A RELIABILITY MODEL FOR "SAFETY SYSTEM-PROTECTED OBJECT" COMPLEX WITH MULTIPLE SAFETY SYSTEMS

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ABSTRACT

The paper presents a new reliability model for "safety system-protected object" complex with multiple safety systems. It is supposed that the complex consists of one protected object and multiple independent safety systems with complex structures. Scheduled periodic inspections of safety systems are also taken into account. Asymptotic estimates of the mean time to accident and the probability of the accident prior to time t are obtained under some assumptions on operation process of the complex.

1 INTRODUCTION

Hazardous facilities use a variety of systems concerned with safety, with safety systems being the most important of those. Safety systems are provided to detect potentially dangerous protected object failures or conditions and to implement appropriate safety actions. Protected object may have several types of hazardous deviations of protected object operation process that require their own safety systems. Some reliability models for the elements of safety systems were introduced by Hansen and Aarø (Aarø & Hansen 1997), Corneliussen and Hokstad (Corneliussen & Hokstad 2003), Høyland and Rausand (Høyland & Rausand 2004). In this paper we propose a different approach to reliability assessment of "safety system-protected object" complex based on asymptotic properties of alternating renewal processes.

In the present study we set out to analyze the reliability of the automated "safety system-protected object" complex with multiple safety systems. Systems of such kind are quite common in the nuclear power engineering, because safety systems of nuclear power plant should employ diversity in the detection of fault sequences and in the initiation of the safety system action to terminate the sequences. We follow Pereguda (Pereguda 2001) in assuming that the operation of the complex can be described using a superposition of alternating renewal processes. Our objective is to provide an asymptotic estimation for such reliability indices as the mean time to accident and the probability of the accident prior to time t.

2 MODEL DESCRIPTION

Let us consider an automated complex of protected object and N safety systems. Safety systems and the protected object are repairable. They are restored to an as-good-as-new state. All failures are supposed to be independent. Let j-th safety system consists of M_j subsystems and k-th subsystem of j-th safety system consists of $C_{j,k}$ elements.

By $\chi_{i,j}$, $i=1,2,\ldots$, $j=1,2,\ldots,N$ denote the time to the i-th protected object failure detected by j-th safety system. Let $\chi_{i,j}$, $i=1,2,\ldots$, $j=1,2,\ldots,N$ be independent random variables and for each fixed j let $\chi_{i,j}$, $i=1,2,\ldots$ be identically distributed random variables with CDF $F_{\chi_j}(t)$. By $\gamma_{i,j}$, $i=1,2,\ldots$, $j=1,2,\ldots,N$ denote the time to the protected object repair after it's i-th failure detected by j-th safety system. Let $\gamma_{i,j}$, $i=1,2,\ldots$, $j=1,2,\ldots,N$ be independent random variables and for each fixed j let $\gamma_{i,j}$, $i=1,2,\ldots$ be identically distributed random variables with CDF $F_{\gamma_j}(t)$. Suppose that moments of the protected object repair are renewal points of the operation process of the complex. Suppose that $F_{\chi_j}(t)$ and $F_{\gamma_j}(t)$ are nonlattice distributions with finite mean. By $\xi_{i,j,k,l}$, $i=1,2,\ldots$, $j=1,2,\ldots,N$, $k=1,2,\ldots,M_j$, $l=1,2,\ldots,C_{j,k}$ denote the time to the i-th failure of the l-th element of the k-th subsystem of the j-th safety system. Let $\xi_{i,j,k,l}$, $i=1,2,\ldots$, $j=1,2,\ldots,N$, $k=1,2,\ldots,M_j$, $l=1,2,\ldots,C_{j,k}$ be independent random variables and for each fixed j, k, l let $\xi_{i,j,k,l}$, $i=1,2,\ldots$ be identically distributed random variables with CDF $F_{\xi_{j,k,l}}(t)$. Suppose that safety system elements are repaired only after corresponding safety subsystem failure is detected. By $\eta_{i,j,k}$, $i = 1,2,\ldots$, $j=1,2,\ldots,N$, $k=1,2,\ldots,M_j$ denote the time to repair of the k-th subsystem of the j-th safety system after it's i-th failure. Let $\eta_{i,j,k}$, $i = 1,2,\ldots$, $j=1,2,\ldots,N$, $k=1,2,\ldots,M_j$ be independent random variables and for each fixed j, k let $\eta_{i,j,k}$, $i = 1,2,\ldots$ be identically distributed random variables with CDF $F_{\eta_{j,k}}(t)$. Suppose that moments of the safety subsystem repair are renewal points of the operation process of the safety subsystem. Suppose that $F_{\xi_{j,k,l}}(t)$ and $F_{\eta_{j,k}}(t)$ are nonlattice distributions with finite mean. A failure of the safety subsystem may be detected immediately or only during scheduled periodic inspections of the safety subsystem. By $T_{j,k}$ denote the period of scheduled inspections of the k-th subsystem of the j-th safety system. By $\theta_{j,k}$ denote the duration of scheduled inspections of the k-th subsystem of the j-th safety system. The safety subsystem may be active or inactive during the inspection. Suppose that each safety system is coherent system (Høyland & Rausand 2004) and each safety subsystem is coherent system. Let $\varphi_{j,k}(x_{j,k,1}, x_{j,k,2}, \ldots x_{j,k,C_{j,k}})$ denote the system structure function of the k-th subsystem of the j-th safety system and let $\psi_j(x_{j,1}, x_{j,2}, \ldots, x_{j,M_j})$ denote the system structure function of the j-th safety system. Let ν be a random number of renewal intervals of the operation process of the complex before an accident. By ω denote the time to accident. An accident takes place when safety systems are unable to detect the protected object failure. Our aim is to estimate the mean time to accident $M\omega$ and the probability $\Pr(\omega \le t)$ of the accident prior to time t.

