ESTIMATION THE SHAPE, LOCATION AND SCALE PARAMETERS OF THE WEIBULL DISTRIBUTION

Dr. Zouaoui Chikr el-Mezouar

●

University of Bechar, Algeria
E-mail: Chikrtime@yahoo.fr

ABSTRACT

In this paper we propose a new estimators of the shape, location and scale parameters of the weibull distribution.

Keyword: Weibull Distribution, Cran's method and method of moments.

1.INTRODUCTION

The shape and scale parameter estimation of weibull distribution within the traditional methods and standard Bayes from work has been studied by Tummala (1980)[5], Ellis and Tummala (1983)[4], Cran (1988)[3], Al-Fawzan (2000)[2], and Al-Nasir (2002)[1].

This paper considers an estimation procedure based on the coefficient of variation, C.V. The recommended use of such estimators, is to provide quick, preliminary estimators of the parameters. Computational experiments on the presented method and comparison with Cran's method are reported.

2- The three-parameter weibull:

Whenever there is a minimum life (a) such that $(T > a)$, the three-parameter weibull may be appropriate. This distribution assumes that no failures will take place to time (a). For this distribution, the cumulative distribution function. C.D.F is given by:

$$F(t) = 1 - \exp\left[-\left(\frac{t-a}{b}\right)^c\right] \quad , t \ge a \quad , a \ge 0 \qquad (2-1)$$

The parameter (a) is called the location parameter. And the (k^{th}) moment is defined by :

$$\mu'_k = a + \frac{b\Gamma\left(1+\frac{1}{c}\right)}{k^{\frac{1}{c}}} \qquad (2-2)$$

In particular, when $k = 1$,

$$\mu_1' = a + b\Gamma\left(1+\frac{1}{c}\right) \qquad (2-3)$$

is the mean time to failure, MTTF of the distribution, and when $k = 2$

$$\mu_2' = a + \frac{b\Gamma\left(1+\frac{1}{c}\right)}{2^{\frac{1}{c}}} \qquad (2-4),$$

and so the variance of this distribution is defined by

$$\sigma^2 = b^2\left\{\Gamma\left(1+\frac{2}{c}\right) - \left[\Gamma\left(1+\frac{1}{c}\right)\right]^2\right\} \qquad (2-5)$$

which is the same as that in the two-parameter model.

3- Estimation of the parameters:

Given the ordered random samples : $t_{(1)} \le t_{(2)} \le \cdots \le t_{(n)}$, the cumulative distribution function .C.D.F can be estimated by :

$$\begin{aligned} S_n(t) &= 0, \quad t < t_{(1)} \\ &= \frac{r}{n}, \quad t_{(r)} \le t \le t_{(r+1)} \quad , r = 1,2,\cdots,n-1 \qquad (3-1) \\ &= 1 \;, \quad t_{(n)} \le t \end{aligned}$$

Ten the population moment, μ_k' is estimated by:

$$\begin{aligned} m_k' &= \int_0^\infty \{1 - S_n(t)\}^k dt \\ &= \sum_{r=0}^{n-1}\left(1-\frac{r}{n}\right)^k \{t_{(r+1)} - t_{(r)}\} \quad , \quad t_{(0)} = 0 \qquad (3-2) \end{aligned}$$

In particular,

$m_1' = \bar{t}$, the sample mean.

Cran (1988)[3] expressed the parameters in terms of the lower order moments as follows:

$$\left.\begin{aligned} a &= \frac{\mu_1'\mu_4' - \mu_2'^2}{\mu_1' + \mu_4' - 2\mu_2'} \ , \\ b &= \frac{\mu_1' - a}{\Gamma\left(1+\frac{1}{c}\right)} \\ &\textit{and} \\ c &= \frac{\ln(2)}{\ln(\mu_1' - \mu_2') - \ln(\mu_2' - \mu_4')} \end{aligned}\right\} \qquad (3-3)$$

Therefore, the moment estimators of $(a),(b)\ and\ (c)$ can be obtained from $(3-3)$ by substituting m_1', m_2' *and* m_4' for μ_1', μ_2' *and* μ_4' respectively and solving.

