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Table of Contents

This paper presents an analysis of the operational process of heat supply system, taking into consideration its reliability. The specific character of the operation of heat-supply systems has been considered in this work. In the process of exploitation of heat-supply systems five operational states have been distinguished, using as a criterion the level of indoor temperature decrease in residential rooms. The method of modelling the reliability of heat-supply systems is worked out. The methodology of determining the overall index of heat-supply system reliability has been presented. The measure of heat-supply system reliability has been taken to be as the scale/quantity of inadequate supply of heat power at a given state. Calculations have been carried out regarding the changeability of exterior conditions for one of the groups of customers – residential users. On the basis of the operational data for the heat supply system with two heat sources, shortfalls of heat power and the probability of their occurrence have been calculated as an application of this methodology.

In the given paper multi-component standby system with renewable elements is considered. For it multi-line closed Markov queuing model for two maintenance operations – replacements and renewals, is constructed and investigated. In this model the numbers of main elements as well as standby ones, also the numbers of replacement units as wall as renewal ones are arbitrary. An economic criterion for dependability planning (structural control) of considered system is introduced, the optimization problem is stated and partially investigated.

The application of the Directive 99/92/EC deals with the safety and health protection of workers potentially exposed to explosive atmospheres and requires the assessment of explosion risks. These can arise by the release of inflammable substances typical of industries classified as major hazards, but they often may be generated in other industries where inflammable materials are handled. Risk assessment of explosive atmospheres is required in both cases, for this purpose, in this article a quantitative approach has been proposed. The paper describes the main aspects of the methodology, based on a probabilistic risk assessment, and finally its application to a case-study.

The radon volume activity in buildings is generally time variable. Its variability is caused by many natural and man-made factors. An example of these factors includes meteorological parameters, soil properties, characteristics of the building construction, properties of water used in the building and also the behavior of inhabitants. These factors can influence each other and also they are related with the exposition of inhabitants.
This article reports a continual indoor radon monitoring and a statistical evaluation of a dataset obtained by the 18 days-long measuring in a house located in the Czech Republic.
The contributions of carefully selected meteorological parameters and human influences were also observed. Results of the observation were divided into two parts (inhabited, uninhabited) and analyzed in relation with the indoor radon concentration. The multiplied linear regression was applied to model obtained datasets.
Results of time series analyses of the continual indoor radon concentration and the meteorological monitoring are presented and discussed as well.

In this paper a problem of a construction of new and practically interesting classes of recursively defined networks, including internet type networks, with sufficiently fast algorithms of calculation of connectivity probability is considered. For this aim recursive and asymptotic formulas of connectivity probability calculation are constructed. Asymptotic formulas are based on assumptions that all network arcs are low reliable or there are high reliable and low reliable arcs in considered network. For example in radial-circle scheme radial arcs may be high reliable and circle arcs – low reliable.

There is investigated a model of a system with the highly reliable elements, where the periods of functioning are changed by the periods of maintenance. The system must be operational only in the periods of functioning although the restoration in these periods is not provided. The system is completely restored in the nearest period of maintenance. Since the elements of system are highly reliable, the reserve of system is rarely exhausted during each period of functioning. Therefore it is possible to use the results, obtained for the systems with fast restoration, for the reliability assessment of system, which is not restorable in the periods of the functioning. The estimations of indices of failure-free performance and maintainability of such systems are obtained.

The paper represents discussion about risk assessment for transhipment system in reduces data condition. As a particular example transhipment system is presented. Article can be treated as first estimation. Future work and objectives are characterized in the end.

Asymptotic formulas for connection probabilities in a rectangular lattice with identical and independent arcs are obtained.. For a small number of columns these probabilities may be calculated by the transfer matrices method. But if the number of columns increases then a calculation complexity increases significantly. A suggested asymptotic method allows to make calculations using a sufficiently simple geometric approach in a general case.

Banking-financial institutions are organizations which might be included in the category of complex systems. Consequently, they can be applied after adaptation and particularization, in the general description and assessment methods of the technical or organizational systems. The banking-financial system faces constrains regarding the functioning continuity. Interruptions in continuity as well as operational incidents represent risks which can lead to the interruption of financial flows generation and obviously of profit. Banking incidents include from false banknote, cloned cards, informatics attacks, false identity cards to ATM attacks. The functioning of banking institutions in an incident-free environment generates concern from both risk assessment and forecasting points of view.

RELIABILITY ASSESSMENT OF HEAT SUPPLY SYSTEMS IN THEIR OPERATIONAL PROCESS

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ABSTRACT

This paper presents an analysis of the operational process of heat supply system, taking into consideration its reliability. The specific character of the operation of heat-supply systems has been considered in this work. In the process of exploitation of heat-supply systems five operational states have been distinguished, using as a criterion the level of indoor temperature decrease in residential rooms. The method of modelling the reliability of heat-supply systems is worked out. The methodology of determining the overall index of heat-supply system reliability has been presented. The measure of heat-supply system reliability has been taken to be as the scale/quantity of inadequate supply of heat power at a given state. Calculations have been carried out regarding the changeability of exterior conditions for one of the groups of customers – residential users. On the basis of the operational data for the heat supply system with two heat sources, shortfalls of heat power and the probability of their occurrence have been calculated as an application of this methodology.

1. INTRODUCTION

Reliability is the primary factor of utility, that is, the ability of a technical system to meet human needs, which directly determines the practical possibilities of realizing the aims of the system (tasks). Even if technical systems are perfect in a functional sense, they become useless if the level of their reliability is not satisfactory - if it is lower than required (Barlow, 1993)]. The reliability of a heat-supply system is closely connected with the reliability of its parts (sub-systems, structures), which determines the quality of the completing tasks by the system after taking into consideration random changes of the functional characteristics of the given system with the existence of computational external conditions (Babiarz, 2002)]. The description of reliability is closely connected with the description of the functioning process of heat-supply systems, considering changeable external conditions. Determining the influence of the conditions on the reliability of heat-supply systems and considering appropriate criteria, is an inseparable part of the analysis of an operational process of the system. Heat-supply systems may be included in the class of relatively complex technical systems. Each of them constitutes a functional whole, divisible into sub-systems, structures and elements connected with the climate, the environment and the demand for heat and are characterized by random changes of their states. The characteristic feature of heat-supply systems is the occurrence of many operational states of the system at various heat loads and efficiencies, determined by a range of random factors affecting the demand for heat. From the standpoint of reliability, the states are described by a combination of damaged and undamaged elements connected with each other by means of appropriate structures. In this work, basic assumptions for modelling and analysis of the reliability of heat-supply system units are presented, considering the diversified operational abilities of heating systems.

2. RELIABILITY STATES OF OPERATIONAL PROCESS IN A HEAT SUPPLY SYSTEM

In the process of the exploitation of heat-supply systems five operational states have been distinguished: (A), (B), (C), (D) i (E) (fig.1), assuming as a criterion the level of indoor temperature decrease in residential rooms T_i [^{0}C] and the time of the duration of interference in heat supply for consumers t_n[h], caused by failure (Babiarz, 2002 & 2003).

The state of complete ability (A) – referred to as the state of operational reliability – the state which determines the situation where the indoor temperature is equal to the computational indoor temperature for the majority of residential rooms $T_{iA} = T_{io} = 20\ ^0$C and there is no interference in heat supply: $Q_A = Q_n$, Q_n[MW] – termed ordered heat power.

The state of partial permissible ability (B) – the state of permissible operational reliability, there are certain limitations in heat supply displaying a decrease in heated rooms temperature to T_i=15^{0}C, taken as a border, at which the human organism is able to function normally. It corresponds with a decrease in heat power supplied to consumers: $\alpha_B Q_n \le Q_B < Q_n$.

The state of partial limited ability (C) – the state of limited operational reliability where considerable difficulties connected with the necessity to make use of other heat sources for heating (electric energy, natural gas etc.) are observed. The heating equipment is only protected against freezing by maintaining a minimal temperature T_{iC} = 8^{0}C due to the supply of heat power: $\alpha_C Q_n \le Q_C < \alpha_B Q_n$.

The state of complete disability (D) – the state of operational unreliability, when border indoor temperature is equal to T_{iD}=0^{0}C and heat power is contained between: $\alpha_D Q_n \le Q_D < \alpha_C Q_n$.

The state of disaster (E) – the state where the water in the central heating system freezes, resulting in damage to the system. There is also a threat to human life as a consequence of loss of heat power supply: $Q_E \le \alpha_D Q_n$.

Changes of heat power value in particular states have been calculated according to introduced factors of heat power decrease: α_B, α_C, α_D . The ordered heat power in a given heat supply system Q_n has been taken as its capacity. It has been determined as production capacity Q_p. The scheme of the above operational states of heat supply system described is given in Figure 1.

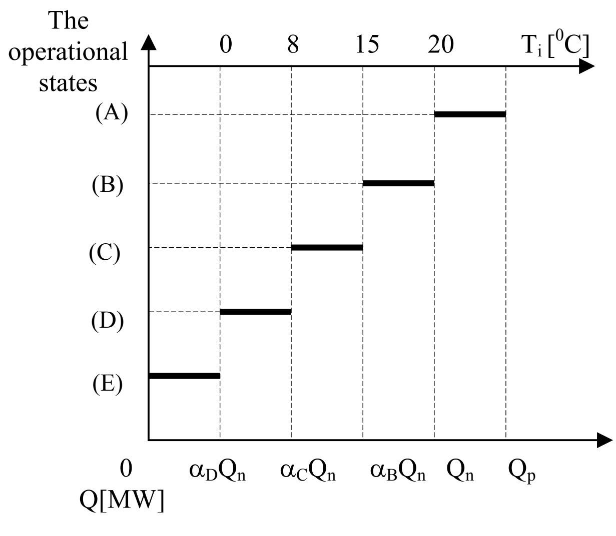

Figure 1. Classification of operational states of heating system depending on heat power Q[MW] and indoor temperature T_i [^{0}C]

Factors of heat power decrease: α_B, α_C, α_D have been calculated from the proportion of heat losses Q_{str} in buildings, considering particular states in different exterior conditions which were determined by the level of outdoor temperature T_e.

Reductions of operational parameters in higher than computational temperature conditions do not always mean the disability of the entire heat supply system. Therefore, the following considerations have been carried out in six variants for different external temperatures T_e (- 20, -15, -10, -5, 0, +5 ^{0}C) and their duration.

$$Q_{srtAx} = k_b \cdot A_b \cdot (T_{iA} - T_{ex}) \tag{1}$$

$$Q_{srtBx} = k_b \cdot A_b \cdot (T_{iB} - T_{ex}) \tag{2}$$

$$Q_{srtCx} = k_b \cdot A_b \cdot (T_{iC} - T_{ex}) \tag{3}$$

$$Q_{srtDx} = k_b \cdot A_b \cdot (T_{iD} - T_{ex}) \tag{4}$$

Where: T_{iA} , T_{iB}, T_{iC}, T_{iD} – indoor temperature appropriate to the state (A), (B), (C), (D); T_{ex} - external temperature taken for x - variant; k_b – overall heat-transfer coefficient of the building; A_b – surface of the cooling division wall in the building;

The factor of permissible heat power decrease in heat supply system α_{Bx} in state (B) for variant x has been determined from the proportion of heat losses in the building in state (B) to heat losses in the state of complete ability (A) given by

$$\alpha_{Bx} = \frac{Q_{strBx}}{Q_{strAx}} = \frac{T_{iB} - T_{ex}}{T_{iA} - T_{ex}} \tag{5}$$

Similarly from the proportion of heat losses in the building residual factors: $\alpha_{Cx,}$ α_{Dx} have been determined: α_{Cx} – the factor of limited heat power decrease in state (C) for variant x; α_{Dx} – the factor of border heat power decrease in state (C) for variant x;

$$\alpha_{Cx} = \frac{Q_{strCx}}{Q_{strAx}} = \frac{T_{iC} - T_{ex}}{T_{iA} - T_{ex}} \tag{6}$$

$$\alpha_{Dx} = \frac{Q_{strDx}}{Q_{strAx}} = \frac{T_{iD} - T_{ex}}{T_{iA} - T_{ex}} \tag{7}$$

Information about heat power supply Q_{ix} in the particular states, demonstrating differences between variants, is presented schematically in Figure 2 in the following formulas:

$$Q_{Ax} \geq Q_n \tag{8}$$

$$\alpha_{Bx} Q_n \leq Q_{Bx} < Q_n \tag{9}$$

$$\alpha_{Cx} Q_n \leq Q_{Cx} < \alpha_{Bx} Q_n \tag{10}$$

$$\alpha_{Dx} Q_n \leq Q_{Dx} < \alpha_{Cx} Q_n \tag{11}$$

$$Q_{Ex} < \alpha_{Dx} Q_n \tag{12}$$

Q_n – nominal heat power determined as ordered heat power, resulting from detailed heat losses for individual conditions.

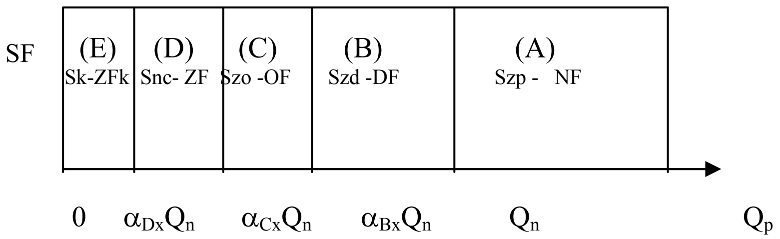

Figure 2. The operational states of heat supply system as a function of heat power

Values of border parameters determining particular states of the system in the considered variants have been calculated according to the formulas (5, 6, 7) and presented graphically in Figure3.

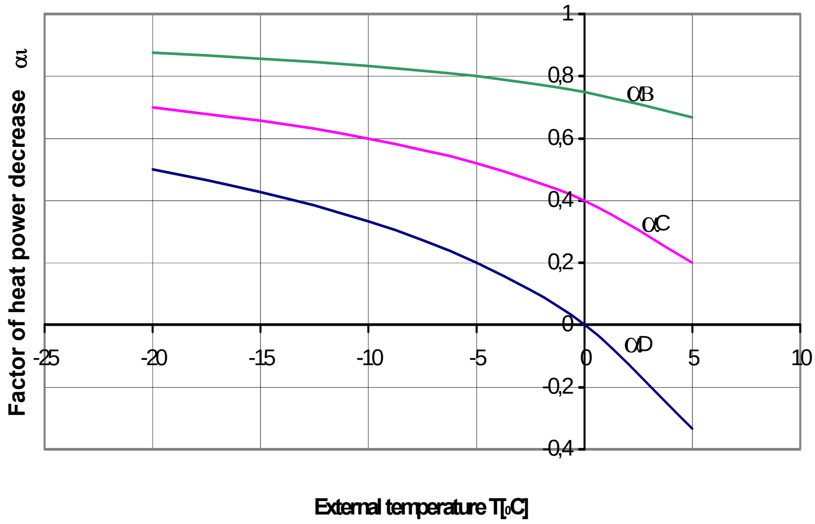

Figure 3. Relationship between factors of heat power decrease and external temperature considering changeable external conditions

Factors of heat power decrease diminish along with an increase in external air temperature.

3. THE METHOD OF RELIABILITY ASSESSMENT

Elements which I have obtained from the division of heat supply system and I can't divide them because they are indivisible in this stage of thinks, I treat as two-state (element operate or completely don't operate) and renewable (we can renew it to work, it can be repaired).
The stationary preparedness index K_{el} determining the probability of finding an element (el) at any time being in the ability state, is taken as a measure of operational reliability of two-state objects.
As a measure of operational reliability of that two-state elements I have taken the stationary preparedness index K_{el} which determine the probability of finding an element (el) in working order at any time, probability that the element will realize its tasks according to its intendment in the way which give to customers possibility of its using without any interruptions, impediment and limits.

$$K_{el} = P(\mathrm{Sz}_{\mathrm{el}}) = \frac{E(t_z)}{E(t_z) + E(t_n)} \tag{13}$$

Stoppage index for element:

$$U_{el} = 1 - K_{el} \tag{14}$$

Two-state objects can remain in numerous operational states depending on the degree of meeting attributed requirements. Owing to the specific character of heat supply systems, it is permitted to operate this system with decreased parameters in specific operational conditions and time (Babiarz 2006).
The measure of reliability is a shortfall of heat power ΔQ_i in i-state for conditions describing variant x.

$$\Delta Q_i = Q_{nix} - Q_i. \tag{15}$$

where:
Q_{nix} – heat power in consider i- state for variant x; Q_i – heat power equivalent to aggregated heat power of heat sources in particular states;

$$Q_{nix} = \varphi_x \cdot Q_n \tag{16}$$

when: φ_x - load factor for variant x;

$$\varphi_x = \frac{T_{io} - T_{ex}}{T_{io} - T_{eo}} \tag{17}$$

The overall index of heat-supply system unreliability U_u has been evaluated. It determines the relation of expected heat power shortage $E(\Delta Q)$ to ordered heat power value Q_n which results from detailed heat loss balances for individual conditions.

$$U_u = \frac{E(\Delta Q)}{Q_n} \tag{18}$$

The overall index of heat-supply system reliability K_u, which is a measure of reliability in a heat supply system operation, can be calculated with the use of the following formula:

$$K_u = 1 - U_u \tag{19}$$

4. AN EXAMPLE OF APPLICATION OF THE DERIVED METHOD

Calculations have been carried out for the heat supply system in the city of Rzeszów, which consists of two interconnected heat sources signed as UZC I and II. Ordered heat power for this system equals Q_n = 407 MW. For the two heat sources y = 5^2 = 25 reliability states have been considered. On the basis of the operational data (Babiarz, 2002), shortfalls of heat power and the probability of their occurrence have been calculated. Results of the calculation of the overall indexes of heat-supply system reliability for one variant determined by the level of external temperature T_e are presented in Table 1.

Heat power value for this variant evaluated according to the formula (16) equals

$Q_{nI} = Q_n$ = 407 MW,

because according to formula (17):

$\varphi_I = 1$

Table 1. Calculation of shortfalls of heat power and the probability of their occurrence in particulars operational states in the city of Rzeszow for variant I

SF	UZC		P_i	Q_i [MW]		ΔQ [MW]	E(ΔQ) [MW]
	I	II		I	II		
1	A	A	0,701053	327,0	80,0	0	0
2	A	B	0,093475	327,0	75,0	5	0,467373
3	A	C	0,049758	327,0	63,0	17	0,845879
4	A	D	0,010851	327,0	52,5	27,5	0,298413
5	A	E	0,000576	327,0	40,0	40	0,023036
6	B	A	0,090406	306,5	80,0	20,5	1,85333
7	B	B	0,012054	306,5	75,0	25,5	0,307385
8	B	C	0,006416	306,5	63,0	37,5	0,240624
9	B	D	0,001399	306,5	52,5	48	0,06717
10	B	E	0,000074	306,5	40,0	60,5	0,004493
11	C	A	0,020378	257,5	80,0	69,5	1,416337
12	C	B	0,002717	257,5	75,0	74,5	0,202433
13	C	C	0,001446	257,5	63,0	86,5	0,125114
14	C	D	0,000315	257,5	52,5	97	0,030598
15	C	E	0,000017	257,5	40,0	109,5	0,001833
16	D	A	0,007182	196,5	80,0	130,5	0,937303
17	D	B	0,000958	196,5	75,0	135,5	0,129763
18	D	C	0,000510	196,5	63,0	147,5	0,075192
19	D	D	0,000111	196,5	52,5	158	0,017566
20	D	E	0,000006	196,5	40,0	170,5	0,001006

21	E	A	0,000233	164,0	40,0	163	0,038058
22	E	B	0,000031	164,0	40,0	168	0,00523
23	E	C	0,000016	164,0	40,0	180	0,002983
24	E	D	0,000003	164,0	40,0	190,5	0,000688
25	E	E	1,91E-07	164,0	40,0	203	3,89E-05
			∑ 1.0				7,091847

Where: A - describes a situation when the heat source (UZC I or II) is in the state of complete ability (NF-Szp), B - describes a situation when the heat source (UZC) is in the state of partial permissible ability (DF-Szd), C - describes a situation when the heat source (UZC) is in the state of partial limited ability (OF-Szo), D - describes a situation when the heat source (UZC) is in the state of complete disability (DF-Snn), E - describes a situation when the heat source (UZC) is in the state of disaster (ZF-Snk).

The overall index of heat-supply system unreliability for the first variant is given as:

$$U_I = \frac{E(\Delta Q)}{Q_n} = \frac{7,0918}{407} = 0,017425$$

The overall index of the heat-supply system reliability for the first variant is given as:

$$K_I = 1 - U_I = 0,982575$$

Calculations of the overall index of the heat-supply system reliability for every variant described by external temperature had been carried out. Results of the calculations of the overall indexes of heat-supply system reliability, accounted for by means of an analysis of states, which can take place in an operation process of heat supply system, are presented graphically in Figure 4.

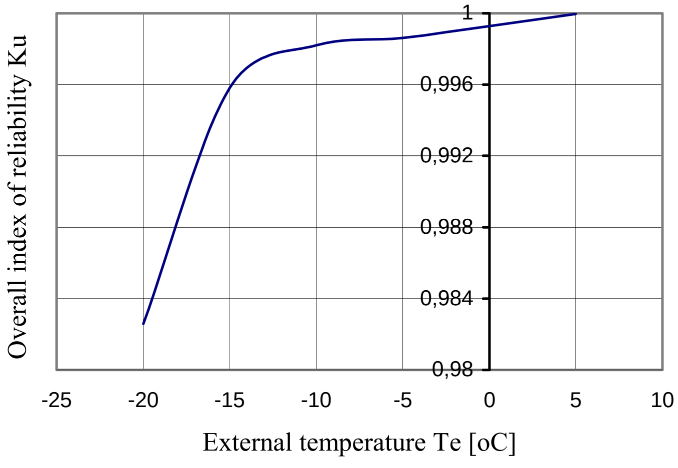

Figure 4. Dependence of overall index of heat-supply system reliability on external temperature

Along with an increase of external air temperature T_e, reliability of the heat supply system increases, whose measure is the overall index of heat-supply system reliability. The index takes a minimal value K_u = 0,982575 for variant I of computational conditions, with T_e = - 20 ^{0}C, and a maximal value K_u = 0,999972 for variant VI, with external temperature T_e = + 5 ^{0}C.

5. SUMMARY

The specific character of the operation of heat-supply systems has been presented in this work. A model describing the functioning of heat-supply system structures in the aspect of reliability has been developed. In the process of the exploitation of heat-supply systems five operational states have been distinguished, assuming as a criterion the level of indoor temperature decrease in residential rooms. The methodology of determining the overall index of heat-supply system reliability has been presented. The measure of heat-supply system reliability has been assumed as the scale/quantity of inadequate supply of heat power at a given state. Calculations have been carried out regarding the changeability of exterior conditions for one of the groups of customers – residential users. During the variant analysis a criterion determined by the level of outdoor temperature was assumed. This way, the multi-state characteristic of a heat-supply system, with reference to its reliability, has been taken into consideration.

Working out of the model of heat-supply system reliability requires taking its complexity, the extent of realization and its systematic treatment into consideration. The method can be used to evaluate the reliability in a heat supply system and to solve a lot of technical problems at the planning stage**.**

REFERENCES

1. Babiarz B.: *A reliability analysis of selected elements in heat supply systems*. Doctoral dissertation. Kraków 2002.
2. Babiarz B.: *An introduction to the assessment of reliability of the heat supply systems.* International Journal of Pressure Vessels and Piping, Volume 83, Nomber 4/2006, p. 230-235, Elsevier 2006.
3. Babiarz B., Rak J.: *The modelling of operational reliability in a heat supply system.* 4 th International Conference of PhD Students, Miskolc 2003.
4. Barlow R. E., Clarotti C. A., Spizzichino F.: *Reliability and decision making*. Chapman & Hall 1993.

MULTI-LINE MARKOV CLOSED QUEUING SYSTEM FOR TWO MAINTENANCE OPERATIONS

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ABSTRACT

In the given paper multi-component standby system with renewable elements is considered. For it multi-line closed Markov queuing model for two maintenance operations – replacements and renewals, is constructed and investigated. In this model the numbers of main elements as well as standby ones, also the numbers of replacement units as wall as renewal ones are arbitrary. An economic criterion for dependability planning (structural control) of considered system is introduced, the optimization problem is stated and partially investigated.

1. INTRODUCTION

The investigation of the problem of construction and investigation of maintenance models for multi-component systems is one of the topical directions in the modern reliability theory [10], [5]. This problem plays a key role in the dependability planning (structural control by economic criteria) of telecommunication and production systems [1], [4]. Let's consider the last statement in detail.

Since the beginning of the second half of the 80ies of the last century considerable changes have taken place in the field of telecommunication, affecting the interests of telecommunication service providers, as well as users and equipment manufacturers.

Here we mean de-monopolization in the field of telecommunication services in almost all developed countries of the world; creation of open market of traditional and new types of service and as a sequence, intensification of competition between telecommunication service providers; strengthening of requirements on the part of users; rapid development and inculcation of new technologies, architectures, etc.

The result of these changes is a considerable increase of interest to the problems of dependability of telecommunication networks and their components.

The essence of this is: low dependability threatens telecommunication service providers with not only possible loss of clients, but also directly affects the economical indices. The refuse of service to users, because of non-serviceability of telecommunication means, is expressed in lost income and frequently in direct losses caused by penal sanctions claimed by users.

Naturally, this problem was in the focus of ITU (International Telecommunications Union). Namely, ITU has issued recommendation E.862 "Dependability planning of telecommunication networks" (Geneva, 1992), which by ITU's assignment was prepared by Swedish experts.

Recommendation E.862 is concerned with models and methods for dependability planning, operation and maintenance of telecommunication networks and the application of these methods to various services in international network.

For all that, this Recommendation gives quite convincing preference to analytical methods for dependability planning, _compared to other methods.

In particular, it is ascertained that the application of the analytical methods gives economically the best-balanced level of dependability, seen from the customer's point of view. This reduces the risk of customer's complaints and loss of business to competitors, as well as the risk of unnecessary investments. It is, therefore, considered to be the best general way of planning for administration, as well as for customers[5].

After this ITU has worked out a lot of documents on the problems of telecommunication service quality and dependability. According to ITU documents Quality of Service (QoS) is the degree of service a provider performs for a client, including the existing agreement relation between them(Service Level Agreement -SLA).

SLA between service providers and users regulates technical, organizational, legal and financial aspects and is becoming an important factor in competition in the developed countries.

A substantial part of SLA is the list of dependability indices and the values the guarantee of their provision being given by the provider. And this is not accidental, as dependability is one of the most important determining factor of quality.

Thus, service provider, i.e., company that leases telecommunication channels has to fundamentally study his possibilities, collect and process statistic data so that to have full idea about the quality of the provided service. Only in this way is it possible to rationally estimate the given guarantees, compensation forms and size he will have to pay in behalf of the client if SLA demands will be broken. In these conditions the optimization of dependability and structure of telecommunication networks and their components acquire particularly great importance.

Described situation, along with other new important developments (circumstances) yet again confirms necessity to construct and research complex systems' reliability and maintenance new, more adequate models.

Simultaneously, until now, models of standby multi-component systems with repairable elements, that would take into account a number of very essential factors, have been developed neither in the mathematical theory of reliability and maintenance nor in the queuing theory, which is the theoretical basis of the reliability models for complex repairable systems.

One of such factors is the length of time, required for the replacement of the failed element in a complex system. The necessity of consideration of this factor has long been emphasized by the leading reliability theory and practice experts [8], [9], [2]. However, only a few simple cases have so far been investigated in this direction. Simultaneously, modern methods of mathematical theory of reliability allow to construct and investigate such models [6], [7].

The matter is that in a majority of cases in which the reliability is investigated, the replacement of the failed element is not regarded as a separate operation.

For relatively simple systems there is no need for making such an assumption, since the replacement operation can be included in the complex maintenance operation, which is a sequence of two operations: repair and replacement.

However, after the failure of some element in renewable multi-element standby systems, the necessity of its replacement by a serviceable standby one comes to the foreground and thus the replacement is quite naturally distinguished as an independent maintenance operation.

In other cases it is supposed that the replacement time length is essentially smaller, than the repair time length, and therefore can be neglected (instant replacement).

But in many systems, especially in complex production ones, the time length of the replacement operation frequently has the same order as the repair time length and may even exceed it. Therefore the assumption about an instant replacement is quite rough. Such an assumption is especially inadmissible in the conditions of a generalized interpretation of the notions of the reliability theory given below.