2 MAIN RESULTS

2.1 Mean time to failure and reliability function

Since the operation process of the complex is a superposition of alternating renewal processes, it follows that

$$\omega = \sum_{i=1}^{\nu-1}\Big(\min(\chi_{i,1}, \chi_{i,2}, \ldots, \chi_{i,N}) + \gamma_{i,1} J_{\chi_{i,1} < \min(\chi_{i,2}, \chi_{i,3} \ldots, \chi_{i,N})} + \gamma_{i,2} J_{\chi_{i,2} < \min(\chi_{i,1}, \chi_{i,3} \ldots, \chi_{i,N})} + \ldots$$
$$+ \gamma_{i,N} J_{\chi_{i,N} < \min(\chi_{i,1}, \chi_{i,2} \ldots, \chi_{i,N-1})}\Big) + \min(\chi_{\nu,1}, \chi_{\nu,2}, \ldots, \chi_{\nu,N}),$$

where J_A is an indicator function of the event A. By α_i denote the time to i-th failure of the protected object. By β_i denote the time to i-th repair of the protected object. We obviously have

$$\alpha_i = \min(\chi_{i,1}, \chi_{i,2}, \ldots, \chi_{i,N})$$

and

$$\beta_i = \gamma_{i,1} J_{\chi_{i,1} < \min(\chi_{i,2}, \chi_{i,3} \ldots, \chi_{i,N})} + \gamma_{i,2} J_{\chi_{i,2} < \min(\chi_{i,1}, \chi_{i,3} \ldots, \chi_{i,N})} + \ldots + \gamma_{i,N} J_{\chi_{i,N} < \min(\chi_{i,1}, \chi_{i,2} \ldots, \chi_{i,N-1})}.$$

Therefore

$$F_\omega(t) = \Pr(\omega \le t) = \Pr\left(\sum_{i=1}^{\nu-1}(\alpha_i + \beta_i) + \alpha_\nu \le t\right)$$

and

$$\Pr(\nu = n) = q(1-q)^{n-1},$$

where q is the probability of accident during a renewal interval $\left[\sum_{i=1}^{r}(\alpha_i + \beta_i), \sum_{i=1}^{r+1}(\alpha_i + \beta_i)\right)$, $\forall r \in \{0,1,2,\ldots\}$. Applying the Laplace-Stieltjes transform to $F_\omega(t)$, we obtain

$$\widetilde{F}_\omega(s) = E\left[e^{-s\omega}\right] = \sum_{n=1}^{\infty} E\left[e^{-s\omega} \mid \nu = n\right]\Pr(\nu = n)$$

where $\widetilde{F}_\omega(s) = \int_0^\infty e^{-st} dF_\omega(t) = E\left[e^{-s\omega}\right]$. We see that

$$E\left[e^{-s\omega} \mid \nu = n\right] = E\left[e^{-s\left(\sum_{i=1}^{\nu-1}(\alpha_i+\beta_i)+\alpha_\nu\right)} \middle| \nu = n\right] = \left(\widetilde{F}_\alpha(s)\right)^n \left(\widetilde{F}_\beta(s)\right)^{n-1}.$$

Note that

$$F_\alpha(t) = 1 - \prod_{j=1}^{N}\left(1 - F_{\chi_j}(t)\right)$$

and

$$F_\beta(t) = \sum_{j=1}^{N} F_{\gamma_j}(t) \int_0^\infty \left(\prod_{\substack{r=1 \\ r \ne j}}^{N}\left(1 - F_{\chi_r}(x)\right)\right) dF_{\chi_j}(x).$$

Finally,

$$\widetilde{F}_\omega(s) = \sum_{n=1}^{\infty}\left(\widetilde{F}_\alpha(s)\right)^n\left(\widetilde{F}_\beta(s)\right)^{n-1} q(1-q)^{n-1} = \frac{q\widetilde{F}_\alpha(s)}{1-(1-q)\widetilde{F}_\alpha(s)\widetilde{F}_\beta(s)}.$$

Since $E[\omega] = -\left.\frac{d\widetilde{F}_\omega(s)}{ds}\right|_{s=0}$, it follows that

$$E[\omega] = E[\alpha] + \frac{1-q}{q}\left(E[\alpha] + E[\beta]\right),$$

where

$$E[\alpha] = \int_0^\infty \left(\prod_{j=1}^{N}\left(1 - F_{\chi_j}(t)\right)\right) dt$$

and

$$E[\beta] = \sum_{j=1}^{N}\int_0^\infty \left(1 - F_{\gamma_j}(t)\right)dt \int_0^\infty \left(\prod_{\substack{r=1 \\ r \ne j}}^{N}\left(1 - F_{\chi_r}(t)\right)\right) dF_{\chi_j}(t).$$

Applying a limit theorem for recurrent point processes with a fixed interarrival time distribution (Kovalenko, Kuznetsov & Pegg 1997) we obtain

$$\Pr\left(\frac{q\omega}{E[\alpha] + E[\beta]} > t\right) \xrightarrow[q \to 0]{} e^{-t}.$$

Therefore

$$\Pr(\omega \le t) \xrightarrow[q \to 0]{} 1 - e^{-\frac{qt}{E[\alpha]+E[\beta]}}.$$

Note that $q \to 0$ for a highly reliable safety system which is the case for most of hazardous facilities.

2.2 Probability of accident during a renewal interval

Applying the law of total probability we obtain

$$q = \sum_{j=1}^{N} q_j \Pr\left(\chi_j < \min\left(\chi_1, \chi_2, \ldots, \chi_{j-1}, \chi_{j+1}, \ldots, \chi_N\right)\right) = \sum_{j=1}^{N} q_j \int_0^{\infty} \left(\prod_{\substack{r=1 \\ r \ne j}}^{N} \left(1 - F_{\chi_r}(t)\right) \right) dF_{\chi_j}(t),$$

where q_j is the probability of accident during a renewal interval due to j-th safety system failure. The accident takes place during i-th renewal interval due to j-th safety system failure if and only if $\chi_{i,j} \in Q_j^-$, where Q_j^- is the set of intervals where the j-th safety system is inactive. Therefore

$$q_j = \int_0^{\infty} \Pr\left(t \in Q_j^-\right) dF_{\chi_j}(t).$$

It is difficult, if at all possible, to obtain explicit relation for $\Pr(t \in Q_j^-)$. Here we use the following approximate relation:

