ELECTRONIC JOURNAL
OF INTERNATIONAL GROUP
ON RELIABILITY

Gnedenko Forum Publications

RELIABILITY: THEORY&APPLICATIONS

JOURNAL IS REGISTERED
IN THE LIBRARY
OF THE U.S. CONGRESS

•

ISSN 1932-2321

•

VOL.1 NO.2 (17)
JUNE, 2010

RELIABILITY
RISK ANALYSIS
MAINTENANCE
SAFETY

Vol.1
No.2 (17)
•
June
2010

San Diego

ISSN 1932-2321

http://www.gnedenko-forum.org/Journal/index.htm

RELIABILITY:
THEORY & APPLICATIONS

Vol.1 No.2 (17),
June, 2010

San Diego
2010

Send your paper

e-Journal ***Reliability: Theory & Applications*** publishes papers, reviews, memoirs, and bibliographical materials on Reliability, Quality Control, Safety, Survivability and Maintenance.

Theoretical papers have to contain new problems, finger practical applications and should not be overloaded with clumsy formal solutions.

Priority is given to descriptions of case studies.

General requirements for presented papers

1. Papers have to be presented in English in MSWord format. (Times New Roman, 12 pt , 1.5 intervals).
2. The total volume of the paper (with illustrations) can be up to 15 pages.
3. A presented paper has to be spell-checked.
4. For those whose language is not English, we kindly recommend to use professional linguistic proofs before sending a paper to the journal.

* * *

The Editor has the right to change the paper title and make editorial corrections.

The authors keep all rights and after the publication can use their materials (re-publish it or present at conferences).

Publication in this e-Journal is equal to publication in other International scientific journals.

Papers directed by Members of the Editorial Boards are accepted without referring.

The Editor has the right to change the paper title and make editorial corrections.

The authors keep all rights and after the publication can use their materials (re-publish it or present at conferences).

Send your papers to

the Editor-in-Chief ,
Igor Ushakov
igorushakov@gmail.com

or

the Deputy Editor,
Alexander Bochkov
a.bochkov@gmail.com

Table of Contents

The reliability growth process applied to a complex system undergoing development and field test involves surfacing failure modes, analyzing the modes, and, in addition to repair, in some cases implementing corrective actions to the surfaced modes. In such a manner, the system configuration is matured with respect to reliability. The conventional procedure of reliability growth implies evaluation of two principal parameters of the NHPP process only for failure rate. Since standard NHPP does not take into account parameters of repairs, it is necessary to develop expanded procedure as the basis for the Availability Growth. It implies evaluation of both: a) the parameters of failure rate and, b) the parameters of repair rate. Authors suggest a model and numerical method to search these parameters.

The problem of the control of the importance of observable laws of change of parameters of reliability (PR) at small statistical data of operating experience or experiment in conditions when the argument has a serial or nominal scale of measurement, concerns to number of the most difficult and insufficiently developed. In particular, at operation of electro installations the important role-played with data on reliability of units of the same equipment, on the reasons of occurrence and character of their damage, law of change PR of the equipment for various classes of a pressure and so forth

The paper presents a new fuzzy reliability model for automated “safety system-protected object” complex. It is supposed that parameters of reliability model and reliability indices are fuzzy variables. Scheduled periodic inspections of safety system are also taken into account. Asymptotic estimates of mean time to accident membership function are proposed.

The paper presents a new reliability model for an automated “safety system-protected object” complex with time redundancy. It is supposed that the time redundancy is caused by a protected object inertia. Scheduled periodic inspections of the safety system are also taken into account. Two-sided estimates of the mean time to accident are proposed.

The paper is devoted to reliability and capability investigation of technological systems, inclusive of development of dynamic reliability model for two-phase product line with buffer storages and multiphase line decomposition

Efficient and fast algorithms of parameters calculation in Burtin-Pittel asymptotic formula for networks with identical and high reliable edges are constructed. These algorithms are applied to a

Bootstrap and resampling methods are the computer methods used in applied statistics. They are types of the Monte Carlo method based on the observed data. Bradley Efron described the bootstrap method in 1979 and he has written a lot about it and its generalizations since then. Here we apply these methods in an empirical Bayes estimation using bootstrap copies of the censored data to obtain an empirical prior distribution.

Continuous-time Markov chains are an important subclass in stochastic processes, which have facilitated many applications in business decisions, investment risk analysis, insurance policy making and reliability modeling. One should be fully aware that the existing continuous-time Markov chains theory is merely a framework under which the random uncertainty governs the phenomena. However, the real world phenomena often reveal a reality in which randomness and vagueness co-exist, and thus probabilistic continuous-time Markov chains modeling practices may be not wholly adequate. In this paper, we define random fuzzy continuous-time Markov chains, explore the related average chance distributions, and propose both a scheme for parameter estimation and a simulation scheme. It is expected that a foundational base can be established for reliability modeling and risk analysis, particularly, repairable system modeling.

The theoretical background and technical information for the program are presented. Further, the components of the program are described and user manual is given.

The paper is concerned with the construction of lower bounds for the reliability of a system when statistical data come from independent tests of its elements. The overview of results known from literature and obtained under the assumption that elements in a system are independent is given. It has been demonstrated using a Monte Carlo experiment that in case when these elements are dependent and when their dependence is described by Clayton and Gumbel copulas these confidence bounds are not satisfactory. New simple bounds have been proposed which in some practical cases perform better than the classical ones.

The paper deals with analysis of ships operation stages in open water areas effected by environmental constraints influencing on ship sea keeping parameters in application to ferry “Stena Baltica” operated in the Baltic Sea between Gdynia and Karlskrona harbors.

In the paper, definitions and theoretical results on system operations process, multi-state system reliability, risk and availability modelling are illustrated by the example of their application to a bulk cargo transportation system operating in Gdynia Port Bulk Cargo Terminal. The bulk cargo transportation system is considered in varying in time operation conditions. The system reliability structure and its components reliability functions are changing in variable operation conditions. The system reliability structures are fixed with a high accuracy. Whereas, the input reliability

In recent years the intensive efforts in developing and producing electronic devices have more and more critical inference in many areas of human activity. Engineering is one of the areas which have been also importantly affected. The paper deals with dependability namely reliability analysis procedure of a highly reliable item. The data on manufacturing and operating of a few hundred thousands pieces of electronic item are available and they are statistically a very important collection/set. However, concerning some items the manufacturing procedure was not checked and controlled accurately. The procedure described in the paper is based on the thorough data analysis aiming at the operating and manufacturing of these electronic elements. The results indicate some behaviour differences between correctly and incorrectly made elements. It was proved by the analysis that dependability and safety of these elements was affected to a certain degree. Although there is a quite big set of data the issue regarding the statistical comparability is very important.

PARAMETER ESTIMATIONS FOR AVAILABILITY GROWTH

Z. Bluvband & S. Porotsky

•

A.L.D. , Tel - Aviv, Israel

e-mail: zigmund@ald.co.il, sergey@ald.co.il

ABSTRACT

The reliability growth process applied to a complex system undergoing development and field test involves surfacing failure modes, analyzing the modes, and, in addition to repair, in some cases implementing corrective actions to the surfaced modes. In such a manner, the system configuration is matured with respect to reliability. The conventional procedure of reliability growth implies evaluation of two principal parameters of the NHPP process only for failure rate. Since standard NHPP does not take into account parameters of repairs, it is necessary to develop expanded procedure as the basis for the Availability Growth. It implies evaluation of both: a) the parameters of failure rate and, b) the parameters of repair rate. Authors suggest a model and numerical method to search these parameters.

1. INTRODUCTION

Accurate prediction and control of reliability plays an important role in the profitability and competitive advantage of a product. Service costs for products within the warranty period or under a service contract are a major expense and a significant pricing factor. Proper spare part stocking and support personnel hiring and training also depend upon good reliability fallout predictions. On the other hand, missing reliability targets may invoke contractual penalties and cost future business.

Telecommunication networks, oil platforms, chemical plants and airplanes consist of a great number of subsystems and components that are all subject to failures. Reliability theory studies the failure behavior of such systems in relation to the failure behavior of their components, which often isn't easier to analyze. There are multiple failure analysis methods in the design and development phase, like FMECA (Failure Mode Effect and Criticality Analysis), FTA (Fault Tree Analysis), ETA (Event Tree Analysis), BFA (Bouncing Failure Analysis), Markov chains, etc. The Analysis of failures, faults and errors from the field (Manufacturing, Test, Operation and Support) is usually performed by FRACAS (Failure Reporting, Analysis and Corrective Action System) using the investigation of the Physical nature of failures and studying the possible causes and roots of the Failures.

Typical task of Reliability Analysis is Reliability Growth Analysis, which deals with failures in the repairable systems. A repairable system is one which can be restored to satisfactory operation by any action, including parts replacements or changes to adjustable settings. When discussing the rate at which failures occur during system operation time (and are then repaired) we will define a Rate Of Occurrence Of Failure (ROCF) or "repair rate".

For systems with repairable failures the standard model is NHPP – Non-Homogeneous Random Poison Process. According this model Amount of Failures into small interval

[T; T + t] is equaled for Rate(T)t. For NHPP Power Law (Crow model, AMSAA model) it is assumed, that

$$Rate(T) = \lambda\beta T^{(\beta-1)}$$

i.e. first failure is according Weibull Distribution, λ and β are Power Law parameters.

For any NHPP process with intensity function Rate(T), the distribution function (CDF) for the inter-arrival time t to the next failure, given a failure just occurred at time T, is given by

$$F(t) = 1 - \exp\left(-\int_0^t R(T+t)dt \right)$$

In particular, for the Power Law the waiting time to the next failure, given a failure at time T, has probability density function (PDF)

$$f(t) = \lambda\beta (T+t)^{\beta-1} \exp\left(-\lambda\left((T+t)^{\beta} - T^{\beta}\right)\right)$$

This NHPP Power Law model really is same as Duane model, for which is assumed, that

$$MTBF_{cumue} = \gamma (t-\delta)^{\alpha}$$

where γ and α are Duane model parameters.

Following expressions are right:

$$\gamma = \frac{1}{\lambda} \quad , \quad \alpha = 1 - \beta$$

Below all models are for NHPP Power Law parameters search, parameters of corresponding Duane model are recalculated according above expressions.

During analysis systems with repairable failures, two main problems are solved:

– Definition of NHPP distribution parameters by means of statistics of failures

– Forecasting of some output criteria (Amount of failures on some period, MTBF, etc.) based on obtained parameters.

This classical task of Reliability Growth Analysis physically may be extended for the Availability Growth Analysis, which assumes, that repairable failures and its restoration are performed due to two factors – failure rate and repair rate [1]. For this task we have to define parameters of "mixed" flows – failures and repairs – instead of single ("continuous") flow for standard NHPP task.

The rest of the article is organized as follows. Availability Growth model as extension of Reliability Growth model is introduced in Chapter 2. First we consider simplest case – single system. Various techniques to solve this model are considered in Chapter 3. In Chapter 4 we present how the Cross-Entropy method can be applied to search parameters of proposed model. The more challenging tasks of Availability Growth are tackled in Chapter 5. In Chapter 6, we show how to get some output estimations of Availability Growth.

2. DEFINITION OF DISTRIBUTION PARAMETERS FOR SINGLE SYSTEM

First consider case of single system. Input statistics of failures and repairs is following: TF[1], TR[1],…, TF[i], TR[i],…, TF[n], TR[n], where

- n is amount of failures
- TF[i] is time of failure number i (failure arrival time – FAT)
- TR[i] is time of finishing of repair number i, i = 1…n

We assume, that both flow of failure and flow of repairs are NHPP processes. So,

$$MTBF(t) = \frac{t^{(1-\beta_f)}}{\lambda_f}$$ – for failure flow

$$MTTR(t) = \frac{t^{(1-\beta_r)}}{\lambda_r}$$ – for repair flow

We have to define parameters λ_f, β_f, λ_r, β_r and for this purpose we will use MLE (Maximum Likelihood Estimations) approach.

Comment. Generally speaking, we can describe failure and/or repair flows by means of some other NHPP Law (e.g. Exponential Law of ROCOF), but usually NHPP Power Law is used.

To define these parameters for flow of failures, we have to consider two different cases:

- Rate of Failures doesn't change during repair.

In this case the deterioration (or reliability growth) of the system during repair is absent (i.e. during repair the failure rate of tire isn't increased, because really it isn't according time, rather according miles). For this case the classical exact Crow formulas [2] are applicable:

$$\beta_f = \frac{n}{\left(n \log(Z[n]) - \sum_{i=1}^{n} \log(Z[i])\right)} \quad , \quad \lambda_f = \frac{n}{\left(Z[n]^{\beta_f}\right)}$$

where Z[i] are "shifted" failure arrivals times and last measurement time (without influence of repair time):

Z[1] = TF[1], Z[i + 1] = Z[i] + (TF[i + 1] – TR[i])

- Rate of Failures changes during repair as usually.

In this case the deterioration of the system during repair is normal (i.e. during repair the failure rate of car is increased according time). For this case the classical Crow formulas are not applicable. Conditional PDF, that i-th failure will be at moment TF[i] in condition, that (i-1)-th repair has finished at moment TR[i-1], is

$$P_f[i] = \lambda_f \beta_f \left(TF[i]^{(\beta_f - 1)}\right) \exp\left(-\lambda_f \left(TF[i]^{\beta_f} - TR[i-1]^{\beta_f}\right)\right) \tag{1}$$

Comment. In this expression for i = 1 we use TR[0] = 0.

$$Negative\ \ Logarifm\ \ Liklihood_{f} = -\sum_{i=1}^{n}\log(P_f[i]) \tag{2}$$

Our goal is to search values of λ_f and β_f such, that *Negative Logarifm Likelihood$_f$* will be minimum.

To define required parameters for flow of repairs, we have to consider only one case – Rate of Repairs changes during repair and non-repair without differences. Formulas will be same, as above. Conditional PDF., that i-th repair will finish at moment TR[i] in condition, that i-th failure was at moment TF[i], is

$$P_r[i] = \lambda_r \beta_r \left(TF[i]^{(\beta_r - 1)}\right)\exp\left(-\lambda_r\left(TR[i]^{\beta_r} - TF[i]^{\beta_r}\right)\right) \tag{3}$$

$$Negative\ \ Logarifm\ \ Liklihood_{r} = -\sum_{i=1}^{n}\log(P_r[i]) \tag{4}$$

Our goal is to search values of λ_r and β_r such, that *Negative Logarifm Likelihood$_r$* will be minimum.

3. COMPARISON OF DIFFERENT GLOBAL OPTIMIZATION APPROACHES

Global Optimization of non-linear function is a common task of a lot of practical problems (supply optimization, text categorization, distribution parameters estimation, etc. and etc.). For example, concerning problem of Parameters Estimation, a Linear Regression model can support only a few cases. It couldn't be used for interval and multiplied censored data, for 3 parameter Weibull estimation, Duane model with multiple systems, Gompertz model, etc. and etc. For this numerous cases we have to search distribution parameters by means of non-linear and non-convex, global optimization – both for MLE and non-linear regression using.

Our task is to search value of **Z**, which provides min G(**Z**)
under constraints $Low_j \le z[j] \le High_j$, j = 1…K, where:

- Z = {z[1],…,z[j],…z[K]} is a set (vector) of parameters
- K is amount of parameters
- Low_j is Low Boundary of Parameter j value (j = 1…K)
- $High_j$ is High Boundary of Parameter j value (j = 1…K)
- G is some Goal Function (analytical-form or, perhaps, table or even algorithm-calculated-form), dependent of vector Z.

To solve this task, two different approaches can be used:

- To write and transform derivatives of Goal Function (e.g., Logarifm-Likelihood for MLE method, Sum of Leased Squares for Non-Linear Regression method, etc.) for each single task, to solve system of non-linear equations, corresponding these situations, to support Global Minimum finding (instead of possible local minimum finding) by means of convex/concave check, etc.
- To use "direct search methods", provided universal search of Global Minimum (without analytical definition of derivatives).

For first approach using we have to define complex analytical expressions for derivatives for each single task. Early usually this approach was used and for each single task it required additional resources both for algorithm developing and software implementation. For example, Quasi-Newton method minimizes the Negative Logarifm Likelihood Function in order to bring partial derivatives to zero. Perhaps, it isn't very hard for simple cases, but for more complex models this approach requires essential additional time.

We propose to use second (universal) approach, which will allow us to search optimal solution not only for single task, but rather for all same situations (LogNormal, Gamma and other distributions, MLE for repairable failures, Non Linear Regression for Gompertz model, etc.), and, generally speaking – for all complex non-convex, multi-extremal optimization tasks. In differ of "derivative" oriented algorithms, the proposed approach will require only one implementation.

For second approach there are developed a lot of methods, based on gradient (or, if a goal function hasn't gradient – on pseudo-gradient) calculation and analysis. But for many real tasks the Goal Function isn't convex, it has many Local Minimums. In these cases such approaches require to know initial point of search, which has to be not far from optimal solution. In such optimization algorithms the initial guesses for the parameters are very crucial. But really we often don't know some information to define this initial point. So, it is impossible to use regular methods (gradients-based).

For Global Optimization Task we propose to use one of the RANDOM SEARCH oriented methods – Cross-Entropy Optimization [3]. It is relatively new random-search oriented approach (for comparison with Genetic Algorithm, implemented as Toolbox on Matlab, or Simulated Annealing Algorithm), but it has provided very good results for several analogous tasks.

4. SHORT DESCRIPTION OF CROSS-ENTROPY ALGORITHM

The method derives its name from the cross-entropy (or Kullback-Leibler) distance - a well known measure of "information", which has been successfully employed in diverse fields of engineering and science, and in particular in neural computation, for about half a century. Initially the Cross-Entropy method was developed for discrete optimization [3], but later was successfully extended for continuous optimization [4]. The Cross-Entropy method is an iterative method, which involves the following two phases:

- Generation of a sample of random data. Size of this data is 500…5000 random vectors of each algorithm steps, amount of steps is 50…100. Generation is performed according to a specified random mechanism.
- Updating the parameters of the random mechanism, on the basis of the data, in order to produce a 'better" sample in the next iteration. Choice of these parameters is performed by means of maximization of Cross-Entropy function. This optimization is performed on the each algorithm step, but in differ on global optimization usually this optimization is performed VERY EASY and FAST, because Cross-Entropy function is convex.

On the first phase we generate sample $Z_1 \ldots Z_V \ldots Z_N$, which has size of N different parameter sets. This generation is performed according common Probability Density Function F(Z) for parameter vector Z, which was calculated on the previously step of the algorithm.

For each v from N (v = 1…N) generated parameter vectors the value of Goal Function is calculated. Then best N_{EL} (N_{EL} = 10…50) parameter vectors Z from all N generated are selected – it is named ELITE part from full sample. This selection is performed according Goal Function values, i.e. parameter vector with number 1 will have minimum value of Goal Function, parameter vector with number 2 will have second value of Goal Function, parameter vector with number N_{EL} will have N_{EL} ordered value of Goal Function.

After this the algorithm calculates new values of the Probability Density Function F(Z) – it is second phase of each algorithm step.

The aim of the new function F(Z) is to maximize Cross-Entropy Function. On the general case the Cross-Entropy Function is following:

$$\sum_{v=1}^{N_{EL}} \ln\{F(Z_V)\}$$

which is Kullback-Leibler probability measure of distance between different Probability Density Functions. In this formula Z_V – value of generated parameter vector on the v-th set of Elite part of current sample.

So, first we have to choice type of PDF to generate random parameter vectors Z. For continuous optimization we can use following types of PDF:

- Beta PDF.
- Normal PDF.
- Double-Exponential PDF
- Etc.

Usage of the Normal PDF F(Z) is advantageous, since in contrast to Beta and Double-Exponential PDFs the Normal PDF allows analytical solution of above task. Other types of PDF involve numerical solution. It is known following analytical solution for Normal PDF parameters (with respect to Mean and Covariance Matrix) of function F(Z):

$$\text{Mean }[j] = \sum_{v=1}^{N_{EL}} \frac{Z_V[j]}{N_{EL}}$$

$$\text{Covariance}[i,j] = \sum_{v=1}^{N_{EL}} (Z_V[i] - \text{Mean}[i]) \frac{(Z_V[j] - \text{Mean}[j])}{N_{EL}} \quad ...i, j = 1...K$$

We have to prevent the too earliest occurrences of the PDF parameter, because in this case optimization is stopped non-correct (PDF will be simply Dirak function!). For this aim instead of simple choice by means of independent current step result analysis we will use smoothed updating procedure:

- $\text{Mean}[j](t) = \alpha\ \text{Mean}_{prel}\ [j](t) + (1 - \alpha)\text{Mean}[j](t-1)$

where:

$\text{Mean}_{prel}\ [j](t)$ – preliminary value of Mean[j], which we had got on current step t, i.e. before smoothed updating,

Mean[j](t) – final value of Mean[j], which we had got on current step t, i.e. after smoothed updating,

Mean[j] (t-1) – final value of Mean[j], which we had got on previously step (t-1),

α – smoothing parameter for Mean updating,

t – step number

- $\text{Cov}[i, j](t) = \zeta(t)\text{Cov}_{prel}\ [i, j](t) + (1 - \zeta(t))\text{Cov}[i, j](t - 1)$,
- $\zeta(t) = \zeta - \zeta(\ (1 - 1/t)^{\gamma}$,

where:

Cov_{prel} [i, j](t) – preliminary value of Covariance[i, j], which we had got on current step t, i.e. before smoothed updating,

Cov[i, j](t) – final value of Covariance[i, j], which we had got on current step t, i.e. after smoothed updating,

Cov[i, j](t-1) – final value of Covariance[i, j], which we had got on previously step (t-1),

ζ and γ – smoothing parameters for Covariance updating.

As seen, for PDF parameter Mean we use fixed smoothing parameter α and for PDF parameter Covariance we use dynamic (dependent of step number) smoothing parameter $\zeta(t)$.

5. SOME EXTENSIONS

5.1 Multiple Systems

In this case the input statistics of failures and repairs will be following: TF[j, 1], TR[j, 1],…, TF[j, i], TR[j, i],…, TF[j, n], TR[j, n], where

- k is amount of systems
- n(j) is amount of failures/repairs on system j
- TF[j, i] is time of failure number i on system number j
- TR[j, i] is time of finishing of repair number i on system number j, i = 1…n(j), j = 1…k.