Namely, the failure of some element is understood as the occurrence of an event when this element cannot execute the definite category of tasks with stated priority. This may be caused not only by the loss of element serviceability, but also by other factors, for example, when an element is switched over to the execution of higher priority tasks, readjustment, heating and so on. Within the framework of such an approach, under the repair time we mean the length of time during which an element is unserviceable in a generalized sense, i.e. is not able to fulfil the above-mentioned flow of tasks. From this generalized interpretation of the notions of failure and repair we easily come to the notion of standby and other notions in a generalized sense. For the background and usefulness of such an interpretation see [3].

2. SYSTEM DESCRIPTION

The basic model in the form of a closed queuing system with two types of service operations is described as follows.

The technical system consists of m main and n standby elements. All elements are identical. It is supposed that for the normal operation of the system, the serviceability of all m main elements is desired. However, if their number is less than m, then the system continues to function but with lower economic effectiveness. The main elements fail with intensity α and the standby ones - with intensity β. A failed main element is replaced by a serviceable standby one if there is such a possibility in the system. In the opposite case (all standby elements are non-serviceable, or the serviceable standby elements are already intended for the replacement of earlier failed main elements) the replacement will be carried out as soon as it becomes possible. The failed elements, both the main and the standby ones, are repaired and become identical to the new ones. There are k and replacement and l repair units in the system. The time lengths of replacement and repair operations have distribution functions $F(x)$ and $G(x)$, respectively(Figure 1).

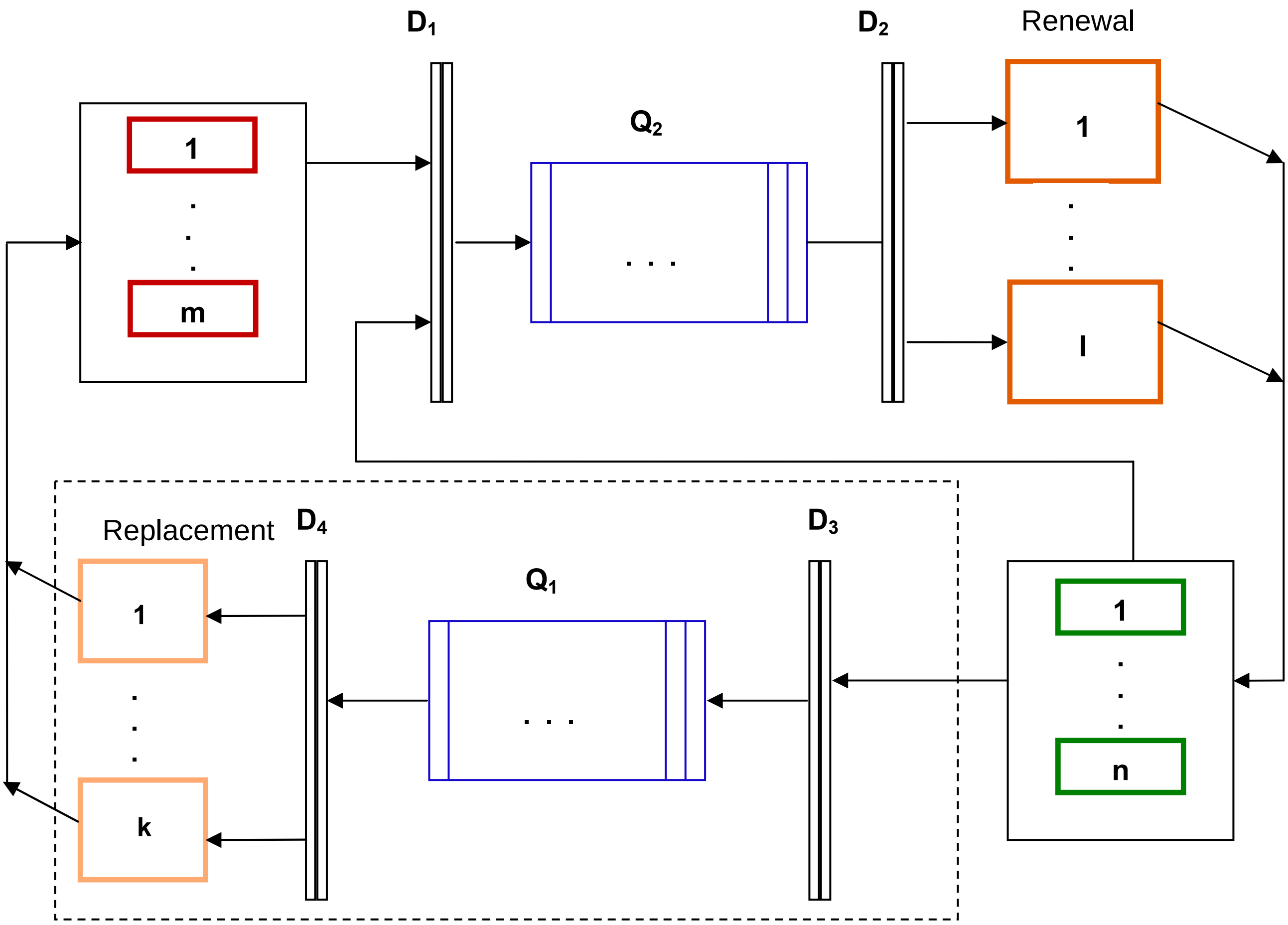

Figure 1.. General Scheme of Closed Queuing System for Replacements and Renewals

D_1, D_2, D_3, D_4 - disciplines of distribution to resources for requests; Q_1 – queue to replacement units; Q_2 – queue to repair units.

Only a few particular cases of the described system have so far been investigated in the reliability and queuing theories, namely:

1. $\boldsymbol{m} = 1$, $\boldsymbol{n} = 1$;
2. $\boldsymbol{m} = 1$, $\boldsymbol{n} = 2$;
3. $\boldsymbol{m} = 2$, $\boldsymbol{n} = 1$;
4. M/M/N, i.e. the repair time length has an exponential distribution, while the replacement time length equals zero (instant replacement);
5. some similar cases have also been investigated.

In the last 6-7 years the specialists of Georgian Technical University have succeeded in making considerable progress in the investigation along these lines. In particular, the models have been constructed and partly investigated for the following cases:

1. ***m***, ***n***, ***k***, ***l*** are arbitrary; the functions $F(x)$ and $G(x)$ are exponential;
2. ***m***, ***n*** and the function $F(x)$ are arbitrary; $\boldsymbol{k} = \boldsymbol{l} = 1$ and the function $G(x)$ is exponential;
3. ***m***, ***n*** and the function $G(x)$ are arbitrary; $\boldsymbol{k} = \boldsymbol{l} = 1$ and function $F(x)$ is exponential;
4. some similar statements have also been considered.

3. THE MATHEMATICAL MODEL

In this section we construct and investigate the mathematical model for the case where ***m, n, k*** and ***l*** are arbitrary. The replacement and repair time lengths have exponential distribution functions with parameters λ and μ respectively.

For describing of considered system we introduce the random processes, which determine the states of considered system at the moment t.

i(t) – the number of elements missed in main group of elements;

j(t) – the number of nonserviceable (failed) elements in the system;

Denote, $p(i, j, t) = P\{i(t) = i;\ j(t) = j\}$, $i = \overline{1,m};\quad j = \overline{0,n+i}$.

We suppose that there exists $\lim\limits_{t\to\infty} p(i, j, t) = p(i, j)$;

Regarding to the functions introduced above we construct a system of usual linear differential first order equations (Kolmogorov equations), which in steady mode transforms into the system of linear algebraic equations. They form four groups of equations describing the following merged states of the considered system:

1. A free state (in the system there are no requests neither for replacement nor for renewal);
2. Replacement (only the replacement operation is carried out in the system);
3. Repair (only the repair operation is carried out in the system);
4. Replacement and repair (both the replacement and the repair operation are carried out in the system).

Hence, in the case $k \le l \le n \le m$ we obtain the mathematical model in steady state mode in the form of the system of linear algebraic equations regarding $p(i, j)$, $i = \overline{1,m};\quad j = \overline{0,n+i}$.

1. Free state

$$(m\alpha + n\beta)P(0,0) = \lambda P(1,0) + \mu P(0,1) \qquad (1.1)$$

2. Replacement

$((m-i)\alpha + n\beta + i\lambda)P(i,0) = (i+1)\lambda P(i+1,0) + \mu P(i,1), 0 < i < k$; (2.1)

$((m-i)\alpha + (n+1-k)\beta + k\lambda)P(i,0) = k\lambda P(i+1,0) + \mu P(i,1), \quad k \le i < m$; (2.2)

$((m+n-k)\beta + k\lambda)P(m,0) = \mu P(m,1)$; (2.3)

3. Renewal

$(m\alpha + (n-j)\beta + j\mu)p(0,j) = \lambda p(1,j) + (j+1)\mu p(0,j+1) + (n-j+1)\beta.p(0,j-1)$,
$1 \le j < l$; (3.1)

$(m\alpha + (m-j)\beta + l\mu)p(0,j) = \lambda p(1,j) + l\mu p(0,j+1) + (n-j+1)\beta.p(0,j-1)$,
$l \le j < n$; (3.2)

$(m\alpha + l\mu)p(0,n) = \lambda p(1,n) + \beta.p(0,n-1)$; (3.3)

4. Replacement and Renewal

$((m-i)\alpha + (n-j)\beta + i\lambda + j\mu)p(i,j) = (m-i+1)\alpha p(i-1,j-1) +$
$+ (n-j+1)\beta P(i,j-1) + (i+1)\lambda p(i+1,j) + (j+1)\mu p(i,j+1), 1 \le i < k, 1 \le j < l$; (4.1)

$((m-i)\alpha + (n-j)\beta + i\lambda + l\mu)p(i,j) = (m-i+1)\alpha p(i-1,j-1) + (i+1)\lambda p(i+1,j) +$
$+ (n-j+1)\beta p(i,j-1) + l\mu p(i,j+1), 1 \le i < k, l \le j \le n$; (4.2)

$((m-j)\alpha + (n+i-j)\mu + l\mu)p(i,j) = p(i-1,j-1)(m-i+1)\alpha +$
$+ (n+i-j+1)\lambda p(i+1,j) + l\mu p(i,j+1), \quad 1 \le i < k, \; n < j < n+i$; (4.3)

$((m-i)\alpha + l\mu)p(i,n+i) = (m-i+1)\alpha p(i-1,n+i-1) + \lambda p(i+1,n+i)$
$1 \le i < k$; (4.4)

$((m-i)\alpha + (n+i-j-k)\beta + l\mu + k\lambda)p(i,j) = (n+i-j+1-k)\beta p(i,j-1) +$
$+ \lambda k p(i+1,j) + (j+1)\mu p(i,j+1), k \le i < m, 1 \le j < l$; (4.5)

$((m-i)\alpha + (n+i-j-k)\beta + l\mu + k\lambda)p(i,j) = (m-i+1)\alpha p(i-1,j-1) +$
$(n+i-j+1)\beta P(i,j-1) + k\lambda P(i+1,j) + l\mu p(i,j+1), \quad k \le i < m, \; l \le j \le n+i-k$; (4.6)

$((m-i)\alpha + (m-i-j)\lambda + l\mu)P(i,j) = (m-i+1)\alpha P(i-1,j-1) + (n+i+1-j)\lambda P(i+1,j) +$
$+ l\mu P(i,j+1), k \le i < m, n+i-k < j < n+i$; (4.7)

$((m-i)\alpha + l\mu)P(i,n+i) = (m-i+1)\alpha p(i-1,n+i-1) + \lambda p(i+1,n+i) \quad k \le i < m$; (4.8)

$((m+n-j-k)\beta + \lambda k + j\mu)p(m,j) = \alpha p(m-1,j-1) +$
$+ (m+n-j+1-k)\beta p(m,j-1) + (j+1)\mu p(m,j+1), 1 \le j < l$; (4.9)

$((m+n-j-k)\beta + \lambda k + l\mu)p(m,j) = \alpha p(m-1,j-1) +$
$+ (m+n-j+1-k)\beta P(m,j-1) + l\mu P(m,j+1), l \le j \le m+n-k$; (4.10)

$((m+n-j)\lambda + l\mu)\mathrm{p}(m,j)\,p(m,j) = \alpha p(m-1,j-1) + l\mu p(m,j+1)$,

m+n-k<j<m+n; (4.11)

$l\mu\mathrm{p}(m,m+n) = \alpha p(m-1,m+n-1)$.

The obtained system together with equilibrium condition: $\sum_{i=0}^{m}\sum_{j=0}^{n+i} p(i,j) = 1$ has, as a rule, unique solution. It's only computational difficulty to find that solution.

In the same way it can be written similar equations for the other arbitrary alignments of m,n,k,l parameters, but we'll not discuss this question here.

4. ECONOMIC ANALYSIS

The system elements as well as maintenance units give different profits depending on the their following states (positions).

For system elements: 1. main; 2. serviceability standby being under replacement; 3. serviceability standby not being under replacement; 4. failed standby being under repair; 5. failed standby not being under repair.

For maintenance units: 6. working replacement; 7. non-working replacement; 8. working repair; 9.non-working repair.

The profit amount per some fixed time interval (time unit) from one system element or one maintenance unit is c_i, $i=\overline{1,9}$ for above-mentioned states. Some of this c_i may be non-positive.

Denote by E_i, $i=\overline{1,9}$ the average number (mathematic expectation) of system elements or maintenance units at the arbitrary time moment in the steady mode of system functioning.

It is not difficult to obtain the expressions for E_i, $i=\overline{1,9}$.

Namely,

$$E_1 = \sum_{i=0}^{m}(m-i)\sum_{j=0}^{n+2} p(i,j);$$

$$E_2 = \sum_{r=1}^{k-1} r\left(\sum_{j=0}^{n} p(r,j) + \sum_{i=r+1}^{m} p(i,n+i-r)\right) + k\sum_{i=k}^{m}\sum_{j=0}^{n+i-k} p(i,j);$$

$$E_3 = \sum_{i=0}^{m}\sum_{j=0}^{n+i}(n+i-j)p(i,j) - E_2;$$

$$E_4 = \sum_{j=0}^{l-1} r\sum_{i=0}^{m} p(i,j) + l\sum_{i=0}^{m}\sum_{j=l}^{n+i} p(i,j);$$

$$E_5 = \sum_{i=0}^{m}\sum_{j=0}^{n+i} jp(i,j) - E_4;$$

$$E_6 = E_2;$$

$$E_7 = k - E_6 = k - E_2;$$

$$E_8 = E_4;$$

$$E_9 = l - E_8 = l - E_4;$$

We introduce the function for economic analysis of the system. This function expresses the profit of examined systems per time unit, taking into account the above-mentioned values.

Initial characteristics of considered systems – ***m***, ***n***, ***k***, ***l***, ***α***, ***β*** and ***λ*** enter into the expression of profit function through $p(i,j)$ - probabilistic characteristics of considered systems. Profit function also depends on $c_1, \dots, c_9$ in the following way $F = F(n,k,l) = \sum_{i=1}^{9} c_i E_i$.

Eventually, the problem of system optimization is stated as problem of mathematical programming (integer programming).

Namely, with those fixed ***m***, ***α***, ***β***, ***λ***, ***μ***, $c_1,\dots, c_9$ for the considered system to select such values of the parameters ***n***, ***k***, ***l*** (optimal numbers of standby elements, replacement units and renewal units) so that the profit function **F** would accept maximum value and to determine this value.

That means the solving of problem of analytical synthesis of multi-element recoverable standby system by economical criterion.

We believe, this result will be very useful for experts working in the field of design, control and management of complex systems.

5. CONCLUSIONS

Main goal of mathematical model, constructed by us, is structural optimization (structural management) of considered system by economic criteria. Since connections between system's elements is fixed, its structural optimization means selection of optimal quantity of standby elements, replacement units and repair units.

From this point of view, intermediate characteristic values p(i,j), introduced by us, are very fruitful: 1) using them it is not difficult to construct mathematical model; 2) system's effectiveness criterion F(n,k,l) is easily expressed by their means.

Finally, structural management problem is brought to integer programming problem, which's solution is only computational difficulty.

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REFERENCES

1. ITU-T Recommendation E.862 (Rev. 1). 1992. Dependability Planning of Telecommunication Networks, Geneva.
2. Jastrebenetsky M.A., Ivanova G.M. 1989. Nadezhnost avtomatizirovannykh system upravlenija tekhnologicheskimi processami. M.: Energoatomizdat.
3. Kakubava R., Khurodze R. 2004. Technical Systems with Structural and Time Redundancy: A Probabilistic Analysis of Their Performance. Automation and Remote Control, vol. 65, no. 5, pp. 825-833 (translated from Avtomatika I Telemekhanika, 2004, no. 5, pp.154-165)
4. Kakubava R., Khurodze R. 2005. Some mathematical aspects of dependability planning of technical systems. Bull. Georgian Acad. Sci., vol. 171, no. 3, pp.250-254.

5. Kakubava R. 2009. New Markovian and semi-Markovian closed queuing systems with two types of service as mathematical models of reliability and maintenance. Extended Abstracts, VI International Conference MMR 2009 - Mathematical Methods in Reliability, M.: RSUOG, pp. 472-474

6. Korolyuk V.S., Limnios N. 2005. Stochastic Systems in Merging Phase Space. World Scientific, Imperial College Press.

7. Limnios N., Oprisan G. 2001. Semi-Markov Processes and Reliability. Birkhauser, Boston.

8. Raykin A. L. 1967. Elementy teorii nadezhnosti dlja proektirovanija tehnicheskih system. M.: Sovetskoye Radio.

9. Ushakov I. A. (Ed.). 1985. Nadezhnost tekhnicheskikh sistem. M.: Radio i Svyaz.

10. Ushakov I. 2006. Reliability: past, present, future. Reliability: Theory & Applications, vol. 1, no. 1, pp. 10-16 (electronic).

RISK ASSESSMENT OF EXPLOSIVE ATMOSPHERES IN WORKPLACES

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ABSTRACT

The application of the Directive 99/92/EC deals with the safety and health protection of workers potentially exposed to explosive atmospheres and requires the assessment of explosion risks. These can arise by the release of inflammable substances typical of industries classified as major hazards, but they often may be generated in other industries where inflammable materials are handled. Risk assessment of explosive atmospheres is required in both cases, for this purpose, in this article a quantitative approach has been proposed. The paper describes the main aspects of the methodology, based on a probabilistic risk assessment, and finally its application to a case-study.

1 INTRODUCTION

In the framework of the General Directive 89/391/CE, the term ATEX (from the French *Atmospheres Explosibles*) is the name commonly given to the framework for controlling explosive atmospheres and the standards of equipment and protective systems used for this purpose. Concerning the control of explosion risk there are two European Directives: Directive 99/92/EC or ATEX 137 and the Directive 94/9/EC or ATEX 95.

According to this legislation the employer has the obligation to prevent the formation of explosive atmospheres adopting all the technical-organizational measures required. When the formation of such inflammable clouds can not be avoided, the ignition must be prevented and the potential damage caused by an explosion must be reduced to the minimum. The employer has to classify the areas in which the formation of explosive atmospheres is possible. Then the *document of evaluation of the risk due to explosive atmospheres* (in this work named *document of evaluation of the ATEX risk*) has to be redacted and, periodically, updated. Such document must undergo to the least requisites fixed by the decree law mentioned before the Italian D.L. 81/08 and must include a section in which the risk is evaluated using specific methodologies. Measures to avoid the formation of explosive atmospheres and ignition sources must also be indicated, finally the characteristics of the equipment used in the workplace must be specified.

Concerning industries classified at major risk, *Safety Reports* include the risk assessment related to the explosions of great magnitude for industries classified as major hazards. The estimation of the risk due to lower magnitude explosions, characterized by lower magnitude, which could potentially involve workplace, is included in the document of the risk assessment of the workplace and in the document of evaluation of the ATEX risk. The approach applied for the explosion risk assessment in the workplaces is generally qualitative (Benintendi et al. 2006) or semi-quantitative (Pezzo et al. 2006). The application of a qualitative method often causes an underestimation of the risk associated with the explosion of flammable clouds arising from small releases, particularly, in confined spaces.

It is important to remember that the evaluation of risk should be completed with the judgement about its acceptability. Some criteria are necessary, the judgement is made on the bases of the experience relative to knowledge of the technology.

2 METHODOLOGICAL APPROACH

In this work a quantitative procedure for the assessment of the explosion risk has been described, it is based on a probabilistic approach. A flow-chart of the procedure is in figure 1.

The application of the procedure requires a detailed knowledge of the system (workplace and activity). The first step of the analysis is the classification of the hazardous areas this can be made by taking into account the quantity of substances released and the probability of release. The risk evaluation will be possible after the quantification of the probability of ignition, the potential damage caused by the explosion and the probability of the presence of workers. The procedure is completed with the evaluation of the acceptability of the risk.

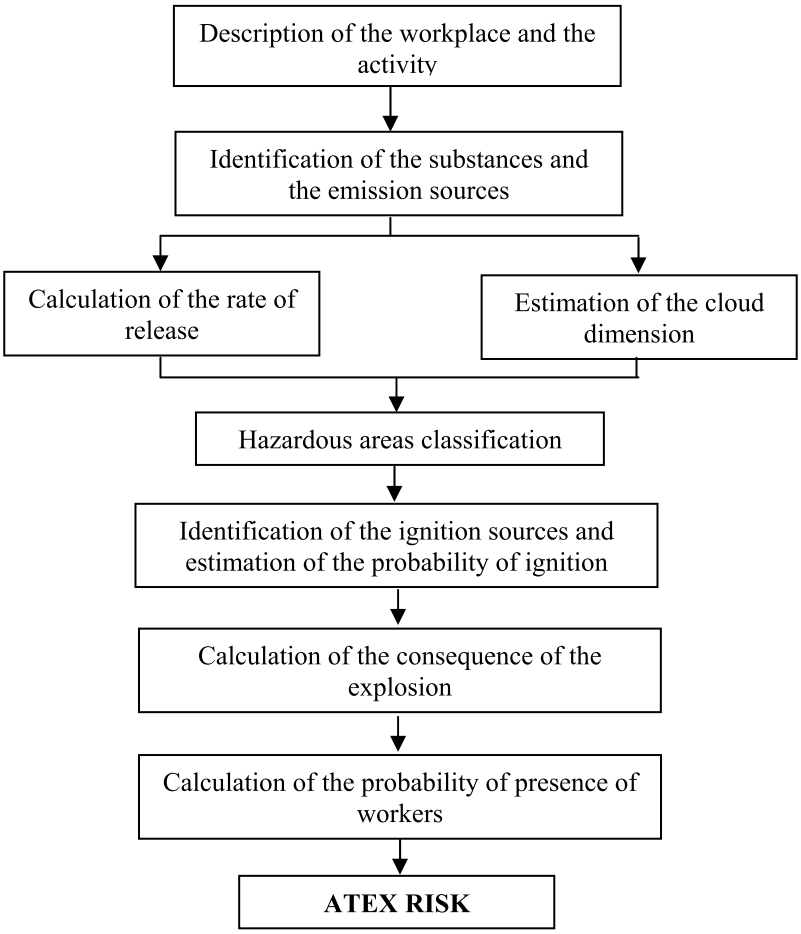

Figure 1. Flow-chart for the evaluation of risk due to the presence of explosive atmospheres.

Some advantages are related to the use of a quantitative approach for risk analysis. It allows an improvement of the overall safety levels, at the same time, in the case of industries at major hazards, the method avoids the underestimation of the risk associated with explosive atmospheres and to identify the correct preventive and protection measures for each case.

2.1 Classification of hazardous areas

An explosive atmosphere is defined as a mixture of inflammable substances with air, under atmospheric conditions. If an ignition has occurred, combustion spreads to the entire unburned mixture. Atmospheric conditions are commonly referred to ambient temperatures and pressures. This means temperatures of – 20 °C to 40 °C and pressures of 0.8 to 1.1 bar.

The classification of the areas has the purpose of establishing the presence of zones characterized by the explosion hazard, in which technical and organizational provisions must be adopted with the aim to make the risk due to the presence of explosive atmospheres negligible. In order to classify the areas, the establishment must be divided into units and it is necessary to define the zones where inflammable substances can be released due to the normal operation of the plant, process deviations or during maintenance activities.

The methodology EN 60079-10 (CEI 31-30, 1996) must be used for zone classification. The method needs to be applied together with two guidelines, Guide CEI 31-35 and Guide CEI 31-35/A, and identifies the hazardous zones of table 1.

Table 1. Probability and duration of explosive atmospheres.

Zone	Probability, P	Duration, t
	Year^{-1}	Hour/year
Zone 0: the presence of an explosive atmosphere is continuous	$P > 0.1$	$t > 1000$ h
Zone 1: an explosive atmosphere is likely to occur during normal operating conditions	$0.1 > P > 10^{-3}$	10 h < t < 1000 h
Zone 2: an explosive atmosphere is unlikely to occur in normal operating conditions or occurs infrequently for short periods of time	$10^{-3} \geq P > 10^{-5}$	0.1 h < t < 10 h

2.2 Probability of presence of ignition sources

An important phase of the risk analysis is the ignition of the sources identification. The standards UNI EN 1127-1 (2001) lists the following main causes of ignition of inflammable atmospheres:

- Hot surfaces;
- Flames and hot gases or particles;
- Mechanical sparks;
- Electrical networks;
- Stray currents;
- Cathode protection;
- Static electricity;
- Lightning;
- Ohmic heating of electric cables;
- Radio-frequency waves;
- Electromagnetic waves;
- Ionizing radiation;
- Ultrasound;

- Adiabatic compression and shock waves;
- Exothermic reactions.

In order to quantify the probability of occurrence of each ignition source, literature data or, preferably, specific studies for the plant under analysis can be used. Methods, such as historical analysis, fault tree analysis, FMEA or FMECA, or specific analytic procedures, could also be applied to assess the probability or the likely effectiveness of each of the ignition sources listed above.

2.3 Consequences of the explosion

The consequences of the explosion must be estimated for each emission source identified through the classification of the areas and for each unit of the establishment. This phase consists in the estimation of the overpressure vs. the distance from the source (the point where ignition occurs). The complexity of the phenomenon would require a fluidynamic study using an appropriate simulation code. The purposes of the work the use of simplified and conservative models which give the pressure peak vs. the distance is sufficient.

Many simplified models for the estimation of the overpressure caused by an explosion are available in Lees 1996 and in the Yellow Book 1997. The most common methods are the *equivalent TNT model* and the *equivalent piston model*, all the methods allow the quantification of the distance where a pressure wave reaches the value of 0.03 bar. The overpressure is the threshold limit causing reversing lesions.

2.4 Presence of workers

The presence of personnel in workplace depends on the number working in the potential damage zone and on their probable of presence. The number of workers involved in a potential explosion can be calculated using the damage zones obtained through the consequence analysis. The probability of presence is calculated according to the worker task (for example shift-workers, head-shifts, maintenance staff). Thus the presence of workers p_w (the parameter is a probability) is calculated using equation (1):

$$p_w = \left(A_i / A_{est} \right) p_i \quad (1)$$

where A_i = impact zone of the explosion; A_{est} = whole area of the establishment; p_i = probability of the presence of personal in the establishment.

3 ATEX RISK

In this work equation (2) has been proposed for the calculation of the risk index associated with the potential presence of explosive atmospheres, R_{ae} (*ATEX risk*):

$$R_{ae} = p_e \cdot p_a \cdot p_w \quad (2)$$

where p_e = probability of release of an inflammable substance from an emission source; p_a = probability of the presence of an ignition source; p_w = presence of workers in the impact area.

4 APPLICATION

The methodology proposed in this paper has been applied to a real establishment. The case study is a petrochemical plant (confidential). The area of the establishment is approximately 400 hectares and consists of 15 manufacturing plants, 10 of auxiliary service, 4 of air pollution protection, 2 of water pollution protection, fire alarm systems, a wide area for the movement of products and general service areas (offices, control room, lunch room, laboratories, etc.). In order to assess the risk the establishment has been divided into 27 units. Figure 2 shows the layout of a unit of the establishment. This is the desulphuration plant, which consists of two reactors, a number of storage tanks, the gas/liquid separation drum, etc.

The quantity of substances released and the probability of formation of an explosive atmosphere have been calculated. Using these data the classification of areas has been carried out. Results of the classification of the areas for a part of the unit of Figure 2 are shown in Figure 3(a). The part under analysis is the gas/liquid separation drum.