$$q_j \approx \int_0^{\infty} \tilde{q}_j dF_{\chi_j}(t),$$

where

$$\tilde{q}_j = \lim_{t \to \infty} \Pr(t \in Q_j^-).$$

It is known (Høyland & Rausand 2004) that the j-th safety system availability at time t is

$$p_j(t) = E\left[\psi_j\left(x_{j,1}(t), x_{j,2}(t), \ldots, x_{j,M_j}(t)\right)\right] = h_j\left(p_{j,1}(t), p_{j,2}(t), \ldots, p_{j,M_j}(t)\right),$$

where $p_{j,k}(t)$ is the availability of the k-th subsystem of the j-th safety system. It can be easily shown that

$$\Pr(t \in Q_j^-) = 1 - h_j\left(\Pr\left(t \in Q_{j,1}^+\right), \Pr\left(t \in Q_{j,2}^+\right), \ldots, \Pr\left(t \in Q_{j,M_j}^+\right)\right),$$

where $Q_{k,j}^+$ is the set of intervals where the k-th subsystem of the j-th safety system is active. Therefore

$$q_j \approx 1 - h_j\left(\tilde{p}_{j,1}, \tilde{p}_{j,2}, \ldots, \tilde{p}_{j,M_j}\right),$$

where

$$\tilde{p}_{j,k} = \lim_{t \to \infty} \Pr\left(t \in Q_{j,k}^+\right).$$

Applying the law of total probability we obtain

$$p_{j,k}(t) = \Pr\left(t \in Q_{j,k}^+\right) = \int_0^{\infty}\int_0^{\infty} \Pr\left(t \in Q_{j,k}^+ \mid \xi_{1,j,k} = x, \eta_{1,j,k} = y\right) dF_{\eta_{j,k}}(y)\, dF_{\xi_{j,k}}(x),$$

where $\xi_{i,j,k}$ is the time to i-th failure of the k-th subsystem of the j-th safety system. Obviously, $\xi_{i,j,k}$, i=1,2,… are identically distributed random variables with CDF $F_{\xi_{j,k}}(t)$ for each fixed j, k. It can be easily shown that

$$F_{\xi_{j,k}}(t) = 1 - h_{j,k}\left(1 - F_{\xi_{j,k,1}}(t), 1 - F_{\xi_{j,k,2}}(t), \ldots, 1 - F_{\xi_{j,k,C_{j,k}}}(t)\right),$$

where

$$h_{j,k}\left(p_{j,k,1}(t), p_{j,k,2}(t), \ldots, p_{j,k,C_{j,k}}(t)\right) = E\left[\varphi_{j,k}\left(x_{j,k,1}(t), x_{j,k,2}(t), \ldots x_{j,k,C_{j,k}}(t)\right)\right].$$

2.3 Safety system without inspections

By definition $p_{j,k}(t)$ is the availability of the k-th subsystem of the j-th safety system. We obviously have

$$p_{j,k}(t) = \iint\limits_{x+y\le t} \Pr\left(t \in Q^{+}_{j,k} \mid \xi_{1,j,k} = x, \eta_{1,j,k} = y\right) dF_{\eta_{j,k}}(y) dF_{\xi_{j,k}}(x) + \iint\limits_{x+y>t} \Pr\left(t \in Q^{+}_{j,k} \mid \xi_{1,j,k} = x, \eta_{1,j,k} = y\right) dF_{\eta_{j,k}}(y) dF_{\xi_{j,k}}(x) = I_1 + I_2.$$

It can be easily shown that

$$I_2 = \iint\limits_{x+y>t} J_{t\in[0,x]} dF_{\eta_{j,k}}(y) dF_{\xi_{j,k}}(x) = 1 - F_{\xi_{j,k}}(t).$$

Since the operation process of the safety system is an alternating renewal process, it follows that

$$I_1 = \iint\limits_{x+y\le t} \Pr\left(t \in Q^{+}_{j,k} \mid \xi_{1,j,k} = x, \eta_{1,j,k} = y\right) dF_{\eta_{j,k}}(y) dF_{\xi_{j,k}}(x) = \iint\limits_{x+y\le t} p_{j,k}(t-x-y) dF_{\eta_{j,k}}(y) dF_{\xi_{j,k}}(x) = \int_0^t p_{j,k}(t-z) dF_{\xi_{j,k}+\eta_{j,k}}(z),$$

where

$$F_{\xi_{j,k}+\eta_{j,k}}(z) = \int_0^z F_{\xi_{j,k}}(z-y) dF_{\eta_{j,k}}(y).$$

Finally,

$$p_{j,k}(t) = 1 - F_{\xi_{j,k}}(t) + \int_0^t p_{j,k}(t-z) dF_{\xi_{j,k}+\eta_{j,k}}(z).$$

This equation is well known as the fundamental renewal equation (Høyland & Rausand 2004). The application of Laplace-Stieltjes transform and tauberian theorems yields

$$\tilde{p}_{j,k} = \lim_{t\to\infty} p_{j,k}(t) = \frac{E[\xi_{j,k}]}{E[\xi_{j,k}] + E[\eta_{j,k}]},$$

where

$$E[\xi_{j,k}] = \int_0^\infty \left(1 - F_{\xi_{j,k}}(t)\right) dt$$

and

$$E[\eta_{j,k}] = \int_0^\infty \left(1 - F_{\eta_{j,k}}(t)\right) dt.$$

Again, this is the well known equation for the limiting availability (Høyland & Rausand 2004).