For definition of λ_f and β_f we have to minimize following Goal Function:

$$Negative\ Logarifm\ Likelihood_f = -\sum_{j=1}^{k}\sum_{i=1}^{n(j)}\log(P_f[j,i]) \tag{5}$$

where P_f [j, i] - Conditional PDF, that i-th failure will be at moment TF[j, i] in condition, that (i -1)-th repair has finished at moment TR[j, (i -1)]. For these conditional PDF-s the expression (1) is applicable without some modifications, we only have to use TF[j,i] instead of TF[i] and TR[j, i] instead of TR[i]. Cross-Entropy Optimization algorithm to search parameters λ_f and β_f also will be exactly same, as for case of single system.

For definition of λ_r and β_r all expressions will be analogous.

5.2 How to take into account End Time and Start Time

Formula (1) assumes, that system starts to operate at time 0, and last measurement corresponds for last failure.

If for some single system j we use non-zero start time TS[j], we have to modify expression for P_f [j, i] for i = 1 – to use TR[j, 0] = TS[j] instead of 0 (see comment under formula (1)).

If for some single system j we use additional end (censored) time TE[j], we have to use additional expression P_f [j, i] for i = n(j) + 1 :

$$P_f[j,n(j)+1] = \exp\left(-\lambda_f\left(TE[j]^{\beta_f} - TR[j,n(j)]^{\beta_f}\right)\right)$$

and for this j to use additional component P_f [j, n(j)+1] on expression (5).

5.3 Definition of un-known parameters δ_f and δ_r

Sometimes initial moments (initializations) of failure rate and repair rate are not zeros (don't confuse with start times of single systems !). Suppose, they are δ_f for failure rate and δ_r for repair rate. In this case instead of t we have to use (t – δ_r) and (t – δ_f) in all formulas of NHPP process. We also have to modify expression (1) – instead of TF[i] and TR[i] to use (TF[i] – δ_f) and (TR[i] – δ_f), to modify expression (3) – instead of TF[i] and TR[i] to use (TF[i] – δ_r) and (TR[i] – δ_r).

If values of parameters δ_f and/or δ_r are unknown, we have to search its by means of minimization of *Negative_Logarifm_Likelihood* – not only for parameters β and λ, but also for parameter δ. To search value of parameter δ, we can use Cross-Entropy Optimization algorithm for modified expressions (2) and (4) (for single system) or expression (4) (for multiple systems).

We also have to note, that MLE approach gets us solution for three parameter optimization only for case β>1 (it is widely known fact for Weibull three parameter search). So, for these situations we have to use some other methods, e.g.:

- To use some non-parametric estimation method (for example, well known MCF approach of Nelson [5]) and based of received results to use Least Squares optimization for thee parameters (β, λ, δ). Least Squares non-linear optimization will be performed by means of Cross-Entropy method.
- Based on defined value of parameter δ to correct values of β and λ by means of MLE optimization using expressions (2) or (4).

6. OUTPUT ESTIMATIONS

Based on obtained parameters we can get some estimations and perform numerical analysis.

For instantaneous values of MTBF and MTTR the following formulas are proved:

- $$MTBF_i(t) = \frac{(t-\delta_f)^{(1-\beta_f)}}{\beta_f \lambda_f}$$
- $$MTTR_i(t) = \frac{(t-\delta_r)^{(1-\beta r)}}{\beta_r \lambda_r}$$

For cumulative values of MTBF and MTTR the following formulas are proved:

- $$MTBF_C(t) = \frac{(t-\delta_f)^{(1-\beta_f)}}{\lambda_f}$$
- $$MTTR_C(t) = \frac{(t-\delta_r)^{(1-\beta r)}}{\lambda_r}$$

It is impossible to obtain analytically the exact expression for instantaneous value of Availability depending of time, but approximately we can assume, that

$$Availability_i(t) \approx \frac{MTBF_i(t)}{MTBF_i(t) + MTTR_i(t)}$$

If $\delta_f = \delta_r = \delta$ (for default $\delta_f = \delta_r = 0$) we can simplify last expression:

$$Availability_i(t) = \frac{1}{1 + \left(\frac{\beta_f \lambda_f}{\beta_r \lambda_r}\right)(t-\delta)^{(\beta_f - \beta_r)}}$$

For cumulative (or mean) value of Availability we use formula

$$Availability_i(t) = \frac{\int_0^t Availability_i(x)dx}{t}$$

It is impossible to obtain analytically the exact expression for cumulative value of Availability depending of time, but approximately we can assume, that

$$Availability_c(t) \approx \frac{MTBF_c(t)}{MTBF_c(t) + MTTR_c(t)}$$

If $\delta_f = \delta_r = \delta$ we can simplify last expression:

$$Availability_c(t) = \frac{1}{1 + \left(\frac{\lambda_f}{\lambda_r}\right)(t-\delta)^{(\beta_f - \beta_r)}}$$

It is evident, that if $\beta_f < \beta_r$, the Instantaneous and Cumulative values of Availability increase depending on time (i.e. we see Availability Growth), although $MTBF_i(t)$ and $MTBF_c(t)$ can be reduced. Otherwise, if $\beta_f > \beta_r$, the Instantaneous and Cumulative values of Availability decrease depending on time (i.e. we see Availability Aging), although $MTBF_i(t)$ and $MTBF_c(t)$ can be increased.

7. CONCLUSION

It is important to recognize, that the Availability parameter should be integrated into general process of a system improvement. But currently the technique of the Reliability Growth doesn't take into account the factor of Availability.

The above described procedure was developed in order to calculate and track the Availability (Dependability) measures based on repair rates', as well as failure rates', modification.

The procedure is based on Cross-Entropy Global Optimization algorithm, which is used to optimize MLE function.

8. REFERENCES

1. Bluvband, Z., 1990, Availability-Growth Aspects of Reliability Growth. *Proceedings Annual Reliability and Maintability Symposium*, p. 522-526
2. Crow L., 1990, Evaluating the Reliability of Repairable Systems. Proceedings Annual Reliability and Maintability Symposium, p. 275- 279
3. Rubinstein R.Y. and Kroese D.P., 2004, The Cross-Entropy Method: A unified approach to Combinatorial Optimization, Monte Carlo Simulation and Machine Learning. Springer-Verlag, New York
4. Kroese D.P., Porotsky S., Rubinstein R.Y. 2006, The Cross-Entropy Method for Continuous Multi-Extremal Optimization. Methodology and Computing in Applied Probability, Vol. 8, No. 3, pp.383–407.
5. Nelson W., 2003, Recurrent Events Data Analysis for Product Repairs, Disease Recurrences, and other Applications. ASA-SIAM.

CONTROL THE IMPORTANCE OBSERVABLE LAWS OF CHANGE RELIABILITY OVER OPERATION

Farhadzadeh E.M., Muradaliyev A.Z., Farzaliyev Y.Z.

●

Azerbaijan Scientific-Research and Design-Prospecting Institute
of Energetic AZ1012, Ave.H.Zardabi-94

e-mail: fem1939@rambler.ru

The problem of the control of the importance of observable laws of change of parameters of reliability (PR) at small statistical data of operating experience or experiment in conditions when the argument has a serial or nominal scale of measurement, concerns to number of the most difficult and insufficiently developed. In particular, at operation of electro installations the important role-played with data on reliability of units of the same equipment, on the reasons of occurrence and character of their damage, law of change PR of the equipment for various classes of a pressure and so forth

Let us agree to name the dependences empirical characteristics (ECh.) changes PR. Calculated on statistical data of operation of law of change ECh. caused by functional and statistical components. From the practical point of view the opportunity essentially is of interest to lower the importance of a statistical component.

To estimate laws of change of functional characteristics (FCh.) it is important for experts since these characteristics allow raise reliability of the equipment with the least expenses, to correct maintenance service, to improve the control of a technical condition, to lower expenses for scheduled repairs and so forth

Thus, the problem consists in establishing influence of casual character of estimations PR on laws of change ECh..

For the decision of this problem we shall enter into consideration following concepts and definitions:

- concept of statistical function of distribution (f.d.) the discrete argument measured in a serial scale or in a scale of names also we shall define this function under the formula:

$$F^*(i+1) = \sum_{\nu=1}^{i} Q_\nu^* = \sum_{\nu=1}^{i} n_\nu / n_\Sigma \qquad (1)$$

$$i = 1,(m_r+1) \;\; F^*(1) = 0; F^*(m_r+1) = 1$$

where: Q_ν^* - estimation of probability of display ν-- version of an attribute (VA); n_Σ - number of displays of a considered attribute; m_r – number VA;

Physically $F^*(i)$ designates probability of display located in the certain order of the first i VA.

- concept hypothetical f.d. In particular, if to assume, that the probability of display of each of m_r VA the same and is equal 1/mr function of uniform distribution can be calculated under the formula:

$$F(i+1) = i / m_r \qquad (2)$$

where $i = (1),\ (m_r+1);\ F(1) = 0;\ F(m_r+1) = 1$;

- concept of statistical function of modeled distribution $F_M^*(i)$. Function $F_M^*(i)$ It is similar on structure of function $F^*(i)$ with that difference, that n_ν with $\nu = 1, m_r$ defined at modeling ECh. on distribution $F(i)$;

- the same, but is defined at modeling ECh. on distribution $F^*(i)$. Let's designate this function as $F_M^{**}(i)$:

- alternative assumptions (hypothesis). Here it is necessary to distinguish two strategies. In the first it supposed, that observable law of change $F^*(i)$ corresponds valid. We shall designate this assumption through $H_{1,1}$. The second assumption consists that $F^*(i)$ casually differs from prospective FCh. and, in particular, $F(i)$ corresponds to the uniform law. We shall designate this assumption through $H_{1,2}$.

Let's consider characteristic examples of this strategy. The computer program representing to experts of the recommendation on increase of reliability of the equipment, according to algorithm of calculation chooses units with number of refusals above average value (initial assumption $H_{1,1}$). However the manufacturer of works has certain doubts regarding objectivity of recommendations. It considers (alternative assumption $H_{1,2}$), that the observable divergence is not enough for objective conclusions. It is necessary to note, that in many publications constructed ECh. form the basis for recommendations on change of reliability.

In the second strategy it is supposed, that there are no serious bases to consider, that reliability of units of object is various. We shall designate this assumption through $H_{2,1}$. At the same time statistical data of operation testify to some divergence of number of refusals of units of object which can be motivated those or other reasons (Let's designate the assumption of not casual character of a divergence $F(i)$ and $F^*(i)$ through $H_{2,2}$). It is obvious, that the problem of a choice of this or that assumption consists in comparison of casual character of realizations of distributions $F(i)$ (or $F^*(i)$) and casual character of a divergence of realizations $F(i)$ (or $F^*(i)$) from $F^*(i)$ (or $F(i)$).

- statistics δ_m, defining the greatest divergence between f.d. We shall distinguish:

- empirical value of the greatest divergence between $F(I)$ and $F^*(I)$. It is calculated under the formula:

$$\delta_{m,Э} = \max\{\delta_{1,Э};\delta_{2,Э},...\delta_{m_r,Э}\}, \quad (3)$$

$$\delta_{i,э} = \left|F(i) - F^*(i)\right|; \quad i=2,(m_r+1) \quad (4)$$

- the greatest divergence between $F^*(i)$ and modeled realizations of this distribution $F_M^{**}(i)$. It is calculated under the formula:

$$\delta_{m,\nu}(H_{1,1}) = \max[\delta_{1,\nu}(H_{1,1});\delta_{2,\nu}(H_{1,1});...\delta_{m_r,\nu}(H_{1,1})] \quad (5)$$

$$\delta_{i,\nu}(H_{1,1}) = \left|F^*(i) - F^{**}_{M,\nu}(i)\right| \quad (6)$$

ν=1, N; i=2, (m_r+1); N-number of iterations of modeling $F^{**}_{M,\nu}(i)$.

- greatest divergence between $F^*(i)$ and modeled on $F(i)$ realizations $F_M^*(i)$. It is calculated under the formula:

$$\delta_{m,\nu}(H_{1,2}) = \max\{\delta_{1,\nu}(H_{1,2});\delta_{2,\nu}(H_{1,2});...\delta_{m_r,\nu}(H_{1,2})\} \quad (7)$$

$$\delta_{i,\nu}(H_{1,2}) = \left|F^*(i) - F^*_{M,\nu}(i)\right| \quad \nu=1, N; \quad i=2, (m_r+1) \quad (8)$$

- greatest divergence between $F(i)$ and modeled on $F(i)$ realizations $F^*_{M,\nu}(i)$. It is calculated under the formula:

$$\delta_{m,\nu}(H_{2,1}) = \max\{\delta_{1,\nu}(H_{2,1});\delta_{2,\nu}(H_{2,1});...\delta_{m_r,\nu}(H_{2,1})\} \quad (9)$$

where:

$$\delta_{i,\nu}(H_{2,1}) = \left|F(i) - F^*_{M,\nu}(i)\right| \quad \nu=1,N; \quad i=1,m_r; \quad (10)$$

- greatest divergence between $F(i)$ and modeled on $F^*(i)$ realizations $F^{**}_{M,\nu}(i)$. It is calculated under the formula:

$$\delta_{m,\nu}(H_{2,2}) = \max\{\delta_{1,\nu}(H_{2,2});\delta_{2,\nu}(H_{2,2});...\delta_{m_r,\nu}(H_{2,2})\} \quad (11)$$

$$\delta_{i,\nu}(H_{2,2}) = \left|F(i) - F^{**}_{M,\nu}(i)\right| \quad \nu=1, N; i=2,(m_r+1); \quad (12)$$

- f.d. $\delta_m(H_{1,1})$, $\delta_m(H_{1,2})$, $\delta_m(H_{2,1})$ и $\delta_m(H_{2,2})$. The analytical kind of these distributions is unknown. We shall define these distributions by a method of statistical modeling.

The graphic illustration of sequence of calculations $F^*[\delta_m(H_{1,1})]$ also $F^*[\delta_m(H_{1,2})]$ is resulted on fig.1, and on fig.2 the graphic illustration of sequence of calculations $F^*[\delta_m(H_{2,1})]$ is resulted and $F^*[\delta_m(H_{2,2})]$

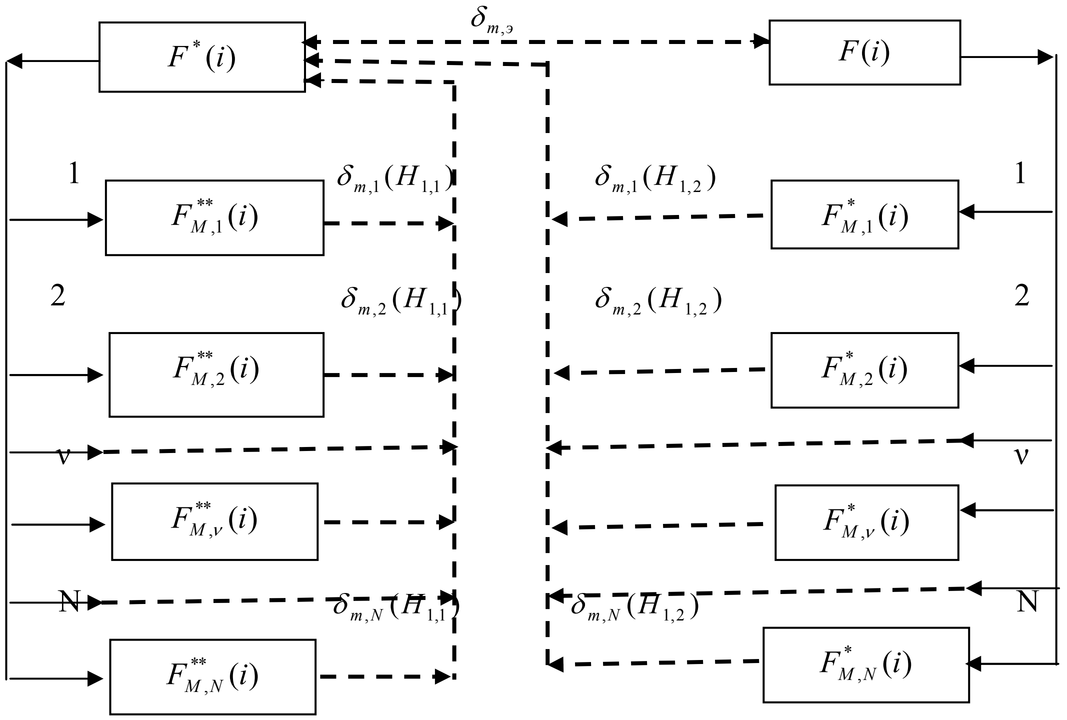


Fig.1. The block diagram of sequence of calculation of distributions $F^*[\delta_m(H_{1,1})]$ and $F^*[\delta_m(H_{1,2})]$

Here by continuous lines the sequence of modeling of distributions $F^{**}_{M,\nu}(i)$ and $F^*_{M,\nu}(i)$, and dotted structure of an estimation of realizations of the greatest deviation of distributions $\delta_{m,\nu}(H_{1,1})$ is shown and $\delta_{m,\nu}(H_{1,2})$.

According to fig.1. The algorithm of calculation $F^*[\delta_m(H_{1,1})]$ also $F^*[\delta_m(H_{1,2})]$ reduced to following sequence of calculations:

1. By program way are modeled n_Σ random numbers with uniform distribution in an interval [0,1]
2. Direct use of these random numbers, especially at small n_Σ leads to essential disorder of probability of occurrence of discrete values δ_m, which kept, and at big enough number of iterations N. As it has been shown in [1] effective method of decrease in a dispersion of distribution $F^*[\delta_m(H_{1,1})]$ as well as $F^*[\delta_m(H_{1,2})]$ application of criterion of Kolmogorov for check of distribution of sample $\{\xi\}_{n_\Sigma}$ to the uniform law is.

According to [2], at factor of trust R calculations spent under the formula:

$$\left(\sqrt{n} + 0{,}12 + \frac{0{,}11}{\sqrt{n}}\right) D_n > C_R \tag{13}$$

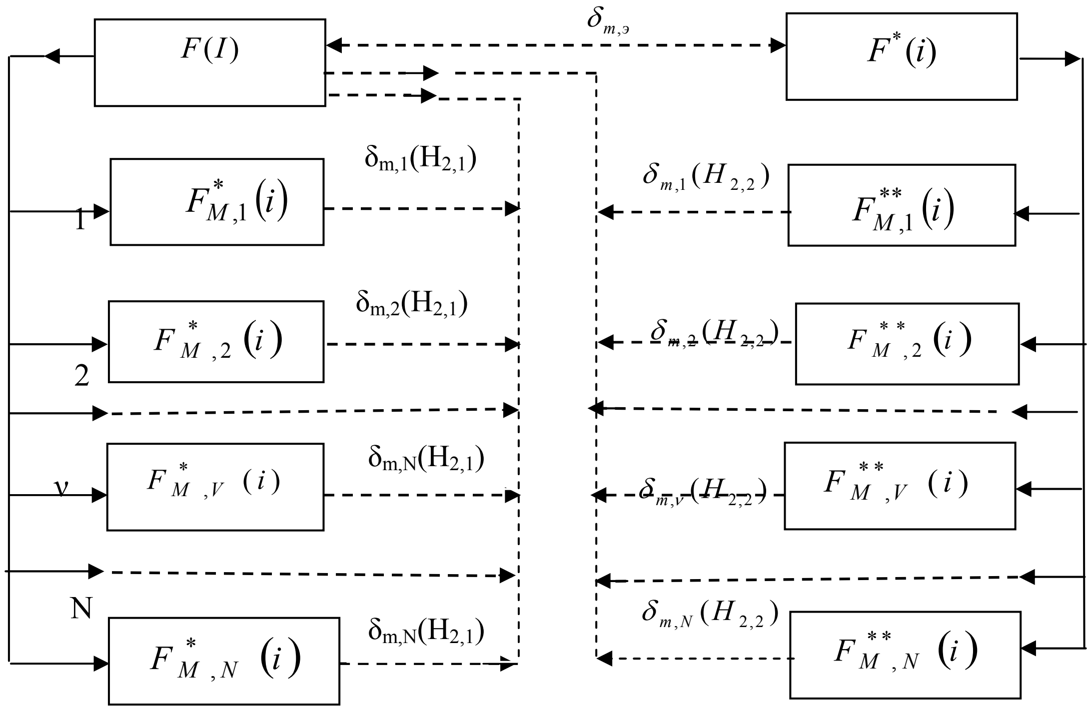


Fig.2. An alternative variant of the block diagram of sequence of calculation of distributions $F^*[\delta_m(H_{2,1})]$ and $F^*[\delta_m(H_{2,2})]$

3. Frequencies of display of each of $m_{r,j}$ VA, by comparison of each of n_Σ random numbers to intervals of distribution $F^*(i)$ under the formula modeled:

$$F^*(i) < \xi \le F^*(i+1) \text{ with } i=1,(m_r+1) \quad (14)$$

4. Under the formula (1) the first realization $F^{**}_{M,1}(i)$ pays off; Decrease in a dispersion of distributions $F^*[\delta_m(H_{1,1})]$ and $F^*[\delta_m(H_{1,2})]$, alongside with application to sample of random variables $\{\xi\}_{n_\Sigma}$ of criterion of Kolmogorov, is reached also by application of a method of the general random numbers. Therefore, if sample $\{\xi\}_{n_\Sigma}$ does not contradict Kolmogorov's criterion, it is remembered;

5. Under formulas (5) and (6) realization $\delta_m(H_{1,1})$ pays off

6. Having repeated 1÷6 (N-1) time, we count N realizations $\delta_m(H_{1,1})$, having arranged which in ascending order we shall receive statistical function of distribution $F^*[\delta_m(H_{1,1})]$.

Calculation statistical function of distribution $F^*[\delta_m(H_{1,2})]$ spent as follows.

7. Random numbers $\{\xi\}_{n_\Sigma}$ in each iteration are not modeled, and undertake from a file $N \cdot n_\Sigma$ of the random numbers, generated at modeling distribution $F^*[\delta_m(H_{1,1})]$.

8. Repeats п.3 with that difference, that comparison is spent under the formula:

$$F(i) < \xi \le F(i+1) \quad (15)$$

where $i = 1,(m_r + 1); F(0) = 0; F(m_r + 1) = 1$;

9. Under the formula similar (1) distribution $F^*_M(i)$ pays off.

10. Under formulas (7) and (8) realization $\delta_m(H_{1,2})$ pays off;

11. Having repeated 8÷11 (N-1) time, we count N realizations $\delta_m(H_{1,2})$, having arranged which in ascending order, we shall receive statistical function of distribution $F^*[\delta_m(H_{1,2})]$.