The next step is the identification of potential ignition sources (SA). Reconnaissance of the workplace and interviews with workers have allowed the exclusion some of the potential sources of ignition listed in Lees 1996. Nine potential sources of ignition have been taken into account: 1) hot surfaces, 2) flames and hot gases or particles, 3) mechanical sparks, 4) electrical system, 5) cathodic protection, 6) static electricity, 7) lightning, 8) electric-overload due to clouds, 9) heating cables. For each SA the likelihood of ignition has been calculated using historical and fault tree analysis or, sometimes, specific calculation procedures. Subsequently, for each SE the effects of an explosion due to the presence of at least one SA have been calculated.

The consequences analysis has been done using the *equivalent mass of TNT model*. The information required for the consequences assessment, using both the method, are the flow rate Q (kg/s) of the released substance and the *distance of permanence* of the cloud d_z (m). To define this last parameter it is necessary to note that after the release the cloud exists for a certain time in the area and, in the presence of ignition sources, can potentially cause an explosion. The *distance of permanence* indicates the maximum dimension of the explosive cloud. The application of the TNT model is very simple, it is based on the estimation of *equivalent mass of TNT* (m_{TNT}) for a certain explosion, using equation (3), and then on the calculation of the distance (x), using a specific correlation such as equation (4):

$$m_{TNT} = \eta \cdot \frac{\Delta H_C}{4{,}196 \cdot 10^6} \cdot m_{cloud} \tag{3}$$

$$x = m^{1/3} e^{\left[3.5031 - 0.724 \ln O_p + 0.0398 (\ln O_p)^2\right]} \tag{4}$$

where m_{TNT} = equivalent mass of TNT (kg); η = yield factor; ΔH_C = enthalpy of combustion of the explosive (kJ/kg); m_{cloud} = mass of the explosive (kg); x = distance (*feet*); O_p = overpressure (*psi*).

The classification of the areas for the unit under analysis has been given in Figure 3(a), the results of the consequence assessment for an emission source are shown in Figure 3(b).

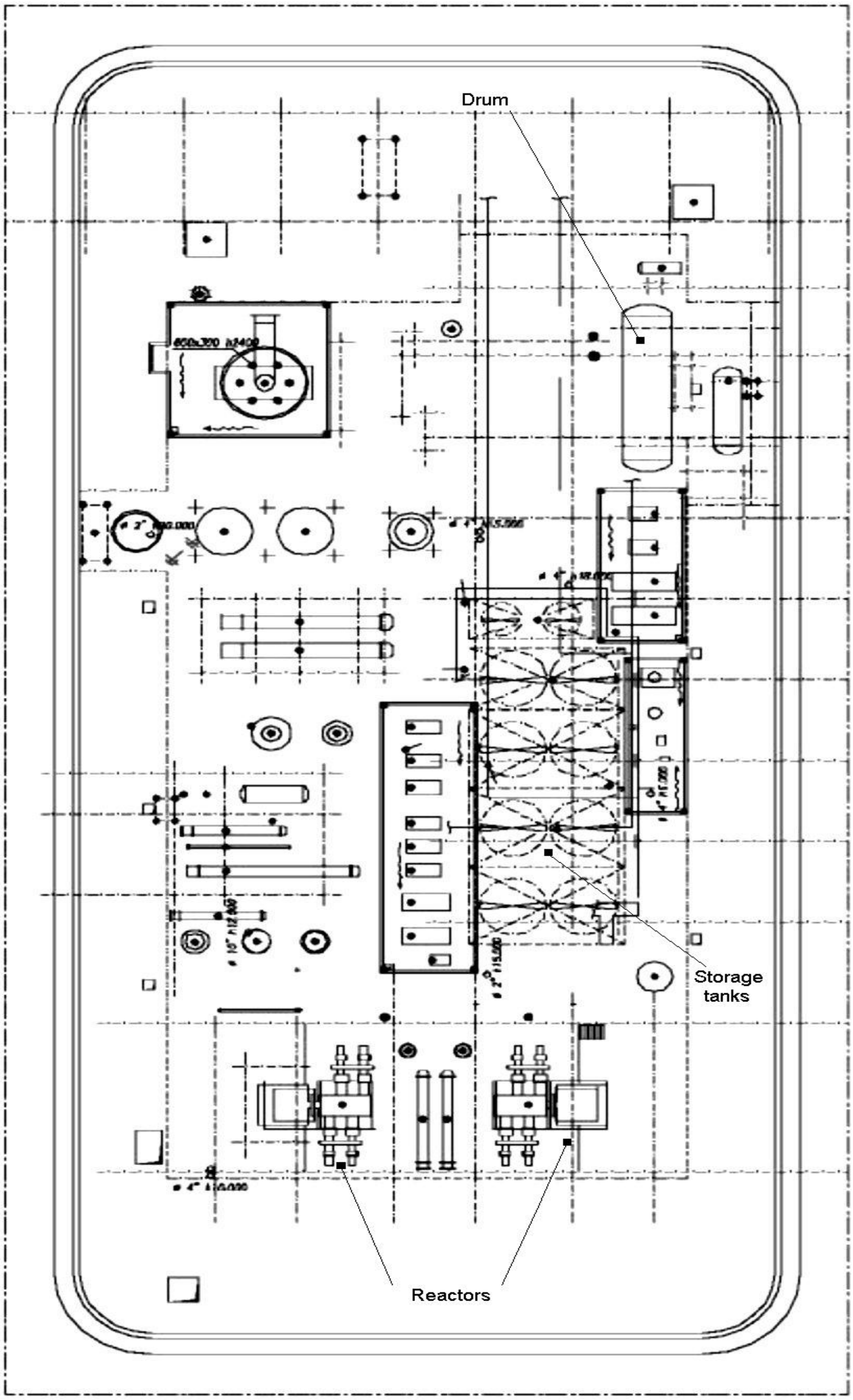

Figure 2. Layout of a unit of the establishment.

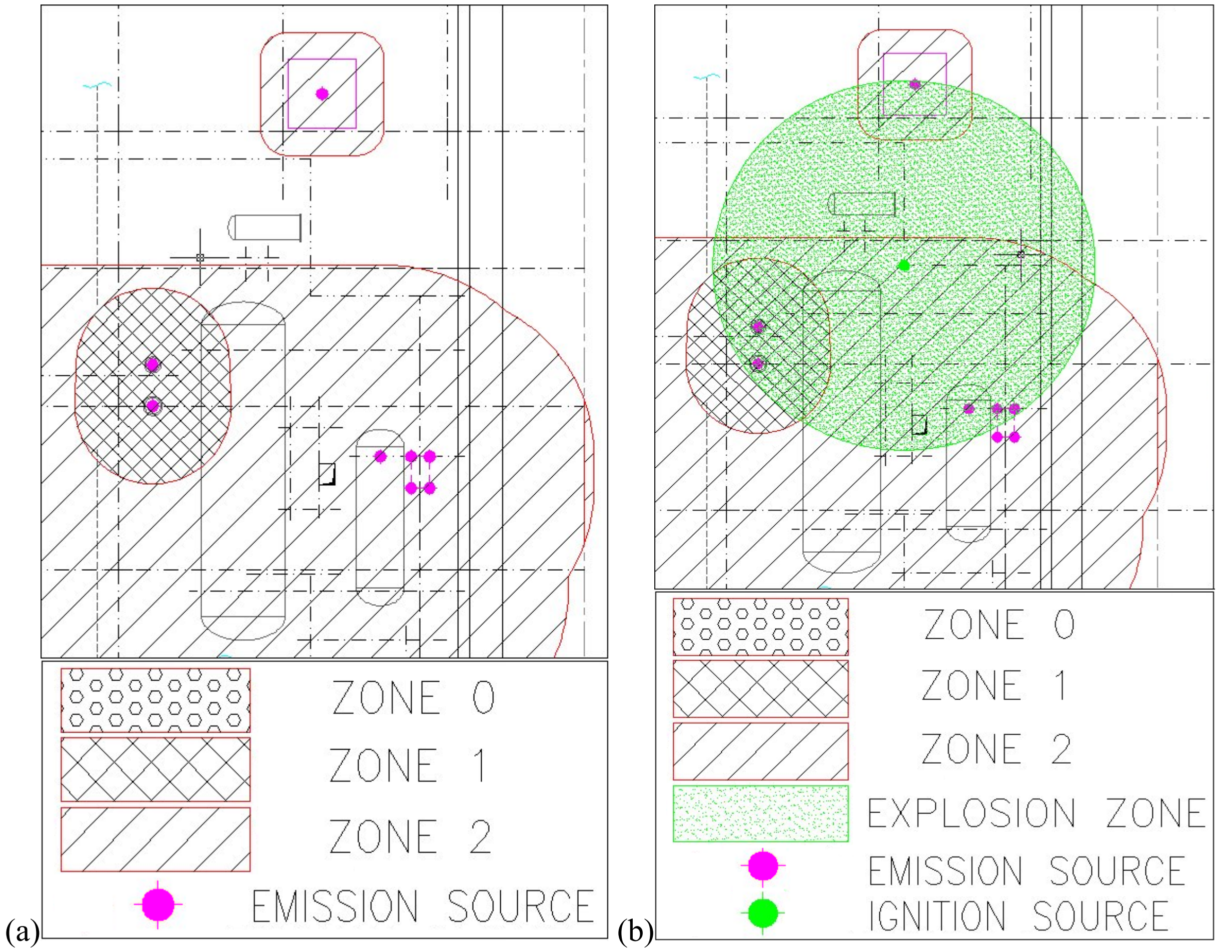

Figure 3. (a) Classification of the areas; (b) Results of consequence analysis.

The presence of personnel has been calculated taking into account three different worker tasks. Table 2 shows the probability of presence for shift workers, foremen and maintenance staff.

Table 2. Probability of presence of workers.

Task	Probability of presence p_i n. hours per day
shift worker	0.91
foreman	0.33
maintenance staff	0.11

Given the values for the probability of release of inflammable substances, of the presence of ignition sources and the presence of workers, the risk has been evaluated according to equation (2).

The graph of Figure 4 shows the values of the index risk R_{ae} calculated for each emission source of the unit of Figure 2. The graph shows how is simplified the identification of critical points where actions to reduce the risk are necessary.

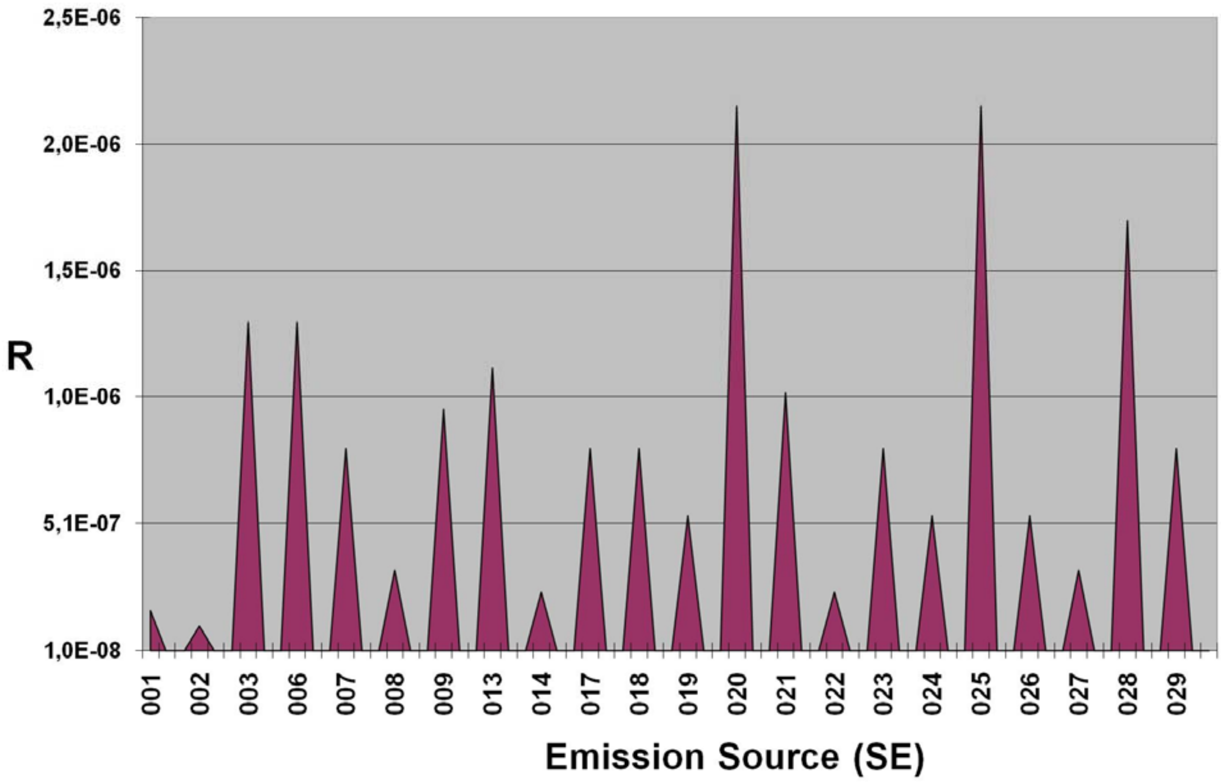

Figure 4. Risk Index (R_{ae}).

5 RISK ACCEPTABILITY

After risk analysis it is necessary to make decisions about existing hazardous or potentially hazardous establishment. Usually these decisions are delegated to organizations with recognized expertise in the area. For existing technology, that expertise will rely on past experience, including incidental statistics (also “near-miss”) for hazardous facilities. This experience must provide threshold limits in order to define the acceptability and the tolerability of the risk.

Taking into account of the typology of the accidents, in this paper the same threshold values used to judge the risk acceptability in industries at major risk have been applied. Explosions analyzed in the Safety Reports are originated by great releases of inflammable substances and, consequently, they impact on large areas. For establishments not at major risk, the explosions have a smaller impact area and involve only the workers. Since both the type of explosions can be studied in the same way it is opportune to uniform the approaches of risk evaluation.

Unfortunately the Italian normative does not defined acceptability criteria for industries classified at major hazard, the risk judgment is made referring to the threshold values of frequency and consequences reported in the Italian D.P.R. 126. Concerning the explosion risk, in this work, it has been proposed to refer to the risk acceptability criterion adopted in the United Kingdom and described in the Italian D.P.C.M. of the 25th February 2005 and in the Guide CEI 31-35/A. The threshold values of risk are:

$R_{ae} < 10^{-6}$ *risk is acceptable*;

$10^{-6} < R_{ae} < 10^{-4}$ *risk must be reduced as low as technically and economically possible*;

$R_{ae} > 10^{-4}$ *risk is not acceptable*.

According to this criterion and this analysis, based on the risk assessment and combined with on-site inspections, the necessity of further implementation of protective and preventive measures can be verified. In order to make this objective a detailed check-list has been drawn.

6 CONCLUSIONS

The proposed methodology permits to identify the critical points in the system (technical and procedural) and decreases the exposure of the workers as low as possible. In particular the quantitative approach allows to not underestimate the risks for the exposed workers. The quantitative evaluation of the explosion risk also allows to obtain an improved effective in the prevention and protection interventions adopted by the company.

A tool for the risk assessment has been created, it permits to repeat the calculations and a faster verifications of the possible improvement of the measures of risk prevention and mitigation for the system under analysis.

Finally through the quantitative analysis it is possible a detailed study of the accidental scenarios due to small releases. For industries at major risk a detailed analysis of such events is essential because they can represent potential sources of domino effects.

REFERENCES

Benintendi, R., Dolcetti G., Grimaz S, & Iorio C.S. (2006). A comprehensive approach to modelling explosion phenomena within ATEX 1999/92/CE. *Chemical Engineering Transactions.* 9, 403-408.

Council Directive 89/391/EEC on the introduction of measures to encourage improvements in the safety and health of workers at work (1989).

Council Directive 94/9/EEC on the approximation of the laws of the Member States concerning equipment and protective systems intended for use in potentially explosive atmospheres (1994).

Council Directive 99/92//EEC on the approximation of the laws of the Member States concerning equipment and protective systems intended for use in potentially explosive atmospheres (1999).

Italian Parliament D.L. 626/94 on the improvement of the safety and health in the workplaces (1994). *In Italian*.

Italian Parliament D.L. 233/03 "Application of the Directive 99/92/CE on the improvement of the safety and healthy in the workplace of workers exposed to explosive atmospheres" (2003). *In Italian*.

Italian Parliament D.P.R. 126 "Application of the Directive 94/9/CE concerning equipment and protective systems intended for use in potentially explosive atmospheres" (1998). *In Italian*.

Italian Parliament D.L. 81/08 concerning the safety and health in the workplaces(2008). *In Italian*.

Italian Parliament D.P.C.M. 25th February 2005 concerning the guidelines for the predisposition of emergency plan (2005). *In Italian*.

ESCIS (1998). *Introduction to Risk Analysis, Approaches and Methods*. Booklet 4.

Guide CEI 31-35. *Guide for the application of the Norm EN 60079-10* (CEI 31-30).

Guide CEI 31-35/A. *Electrical apparatus for explosive atmospheres – Guide for the application of the Norm EN 60079-10 (CEI 31-30) Classification of hazardous zones*.

Lees, F.P. (1996). *Loss Prevention in the Process Industries*. Elsevier, London.

Norm EN 60079-10 (1996) - *Electrical Apparatus for Explosive Gas Atmospheres. Classification of Hazardous Areas*.

Pezzo, T., Vannucci, D. & Uguccioni, G. (2006) Semiquantitative Analysis for Assessment of Explosion Risk for Workers - Application of the Requirements of the ATEX Directive. *Chemical Engineering Transactions* 9, 403-408.

Standard UNI EN 1127-1 (2001). *Explosion prevention and protection. Basic concepts and methodology*.

TNO (1997). *Methods for the calculation of physical effects due to releases of hazardous materials (Yellow Book)*. The Hauge.

STATISTICAL MODELLING OF INDOOR RADON CONCENTRATION USING METEOROLOGICAL PARAMETERS

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ABSTRACT

The radon volume activity in buildings is generally time variable. Its variability is caused by many natural and man-made factors. An example of these factors includes meteorological parameters, soil properties, characteristics of the building construction, properties of water used in the building and also the behavior of inhabitants. These factors can influence each other and also they are related with the exposition of inhabitants. This article reports a continual indoor radon monitoring and a statistical evaluation of a dataset obtained by the 18 days-long measuring in a house located in the Czech Republic.

The contributions of carefully selected meteorological parameters and human influences were also observed. Results of the observation were divided into two parts (inhabited, uninhabited) and analyzed in relation with the indoor radon concentration. The multiplied linear regression was applied to model obtained datasets. Results of time series analyses of the continual indoor radon concentration and the meteorological monitoring are presented and discussed as well.

INTRODUCTION

One of a natural radionuclide that commonly occurs in rocks is uranium U-238 [[8]]. It is evident that radon rising by radioactive decay from uranium is steadily generated in natural conditions. Considering the human life span, the radon is in principle a stationary source of irradiation.

The radon volume concentration in a building is not stable; it can vary in time because the radon concentration is influenced by many factors e.g. meteorological, structural parameters [[3], [8], [9], [10], [11]]. These are several reasons why it is not straightforward to describe the radon behaviour in houses and exactly determine the indoor radon concentration. The concentration of radon in homes and other buildings varies substantially from one area to another, from one structure to another, and even within the same structure [[2], [3], [8]]. Likewise, in the same building, there is often a substantial variation with time on various temporal scales, i.e., season to season, week to week, and on a daily or hourly basis [[2], [6], [9], [11]].

Understanding the nature and origin of such variability is important as a basis for evaluating the range of the radon concentration problem, interpreting monitored data and for formulating of effective strategies for control.

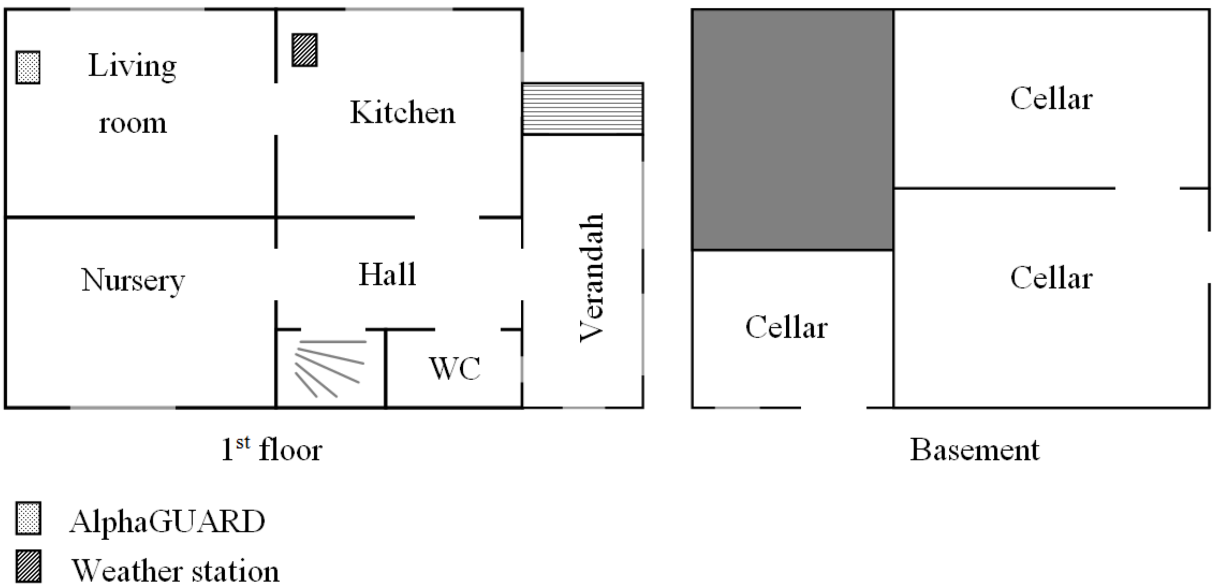
In this contribution, we try to find a relationship between the indoor radon concentration and the current meteorological factors. The aim is also to find factors, which mostly influence the indoor radon concentration and to develop a model for describing the behavior of radon concentration depending on selected factors in the given room. We tried to find how selected meteorological factors influence the indoor radon concentration by using the multiple linear regressions techniques. The most important meteorological factors and parameters of a reduced linear model characterizing the

radon concentration were statistically estimated. Finally, in order to verify the obtained model, we compared measured and estimated data.
This paper is organised as follows. In the first section, we give an overview of the time period of the measurements, the construction of the measured house and a description of used tools for measuring radon concentrations and its location within the house. In the second section, we describe the obtained measured datasets and its modification, statistical analysis, tables and figures of results of the applied analysis and the best alternative model of radon concentrations in the selected room. In the third section, we discuss the selection of the best, statistically reduced models. We also analyse an effect of (non)presence of human to the ability of the model to correctly predict the radon concentration. The summary of interesting results and the future work are discussed in conclusion.

MATERIALS AND METHODS

The description of the measured house

The measurements were carried in one family house (see Figure 1.). The experimental building is situated in the village Strašín in the southwest region of the Czech Republic near the Šumava Mountains. The house was built in 1930. It is a masonry and partly cellar construction which is situated on a steep land on the edge of the village. The house has two cellars (with separate outdoors), three rooms on the first floor and one attic that was closed during the measurements. All rooms are ventilated by natural means and two of all is heated by stoves.



The ground plan of measured house

The building is connected to the public water pipe-line as required by Czech law, so the water source of radon is eliminated [1]. The measured gamma dose rate in contact with construction materials was very close to the natural background. It indicates that the building materials do not contain any natural radionuclide. So we suppose that the main source of indoor radon is the soil gas. The average value of radon concentration in soil was 20 kBq/m^3.
The studied room (living room) is located on the north side of the building and joint with the adjacent heated room (kitchen); see Figure 1.
There was a cinder filling under the wooden floor of the experimental room. The floor was covered by a carpet. The volume of the experimental room was 30 m^3.
The house was selected on the basis of screening measurements performed by the Czech National Radiation Protection Institute in Prague, the Czech Republic.

The description of the equipment

We have chosen the continual method of monitoring for the evaluation of radon concentration in context of the selected meteorological factors. The Alphaguard radon monitor has been used for continuous detection of the radon concentration [[5]]. It has an ionization chamber and uses an alpha spectroscopy for the radon detection. The two

common isotopes of radon (i.e., Rn-222 and Rn-220) are identified through their respective energies from the alpha decays. The signal generated from the alpha detection is converted to a digital output. The AlphaGUARD was operated in the diffusion mode at a 60-min cycle.
The actual meteorological conditions were recorded by the wireless weather station Vantage Pro2™ [[4]]. Sensors for detecting outside conditions were located southeast of the building and the indoor station was placed in the adjacent room constantly connected with the experimental room.

Results and Discussion

Time series of the indoor radon concentration, the indoor climate parameters including temperature, humidity and atmospheric pressure in the measured room were obtained by the radon monitor AlphaGUARD.
The radon monitoring started 3rd October at 5 PM and finished 20th October at 3 PM. The values were recorded every 60 minutes so each time series has 407 values. In order to stabilize the radon monitor and the indoor climate, the first two values (3rd October at 5 PM and 6 PM) were eliminated from analyses. The final analysing time series starts at 3rd October at 7 PM and ends 20th October at 3 PM. The total number of analysed values is 405. We compared these time series with ones obtained by the weather station Vantage Pro2™. The weather station has recorded time series of the following meteorological parameters:

- indoor/outdoor temperature,
- indoor/outdoor relative air humidity,
- indoor/outdoor barometric pressure,
- indoor/outdoor dew-point,
- wind direction and velocity,
- rainfall.

The weather station operated during the same time as the AlphaGUARD. The values of the weather station were recorded every 30 minutes, so the total number of recorded values was 813. Because of the different sampling intervals of the weather station from one of the AlphaGUARDs, we have modified time series of meteorological parameters to the one-hour interval by the averaging; we used the arithmetic averages of last two values corresponding to 30 minutes interval values according to the equation 1:

$$x_i = \frac{x_{2i} + x_{2i-1}}{2} \text{, for } i = 1, 2 \ldots 406 \qquad (1)$$

We eliminated the first value for the same reasons as for the monitoring of radon. Finally, the adjusted time series of meteorological parameters had 405 values.

Monitoring of the human influence

For reasons of known influence of human activities (such as heating or ventilation), we divided each time series into 2 parts – inhabited and uninhabited. The object was 11 days inhabited and 7 days uninhabited. Although an inhabitant left the house 15th October at 9:10 AM, the second "Uninhabited" part starts on October 15 at 7:00 PM, in order to stabilize indoor climate and regression models after the person has left. For each measured parameter, we finally obtained time series of 291 values for the first "Inhabited" part and time series of 114 values for the second "Uninhabited" part.

Statistical analysis of measured data

In our contribution we try to find a relationship between the indoor radon concentration (dependent variable) and several measured meteorological parameters (independent variables). The statistical analyses of these data sets were done using multiple linear regression techniques [[7]]. The general purpose of multiple regressions is to learn more about relationships between several independent (predictor) variables and the dependent (criterion) variable. The statistical analyses were provided by the Statgraphics Plus 5.1 software.
Because of different effects of radon concentration in the case of inhabited and uninhabited time, we selected various independent parameters, see Tables 1 - 2. Tables shows the results of fitting of various multiple regression models, in order to describe the relationship between indoor radon concentration and 13 predictor variables in case of "Inhabited part" and 7 predictor variables in case of "Uninhabited part". Models were fitted by various combinations of meteorological parameters. The tabulated statistics include the mean squared error (MSE), the adjusted and unadjusted R-Squared values and appropriate parameters.
A linear regression technique uses the estimated MSE to determine the statistical significance of factors under the study. If an MSE is zero, it means that the estimator predicts observations with perfect accuracy.

The *R-squared* coefficient of determination is a statistical measure of how well the regression model approximates the real data-points. The values can vary from 0 to 1. If *R-squared* = 1, the regression curve perfectly fits the observed data [[7]]. The adjusted R-Squared statistic measures the proportion of the variability in the indoor radon concentration which is explained by the model. The explanation of this statistic is almost the same as *R-squared* but it penalizes the statistic as extra variables included in the model. Of course, larger values of adjusted R-Squared correspond to smaller values of the mean squared error.

