems and Processes.* WP6 – Task 6.1. Methods-algorithms for evaluating unknown parameters of system operation processes, reliability and safety models. WP6 - Task 6.1.2 – 16.03.2009. Poland-Singapore Joint Research Project, 2007-2010.

Grabski, F. 2002. *Semi-Markov Models of Systems Reliability and Operations Analysis*. Warsaw: Systems Research Institute, Polish Academy of Science.

Kolowrocki, K., Soszynska, J. 2008. *A general model of technical systems operation processes related to their environment and infrastructure*. WP 2 – Task 2.1 – 31.05.2008. Poland-Singapore Joint Research Project, 2007-2010.

Soszynska, J., Kołowrocki, K., Kamiński, P., Jurdziński, M., Guze, S., Milczek, B., Golik, P. 2009. *Data Mining for Identification and Prediction of Safety and Reliability Characteristics of Complex Industrial Systems and Processes.* WP6– Task 6.2. Preliminary safety and risk analysis and evaluation of a Stena Baltica ferry in variable operation conditions. Sub-Task 6.2.5 - Sub-Task 6.2.5 – 16.03.2009. Poland-Singapore Joint Research Project, 2007-2010.

Kolowrocki, K., Kwiatuszewska- Sarnecka, B. 2009. *Data Mining for Identification and Prediction of Safety and Reliability Characteristics of Complex Industrial Systems and Processes.* WP6 – Task 6.1. Methods-algorithms for evaluating unknown parameters of system operation processes, reliability and safety models – a preliminary study. WP6 - Task 6.1.1 – 16.03.2009. Poland-Singapore Joint Research Project, 2007-2010.