The algorithm of calculation $F^*[\delta_m(H_{2,1})]$ also $F^*[\delta_m(H_{2,2})$ is practically similar to the above-stated with that difference, that in p.3 modeling is spent under the formula (15), and in under the formula (14).

- Concepts initial (G_1) and alternative (G_2) hypotheses.

We shall agree that if between estimations of a population mean of realizations of the greatest divergence of distributions the below-mentioned inequality takes place:

$$M^*[\delta_m(H_{1,1})] < M^*[\delta_m(H_{1,2})], \text{ то } G_1=H_{1,1};\ G_2=H_{1,2} \quad (16)$$

where:

$$M^*[\delta_m(H_{1,1})] = \sum_{j=1}^{N} \delta_{m,j}(H_{1,1})/N$$

$$M^*[\delta_m(H_{1,2})] = \sum_{j=1}^{N} \delta_{m,j}(H_{1,2})/N$$

If the inequality looks like

$$M^*[\delta_m(H_{1,2})] > M^*[\delta_m(H_{1,2})], \text{ то } G_1=H_{1,2};\ G_2=H_{1,1} \quad (17)$$

- we shall define the distributions reflecting a mistake of the first sort $\alpha[\delta_m(G_1)]$ and the second sort $\beta[\delta_m(G_2)]$ under formulas:

If the parity (12) is fair:

$$\left.\begin{aligned} \alpha^*[\delta_m(G_1)] &= 1 - F^*[\delta_m(H_1)] \\ \beta^*[\delta_m(G_2)] &= F^*[\delta_m(H_2)] \end{aligned}\right\} \quad (18)$$

If the parity (13), is fair

$$\left.\begin{aligned} \alpha^*[\delta_m(G_1)] &= 1 - F^*[\delta_m(H_2)] \\ \beta^*[\delta_m(G_2)] &= F^*[\delta_m(H_1)] \end{aligned}\right\} \quad (19)$$

Distributions $\alpha^*[\delta_m(G_1)]$ also $\beta^*[\delta_m(G_2)]$ are necessary for definition of critical values $\delta_m^*(\alpha_k)$ and $\delta_m(\beta_k)$, where α_k and β_k – a significance value of mistakes of the first and second sort.

However, as distributions $\alpha^*[\delta_m(G_1)]$ also $\beta^*[\delta_m(G_2)]$ are discrete, direct definition $\delta_m^*(\alpha_k)$ and $\delta_m^*(\beta_k)$ appears impossible. The values α_k accepted in an engineering practice and β_k, equal 0,1 or 0,05 in the list of discrete values of distributions $\alpha^*[\delta_m(G_1)]$ and $\beta^*[\delta_m(G_2)]$, as a rule, are absent. As critical values of statistics for mistakes of the first and second sort values $\delta_m(\alpha \le \alpha_k)$ and $\delta_m(\beta \le \beta_k)$, corresponding their nearest smaller values get out. We shall designate them through $\delta_m(\alpha_k^\partial)$, $\delta_m(\beta_k^\partial)$ where the index «∂» will mean the valid critical values of mistakes of the first and second sort.

- choice of one of two assumptions (H_1 and H_2) is spent by the control of performance below-mentioned of some heuristic restrictions [3]:
- it is considered, that if minimal (optimum from the point of view of a minimum of mistakes of the first and second sort) risk of the erroneous decision less than admissible (critical) value γ_K the divergence of assumptions H_1 and H_2 is essential, and the preference is given hypothesis G_1 if $\delta_{m,э}^*$ does not exceed optimum value $\delta_{m,opt}^*$ (corresponding γ_{opt}). Otherwise, i.e. when $\delta_{m,э}^* > \delta_{m,opt}^*$, the preference is given hypothesis G_2;
- it is considered, that if $\delta_{m,э}^*$ more or it is equaled $\delta_m(\alpha_k^\partial)$, hypothesis G_1 should be rejected;
- it is considered, that if $\delta_{m,э}^*$ less or it is equaled $\delta_m(\beta_k^\partial)$, hypothesis G_2 should be rejected;
- it is considered, that if $M^*[\delta_m(G_1)] > \delta_m(\beta_k^\partial)$, and $M^*[\delta_m(G_2)] < \delta_m(\alpha_k^\partial)$ the preference is given hypothesis G_1. If $M^*[\delta_m(G_1)] \le \delta_m(\beta_k^\partial)$, and $M^*[\delta_m(G_2)] \ge \delta_m(\alpha_k^\partial)$ the preference is given hypothesis G_2.

If $M^*[\delta_m(G_1)] \le \delta_m(\beta_k^{\partial})$, and $M^*[\delta_m(G_2)] < \delta_m(\alpha_k^{\partial})$ or $M^*[\delta_m(G_1)] > \delta_m(\beta_k^{\partial})$, and $M[\delta_m(G_2)] \ge \delta_m(\alpha_k^{\partial})$, that management transferred by blocks of an estimation of risk of the erroneous decision;
- it is considered, that if the risk of the erroneous decision $\gamma > \gamma_k$, however risk of the erroneous decision of hypothesis G_1 does not exceed, the preference is given hypothesis G_2. If the risk of the erroneous decision of hypothesis G_2 does not exceed, the preference given G_1;
- it is considered, that if $\gamma > \gamma_k$ also risks of erroneous decisions at deviation G_1 and G_2 are great enough, it is required to reconsider classification of data and in particular, to reduce number VA m_r.

Practical use of the developed program model is preceded with a stage of its research. The basic purpose thus is the control of adequacy of the decision over possible changes of initial data. As adequacy of the decision, we shall understand ability of recognition on ECh. conformity (or discrepancies) probabilities of display VA to the uniform law provided that functional component ECh. us is known. This quality monitoring is called as a method of the decision of "a return problem» and is realized under the special program on the computer. The integrated block diagram of algorithm of the control of adequacy we shall result on fig.3.

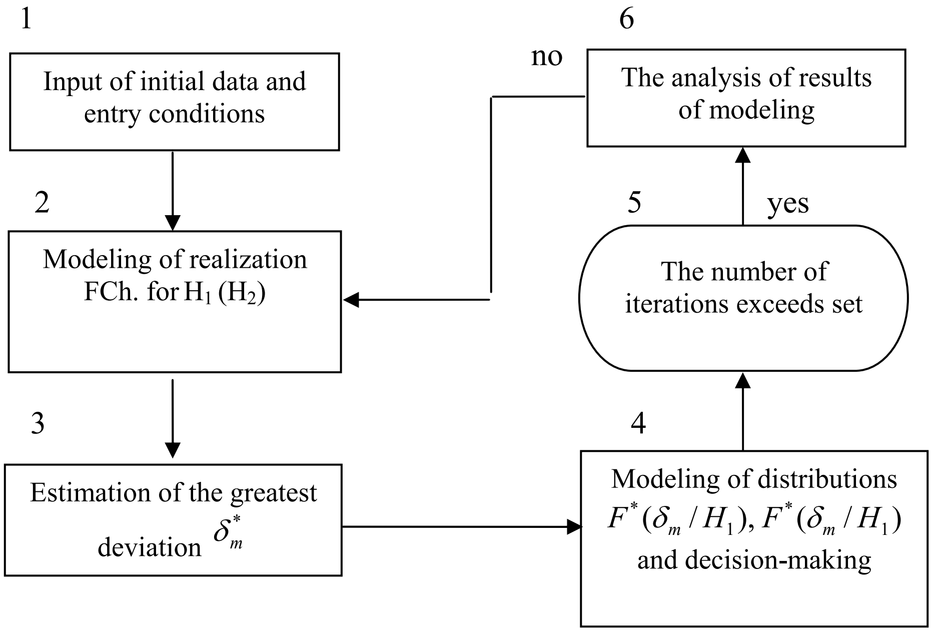


Fig. 3. The integrated block diagram of the control of adequacy of the decision.

Analysis ECh. following questions have been considered:
1. The result of the decision how much will change at modeling distributions of the greatest deviation on algorithm of the schemes represented on fig. 1 and 2.
2. What basic requirements are shown to methodology of calculation $F^*[\delta_m(H_1)]$ and $F^*[\delta_m(H_2)]$ with the purpose of their subsequent joint consideration? As distributions $F^*[\delta_m(H_1)]$ also $F^*[\delta_m(H_2)]$ are discrete, the essence of requirements should is reduced to identity of levels of digitization.
3. How the result of the decision will change at arrangement VA in decreasing order (increases) of probability of their display for nominal scale VA? In practice the arrangement of these VA is made subjectively. Thus any arrangement is supposed.
4. How the significance value of criterion of Kolmogorov (affects at the control of conformity software sold random numbers to the uniform law of distribution) on result of calculation of distributions $\alpha(i)$ and $\beta(i)$? It is obvious, that the significance value less, the probability of a mistake of the second sort is more. In our case, this probability reflects an opportunity of

conformity of distribution of random numbers to the law distinct from uniform. With reduction of number of realizations this probability grows.
5. How check of assumptions increase in number VA influences result? In particular, how inclusion in list VA of a version affects, the number of which cases of display is equal to zero?
6. What influence renders on results of calculation the account of parities of average values статистик $\delta_m(H_1)$ and $\delta_m(H_2)$?

Results of calculations have allowed establishing:
1. At the fixed number VA, equal m_r, and number of casual events (for example, refusals) n_Σ:
- distribution $F[\delta_m(\Gamma_1)]$ does not depend on laws of change as $F(i)$, and $F^*(i)$;
- at fixed f.d. $F(i)$ and $F^*(i)$ distribution $F[\delta_m(G_2)]$ for $\delta_m(G_2)$, calculated under formulas (7) and (8) or under formulas (11) and (12) one and too;
2. If to assume, that casual character of realizations $F_M^*(i)$ is rather calculated $F(i)$ under the formula:

$$\delta_{m,\nu}(H_1) = \max\{\delta_{1,\nu}(H_1); \delta_{2,\nu}(H_1); \ldots \delta_{m_r,\nu}(H_1)\} \tag{20}$$

$$\delta_{i,\nu}(H_1) = \left|F(i) - F^*_{M,\nu}(i)\right|; \quad \nu=1, N;\ i=1, (m_r+1);$$

and rather $F^*(I)$ - under the formula:

$$\delta_{m,\nu}(H_2) = \max\{\delta_{1,\nu}(H_2); \delta_{2,\nu}(H_2); \ldots \delta_{m_r,\nu}(H_2)\} \tag{21}$$

$$\delta_{i,\nu}(H_2) = \left|F^*(i) - F^*_{M,\nu}(i)\right|; \quad \nu=1, N;\ i=1, (m_r+1);$$

That this way of calculation $F^*[\delta_m(H_1)]$ also $F^*[\delta_m(H_2)]$ will lead to that the number of identical digitization of distributions $F^*[\delta_m(H_1)]$ and $F^*[\delta_m(H_2)]$ will be equal many cases to zero, i.e. joint consideration of these distributions will appear impossible. Really. Under the formula (20)

$$\delta_{i,\nu}(H_1) = \left|\frac{i}{m_r} - \frac{n_{i,\nu}(H_1)}{n_\Sigma}\right| \tag{22}$$

If to consider, that the size $\delta^*_{i,\nu}(H_2)$ (see the formula 21) is equal

$$\delta^*_{i,\nu}(H_2) = \left|\frac{n_{i,(\nu+1)}(H_1)}{n_\Sigma} - \frac{n_{i,\nu}(H_2)}{n_\Sigma}\right|, \tag{23}$$

that is easy for noticing, that if m_r and n_Σ have no same factors values $\delta^*_{i,\nu}(H_1)$ and $\delta^*_{i,\nu}(H_2)$ with $\nu = 1, N$ will differ.
3. If to lead modeling realization ECh. of probability display on $F(i)$, corresponding the uniform law, m_r VA at n_Σ "experiences" and to check up the assumption of a casual divergence of distributions $F(i)$ and $F^*(i)$ (hypothesis H_1) the method resulted above and criterion it will appear, that at initial casual arrangement VA and at the set significance value, hypothesis H_1, as one would expect, proves to be true. If now to place estimations of probability of display m_r VA in ascending order (or decrease) application of criterion testifies that observable law of change ECh. is not casual. The important practical conclusion from here follows: arrangement VA on experimental data at a nominal scale of change should be casual. Casual character of accommodation is provided with application of a method of Monte-Carlo;
4. The disorder of values of distributions $\alpha^*(i)$ and $\beta^*(i)$ in points of digitization $i = 1, m_r$ with growth of a significance value of criterion of Kolmogorov up to (0,4-0,6) nonlinear decreases. At the subsequent decrease in significance value influence of a deviation of random numbers from the uniform law on disorder $\alpha^*(i)$ also $\beta^*(i)$ becomes invariable small. Some increase in duration of calculation at $\alpha_k = 0{,}6$ is completely compensated decrease in number of iterations;
5. More detailed display of versions of an analyzed attribute, for example, units of the equipment, leads to that data becomes insufficiently for a choice of one of two assumptions. This conclusion is

put forward, if VA with $Q_i^* = 0$ are distributed on m_r VA casually. If all VA with $Q_i^* = 0$ are concentrated in one group, the conclusion about not casual divergence of estimations of probability of display VA follows. Hence, there is some optimum number VA at which functional characteristic shown precisely enough;

6. If estimations of average value of the maximal deviations (δ_m) of distributions $F^*(\delta_m / H_1)$ also $F^*(\delta_m / H_2)$ are practically equal, result of check of hypotheses H$_1$ and H$_2$ is the conclusion that the preference should given hypothesis H$_1$. Practical equality of estimations $\delta^*_{m,av}(H_1)$ also $\delta^*_{m,av}(H_2)$ causes necessity of application of criterion. A condition of the consent with casual character of change ECh. is the inequality:

$$\Delta\delta_{m,cp} = \left|\delta^*_{m,av}(H_1) - \delta^*_{m,av}(H_2)\right| \le \varepsilon = 1/n_\Sigma \qquad (24)$$

As a practical example, we shall consider results of check of the assumption of not casual character ECh. change of the importance of units of switches 110 кv (hypothesis H$_1$).

Experimental data are borrowed from work [2] and resulted in table 2. Here results of modeling of number of displays of each of VA for switches with Un=110 кv $(m_r = 9$ and $n_\Sigma = 29)$ provided that theoretical probabilities of refusals of each of units are equal are resulted. Calculations show, that assumption H1 cannot be accepted. As follows from table 2, the greatest number of refusals equal 8, observable on experimental data is observed at modeling and at other units of switches, that indirectly confirms results of check of hypothesis H$_1$.

Table 2

Comparative estimation of experimental and modeled structure of refusals actually switches with drive U$_н$=110 кv

The damaged element and the reason of refusals	Experim. Data	Data of modeling				
		1	2	3	4	5
Drive	4	3	**8**	6	6	1
Arc extinguisher camera	4	4	2	**7**	2	3
Separator	2	0	2	1	0	2
Inputs	1	1	2	0	3	0
Supporting-pivotal insulation	3	**8**	1	3	2	5
Consolidation	3	2	2	2	1	2
Case of management	**8**	6	**8**	5	5	**13**
Not classified attributes	2	2	2	2	**9**	4
The obscure reasons	3	4	3	4	2	0

Conclusion.

The method, algorithm and the program for MSDB PARADOX. Is developed, allowing to estimate objectivity of observable laws of change parameters of reliability the equipment on retrospective data

Literature.

1. Farhadzadeh E.M., Muradaliyev A.Z., Farzaliyev Y.Z. Method of decrease in a dispersion of imitating model. News NSA of Azerbaijan. A series of physic technical and mathematical sciences, №1, 2008
2. Averill M.Lou, V.Devid Kelton. Imitating modelling. Classics CS, 3 pub. – the joint venture: Peter; Kiev: Publishing group BHV, 2004 847 with: silt.
3. Farhadzadeh E.M., Muradaliyev A.Z., Farzaliyev Y.Z., Ismailova S.M. Perfection of system of an estimation and comparison of parameters of reliability of objects of industrial power. Kazan: News of High schools. Problems of Power 1-2, 2008.

A FUZZY RELIABILITY MODEL FOR "SAFETY SYSTEM-PROTECTED OBJECT" COMPLEX

A. I. Pereguda, D. A. Timashov

•

Obninsk Institute for Nuclear Power Engineering, Obninsk, Russia

e-mail: pereguda@iate.obninsk.ru

ABSTRACT

The paper presents a new fuzzy reliability model for automated "safety system-protected object" complex. It is supposed that parameters of reliability model and reliability indices are fuzzy variables. Scheduled periodic inspections of safety system are also taken into account. Asymptotic estimates of mean time to accident membership function are proposed.

1 INTRODUCTION

Many researchers applied the concept of fuzzy reliability on various systems (Cai et al 1991, Cai et al. 1993, Cai 1996, Chen & Mon 1993, Onisava & Kacprzyk 1995, Utkin & Gurov 1995, Verma et al. 2007). Those researches are based on possibility instead of probability assumption or fuzzy state instead of binary state assumption. This paper presents a slightly different concept of fuzzy reliability. It is supposed that parameters of the reliability model are fuzzy variables. According to the random fuzzy variables theory presented by Liu (Liu 2002) reliability indices in this case are also fuzzy variables. In order to develop a complete practical methodology of the fuzzy reliability assessment we also consider some aspects of the fuzzy parameter estimation and numerical methods of the fuzzy arithmetic.

In the present study we set out to analyze the reliability of the automated "safety system-protected object" complex. Systems of such kind are quite common in the nuclear power engineering. We follow Pereguda (Pereguda 2001) in assuming that the operation of the complex can be described using a superposition of alternating renewal processes. We also utilize the concept of a random fuzzy renewal process (Shen et al. 2008, Zhao et al. 2006). Our objective is to provide an asymptotic estimation for the mean time to accident membership function.

2 MODEL DESCRIPTION

Let us consider an automated complex of safety system and protected object. The safety system and the protected object are repairable. They are restored to an as-good-as-new state. It is assumed that failures of the safety system can be detected only during periodic inspections of the safety system. All failures are supposed to be independent.

Let $\{F_\chi(t;\boldsymbol{\lambda}(\theta)), \theta \in \Theta\}$ be a family of probability distributions on the probability space (Ω, A, P) with a common fuzzy parameters vector **λ** on the credibility space (Θ, Π, Cr) which induces a joint membership function, $\mu_{\boldsymbol{\lambda}}(\mathbf{x})$, then χ is a random fuzzy variable (Guo et al. 2007). For example, χ is an exponentially distributed random fuzzy variable, χ~EXP(λ) if

$$F_\chi(t;\lambda(\theta)) = \begin{cases} 1 - e^{-\lambda(\theta)t}, \; if \;\; t \geq 0 \\ 0, \; otherwise. \end{cases}$$

If χ is a random fuzzy variable defined on the credibility space (Θ, Π, Cr) than the probability $\Pr(\chi \in A)$ is a fuzzy variable for any Borel set $A \subseteq R$ and the expected value $E[\chi]$ is a fuzzy variable provided that $E[\chi(\theta)]$ is finite for each $\theta \in \Theta$ (Liu 2002).

By χ_i, i = 1,2,… denote the time to the i-th protected object failure. Let χ_i, i = 1,2,… be independent and identically distributed (i.i.d) random fuzzy variables (Li & Liu 2006) with CDF $F_\chi(t; \boldsymbol{\lambda}_\chi(\theta))$. By γ_i, i = 1,2,… denote the time to the protected object repair after it's i-th failure. Let γ_i, i = 1,2,… be i.i.d. random fuzzy variables with CDF $F_\gamma(t; \boldsymbol{\lambda}_\gamma(\theta))$. Suppose that moments of the protected object repair are renewal points of the operation process of the complex. By ξ_i, i = 1,2,… denote the time to the i-th failure of the safety system. Let ξ_i, i = 1,2,… be i.i.d. random fuzzy variables with CDF $F_\xi(t; \boldsymbol{\lambda}_\xi(\theta))$. By η_i, i = 1,2,… denote the time to the safety system repair after it's i-th failure. Let η_i, i = 1,2,… be i.i.d. random fuzzy variables with CDF $F_\eta(t; \boldsymbol{\lambda}_\eta(\theta))$. Suppose that moments of the safety system repair are renewal points of the operation process of the safety system. By T denote the period of scheduled inspections of the safety system. By δ denote the duration of scheduled inspections of the safety system. The safety system is inactive during the inspection. By ν denote the number of renewal intervals before the accident. Let ν be an integer random fuzzy variable. By ω denote the time to accident. An accident takes place when the protected object fails during the period of the safety system inactivity. Our aim is to estimate the membership function $\mu_{M\omega}(y)$ of the mean time to accident.

2 MAIN RESULTS

Since the operation process of the complex is a superposition of alternating renewal processes, it follows that

$$\omega = \sum_{i=1}^{\nu-1} (\chi_i + \gamma_i) + \chi_\nu .$$

Taking into account the fact that E[$\omega(\theta)$], E[$\chi_i(\theta)$], E[$\gamma_i(\theta)$], E[$\xi_i(\theta)$], E[$\eta_i(\theta)$] and E[$\nu(\theta)$] are crisp variables for each fixed $\theta \in \Theta$ we obtain

$$E[\omega(\theta)] = E\left[\sum_{i=1}^{\nu(\theta)-1} (\chi_i(\theta) + \gamma_i(\theta)) + \chi_{\nu(\theta)}(\theta)\right]$$

for each fixed $\theta \in \Theta$. Since all random fuzzy variables of interest are independent it follows that

$$\Pr\left(\sum_{i=1}^{\nu(\theta)-1} (\chi_i(\theta) + \gamma_i(\theta)) + \chi_{\nu(\theta)}(\theta) \ge r\right)$$

$$= \sum_{k=1}^{\infty} \Pr(\nu(\theta) = k) \Pr\left(\sum_{i=1}^{k-1} (\chi_i(\theta) + \gamma_i(\theta)) + \chi_k(\theta) \ge r\right).$$

Note that

$$E[\omega(\theta)] = \int_0^\infty \Pr\left(\sum_{i=1}^{\nu(\theta)-1} (\chi_i(\theta) + \gamma_i(\theta)) + \chi_{\nu(\theta)}(\theta) \ge r\right) dr$$

$$= \sum_{k=1}^{\infty} \Pr(\nu(\theta) = k) E\left[\sum_{i=1}^{k-1} (\chi_i(\theta) + \gamma_i(\theta)) + \chi_k(\theta)\right].$$

Since all random fuzzy variables of interest are i.i.d. it follows that

$$E[\omega(\theta)] = \sum_{k=1}^{\infty} \Pr(\nu(\theta) = k) E[k\chi(\theta) + (k-1)\gamma(\theta)] = E[\nu(\theta)\chi(\theta) + (\nu(\theta) - 1)\gamma(\theta)].$$

Therefore

$$E[\omega] = E[\nu\chi + (\nu - 1)\gamma].$$

Taking into account the fact that all random fuzzy variables of interest are independent we obtain

$$E[\omega(\theta)] = \frac{1}{q(\theta)} E[\chi(\theta)] + \frac{1-q(\theta)}{q(\theta)} E[\gamma(\theta)]$$

$$= \frac{1}{q(\theta)} \int_0^\infty \left(1 - F_\chi\left(t; \boldsymbol{\lambda}_\chi(\theta)\right)\right) dt + \frac{1-q(\theta)}{q(\theta)} \int_0^\infty \left(1 - F_\gamma\left(t; \boldsymbol{\lambda}_\gamma(\theta)\right)\right) dt$$

for each fixed $\theta \in \Theta$, where $q(\theta)$ is the probability of the accident during a renewal interval.