Models with the largest R-Squared values for "Inhabited Part" of Time Series (left) and explanations of parameters (right)

MSE	R-Squared	Adjusted R-Squared	Included Variables
16691.1	34.7539	33.6092	CDELM
16955.0	33.7222	32.5594	BCDFL
17006.1	33.5225	32.3562	CDEKL
17017.9	33.4762	32.3091	CDGLM
17027.1	33.2066	32.2724	CDEL
17032.6	33.4188	32.2507	BDELM
17073.8	33.0235	32.0868	CDLM
17118.3	32.849	31.9098	DELM
17359.2	31.9039	30.9515	CDKL
17361.7	31.656	30.9416	CDL

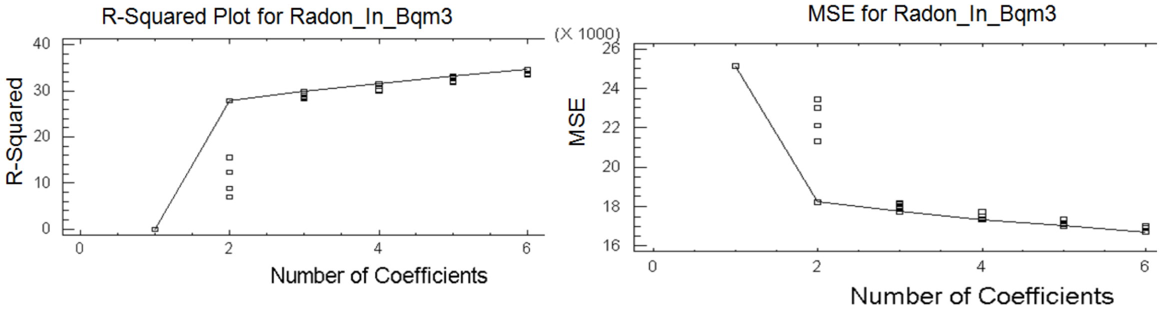
Signification	Explanation
A=DewingPIn_oC	Dewing Point in the Kitchen
B=DewingPOut_oC	Dewing Point Outdoor
C=Hour	Hours of a Day
D=HumIn_proc	Relative Humidity in the Kitchen
E=HumIn_procAG	Relative Humidity in the Measured Room
F=HumOut_proc	Relative Humidity Outdoor
G=P_hPa	Atmospheric Pressure Outdoor
H=P_hPa_AG	Atmospheric Pressure in the Measured Room
I=Rainfall_mm	Rainfall
J=Tin_oC	Temperature in the Kitchen
K=Tin_oC_AG	Temperature in the Measured Room
L=Tout_oC	Temperature Outdoor
M=wind_invms	Wind Velocity

Models with the largest R-Squared values for "Uninhabited Part" of Time Series (left) and explanations of parameters (right)

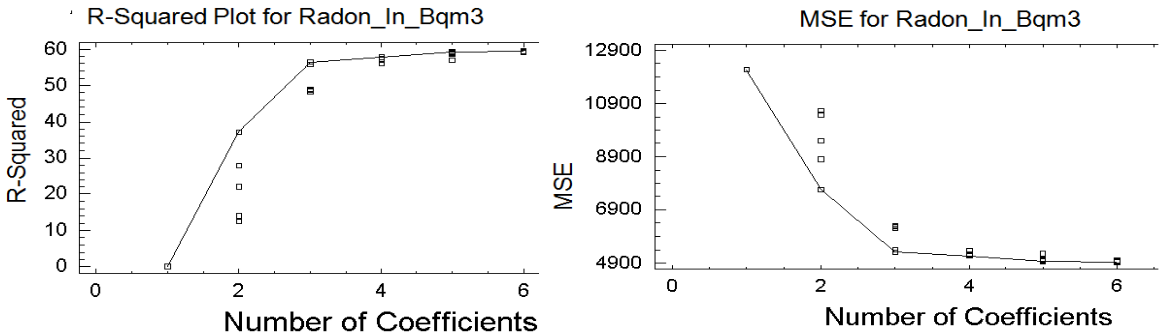
MSE	R-Squared	Adjusted R-Squared	Included Variables
4937.2	61.2712	59.4782	ABCDG
4950.1	61.1701	59.3724	ABCDF
4955.67	60.7665	59.3267	ACDE
4966.34	61.0427	59.2391	ABCDE
4972.7	60.9928	59.1869	ACDFG
4977.77	60.5915	59.1453	ACDF
4983.36	60.9092	59.0994	ACDEG
5015.33	60.2941	58.837	ABCD
5052.4	60.0007	58.5328	ACDG
5154.82	58.8154	57.6922	ACD

Signification	Explanation
A=DewingPIn_oC	Dewing Point in the Kitchen
B=DewingPOut_oC	Dewing Point Outdoor
C=HumIn_procAG	Relative Humidity in the Measured Room
D=HumOut_proc	Relative Humidity Outdoor
E=P_hPa	Atmospheric Pressure Outdoor
F=P_hPa_AG	Atmospheric Pressure in the Measured Room
G=Tdiff_oC	Difference of Temperature in the Measured Room and Outdoor

Figure 2. and Figure 3. show selected statistical models which give the largest adjusted *R-Squared* values for "Inhabited Part" and "Uninhabited Part" of time series, respectively.



The Comparison of R-Squared (left) and Mean Squared Error (right) with the Number of Coefficients for “Inhabited Part” of Time Series



The Comparison of R-Squared and Mean Squared Error (MSE) with the Number of Coefficients for “Uninhabited Part” of Time Series

Depending on the size of the *R-Squared* and MSE (Tables 1., 2. and Figures 2., 3.) we have chosen the favourite models, which has the highest *R-Squared* and the lowest MSE.

The best model for the “inhabited part” of time series

The best model for the “inhabited part” of time series contains 5 variables: hours of a day (*Hour*), relative humidity in the kitchen (*HumIn_proc*), relative humidity in the kitchen (*HumIn_procAG*), temperature outdoor (Tout_oC) and wind velocity (wind_invms). Table 3. shows the estimates of variables, its errors of the estimate and corresponding P-values. Since the P-value is less than 0.05, there is an indication of possible serial correlations. In determining whether the model can be simplified, notice that the highest P-value on the independent variables is 0.0098, belonging to the wind velocity (*wind_invms*). Since the P-value is less than 0.01, the highest order term is statistically significant at the 99% confidence level and no simplification of the model is statistically recommended.

The Best Alternative Model of the “Inhabited Part” of Time Series.

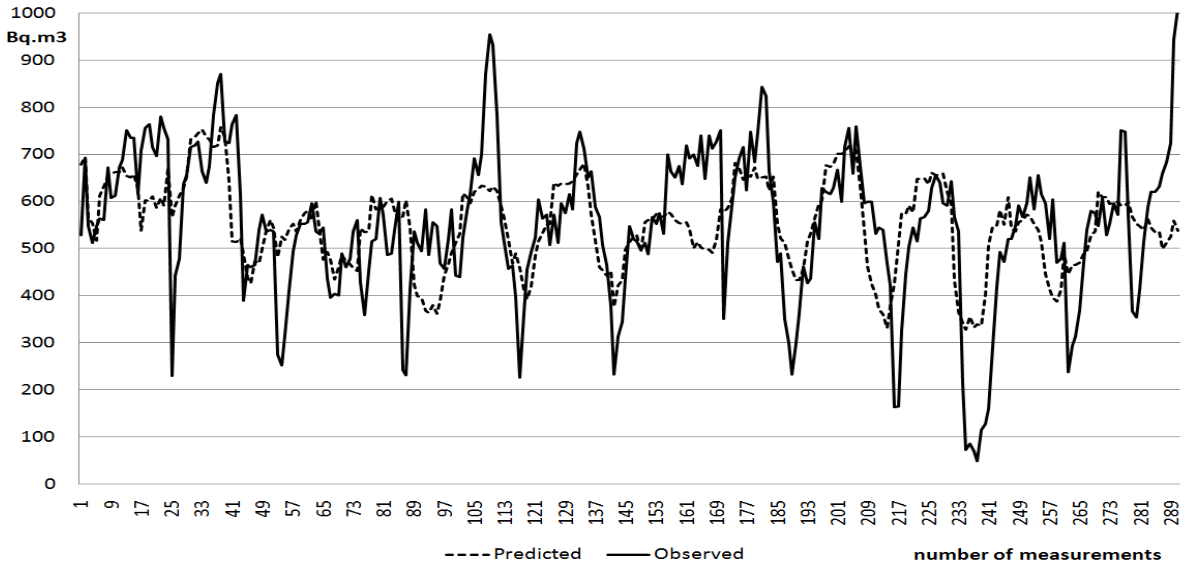
Parameter	Estimate	Error	P-Value
CONSTANT	610.157	228.463	0.0080
Hour	-3.39318	1.17631	0.0042
HumIn_proc	15.9026	3.28874	0.0000
HumIn_procAG	-12.1664	4.42536	0.0064
Tout_oC	-18.858	1.96029	0.0000
wind_invms	77.8566	29.9478	0.0098

The R-Squared statistic (Table 1.) indicates that the model of the “inhabited part” explains 34.75% of the variability of the indoor radon concentration. The adjusted R-squared statistic, which is more suitable for comparing models with different numbers of independent variables, is 33.60%. The standard error of the estimate (Table 1.) shows the standard deviation of the residuals to be 129.194.

The output of the analysis is a multiple linear regression model, which describes the relationship between the indoor radon concentration (*Radon_In_Bqm3*) and five independent variables. The expression of the fitted model is:

$$Radon\_In\_Bqm3=610.157-3.39318*Hour+15.9026*HumIn\_proc-12.1664*HumIn\_procAG-18.858*Tout\_oC+77.8566*wind\_invms \quad (2)$$

Figure 4. shows how the fitted model follows the shape of the observed values of indoor radon concentration.



The Comparison of the Predicted and Measured Values of “Inhabited Part” of Time Series.

The best model for the “uninhabited part” of time series

The best model for the “uninhabited part” of the time series was estimated under the value of the R-Squared and the mean squared error as before. By analysing Table 2. and Figure 3., the best model for the “uninhabited part” of time series contains these five variables: dewing point in the kitchen (*DewingPIn_oC*), dewing point outdoor (*DewingPOut_oC*), relative humidity in measured room (*HumIn_procAG*), relative humidity outdoor (*HumOut_proc*), difference of temperature in the measured room and outdoor (*Tdiff_oC*), see Table 4.

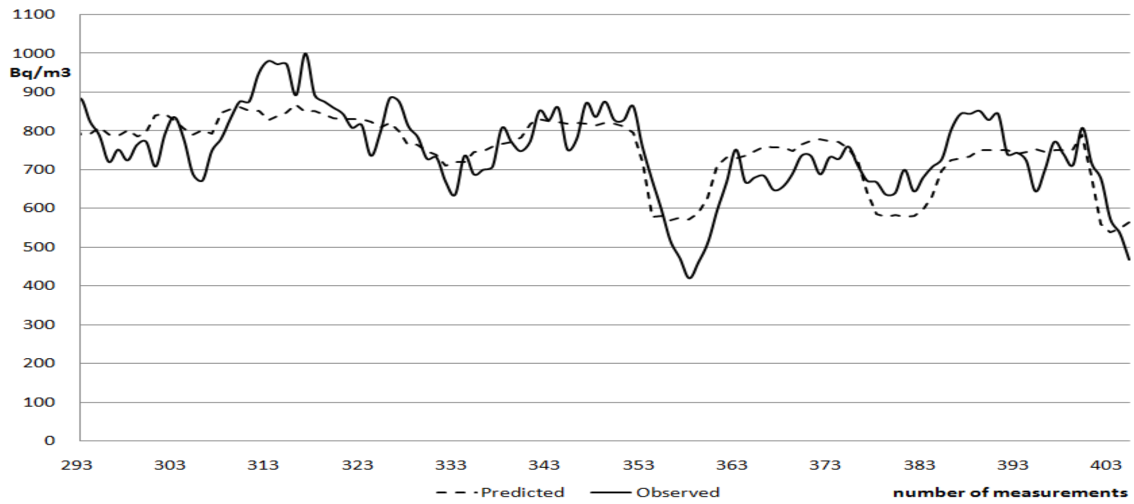
The Best Alternative Model of the “Uninhabited Part” of Time Series.

Parameter	Estimate	Error	P-Value
CONSTANT	-1036.0	459.595	0.0262
DewingPIn_oC	60.416	20.9351	0.0047
DewingPOut_oC	-36.7471	19.5222	0.0625
HumIn_procAG	15.7088	5.60164	0.0060
HumOut_proc	10.0782	3.62535	0.0064
Tdiff_oC	-30.3273	18.3721	0.1017

The R-Squared statistic (Table 2.) indicates that the model as fitted explains 61.27% of the variability of the indoor radon concentration. The adjusted R-squared statistic, which is more suitable for comparing models with different numbers of independent variables, is 59.48%.
The expression of the fitted model for the “uninhabited part” is:

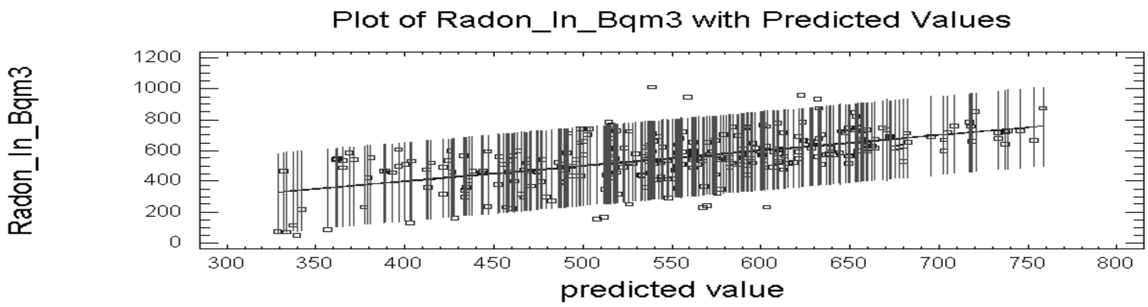
$$Radon\_In\_Bqm3 = -1036.0 + 60.416*DewingPIn\_oC - 36.7471*DewingPOut\_oC+ 15.7088*HumIn\_procAG + 10.0782*HumOut\_proc - 30.3273*Tdiff\_oC \quad (3)$$

Figure 5. shows how the fitted model follows the shape of the observed values of indoor radon concentration.

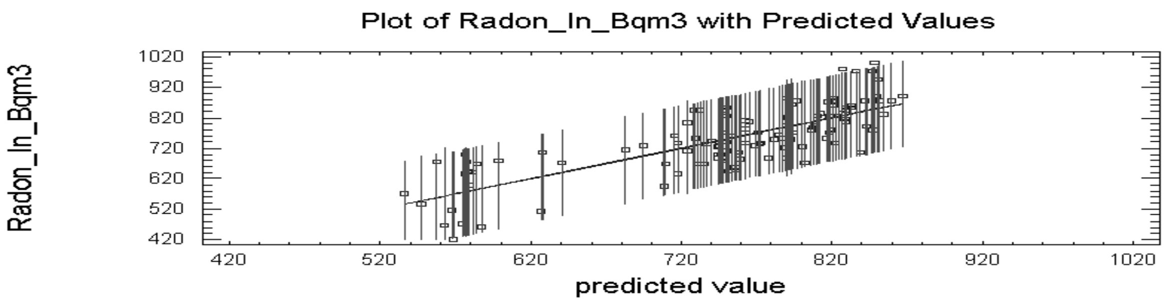


The Comparison of the Predicted and Measured Values of "Uninhabited Part" of Time Series

Figure 6. and Figure 7. show limits for predicted values at 95 % confidence level. Confidence intervals show how precisely the model estimates the observed values given the amount of available measured data and the measured noise. Figures also show statistically unusual observations (outliers). It can be caused by the errors of measuring equipments, the statistical fluctuation of the radioactive decay, stochastic characteristics of the building structure, the duration of measurements and by limitations of the used multiple linear regression models.
In the case of „inhabited part" of time series (see Figure 6.) we can observe more unusual observations. It approves the presence of human and his hardly predictable activities (heating, ventilation, …), which significantly influence the indoor radon concentration.



Limits for predicted values at 95 % confidence level – "inhabited part" of time series



Limits for predicted values at 95 % confidence level – "uninhabited part" of time series

Conclusion

In this paper, we measured and statistically analysed a relationship between the indoor radon concentration and various meteorological parameters. Because of human activities, which significantly influence the indoor radon concentration, we separated the dataset into 2 parts: "inhabited" and "uninhabited". The statistical analyses of these datasets were realized by the multiple linear regression techniques. The "uninhabited" part of the measured data can be quite reliably predicted by the multiple linear models (R-Squared ~ 61%). On the other hand, the R-Squared of the "inhabited" part is

only 34.7 % and the model is characterized by the mean squared error, which is approximately 16691.1/ 4937.2 ~ 3.40 times larger than the mean squared error of the model belonging to the "uninhabited" part.
Surprisingly, the adjusted models of the indoor radon concentration in the selected room were in both cases determined i.a. by the humidity and in the "uninhabited" case by the dew point. The known influence of temperature differences was not statistically proved for the "inhabited" case. It could be caused by human activities, especially by the frequent ventilation (also during nights) or heating by the stove: The stove could exhaust the indoor air by radon.
The large value of MSE of the "inhabited" part suggests unpredictable human activities. The human activity parameter was included indirectly by the form of timing (hours of a day). The description of uncertainties of the human activities, which significantly influence the radon concentration (awake, sleeping, …), will be the subject of the future research.
Analyses of radon concentrations are now connected to one selected room of the partly basement family house. The datasets of monitoring of the rest of the building will be available and the analogous analyses for remaining rooms are also planned.
In the future work, it would also be interesting to deeply analyse relationships among meteorological parameters, in order to improve and/or to simplify the developed statistical models of radon concentration in the room. We are just testing nonlinear models using the software Eureqa [12].

Acknowledgments

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References

[1] Act No. 18/1997 On Peaceful Utilisation of Nuclear Energy and Ionising Radiation (the Atomic Act) and on Amendments and Alterations to Some Acts. The Parliament of the Czech Republic.
[2] Bossew, P. & Lettner, H. 2007. Investigations on indoor radon in Austria, Part 1: Seasonality of indoor radon concentration. *Journal of Environmental Radioactivity* 98: 329 – 345.
[3] Cothern, C. R. & Smith, Jr. J.E. (ed) 1988. *Environmental Radon*. New York: Springer.
[4] Davis-Weather: Vantage Pro2™ Weather Stations [online]. Davis Instruments, Hayward, California. Available from: www.davisnet.com/weather/products/vantage2.asp [Accessed 9th June 2009]
[5] Genitron Instruments - Products: Radon Monitoring with AlphaGUARD [online]. Saphymo GmbH, Frankfurt a.M, Germany. Available from: http://www.genitron.de/download/ag_fb_gb_01_a4.pdf [Accessed 8th June 2009]
[6] Karpinska, M., Mnich, Z., Kapała, J., Antonowicz, K. & Przestalski, M. 2005. Time changebility in radon concentration in one-family dwelling houses in the northeastern region of Poland. *Radiation Protection Dosimetry* 113: 300 – 307.
[7] Montgomery, D. C. & Runger, G.C. 2003. *Applied Statistics and Probability for Engineers*. New York: John Wiley & Sons.
[8] Mueller Associates, Inc., SYSCON Corporation, Brookhaven National Laboratory 1988. *Handbook of Radon In Buildings: Detection, Safety, and Control*. New York: Hemisphere Publishing Corporation.
[9] Nazaroff, W.W., Feustel, H., Nero, A.V., Revzan, K.L., Grimsrud, D.T., Essling, M.T. & Toohey R-E 1984. *Radon transport into a single-family house with a basement* [online]. Paper LBL-16572, Lawrence Berkeley National Laboratory, University of Carolina. Available from: http://repositories.cdlib.org/cgi/viewcontent.cgi?article=3915&context=lbnl [Accessed 23rd January 2009].
[10] Nazaroff, W.W. & Nero, A.V. 1988. *Radon and its decay products in indoor air*. New York: John Wiley & Sons Inc.
[11] Nero, A.V., Gadgil, A.J., Nazaroff, W.W. & Revzan, K.L. 1990. *Indoor Radon and Decay Products: Concentrations, Causes, and Control Strategies* [online]. Technical Report, Paper LBL—27798, Lawrence Berkeley National Laboratory, University of Carolina, Washington, D.C. Available from: http://www.osti.gov/energycitations/product.biblio.jsp?osti_id=6318312 [Accessed 1st March 2009].
Schmidt, M. & Lipson, H. 2009. Distilling Free-Form Natural Laws from Experimental Data. *Science* 324, no. 5923: 81 – 85.

CALCULATION OF CONNECTIVITY PROBABILITY IN RECURSIVELY DEFINED RANDOM NETWORKS

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ABSTRACT

In this paper a problem of a construction of new and practically interesting classes of recursively defined networks, including internet type networks, with sufficiently fast algorithms of calculation of connectivity probability is considered. For this aim recursive and asymptotic formulas of connectivity probability calculation are constructed. Asymptotic formulas are based on assumptions that all network arcs are low reliable or there are high reliable and low reliable arcs in considered network. For example in radial-circle scheme radial arcs may be high reliable and circle arcs – low reliable.

1 INTRODUCTION

Recursively defined networks and structures are investigated in reliability theory for a long time. For an example it is possible to consider parallel-sequential connections [1,2] defined as follows. The class K_1 of parallel-sequential connections contains independently working elements A. If the ports $A,B \in K_1$ and their arcs sets do not intersect then the parallel connection $(A||B) \in K_1$ and the sequential connection $(A \to B) \in K_1$.

Another example is represented in [3] where in frames of logic-probability approach [4] a class of logic forms with independent Boolean random variables is constructed. This class may be considered as recursively defined class K_2 which contains all independent Boolean random variables and if logic forms $A,B \in K_2$, and their sets of variables do not intersect then the conjunction $(A \wedge B) \in K_2$, the disjunction $(A \vee B) \in K_2$ and the negation $\left(\overline{A}\right) \in K_2$. Here we formally put $\left(\overline{\left(\overline{A}\right)}\right) = A$. This class of logic forms has found manifold economical applications in risk and efficiency control.

For recursively defined networks which consist of equally reliable arcs and which describe different physical systems there are methods of a reliability calculation based on a definition of a root of a high power polynomial. These methods have polynomial complexity [5]. Analogous problem is considered [6] for recursively defined networks of "consecutive-k-out-of-n" type.

It is easy to establish that a calculation of the port $A \in K_1$ reliability (a probability that there is a working way between initial and final nodes of A) and a calculation of a reliability of a logic system characterized by a logic form $A \in K_2$ (a probability of an event (A=1)) it is necessary a number of arithmetic operations equal to a number of brackets in the form A representation.

So it is naturally to put a question, is it possible to construct new and practically interesting classes of recursively defined networks, including internet type networks [7], with sufficiently fast algorithms of calculation of connectivity probability.

In this paper this question receives a positive respond. Recursive and asymptotic formulas of connectivity probability calculation are constructed. Asymptotic formulas are based on assumptions that all arcs are low reliable or there are high reliable and low reliable arcs in considered network. For example in radial-circle scheme radial arcs may be high reliable and circle arcs – low reliable.

It is worthy to remark that there are results [8] - [10] mathematically interesting and rich in content to construct upper bounds of connectivity probabilities in networks of general type. These bounds are based on a construction of a maximal system of network frameworks. But in special recursively defined networks it is possible to obtain fast algorithms of reliability calculation without construction of network frameworks. Such approach allows accelerating reliability calculations significantly.

2 RECURSION CREATED BY GLUEING OF NETWORKS IN SINGLE NODE

Suppose that $\Gamma_1',...,\Gamma_l'$ are networks with no intersected sets of arcs and B′ is a family of networks consisting of sequences of independent copies of $\Gamma_1',...,\Gamma_l'$. Consider a recursively defined class B (of networks) with a family of generators $B' \subset B$ and assume that $r(\Gamma)=0$, $\Gamma\in B'$.

Suppose that networks $\Gamma\in B$, $\Gamma'\in B'$ have finite sets of nodes U, U′ and no intersected sets of arcs W, W′. Then the network $\Gamma \overset{u}{\otimes} \Gamma'$ constructed by a glueing of networks Γ, Γ′ in a single node also belongs to class B and $r\left(\Gamma \overset{u}{\otimes} \Gamma'\right) = r(\Gamma)+1$. Here $r(\Gamma)$ may be considered as a number of glueings generating the network Γ and $r(\Gamma)$ does not exceed a number of arcs in Γ.

Example 1. The network Γ is radial-circle with n nodes 1,…,n on a circle with a center 0 if it has the nodes set U={0,1,…,n} and the arcs set W={(01),…,(0n), (12),(23),…,(n-1)n,(n1)}. Suppose that the networks $\Gamma_1',...,\Gamma_l'$ are radial-circle. Construct from them the family of generators B′ and introduce the recursively defined class of networks B obtained by a glueing procedure $\Gamma \overset{u}{\otimes} \Gamma'$, $\Gamma\in B$, $\Gamma'\in B'$ with the reserve that their joint node is a center of the radial-circle network Γ′. The network $\Gamma\in B$ may be considered following [7] as a network of internet type.

We shall interest in the probability $\pi(\Gamma)$ of the network $\Gamma\in B$ connectivity. It is the probability that there is a family of working arcs in the network Γ which are connected with all its nodes and arcs of this family create a connected deterministic network.

Lemma 1. For $\Gamma\in B$, $\Gamma'\in B'$ the following recursive formula is true:

$$\pi\left(\Gamma \overset{u}{\otimes} \Gamma'\right) = \pi(\Gamma)\pi(\Gamma'). \tag{1}$$

Proof. Indeed if the network $\Gamma \overset{u}{\otimes} \Gamma'$ is connected then the networks Γ, Γ′ are connected also. Vice versa if the networks Γ, Γ′ are connected then the network $\Gamma \overset{u}{\otimes} \Gamma'$ is connected also. From the class B definition we have that the probability of the network $\Gamma \overset{u}{\otimes} \Gamma'$ connectivity equals the product $\pi(\Gamma)\ \pi(\Gamma')$.

Denote $N(\Gamma)$ a number of arithmetical operations necessary to calculate the connectivity probability $\pi(\Gamma)$, $\Gamma\in B$, $N' = \sum_{1\le j\le l} N\left(\Gamma_j'\right)$.

Theorem 1. For any $\Gamma\in B$ the inequality

$$N(\Gamma) \le N' + r(\Gamma) \tag{2}$$

is true.

Proof. It is obvious that for $\Gamma\in B'$ the inequality (2) takes place. Suppose that the formula (2) is true for the network $\Gamma\in B$, check this formula for the superposition $\Gamma \overset{u}{\otimes} \Gamma'$, $\Gamma'\in B'$. From the definition of N' and from the recursive formula (1) obtain that

$$N\left(\Gamma\overset{u}{\otimes}\Gamma'\right)\le N(\Gamma)+1\le N'+r(\Gamma)+1=N'+r\left(\Gamma\overset{u}{\otimes}\Gamma'\right).$$

Remark 1. Theorem 1 gives linear by a number of arcs complexity of a connectivity probability calculation for networks from recursively defined class B.

Remark 2. Analogously to a definition of a connectivity probability $\pi(\Gamma)$ it is possible to introduce a probability $\psi(\Gamma)$ that there is closed working way through all nodes of Γ. Repeating the inequality (2) proof it is possible to obtain the inequality

$$K(\Gamma)\le K'+r(\Gamma), \tag{3}$$

where $K(\Gamma)$ is a number of arithmetical operations necessary to calculate $\psi(\Gamma)$, $\Gamma\in B$, $K'=\sum_{1\le j\le l}K\left(\Gamma'_j\right)$. A formulation of $\psi(\Gamma)$ calculation problem is similar to the travelling salesman problem.

For the network $\Gamma\in B$ analogously to Floid and Steinberg problem [11] find a family of probabilities $P_\Gamma=\{P_\Gamma(u_0,v_0) : u_0\ne v_0, u_0,v_0\in U\}$ which consists of $l(\Gamma)(l(\Gamma)-1)/2$ elements. Here $l(\Gamma)$ is a number of nodes in the network Γ. For a definition of the family P_Γ it is convenient to use the recursive formulas

$$P_{\Gamma\overset{u}{\otimes}\Gamma'}(u_0,v_0)=\begin{cases}P_\Gamma(u_0,v_0),\ u_0,v_0\in U,\\ P_{\Gamma'}(u_0,v_0),\ u_0,v_0\in U',\\ P_\Gamma(u_0,v_0)P_{\Gamma'}(u_0,v_0),\ u_0\in U,\ v_0\in U'.\end{cases} \tag{4}$$

Denote $N(\Gamma)$ a number of arithmetical operations necessary to calculate P_Γ, $\Gamma\in B$, $N=\sum_{1\le j\le l}N\left(\Gamma'_j\right)$.