2.4 Safety system with inspections, safety system is inactive during inspection

Let us again write the availability of the k-th subsystem of the j-th safety system as the sum of the following two expressions:

$$p_{j,k}(t) = \iint\limits_{\tau_{j,k}(x,y)\le t} \Pr\left(t \in Q^{+}_{j,k} \mid \xi_{1,j,k} = x, \eta_{1,j,k} = y\right) dF_{\eta_{j,k}}(y) dF_{\xi_{j,k}}(x) + \iint\limits_{\tau_{j,k}(x,y)>t} \Pr\left(t \in Q^{+}_{j,k} \mid \xi_{1,j,k} = x, \eta_{1,j,k} = y\right) dF_{\eta_{j,k}}(y) dF_{\xi_{j,k}}(x) = I_1 + I_2,$$

where $\tau_{j,k}(\xi_{1,j,k},\eta_{1,j,k}) = \left(\left\langle \frac{\xi_{1,j,k}}{T_{j,k}+\theta_{j,k}} \right\rangle + 1\right)\left(T_{j,k}+\theta_{j,k}\right) + \eta_{1,j,k}$ is the length of the renewal interval of the k-th subsystem of the j-th safety system operation process and $\langle x \rangle$ is an integer part of x. We see that

$$I_2 = \iint\limits_{\tau_{j,k}(x,y)>t} \left(\sum_{r=0}^{\left\langle \frac{x}{T_{j,k}+\theta_{j,k}} \right\rangle - 1} J_{t\in\left[r\left(T_{j,k}+\theta_{j,k}\right),\, r\left(T_{j,k}+\theta_{j,k}\right)+T_{j,k}\right)} + J_{t\in\left[\left\langle \frac{x}{T_{j,k}+\theta_{j,k}} \right\rangle \left(T_{j,k}+\theta_{j,k}\right),\, x\right)} \right) dF_{\eta_{j,k}}(y)\, dF_{\xi_{j,k}}(x).$$

It can be easily shown that

$$I_2 = \left(1 - F_{\xi_{j,k}}(t)\right) - \sum_{r=1}^{\infty}\left(1 - F_{\xi_{j,k}}\left(r\left(T_{j,k}+\theta_{j,k}\right)\right)\right)\left(J_{(r-1)\left(T_{j,k}+\theta_{j,k}\right)+T_{j,k}\le t} - J_{r\left(T_{j,k}+\theta_{j,k}\right)\le t}\right) = F_{\zeta_{j,k}}(t) - F_{\xi_{j,k}}(t),$$

where

$$F_{\zeta_{j,k}}(t) = 1 - \sum_{r=1}^{\infty}\left(1 - F_{\xi_{j,k}}\left(r\left(T_{j,k}+\theta_{j,k}\right)\right)\right)\left(J_{(r-1)\left(T_{j,k}+\theta_{j,k}\right)+T_{j,k}\le t} - J_{r\left(T_{j,k}+\theta_{j,k}\right)\le t}\right).$$

Note that

$$I_1 = \iint\limits_{\tau_{j,k}(x,y)\le t} \Pr\left(t \in Q^{+}_{j,k} \mid \xi_{1,j,k} = x, \eta_{1,j,k} = y\right) dF_{\eta_{j,k}}(y)\, dF_{\xi_{j,k}}(x) =$$

$$= \iint\limits_{\tau_{j,k}(x,y)\le t} p_{j,k}\left(t - \tau(x,y)\right) dF_{\eta_{j,k}}(y)\, dF_{\xi_{j,k}}(x) = \int_0^t p_{j,k}(t-z)\, dF_{\tau_{j,k}\left(\xi_{j,k},\eta_{j,k}\right)}(z),$$

where $F_{\tau_{j,k}\left(\xi_{j,k},\eta_{j,k}\right)}(z) = \Pr\left(\tau_{j,k}\left(\xi_{j,k},\eta_{j,k}\right) \le t\right)$. Therefore

$$p_{j,k}(t) = F_{\zeta_{j,k}}(t) - F_{\xi_{j,k}}(t) + \int_0^t p_{j,k}(t-z)\, dF_{\tau_{j,k}\left(\xi_{j,k},\eta_{j,k}\right)}(z).$$

Applying the same technique as above we get the following estimation:

$$\tilde{P}_{j,k} = \lim_{t\to\infty} p_{j,k}(t) = \frac{E\left[\xi_{j,k}\right] - E\left[\zeta_{j,k}\right]}{E\left[\tau_{j,k}\left(\xi_{j,k},\eta_{j,k}\right)\right]},$$

where

$$E\left[\zeta_{j,k}\right] = \theta_{j,k} E\left[\left\langle \frac{\xi_{j,k}}{T_{j,k}+\theta_{j,k}} \right\rangle\right] = \theta_{j,k}\sum_{r=1}^{\infty} r\left(F_{\xi_{j,k}}\left((r+1)\left(T_{j,k}+\theta_{j,k}\right)\right) - F_{\xi_{j,k}}\left(r\left(T_{j,k}+\theta_{j,k}\right)\right)\right),$$

$$E\left[\xi_{j,k}\right] = \int_0^{\infty}\left(1 - F_{\xi_{j,k}}(t)\right)dt,$$

$$E\left[\tau_{j,k}\left(\xi_{j,k},\eta_{j,k}\right)\right] = \int_0^{\infty}\left(1 - F_{\eta_{j,k}}(t)\right)dt +$$

$$+ (T_{j,k}+\theta_{j,k})\left(1 + \sum_{r=1}^{\infty} r\left(F_{\xi_{j,k}}\left((r+1)\left(T_{j,k}+\theta_{j,k}\right)\right) - F_{\xi_{j,k}}\left(r\left(T_{j,k}+\theta_{j,k}\right)\right)\right)\right).$$