Let $Q^+(\theta)$ be the set of intervals where the safety system is active and let $Q^-(\theta)$ be the set of intervals where safety system is inactive. We obviously have

$$q(\theta) = \int_0^\infty \Pr\left(t \in Q^-(\theta)\right) dF_\chi\left(t; \boldsymbol{\lambda}_\chi(\theta)\right)$$

for each fixed $\theta \in \Theta$. Note that

$$\Pr\left(t \in Q^-(\theta)\right) = 1 - \Pr\left(t \in Q^+(\theta)\right) = 1 - \Pr^+(t;\theta).$$

Applying the law of total probability we obtain

$$\Pr^+(t;\theta) = \int_0^\infty \int_0^\infty \Pr\left(t \in Q^+(\theta) \mid \xi(\theta) = x, \eta(\theta) = y\right) dF_\eta\left(y; \boldsymbol{\lambda}_\eta(\theta)\right) dF_\xi\left(x; \boldsymbol{\lambda}_\xi(\theta)\right).$$

Since the operation process of the safety system is an alternating renewal process, it follows that

$$\Pr^+(t;\theta) = \iint_{\tau_{SS}(x,y) \le t} \Pr\left(t \in Q^+(\theta) \mid \xi(\theta) = x, \eta(\theta) = y\right) dF_\eta\left(y; \boldsymbol{\lambda}_\eta(\theta)\right) dF_\xi\left(x; \boldsymbol{\lambda}_\xi(\theta)\right)$$

$$+ \iint_{\tau_{SS}(x,y) > t} \Pr\left(t \in Q^+(\theta) \mid \xi(\theta) = x, \eta(\theta) = y\right) dF_\eta\left(y; \boldsymbol{\lambda}_\eta(\theta)\right) dF_\xi\left(x; \boldsymbol{\lambda}_\xi(\theta)\right) = I_1 + I_2,$$

where $\tau_{SS}(\xi,\eta) = \left(\left\langle \frac{\xi}{T+\delta} \right\rangle + 1\right)(T+\delta) + \eta$ is the length of the renewal interval of the safety system operation process and $\langle x \rangle$ is an integer part of x. We see that

$$I_2 = \iint_{\tau_{SS}(x,y) > t} \left(\sum_{m=0}^{\left\langle \frac{x}{T+\delta} \right\rangle - 1} J_{t \in [m(T+\delta), m(T+\delta)+T)} + J_{t \in \left[\left\langle \frac{x}{T+\delta} \right\rangle (T+\delta), x\right)} \right) dF_\eta\left(y; \boldsymbol{\lambda}_\eta(\theta)\right) dF_\xi\left(x; \boldsymbol{\lambda}_\xi(\theta)\right),$$

where J_A is an indicator function of the event A. It now follows that

$$I_2 = \left(1 - F_\xi\left(t; \boldsymbol{\lambda}_\xi(\theta)\right)\right) - \sum_{m=1}^\infty \left(1 - F_\xi\left(m(T+\delta); \boldsymbol{\lambda}_\xi(\theta)\right)\right)\left(J_{(m-1)(T+\delta)+T \le t} - J_{m(T+\delta) \le t}\right)$$

$$= F_\zeta\left(t; \boldsymbol{\lambda}_\xi(\theta)\right) - F_\xi\left(t; \boldsymbol{\lambda}_\xi(\theta)\right),$$

where

$$F_\zeta\left(t; \boldsymbol{\lambda}_\xi(\theta)\right) = 1 - \sum_{m=1}^\infty \left(1 - F_\xi\left(m(T+\delta); \boldsymbol{\lambda}_\xi(\theta)\right)\right)\left(J_{(m-1)(T+\delta)+T \le t} - J_{m(T+\delta) \le t}\right).$$

Note that

$$I_1 = \iint_{\tau_{SS}(x,y) \le t} \Pr^+\left(t - \tau_{SS}(x,y); \theta\right) dF_\eta\left(y; \boldsymbol{\lambda}_\eta(\theta)\right) dF_\xi\left(x; \boldsymbol{\lambda}_\xi(\theta)\right) = \int_0^t \Pr^+(t - z; \theta) dF_{\tau_{SS}}\left(z; \boldsymbol{\lambda}_\eta(\theta), \boldsymbol{\lambda}_\xi(\theta)\right).$$

Finally,

$$\Pr^+(t;\theta) = f(t;\theta) + \int_0^t \Pr^+(t - z; \theta) dF_{\tau_{SS}}\left(z; \boldsymbol{\lambda}_\eta(\theta), \boldsymbol{\lambda}_\xi(\theta)\right),$$

where $f(t;\theta) = F_\zeta\left(t; \boldsymbol{\lambda}_\xi(\theta)\right) - F_\xi\left(t; \boldsymbol{\lambda}_\xi(\theta)\right)$ and $F_{\tau_{СБ}}\left(z; \boldsymbol{\lambda}_\eta(\theta), \boldsymbol{\lambda}_\xi(\theta)\right) = P\left(\left(\left\langle \frac{\xi(\theta)}{T+\delta} \right\rangle + 1\right)(T+\delta) + \eta(\theta) \le z\right).$

The application of Laplace-Stieltjes transform and tauberian theorems yields

$$\lim_{t\to\infty}\Pr^+(t;\theta)=\frac{E[\xi(\theta)]-E[\zeta(\theta)]}{E[\tau_{SS}(\theta)]},$$

where

$$E[\zeta(\theta)]=\delta E\left[\left\langle\frac{\xi(\theta)}{T+\delta}\right\rangle\right]=\delta\sum_{k=1}^{\infty}k\Big(F_\xi\big((k+1)(T+\delta);\boldsymbol{\lambda}_\xi(\theta)\big)-F_\xi\big(k(T+\delta);\boldsymbol{\lambda}_\xi(\theta)\big)\Big),$$

$$E[\xi(\theta)]=\int_0^\infty\big(1-F_\xi(t;\boldsymbol{\lambda}_\xi(\theta))\big)dt,$$

$$E[\tau_{SS}(\theta)]=\int_0^\infty\big(1-F_\eta(t;\boldsymbol{\lambda}_\eta(\theta))\big)dt+(T+\delta)\left(1+\sum_{k=1}^{\infty}k\Big(F_\xi\big((k+1)(T+\delta);\boldsymbol{\lambda}_\xi(\theta)\big)-F_\xi\big(k(T+\delta);\boldsymbol{\lambda}_\xi(\theta)\big)\Big)\right).$$

Therefore

$$q(\theta)\approx 1-\frac{E[\xi(\theta)]-E[\zeta(\theta)]}{E[\tau_{SS}(\theta)]}.$$

Taking into account the definition of a function of fuzzy variables and Zadeh Extension Principle we obtain

$$\mu_{E[\omega]}(y)=\sup_{y=f(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4)}\min\big(\mu_{\boldsymbol{\lambda}_\chi}(\mathbf{x}_1),\mu_{\boldsymbol{\lambda}_\gamma}(\mathbf{x}_2),\mu_{\boldsymbol{\lambda}_\xi}(\mathbf{x}_3),\mu_{\boldsymbol{\lambda}_\eta}(\mathbf{x}_4)\big),$$

where

$$f(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4)=\frac{1}{q(\mathbf{x}_3,\mathbf{x}_4)}\int_0^\infty\big(1-F_\chi(t;\mathbf{x}_1)\big)dt+\frac{1-q(\mathbf{x}_3,\mathbf{x}_4)}{q(\mathbf{x}_3,\mathbf{x}_4)}\int_0^\infty\big(1-F_\gamma(t;\mathbf{x}_2)\big)dt,$$

$$q(\mathbf{x}_3,\mathbf{x}_4)\approx 1-\frac{E[\xi(\mathbf{x}_3)]-E[\zeta(\mathbf{x}_3)]}{E[\tau_{SS}(\mathbf{x}_3,\mathbf{x}_4)]},$$

$$E[\zeta(\mathbf{x}_3)]=\delta\sum_{k=1}^{\infty}k\Big(F_\xi\big((k+1)(T+\delta);\mathbf{x}_3\big)-F_\xi\big(k(T+\delta);\mathbf{x}_3\big)\Big),$$

$$E[\xi(\mathbf{x}_3)]=\int_0^\infty\big(1-F_\xi(t;\mathbf{x}_3)\big)dt,$$

$$E[\tau_{SS}(\mathbf{x}_3,\mathbf{x}_4)]=\int_0^\infty\big(1-F_\eta(t;\mathbf{x}_4)\big)dt+(T+\delta)\left(1+\sum_{k=1}^{\infty}k\Big(F_\xi\big((k+1)(T+\delta);\mathbf{x}_3\big)-F_\xi\big(k(T+\delta);\mathbf{x}_3\big)\Big)\right).$$

Therefore it is now possible to estimate the fuzzy mean time to accident. In order to perform defuzzyfication we use expected value operator suggested by Liu (Liu & Liu 2003):

$$E[\omega]=\frac{1}{2}\int_0^\infty\left(\sup_{y\ge r}\mu_{M\omega}(y)+1-\sup_{y<r}\mu_{M\omega}(y)\right)dr.$$

It is clearly evident that the most difficult part in the proposed methodology is the evaluation of $\mu_{M\omega}(y)$ according to Zadeh Extension Principle. To overcome these difficulties one should use a suitable numerical method. We suggest to use General Transformation Method (Hanss 2005) for this task. This method consists of a decomposition of input fuzzy variables, a transformation of the input intervals, an evaluation of the model and a retransformation of the output array. Another important issue is the fuzzy parameter estimation. We suggest to use fuzzy estimators developed by Buckley (Buckley 2006). These estimators are based on confidence intervals and allow the estimation of the membership function of the distribution parameter from the sample data.

Consider now the following example. Suppose $\chi\sim$EXP(λ_χ), $\gamma\sim$EXP(λ_γ), $\xi\sim$EXP(λ_ξ), $\eta\sim$EXP(λ_η). Therefore

$$\mu_{E[\omega]}(y)=\sup_{y=f(x_1,x_2,x_3,x_4)}\min\big(\mu_{\lambda_\chi}(x_1),\mu_{\lambda_\gamma}(x_2),\mu_{\lambda_\xi}(x_3),\mu_{\lambda_\eta}(x_4)\big),$$

where

$$f(x_1, x_2, x_3, x_4) = \frac{1}{q(x_3, x_4)} \frac{1}{x_1} + \frac{1 - q(x_3, x_4)}{q(x_3, x_4)} \frac{1}{x_2},$$

$$q(x_3, x_4) \approx 1 - \frac{E[\xi(x_3)] - E[\zeta(x_3)]}{E[\tau_{SS}(x_3, x_4)]},$$

$$E[\zeta(x_3)] = \delta \frac{e^{-x_3(T+\delta)}}{1 - e^{-x_3(T+\delta)}}, \quad E[\xi(x_3)] = \frac{1}{x_3},$$

$$E[\tau_{SS}(x_3, x_4)] = \frac{1}{x_4} + (T + \delta)\left(1 + \frac{e^{-x_3(T+\delta)}}{1 - e^{-x_3(T+\delta)}}\right).$$

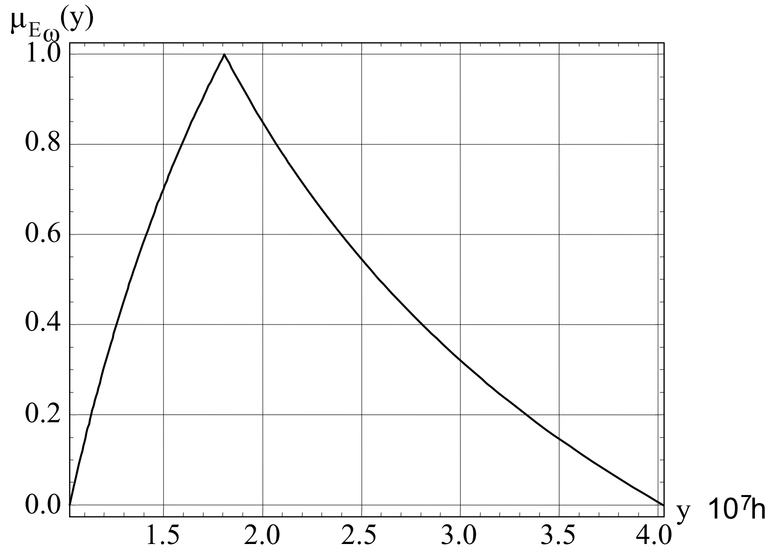


Figure 1. Membership function of the mean time to accident.

Let all fuzzy parameters be triangular fuzzy variables: $\mu_{\lambda_\chi}(x) = \Delta(1\times10^{-6}h^{-1}, 1.5\times10^{-6}h^{-1}, 2\times10^{-6}h^{-1})$, $\mu_{\lambda_\gamma}(x) = \Delta(1h^{-1}, 1.5h^{-1}, 2h^{-1})$, $\mu_{\lambda_\chi}(x) = \Delta(1\times10^{-4}h^{-1}, 1.5\times10^{-4}h^{-1}, 2\times10^{-4}h^{-1})$, $\mu_{\lambda_\eta}(x) = \Delta(1h^{-1}, 1.5h^{-1}, 2h^{-1})$, T = 500h, δ = 0.1h. Membership function of the mean time to accident is shown on Figure 1.

3 CONCLUSIONS

The proposed model permits to assess the reliability of one specific class of technological systems with fuzzy parameters. In particular the suggested approach allows to evaluate the membership function of the mean time to accident for the "safety system-protected object" complex. The proposed approach allows to take into account the uncertainty of reliability model parameters and reliability indices. The solution obtained is useful for reliability assessment of nuclear power plants and similar dangerous technological objects.

REFERENCES

Buckley, J.J. 2006. *Fuzzy Probability and Statistics.* Springer-Verlag.

Cai, K.Y., Wen, C.Y. & Zhang, M.L. 1991. Fuzzy variables as a basis for a theory of fuzzy reliability in the possibility context. *Fuzzy Sets Syst.* 42: 145-172.

Cai, K.Y., Wen, C.Y. & Zhang, M.L. 1993. Fuzzy states as a basis for a theory of fuzzy reliability. *Microelect. Reliabil.* 33: 2253-2263.

Cai, K.Y. 1996. *Introduction to Fuzzy Reliability*. Norwell: Kluwer Academic Publishers.

Chen, C.-H. & Mon, L. 1993. Fuzzy system reliability analysis by interval of confidence. *Fuzzy Sets Syst.* 56: 29-35.

Guo, L., Zhao, R.Q., Guo, D. & Dunne, T. 2007. Random fuzzy variable modelling on repairable system. *Journal of Uncertain Systems* 1, no. 3: 222-234.

Hanss, M. 2005. Applied Fuzzy Arithmetic: An Introduction with Engineering Applications. Springer-Verlag.

Li, X. & Liu, B. 2006. New independence definition of fuzzy random variable and random fuzzy variable. *World Journal of Modelling and Simulation* 2, no. 5: 338-342.

Liu, B. 2002. *Theory and Practice of Uncertain Programming.* Physica Verlag Heidelberg.

Liu, Y. & Liu, B. 2003. Expected value operator of random fuzzy variable and random fuzzy expected value models. *Int. J. Uncertainty, Fuzziness Knowl.-Based Syst.* 11: 195-215.

Onisava, T. & Kacprzyk, J. 1995. *Reliability and safety analysis under fuzziness.* Physica Verlag Heidelberg.

Pereguda, A.I. 2001. Calculation of the Reliability Indicators of the System Protected Object-Control and Protection System. *Atomic Energy* 90: 460-468.

Shen, Q., Zhao, R. & Tang, W. 2008. Random fuzzy alternating renewal processes. *Soft Computing – A Fusion of Foundations, Methodologies and Applications* 13, no. 2: 139-147.

Utkin, L.V. & Gurov, S.V. 1995. A general formal approach for fuzzy reliability analysis in the possibility context. *Fuzzy Sets Syst.* 83: 203-213.

Verma, A.K. et al. 2007. *Fuzzy-Reliability Engineering: Concepts and Applications.* Narosa Publishing House.

Zhao, R., Tang, W. & Yun, H. 2006. Random fuzzy renewal process. *European Journal of Operational Research* 169: 189-201.

AN ADVANCED RELIABILITY MODEL FOR AUTOMATED "SAFETY SYSTEM-PROTECTED OBJECT" COMPLEX WITH TIME REDUNDANCY

A. I. Pereguda, D. A. Timashov

•

Obninsk Institute for Nuclear Power Engineering, Obninsk, Russia

e-mail: pereguda@iate.obninsk.ru

ABSTRACT

The paper presents a new reliability model for an automated "safety system-protected object" complex with time redundancy. It is supposed that the time redundancy is caused by a protected object inertia. Scheduled periodic inspections of the safety system are also taken into account. Two-sided estimates of the mean time to accident are proposed.

1 INTRODUCTION

Redundancy is a widely used and widely referenced concept. Time redundancy means that some excess time is available after the system fault. It is possible to prevent an accident during this period. Such kind of redundancy may arise by design or as a natural byproduct of design. There are some methods available for the estimation of reliability indices of systems with time redundancy (Gnedenko & Ushakov 1995). But there is a lack of reliability models for automated "safety system-protected object" complex with the time redundancy caused by a protected object inertia. Systems of such kind are quite common in the nuclear power engineering due to an inertia of physical processes in the reactor core. This natural redundancy is seldom acknowledged and exploited. In the present study we set out to analyze the reliability of such system. We follow Pereguda (Pereguda 2001) in assuming that the operation of the complex can be described using a superposition of alternating renewal processes. Our objective is to provide an asymptotic estimation for the mean time to accident.

2 MODEL DESCRIPTION

Let us consider an automated complex of a safety system and a protected object. The safety system and the protected object are repairable. They are restored to an as-good-as-new state. It is assumed that safety system failures can be detected only during periodic inspections of the safety system. All failures are supposed to be independent. Safety system consists of two subsystems: the temperature subsystem and the power subsystem. If the power subsystem fails then the temperature subsystem is still able to prevent an accident. By χ_i, $i = 1,2,\ldots$ denote the time to the i-th protected object failure due to the increased power level. Let χ_i, $i = 1,2,\ldots$ be independent and identically distributed (i.i.d) random variables with CDF $F_\chi(t)$. By γ_i, $i = 1,2,\ldots$ denote the time to the protected object repair after it's i-th failure due to the increased power level. Let γ_i, $i = 1,2,\ldots$ be i.i.d. random variables with CDF $F_\gamma(t)$. Suppose that moments of the protected object repair after it's failure due to the increased power level are renewal points of the operation process of the complex. By δ_i denote the time between i-th protected object failure due to the increased power level and the subsequent failure due to the increased temperature. Let δ_i, $i = 1,2,\ldots$ be i.i.d. random variables with CDF $F_\delta(t)$. Thus the power safety subsystem may prevent an accident during the $[\chi_i, \chi_i + \delta_i)$ interval. Alternatively the temperature safety subsystem may prevent an accident at $\chi_i + \delta_i$. By α_i, $i = 1,2,\ldots$

denote the time to the protected object repair after such an event. Let α_i, i = 1,2,… be i.i.d. random variables with CDF $F_\alpha(t)$. Suppose that moments of the protected object repair after it's failure due to the increased power level and subsequent increased temperature are renewal points of the operation process of the complex. By φ_i, i = 1,2,… denote the time to the i-th protected object failure due to the increased temperature. Let φ_i, i = 1,2,… be independent and identically distributed (i.i.d) random variables with CDF $F_\varphi(t)$. By ψ_i, i = 1,2,… denote the time to the protected object repair after it's i-th failure due to the increased power level. Let ψ_i, i = 1,2,… be i.i.d. random variables with CDF $F_\psi(t)$. Suppose that moments of the protected object repair after it's failure due to the increased temperature are renewal points of the operation process of the complex. By ξ_i^p, i = 1,2,… denote the time to the i-th failure of the power safety subsystem. Let ξ_i^p, i = 1,2,… be i.i.d. random variables with CDF $F_{\xi^p}(t)$. By η_i^p, i = 1,2,… denote the time to the power safety subsystem repair after it's i-th failure. Let η_i^p, i = 1,2,… be i.i.d. random variables with CDF $F_{\eta^p}(t)$. Suppose that moments of the power safety subsystem repair after it's failure are renewal points of the operation process of the power safety subsystem. By T^p denote the period of scheduled inspections of the power safety subsystem. By θ^p denote the duration of scheduled inspections of the power safety subsystem. By ξ_i^t, i = 1,2,… denote the time to the i-th failure of the temperature safety subsystem. Let ξ_i^t, i = 1,2,… be i.i.d. random variables with CDF $F_{\xi^t}(t)$. By η_i^t, i = 1,2,… denote the time to the temperature safety subsystem repair after it's i-th failure. Let η_i^t, i = 1,2,… be i.i.d. random variables with CDF $F_{\eta^t}(t)$. Suppose that moments of the temperature safety subsystem repair after it's failure are renewal points of the operation process of the temperature safety subsystem. By T^t denote the period of scheduled inspections of the power safety subsystem. By θ^t denote the duration of scheduled inspections of the power safety subsystem. The safety system is inactive during the inspection. By ω denote the time to accident. Our aim is to estimate the mean time to accident $E[\omega]$.