Theorem 2. For any $\Gamma\in B$ the following inequality is true:

$$N(\Gamma)\le\frac{l(\Gamma)(l(\Gamma)-1)}{2}+N. \tag{5}$$

From the formulas (5) we have that

$$\lim_{l(\Gamma)\to\infty}\frac{2N(\Gamma)}{l(\Gamma)(l(\Gamma)-1)}=1. \tag{6}$$

So in asymptotic $l(\Gamma)\to\infty$ a calculation of $P_\Gamma(u_0,v_0)$ a for single pair of nodes $u_0\ne v_0$ it is necessary no more than unit arithmetical operation·

Proof. It is obvious that for $\Gamma\in B'$ the inequality (5) is true. Suppose that this inequality takes place for the network $\Gamma\in B$, prove this inequality for the superposition $\Gamma\overset{u}{\otimes}\Gamma'$, where $\Gamma'\in B'$, for a simplicity denote $l=l(\Gamma)$, $l'=l(\Gamma')$, $L=l(\Gamma\overset{u}{\otimes}\Gamma')=l+l'-1$.

Then from the formula (4) obtain

$$N\left(\Gamma\overset{u}{\otimes}\Gamma'\right)\le N(\Gamma)+(l-1)(l'-1)\le N'+\frac{l(l-1)}{2}+(l-1)(l'-1)\le N'+\frac{L(L-1)}{2}.$$

3 ASYMPTOTIC FORMULAS

Consider Example 1 in which the set of generators consists of radial-circle schemes Γ satisfying the following asymptotic condition. Suppose that each arc $w\in W$ independently on other arcs work with the probability p_w, $0<p_w<1$, $w\in W$ and for $h\to 0$ reliabilities of circle arcs

$$\begin{aligned}&p_{u_1,u_2}=p_{u_1,u_2}(h)\to 0,\ \ p_{u_2,u_3}=p_{u_2,u_3}(h)\to 0,...,\\ &p_{u_{n-1},u_n}=p_{u_{n-1},u_n}(h)\to 0,\ \ p_{u_n,u_1}=p_{u_n,u_1}(h)\to 0.\end{aligned} \tag{7}$$

The reliabilities of radial arcs $(u*,u_1),\ldots, (u*,u_n)$ are positive and do not depend on the parameter h

$$p_{u*,u_1} = const > 0,\ldots, p_{u*,u_n} = const > 0. \qquad (8)$$

Remark that the conditions (7), (8) are taken from manifold observations of real systems in which reliabilities of radial arcs are significantly larger than reliabilities of circle arcs.

Theorem 3. If the conditions (7), (8) are true than

$$\pi(\Gamma) \to \prod_{i=1}^{n} p_{u*,u_i},\ h \to 0. \qquad (9)$$

Proof. Denote the event A that the network Γ is connected. Then there is a family of working arcs $W* \subseteq W$ so that a set of their edges coincides with U and for any pair of nodes from U there is a way R with arcs from $W*$. Define the event $B \subseteq A$ that all radial arcs $(u*,u_1),\ldots, (u*,u_n)$ work. Introduce the event C that there is failing radial arc and the event D that there is working circle arc. It is obvious that

$$A \setminus B \subseteq C \cap D$$

and consequently

$$P(A) - P(B) \le P(C)P(D). \qquad (10)$$

From the events A,B,C,D definition obtain that for $h \to 0$ we have

$$P(A) = \pi(\Gamma),\ P(B) = \prod_{i=1}^{n} p_{u*,u_i}\ \ P(C \cap D) = P(C)P(D) \le P(D) \le \sum_{i=1}^{n} p_{u*,u_i} \to 0. \qquad (11)$$

Put the formulas (11) into the inequality (10) and obtain the asymptotic formula (9).

Remark 3. Repeating Theorem 3 proof it is easy to spread its statement onto a network consisting of center and circle nodes in which center is connected with all circle nodes and radial arcs reliabilities satisfy the condition (8). Circle nodes may be connected with each other by arbitrary (not only circle) arcs which satisfy the condition (7).

Remark 4. Suppose that the network Γ with the nodes set U and with the arcs set W satisfies the following condition. There is the subset W_1 in the set W which creates connected sub network Γ_1 of the network Γ with the nodes set $U_1=U$. And a deleting of any arc from the network Γ_1 makes it disconnected or the equality $U_1=U$ becomes not true. Then if the reliabilities p_w, $w \in W$, satisfy the conditions

$$p_w = const > 0, w \in W_1,\ p_w = p_w(h) \to 0,\ h \to 0,\ w \in W \setminus W_1,$$

then we have

$$\pi(\Gamma) \to \prod_{w \in W_1} p_w,\ h \to 0.$$

Consider the port Γ with fixed initial and final nodes u^*,v^*, finite set of nodes U and finite set of arcs W. Suppose that the set W consists of no intersected subsets W_1, W_2 and for any $w \in W_1$, $p_w(h) \equiv p_w > 0$ and for any $w \in W_2$ $p_w(h) \to 0$, $h \to 0$. Denote $\Re = \{R_1,\ldots R_n\}$ a family of all acyclic ways between the nodes u^*, v^*.

Theorem 4. If the following formulas are true

$$R_1 \subseteq W_1,\ R_2 \cap W_2 \ne Щ,\ldots, R_n \cap W_n \ne Щ \qquad (12)$$

then the probability that the nodes u^*,v^* are connected in the port Γ satisfies the formula

$$P_\Gamma(u^*,v^*) \to \prod_{w \in R_1} p_w,\ h \to 0. \qquad (13)$$

Proof. Denote U_R the event that all arcs from the way R work then

$$P_\Gamma(u^*,v^*) \quad P \quad \underset{1\ i\ m}{} U_{R_i} \quad ,$$

$$\sum_{i=1}^{m} P(U_{R_i}) - \sum_{1\le i<k\le m} P(U_{R_i}U_{R_k}) \le P_\Gamma(u^*,v^*) \le \sum_{i=1}^{m} P(U_{R_i}), \quad (14)$$

From the sets W_1, W_2 definition for $h\to 0$

$$P(U_{R_1}) \equiv \prod_{w\in R_1} p_w,\ P(U_{R_k}) = 0(1),\ 1<k\le m,$$
$$P(U_{R_i}U_{R_k}) = 0(1),\ 1\le i\ne k\le m,\ h\to 0. \quad (15)$$

The formulas (14), (15) lead to (13).

Remark 5. If the following conditions are true

$$R_1 \subseteq W_1, 1\le i\le p<n,\ R_j \cap W_2 \ne Щ,\ p<j\le n, \quad (16)$$

then the connection probability of the nodes u^*,v^* in the port Γ satisfies the formula

$$P_\Gamma(u^*,v^*) \to P\left(\bigcup_{1\le i\le p} U_{R_i}\right),\ h\to 0. \quad (17)$$

Remark 6. Consider the radial-circle scheme described in Example 1 and satisfying the conditions (7), (8). Here the set W_1 consists of radial arcs and the set W_2 – from circle arcs. If the node u^* coincides with the center and v^* coincides the circle node u_i, $i\in\{1,\dots,n\}$ then the way R_1 consists of the arc (u_*,u_i). If the nodes $u_*=u_i$, $v_*=u_k$, $1\le i\ne k\le m$ are circle then the way R_1 consists of the arcs (u_*,u_i), (u_*,u_k). Consequently in the conditions *(7), (8)* we obtain the formulas

$$P_\Gamma(u_*,u_i) \to p_{u_*,u_i},\ P_\Gamma(u_i,u_k) \to p_{u_*,u_i}p_{u_*,u_k},\ h\to 0,\ 1\le i\ne k\le n. \quad (18)$$

4 NUMERICAL EXPERIMENT

Consider the radial-circle scheme with 6 circle nodes and with arcs reliabilities

p(01)=0.993515, p(02)=0.99727, p(03)=0.995938, p(04)=0.980191,
p(05)=0.98099, p(06)=0.990262, p(12)=0.0132823, p(23)=0.00489211,
p(34)=0.010295, p(45)=0.00573119, p(56)=0.0180407, p(61)=0.0034061.

Denote by π*, π** estimates of connectivity probability obtained by the asymptotic formula (9) and by the Monte-Carlo method with 100000 random realizations of the radial-circle scheme. Numerical experiment shows that

$$\pi^* = 0.955411,\ \pi^{**} = 0.95552,\ \left|\frac{\pi^{**}}{\pi^*}-1\right| = 1.14*10^{-4},\ \left|\frac{1-\pi^{**}}{1-\pi^*}-1\right| = 0.0025,$$

and calculation time for π^* is more than 10000 times smaller than calculation time for π^{**}.

Suppose now that

p(01)=0.91, p(02)=0.92, p(03)=0.93, p(04)=0.94, p(05)=0.95, p(06)=0.96,
p(12)=0.0132823, p(23)=0.00489211, p(34)=0.010295,
p(45)=0.00573119, p(56)=0.0180407, p(61)=0.0034061.

Then numerical experiment gives us the estimates

$$\pi^* = 0.667475,\ \pi^{**} = 0.67149,\ \left|\frac{\pi^{**}}{\pi^*}-1\right| = 0.00597957,$$

with the same difference of calculation times for the estimates π*, π**.

Denote $\overline{P}^* = \left\|\overline{P}^*(u_i,u_j)\right\|_{i,j=0}^6$ and put $\overline{P}^{**} = \left\|\overline{P}^{**}(u_i,u_j)\right\|_{i,j=0}^6$ the matrices of the connection probabilities estimates using the formula (18) and using the Monte-Carlo method with 100000 random realizations. Numerical experiment shows that calculation time of the matrix $\overline{P}^*$ is approximately 20000 times smaller than the calculation time of the matrix $\overline{P}^{**}$. Denote $A = \left\|A(u_i,u_j)\right\|_{i,j=0}^6$, where

$$A(u_i,u_j) = \left|\overline{P}^{**}(u_i,u_j)/\overline{P}^*(u_i,u_j) - 1\right|.$$

The matrixes $\overline{P}^{*}, \overline{P}^{**}, A$ are represented only by over diagonal elements because they are symmetric and have zero diagonal elements. Calculation results are following:

P*=

–	0.99351	0.99727	0.995938	0.980191	0.98099	0.990262
–	–	0.99080	0.989479	0.973834	0.974628	0.98384
–	–	–	0.993219	0.977515	0.978312	0.987559
–	–	–	–	0.976209	0.977005	0.98624
–	–	–	–	–	0.961558	0.970646
–	–	–	–	–	–	0.971437
–	–	–	–	–	–	–

,

P**=

–	0.99403	0.99740	0.99602	0.9801	0.98201	0.99026
–	–	0.99147	0.99014	0.9742	0.97622	0.98435
–	–	–	0.99348	0.9776	0.9795	0.9877
–	–	–	–	0.9762	0.97822	0.98634
–	–	–	–	–	0.96278	0.9706
–	–	–	–	–	–	0.97254
–	–	–	–	–	–	–

,

A=

–	0.00051836	0.00013036	0.000082334	0.00008264	0.0010398	0.00000202
–	–	0.00067349	0.000667682	0.00039589	0.0016332	0.00051822
–	–	–	0.000262692	00.00015849	0.0012144	0.00014320
–	–	–	–	0.00003128	0.0012434	0.00010185
–	–	–	–	–	0.0012713	0.00004729
–	–	–	–	–	–	0.00113531
–	–	–	–	–	–	–

References

1. Barlow R. E., Proschan F. Mathematical Theory of Reliability. *London and New York: Wiley*, 1965.

2. Ushakov, I.A. et al. *Reliability of technical systems: Handbook.* 1985. Moscow: Radio and Communication. (In Russian).

3. Solojentsev E.D. Specific of logic-probability risk theory with groups of incompatible events. Automatics and remote control, 2003, № 7, pp. 187-203. (In Russian)

4. Riabinin I.A. Reliability and safety of systems with complex structure. Spb.: edition of Sankt-Petersberg University, 2007. (In Russian).

5. Tanguy C. Exact solutions for the two-terminal Reliability of recursive structures: a few directions. *MMR 2009 - Mathematical methods in reliability*. Moscow, 2009, pp. 220-224.

6. Cui L., Zhao X. Recursive Equations of Reliability for Linear Consecutive-k-out-of-n: F Systems with Sparse d. *MMR 2009 - Mathematical methods in reliability*. Moscow, 2009, p. 45.

7. Ball M. O., Colbourn C. J., Provan J. S. Network Reliability. In Network Models. Handbook of Operations Research and Management Science. *Elsevier. Amsterdam*, 1995, vol. 7, pp. 673-762.

8. Polesskiy V.P. Estimates of connectivity probability of random graph. Problems of information transmission, 1990, vol. 26, № 1, pp. 90-98. (In Russian).

9. Polesskiy V.P. Low estimates of connectivity probability for some classes of random graphs. Problems of information transmission, 1993, vol. 29, № 2, pp. 85-95. (In Russian).

10. Polesskiy V.P. Low estimates of connectivity probability in random graphs generated by doubly-connected graphs with fixed base spectrum. Problems of information transmission, 1992, vol. 28, № 2, pp. 86-95. (In Russian).

11. Floid R.W., Steinberg L. An adaptive algorithm for spatial greyscale. SID 75 Digest, 1975, pp. 36-37.

RELIABILITY ASSESSMENT OF SYSTEMS WITH PERIODIC MAINTENANCE UNDER RARE FAILURES OF ITS ELEMENTS

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Abstract

There is investigated a model of a system with the highly reliable elements, where the periods of functioning are changed by the periods of maintenance. The system must be operational only in the periods of functioning although the restoration in these periods is not provided. The system is completely restored in the nearest period of maintenance. Since the elements of system are highly reliable, the reserve of system is rarely exhausted during each period of functioning. Therefore it is possible to use the results, obtained for the systems with fast restoration, for the reliability assessment of system, which is not restorable in the periods of the functioning. The estimations of indices of failure-free performance and maintainability of such systems are obtained.

1. Introduction and motivation

There is examined a system with periodic maintenance. The periods of functioning of the system, are changed by the periods, when the system is turned off and there is produced its maintenance and the restoration of all failed elements. The elements of the system fail only in the period of functioning, and their restoration is produced in the period of maintenance. User is interested in the high reliability of the system in the periods of functioning.

Examples of such systems are aircrafts, reusable spacecrafts, etc. So aircrafts in a flight are carrying out their flight mission. After landing their complete maintenance is carried out. After this, the aircrafts are ready to the fulfillment of a new flight mission. Another example is a nuclear power plant where users as a rule have no access or a limited access to its reactor compartment between periods of planned maintenance. At the time of the maintenance nuclear reactor can be adjusted and renewed to continue its mission.

For these systems the periods of functioning, where the system works and fulfills the assigned functions, are changed by the periods of maintenance, when the system is turned off and the maintenance is produced. The restoration of all failed elements as well as the elimination of all faultinesses, accumulated during the functioning period, is produced only in the second period and it manages to end up to the next functioning period. In the second period the reliability of operational elements is not changed. After gluing together the functioning periods of the system, we will obtain the model of the system with the periodic instantaneous maintenance.

In the work there is given assessment of the indices of failure-free performance and maintainability under the condition that elements of system fail rarely in the functioning period.

2. Model description

The duration of the periods of the time from initial moment of the functioning of the system till the nearest maintenance period or between the end of the period of maintenance and the beginning of the following period of maintenance is a random variable ξ with distribution function (DF) $P\{\xi < x\} = \Phi(x)$ and mean value $M\xi = T$.

The elements of the system can fail only in the functioning periods of the system, and their restoration is produced only in the nearest interval of maintenance. The failure of the system occurs when between the first failure of some element of the system and the nearest interval of maintenance there is completely exhausted the entire reserve of the system, including a time reserve, if it exists. The failures of elements and the failure of the system are instantly detected.

If the reserve of the system is spent not completely in the period of functioning, then the failure of the system is not observed, the reserve of the system is restored in the nearest interval of maintenance and the system is derived for the new period of functioning.

If the failures of elements do not occur in a certain period of the functioning, then the reliability of the system in the nearest period of maintenance is not changed.

The intervals of functioning can have different duration in sense of their mean value. Since in the intervals of functioning the system is not restored, the failure of the system is more probable in the more prolonged interval of functioning. After taking the worst (most prolonged in average) interval of functioning we can obtain the estimation of the worst reliability of the system. Therefore without limiting the generality we will consider that all intervals of functioning are distributed equally and correspond to the most prolonged by its mean value interval of functioning.

We will examine the highly reliable systems, in which the failures of elements in each interval of functioning occur rarely. This in particular means that the product of the maximum of summary failure rate of the system elements by the maximum duration of the functioning interval is considerably less than one. Actually this case corresponds to practice.

We are interested to study the behavior of the system only in the summary interval of functioning, obtained as a result of ignoring all intervals of maintenance. This summary interval can be considered as a new time axis with the point flow *T*. Each point of the flow corresponds to the interval of the maintenance, where the system is restored instantly and completely.

On this time axis it is possible to indicate the moments of the failures of the elements of the system. After the first failure of an element in any functioning interval an interval of malfunction (*IM*) is begun. *IM* ends at the moment of the beginning of the nearest interval of maintenance. Let us name an *IM* as a failure *IM*, if the failure of system occurs in it. Let us name a point of flow *T* as a failure point if it is contiguous with a failure *IM*. After such a failure point the system is fully restored (!).

We have a point flow *T* with rarefaction where some points may be failure points with probability converged to zero. But this means that there is possible to use an asymptotic approach for reliability assessment of the system.

Since on the functioning intervals the restorations of elements are not produced the failure of the system in *IM* is developed only along the monotonic path. Monotonic path is such a trajectory of the failure of the system on which from the moment of the failure of the first element in *IM* (the beginning of *IM*) and to the moment of the failure of the system in this *IM* none restorations were finished.

Let us estimate the behavior of the system to the first failure of the system and between two adjacent failures of the system, when the elements of the system are highly reliable. This means that the probability of the failure of system in any period of functioning is small and more than zero, which corresponds to the conditions for fast restoration, introduced and studied in [1] and [2].

Thus the estimation of the reliability of system with the periodic maintenance, where there are no restorations on the intervals of functioning, can be and will be actually produced with using an apparatus, developed for the systems with the fast restoration. It sounds paradoxically, but this is actually so because the elements of the system are highly reliable, the periods of functioning have relatively small duration and the point interval of maintenance follows them.

3. Mathematical formulation of problem

Present article rest upon the results of the works of Genis [1] and [2]. In the work [1] there is given the general approach to the estimation of the reliability of systems with fast restoration. In the work [2] there is given the more detailed determination of the criterion of fast restoration.

For convenience of the reader let us give some details of the systems, already described in [1] and [2], which are used in this work. System consists of *n* elements. Each element of the system can be only in the operational or inoperative state. Each operational element can be located in the loaded or unloaded regime (lightened regime for the brevity is omitted). Let $F_i(x)$, $f_i(x)$, и m_i are accordingly the distribution function, the density of distribution (*DD*), and the mean value of the time of failure-free operation of the *i*-th element in the system, $m_i < \infty$, $i \in \overline{1,n}$.

Within the framework of this work we will examine the systems of the 1-st and the 3-d types, described in [1]. Systems of the 1-st type are systems with the exponentially distributed times of failure-free operation of elements. Systems of the 3-d type are the systems, whose elements have a limited *DD* of the failure-free operation time; there is required also, that these *DDs* in zero are not equal to zero, $f_i(0) = c_i \neq 0$, $i \in \overline{1,n}$.

There are no limitations to the structure of the system. There is assigned the criterion of the system's failure, which can include and the condition of time redundancy. The state of the elements of system at the moment z is described by the vector $\vec{v}(z) = \{v_1(z),...,v_n(z)\}$, where each component can take the values of $\{0, 1, \dots, n\}$. Number 0 corresponds to failed elements; numbers from 1 to n correspond to operational elements. These numbers make it possible to unambiguously assign the order of the replacement. Vector $\vec{v}(z)$ helps to estimate the reliability of a concrete system.

Let E is the set of the states of the system, $\{\vec{v}(z)\} = E = E_+ \cup E_-$, where E_+ is the area of the fully operational, and E_- is the area of the before failure states of the system. The system is considered as before failed at the moment z if $\vec{v}(z) \in E_-$ and failed if its malfunction lasts time not smaller then $\eta, P\{\eta < x\} = H(x)$. In the absence of the time reserve ($\eta \equiv 0$) the area of the before failed states of system is converted into the area of the failures of the system. All elements of the system were new and functioned properly at the initial moment of time.

Let $\vec{b}$ is a certain state vector of elements of the system directly before *IM*, and $\vec{b}^N$ is the state vector of the elements of the system on the same *IM* immediately after the moment of passing the state vector of system from the area E_+ into the area E_-. Let us name the way π leading from $\vec{b} \in E_+$ into the state $\vec{b}^N \in E_-$ on the *IM* the sequence of the state vectors of elements, beginning from the vector $\vec{b}$, which directly precede the beginning of the *IM*, and ending with the vector $\vec{b}^N$, which corresponds to the first before failed state of the system on this *IM*.

The path length is equal to the number of elements, which failed on this way. Let us name way monotonic, if on it there are no restorations of elements. Let us name monotonic way minimal for $\vec{b}$ if its length $l(\vec{b})$ equals to the minimum of the path lengths, which lead from $\vec{b}$ to E_-. Then the minimum number of elements, whose failure can cause the system's failure, equals $s = \min l(\vec{b})$ on $\vec{b} \in E_+$.

The value $s \geq 2$ corresponds to a fault-tolerant system with a structural reserve. If $s = 1$ a failure of some element may lead to a system failure unless there is not introduced a time reserve that will not be exhausted till the nearest moment of a maintenance period.

It is considered that the system works in the conditions of the fast restoration. Practically this means that the mean time of the interval of functioning is substantially less than the mean time between any two failures of elements in the system (see section 4).

Let in the steady-state operation section of work with the restoration discipline d_1 with the straight order of maintenance and *one* repair unit, $\beta(d_1,1)$ is the estimation of the system's failure rate taking into account only monotonic paths of the failure (but in the system with periodic maintenance there are no other paths), $\tau''(d_1,1)$ is the random system's restoration time after failure, $T_R(d_1,1)$ is the mean value of this time, $K_A(d_1,1)$ is the availability function of the system. In all of these indices the first parameter is the type of restoration discipline and the second one is the number of restoration units in the system.

The problem consists in estimating of the indices of failure-free performance and maintainability of the described system under the conditions of the fast restoration.

A system with the periodic maintenance fails, if the reserve of the system is completely exhausted during the period between two maintenances. In this case the failure of the system occurs only along the monotonic paths.

Let DF $\Phi(x)$ is absolutely continuous. In the steady-state operating conditions of the system DF of time from the beginning of IM to the nearest moment of the system's maintenance is equal to

$$A(x) = \int_0^x \overline{\Phi}(u)du / T. \qquad (3.1)$$

For the failure of the system during an *IM* there must be spent the reserve of the system. Therefore the behavior of the described system is analogous to the behavior of the system of section 2 [1] with restoration discipline $d = d_1$ with the straight order of maintenance FIFO, one repair unit $k = 1$, and with identical for all elements distribution function of the restoration time $G_i(x) = G(x) = A(x)$,

$i=\overline{1,n}$, with the additional assumption that only monotonic paths of the system's failure are permitted. Indeed, for such repairable system the first failed element will not be restored during *IM* time, in the same time the reserve of system should be spent, and under the conditions of fast restoration the failure of system will be developed along the monotonic path.

In [1] and [2] for establishing the criteria of the system's behavior and formulating the results of the study there were used the indices of failure-free performance of the elements m_i and c_i, *DF* of restoration time *G(x)* (if it is identical for all elements) and "shifted" for the time η moments *DF G(x)*

$$m_R^{(k)}(\eta)=\int_0^\infty\int_0^\infty kx^{k-1}\overline{G}(x+u)dxdH(u),\ \ m_R^{(k)}=m_R^{(k)}(0)\,,\ m_R=m_R^{(1)}, \qquad (3.2)$$

where for any *DF* $\Gamma(x)$ $\overline{\Gamma}(x)=1-\Gamma(x)$.

In the present work let us replace $G(x)$ by $A(x)$ from (3.1) and let us introduce the designations

$$\tilde{m}_R^{(k)}(\eta)=\int_0^\infty\int_0^\infty kx^{k-1}\overline{\Phi}(x+u)dxdH(u),\ \ \tilde{m}_R^{(k)}=\tilde{m}_R^{(k)}(0). \qquad (3.3)$$

The following lemma makes it possible to pass from (3.2) to (3.3).

Lemma 3.1. For any numbers $k\ge 1$

$$m_R^{(k)}(\eta)=\frac{1}{(k+1)T}\tilde{m}_R^{(k+1)}(\eta)\,, \qquad (3.4)$$

if $\tilde{m}_R^{(k+1)}(\eta)$ exists.

The proof of Lemma 3.1 is given in the Appendix.

4. Asymptotic approximation. Criterion of the fast restoration

Let *s* is the minimum number of elements, whose failure can cause the malfunction of the system; $\overline{\lambda}$ and $\underline{\lambda}$ are the maximum and the minimum failure rates of elements in the operational system [1].

Let us say that the condition of the fast restoration is satisfied for the system if $\underline{\lambda}>0$,

$$\alpha=[\overline{\lambda}\ m_R^{(s)}\ /\ (m_R)^{s-1}]=[\overline{\lambda}(\tilde{m}_R^{(s+1)}/(s+1)T)/(\tilde{m}_R^{(2)}/2T)^{s-1}]\to 0, \qquad (4.1)$$

and for all *DF* $F_i(x)$, $i=\overline{1,n}$, the limited distribution densities must exist [1, 2].

In the practically important cases $m_R^{(s)}\le C\,(m_R)^s$, where *C* is a certain constant. In these cases the condition for fast restoration (4.1) is reduced to

$$\alpha = \underline{\lambda} m_R = \underline{\lambda} \tilde{m}_R^{(2)} / 2T \to 0 . \qquad (4.2)$$

For the fast restoration it is actually required that the probability of the system's failure on the period of functioning would be more than zero (it is enough to demand $\underline{\lambda} > 0$), and the probability of the system's failure on *IM* converges to zero ($\alpha \to 0$).

5. Estimation of the indices of failure-free performance

Let $\tau_i(t)$ is the interval from the moment *t* to the nearest moment of the system's failure after moment *t*. Details of the determination of time to the first failure of the system $\tau_1(t)$ and time between (*j*-1)-th and *j*–th system's failures $\tau_j(t), j \geq 2,$ are given in section 5 of [1].

Taking into account the theorem 5.3 of [1] we will obtain that for the dicussed system it is fullfilled

Corollary 5.1. In the steady-state operating conditions of the system with periodic maintenance and with structural reserve for $i \geq 1$ under the conditions of the fast restoration $\underline{\lambda} > 0$ and $\alpha \to 0$

$$P\{\tau_i(t) \geq x\} \approx \exp\{-\beta(d_1,1)x\}. \qquad (5.1)$$

Let us name the intervals between the end of one *IM* and the beginning of the next *IM* in the new time axis as intervals of operability (*IO*). Let from $\vec{v}(z) = \vec{b} \in E_+$, $z \in$ *IO*, there are possible minimal paths $l = l(\vec{b})$, leading into the certain state $\vec{b}^j \in E_-$. Every of state $\vec{b}^j$ is characterized by the set l of numbers of failed elements, belonging to the set $J = J_+ \cup J_-$, where J_+ and J_- are accordingly the sets of the numbers of those elements, which in the state $\vec{b}$ were located in the loaded and unloaded regime. Let the set of minimum paths leading from $\vec{b}$ into $\vec{b}^j$ is Π^j. Let

$$\Lambda^j = \prod_{k \in J_-} c_k \prod_{i \in J_+} 1/m_i . \qquad (5.2)$$

Let $R_j(\vec{b})$ is the number of minimal paths of the length $l = l(\vec{b}),$ leading from $\vec{b}$ into $\vec{b}^j \in E_-$; *s* is the minimum number of elements, whose failure can cause the before failure system state; $\lambda(\vec{b})$ and $q^{(\vec{b})}$ are respectively the summary failure rate of elements and the probability of the occurrence of a failure *IM* in the steady-state operating conditions of the system in condition that the system directly before *IM* was in the state $\vec{b}$.