2.5 Safety system with inspections, safety system is active during inspection

Using the same method as above we obtain

$$p_{j,k}(t)=\iint\limits_{\tau_{j,k}(x,y)\le t}\Pr\left(t\in Q^{+}_{j,k}\mid \xi_{1,j,k}=x,\eta_{1,j,k}=y\right)dF_{\eta_{j,k}}(y)dF_{\xi_{j,k}}(x)+$$

$$+\iint\limits_{\tau_{j,k}(x,y)>t}\Pr\left(t\in Q^{+}_{j,k}\mid \xi_{1,j,k}=x,\eta_{1,j,k}=y\right)dF_{\eta_{j,k}}(y)dF_{\xi_{j,k}}(x)=I_1+I_2,$$

where $\tau_{j,k}(\xi_{1,j,k},\eta_{1,j,k})=\left(\left\langle\dfrac{\xi_{1,j,k}}{T_{j,k}+\theta_{j,k}}\right\rangle+1\right)\left(T_{j,k}+\theta_{j,k}\right)+\eta_{1,j,k}$ is the length of the renewal interval of the k-th subsystem of the j-th safety system operation process. It is clear that

$$I_2=\iint\limits_{\tau_{j,k}(x,y)>t}J_{t\in[0,x]}dF_{\eta_{j,k}}(y)dF_{\xi_{j,k}}(x)=1-F_{\xi_{j,k}}(t)$$

and

$$I_1=\iint\limits_{\tau_{j,k}(x,y)\le t}\Pr\left(t\in Q^{+}_{j,k}\mid \xi_{1,j,k}=x,\eta_{1,j,k}=y\right)dF_{\eta_{j,k}}(y)dF_{\xi_{j,k}}(x)=$$

$$=\iint\limits_{\tau_{j,k}(x,y)\le t}p_{j,k}(t-\tau(x,y))dF_{\eta_{j,k}}(y)dF_{\xi_{j,k}}(x)=\int_0^t p_{j,k}(t-z)dF_{\tau_{j,k}(\xi_{j,k},\eta_{j,k})}(z),$$

where $F_{\tau_{j,k}(\xi_{j,k},\eta_{j,k})}(z)=\Pr\left(\tau_{j,k}\left(\xi_{j,k},\eta_{j,k}\right)\le t\right)$. And once again we obtain fundamental renewal equation

$$p_{j,k}(t)=1-F_{\xi_{j,k}}(t)+\int_0^t p_{j,k}(t-z)dF_{\tau_{j,k}(\xi_{j,k},\eta_{j,k})}(z).$$

Therefore

$$\tilde{p}_{j,k}=\lim_{t\to\infty}p_{j,k}(t)=\frac{E\left[\xi_{j,k}\right]}{E\left[\tau_{j,k}\left(\xi_{j,k},\eta_{j,k}\right)\right]},$$

where

$$E\left[\xi_{j,k}\right]=\int_0^\infty\left(1-F_{\xi_{j,k}}(t)\right)dt,$$

$$E\left[\tau_{j,k}\left(\xi_{j,k},\eta_{j,k}\right)\right]=\int_0^\infty\left(1-F_{\eta_{j,k}}(t)\right)dt+$$

$$+(T_{j,k}+\theta_{j,k})\left(1+\sum_{r=1}^{\infty}r\left(F_{\xi_{j,k}}\left((r+1)\left(T_{j,k}+\theta_{j,k}\right)\right)-F_{\xi_{j,k}}\left(r\left(T_{j,k}+\theta_{j,k}\right)\right)\right)\right).$$

3 CASE STUDY

Consider the following example. Suppose that complex consists of 5 safety systems and one protected object.

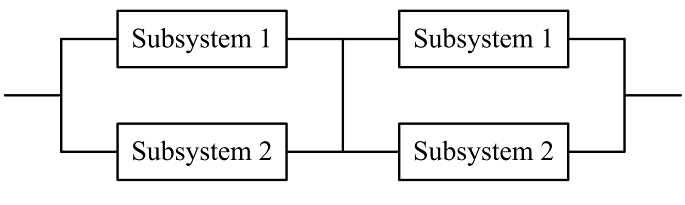

Figure 1. Reliability block diagram of the first safety system.

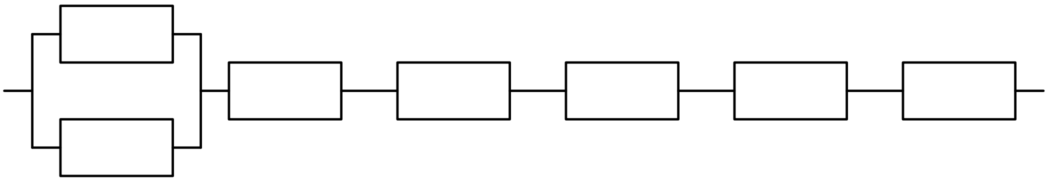

Figure 2. Reliability block diagram of Subsystem 1.

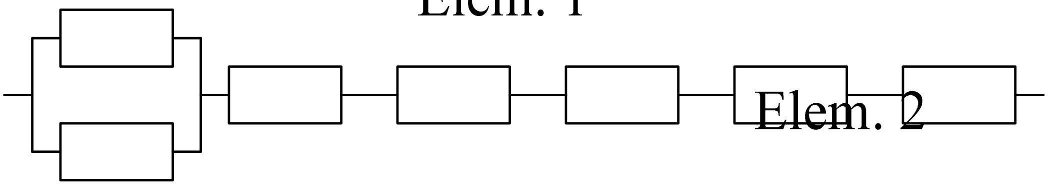

Figure 3. Reliability block diagram of Subsystem 2.

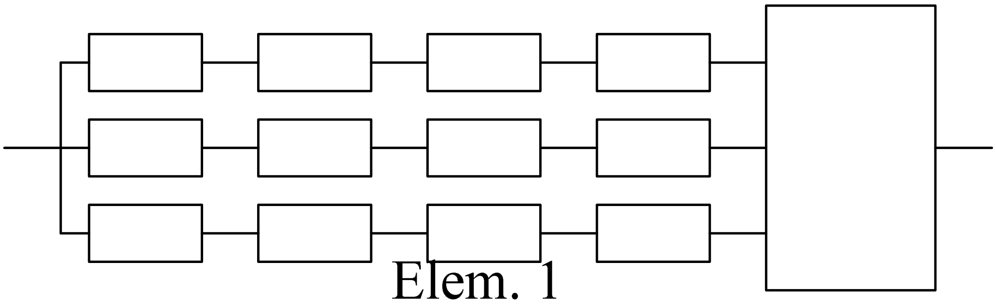

Figure 4. Reliability block diagram of the second safety system.