2 MAIN RESULTS

Since the operation process of the complex is a superposition of alternating renewal processes, it follows that

$$\omega = \sum_{i=1}^{\nu-1} \sigma_i + \sigma'_\nu$$

where

$$\sigma_i = \min(\chi_i, \varphi_i) + \left((\beta_i + \gamma_i)J_{B_i} + (\delta_i + \alpha_i)J_{\overline{B}_i}\right)J_{\chi_i \le \varphi_i} + \psi_i J_{\varphi_i < \chi_i}$$

and

$$\sigma'_i = \min(\chi_i, \varphi_i) + \delta_i J_{\chi_i \le \varphi_i} .$$

By β_i we denote the interval between the protected object failure due to the increased power level and the activation of the power safety subsystem. Note that $0 \le \beta_i < \delta_i$. By B_i we denote the event that the power safety subsystem was activated in the $[\chi_i, \chi_i + \delta_i)$ interval. By $\overline{B}_i$ we denote the event that the power safety subsystem was not activated in the $[\chi_i, \chi_i + \delta_i)$ interval. J_B is an indicator function of the event B.

We obviously have

$$F_\omega(t) = \Pr(\omega \le t) = \Pr\left(\sum_{i=1}^{\nu-1}\sigma_i + \sigma'_\nu \le t\right).$$

Applying the Laplace-Stieltjes transform to $F_\omega(t)$, we obtain

$$\tilde{F}_\omega(s) = E\left[e^{-s\omega}\right] = \sum_{n=1}^{\infty} E\left[e^{-s\omega} \mid \nu = n\right]\Pr(\nu = n)$$

where $\tilde{F}_\omega(s) = \int_0^\infty e^{-st}dF_\omega(t) = E\left[e^{-s\omega}\right]$, $\Pr(\nu = n) = q(1 - q)^{n-1}$ and q is the probability of an accident during a renewal interval. We see that

$$E\left[e^{-s\omega} \mid \nu = n\right] = E\left[e^{-s\left(\sum_{i=1}^{\nu-1}\sigma_i + \sigma'_\nu\right)} \middle| \nu = n\right] = \left(\tilde{F}_\sigma(s)\right)^{n-1}\tilde{F}_{\sigma'}(s).$$

Therefore

$$\tilde{F}_\omega(s) = \sum_{n=1}^{\infty}\left(\tilde{F}_\sigma(s)\right)^{n-1}\tilde{F}_{\sigma'}(s)q(1-q)^{n-1} = \frac{q\tilde{F}_{\sigma'}(s)}{1-(1-q)\tilde{F}_\sigma(s)}$$

Since $E[\omega] = -\left.\frac{d\tilde{F}_\omega(s)}{ds}\right|_{s=0}$, it follows that

$$E[\omega] = E[\sigma'] + \frac{1-q}{q}E[\sigma].$$

Variable β has an unknown distribution. Therefore variable σ also has an unknown distribution. Using stochastic ordering (Stoyan, 1983), we get the following estimation

$$E[\sigma'] + \frac{1-q}{q}\Big(E[\min(\chi,\varphi)] + \big(E[\gamma]\Pr(B) + (E[\delta] + E[\alpha])\Pr(\overline{B})\big)\Pr(\chi \le \varphi)\Big) + E[\psi]\Pr(\varphi < \chi) \le E[\omega] \le$$

$$\le E[\sigma'] + \frac{1-q}{q}\Big(E[\min(\chi,\varphi)] + \big((E[\delta] + E[\gamma])\Pr(B) + (E[\delta] + E[\alpha])\Pr(\overline{B})\big)\Pr(\chi \le \varphi)\Big) + E[\psi]\Pr(\varphi < \chi),$$

where

$$E[\sigma'] = E[\min(\chi,\varphi)] + E[\delta]\Pr(\chi \le \varphi).$$

By U_n denote the moment of the n-th failure of the power safety subsystem. By V_n denote the moment of the n-th repair of the power safety subsystem. Then the corresponding accident takes place when

$$U_n \le \chi < V_n - \delta,$$
$$\delta \le V_n - U_n$$

or when

$$V_{n-1} + T^p \le \chi < V_{n-1} + (T^p + \theta^p) - \delta;$$
$$V_{n-1} + (T^p + \theta^p) + T^p \le \chi < V_{n-1} + 2(T^p + \theta^p) - \delta;$$
$$\ldots$$
$$V_{n-1} + \left(\left\langle\frac{\xi_n}{T^p + \theta^p}\right\rangle - 1\right)(T^p + \theta^p) + T^p \le \chi < V_{n-1} + \left\langle\frac{\xi_n}{T^p + \theta^p}\right\rangle(T^p + \theta^p) - \delta;$$
$$\delta < \theta^p$$

where $\langle x \rangle$ is an integer part of x.

Since the operation process of the safety system is an alternating renewal process, it follows that

$$U_n = \sum_{i=1}^{n}\xi_i^p + \sum_{i=1}^{n-1}\left((T^p + \theta^p) - \left\{\frac{\xi_i^p}{T^p + \theta^p}\right\}(T^p + \theta^p)\right) + \sum_{i=1}^{n-1}\eta_i^p,$$

$$V_n = \sum_{i=1}^{n} \xi_i^p + \sum_{i=1}^{n} \left((T^p + \theta^p) - \left\{ \frac{\xi_i^p}{T^p + \theta^p} \right\} (T^p + \theta^p) \right) + \sum_{i=1}^{n} \eta_i^p ,$$

where $\{x\}$ is a fractional part of x. Taking into account the condition of accident, we obtain:

$$\Pr(\overline{B}) = \sum_{n=1}^{\infty} \int_0^{\infty} \left(E\left[J_{U_n \le x < V_n - \delta} J_{\Delta_n > 0} + \sum_{i=1}^{\left\langle \frac{\xi_n^p}{T^p + \theta^p} \right\rangle} J_{V_{n-1} + (i-1)(T^p + \theta^p) + T^p \le x < V_{n-1} + i(T^p + \theta^p) - \delta} J_{\zeta > 0} \right] \right) dF_{\chi}(x)$$

where

$$\Delta_n = \eta_n + \varepsilon_n - \delta,$$

$$\varepsilon_n = T^p + \theta^p - \left\{ \frac{\xi_n^p}{T^p + \theta^p} \right\} (T^p + \theta^p),$$

$$\zeta = \theta^p - \delta$$

It now follows that

$$\Pr(\overline{B}) = \sum_{n=1}^{\infty} \int_0^{\infty} \Pr\big(\min(U_n, (U_n + \Delta_n)) \le x\big) dF_{\chi}(x) - \sum_{n=1}^{\infty} \int_0^{\infty} \Pr\big(U_n + \Delta_n \le x\big) dF_{\chi}(x) +$$

$$+ \sum_{n=1}^{\infty} \int_0^{\infty} E\left[\sum_{i=1}^{\left\langle \frac{\xi_n^p}{T^p + \theta^p} \right\rangle} \Pr\big(\min((V_{n-1} + i(T^p + \theta^p) - \theta^p), (V_{n-1} + i(T^p + \theta^p) - \theta^p + \zeta)) \le x\big) \right] dF_{\chi}(x) -$$

$$- \sum_{n=1}^{\infty} \int_0^{\infty} E\left[\sum_{i=1}^{\left\langle \frac{\xi_n^p}{T^p + \theta^p} \right\rangle} \Pr\big(V_{n-1} + i(T^p + \theta^p) - \theta^p + \zeta \le x\big) \right] dF_{\chi}(x) = q_1 + q_2 .$$

Note that

$$q_1 = \sum_{n=1}^{\infty} \int_0^{\infty} \int_0^{\infty} \Big((F_{\xi^p} * (F_{\xi^p} * F_{\eta^p} * F_{\varepsilon})^{*(n-1)})(x) - (F_{\xi^p} * (F_{\xi^p} * F_{\eta^p} * F_{\varepsilon})^{*(n-1)})(x - y) \Big) dF_{\Delta}(y) dF_{\chi}(x) ,$$

where $F_{\xi^p} * F_{\eta^p}(t) = \int_0^t F_{\xi^p}(t - z) dF_{\eta^p}(z)$ and $F^{*(2)}(t) = F * F(t)$. Equivalently

$$q_1 = \int_0^{\infty} \int_0^{\infty} (H_0(x) - H_0(x - y)) dF_{\Delta}(y) dF_{\chi}(x)$$

where $H_0(x) = \sum_{n=1}^{\infty} F_{\xi^p} * (F_{\xi^p} * F_{\eta^p} * F_{\varepsilon})^{*(n-1)}(x)$. Furthermore

$$q_2 = \sum_{n=1}^{\infty} \int_0^{\infty} \int_0^{\infty} E\left[\sum_{i=1}^{\left\langle \frac{\xi_n^p}{T^p + \theta^p} \right\rangle} \Big(\Pr\big(V_{n-1} + i(T^p + \theta^p) - \theta^p \le x\big) - \Pr\big(V_{n-1} + i(T^p + \theta^p) - \theta^p \le x - y\big) \Big) \right] dF_{\zeta}(y) dF_{\chi}(x) .$$

In other notation,

$$q_2 = \int_0^{\infty} \int_0^{\infty} E\left[\sum_{i=1}^{\left\langle \frac{\xi^p}{T^p + \theta^p} \right\rangle} (H_{0i}(x) - H_{0i}(x - y)) \right] dF_{\zeta}(y) dF_{\chi}(x) ,$$

where $H_{0i}(x)=\sum_{n=1}^{\infty}F_{2i,n}(x)$ and $F_{2i,n}(x)=\Pr(V_{n-1}+i(T^p+\theta^p)-\theta^p\le x)$. The application of renewal limit theorems (Rausand & Høyland 2004) yields

$$q_1\approx\frac{1}{E\left[\eta^p\right]+(T^p+\theta^p)+(T^p+\theta^p)E\left[\left\langle\frac{\xi^p}{T^p+\theta^p}\right\rangle\right]}\int_0^{\infty}ydF_{\Delta}(y),$$

$$q_2\approx\frac{E\left[\left\langle\frac{\xi^p}{T^p+\theta^p}\right\rangle\right]}{E\left[\eta^p\right]+(T^p+\theta^p)+(T^p+\theta^p)E\left[\left\langle\frac{\xi^p}{T^p+\theta^p}\right\rangle\right]}\int_0^{\infty}ydF_{\zeta}(y).$$

Finally,

$$\Pr(\overline{B})\approx\frac{1}{E[\eta^p]+(T^p+\theta^p)+(T^p+\theta^p)E\left[\left\langle\frac{\xi^p}{T^p+\theta^p}\right\rangle\right]}\left(\int_0^{\infty}ydF_{\Delta}(y)+E\left[\left\langle\frac{\xi^p}{T^p+\theta^p}\right\rangle\right]\int_0^{\infty}ydF_{\zeta}(y)\right).$$

The Monte-Carlo method can be used to estimate $\int_0^{\infty}ydF_{\Delta}(y)$ and $\int_0^{\infty}ydF_{\zeta}(y)$:

$$\Pr(\overline{B})\approx\frac{1}{E\left[\eta^p\right]+(T^p+\theta^p)+(T^p+\theta^p)E\left[\left\langle\frac{\xi^p}{T^p+\theta^p}\right\rangle\right]}\times$$
$$\times\left(E\left[\max\left(\eta^p+T^p+\theta^p-\left\{\frac{\xi^p}{T^p+\theta^p}\right\},0\right)\right]+E\left[\left\langle\frac{\xi^p}{T^p+\theta^p}\right\rangle\right]E\left[\max\left(\theta^p-\delta,0\right)\right]\right).$$

Note that

$$\Pr(B)=1-\Pr(\overline{B}).$$

We obviously have

$$q=q^{pt}\Pr(\chi\le\varphi)+q^t\Pr(\chi>\varphi),$$

where q^{pt} is the probability of failure of both safety subsystems and q^t is the probability of failure of the temperature safety subsystem. Furthermore

$$q^{pt}=\Pr(\overline{B})q^t.$$

Using the same technique as earlier we obtain the following estimation of q^t

$$q^t\approx1-\frac{E\left[\xi^t\right]-\theta^tE\left[\left\langle\frac{\xi^t}{T^t+\theta^t}\right\rangle\right]}{E\left[\eta^t\right]+(T^t+\theta^t)+(T^t+\theta^t)E\left[\left\langle\frac{\xi^t}{T^t+\theta^t}\right\rangle\right]}.$$

Therefore we managed to estimate all variables necessary to evaluate mean time to accident. Though some of them should be evaluated numerically the required techniques are pretty much straightforward.

3 CONCLUSIONS

The proposed model permits to assess the reliability of one specific class of technological systems with time redundancy. In particular the suggested approach allows to evaluate the mean time to accident for the "safety system-protected object" complex. The proposed approach allows to not underestimate the reliability of the complex with time redundancy. The solution obtained is useful for the reliability assessment of nuclear power plants and similar dangerous technological objects.

REFERENCES

Gnedenko, B.V., Ushakov, I.A. 1995. *Probabilistic Reliability Engineering*. John Wiley & Sons, Inc.

Pereguda, A.I. 2001. Calculation of the Reliability Indicators of the System Protected Object-Control and Protection System. *Atomic Energy* 90: 460-468.

Rausand, M., Høyland, A. 2004 *System Reliability Theory: Models, Statistical Methods and Applications*. John Wiley & Sons, Inc.

Stoyan, D. 1983 *Comparison Methods for Queues and Other Stochastic Models*. Wiley-Interscience.

RELIABILITY AND CAPABILITY MODELING OF TECHNOLOGICAL SYSTEMS WITH BUFFER STORAGE

Armen S. Stepanyants, Valentina S. Victorova

●

Institute of Control Science, Russian Academy of Sciences
117997 Moscow, Profsouznay 65
E-mail: ray@ipu.ru, viktorova_v@mail.ru

ABSTRACT

The paper is devoted to reliability and capability investigation of technological systems, inclusive of development of dynamic reliability model for two-phase product line with buffer storages and multiphase line decomposition

Key words: Reliability analysis, markov process, multiphase systems, multi-flow structure decomposition

1 INTRODUCTION

Multiphase systems are the systems where technological process and supporting equipment are divided into sections referred as phases. One of the approaches to improving reliability and capability is to include into multiphase system time redundancy using buffer storages. When failure of input section equipment occurs buffer storage ensures uninterrupted technological process in output sections. Valid choice of placement location and capacity of buffer storages is impossible without reliability modeling and analysis of system projects alternatives. Common prediction models of multiphase systems describe only single-flow structures and suppose absolute reliability of buffer storage (*Cherkesov 1974*). In this paper we suggest analytical method for calculation reliability and capability of multiphase systems based on two-parameter markov process. The prediction model takes into account different ratio of input and output devices capability and unreliable buffers. The model decomposition technique is developed. This makes it possible to analyze multi-flow systems with tree-type structures. Procedure of construction state space and transition graph of the two-parameter markov process is created. The procedure is founded on selection of state subsets, corresponding to intermediate and marginal (maximum or minimum) level of resource (inventory) in buffer, and generation of boundary and limiting transition. Process of generation of difference equation and boundary condition are described.

2 TWO-PHASE SYSTEM DESCRIPTION

Schema of single-flow two-phase system with input (1) and output (2) processing devices and transient buffer (3) is shown in **Figure 1**.

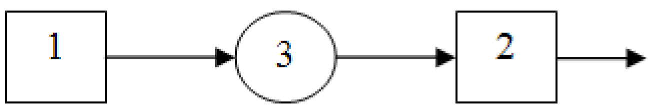


Figure 1. Single-flow two-phase system with buffer storage.

Each processing devices is characterized by capability q_i, failure rate λ_i, recovery rate μ_i; buffer is characterized by capacity z ($0 \le z \le z_M$), failure rate λ_H, recovery rate μ_H . Let us denote the state of markov graph for two-phase system by three-digit binary code. The first two digits indicate the states of the devices and the third digit indicates the buffer state. Digit 1 indicates that the state of device (buffer) is good, 0 – is failed.

Reliability behavior of the system depends on inventory level in the buffer:

- zero level (z = 0); we will designate zero level subset of markov reliability model state set as G
- maximum level ($z = z_M$); we will designate maximum level subset of markov reliability model state set as V
- intermediate level ($0 < z < z_M$); we will designate intermediate level subset of markov reliability model state set as W

3 METHODOLOGY OF TWO-PARAMETER MARKOV MODEL CONSTRUCTION

Let us define the markov model construction sequence:

1. Definition of all possible states for subsets G, V, W
2. Analysis of the states in compliance with characteristics of performance and failures, removing the states which can not stand in given subset and which have not transition from another states
3. Determination of the states which have marginal (limiting) transitions from another states (these are transitions from subset W into V and G, assignable with buffer inventory level maximization (minimization). Marginal transitions are indicated as dotted line.
4. Determination of boundary transitions from subsets V and G into subset W. These transitions exist for the states in subsets V and G, for which failure or recovery of the system devices result in buffer marginal inventory level decrease (increase). Boundary transitions are also indicated as dotted arc, waited with appropriate failure (recovery) rate.

After markov graph construction we can define mathematical model of the system. Let us denote state probability for subset W as P(z,t) and for subsets V and G as $F(z_M,t)$ and F(0,t) respectively. Now we can set up difference equation for characteristic states of the system. Characteristic states are the following:

1. The states which have input and output transitions in the range of one subset
2. The states which have input limiting transitions
3. The states which have output boundary transitions (equations for these states determine boundary conditions)

Figure 2 shows graphs with characteristic state α_i and input (output) transition. Graph I shows transitions in the range of one subset. Graph II shows boundary transition.

Difference equation for case I (transitions in the range of one subset) is of the form

$$P_{\alpha i}(z, t+\Delta t) - P_{\alpha i}(z \pm \Delta z_i, t) = -(\Delta t \sum_{i=1}^{m} \psi_i) \cdot P_{\alpha i}(z \pm \Delta z_i, t) + \Delta t \sum_{1}^{n} \cdot \varphi_i \cdot P_i(z \pm \Delta z_i, t) \tag{1}$$

Partial differential equation is:

$$q_{\alpha i} \cdot \frac{\partial P_{\alpha i}(z,t)}{\partial z} + \frac{\partial P_{\alpha i}(z,t)}{\partial t} = -(\sum_{i=1}^{m} \psi_i) \cdot P_{\alpha i}(z,t) + \sum_{i=1}^{n} \varphi_i \cdot P_i(z,t) \tag{2}$$

Let us consider stationary area and take into account the fact that $\frac{\partial P(z,t)}{\partial t} = 0$ when $t \to \infty$.

Then

$$q_{\alpha i} \cdot \frac{\partial P_{\alpha i}(z)}{\partial z} = -(\sum_{i=1}^{m} \psi_i) \cdot P_{\alpha i}(z) + \sum_{i=1}^{n} \varphi_i \cdot P_i(z) \tag{3}$$

Under (3) we can formulate the following rule for setting up differential equation for any state with transitions in the range of one subset.

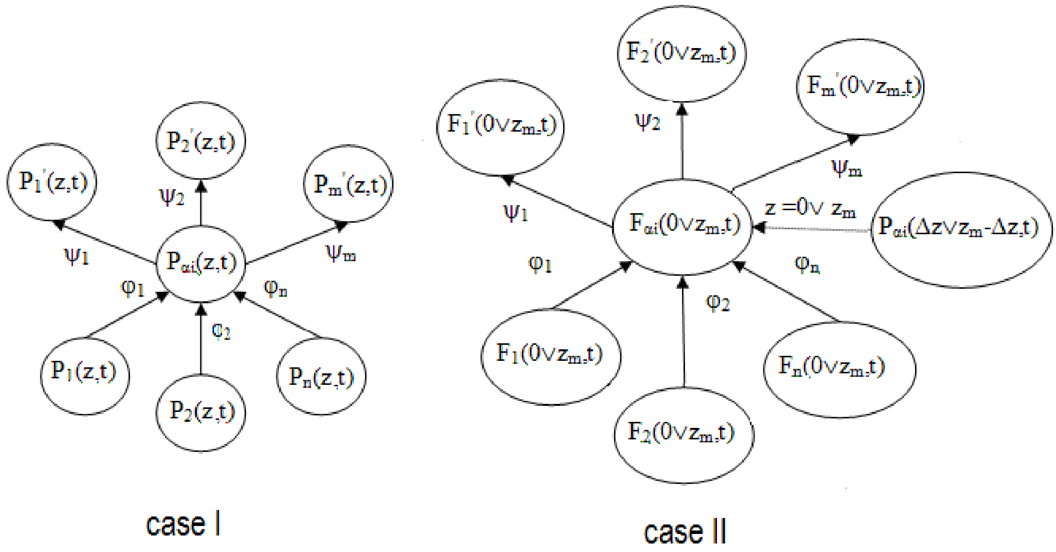


Figure 2. Graphs for case I (transitions in area of one state subset) and case II (limiting transition).

Rule 1. Derivative of state probability with respect to buffer inventory level (z) multiplied by rate of level change ($q_{\alpha i}$) is equal to product of state probability by sum of output transition rates, signed with minus, plus sum of product of input transition rate by probability of state from which transition is done.

Similarly we get differential equation for the case II. Here state α_i in the range of one subset has input transitions with rate φ_i, output transitions with rate ψ_I and limiting transition from subset W (z=0 or z=z_m).

$$\frac{\partial F_{\alpha i}(0 \vee z_m, t)}{\partial t} = \sum_{i=1}^{n} \varphi_i F_i(0 \vee z_m, t) - (\sum_{i=1}^{m} \psi_i) \cdot F_{\alpha i}(0 \vee z_m, t) + |q_{\alpha i}| \cdot P_{\alpha i}(0 \vee z_m, t) \qquad (4)$$

For stationary area ($t \to \infty$) we have algebraic equation

$$(\sum_{i=1}^{m} \psi_i) \cdot F_{\alpha i}(0 \vee z_m) = \sum_{i=1}^{n} \varphi_i F_i(0 \vee z_m) + |q_{\alpha i}| \cdot P_{\alpha i}(0 \vee z_m) \qquad (5)$$

Then it is possible to formulate the rule for states with input limiting transition.