Then there is valid

Theorem 5.1. Under the conditions of the fast restoration with $\underline{\lambda} > 0$ and $\alpha \to 0$ in the steady-state operating conditions of the system with the periodic maintenance and structural reserve

$$\lambda(\vec{b}) q^{(\vec{b})} \approx \frac{1}{T} \sum_{\vec{b}^j \in E_-} R_j(\vec{b}) \Lambda^j \tilde{m}_R^{(l)}(\eta) / l! \qquad (5.3)$$

The prove of Theorem 5.1 is given in the Appendix.

Observation 5.1. In theorem 5.1 there is possible to remove the requirement of absolute continuity of DF $\Phi(x)$. Using an apparatus for a study of rare event in the regenerating process [3], it is possible to obtain estimation (5.3) after requiring existence of finite first and second moments for the period ξ. This makes it possible to extend the theorem 5.1 to the case, when $\xi \equiv T$, that has a great practical value.

In the section 8 of [1] it is shown that under the conditions of the fast restoration the estimation of the indices of the reliability of a complex system can be brought to the estimations of the indices of the reliability of its series-connected in the sense of the reliability schemes p out of m, calculated under the assumption of their autonomous operation. The scheme p out of m has m elements. Its malfunction occurs with the failure of not less then p elements from m, $p \le m$, and its failure begins when the malfunction of scheme lasts not less than time $\eta, P\{\eta < x\} = H(x)$. Therefore within the framework of this article we will consider that the system is a scheme p out of m, which failure rate is β_p.

Using (5.3), there is possible to obtain the estimations $\beta_p = \beta_p(d_1,1)$ for the system p out of n with the periodic maintenance for various types of structural reserve.

In particular let us examine schemes n out of n, corresponding to the parallel in the sense of reliability connection of elements. The scheme becomes defective, if all n elements fail.

Corollary 5.2. For the system n out of n with the periodic maintenance and the loaded reserve

$$\beta_n \approx \tilde{m}_R^{(n)}(\eta)/T\prod_{i=1}^{n} m_i. \qquad (5.4)$$

System n out of n with the unloaded reserve consists of the 1 basic element used in the loaded regime, and (n - 1) reserve elements, which are located in the unloaded regime. With the failure of the basic element its place takes the element from the unloaded reserve, which stayed in the reserve the greatest time. The malfunction of system begins when the basic element fails and there is absent a reserve element that is capable to replace the failed one.

Corollary 5.3. For the system n out of n with the periodic maintenance and the unloaded reserve

$$\beta_n \approx \tilde{m}_R^{(n)}(\eta)\sum_{k=1}^{n}\prod_{i\neq k} c_i / Tn!\sum_{j=1}^{n} m_j. \qquad (5.5)$$

6. Indices of maintainability and availability function

There is examined the system, described above, with $\eta \equiv h = const$. In this case

$$\tilde{m}_R^{(k)}(y) = \int_0^\infty k z^{k-1} \overline{\Phi}(z+y) dz .$$

For the system n out of n with the periodic maintenance and the loaded reserve
(the parallel in the sense of reliability connection of the elements)

$$P\{\tau^{''} \geq x\} \approx \tilde{m}_R^{(n)}(h+x) / \tilde{m}_R^{(n)}(h), \quad (6.1)$$

$$T_R \approx \tilde{m}_R^{(n+1)}(h) / (n+1)\tilde{m}_R^{(n)}(h), \quad (6.2)$$

$$K_A \approx 1 - \tilde{m}_R^{(n+1)}(h) / (n+1) T \prod_{i=1}^{n} m_i . \quad (6.3)$$

For the system n out of n with the periodic maintenance and the unloaded reserve the estimations of $P\{\tau^{''} \geq x\}$ and T_R are the same, as for the system with the loaded reserve, but the estimation of availability function takes the form

$$K_A \approx 1 - \tilde{m}_R^{(n+1)}(h) \sum_{i=1}^{n} \prod_{j \neq i} c_j / (n+1)! T \sum_{k=1}^{n} m_k . \quad (6.4)$$

Summary

The received results allow the developing of a practical methodology of reliability assessment of systems described in the first section of this article. Together with technologist who knows the system very good it should be established the criteria of the system failure. After that the system's reliability assessment is provided. If results are not satisfactory there is introduced an additional reserve in the system and reliability assessment is repeated. The process is continued until satisfactory results are obtained.

Appendix

Proof of the Lemma 3.1. Taking into account (3.1) for any value $u \geq 0$

$$\overline{G}(x+u) = \frac{1}{T}\int_{x+u}^{\infty} \overline{\Phi}(y)dy = \frac{1}{T}\int_{x}^{\infty} \overline{\Phi}(y+u)dy \, .$$

In this case for any integer $k \geq 1$

$$\int_0^{\infty} kx^{k-1}\overline{G}(x+u)dx = \frac{1}{T}\int_0^{\infty} kx^{k-1}\int_x^{\infty} \overline{\Phi}(y+u)dydx \, .$$

After replacing the order of integration for 0< x< ∞ and x<y< ∞ to 0<y< ∞ and 0<x<y, and then replacing the variable of integration y to x we will obtain

$$\int_0^{\infty} kx^{k-1}\overline{G}(x+u)dx = \frac{1}{T}\int_0^{\infty} \overline{\Phi}(y+u)\int_0^{y} kx^{k-1}dxdy = \frac{1}{(k+1)T}\int_0^{\infty}(k+1)x^k\overline{\Phi}(x+u)dx \, . \quad \text{(A.1)}$$

After adding to (A.2) integration by value *u* with measure *H(u)* we will obtain the assertion (3.4) of Lemma 3.1.

Proof of the Theorem 5.1. We will use the Theorem 6.1 of [1] taking into account that the discipline of the restoration in the system is d_1, the number of repair units is *k* = 1, and for any element i_1 failed the first on *IM* the DF $G_{i_1}(x) = G(x) = A(x)$. Then we will obtain that from (6.4) of [1] follows

$$\lambda(\vec{b})q^{(\vec{b})} \approx \sum_{\vec{b}^j \in E_-} \Lambda^j \sum_{\pi \in \Pi^j} \int_0^{\infty}\int_0^{\infty} \frac{x^{l-2}}{(l-2)!}\overline{G}(x+u)dxdH(u) \, . \quad \text{(A.2)}$$

Since for any path $\pi \in \Pi^j$ the integral in (A.2) is the same and it is equal to $m_R^{(l-1)}(\eta)/(l-1)!$, and the number of minimal paths in Π^j equals $R_j(\vec{b})$, then

$$\lambda(\vec{b})q^{(\vec{b})} \approx \frac{1}{(l-1)!}\sum_{\vec{b} \in E_-} R_j(\vec{b})\Lambda^j m_R^{(l-1)}(\eta) \quad \text{(A.3)}$$

After applying assertion of lemma 3.1 to (A.3) we will obtain assertion (5.3) of theorem 5.1.

REFERENCES

1. Genis, Y. On Reliability of Renewal Systems with Fast Repair. Journal of International Group on Reliability, Reliability: Theory & Applications, No. 3 (Vol. 1), pp. 31 – 47, September 2006. ISSN 1932 – 2321.
2. Genis, Y. Reliability and Risk Assessment of Systems of Protection and Blocking with Fast Restoration. Journal of International Group on Reliability, Reliability & Risk Analysis: Theory & Application, No. 1 (Vol. 1) 2008, pp 41 – 57, ISSN 1932 – 2321.
3. Genis, Y. Stochastic-system behavior described by semi-renewal process. Automation and Remote Control, 40(5), 668-681. (1979).

FUNDAMENTAL RISK ASSESSMENT IN EXAMPLE OF TRANSSHIPMENT SYSTEM

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ABSTRACT

The paper represents discussion about risk assessment for transhipment system in reduces data condition. As a particular example transhipment system is presented. Article can be treated as first estimation. Future work and objectives are characterized in the end.

1 INTRODUCTION

Container transport is permanently growing way of goods transportation. Even during the time of global crisis, transportation in these load units still increase (in Poland more than 5%). Translocation of goods is part of logistics chain, where one of important factor is reliability. The reliability model of combined transport was presented during conference of ESREL. There were calculations of transitions between reliability and maintenance states in that paper. However the problem of reliability can be also considered from risk assessment point of view. Researches proved that one of important link in this chain is transshipment. The article shows fundamental risk assessment of transshipment system as container terminal in reduced data conditions.

2. RISK ASSESSMENT – INITIAL SET

The paper was prepared according to [3]. In general "risk is something that may badly affects your system". In particular example risk is meant as decrease of transshipment ability on terminal caused by adverse circumstances; in result the system is not able to serve all arriving containers. It was assumed that bad transshipment conditions are affected by crane's break downs. Naturally, stops can be introduced by i.e. some atmospheric reasons, some bad managing decisions, etc., but according to assumptions, that all bad what may happen is caused by break downs.
Risk assessment is in our case based onto finding three factors – risk contributors:

- probability of occurrence - v_{oi},
- impact V_i,
- probability of "discover ability" v_d ,

and all the factors must be given for *i-th*-element of the system.
When prepare standard risk assessment, character (or just some information) of break down must be known. Not only information that machine is broken, but also time of failure, time of repair, probability of occurrence, etc. Than for each machine we can prepare data like in table1.

Table 1. Desired data for machine

No.	Broken element	Probability of occurrence	Time of repair	Probability of "discovery"
1	element 1	p_1	t_1	p_{d1}
2	element 2	p_2	t_2	p_{d2}
..	…	…	…	…
n		p_n	t_n	p_{dn}

In our case we don't have all information about. We have just information given in table 2.
Value of impact can be characterized by scale number according to intervals defined by number of not transshipped containers. For determination of the probability of occurrence of the failure mode, besides published information regarding the failure rate, it is very important to consider the operational profile (environmental, mechanical, and/or electrical stresses applied) of each component that contribute to its probability of occurrence. This is because the component failure rates, and consequently failure rate of the failure mode under consideration, in most cases increase proportionally with the increase of applied stresses with the power law relationship or exponentially. Probability of occurrence of the failure modes for the design can be estimated from [3]:

– data from the component life testing,
– available databases of failure rates,
– field failure data,
– failure data for similar items or for the component class.

Discover ability likelihood can be also defined by scale according to intervals. The proposal represented by a scale is presented in table 2. The scale was determined according to the perspective of the entrepreneurs and the technicians who are responsible for corrective maintenance.

Table 2. Discover ability scale

Discover ability	Character	Value
Minor	chance to discovered a failure manifestation before every new motion	1
Major	chance to discovered a failure manifestation before a fault is not sure at all	2
Critical	chance to discover a failure manifestation is hard	3
Catastrophic	chance to discover a failure manifestation. is impossible	4

One of the methods of quantitative determination of criticality is the Risk Priority Number, RPN [3]. Risk is here evaluated by a subjective measure of the severity of the effect and an estimate of the expected probability of its occurrence for a predetermined time period assumed for analysis. In some cases where these measures are not available, it may become necessary to refer to a simpler form of a non-numeric FMEA.
Risk priority number (RPN) can be obtain by following formula

$$RPN_i = v_{oi} \cdot V_i \cdot v_d \tag{1}$$

where:
value of probability of occurrence - v_{oi},

value of impact V_i,
value of likelihood of discover ability v_d,

Obtained values are compared to scale, that describe the priority of the event affecting the system/element of the system. The problem is to find adequate thresholds of risk level. The acceptance of RPN may be linked with hurt understood as looses of money. Normally there is threshold of permissive looses. Its contributory factors are:
money not earn because of decrease of container transshipment number,
penalty coursed by breach of delivery contract,

- cost of repairs,
- other reasons.

Thresholds can be assigned from technical point of view. Mechanical systems are designed and prepared to run with load of 80%. Greater usage (even 100%) is naturally possible, but makes, those elements of machines (systems) wear faster than average. I.e. on container terminals transshipment machines are prepared to handle 42 tons, but heavy loaded container weight 32 tones. Typical container crane (rail crane) can pick up 40 tons – 32 tons – it means 80% of total handle capacity.
Transshipment ability is given according to formula [1]:

$$W = \frac{\beta \cdot T}{t_c}, \qquad (2)$$

where:
T – time in a day, when terminal is open [min],
t_c – time of one cycle of transshipment [min],
β – work time efficiency rate.

Work efficiency rate gives information in how many percent of period of time, the machine is really loaded (at work). This number is usually presented as a work efficiency rate with values from 0 to 1. In practice β factor is in interval (0.6 – 0.8).

3. THE CASE

3.1. System introduction

The system consists of 4 cranes on container terminal. Terminal is open 12h a day. Transshipment abilities of each machine calculated according to formula (2) are presented in table 3. If provider can make about 184 movements per day, and there is access of 150 containers, than system can make 34 movements more that is required. Decrease of transshipment ability on 20% means, that system can not operate 37 containers. According to terminal owner it was assumed that minor impact, acceptable by terminal management, is decrease of transshipment ability on 45 containers.

Table 3. Cranes transshipment abilities

	Crane 1	Crane 2	Crane 3	Crane 4
t_c [min]	9	9	12	15
T 12h/day [min]	720	720	720	720
β	0.64	0.71	0.73	0.72
Trans-shipment ability [cont.]	51.2 = 51	56.8 = 56	43.8 = 43	34.56 = 34

That number was consistently use to assumed numbers as a scale of impact. The scale of impact is presented in table 4.

Table 4. Scale of impact

Type of impact	Number of not transshipped containers	Value
Minor	0-45	1
Major	46-90	2
Critical	91-135	3
Catastrophic	136-	4

One of the problems in particular example is assessing discover ability likelihood. According to formula (1), to calculate risk priory number, value of ability likelihood must be known. This factor also can be given from scale, which describes if each failure could be discovered before affect the system. There is no information about it in the case. Research materials don't include information about detecting probable failures. Consequently, the factor must be assumed.
Discover ability scale can be also found on 4 level's, where value of 1 mean easy recognize coming failure, and value of 4 absolutely inability of coming failure discover. The mean value of four level scale is 2.5. This value was taken to each calculation as discover likelihood.

Scale of impact has influence on thresholds of RPN acceptance. Thresholds are assumes:

1. operational ability lowered not more that 20% (transshipment ability greater that 147 containers),
2. operational ability lowered on 21 – 40 % (transshipment ability between 109 - 146 containers),
3. operational ability lowered on 41 – 60 % (transshipment ability between 108 - 72 containers),
4. operational ability lowered more than 61 % (transshipment ability less than 72 containers),.

Naturally it can calculate "how much costs 1%". Total operational ability is 184 containers/day, so 1% is 1.84. When one transshipment costs 24EUR (typical price for one transshipment), than 1% is 1.84x24EUR = 44.16EUR. Than thresholds from the financial point of view are:

1. total looses less than 20x44.16EUR = 883.2 ~ 883EUR,
2. total looses between 883 – 1760EUR,
3. total looses between 1760 – 2650EUR,
4. more than 2650 EUR.

Due to following calculations container terminal management estimated, that acceptable conditions are result of:

- probability of crane's failure 0.04,
- discover likelihood value of 2.5,
- money looses less than 883 EUR – decrease of transshipment ability less than 20%, consequently impact value of 1.

It makes, that acceptable level of RPN is 0.1.
As completely unacceptable RPN follows from

- probability of failure more than 0.1
- discover ability value of 2.5,
- impact value of 4.

Than unacceptable RPN is 1 or more.
According to presented assumption, taking into account system's conditions values of RPN can be calculated due to formula (1).
Table 5 shows numbers of RPN when only one machine breaks down.
As it can be seen the highest RPN value is prescribed to machine 3, where simple break down makes value of 0.2. Rest of machines makes RPN on half lower. Even if focus on the slowest machine – the 4th and the most efficient – the 2nd, both of them obtain RPN on level 0.1, whitch is still acceptable.
The simplest scenario is when only one machine breaks down. Examples of various break downs should be taken into account.

Table 5. RPN values when one crane is broken down

	Crane 1	Crane 2	Crane 3	Crane 4
Machine trans-shipment ability	51	56	43	34
Trans-shipment ability of system	133	128	141	150
Value of impact	2	2	2	1
Failure probability	0.02	0.02	0.04	0.04
RPN	0.1	0.1	0.2	0.1

3.2. Scenarios

All machines may break down one after another or at the same time. It was assumed that reparation of machines 1 and 2 last exactly 3 days, machines 3 and 4 – 2 days.
When there is assumption about time of repair than the total looses are result from combination of break downs and total time of reparation of each machine. But each day the risk will be different, because transhipment ability is different for each machine. So also sequence of break downs is important.

When two machines break down, there are 30 combinations of system delays. When three, then the number of reparation scenarios is 208.

In adverse situation 4 machines can stop one by one, and total time of reparation is 7 days or all machines stop at the same time. First situation is presented in table 6.

In case of the longest disturbs RPN value changes in following days. The highest value fall due to fifth day of disturbs. RPN value is 0.45, against 0.1 on 1st, 2nd and 7th day. Total looses caused by decrease of transshipment is almost 5700 EUR. It is important to point, that although four machines break down, RPN value during 4 days is on acceptable level. RPN is far from unacceptable value.

Other scenario is presented in table 7. Four machines break down day after another. The lowest transshipment ability is on 3rd day, only 34 containers. The highest impact value is also on 3rd day – 3 in scale. In this case RPN achieves higher value – 0.6. Total looses are 7320 EUR.

There is another scenario of break downs presented in table 8. In this example two faster cranes break down at the same time. In third day of reparation another crane stops. In this case value of impact is 3and lasts also 3 days. The greatest RPN is 0.6. Total looses are 7320 EUR.

Table 6. Model of the longest disturbs of system

Crane 1	51	51	51				
Crane 2			56	56	56		
Crane 3					43	43	
Crane 4						34	34
Day of reperation	1	2	3	4	5	6	7
Decrese of trans-shipment	51	51	107	56	99	77	34
Impact value	2	2	3	2	3	2	1
p-ty 1st	0.02	0.02	0.02	0	0	0	0
p-ty 2nd	0	0	0.02	0.02	0.02	0	0
p-ty 3rd	0	0	0	0	0.04	0.04	0
p-ty 4th	0	0	0	0	0	0.04	0.04
RPN	0.1	0.1	0.3	0.1	0.45	0.4	0.1
Looses [EUR]*	408	408	1752	528	1560	1032	0
total looses [EUR]*	408	816	2568	3096	4656	5688	5688

* - 24 EUR is assumed cost of one transshipment

Table 7. Scenario of 5 days of disturbs – first variant

Machine 1	51	51	51		
Machine 2		56	56	56	
Machine 3			43	43	
Machine 4				34	34
Day of reperation	1	2	3	4	5
Decrese of trans-shipment	51	107	150	133	34
Impact value	2	2	3	3	1
p-ty 1st	0.02	0.02	0.02	0	0
p-ty 2nd	0	0.02	0.02	0.02	0.02
p-ty 3rd	0	0	0.04	0.04	0.04
p-ty 4th	0	0	0	0.04	0.04
RPN	0.1	0.2	0.6	0.75	0.25
Looses [EUR]*	408	1752	2784	2376	0
total looses [EUR]*	408	2160	4944	7320	7320

* - 24 EUR is assumed cost of one transshipment

There is more difficult situation presented in table 9. Four machines are broken at the same time. In this case RPN is the biggest in compare to scenarios presented before – the lowest 0.8, the greatest 1.2. The screnario presents situation, when RPN crosses value of acceptance.During time of

disturbs ipact value achieves the highest number of 4. Calculated looses in two days are greater than in total time of disturbs in scenario from table 6. Total money loose is almost 9000 EUR.
Only the most harmful scenarios were presented in the paper. According presented basic calculations it can be seen, that failure of all machines in assumed conditions may occur different impact and brings various money looses. When all operating machines break down and time of disturbs is shorter and RPN and money looses are greater. In fact break down of all machines is hardly probable. Path of machine's break downs is presented in figure 1.

Table 8. Scenario of 5 days of disturbs – second variant

Machine 1	51	51	51		
Machine 2	56	56	56		
Machine 3			43	43	
Machine 4				34	34
Day of reperation	1	2	3	4	5
Decrese of trans-shipment	107	107	150	77	34
Impact value	3	3	3	2	1
p-ty 1st	0.02	0.02	0.02	0	0
p-ty 2nd	0.02	0.02	0.02	0.02	0.02
p-ty 3rd	0	0	0.04	0.04	0.04
p-ty 4th	0	0	0	0.04	0.04
RPN	0.3	0.3	0.6	0.5	0.25
Looses [EUR]*	1752	1752	2784	1032	0
total looses [EUR]*	1752	3504	6288	7320	7320

Probability that two machines break down at the same time is 0.0104. Probability of break down of three machines at the same time is 0.000576, so probability of damage of four machines at the same time can be practically overlooked.

4. CONCLUSIONS AND FUTURE WORK

In particular example of system the analysis shown that keeping long duration of disturbs makes risk priory number and consequences lower. In case of 7 days of disturbs the maximum RPN is 0.45, when disturbs last only 3 day, RPN 1.2. Lower impact is possible when less number of transshipment machines break down at the same time. According to this, financial consequences are also not so harmful. When all machines are out of order looses are on level of 8000 EUR, if they break down one after another looses are reduced on even 40%.
This article shows initial risk assessment in reduced data condition. Lack of data made, that the authors couldn't prepare full risk assessment. There is also no links to risk management, no indications to avoid risk situations in article. It can be said, that presented assessment is founded on boundary assumptions. A future objective is to clarify and define possible break downs of machines. The classification will be prepared for each machine.

Table 9. Variant of total break down of system

Machine 1	51	51	51
Machine 2	56	56	56
Machine 3		43	43
Machine 4	34	34	
Day of reperation	1	2	3
Decrese of transshipment	141	184	150
Impact value	4	4	4
p-ty 1st	0.02	0.02	0.02
p-ty 2nd	0.02	0.02	0.02
p-ty 3rd	0	0.04	0.04
p-ty 4th	0.04	0.04	0
RPN	0.8	1.2	0.8
Looses [EUR]*	2568	3600	2784
total looses [EUR]*	2568	6168	8952

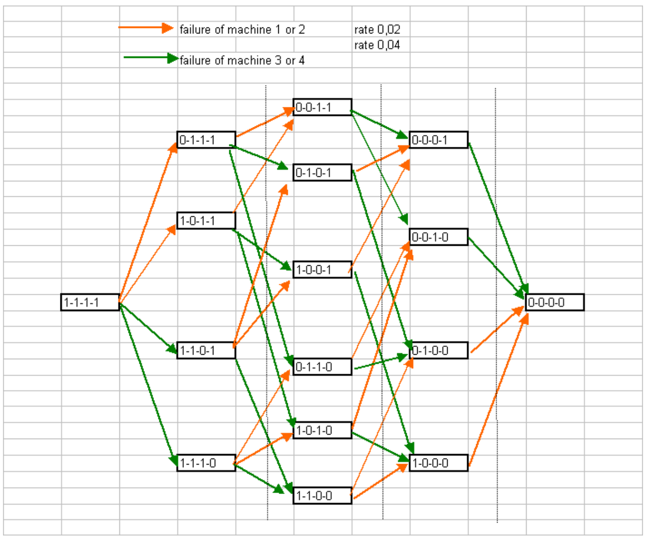

Figure 1. Paths of break downs

Other objective is to asses consequences and impact of disturbs. These recognize requires time of data collection and researches carried on real system. Cutting-edge result is to qualify failures, impact and occurrences probabilities and their function distributions. It should be taken into account, that machines working on terminal are exposing on processes like aging and different types of forces, conditions, ect. It is very important to recognize failure mechanisms, to solve one of the most important weaknesses of the article – lacks of information about value of likelihood of discovers ability. Future work is connected with detailing characteristics of both discover abilities in response to failures and description, more precise classification of failures which may occur. This goal enables to precise the approaches for discover ability and consequences of possible failure.

Acknowledgements

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REFERENCES

[1] Jakubowski L.: Technologies of loading works (in polish). Oficyna Wydawnicza Politechniki Warszawskiej. Warszawa, 2003.

[2] Zajac M.: Application of five-phases model of reliability of combined transportation system. ESREL 2006, Lisbona, 2006.

[3] IEC 60812 ed. 2: Analysis technique for system reliability – Procedure for failure mode and effect analysis (FMEA)

ASYMPTOTIC ANALYSIS OF LATTICE RELIABILITY

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ABSTRACT

Asymptotic formulas for connection probabilities in a rectangular lattice with identical and independent arcs are obtained.. For a small number of columns these probabilities may be calculated by the transfer matrices method. But if the number of columns increases then a calculation complexity increases significantly. A suggested asymptotic method allows to make calculations using a sufficiently simple geometric approach in a general case.

INTRODUCTION

A calculation of connection probabilities in a random graph is a complex problem. In a general case it demands a number of arithmetical operations which increases as a geometric progression with a number of arcs in the graph [1], [2]. So this problem is very important in the reliability theory. It attracts special interest of physicists [3], [4] if we consider a random lattice with identical arcs.

A main approach to this problem solution is is in an application of the transfer matrices. In this method it is necessary to obtain recurrent formulas (by a length of the lattice). But a dimension of the transfer matrices increases sufficiently fast with a width of the lattice.

So an idea to construct an alternative approach origins. In this paper this approach is based on a suggestion that a work probability or a failure probability of arcs is small. These assumptions allow to obtain asymptotic formulas in a form of sums of work probabilities for ways or of failure probabilities for cross sections with minimal numbers of arcs.

A determination of such asymptotes becomes sufficiently simple though bulky enumerative problem of the graph theory.

1 MAIN DESIGNATIONS

Suppose that $\Gamma=\{U,W\}$ is the no oriented graph with the finite nodes set U, with the finite arcs set $W=\{w=(u,v),\ u,v\in U\}$ and with the fixed initial and final nodes $u_0,v_0\in U$. Denote by $\mathcal{R}=\{R_1,\ldots,R_m\}$ the set of all acyclic ways R between the nodes u_0, v_0 and the set $\mathcal{L}=\{L\}$ of all cross sections which are defined by the formulas

$\mathcal{A}=\{A\subset U,\ u_0\in A, v_0\notin A\}$, $L=L(A)=\{w=(u,v),\ u\in A, v\in U\setminus A\}$,

$\mathcal{L}=\{L(A),\ A\in\mathcal{A}\}$. An each arc $w\in W$ works with the probability p, $0<p<1$, $\overline{p}=1-p$, independently on all other arcs.

Denote $P_\Gamma=P_\Gamma(p_w,\ w\in W)$ the probability that there is a working way between the nodes u_0, v_0 in the graph Γ and designate by $\overline{P}=1-P_\Gamma$ the failure probability of this graph. Suppose that U_R is the event that all arcs in the way R work and V_L the event that all arcs in the cross section L fail. From the definition is it easy to obtain that

$$P_\Gamma=P\left(\bigcup_{R\in\mathcal{R}}U_R\right),\ \overline{P_\Gamma}=P\left(\bigcup_{L\in\mathcal{L}}V_L\right) \qquad (1)$$

2 ASYMPTOTIC FORMULAS

From the first equality in (1) obtain:

$$\sum_{i=1}^{m} P\left(U_{R_i}\right) - \sum_{1\le i<k\le m}^{m} P\left(U_{R_i}U_{R_k}\right) \le P_\Gamma \le \sum_{i=1}^{m} P\left(U_{R_i}\right).$$

Consequently if the condition $p(h)\to 0$, $h\to 0$, is true then

$$P_\Gamma \sim \sum_{i=1}^{m} P\left(U_{R_i}\right) = \sum_{i=1}^{m} \prod_{w\in R_i} p(h),\ h\to 0, \qquad (2)$$

And the relative error of the asymptotic formula (2) is

$$A_\Gamma = \left|\frac{P_\Gamma}{\sum_{i=1}^{m} P\left(U_{R_i}\right)} - 1\right| \le mp(h)\to 0,\ h\to 0. \qquad (3)$$

Denote $\mathcal{L}_1 = \{L_1,\ldots,L_n\}$ the set of all minimal (by a number of arcs) cross sections from the family $\mathcal{L}$. The second formula in (1) and the family $\mathcal{L}_1$ definition lead to the equality

$$\overline{P_\Gamma} = P\left(\bigcup_{L\in\mathcal{L}_1} V_L\right). \qquad (4)$$

From the formula (4) using an induction by n obtain the inequalities

$$\sum_{i=1}^{n} P\left(V_{L_i}\right) - \sum_{1\le i<k\le n} P\left(V_{L_i}V_{L_k}\right) \le \overline{P_\Gamma} \le \sum_{i=1}^{n} P\left(V_{L_i}\right).$$

So if the condition $\overline{p}(h)\to 0$, $h\to 0$, is true then

$$P_\Gamma \sim \sum_{i=1}^{n} P\left(V_{L_i}\right) = \sum_{i=1}^{n} \prod_{w\in L_i} \overline{p}(h),\ h\to 0, \qquad (5)$$

And the relative error of the asymptotic formula (5) is

$$\overline{A}_\Gamma = \left|\frac{\overline{P}_\Gamma}{\sum_{i=1}^{n} P\left(V_{L_i}\right)} - 1\right| \le n\overline{p}(h)\to 0,\ h\to 0. \qquad (6)$$

3 LOW RELIABLE ARCS

Consider the finite lattice with the size $(n\_+n+n_+)\times(m\_+m+m_+)$ and fix the initial node (0,0) and the final node (n,m) in the internal rectangular S. The nodes $(-n,-m\_)$, $(n+ n_+,\ m +m_+)$ are extreme for the rectangular S' which contains S. Suppose that $p(h)= h$, l_i is the number of arcs in the way R_i , then $P(U_{R_i}) = h^{l_i}$ and from the formula (2) obtain

$$P_\Gamma \sim \sum_{i=1}^{m} h^{l_i} \sim ah^b,$$

where $b = \min_{1\le i\le m} l_i$, a is the number of the ways which contain b arcs . It is easy to obtain obvious that $b=m+n$, $a = C_{m+n}^{m}$.