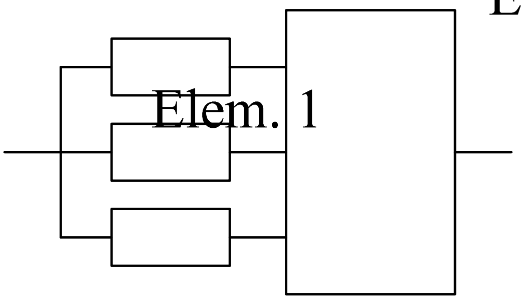

Figure 5. Reliability block diagram of the third safety system.

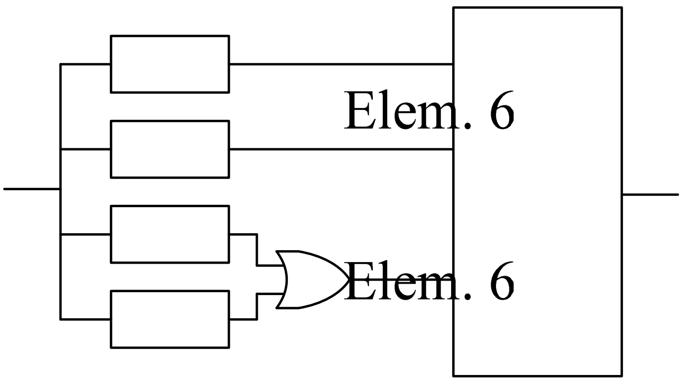

Figure 6. Reliability block diagram of the fourth safety system.

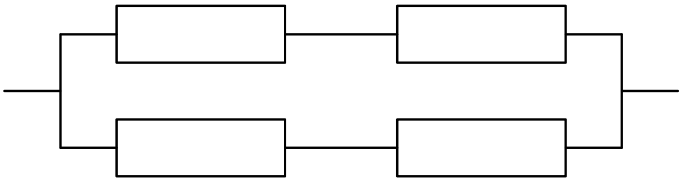

Figure 7. Reliability block diagram of the fifth safety system.

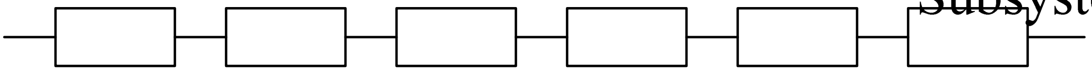

Figure 8. Reliability block diagram of Subsystem 3.

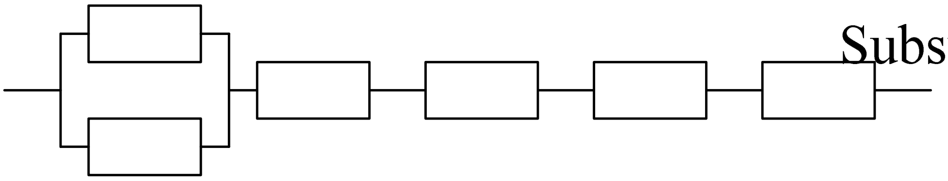

Figure 9. Reliability block diagram of Subsystem 4.

Reliability block diagrams of safety systems are shown on Figures 1 through 9. We obviously have N=5, M_1=2, M_2=1, M_3=1, M_4=1, M_5=2. It can be easily shown that

$$h_1(t) = \left(p_{1,1}(t) + p_{1,2}(t) - p_{1,1}(t)p_{1,2}(t)\right)^2,$$

$$h_2(t) = 3\left(p_{2,1}(t)\right)^2 - 2\left(p_{2,1}(t)\right)^3, \quad h_3(t) = 3\left(p_{3,1}(t)\right)^2 - 2\left(p_{3,1}(t)\right)^3,$$

$$h_4(t) = \left(p_{4,1}(t)\right)^2 + 2p_{4,1}(t)\left(2p_{4,1}(t) - \left(p_{4,1}(t)\right)^2\right)\left(1 - p_{4,1}(t)\right),$$

$$h_5(t) = 1 - \left(1 - p_{5,1}(t)p_{5,2}(t)\right)^2.$$

Therefore $C_{1,1}$=4, $C_{1,2}$=4, $C_{2,1}$=4, $C_{3,1}$=1, $C_{4,1}$=1, $C_{5,1}$=6, $C_{5,2}$=3. We see that

$$h_{1,1}(t) = \left(2p_{1,1,1}(t) - \left(p_{1,1,1}(t)\right)^2\right)\left(p_{1,1,2}(t)\right)^3 p_{1,1,3}(t)p_{1,1,4}(t),$$

$$h_{1,2}(t) = \left(2p_{1,2,1}(t) - \left(p_{1,2,1}(t)\right)^2\right)\left(p_{1,2,2}(t)\right)^3 p_{1,2,3}(t)p_{1,2,4}(t),$$

$$h_{2,1}(t) = p_{2,1,1}(t)p_{2,1,2}(t)p_{2,1,3}(t)p_{2,1,4}(t),$$

$$h_{3,1}(t) = p_{3,1,1}(t), \quad h_{4,1}(t) = p_{4,1,1}(t),$$

$$h_{5,1}(t) = p_{5,1,1}(t)p_{5,1,2}(t)p_{5,1,3}(t)p_{5,1,4}(t)p_{5,1,5}(t)p_{5,1,6}(t),$$

$$h_{5,2}(t) = \left(2p_{5,2,1}(t) - \left(p_{5,2,1}(t)\right)^2\right)\left(p_{5,2,2}(t)\right)^3 p_{5,2,3}(t).$$

Let $F_{\chi_j}(t) = EXP\left(t;\lambda_{\chi_j}\right)$, j=1,2,…,N, $F_{\gamma_j}(t) = EXP\left(t;\lambda_{\gamma_j}\right)$, j=1,2,…,N, $F_{\xi_{j,k,l}}(t) = EXP\left(t;\lambda_{\xi_{j,k,l}}\right)$, j=1,2,…,N, k=1,2,…,M_j, l=1,2,…,$C_{j,k}$ and $F_{\eta_{j,k}}(t) = EXP\left(t;\lambda_{\eta_{j,k}}\right)$, j=1,2,…,N, k=1,2,…,M_j, where

$$EXP(t;\lambda) = \begin{cases} 1 - e^{-\lambda t}, \mathit{if}\ t \geq 0 \\ 0, \mathit{otherwise}. \end{cases}$$