Rule 2. Probability of considering state multiplied by sum of output transition rate is equal to sum of transition probabilities from other states to given state and probability of limiting transition. Probability of limiting transition is probability of state from which transition is done multiplied by absolute value of rate of level change.

Boundary condition occurs when transition exists from states of subsets V and G into states of subset W:

$$\begin{aligned} \psi \cdot F_{\alpha i}(z_m, t) &= P_{\alpha i}(z_m, t) \cdot |q_{\alpha i}| \\ \varphi \cdot F_{\alpha i}(0, t) &= P_{\alpha i}(0, t) \cdot |q_{\alpha i}| \end{aligned} \qquad (6)$$

Stationary boundary condition is:

$$\psi \cdot F_{\alpha i}(z_m) = P_{\alpha i}(z_m) \cdot |q_{\alpha i}|$$
$$\varphi \cdot F_{\alpha i}(0) = P_{\alpha i}(0) \cdot |q_{\alpha i}| \qquad (7)$$

4 MARKOV RELIABILITY MODEL FOR TWO-PHASE SYSTEM

Proceeding from rules and equations of previous section one can construct reliability models for two-phase single-flow system. Models were constructed for three alternatives of relationship of processing devices capability (q_1=q_2=q; $q_1 > q_2$; $q_1 < q_2$).

4.1 Model for equality of input and output capability

Markov graph for equality of capability of input and output processing devices (q_1=q_2=q) is shown on **Figure 3**.

System of partial differential equation is:

$$
\begin{aligned}
&-q \cdot \frac{\partial P_{011}(z,t)}{\partial z} + \frac{\partial P_{011}(z,t)}{\partial t} = -(\mu_1 + \lambda_2 + \lambda_H) \cdot P_{011}(z,t) + \lambda_1 \cdot P_{111}(z,t) + \mu_2 \cdot P_{001}(z,t) + \mu_H \cdot P_{010}(z,t)\\
&q \cdot \frac{\partial P_{101}(z,t)}{\partial z} + \frac{\partial P_{101}(z,t)}{\partial t} = -(\mu_2 + \lambda_1 + \lambda_H) \cdot P_{101}(z,t) + \lambda_2 \cdot P_{111}(z,t) + \mu_1 \cdot P_{001}(z,t) + \mu_H \cdot P_{100}(z,t)\\
&\frac{\partial P_{111}(z,t)}{\partial t} = -(\lambda_1 + \lambda_2 + \lambda_H) \cdot P_{111}(z,t) + \mu_1 \cdot P_{011}(z,t) + \mu_2 \cdot P_{101}(z,t) + \mu_H \cdot P_{110}(z,t)\\
&\frac{\partial P_{001}(z,t)}{\partial t} = -(\mu_1 + \mu_2) \cdot P_{001}(z,t) + \lambda_2 \cdot P_{011}(z,t) + \lambda_1 \cdot P_{101}(z,t)\\
&\frac{P_{010}(z,t)}{\partial t} = -(\mu_1 + \mu_H) \cdot P_{010}(z,t) + \lambda_H \cdot P_{011}(z,t)\\
&\frac{\partial P_{100}(z,t)}{\partial t} = -(\mu_2 + \mu_H) \cdot P_{100}(z,t) + \lambda_H \cdot P_{101}(z,t)\\
&\frac{\partial P_{110}(z,t)}{\partial t} = -\mu_H \cdot P_{110}(z,t) + \mu_1 \cdot P_{010}(z,t) + \mu_2 \cdot P_{100}(z,t) + \lambda_H \cdot P_{111}(z,t)\\
&\frac{\partial F_{111}(0,t)}{\partial t} = -(\lambda_1 + \lambda_2 + \lambda_H) \cdot F_{111}(0,t) + \mu_1 \cdot F_{011}(0,t) + \mu_H \cdot F_{110}(0,t)\\
&\frac{\partial F_{011}(0,t)}{\partial t} = -\mu_1 \cdot F_{011}(0,t) + \lambda_1 \cdot F_{111}(0,t) + q \cdot P_{011}(0,t)\\
&\frac{\partial F_{110}(0,t)}{\partial t} = -\mu_H \cdot F_{110}(0,t) + \lambda_H \cdot F_{111}(0,t)\\
&\frac{\partial F_{111}(z_m,t)}{\partial t} = -(\lambda_1 + \lambda_2 + \lambda_H) \cdot F_{111}(z_m,t) + \mu_2 \cdot F_{101}(z_m,t) + \mu_H \cdot F_{110}(z_m,t)\\
&\frac{\partial F_{101}(z_m,t)}{\partial t} = -\mu_2 \cdot F_{101}(z_m,t) + \lambda_2 \cdot F_{111}(z_m,t) + q \cdot P_{101}(z_m,t)\\
&\frac{\partial F_{110}(z_m,t)}{\partial t} = -\mu_H \cdot F_{110}(z_m,t) + \lambda_H \cdot F_{111}(z_m,t)
\end{aligned}
\qquad (8)
$$

Boundary condition:

$$q \cdot P_{011}(z_m,t) = \lambda_1 \cdot F_{111}(z_m,t)$$
$$q \cdot P_{101}(0,t) = \lambda_2 \cdot F_{111}(0,t) \quad (9)$$

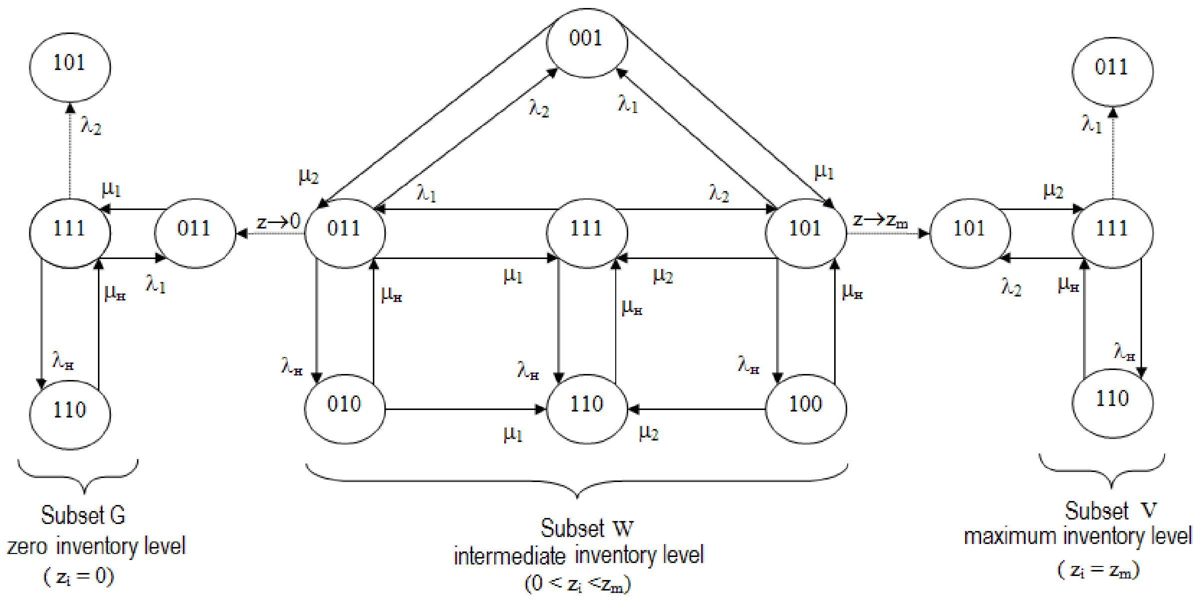


Figure 3. Markov graph for two-phase system ($q_1=q_2=q$).

At stationary area ($t \to \infty$) system (8) turns into the system of differential-algebraic equation:

$$-q \cdot P'_{011}(z) = -(\mu_1+\lambda_2+\lambda_H)\cdot P_{011}(z)+\lambda_1\cdot P_{111}(z)+\mu_2\cdot P_{001}(z)+\mu_H\cdot P_{010}(z)$$
$$q \cdot P'_{101}(z) = -(\mu_2+\lambda_1+\lambda_H)\cdot P_{101}(z)+\lambda_2\cdot P_{111}(z)+\mu_1\cdot P_{001}(z)+\mu_H\cdot P_{100}(z)$$
$$0 = -(\lambda_1+\lambda_2+\lambda_H)\cdot P_{111}(z)+\mu_1\cdot P_{011}(z)+\mu_2\cdot P_{101}(z)+\mu_H\cdot P_{110}(z)$$
$$0 = -(\mu_1+\mu_2)\cdot P_{001}(z)+\lambda_2\cdot P_{011}(z)+\lambda_1\cdot P_{101}(z)$$
$$0 = -(\mu_1+\mu_H)\cdot P_{010}(z)+\lambda_H\cdot P_{011}(z)$$
$$0 = -(\mu_2+\mu_H)\cdot P_{100}(z)+\lambda_H\cdot P_{101}(z)$$
$$0 = -\mu_H\cdot P_{110}(z)+\mu_1\cdot P_{010}(z)+\mu_2\cdot P_{100}(z)+\lambda_H\cdot P_{111}(z)$$
$$0 = -(\lambda_1+\lambda_2+\lambda_H)\cdot F_{111}(0)+\mu_1\cdot F_{011}(0)+\mu_H\cdot F_{110}(0)$$
$$0 = -\mu_1\cdot F_{011}(0)+\lambda_1\cdot F_{111}(0)+q\cdot P_{011}(0) \quad (10)$$
$$0 = -\mu_H\cdot F_{110}(0)+\lambda_H\cdot F_{111}(0)$$
$$0 = -(\lambda_1+\lambda_2+\lambda_H)\cdot F_{111}(z_m)+\mu_2\cdot F_{101}(z_m)+\mu_H\cdot F_{110}(z_m)$$
$$0 = -\mu_2\cdot F_{101}(z_m)+\lambda_2\cdot F_{111}(z_m)+q\cdot P_{101}(z_m)$$
$$0 = -\mu_H\cdot F_{110}(z_m)+\lambda_H\cdot F_{111}(z_m)$$

We use the following boundary and normalizing condition when solving system (10):

$$q \cdot P_{011}(z_m) = \lambda_1 \cdot F_{111}(z_m)$$
$$q \cdot P_{101}(0) = \lambda_2 \cdot F_{111}(0) \qquad (11)$$
$$\sum_{ijk} \int_0^{z_m} P_{ijk}(z)\partial z + \sum_{ijk} F_{ijk}(z_m) + \sum_{ijk} F_{ijk}(0) = 1$$

4.2 Model for unequal input and output capability ($q_1 > q_2$)

Markov graph for unequal capability of input and output processing devices ($q_1 > q_2$) is shown on **Figure 4**.

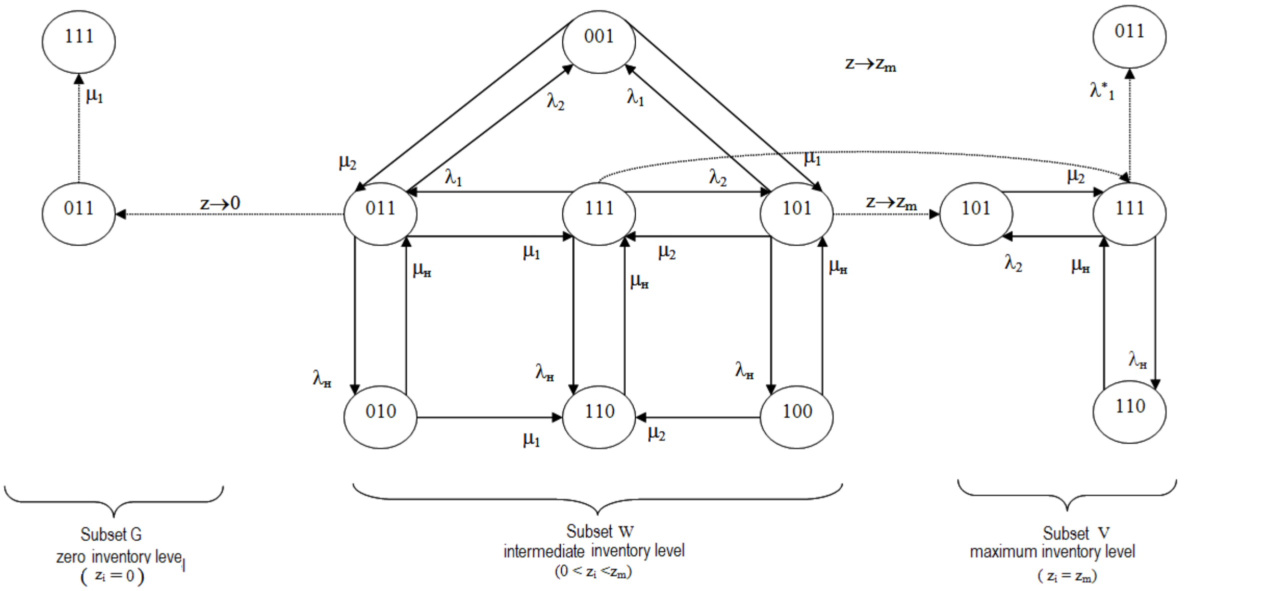


Figure 4. Markov graph for two-phase system ($q_1 > q_2$).

Let us directly consider stationary area ($t \to \infty$) and system of differential-algebraic equation:

$$-q_2 \cdot P'_{011}(z) = -(\mu_1 + \lambda_2 + \lambda_H) \cdot P_{011}(z) + \lambda_1 \cdot P_{111}(z) + \mu_2 \cdot P_{001}(z) + \mu_H \cdot P_{010}(z)$$
$$q_1 \cdot P'_{101}(z) = -(\mu_2 + \lambda_1 + \lambda_H) \cdot P_{101}(z) + \lambda_2 \cdot P_{111}(z) + \mu_1 \cdot P_{001}(z) + \mu_H \cdot P_{100}(z)$$
$$(q_1 - q_2) \cdot P'_{111}(z) = -(\lambda_1 + \lambda_2 + \lambda_H) \cdot P_{111}(z) + \mu_1 \cdot P_{011}(z) + \mu_2 \cdot P_{101}(z) + \mu_H \cdot P_{110}(z)$$
$$0 = -(\mu_1 + \mu_2) \cdot P_{001}(z) + \lambda_2 \cdot P_{011}(z) + \lambda_1 \cdot P_{101}(z)$$
$$0 = -(\mu_1 + \mu_H) \cdot P_{010}(z) + \lambda_H \cdot P_{011}(z)$$
$$0 = -(\mu_2 + \mu_H) \cdot P_{100}(z) + \lambda_H \cdot P_{101}(z) \qquad (12)$$
$$0 = -\mu_H \cdot P_{110}(z) + \mu_1 \cdot P_{010}(z) + \mu_2 \cdot P_{100}(z) + \lambda_H \cdot P_{111}(z)$$
$$0 = -\mu_1 \cdot F_{011}(0) + q_2 \cdot P_{011}(0)$$
$$0 = -(\lambda_1 + \lambda_2 + \lambda_H) \cdot F_{111}(z_m) + \mu_2 \cdot F_{101}(z_m) + \mu_H \cdot F_{110}(z_m) + (q_1 - q_2) \cdot P_{111}(z_m)$$
$$0 = -\mu_2 \cdot F_{101}(z_m) + \lambda_2 \cdot F_{111}(z_m) + q_1 \cdot P_{101}(z_m)$$
$$0 = -\mu_H \cdot F_{110}(z_m) + \lambda_H \cdot F_{111}(z_m)$$

Boundary and normalizing condition:

$$
\begin{aligned}
& q \cdot P_{011}(z_m) = \lambda_1^* \cdot F_{111}(z_m) \\
& (q_1 - q_2) \cdot P_{111}(0) = \mu_1 \cdot F_{011}(0) \\
& \sum_{ijk} \int_0^{z_m} P_{ijk}(z)\partial z + \sum_{ijk} F_{ijk}(z_m) + \sum_{ijk} F_{ijk}(0) = 1
\end{aligned}
\tag{13}
$$

4.3 Model for unequal input and output capability ($q_1 < q_2$)

Markov graph for unequal capability of input and output processing devices ($q_1 < q_2$) is shown on **Figure 5**.

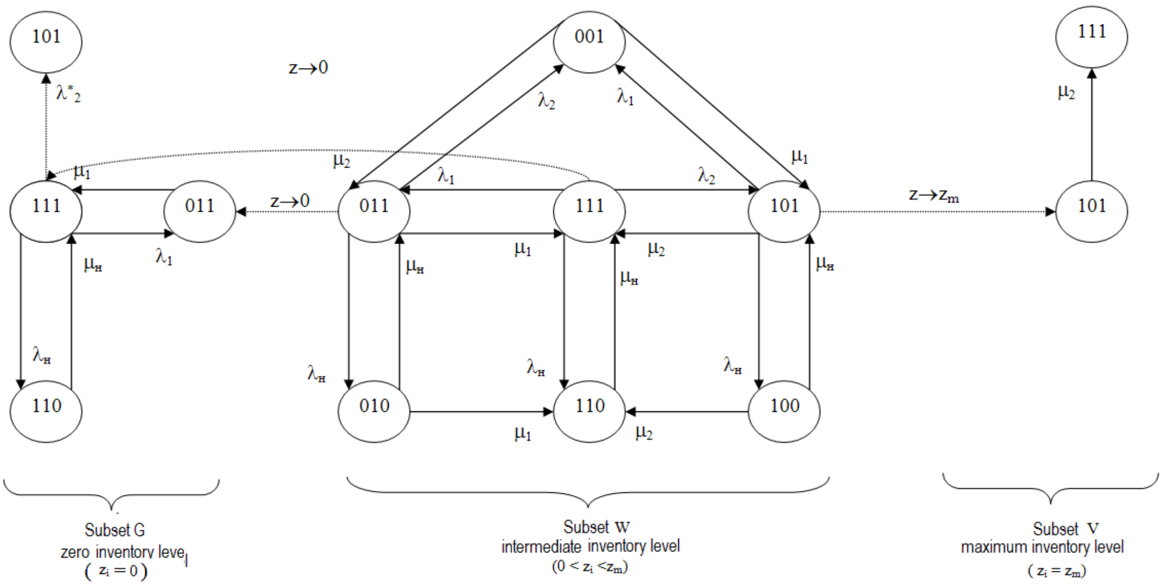


Figure 5. Markov graph for two-phase system ($q_1 < q_2$)

System of differential-algebraic equation $(t \to \infty)$:

$$
\begin{aligned}
& -q_2 \cdot P'_{011}(z) = -(\mu_1 + \lambda_2 + \lambda_{\text{н}}) \cdot P_{011}(z) + \lambda_1 \cdot P_{111}(z) + \mu_2 \cdot P_{001}(z) + \mu_{\text{н}} \cdot P_{010}(z) \\
& q_1 \cdot P'_{101}(z) = -(\mu_2 + \lambda_1 + \lambda_{\text{н}}) \cdot P_{101}(z) + \lambda_2 \cdot P_{111}(z) + \mu_1 \cdot P_{001}(z) + \mu_{\text{н}} \cdot P_{100}(z) \\
& -(q_2 - q_1) \cdot P'_{111}(z) = -(\lambda_1 + \lambda_2 + \lambda_{\text{н}}) \cdot P_{111}(z) + \mu_1 \cdot P_{011}(z) + \mu_2 \cdot P_{101}(z) + \mu_{\text{н}} \cdot P_{110}(z) \\
& 0 = -(\mu_1 + \mu_2) \cdot P_{001}(z) + \lambda_2 \cdot P_{011}(z) + \lambda_1 \cdot P_{101}(z) \\
& 0 = -(\mu_1 + \mu_{\text{н}}) \cdot P_{010}(z) + \lambda_{\text{н}} \cdot P_{011}(z) \\
& 0 = -(\mu_2 + \mu_{\text{н}}) \cdot P_{100}(z) + \lambda_{\text{н}} \cdot P_{101}(z) \\
& 0 = -\mu_{\text{н}} \cdot P_{110}(z) + \mu_1 \cdot P_{010}(z) + \mu_2 \cdot P_{100}(z) + \lambda_{\text{н}} \cdot P_{111}(z) \\
& 0 = -(\lambda_1 + \lambda_2^* + \lambda_{\text{н}}) \cdot F_{111}(0) + \mu_1 \cdot F_{011}(0) + \mu_{\text{н}} \cdot F_{110}(0) + (q_2 - q_1) \cdot P_{111}(0) \\
& 0 = -\mu_1 \cdot F_{011}(0) + \lambda_1 \cdot F_{111}(0) + q_2 \cdot P_{011}(0) \\
& 0 = -\mu_{\text{н}} \cdot F_{110}(0) + \lambda_{\text{н}} \cdot F_{111}(0) \\
& 0 = -\mu_2 \cdot F_{101}(z_m) + q_1 \cdot P_{101}(z_m)
\end{aligned}
\tag{14}
$$

Boundary and normalizing condition:

$$
\begin{aligned}
&q_1 \cdot P_{101}(0) = \lambda_2^* \cdot F_{111}(0) \\
&(q_2 - q_1) \cdot P_{111}(z_m) = \mu_2 \cdot F_{101}(z_m) \\
&\sum_{ijk} \int_0^{z_m} P_{ijk}(z)\partial z + \sum_{ijk} F_{ijk}(z_m) + \sum_{ijk} F_{ijk}(0) = 1
\end{aligned}
\tag{15}
$$

Computer-oriented procedure was developed for analytical solving systems (10), (12), (14). In accordance with this procedure at first one have to obtain probability density function $P_{101}(z)$:

$$P_{101}(z) = P_{101}(0) \cdot e^{\frac{a}{q}z}, \tag{16}$$

where $a = -(\lambda_1 + \mu_2 + \lambda_н) + \lambda_2 b + \frac{\mu_1(\lambda_1 + \lambda_2)}{\mu_1 + \mu_2} + \frac{\mu_н \lambda_н}{\mu_2 + \mu_н}$

Further probability $F_{101}(z)$ is defined via density function

$$F_{101}(z) = \int_0^{z_m} P_{101}(z)dz = \frac{q}{a} \cdot P_{101}(0)\left[e^{\frac{a}{q}z_m} - 1\right] = C_1 \cdot H_1. \tag{17}$$

Then each i[th] unknown $F_{ijk}(z)$ is represented as product of invariable and variable actors $C_1 \cdot H_i$, where H_i recursively calculated from H_{i-1}, and C_1 is calculated from normalizing condition (11,13,15).