4 HIGH RELIABLE ARCS

Suppose that $\overline{p}(h) = h$, l_i is the number of arcs in the cross section L_i, then $P(V_{L_i}) = h^{l_i}$ and from the formula (2) obtain

$$\overline{P}_\Gamma \sim \sum_{i=1}^{m} h^{l_i} \sim ch^d,$$

where $d = \min_{1\le i\le m} l_i$, c is the number of cross sections which have d arcs.

Consider the following cases represented on figures with the same numbers:

1) $n\_= m\_= n_+= m_+=0$; 2) $n\_= m\_=m_+=0,\ n_+>0$; 3) $n\_= m\_=0,\ n_+>0,\ m_+>0$;

4) $m\_=m_+=0$, $n_+>0$, $n\_>0$; 5) $m\_=0$, $n\_>0$, $n_+>0$, $m_+>0$; 6) $n\_>0$, $m\_=n_+=0$, $m_+=0$;
7) $m\_>0$, $n\_>0$, $n_+>0$, $m_+>0$.

In the case 1) internal and external rectangular coincide: $S=S'$, in the cases 2) - 7) the inclusion $S\subset S'$ takes place.

Remark that listed cases do not describe all possible situations. For example an analog of the condition $n\_=m\_=m_+=0$, $n_+>0$ (see the case 2) may be the condition $n\_=m\_=n_+=0$, $m_+>0$. But it is simple to check that all possible arrangements may be reduced to listed ones after a replacement of + by – and visa versa or after a tumbling of the lattice S on ninety degrees to the left or to the right.

The considered lattice may be interpreted as an oriented graph in which the arcs (u,v), (v,u) belong or do not belong to the graph simultaneously. So from the Ford - Falkerson theorem about an equality of a maximal flow and a minimal ability to handle of cross sections [5, гл. I] it is easy to obtain the inequality $d\le\min(a,b)$ where a is the number of arcs outgoing from the initial node and b is number of arcs incoming to the final node. This inequality in the listed cases transforms into the formulas:

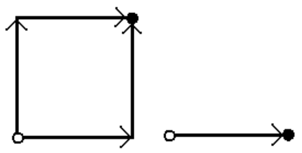

Figure 1. On the left $d\le 2$, $m>0$, on the right $d\le 1$, $m=0$.

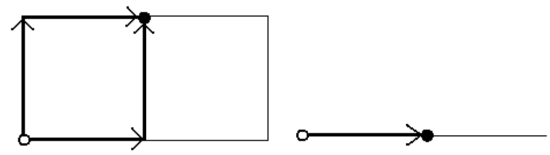

Figure 2. On the left $d\le 2$, $m>0$, on the right $d\le 1$, $m=0$.

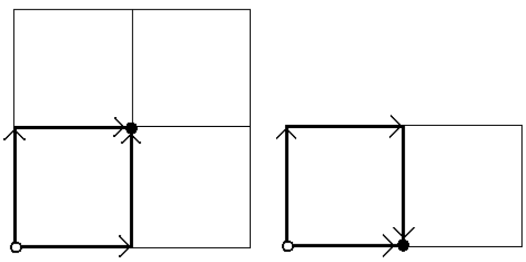

Figure 3. $d=2$, on the left $m>0$, on the right $m=0$.

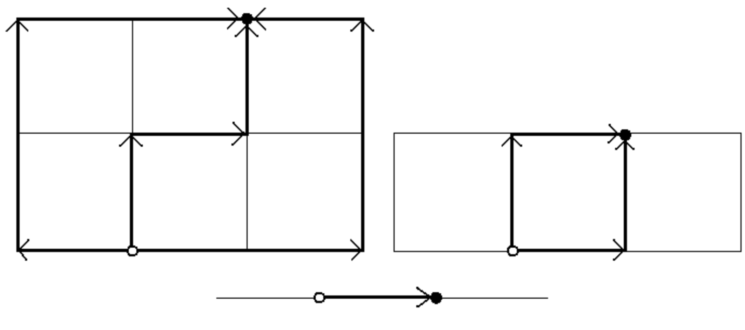

Figure 4. On the left above; $d\le 3$, $m>1$, on the right above $d\le 2$, $m=1$, below $d\le 1$, $m=0$.

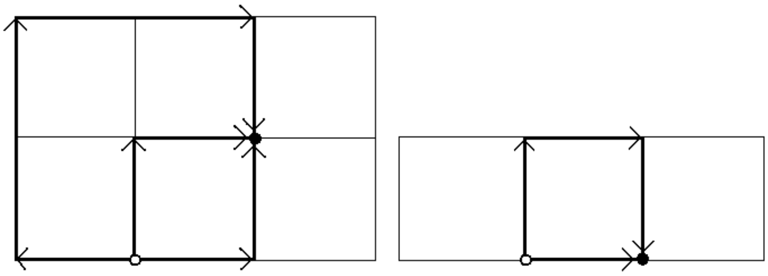

Figure 5. On the left $d\le 3$, $m>0$, on the right $d\le 2$, $m=0$.

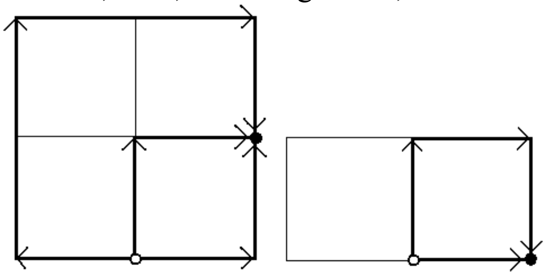

Figure 6. On the left $d\le 3$, $m>0$, on the right $d\le 2$, $m=0$.

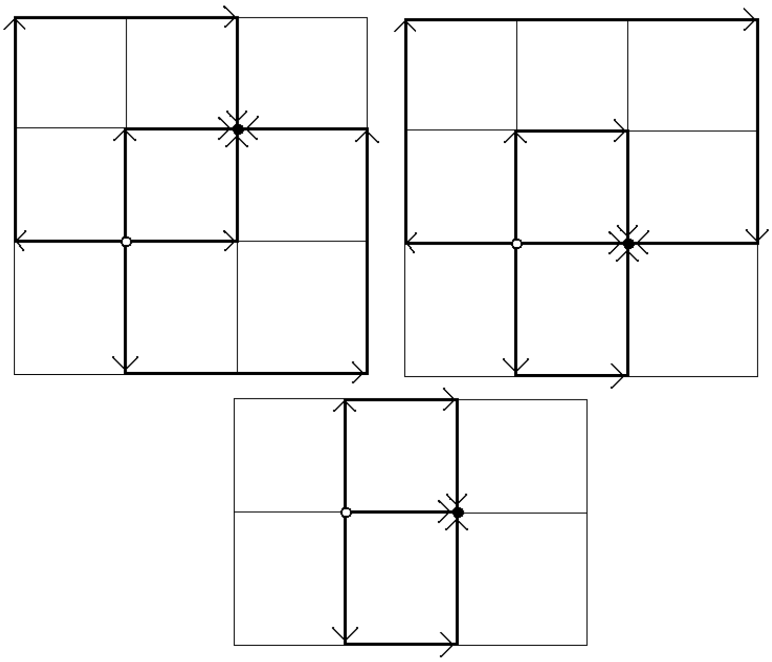

Figure 7. On the left above $d\le 4$, $m>0$, on the right above $d\le 4$, $m=0$, $m_+ + m\_ >2$, below $d\le 3$, $m=0$, $m_+ + m\_ =2$.

Then choosing load arcs as marked on these figures and unload arcs as all others it is possible to transform obtained inequalities into equalities.:

1), 2), $d=1+I(m>0)$; 3) $d=2$; 4) $d=3I(m>0)+2I(m=1)+I(m=0)$; 5), 6) $d=3I(m>0)+2I(m=0)$;

7) $d=4I(m>0)+4I(m=0,\ m_- + m_+ >2)+3I(m=0,\ m_- + m_+ =2)$.

Calculate now the asymptotic constant c. For this purpose show on the following figures all possible types of cross sections with minimal number of arcs.

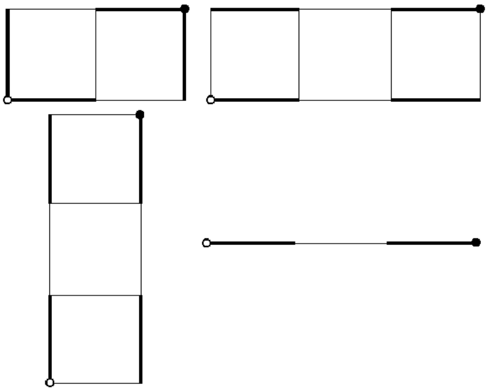

Figure 1a. On the left above $m>0$, on the right above $m=1$, on the left below $m>0$, $n=1$, on the right below $m=0$

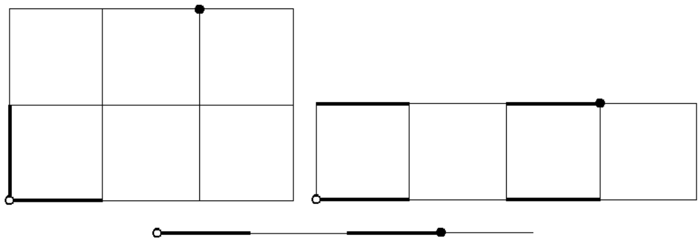

Figure 2a. On the left above $m>0$, on the right above $m=1$, below $m=0$

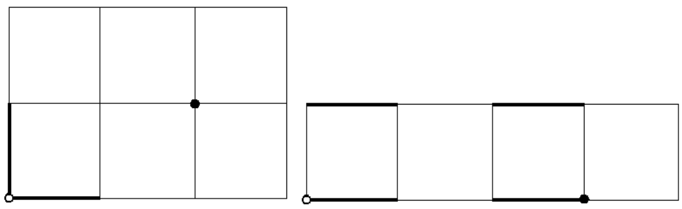

Figure 3a. On the left $m>0$, on the right $m=0$

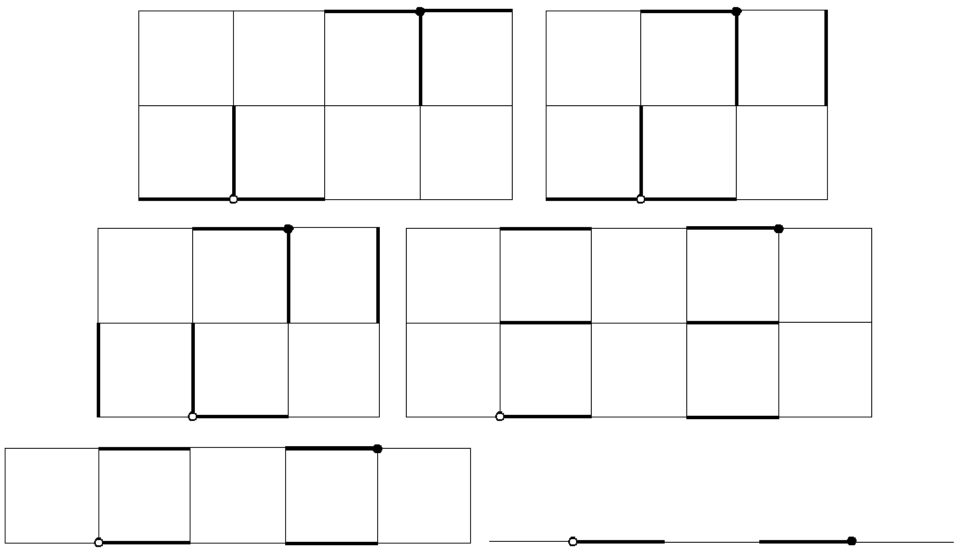

Figure 4a. Overhead to the left $m>0$, to the right $m>0$, $n_+=1$, middle to the left $m>0$, $n_+= n_-=1$, to the right $m=2$, bottom to the left $m=1$, to the right $m=0$

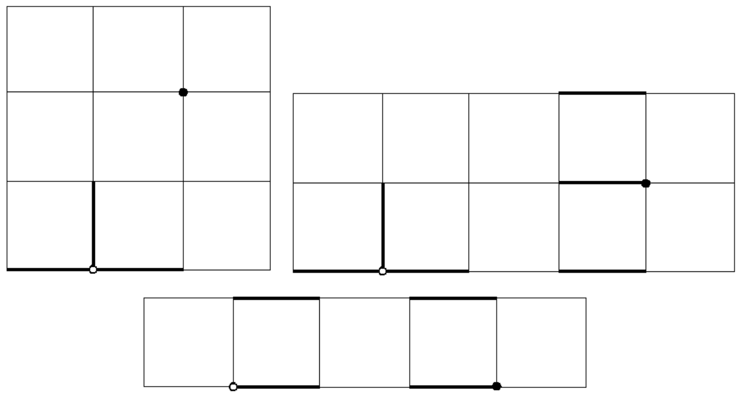

Figure 6a. Overhead to the left $m>0$, to the right $m=1$, $m_+=1$, bottom $m=0$, $m_+=1$

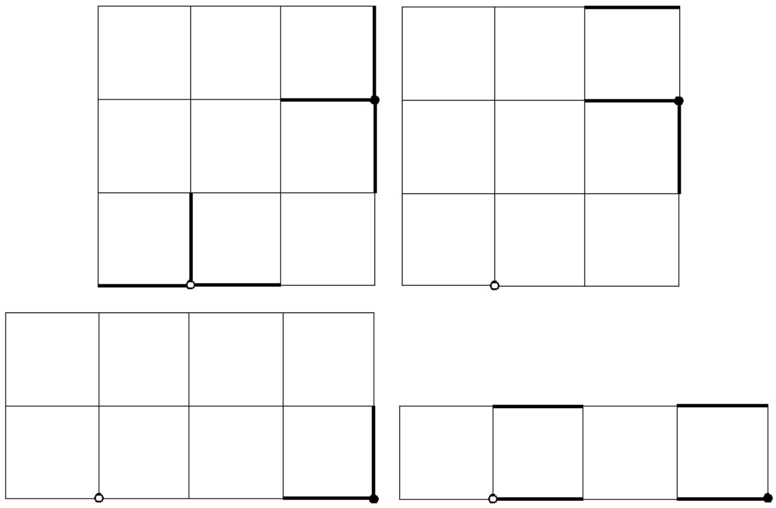

Figure 5a. Overhead to the left $m>0$, to the right $m>0$, $n_+=1$, bottom to the left $m=0$, $m_+>1$, to the right $m=0$, $m_+=1$

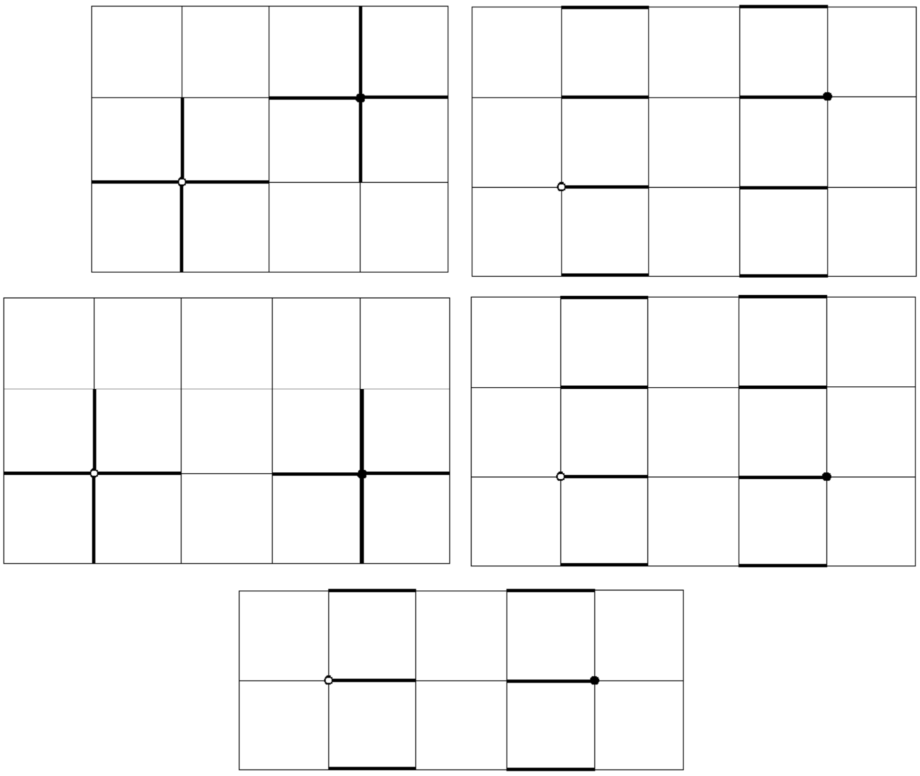

Figure 7a. Overhead to the left $m>0$, to the right $m=1$, $m_{+}+m_{-}=1$, middle to the left $m=0$, to the right $m=0$, $m_{+}=2$, $m_{-}=12$, bottom $m=0$, $m_{-}=m_{+}=1$

Using these figures it is easy to obtain the following equalities:

1) $c=2I(m>0)+nI(m=0,1)+mI(n=1)$; 2) $c=I(m>0)+nI(m=0,1)$; 3) $c=1+nI(m=0,m_{+}=1)$;

4) $c=I(m\bullet\ 2)[2I(n_{+},\ n_{-}>1)+3I(n_{+}>1,\ n_{-}=1$ или $n_{+}=1,\ n_{-}>1)+4I(n_{+}=n_{-}=1)]+nI(m=0,1,2)$;

5) $c=2I(m>0)+I(m=0)[1+nI(m_{+}=1)]$; 6) $c=I(m>0)+nI(m=0)$;

7) $c=2I(m>1)+I(m=1)[2+nI(m_{+}+m_{-}=2)]+I(m=0)[2I(m_{+}+m_{-}>2)+nI(m_{+}+m_{-}=2,3)]$.

CONCLUSION

As a result an initial asymptotic problem of a connection probabilities calculation is divided into a few of comparably simple geometric – combinatorial problems. A main difficulty of this solution is in a choice of this division.

REFERENCES

1. Barlow, R.E. and Proschan, F. 1965. *Mathematical Theory of Reliability.* London and New York: Wiley.

2. Ushakov, I.A. et al. *Reliability of technical systems: Handbook.* 1985. Moscow: Radio and Communication. (In Russian).

3. Tanguy, C. What is the probability of connecting two points? 2007. *J. Phys. A: Math. Theor.* Vol. 40: 14099-14116.

4. Tanguy, C. Asymptotic dependence of average failure rate and MTTF for a recursive meshed network architecture. 2009. *Reliability and risk analysis: theory and applications.* Vol. 2 (13), part 2: 45-54.

5. Ford, L.R., Fulkerson, D.R. *Flows in networks.* 1962. New Jersey. Princeton: Princeton university press.

THE RISK OF OPERATIONAL INCIDENTS IN BANKING INSTITUTIONS

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ABSTRACT

Banking-financial institutions are organizations which might be included in the category of complex systems. Consequently, they can be applied after adaptation and particularization, in the general description and assessment methods of the technical or organizational systems. The banking-financial system faces constrains regarding the functioning continuity. Interruptions in continuity as well as operational incidents represent risks which can lead to the interruption of financial flows generation and obviously of profit. Banking incidents include from false banknote, cloned cards, informatics attacks, false identity cards to ATM attacks. The functioning of banking institutions in an incident-free environment generates concern from both risk assessment and forecasting points of view.

Key words: operational risk, banking reliability, complex systems, incident probability, the risk of functioning interruption.

1 THE ISSUE STATEMENT

Usually, an incident-free performance is expressed by the reliability term. The reliability represents a qualitative characteristic of systems in the largest meaning of the term. Reliability is measured as probability of success. Specific to the numerical expressions of reliability is the fact that the main indicator is a probability, and therefore a positive number between 0 and 1.

In probabilistic expression, the risk of incidents has the following synthetic form:

$$I(t) = 1 - R(t) \tag{1}$$

This is a complementary value for the probability of performance without incidents, which is the operational reliability of the bank:

$$R(t) = P\{T \geq t\} \tag{2}$$

reliability which is expressed through a probability that measures the chance that the time T of functioning without incidents surpasses a previously established period t. We can say about $R(t)$ that:

- it is a decreasing function, and therefore reliability decreases in time;
- for $t=0$, $R(t)=1$, so at the starting point, the bank must be in perfect performance;
- for $t\rightarrow\infty$, $R(t)=0$, which means that for very long periods of time, reliability tends to become null;

Since reliability measures the probability of a system to fulfil its functions in a certain time period, time must be introduced as condition element in estimating reliability.

An important element is represented by the intensity or rate of banking incidents:

$$\lambda(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{R(t)} \tag{3}$$

where $f(t)$ is the probability density and $F(t)$ the distribution function (the probability of a banking incident), determined after testing the incident distribution law for the analysed banking system.

The probability density is related to an important indicator of the performance of a banking system, and that is the "distribution function of time without operational incidents" $F(t;\theta_i)$, which is defined as a probability of the operational malfunctioning in a bank:

$$F(t;\theta_i) = Prob\{T < t\} \tag{4}$$

The distribution function can be considered as "unreliability function" or "risk function" because there is an obvious relation:

$$Prob\{T < t\} + Prob\{T \geq t\} = 1 \tag{5}$$

The specific indicators of banking risk and reliability are presented in Table 1.

Table 1. The main indicators of risk and reliability

No.	Name	Calculus relation
1	The distribution function of performance non – incident time (T)	$F(t) = Prob\{T < t\}$
2	The probability density of performance non-incident time	$f(t) = F_t^{'}(t) = \left(\frac{dF(t)}{dt}\right)$
3	Reliability function (non-incident)	$R(t) = P\{T \geq t\}$
4	The average period of functioning without incidents	$E(T) = \int_0^\infty t \cdot f(t)dt$
5	The variance of functioning time without incidents	$Var(T) = E(T^2) - [E(T)]^2$
6	Variation coefficient	$CV(T) = \frac{\sqrt{Var(T)}}{E(T)}$

2 OPERATIONAL RISK INDICATORS

Risk measurement can be achieved through two major groups of indicators:
- Indicators based on descriptive statistics methods (non-parametric indicators)
- Indicators based on probabilistic models (parametric indicators).

2.1 Non-parametric indicators

They are indicators determined based on observation data, more precisely based on information collected on a period of time, for a bank portfolio.

Among these indicators we can find:

1. The relative frequency of "incidents" in banks:

$$\hat{f}(t_i) = \frac{r_i}{\sum_{i=1}^{m} r_i} \tag{6}$$

where r_i – is the number of banking incidents.

Based on these relative frequencies we now calculate

2. Cumulated relative frequency of incidents

$$\hat{F}(t_i) = \frac{1}{N}\sum_{j=1}^{i} r_i \tag{7}$$

which expresses the weight of banking units with incidents until the end of interval *i*; its value is increasing and equals 1 at the last interval of the series.

3. The relative frequency of non-incident banking units, is determined as complement to 1 of the cumulative relative frequency of incidents:

$$\hat{R}(t_i) = 1 - \hat{F}(t_i) = \frac{N_i}{N} \tag{8}$$

4. The average frequency (number) of incidents in a period of observation is determined as the total number of incidents $N = \sum_{i=1}^{m} r_i$ divided to the amount of non-incidents time of all banking units in the sample $\sum_{i=1}^{m} t_i \cdot r_i$.

$$\bar{f} = \frac{\sum_{i=1}^{m} r_i}{\sum_{i=1}^{m} t_i \cdot r_i} \text{ or } \bar{r} = \frac{\sum_{i=1}^{m} t_i \cdot r_i}{\sum_{i=1}^{m} t_i} \tag{9}$$

5. The mean time between failures (mean time of functioning without operational incidents) – MBTF is :

$$\bar{t} = \frac{\sum_{i=1}^{m} t_i \cdot r_i}{\sum_{i=1}^{m} r_i} = \frac{\sum_{i=1}^{m} t_i \cdot r_i}{N} \tag{10}$$

Obviously, there is an inverse relation between MTGF and *f*, MTBF = *1/f*.

MTBF is a direct indicator, because its measurement is directly proportional to the degree of reliability: a higher degree of reliability means a higher MTBF, and the other way around.

6. Incidents density, determined by dividing the number of incidents registered in an observation interval to the length of that interval. In the case in which time intervals are equal during the entire length of the series, the indicator becomes analogous to the experimental distribution density.

7. Incident rate

This indicator shows the weight of incidents during an observation time period as against to the existing one at the beginning of that period,

$$\lambda(t) = \frac{r_i}{\Delta t \cdot (n - r_i)} \tag{11}$$

where $N_i = n - r_i$ is the number of units where no incidents occurred at the beginning of interval *i*. If the product functions in stationary regime, the incident rate for the entire sample equals the average frequency of incidents.

2.2 Parametric indicators. Statistic models generation for non-incidents operational time

In his monumental paper "Encyclopaedia of Statistical sciences" the well-known statisticians N.L. Johnson and S. Kotz (Johnson & Kotz 1983) state that one of the basic preoccupations of classical statistics was, and still is, the finding of statistic models generation mechanism, in other words finding distribution functions or probability densities.

Accordingly, they enumerate five systems of "frequency curves": Pearson (as differential equation), Gram-Charlier-Edgeworth (as series development), Burr (as differential equation), Johnson (as normality transformation), and Turkey (special transformation).

Also worth mentioning are – in chronological order – the McKay system (1932), based on the use of Bessel functions, the Perks system (1932), which generates a very wide range of logistic distributions, the Darmois (1935) and Koopman (1936) systems regarding exponential distributions, the Gnedenko system (1943) using extreme value distributions, the Toranzos (1952) system, a generalization of the Pearson and Mathai-Saxena systems (1966) based on the generalized hypergeometric function.

In 1982, M. A. Savageau (Savageau 1982) proposes a system of differential equations in which the unknown variables are two densities f_1 and f_2:

$$\begin{cases} \dfrac{df_1}{dx} = a_1 \cdot f_1^{\alpha_{11}} \cdot f_2^{\alpha_{12}} - b_1 \cdot f_1^{\beta_{11}} \cdot f_2^{\beta_{12}} \\ \dfrac{df_2}{dx} = a_2 \cdot f_1^{\alpha_{21}} \cdot f_2^{\alpha_{22}} - b_2 \cdot f_1^{\beta_{21}} \cdot f_2^{\beta_{22}} \end{cases} \tag{12}$$

Where a_1, a_2, b_1, b_2 are real negative numbers and α_{11}, α_{12} etc. are random real numbers which provide known densities through various particularizations.