Since the estimator of (a) is inadmissible by being negative or by exceeding $t_{(1)}$, hence we can use the alternative estimator:

$$\hat{a} = t_{(1)} - \frac{\hat{b}\Gamma\left(1+\frac{1}{\hat{c}}\right)}{n^{\frac{1}{\hat{c}}}} \qquad (3-4)$$

We propose, the coefficient of variation, to get an expression which is a function of (c) only , i.e,

$$C.V. = \frac{\sqrt{\mu_2 - \mu_1^2}}{\mu_1 - t_{(1)}} = \frac{\sqrt{\Gamma\left(1+\frac{2}{c}\right) - \left[\Gamma\left(1+\frac{1}{c}\right)\right]^2}}{\Gamma\left(1+\frac{1}{c}\right)\left(1-\frac{1}{n^{\frac{1}{c}}}\right)} \qquad (3-5)$$

Now, we can form a table for various $(C.V.)$ by using $(3-5)$ for different (c) values.

In order to estimate $(c)\ and\ (b)$, we calculate the coefficient of variation $(C.V.)$ of the data and comparing with $(C.V.)$ using the table to estimate the shape parameter (c) , i.e

$$C.\hat{V}. = \frac{\sqrt{\Gamma\left(1+\frac{2}{\hat{c}}\right) - \left[\Gamma\left(1+\frac{1}{\hat{c}}\right)\right]^2}}{\Gamma\left(1+\frac{1}{\hat{c}}\right)\left(1-\frac{1}{n^{\frac{1}{\hat{c}}}}\right)} \qquad (3-6)$$

substituting , the scale parameter (b) can then be estimated.

4- simulation results:

The objective of our experiments is to compare the proposed estimators with Cran's estimators. We have generated random samples with known parameters for different sample sizes. To be able to compare, we calculated the mean-squared-error (MSE) for each method , and the table 1, shows the complete results.

Table (1) : Comparison between proposed method and Cran's method (R=1000)

Sample size (n)	parameters	proposed	Cran	The Best
		MSE	MSE	
10	$a = 2$	15.9134	333.2995	proposed
	$b = 4$	17.4373	333.1223	Proposed
	$c = 2$	10.3747	237.5663	Proposed
25	$a = 2$	1.3287	10.8445	Proposed
	$b = 4$	1.8016	11.3442	Proposed
	$c = 2$	1.0492	8.5543	Proposed
50	$a = 2$	0.2506	0.3278	Proposed
	$b = 4$	0.4193	0.5162	Proposed
	$c = 2$	0.2041	0.3486	Proposed
100	$a = 2$	0.0806	0.0914	Proposed
	$b = 4$	0.1543	0.1678	Proposed
	$c = 2$	0.0706	0.1173	proposed

5-Conclusion:

In this paper, we have presented both Cran's method and proposed method (using the coefficient of variation) for estimating the three-parameter weibull distribution. It has been shown from the computational results that the method which gives the best estimates is the proposed method.

References:

1. Al-Fawzan , M.A (2000) : "Methods for Estimating parameters of the Weibull distribution" King Abdulaziz city for science and technology Saudi Arabia.
2. Cran G.W. (1988) : "Moment Estimators for the 3-parameter Weibull Distribution" IEEE TRANSACTIONS ON RELIABILITY , vol. 37, N^0.4.

3. Ellis , W.C. and Tummala, V.M.R (1986) : “Minimum Expected Loss Estimators of the shape and scale parameters of the Weibull Distribution” IEEE TRANSACTIONS ON RELIABILITY , vol. R-35, N^0.2.
4. Tummala, V.M.R (1980) : “Minimum Expected loss estimators of reliability and shape parameter of Weibull distibution” Industrial Mathematics , the Industrial Mathematics Sciety, vol. 30, part I pp.61-67.