Suppose that failures of all safety subsystem are detected only during scheduled periodic inspections of the safety subsystem and safety subsystems are active during an inspection. Therefore

$$E[\omega] = E[\alpha] + \frac{1-q}{q}\left(E[\alpha] + E[\beta]\right)$$

and

$$\Pr(\omega \le t) \xrightarrow[q\to 0]{} 1 - e^{-\frac{qt}{E[\alpha]+E[\beta]}},$$

where

$$E[\alpha] = \frac{1}{\lambda_{\chi_1} + \lambda_{\chi_2} + \lambda_{\chi_3} + \lambda_{\chi_4} + \lambda_{\chi_5}},$$

$$E[\beta] = \frac{1}{\lambda_{\chi_1} + \lambda_{\chi_2} + \lambda_{\chi_3} + \lambda_{\chi_4} + \lambda_{\chi_5}} \left(\frac{\lambda_{\chi_1}}{\lambda_{\gamma_1}} + \frac{\lambda_{\chi_2}}{\lambda_{\gamma_2}} + \frac{\lambda_{\chi_3}}{\lambda_{\gamma_3}} + \frac{\lambda_{\chi_4}}{\lambda_{\gamma_4}} + \frac{\lambda_{\chi_5}}{\lambda_{\gamma_5}} \right),$$

$$q = \frac{q_1\lambda_{\chi_1} + q_2\lambda_{\chi_2} + q_3\lambda_{\chi_3} + q_4\lambda_{\chi_4} + q_5\lambda_{\chi_5}}{\lambda_{\chi_1} + \lambda_{\chi_2} + \lambda_{\chi_3} + \lambda_{\chi_4} + \lambda_{\chi_5}},$$

$$q_1 \approx 1 - \left(\tilde{p}_{1,1} + \tilde{p}_{1,2} - \tilde{p}_{1,1}\tilde{p}_{1,2}\right)^2,$$

$$q_2 \approx 1 - 3\tilde{p}_{2,1}^2 + 2\tilde{p}_{2,1}^3, \quad q_3 \approx 1 - 3\tilde{p}_{3,1}^2 + 2\tilde{p}_{3,1}^3,$$

$$q_4 \approx 1 - \tilde{p}_{4,1}^2 - 2\tilde{p}_{4,1}\left(2\tilde{p}_{4,1} - \tilde{p}_{4,1}^2\right)\left(1 - \tilde{p}_{4,1}\right), \quad q_5 \approx \left(1 - \tilde{p}_{5,1}\tilde{p}_{5,2}\right)^2,$$

$$\tilde{p}_{j,k} = \frac{E[\xi_{j,k}]}{E[\tau_{j,k}(\xi_{j,k},\eta_{j,k})]}, \ j=1,2,\ldots,N,\ k=1,2,\ldots,M_j,$$

$$E[\xi_{1,1}] = \frac{2}{\lambda_{\xi_{1,1,1}} + 3\lambda_{\xi_{1,1,2}} + \lambda_{\xi_{1,1,3}} + \lambda_{\xi_{1,1,4}}} - \frac{1}{2\lambda_{\xi_{1,1,1}} + 3\lambda_{\xi_{1,1,2}} + \lambda_{\xi_{1,1,3}} + \lambda_{\xi_{1,1,4}}},$$

$$E[\xi_{1,2}] = \frac{2}{\lambda_{\xi_{1,2,1}} + 3\lambda_{\xi_{1,2,2}} + \lambda_{\xi_{1,2,3}} + \lambda_{\xi_{1,2,4}}} - \frac{1}{2\lambda_{\xi_{1,2,1}} + 3\lambda_{\xi_{1,2,2}} + \lambda_{\xi_{1,2,3}} + \lambda_{\xi_{1,2,4}}},$$

$$E[\xi_{2,1}] = \frac{1}{\lambda_{\xi_{2,1,1}} + \lambda_{\xi_{2,1,2}} + \lambda_{\xi_{2,1,3}} + \lambda_{\xi_{2,1,4}}}, \quad E[\xi_{3,1}] = \frac{1}{\lambda_{\xi_{3,1,1}}}, \quad E[\xi_{4,1}] = \frac{1}{\lambda_{\xi_{4,1,1}}},$$

$$E[\xi_{5,1}] = \frac{1}{\lambda_{\xi_{5,1,1}} + \lambda_{\xi_{5,1,2}} + \lambda_{\xi_{5,1,3}} + \lambda_{\xi_{5,1,4}} + \lambda_{\xi_{5,1,5}} + \lambda_{\xi_{5,1,6}}},$$

$$E[\xi_{5,2}] = \frac{2}{\lambda_{\xi_{5,2,1}} + 3\lambda_{\xi_{5,2,2}} + \lambda_{\xi_{5,2,3}}} - \frac{1}{2\lambda_{\xi_{5,2,1}} + 3\lambda_{\xi_{5,2,2}} + \lambda_{\xi_{5,2,3}}},$$

$$E[\tau_{1,1}(\xi_{1,1},\eta_{1,1})] = \frac{1}{\lambda_{\eta_{1,1}}} + (T_{1,1} + \theta_{1,1})\left(1 + \frac{2e^{-(\lambda_{\xi_{1,1,1}} + 3\lambda_{\xi_{1,1,2}} + \lambda_{\xi_{1,1,3}} + \lambda_{\xi_{1,1,4}})(T_{1,1}+\theta_{1,1})}}{1 - e^{-(\lambda_{\xi_{1,1,1}} + 3\lambda_{\xi_{1,1,2}} + \lambda_{\xi_{1,1,3}} + \lambda_{\xi_{1,1,4}})(T_{1,1}+\theta_{1,1})}} - \right.$$
$$\left. - \frac{e^{-(2\lambda_{\xi_{1,1,1}} + 3\lambda_{\xi_{1,1,2}} + \lambda_{\xi_{1,1,3}} + \lambda_{\xi_{1,1,4}})(T_{1,1}+\theta_{1,1})}}{1 - e^{-(2\lambda_{\xi_{1,1,1}} + 3\lambda_{\xi_{1,1,2}} + \lambda_{\xi_{1,1,3}} + \lambda_{\xi_{1,1,4}})(T_{1,1}+\theta_{1,1})}}\right),$$