For example, $a_1 = 1, a_2 = 1, b_1 = 1, b_2 = 0, \alpha_{11} = 1, \alpha_{12} = -1, \beta_{11} = 1, \beta_{12} = -1, \alpha_{21} = 0,$ $\alpha_{22} = 0, \beta_{21} = 0, \beta_{22} = 0$ leads us to:

$$\frac{df_1}{dx} = \frac{f_1}{f_2} - f_1 \cdot f_2 \text{ and } \frac{df_2}{dx} = 1,$$

where $f_2 = x$ and therefore:

$$\frac{df_1}{dx} = \frac{1}{x} \cdot f_1 - x \cdot f_1 \text{ or } \frac{df_1}{f_1} = \left(\frac{1}{x} - x \right) \cdot dx,$$

which through integrations leads us to the density

$$f_1(x) = \exp\left\{ \int \left(\frac{1}{x} - x \right) dx \right\} = \exp\left\{ \ln x - \frac{x^2}{2} \right\} = x \cdot \exp\left\{ -\frac{x^2}{2} \right\}, x \geq 0$$

which is precisely the reduced Rayleigh density.

The majority of the proposed systems are in fact differential equations in which the unknown variable is either F (the distribution function) or f (the probability function).

Karl Pearson's classical system (Pearson 1895) is given by the separate values differential equation:

$$\frac{df}{dx} = \frac{(x - c_0) \cdot f}{c_1 + c_2 x + c_3 x^2}$$

or

$$\frac{df}{f} = \frac{(x - c_0)}{c_1 + c_2 x + c_3 x^2} \cdot dx \tag{13}$$

In which various particularizations of the c_i coefficients lead to the majority of the classical models.

Thus, if $c_0 = \mu, c_1 = -\sigma^2, c_2 = c_3 = 0,$ where $\mu \in R$, $\sigma > 0,$ for $x \in R$**,** we have:

$$\frac{df}{f} = \frac{x - \mu}{-\sigma^2} dx \quad \text{or} \quad f = A \cdot \exp\left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\}$$

where A is the normalization factor $\left(1/\sigma\sqrt{2\pi}\right)$, and therefore we have the Gauss-Laplace density.

For $c_0 = k/\theta, c_1 = c_3 = 0, c_2 = 1/2, (k, \theta > 0)$ we have

$$f = \frac{1}{\theta^{k+1}\Gamma(k+1)} \cdot x^k \cdot \exp\left\{-\frac{x}{\theta}\right\}, x \geq 0, \theta, k > 0 \quad (14)$$

which is the well-known Gamma model.

In general, the density expression depends on the roots of the denominator $c_1 + c_2 x + c_3 x^2$ and as can be seen in the two examples presented, the c_i coefficients are various moments of the distribution. The complete list of the "Pearson curves" can be consulted in "Distributions in Statistics. Continuous Univariate Distributions" (Johnson & Kotz 1970).

The Gram-Charlier-Edgeworth system (Cohen 1998) is based on a theorem which assures the possibility of writing the $f(x)$ density of a random continuous variable X as an infinite series of terms depending on $f_0(x)$ - the density of the standard normal variable:

$$f_0(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{x}{2}\right\} \quad (15)$$

and certain coefficients. The Gram-Charlier-Edgewords decomposition uses Hermite polynoms. F. Y. Edgeworth (Perote & Del Brio 2006) simplified to a point the Gram-Charlier series, proposing the following expression for the density approximation:

$$f(x) = \frac{1}{\sqrt{2\pi}}\left[1 - \frac{1}{6}\sqrt{\beta_1}\left(x^2 - 3x\right) + \frac{1}{24}(\beta_2 - 3)\cdot\left(x^4 - 6x^2 + 3\right) + \right.$$
$$\left. + \frac{1}{72}\beta_1\left(x^5 - 10x^3 + 14x\right)\right] \cdot \exp\left\{-\frac{x^2}{2}\right\} \quad (16)$$

where $\sqrt{\beta_1}$ and $\sqrt{\beta_2}$ are asymmetry and excess coefficients.

Irving W. Burr (Burr 1942) proposes a relatively malleable system through a differential equation, in which the unknown variable is given by this distribution function

$dF = F(1/F)g(x)dx$

where $0 \leq F \leq 1$, and $g(x)$ is a non-negative function, chosen conveniently.

Although the Burr system provides twelve explicitly expressed distribution functions, it seems that only one of these functions made a name for itself in statistics. It's the "Burr distribution":

$$F(x;c,k) = 1 - \left(1 + x^c\right)^{-k}, x \geq 0, c, k > 0 \quad (17)$$

1. A general model

In the followings, we will propose a general differential equation, which has the interesting property that depending on the particular choices of the implicated elements, it provides the distribution function or the probability density, which neither one of the other systems does.

The differential equation is:

$$\frac{d\varphi}{dx} = a(x)\cdot\varphi^{\alpha} + b(x)\cdot\varphi^{\beta}, \quad (18)$$

where φ is a strictly positive real function, for x usually between $(0, +\infty)$, $a(x)$ and $b(x)$ are continuous functions defined on R, and α, β two random numbers.

We will demonstrate that this equation provides distribution functions or density functions, by case. First of all, let us notice that for

$b(x) = -a(x), \quad \alpha = 1$ and $\beta = 2$

we have

$$\frac{d\varphi}{dx} = \varphi(1-\varphi)\cdot a(x), \quad (19)$$

which is precisely the Burr model; therefore, this equation can be considered a natural generalization of the Burr model.

In the international literature, there are procedures which generate functions for laws such as the exponential law, power, normal, log-normal etc.

In the followings, we develop a highly general law, the Weibull law, which involves other models as particular cases.

2. The Weibull Model

If we chose $a(x)=b(x)=\frac{k}{\theta}x^{k-1}, \theta, k>0, \alpha=0, \beta=1$ and determine

$$\frac{d\varphi}{dx}=(1-\varphi)\frac{k}{\theta}x^{k-1} \quad \text{or} \quad \frac{d\varphi}{1-\varphi}=\frac{k}{\theta}x^{k-1},$$

meaning that:

$$-\ln(1-\phi)=\frac{x^k}{\theta}, \quad \text{so that} \quad 1-\phi=\exp\left\{-\frac{x}{\theta}\right\}$$

therefore

$$\varphi(x;\theta,k)=1-\exp\left\{\frac{-x^k}{\theta}\right\}, \quad x\geq 0, \quad k>0 \tag{20}$$

which is the well-known Weibull distribution.

Let us look at some historic elements: in 1939, the Swedish engineer Waloddi Weibull, professor at the Royal Institute of Technology in Stockholm (KTH - Kungl Tekniska Högskolan), proposed a statistical model to describe the spread manner of the observed values during experiments regarding the resistance of materials.

He started from the idea that the "fatigue" of the materials cannot be described realistically by the exponential model, since its failure rate does not depend explicitly on time. The model can also be used in the case of banking incidents, due to its general virtues.

In order to avoid this fault of the exponential case, Weibull takes into consideration the following distribution:

$$X: F_x(x;\gamma,\theta,k)=1-\exp\left\{-\left(\frac{x-\gamma}{\theta}\right)^k\right\}, \tag{21}$$

where $x\geq\gamma>0, \quad \theta,k>0$. For the particular case of $\gamma=0$ and $k=1$, it contains the exponential model, while for $\gamma=0$ and $k=2$, Lord Rayleigh's model (1842-1919).

Let us observe that for Rayleigh's case

$$F_x(x;0,\theta,2)=1-\exp\left\{-\left(\frac{x}{\theta}\right)^2\right\}, x\geq 0, \quad \theta>0 \tag{22}$$

the incident intensity is a linear time function $z(x)=\frac{2}{\theta}\cdot x$ and that it's obvious that if $x_1<x_2$ than $z(x_1)<z(x_2)$, and therefore the respective indicator is directly proportional to time. This aspect makes the Rayleigh model be an efficient describer of usual incidents.

Weibull's papers were published in a small journal at the time (The Swedish Technical Academy Annals - Ingeniörs Vetenskaps Akademieus Handligar), and remained without immediat echo. Only after the Second World War, in 1951 more precisely, did Weibull re-start his research through the American magazine, Journal of Applied Mechanics, under the title "A statistical distribution of wide applicability", which started a real "Weibull euphoria".

Later on, in 1977, Weibull himself commenced a bibliographic research and identified a number of 1019 articles and 36 book titles which dealt with various aspects of the model. Weibull had already expressed his intentions of completing this study through the investigation of other publications besides the ones in English, but his death, in 1979, put an end to these plans.

An important explanation is imposed: Weibull has the merit of highlighting the wide range of applications of the model, but theoretically, it was deduced by the French mathematician Maurice Fréchet back in 1927 (Fréchet 1927).

The British statisticians R. A. Fisher and L. H. Tippet deduce it in a study concerning limiting forms of the extreme statistics (Fisher & Tippet 1928).

The Russian academician B. V. Gnedenko also studies the model thoroughly through these limiting distributions (Gnedenko 1943).

All these papers were purely mathematical, and they didn't have the necessary impact among practicians, while the Weibull Model entered and remained in the post-war scientific literature definitely related to the name of the Swedish engineer.

In Romanian translation, the first large paper to present a study in the field was due to the Russian authors Gnedenko B. V., Beleaev I.K., Soloviev A. D. – "Mathematical methods in the safety theory", together with spreading the term of "safety" (faith, trust, solidity, the Russian надежность) (Gnedenko & al. 1968).

In the francophone literature, also accessible in Romania, the monographic paper of two French engineers benefited from a certain spreading: "Fiabilité et Statistique Previsionelles: Méthodes de Weibull" (Pollard & Rivoire 1971). They were also the ones to introduce the expression "Weibull Method", taken by the Japanese under the term "Waiburumokei" and by the Russians as "Pravila Veibulla - Правила Veibulla", meaning Weibull's Rule.

Thus considering the bi-parametric distribution

$$X : F_x(x;\theta,k) = 1 - \exp\left\{-\left(\frac{x}{\theta}\right)^k\right\}, \quad x \geq 0, \quad \theta, k > 0 \tag{23}$$

to which the following density corresponds

$$X : f_x(x;\theta,k) = k\theta^{-k} x^{k-1} \exp\left\{-\left(\frac{x}{\theta}\right)^k\right\}, \quad x \geq 0, \quad \theta, k > 0 \tag{24}$$

For this form

$E(X) = \theta \cdot \Gamma(1 + 1/k)$ and $-Var(X) = \theta^2 \cdot \left[\Gamma(1 + 2/k) - \Gamma^2(1 + 1/k)\right]$

$X_{me} = \theta(\ln 2)^{1/k}$ - the median and $X_{mo} = \theta[(k/1)/k]^{1/k}$ - the mode $(k > 1)$.

If the *k* parameter is relatively large, than we can utilize the approximations:

$E(X) \approx 1 - 057722/\theta + 0{,}98905/\theta^2$ and $Var(X) \approx 1{,}64493/\theta^2$.

We also have $0{,}885 \cdot \theta < E(X < \theta)$ and $F_x(\theta) \approx 0{,}63$.

The variation coefficient $CV(X)$ only depends on the *k* parameter

$$CV(X) = \left[\frac{\Gamma(1 + 2/k)}{\Gamma^2(1 + 1/k)} - 1\right]^{1/2}$$

Estimating the model parameters through the ordinary least squares method, or the maximum likelihood method, are the most common procedures.

A synthesis of the Weibull Method was presented in Romanian (Isaic-Maniu 1983).

The Weibull Function Study. The frequency function or the Weibull operational risk function has a general form:

$$F(t) = 1 - R(t) = \exp\left[-\left(\frac{t-\gamma}{\theta}\right)^k\right] \tag{25}$$

Let us notice in figure 1, that if $t = \gamma$, then the value of this function is 1, and that the function is decreasing for $t > \gamma$. Also, if $t \to \infty$, then $R(t) \to 0$. $R(t)$ decreases monotonously and "abruptly" for $0 < k < 1$, it is convex and decreases less abruptly for the same *k*, but for a higher θ than in the previous case.

For *k* = 1 and θ established, $R(t)$ is monotonously decreasing and is convex (this is in fact the exponential case). For $k > 1$, $R(t)$ decreases if *t* decreases. The value of $R(t)$ for a mission of $t = \theta + \gamma$ (let's say, hours) is always 0,368, which is e^{-1} (the points marked with • in figure 1).

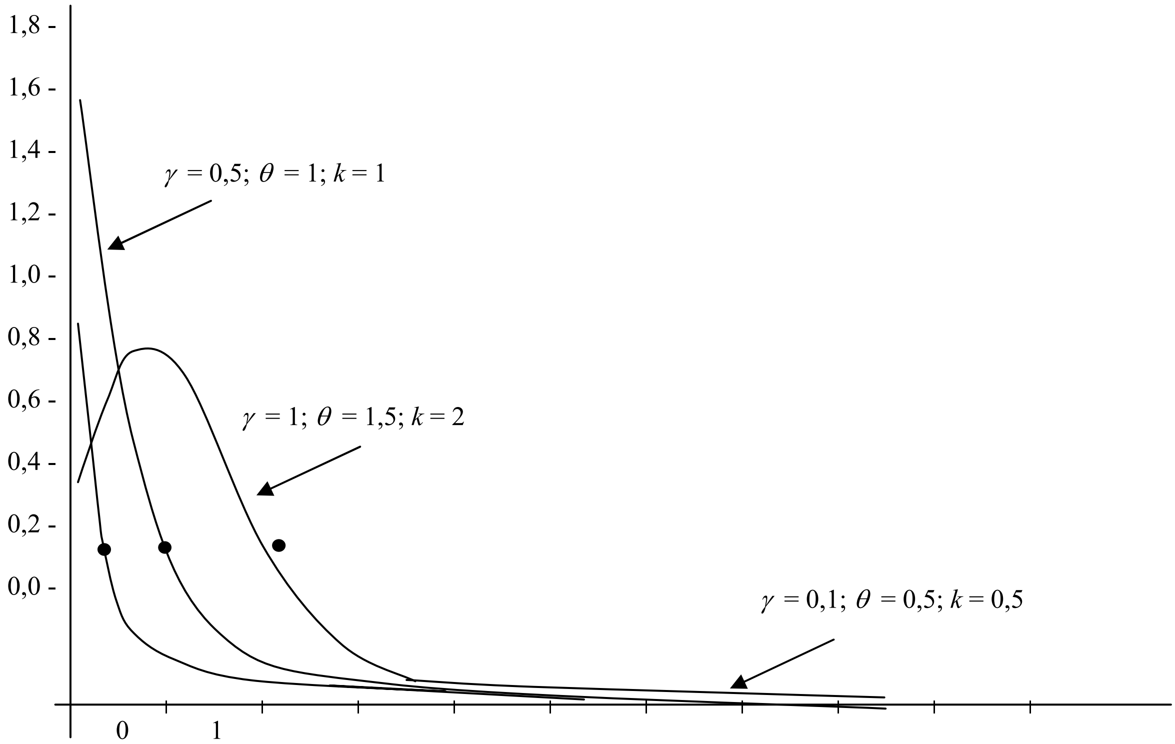

Figure 1. Weibull Reliability Function

An extremely useful indicator is the "conditional reliability function". Assuming that a banking system already functioned without incidents for *t* hours, then it is important to find out the reliability of a mission, another temporal interval of t_0 hours, which is $R(t;t_0)$.

It is known that

$$R(t;t_0) = \frac{R(t+t_0)}{R(t)},$$

where $R(t;t_0)$ is the reliability for a new mission (operation) of t_0 hours, being given the fact that the system functioned without incidents for *t* hours.

Therefore, in the Weibull case

$$R(t;t_0)=\frac{\exp\left[-\frac{(t+t_0-\gamma)^k}{\theta}\right]}{\exp\left[-\frac{(t-\gamma)^k}{\theta}\right]}=\exp\left\{-\left[\left(\frac{t+t_0-\gamma}{\theta}\right)^k-\left(\frac{t-\gamma}{\theta}\right)^k\right]\right\}$$

The t_R reliable life for a system with a given reliability $R(t_R)$, which begins functioning at moment zero, is obtained from:

$$R(t_R)=\exp\left[-\left(\frac{t_R-\gamma}{\theta}\right)^k\right] \tag{26}$$

through logarithmation

$$t_R=\gamma+\theta\{-\ln[R(t_R)]\}^{1/k}$$

This is given by the life period in which the system will operate without incidents with a given probability $R(t_R)$.

If $R(t_R)$ = 50%, then t_R is precisely the "median interval".

Estimating the model parameters based on truncated observations. Because of the fact that the most frequently encountered situation is not based on total observations but on partial ones, whose lengths are interrupted, this being a distinct situation from that of technical systems, observed till exhaustion, we shall consider this case, applicable to banking systems observable for a period of time, followed by assessment.

Thus, if we consider X_0 – the time when observations are interrupted, and $X_{(r)}$ the moment of the ***r***th incident, the likelihood function for this incident variant is described as:

$$L=f_{X_x}\left(X_{\gamma_0+1,\ldots,}X_\gamma\right)$$

We obtain

$$L=\frac{n!}{(n-r)!r_0}\left(\frac{\beta}{\theta}\right)^{r-r_0}\cdot\prod_{i=r_0+1}^{r}\left(\frac{x_i-\gamma}{\theta}\right)^{\beta-1}\cdot\exp\left[-\sum_{i=r_0+1}^{r}\left(\frac{x_i-\gamma}{\theta}\right)^{\beta}-(n-r)\left(\frac{x_i-\gamma}{\theta}\right)^{\beta}\right]$$
$$\left\{1-\exp\left[-\left(\frac{t_{r_i+1}-\gamma}{\theta}\right)^{\beta}\right]\right\}^{r_0} \tag{27}$$

Calculating the derivatives of the logarithm for the previous function, relative to the three parameters, we have

$$\frac{\partial\ln L}{\partial\theta}=\frac{(r-r_0)\beta}{\theta}+\beta\frac{\sum_{i=r_0+1}^{r}(x_i-\gamma)^{\beta}}{\theta^{\beta+1}}+\frac{\beta(n-r)(x_r-\gamma)^{\beta}}{\theta^{\beta+1}}-$$
$$-\frac{\beta\left(x_{r_0+1}-\gamma\right)^{\beta}}{\theta^{\beta+1}}\cdot\frac{r_0\exp\left[-\left(x^{r_0+1}-\gamma\right)^{\beta}/\theta^{\beta}\right]}{1-\exp\left[-\left(x^{r_0+1}-\gamma\right)^{\beta}/\theta^{\beta}\right]} \tag{28}$$

for θ, and then for the form parameter

$$\frac{\partial\ln L}{\partial\theta}=(r-r_0)\left(\frac{1}{\beta}-\ln\theta\right)+\sum_{i=r_0+1}^{r}\ln(x_i-\gamma)-\sum_{i=r_0+1}^{r}\left(\frac{x_i-\gamma}{\theta}\right)\ln\left(\frac{x_i-\gamma}{\theta}\right)-$$
$$-(n-r)\left(\frac{x_r-\gamma}{\theta}\right)^{\beta}\ln\left(\frac{x_r-\gamma}{\theta}\right)+r_0\left(x_{r_0+1}-\gamma\right)^{\beta}\ln\left(\frac{x_{r_0+1}-\gamma}{\theta}\right) \tag{29}$$

and, finally, for the γ parameter

$$\frac{\partial \ln L}{\partial \gamma} = (1-\beta)\sum_{i=r_0+1}^{r}(x_i-\gamma)^{-1} + \frac{\beta}{\theta^\beta}\sum_{i=r_0+1}^{r}(x_i-\gamma)^{\beta-1} + (n-r)\frac{\beta}{\theta^\beta}(x_r-\gamma)^{\beta-1}$$
$$-\beta_{r_0}\left(x_{r_0+1}-\gamma\right)^{\beta-1}\cdot\frac{\exp\left[-\left(x_{r_0+1}-\gamma\right)^\beta/\theta^\beta\right]/\theta^\beta}{1-\exp\left[-\left(x_{r_0+1}-\gamma\right)^\beta/\theta^\beta\right]/\theta^\beta} \qquad (30)$$

3 CHARACTERIZING OPERATIONAL BANKING INCIDENTS RISK

In the period July 1 – September 30 2009, the frequency of banking incidents was registered for a group of three banks with a total of 656 operative units.

The observation time values were transformed in standard work hours (8 hours), while the observations and results registered over the incidents were grouped in six day intervals (48 effective hours of operation), the final results being presented as a series in table 2. The banking incidents observations (false banknotes, cloned cards, false documents, informatics system hacking, the devastation of ATMs etc) concerned three banks in different categories of size:

1. Strong bank with 430 operative units.
2. Middle bank with 180 operative units
3. Small bank with 46 operative units.

Table 2. Banking incident distribution

No.	Intervals of observation	Operational Incidents by bank type			Total operational incidents
	(hours)	(k_1) 1	(k_2) 2	(k_3) 3	K Total
1	0 - 48	36	19	3	58
2	48 - 96	48	23	7	78
3	96 - 144	38	28	10	76
4	144 - 192	60	30	11	101
5	192 - 240	75	36	10	129
6	240 - 288	42	42	11	95
7	288 - 336	47	35	7	89
8	336 - 384	40	34	8	82
9	384 - 432	46	28	12	86
10	432 - 480	51	34	11	96
11	480 - 528	42	31	11	84
12	528 - 576	39	29	8	76
13	576 - 624	49	12	6	67
14	624 - 672	21	16	6	43
TOTAL		$\Sigma k_1 = 634$	$\Sigma k_2 = 397$	$\Sigma k_3 = 121$	$\Sigma k = 1160$

The total operational incidents, by hourly intervals, and by category of bank, are presented in table 2; both non-probabilistic indicators and indicators calculated based on a previously validated model shall be determined.

The mean of the overall operational incidents, by type of bank was:

$\overline{k} = 82,85$ - total mean

$\overline{k}_1 = 45,29; \overline{k}_2 = 28,36; \overline{k}_3 = 82,85$

The average number of operational incidents by operational unit is:

$\overline{r} = 1,76$ - general rate

$\overline{r}_1 = 1,47; \overline{r}_2 = 2,21; \overline{r}_3 = 2,63$

4 CONCLUSIONS

The main conclusion that can be drawn from interpreting the results concerns the high reliability of large banks, as the number of operational incidents is lower.

Establishing operational reliability indicators requires achieving a probabilistic model regarding the occurrence of incidents.

The relatively high frequency of the incidents excludes the Poisson model (for the incidents frequency) or the exponential model (for the time period when events occur) and indicates as viable the Weibull model, which includes as particular cases other models, through its generalizing qualities.

For the data in table 2 regarding the operational incidents distribution, the calculations lead to a mean of the operational time $\overline{t} = 323,35$ hours (in operational hours), with a standard deviation of $\hat{s} = 289,3$ hours.

Applying the ordinary least squares method for the Weibull model parameters, in the tri-parametric variant, we have:

$\hat{\theta} = 357,65$ hours

$\gamma = 6,58$ hours

$\beta = 2,1$

The distribution function for the operational reliability model, for t = 240 hours, it is considered that the effective work time in a month is:

$$\hat{F}(t) = 1 - \exp\left[-\left(\frac{t - 6,58}{357,65}\right)^{2,1}\right] = 0,335$$

The reliability function for an operational monthly interval:

$$\hat{R}(t) = 1 - \exp\left[-\left(\frac{t - 6,58}{357,65}\right)^{2,1}\right] = 0,6648$$

The intensity of the banking operational incidents:

$$\hat{h}(t) = \frac{2,1(240 - 6,58)^{2,1-1,0}}{357,65^{2,1}} = 0,00367$$

The final conclusion concerns the low reliability of the system of the three banks under observation. With 66.5% chance of surpassing the 240 hour period of functioning without operational incidents, the system of the three banks proves to be extremely vulnerable.

The calculations were determined on the system of the three banks, and for smaller banks, reliability proves to be about 0.6 times lower as against to larger banks.

This fact demands increased preventive measures for this group of banks, parallel to a general preoccupation of reducing operational incidents.

REFERENCES

Bârsan-Pipu, N., Isaic-Maniu, Al. & Vodă, V. Gh. 1999: *Defectarea. Modele statistice cu aplicații*, Bucharest, Editura Economica.

Bârsan-Pipu, N. & Popescu, I. 2003. *Managementul riscului. Concepte. Metode. Aplicații.,* Braşov, Editura Universității Transilvania.

Blischke, W., R. & Murthz, D. N. P. 2000. *Reliability. Modeling, Prediction, Oportunization.*, New York, John Wiley and Sons Inc.

Burr, I. W. 1942. Cumulative frequency functions, *Annals of Mathematics and Statistics*, 13: pp. 215-232

Cohen, L. 1998. *Multidimensional Systems and Signal Processing*, vol. 9, Issue 4

Deming, W. E. 1965. Principles of professional statistical practice. *Annals of Mathematical Statistics*, vol. 36, no. 6: page 1883 - 1900.

Dorin, Al. C., Isaic-Maniu, Al. & Vodă, V. Gh. 1994. *Probleme statistice ale fiabilității. Aplicații în cazul sculelor aşchietoare,* Bucharest, Editura Economica.

Dragan, I. 2006. *Caracterizarea statistică a riscului*, Bucharest, Editura Academy of Economic Studies.

Fréchet, M. 1927. Sur la loi de probabilité de l'écart maximum, *Annales de la Société Polonaise de Mathematique,* vol. 6: pp. 93-116.

Fisher, R. A. & Tippet, L. H. 1928. Limiting forms of the frequency distributions of the largest or smallest member of a sample, *Proc. Cambridge Philos. Soc.*, 24: 180-190.

Gertsbakh, I. 2000. *Reliability Theory with Applications to Preventive Maintenance*, Berlin, Springer – Vwerlag.

Gnedenko B. V. 1943. Sur la distribution limite du terme maximum d'une serie aleatorie*, Annalia Mathematica* 44: pp. 423-453.

Gnedenko, B. V., Beleaev I. K. & Soloviev A. D. 1968. *Metode matematice în teoria siguranței,* Bucharest, Editura Tehnică.

Isaic-Maniu, Al. 1983. *Metoda Weibull. Aplicații.* Bucharest, Editura Academiei Române (LCCN Permalink: http://lccn.loc.gov/85134196; The Library of Congress USA).

Isaic-Maniu, Al. & Vodă, V., Gh. 1997. *Manualul Calității*, Bucharest, Editura Economica.

Isaic-Maniu, Al. & Vodă, V., Gh 2005. On a Homographic Distribution Function, *Economic Computation and Economic Cybernetics Studies and Research*, no. 1-4: pp. 11-18.

Isaic-Maniu, Al. & Vodă, V., Gh 2008. Some Comments on an Entropy – Like Transformation of Soleha and Sewilam, *Economic Computation and Economic Cybernetics Studies and Research*, no. 1-2: pp. 5-11.

Isaic-Maniu, Al. & Vodă, V., Gh 2008. Modelul invers Weibull. Aplicatii, *Economic Computation and Economic Cybernetics Studies and Research,* 2: 29-37.

Isaic-Maniu, Al. & Vodă, V., Gh 2009. Some Inferences on the Ratio Average Lifetime / Testing Time in Acceptance Sampling Plans for Reliability Inspection, *Risk Analysis: Theory & Application, Electronic Journal of International Group on Reliability,* Vol.2, No.3 (14): pp. 51-58

Isaic-Maniu, Al. & Vodă, V., Gh 2009. The Poisson Property in the Case of Some Continuous Distributions, *Revista Romana de Statistica*, no. 2: 56-64.

Johnson, N. L. & Kotz, S. 1970. *Distributions in Statistics. Continuous Univariate Distributions*, vol. 1, Boston, Houghton-Mifflin.

Johnson, N. L. & Kotz, S. 1983. *Encyclopedia of Statistical Sciences*, New York, John Wiley and Sons.

Pearson, K. 1895. *Contributions to the Mathematical Theory of Evolution. Skew Variation in Homogeneous Material.*, London, Phil. Trans. Royal Society

Perote, J. & Del Brio, E. B. 2006. *Multivariate Gram-Charlier Densities*, Universidad Rey Juan Carlos. Dept. de Economía Aplicada y Fundamentos del Análisis Económico, Fac. de Ciencias Jurídicas y Sociales

Pollard, A. & Rivoire, C. 1971. *Fiabilité et Statistique Previsionelles: Méthodes de Weibull*, Paris, Eyrolles.

Savageau, M. A. 1982. A Suprasystem of Probability Distributions, *Biomedical Journal* 24: pp. 323-330

Soleha, El-Moniem, M. & Sewilam, A., Imam 2007. Generalized Rayleigh Distribution Revisited, http://interstat.statjouruals.net/YEAR/2007/abstrats/0702006.php-3k-z.

Vodă, V. Gh. 2009. Some Comments on Statistical Risks, *Risk Analysis: Theory & Application, Electronic Journal of International Group on Reliability,* Vol.2: pp.121-127

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