Stationary availability $K_r(z)$ and mathematical expectation of capability C(z) of two-phase system are

$$K_r(z)=F_{111}(z)+F_{011}(z)+F_{111}(0)+F_{111}(z_m); \quad C(z)=K_r(z)\cdot q. \tag{18}$$

5 RELIABILITY AND CAPABILITY ANALYSIS OF MULTIPHASE SYSTEM

Multiphase systems are aggregate of two-phase systems. Examples of multiphase multiflow systems, specified in graphical editor of software implemented described above models, are shown in **Figure 6**.

The procedure of calculation estimate of availability of multiphase system includes the following steps:

1. Pick out the triplet (buffer, input device, output device) with minimum buffer capacity
2. Calculate availability and average capability indexes (18) for evolved triplet via appropriate models (10, 12, 14)
3. Replace the triplet by one processing device with equivalent availability and capability calculated on previous step
4. Repeat steps 1-3 until all multiphase structure will be represented by one equivalent device

It was shown in (*Victorova 2009)* that above procedure ensures derivation of availability low estimate.

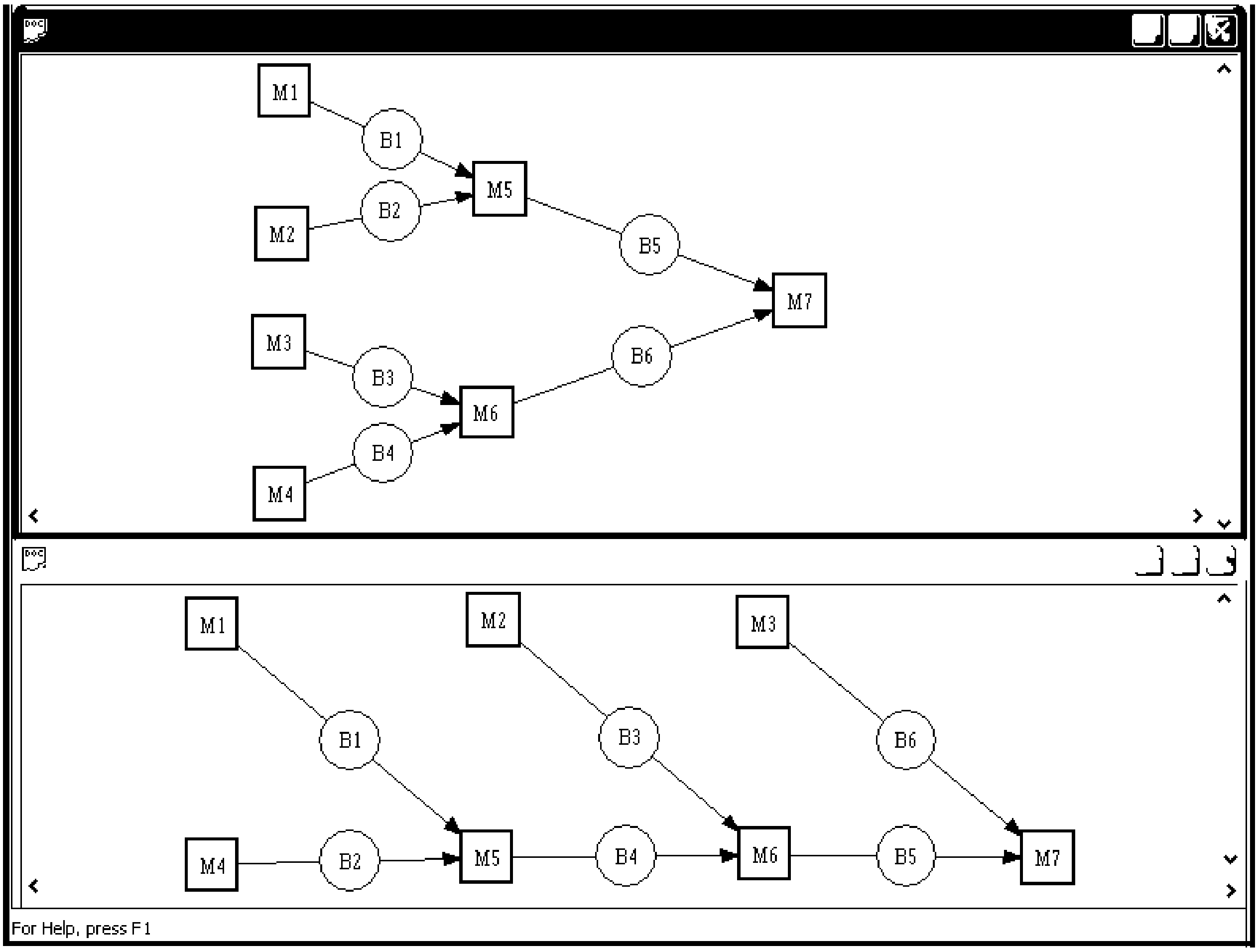


Figure.6. Screen shot of software for reliability and capability analysis of multiphase systems.

6 CONCLUSION

For adequate reliability and capability modeling of technological systems it is necessary to take into account unreliability of buffer storages. Statistical analysis of failure data of buffer storages shows failure rate growth with increasing capacity. On assumption of absolute buffer reliability one can make pitfall about continuous capability growth with increasing capacity (see upper curves on **Figure 7**). Analysis based on the models suggested in this paper shows that inflection point exist on the curve of capability as function of capacity. After this point one can observe decrease of reliability and capability of multiphase systems as it is shown on lower curves of **Figure 7**.

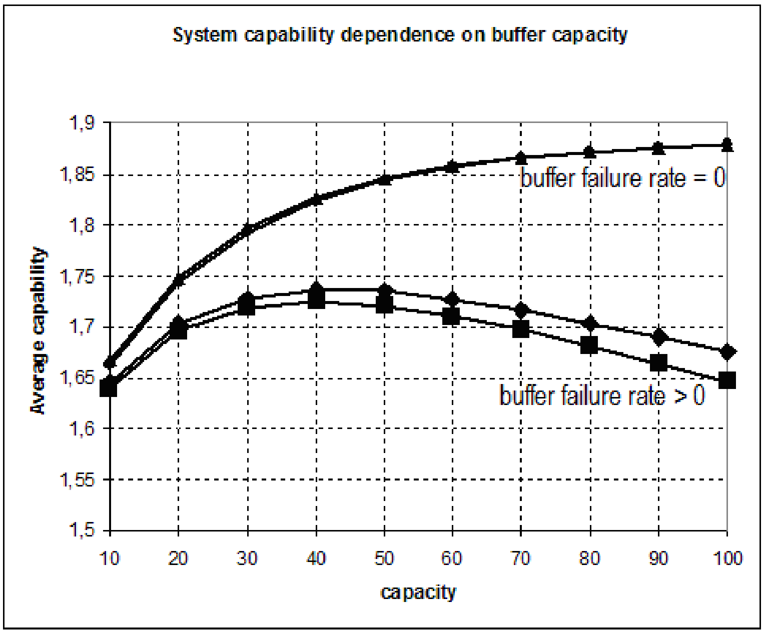


Figure 7. Multiphase system capability dependence on buffer storage capacity.

7 REFERENCES

Cherkesov, G.N. (1974). *Reliability of Technical Systems with Time Redundancy*, Sovetskoe Radio: Moscow.

Victorova V.C. (2009). *Aggregation of reliability and safety models of complex technical systems*, Abstract of Dr.Sci thesis, Institute of Control Sciences RAS: Moscow

COMPARISON ANALYSIS OF RELIABILITY OF NETWORKS WITH IDENTICAL EDGES

G.Tsitsiashvili

•

640041, Russia, Vladivostok, Radio str. 7, IAM, FEB RAS,

e-mail: guram@iam.dvo.ru,

ABSTRACT

Efficient and fast algorithms of parameters calculation in Burtin-Pittel asymptotic formula for networks with identical and high reliable edges are constructed. These algorithms are applied to a procedure of a comparison of networks obtained from a radial-circle network by a cancelling of some edges or their collapsing into nodes or by a separate reservation of these edges.

1 INTRODUCTION

A problem of a calculation of connection probabilities between nodes of a random graph is important and complicated problem in the probability theory. In general case it demands a number of arithmetical operations which increases as a geometric progression dependently on a number of the graph edges (Barlow et al. 1962), (Ushakov et al. 1985). So a construction of special methods of its complexity decreasing is actual in the reliability theory.

For random graphs with identical and high reliable edges there is the Burtin-Pittel asymptotic formula (see for example (Getsbakh, 2000)) which expresses a probability of a disconnection between initial and final edges via a minimal number d of edges in the graph cuts and a number c of cuts with d edges. Apparently one of the most convenient methods of a reliability calculation for graphs with identical edges is based on topological invariants (private communication of I. Gertsbakh). This method allows to calculate a network reliability directly as a polynomial of an edge reliability using the Monte-Carlo method. It allows to calculate integer coefficients c, d also. But the Monte-Carlo method is approximate and may give with a small probability a unit mistake in the coefficients definition. Such mistake may influence significantly on an accuracy of the Burtin-Pittel asymptotic formula.

This circumstance makes necessary to return to accuracy algorithms of Ford and Fulkerson (Ford et al. 1965) to calculate main asymptotic parameter d. In some cases these algorithms may be significantly simplified by a construction of some sufficient conditions in the Ford-Fulkerson problem. These conditions allow to define an influence of different connections – horizontal and vertical on a network reliability. As a result it is possible to find significant changes of the network reliability by its structural transformations like a cancelling of edges or their collapsing into nodes or their separate reservation. These considerations are made on an example of a radial-circle network with a few concentric circles.

Suppose that $\Gamma = \{U, W\}$ is no oriented graph with the finite set U of nodes and the finite- set $W = \{w = (u, v),\ u, v \in U\}$. Assume that each edge $w \in W$ fails with the probability $\overline{p}_w = 1 - p_w$, $0 < \overline{p}_w < 1$, independently on all other edges. Denote $\overline{P}_\Gamma$ the disconnection (or failure probability) of the graph Γ.

Theorem 1. If $\overline{p}_w(h) = h$, $w \in W$, then the Burtin-Pittel formula is true:

$$\overline{P}_\Gamma \sim ch^d,\ h \to 0. \quad (1)$$

2 NECESSARY VOLUME OF SEPARATE RESERVE

Suppose now that $\overline{p}_w(h) \equiv q = const$, $0<q<1$, $w \in W$, and each graph edge has k-fold reserve. Such scheme of a separate reservation is analyzed detailed in [1]. Denote the graph Γ with k-fold reserve of each edge by Γ_k and put $N(\varepsilon)=\min\left(n:\overline{P}_{\Gamma_k}<\varepsilon\right)$, $0<\varepsilon<1$.

Theorem 2. The following asymptotic formula takes place:

$$N(\varepsilon) \sim \frac{\ln \varepsilon}{d \ln q}, \ \varepsilon \to 0. \tag{2}$$

Proof: In the theorem 2 conditions the asymptotic formula (1) has the form

$$\overline{P}_{\Gamma_k} \sim c q^{kd}, \ k \to \infty. \tag{3}$$

Consequently for any $\delta>0$ there is $N_1=N_1(\delta)$ so that for $k>N_1(\delta)$ the following inequality is true:

$$c q^{kd}(1-\delta) \le \overline{P}_{\Gamma_k} \le c q^{kd}(1+\delta). \tag{4}$$

Fix δ and suppose that $\varepsilon<\overline{P}_{\Gamma_{N_1(\delta)}}$ then from the formula (4) obtain the inequalities

$$c q^{N(\varepsilon)d}(1-\delta) \le \varepsilon \le c q^{(N(\varepsilon)-1)d}(1+\delta)$$

and so

$$\ln c + N(\varepsilon) d \ln q + \ln(1-\delta) \le \ln \varepsilon \le \ln c + (N(\varepsilon)-1) d \ln q + \ln(1+\delta).$$

Consequently obtain

$$\frac{\ln c + \ln(1-\delta)}{\ln \varepsilon} + \frac{N(\varepsilon) d \ln q}{\ln \varepsilon} \ge 1 \ge \frac{\ln c + \ln(1+\delta) - d \ln q}{\ln \varepsilon} + \frac{N(\varepsilon) d \ln q}{\ln \varepsilon}$$

and as a result the formula (2) takes place.

Remark that the asymptotic formula (1) for the failure probability $\overline{P}_\Gamma$ includes the constants c,d whereas the asymptotic formula (2) for the necessary volume of the separate reserve $N(\varepsilon)$ includes only the single constant d. To define the constant d it is sufficient to use the Ford-Falkerson algorithm (Ford et al. 1965) with a complexity proportional a cube of the edges number (Kormen et al. 2004). An accuracy calculation of the constant c is much heavier and reduces to NP - complete problem.

Theorem 3. The following inequality is true:

$$N(\varepsilon) \le \frac{|\ln(\varepsilon/n)|}{|\ln q|} + 1. \tag{5}$$

Proof. Indeed the probability P_{Γ_k} satisfies the inequality

$$P_{\Gamma_k} \ge \left(1-q^k\right)^n \ge 1 - n q^k$$

which leads to the formula (5).

Denote by Γ^k the parallel connection of k independent copies of the graph Γ in initial and in final nodes and designate $M(\varepsilon)=\inf\left(k: P_{\Gamma^k} \ge 1-\varepsilon\right)$ the necessary volume of Γ block reserve. If the graph Γ contains r acyclic ways from the initial to the final node and the minimal number of edges in these ways is l then $M(\varepsilon) \ge \frac{1-\varepsilon}{r p^l}$. Suppose that the graph Γ is a sequential connection of n edges then it is easy to obtain that

$$M(\varepsilon) \ge \frac{1-\varepsilon}{p^n}. \tag{6}$$

In the formula (6) a low bound of $M(\varepsilon)$ is an increasing by n geometric progression. Analogous low bound may be obtained for a port Γ in which initial node u is connected with m nodes on the first floor, each node of the first floor is connected with s among m nodes of the second floor, and so on. All nodes on the l-th floor are connected with final node v, all edges are directed from u to v. Such graph is called channel graph and is analyzed detailed in (Harms et al. 1995). The formulas (5), (6) allow to establish a strong asymptotic difference between the separate reserve and the block reserve. Comparison theorems of reliabilities of ports with these reserves have been established in (Barlow et al, 1962).

3 RADIAL-CIRCLE SCHEME

Consider a radial-circle graph Γ with m concentric circles, l radiuses and with independent edges which have failure probability $\overline{p}_w = h$.

3.1 Influence of structural changes on asymptotic of failure probability

Consider a probability $\overline{P}_\Gamma$ of a disconnection between the center u and the node v on internal circle of Γ. Say that circle edges provide horizontal connections and radial edges provide vertical connections in the graph Γ. Our problem is to analyze an influence of horizontal connections on the probability $\overline{P}_\Gamma$ if edges are high reliable and their failure probability $h \to 0$.

For a simplicity suppose that $l > 3$ then it is easy to find $d = 3$, $c = 1$ and the single cut with $d = 3$ edges consists of edges which end in the node v. As a result obtain that $\overline{P}_\Gamma \sim h^3$, $h \to 0$.

Suppose now that all circle edges have single reliability and so these edges may be collapsed into nodes. Then the graph Γ transforms in to a sequential connection of m parallel connections of l edges. Simple calculations show that then $d = l$, $c=m$ and consequently $\overline{P}_\Gamma \sim mh^l$, $h \to 0$.

Suppose that all circle edges have zero reliability and so may be cancelled. The there is a single way between the nodes u,v which contains l edges. Consequently $d = 1$, $c=m$ and so $\overline{P}_\Gamma \sim mh$, $h \to 0$.

Suppose that two circle edges connected with the node u have r-fold reserve, $2r + 1 < l$. Then we obtain $d = 2r + 1$, $c = 1$, $\overline{P}_\Gamma \sim h^{2r+1}$, $h \to 0$.

Consequently manipulations with a replacement of circle edges (which characterize horizontal connections) by absolutely reliable or absolutely unreliable or by r-fold reserves may influence significantly on the probability $\overline{P}_\Gamma$.

3.2 Accelerated algorithm of the constant d calculation

The constant d calculation in general case has a cubic complexity and it is sufficiently large for all possible pairs of nodes. So a problem is to simplify this algorithm for some families of graphs. In (Tsitsiashvili, 2010) such simplification is made for a rectangle on a lattice. In this paper a radial-circle graph Γ with m concentric circles and l radiuses is considered.

Using the Ford-Falkerson theorem about an equality of maximal flow and minimal ability to handle of cuts (Ford et al. 1962) it is easy to obtain the inequality

$$d \le \delta\ ,\ \delta = \min(\alpha_u, \alpha_v)\ . \tag{7}$$

Here α_u is the number of edges connected with the initial node u and α_v is the number of edges connected with the final node v. From the Ford-Falkerson theorem and from the inequality (7) we have that

$$d = \delta$$

if there are d independent (no intersected by edges) ways between the nodes u,v.

Use this sufficient condition to calculate the constant d. Denote C_u, C_v the circles which contain the nodes u,v and put R_u, R_v radiuses which contain u,v appropriately. Without a restriction of a generality suppose that C_u borders C_v.

The constant d calculation. Suppose that $l > 2$.

1) If the nodes u,v belong to an external circle then $\delta = 3$ and between u,v there are the following independent: two ways on the internal circle and a way via the center (Fig. 1) consequently $d = 3$.

2) If the nodes u,v belong to an internal circle then $\delta = 4$ and between u,v there are the following independent ways: two of them on the same circle, one way via the center and one way via external circle (Fig. 2). In this case we have $d = 4$.

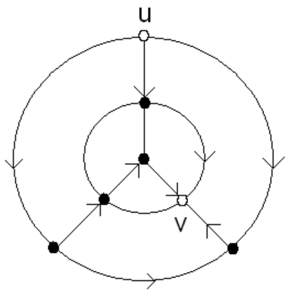


Figure 1.

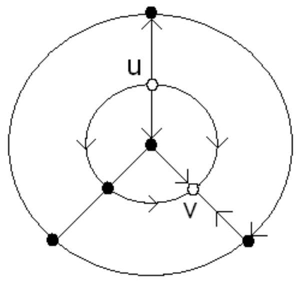


Figure 2.

3) Suppose that u belongs to external circle and v to internal circle then $\delta = 3$. If u,v belong to different radiuses then between u,v there are the following independent ways: the first way is from u along R_u to C_v and then to v, the second way is along C_u to R_v and then to v, the third way is along C_u to a radius which does not coincide with R_u, R_v then to the center and then along R_v to v (Fig. 3a) consequently $d = 3$. If u,v belong to the same radius then there are the following independent ways between u,v: the radial way, two ways along the external circle to some radius then to the center and then along radius to v (Fig. 3b), consequently $d = 3$.

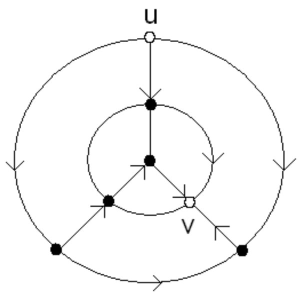


Figure 3a.

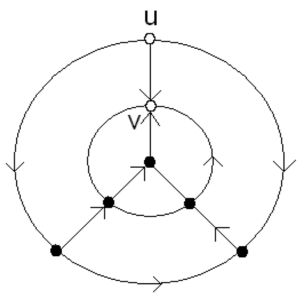


Figure 3b.

4) Suppose that the nodes u,v belong to internal circles then $\delta = 4$. Consider the case when these nodes belong to different radiuses.

If $l = 3$ then it is possible to construct the cut between u,v which consists of edges with ends on C_u and place on different radiuses (Fig. 4a) consequently $d \le 3$. It is easy to prove that between u,v there are not cuts with two edges consequently $d = 3$.

If $l > 3$ then $\delta = 4$ and it is possible to find four independent ways between u,v (Fig. 4b) consequently $d = 4$.

If the nodes u,v belong to the same radius then analogously it is possible to obtain that for $l = 3$ we have $d = 3$, and for $l > 3$ we have $d = 4$.

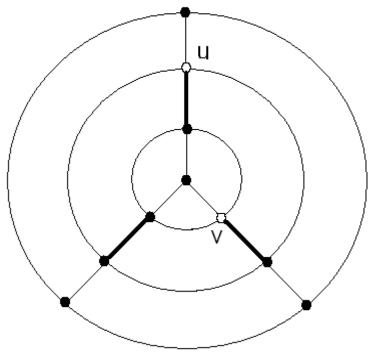


Figure 4a.

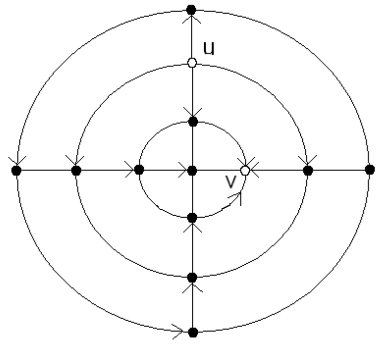


Figure 4b.

The constant c calculation.

Suppose that the node v is the center and the node u is on the m_1-th concentric (from the center) circle, $m_1 \le m$. If $l = 2$ then $c = m_1$. If $l = 3$ then for $m_1 < m$ we have $c = m_1$ and for $m_1 = m$ we have $c = m_1 + 1$. Suppose that $l = 4$ then for $m_1 < m$ we have $c = m_1 + 1$ and for $m_1 = m$ we have $c = 1$. Suppose that $l > 4$ then $c = 1$.

4 CONCLUSION

Remark that an application of Burtin-Pittel asymptotic formula for networks with identical and high reliable edges is sufficiently complicated procedure independently on considered (mainly heuristic arguments). So it is necessary to find some more arguments for the example of radial-circle network with a few concentric circles and radial edges with positive and fixed reliability and low reliable circle edges. This network may be considered using statements from (Tsitsiashvili et al. 2010)].

Consider the port Γ with fixed initial and final nodes u, v and the finite sets of nodes U and edges W. Suppose that the set W consists of no intersected subsets W_1, W_2 where $p_w(h) \equiv p_w > 0,\ w \in W_1$, and for $w \in W_2$ we have $p_w(h) \to 0,\ h \to 0$. Denote $\mathcal{R} = \{R_1, ..., R_n\}$ the set of all acyclic ways between u, v.