$$E[\tau_{1,2}(\xi_{1,2},\eta_{1,2})] = \frac{1}{\lambda_{\eta_{1,2}}} + (T_{1,2} + \theta_{1,2})\left(1 + \frac{2e^{-(\lambda_{\xi_{1,2,1}} + 3\lambda_{\xi_{1,2,2}} + \lambda_{\xi_{1,2,3}} + \lambda_{\xi_{1,2,4}})(T_{1,2}+\theta_{1,2})}}{1 - e^{-(\lambda_{\xi_{1,2,1}} + 3\lambda_{\xi_{1,2,2}} + \lambda_{\xi_{1,2,3}} + \lambda_{\xi_{1,2,4}})(T_{1,2}+\theta_{1,2})}} - \right.$$
$$\left. - \frac{e^{-(2\lambda_{\xi_{1,2,1}} + 3\lambda_{\xi_{1,2,2}} + \lambda_{\xi_{1,2,3}} + \lambda_{\xi_{1,2,4}})(T_{1,2}+\theta_{1,2})}}{1 - e^{-(2\lambda_{\xi_{1,2,1}} + 3\lambda_{\xi_{1,2,2}} + \lambda_{\xi_{1,2,3}} + \lambda_{\xi_{1,2,4}})(T_{1,2}+\theta_{1,2})}}\right),$$

$$E[\tau_{2,1}(\xi_{2,1},\eta_{2,1})] = \frac{1}{\lambda_{\eta_{2,1}}} + (T_{2,1} + \theta_{2,1})\left(1 + \frac{e^{-(\lambda_{\xi_{2,1,1}} + \lambda_{\xi_{2,1,2}} + \lambda_{\xi_{2,1,3}} + \lambda_{\xi_{2,1,4}})(T_{2,1}+\theta_{2,1})}}{1 - e^{-(\lambda_{\xi_{2,1,1}} + \lambda_{\xi_{2,1,2}} + \lambda_{\xi_{2,1,3}} + \lambda_{\xi_{2,1,4}})(T_{2,1}+\theta_{2,1})}}\right),$$

$$E[\tau_{3,1}(\xi_{3,1},\eta_{3,1})] = \frac{1}{\lambda_{\eta_{3,1}}} + (T_{3,1} + \theta_{3,1})\left(1 + \frac{e^{-\lambda_{\xi_{3,1,1}}(T_{3,1}+\theta_{3,1})}}{1 - e^{-\lambda_{\xi_{3,1,1}}(T_{3,1}+\theta_{3,1})}}\right),$$

$$E\left[\tau_{4,1}\left(\xi_{4,1},\eta_{4,1}\right)\right]=\frac{1}{\lambda_{\eta_{4,1}}}+\left(T_{4,1}+\theta_{4,1}\right)\left(1+\frac{e^{-\lambda_{\xi_{4,1,1}}\left(T_{4,1}+\theta_{4,1}\right)}}{1-e^{-\lambda_{\xi_{4,1,1}}\left(T_{4,1}+\theta_{4,1}\right)}}\right),$$

$$E\left[\tau_{5,1}\left(\xi_{5,1},\eta_{5,1}\right)\right]=\frac{1}{\lambda_{\eta_{5,1}}}+\left(T_{5,1}+\theta_{5,1}\right)\left(1+\frac{e^{-\left(\lambda_{\xi_{5,1,1}}+\lambda_{\xi_{5,1,2}}+\lambda_{\xi_{5,1,3}}+\lambda_{\xi_{5,1,4}}+\lambda_{\xi_{5,1,5}}+\lambda_{\xi_{5,1,6}}\right)\left(T_{5,1}+\theta_{5,1}\right)}}{1-e^{-\left(\lambda_{\xi_{5,1,1}}+\lambda_{\xi_{5,1,2}}+\lambda_{\xi_{5,1,3}}+\lambda_{\xi_{5,1,4}}+\lambda_{\xi_{5,1,5}}+\lambda_{\xi_{5,1,6}}\right)\left(T_{5,1}+\theta_{5,1}\right)}}\right),$$

$$E\left[\tau_{5,2}\left(\xi_{5,2},\eta_{5,2}\right)\right]=\frac{1}{\lambda_{\eta_{5,2}}}+\left(T_{5,2}+\theta_{5,2}\right)\left(1+\frac{2e^{-\left(\lambda_{\xi_{5,2,1}}+3\lambda_{\xi_{5,2,2}}+\lambda_{\xi_{5,2,3}}\right)\left(T_{5,2}+\theta_{5,2}\right)}}{1-e^{-\left(\lambda_{\xi_{5,2,1}}+3\lambda_{\xi_{5,2,2}}+\lambda_{\xi_{5,2,3}}\right)\left(T_{5,2}+\theta_{5,2}\right)}}-\right.$$
$$\left.-\frac{e^{-\left(2\lambda_{\xi_{5,2,1}}+3\lambda_{\xi_{5,2,2}}+\lambda_{\xi_{5,2,3}}\right)\left(T_{5,2}+\theta_{5,2}\right)}}{1-e^{-\left(2\lambda_{\xi_{5,2,1}}+3\lambda_{\xi_{5,2,2}}+\lambda_{\xi_{5,2,3}}\right)\left(T_{5,2}+\theta_{5,2}\right)}}\right).$$

4 CONCLUSIONS

The proposed model permits to assess the reliability of the "safety system-protected object" complex with multiple safety systems. In particular the suggested approach allows to evaluate such reliability indices as the mean time to accident and the probability of the accident prior to time *t*. The proposed approach allows to take into account the structure of safety systems and scheduled periodic inspections of safety systems. The solution obtained is useful for reliability assessment of nuclear power plants and similar dangerous technological objects.

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