Theorem 4. If $R_1 \subseteq W_1$, $R_2 \cap W_2 \ne \varnothing, ..., R_n \cap W_2 \ne \varnothing$ then the connection probability

$$P_\Gamma(u,v) \to \prod_{w \in R_1} p_w,\ h \to 0.$$

Using this theorem it is easy to obtain that $P_\Gamma(u,v)$ may be approximated by a product of reliabilities of radial edges which belong to acyclic way between the nodes u, v. An accuracy and a performance of this approximation for the network with a single circle is discussed in (Tsitsiashvili et al. 2010).

The author thanks I Gerstbakh for useful discussions. Supported by Far Eastern Branch of RAS, projects 09-1-П2-07, 09-1-OMN-07.

REFERENCES

1. Barlow R.E., Proschan F. 1965. *Mathematical Theory of Reliability*. London and New York: Wiley.
2. Ford L.R., Fulkerson D.R. 1962. *Flows in networks*. Princeton university press. Princeton, New Jersey.
3. Gertsbukh I. 2000. *Theory with Applications to Preventive Maintenance*. Springer Verlag.
4. Harms D.D., Kraezl M. Colbourn C.J., Devitt J.S. Network reliability. Experiments with a Symbolic Algebra Environment. 1995. CRC Press. Inc.

5. Kormen T., Leizerson Ch., Rivest R. 2004. *Algorithmes: construction and analysis*. Moscow: Laboratory of basic knowledges.
6. Tsitsiashvili G.Sh. 2010. Asymptotic analysis of lattice reliability. *Reliability: Theory and Applications*. № 1: 65-70 pp.
7. Tsitsiashvili G.Sh., Losev A.S. 2010. Calculation of connectivity probability in recursively defined random networks. *Reliability: Theory and Applications*. V. 1 (16), № 1: 40-46 pp.
8. Ushakov I.A. et al. 1985. *Reliability of technical systems: Handbook.* Moscow: Radio and Communication.

ASYMPTOTIC FORMULAS IN DISCRETE TIME RISK MODEL WITH DEPENDENCE OF FINANCIAL AND INSURANCE RISKS

G.Tsitsiashvili

●

640041, Russia, Vladivostok, Radio str. 7, IAM, FEB RAS,

e-mail: guram@iam.dvo.ru

ABSTRACT

Asymptotic formulas for a ruin probability in discrete time risk model with a dependence of financial and insurance risks are obtained. These formulas are constructed in a suggestion which is adequate to economical crisis: the larger is a financial risk the larger is an insurance risk.

1. INTRODUCTION

In this paper we obtain asymptotic formulas for a ruin probability in discrete time risk model with a dependence of financial and insurance risks. Earlier simple asymptotic formulas for the ruin probability in a case of independent financial and insurance risks have been obtained in [1]. More complicated cases with special restrictions on insurance risks dependence are considered for example in [2,3]. Nevertheless until recently an asymptotic analysis of risk model with a dependence of insurance and financial risks is not made. But in modern period of strong economical crisis such dependence may be recognized easily in different large anthropogenic catastrophes. So a problem to analyze asymptotically this dependence is actual now.

In this paper we consider special model of insurance and financial risks dependence based on suggestion that a financial risk has a finite number of meanings and for each meaning an insurance risk has its own distribution. Then Pareto-tailed and Weibull-tailed asymptotics of insurance risks distributions are considered. In frames of this suggestion we assume that the larger is the financial risk the larger is the insurance risk. This stochastic modeling approach allows to obtain new asymptotic formulas for ruin probability in risk models.

2 PRELIMINARIES

Classes of distributions. Throughout, for a given random variable (r.v.) X concentrated on $(-\infty,\infty)$ with a distribution function (d.f.) F then its right tail $\overline{F}(x)=P(X>x)$. For two d.f.'s F_1 and F_2 concentrated on $(-\infty,\infty)$ we write by $F_1*F_2(x)$ the convolution of F_1 and F_2 and write by $F_1^{*2}=F_1*F_1$ the convolution of F_1 with itself. All limiting relationships, unless otherwise stated, are for $x\to\infty$. Let $a(x)\geq 0$ and $b(x)>0$ be two infinitesimals, satisfying

$$l^-\leq\liminf_{x\to\infty}\frac{a(x)}{b(x)}\leq\limsup_{x\to\infty}\frac{a(x)}{b(x)}\leq l^+.$$

We write $a(x)=O(b(x))$, if $l^+<\infty$ and $a(x)\sim b(x)$ if $l^+=l^-=1$.

Introduce the following classes of d.f.`s concentrated on $[0,\infty)$:

$$\mathcal{S}=\left\{F(x):\lim_{x\to\infty}\frac{\overline{F^{*2}}(x)}{\overline{F}(x)}=2\right\},\ \mathcal{L}=\left\{F(x):\forall t\ \lim_{x\to\infty}\frac{\overline{F}(x-t)}{\overline{F}(x)}=1\right\},$$

$\mathcal{R}_{-\alpha}=\left\{F(x):\forall\theta>0\ \lim_{x\to\infty}\frac{\overline{F}(\theta x)}{\overline{F}(x)}=\theta^{-\alpha}\right\}$, $0<\alpha<\infty$, $\mathcal{R}=\bigcup_{0<\alpha<\infty}\mathcal{R}_{-\alpha}$. $\mathcal{S}$ is called the class of subexponential d.f.`s. $\mathcal{L}$ is called the class of long tailed d.f.`s. $\mathcal{R}$ (or $\mathcal{R}_{-\alpha}$) is called the class of regular varying d.f.`s (with index α). More generally, d.f. F concentrated on $(-\infty,\infty)$ is also said to belong to these classes if its right-hand distribution $\overline{F}(x) = F(x)\square(x>0)$ does.

Proposition 1. The classes $\mathcal{R}, \mathcal{S}, \mathcal{L}$ satisfy the formula [4] $\mathcal{R} \subset \mathcal{S} \subset \mathcal{L}$. If for some $a,b, 0<a, 0<b<1$ d.f. F satisfies the equivalence $\overline{F}(t) \sim \exp\left(-at^b\right)$, $t \to \infty$ then $F \in \mathcal{S}$.

Proposition 2. Let F_1 and F_2 be two d.f.'s concentrated on $(-\infty,\infty)$. If $F_2 \in \mathcal{L}$, $F_1 \in \mathcal{S}$ and $\overline{F}_2(x) = O\left(\overline{F}_1(x)\right)$, then [1, Lemma 3.2] $F_1 * F_2 \in \mathcal{S}$ and $\overline{F_1 * F_2}(x) \sim \overline{F_1}(x) + \overline{F_2}(x)$.

3 DISCRETE TIME RISK MODEL AND ITS PROPERTIES

Consider discrete time risk model (with annual step) with initial capital $x, x \geq 0$ and nonnegative losses

$$Z_n,\ n=1,2,\ldots,\ P(Z_n < t) = F(t).$$

Suppose that income A_n, $n=1,2,\ldots$ to end of n-th year is defined as difference between unit premium sum and loss $A_n = 1 - Z_n$. Assume that $R_n > 1$ is inflation factor from $n-1$ to n year, $n=1,2,\ldots$. In [5] $X_n = -A_n$ is called insurance risk and $Y_n = R_n^{-1}$ is called financial risk.

Suppose that the following condition is true:

(A). $\{(A_n, R_n), n \geq 1\}$ is sequence of independent and identically distributed random vectors (i.i.d.r.) vectors

$$S_0 = x,\ S_n = R_n S_{n-1} + A_n,\ n = 1,2\ldots \tag{1}$$

In this model with initial capital x ruin time is defined by formula

$$\tau(x) = \inf\{n = 1,2,\ldots : S_n \leq 0 | S_0 = x\},$$

and finite time ruin probability $\psi(x,n)$ - by formula

$$\psi(x,n) = P(\tau(x) \leq n).$$

So the sum S_n money accumulated by insurance company to n- th year end satisfies recurrent formula

$$S_0 = x\ ,S_n = x\prod_{j=1}^{n} B_j + \sum_{i=1}^{n} A_i \prod_{j=i+1}^{n} B_j\ ,\ \ n = 1,2\ldots, \tag{2}$$

where $\prod_{j=n+1}^{n} = 1$ by convention.

According to the notation above, we can rewrite the discounted value of the surplus S_n in (2) as

$$\tilde{S}_0 = x, \qquad \tilde{S}_n = S_n \prod_{j=1}^{n} Y_j = x - \sum_{i=1}^{n} X_i \prod_{j=1}^{i} Y_j = x - W_n.$$

Hence, we easily understand that, for each $n = 0,1,\ldots$,

$$\psi(x,n) = \mathrm{P}(U_n > x), \tag{3}$$

where

$$U_n = \max\left\{0, \max_{1\leq k\leq n} W_k\right\},\ \text{with } U_0 = 0. \tag{4}$$

Define another Markov chain as

$$V_0 = 0,\quad V_n = Y_n(0, X_n + V_{n-1}),\quad n = 1,2\ldots. \tag{5}$$

Theorem 1. Suppose that the condition **(A)** is true.

1. Random variables U_n and V_n coincide by distribution

$$U_n \overset{d}{=} V_n, \ n = 0,1,... \tag{6}$$

2. Equality

$$\psi(x,n) = P(V_n \geq x) \tag{7}$$

is true.

Proof: The result (6) is trivial for the case when $n = 0$. Now we aim at (6) for each $n = 1,2...$. Let $n \geq 1$ be fixed. It is easy to obtain the equality

$$U_n = \max\left\{0, \max_{1\le k\le n} \sum_{i=1}^{k} X_i \prod_{j=1}^{i} Y_i\right\} = T_n((X_1,Y_1),...,(X_n,Y_n)).$$

Here T_n is a deterministic function. From the condition **(A)** we obtain that

$$((X_1,Y_1),...,(X_n,Y_n)) \overset{(d)}{=} ((X_n,Y_n),...,(X_1,Y_1)).$$

and consequently

$$U_n = T_n((X_1,Y_1),...,(X_n,Y_n)) \overset{(d)}{=} T_n((X_n,Y_n),...,(X_1,Y_1)) =$$

$$= \max\left\{0, \max_{1\le k\le n} \sum_{i=1}^{k} X_{n+1-i} \prod_{j=1}^{i} Y_{n+1-j}\right\} = \max\left\{0, \max_{1\le k\le n} \sum_{i^*=n+1-k}^{n} X_{i_*} \prod_{j^*=i^*}^{n} Y_{j^*}\right\} =$$

$$= \max\left\{0, \max_{1\le k^*\le n} \sum_{i^*=k^*}^{n} X_{i_*} \prod_{j^*=i^*}^{n} Y_{j^*}\right\} = \tilde{V}_n. \tag{8}$$

Here

$$\tilde{V}_n = Y_n\left(0, X_n + \tilde{V}_{n-1}\right)^+, \ n = 1,2...,$$

which is just the same as (5). So we immediately conclude that $\tilde{V}_n = V_n$ for each $n = 1,2...$. Finally, it follows from (8) that (6) holds for each $n = 1,2...$. The formula (7) is a sequence of the formulas (3), (6). This ends the proof of Theorem 1.

Remark 1. Theorem 1 proof practically repeats the proof of [1,Theorem 2.1]. A single difference is that the condition of r.v.`s $X_1,Y_1,...,X_n,Y_n$ independence is replaced by more weak condition **(C).**

Introduce the finite set $Q = \{1,...,m\}$ and for any $q \in Q$ define d.f.`s $F_q(t)$ and i.i.d.r.v.`s $X_{n,q}$, $P\left(X_{n,q} > t\right) = \overline{F}_q(t)$ and positive constants R_q, $R_1 < R_q$, $q \neq 1$, $q \in Q$. Suppose that $0 < p_q < 1$, $\sum_{q\in Q} p_q = 1$.

(B). Random vector (X_n,Y_n) satisfies the condition

$$P\left((X_n,Y_n) = \left(X_n^{(q)}, \frac{1}{R^{(q)}}\right)\right) = p_q, \ q \in Q. \tag{9}$$

From the formula (2) and the condition (9) we have

$$P(V_n > t) = \sum_{q\in Q} p_q P\left(X_n^{(q)} + V_{n-1} > R^{(q)} t\right), \ t > 0. \tag{10}$$

(C). Suppose that $F_q(t) \in S$, $q \in Q$ and for any $q_1, q_2 \in Q$, $q_1 \neq q_2$ one of the following equalities is true

$$\overline{F_{q_1}}(t) = O\left(\overline{F_{q_2}}(t)\right) \text{ or } \overline{F_{q_2}}(t) = O\left(\overline{F_{q_1}}(t)\right) \tag{11}$$

4 ASYMPTOTIC ANALYSIS OF RUIN PROBABILITY

Theorem 2. If the conditions **(A), (B), (C)** are true then for $t \to \infty$

$$P(V_n > t) \sim \sum_{q_1 \in Q} p_{q_1} \overline{F}_{q_1}\left(R^{(q_1)}t\right) + \sum_{q_1, q_2 \in Q} p_{q_1} p_{q_2} \overline{F}_{q_1}\left(R^{(q_1)}R^{(q_2)}t\right) + \ldots + \\ + \sum_{q_1,\ldots,q_n \in Q} p_{q_1} \cdot \ldots \cdot p_{q_n} \overline{F}_{q_1}\left(R^{(q_1)} \cdot \ldots \cdot R^{(q_n)}t\right). \quad (12)$$

Proof: Suppose that $n = 1$ then

$$P(V_1 > t) = \sum_{q \in Q} p_q P\left(X_1^{(q)} > R^{(q)}t\right) = \sum_{q \in Q} p_q \overline{F}_{q_1}\left(R^{(q)}t, t > 0\right).$$

So for $n = 1$ the asymptotic formula (12) is true. Suppose that the formula (12) takes place for fixed n. Then from the formula (10) we obtain

$$P(V_{n+1} > t) = \sum_{q_{n+1} \in Q} p_{q_{n+1}} P\left(X_{n+1}^{(q_{n+1})} + V_n > R^{(q_{n+1})}t\right), t > 0 .$$

So from the formula 12) and from Propositions 1, 2 and from the conditions **A), (B), (C)** we have for $t \to \infty$ that

$$P(V_{n+1} > t) \sim \sum_{q_{n+1} \in Q} p_{q_{n+1}} \left[\sum_{q_1 \in Q} p_{q_{n+1}} \overline{F}_{q_1}\left(R^{(q_{n+1})}R^{(q)_1}t\right) + \sum_{q_1, q_2 \in Q} p_{q_1} p_{q_2} \overline{F}_{q_1}\left(R^{(q_{n+1})}R^{(q_1)}R^{(q_2)}t\right) + \ldots + \right.$$

$$\left. + \sum_{q_1,\ldots,q_n \in Q} p_{q_1} \cdot \ldots \cdot p_{q_n} \overline{F}_{q_1}\left(R^{(q_{n+1})}R^{(q_1)} \cdot \ldots \cdot R^{(q_n)}t\right) + F_{q_{n+1}}\left(R^{(q_{n+1})}t\right) \right] =$$

$$= \sum_{q_{n+1}, q_1 \in Q} p_{q_{n+1}} p_{q_1} \cdot \overline{F}_{q_1}\left(R^{(q_{n+1})}R^{(q_1)}t\right) + \sum_{q_{n+1}, q_1, q_2 \in Q} p_{q_{n+1}} p_{q_1} p_{q_2} \cdot \overline{F}_{q_1}\left(R^{(q_{n+1})}R^{(q_1)}R^{(q_2)}t\right) + \ldots +$$

$$+ \sum_{q_{n+1}, q_1, \ldots, q_n \in Q} p_{q_{n+1}} p_{q_1} \cdot \ldots \cdot p_{q_n} \cdot \overline{F}_{q_1}\left(R^{(q_{n+1})}R^{(q_1)} \cdot \ldots \cdot R^{(q_n)}t\right) + \sum_{q_{n+1} \in Q} p_{q_{n+1}} F_{q_{n+1}}\left(R^{(q_{n+1})}t\right) =$$

$$= \sum_{q_1 \in Q} p_{q_1} \overline{F}_{q_1}\left(R^{(q_1)}t\right) + \sum_{q_1, q_2 \in Q} p_{q_1} p_{q_2} \overline{F}_{q_1}\left(R^{(q_1)}R^{(q_2)}t\right) + \ldots +$$

$$+ \sum_{q_1, \ldots, q_n, q_{n+1} \in Q} p_{q_1} \cdot \ldots \cdot p_{q_{n+1}} \cdot \overline{F}_{q_1}\left(R^{(q_1)} \cdot \ldots \cdot R^{(q_{n+1})}t\right).$$

Last equality is obtained by a replacement of indexes $1,\ldots,n+1$ in its summands. So the formula (12) is proved for index $n+1$ also.

Consider the following asymptotic conditions for $t \to \infty$.

(D$_1$). There are positive numbers c_q, α_q, $q \in Q$, $\alpha_1 < \alpha_q$, $1 < q \le m$ so that $\overline{F}_q(t) \sim c_q t^{-\alpha_q}$

(D$_2$). There are positive numbers c_q, $q \in Q$, α so that $\overline{F}_q(t) \sim c_q t^{-\alpha}$

(D$_3$). There are positive numbers c_q, β_q, $q \in Q$, $\beta_1 < \beta_q$, $1 < q \le m$ so that $\overline{F}_q(t) \sim \exp\left(-c_q t^{\beta_q}\right)$.

(D$_4$). There are positive numbers c_q, $q \in Q$, β so that $\overline{F}_q(t) \sim\sim \exp\left(-c_q t^{\beta}\right)$

It is easy to prove that the family $F_q(t)$, $q \in Q$ under each of the conditions **(D$_1$)**, **(D$_2$)**, **(D$_3$)**, **(D$_4$)** satisfies the condition **(C)**.

In the condition **(D$_1$)** the formula (12) may be represented in the following form

$$\psi(t,n) \sim \sum_{k=1}^{n} p_1^k \overline{F}_1\left(R_1^k t\right) \sim c_1 t^{-\alpha_1} \sum_{k=1}^{n} \frac{p_1^k}{R_1^{k\alpha}}$$

and consequently

$$\psi(t,n) \sim \begin{cases} c_1 t^{-\alpha_1} \dfrac{1-p_1^{n+1} R_1^{-(n+1)\alpha}}{1-p_1 R_1^{-\alpha}}, & p_1 R_1^{-\alpha} \neq 1 \\ n c_1 t^{-\alpha_1}, & p_1 R_1^{-\alpha} = 1. \end{cases} \tag{13}$$

In the condition **(D$_2$)** the formula (12) may be represented in the following form

$$\psi(t,n) \sim t^{-\alpha}\left[\sum_{q_1\in Q} c_{q_1} p_{q_1} R_{q_1}^{-\alpha} + \sum_{q_1,q_2\in Q} c_{q_1} p_{q_1} p_{q_2} R_{q_1}^{-\alpha} R_{q_2}^{-\alpha} + \right.$$
$$\left. + \sum_{q_1,\dots,q_n\in Q} c_{q_1} p_{q_1}\cdot\ldots\cdot p_{q_n} R_{q_1}^{-\alpha}\cdot\ldots\cdot R_{q_n}^{-\alpha} + \right] = t^{-\alpha} S_1 \frac{1-S_2^n}{1-S_2}. \tag{14}$$

with

$$S_1 = \sum_{q_1\in Q} c_{q_1} p_{q_1} R_{q_1}^{-\alpha},\ S_1 = \sum_{q_1\in Q} c_{q_1} p_{q_1} R_{q_1}^{-\alpha}$$

In the condition **(D$_3$)** the formula (12) may be represented in the following form

$$\psi(t,n) \sim \sum_{k=1}^{n} p_1^k F_1\left(R_1^k t\right) = \sum_{k=1}^{n} p_1^k \exp\left(-c_1\left(R_1^k t\right)^{\beta_1}\right)$$

and so

$$\psi(t,n) \sim \begin{cases} p_1^n\left(-c_1\left(R_1^n t\right)^{\beta_1}\right), R_1 < 1, \\ p_1\left(-c_1\left(R_1 t\right)^{\beta_1}\right), R_1 > 1, \\ \exp\left(-c_1\left(R_1 t\right)^{\beta_1}\right)\dfrac{1-p_1^{n+1}}{1-p_1}, R_1 = 1. \end{cases} \tag{15}$$

In the condition **(D$_4$)** the formula (12) may be represented in the following form

$$\psi(t,n) \sim \sum_{k=1}^{n}\left[\sum_{q_1,\dots,q_k\in Q} p_{q_1}\cdot\ldots\cdot p_{q_k} \exp\left(-c_{q_1}\left(R_{q_1}\cdot\ldots\cdot R_{q_k} t\right)^{\beta}\right)\right]. \tag{16}$$

Suppose that there is the constant q' satisfying the inequalities

$$c_{q'} R_{q'}^{\beta} < c_{q'} R_q^{\beta},\ q \neq q',\ q \in Q.$$

The equivalences (16) may be rewritten as follows

$$\psi(t,n) \sim p_{q'} \sum_{k=1}^{n} p_1^{k-1} \exp\left(-c_{q'}\left(R_{q'} R_1^{k-1} t\right)^{\beta}\right).$$

and so

$$\psi(t,n) \sim \begin{cases} p_{q'} p_1^{n-1} \exp\left(-c_{q'}\left(R_{q'} R_1^{n-1} t\right)^{\beta}\right), R_1 < 1, \\ p_{q'} \exp\left(-c_{q'}\left(R_{q'} t\right)^{\beta}\right), R_1 < 1, \\ p_{q'} \dfrac{1-p_1^n}{1-p_1} \exp\left(-c_{q'}\left(R_{q'} t\right)^{\beta}\right), R_1 = 1. \end{cases} \tag{17}$$

5 CONCLUSION

A comparison of the asymptotic formulas (13), (14), (15), (17) with the results of [1] shows that a dependence of financial and insurance risks introduces significant changes into asymptotic formulas for ruin probability of discrete time risk model.

REFERENCES

1. Tang Q.; Tsitsiashvili G. 2003. Precise estimates for the ruin probability in finite horizon in a discrete-time model with heavy-tailed insurance and financial risks. *Stochast.Process. Appl.* Vol. 108, no. 2, 299--325.3.
2 Tang Q.; Tsitsiashvili G. 2003. Randomly Weighted Sums of Subexponential Random Variables with Application to Ruin Theory. *Extremes.* Vol. 6, pp. 171-188.
3. Tang Q., Wang G.,_ Yuen K.C. Uniform tail asymptotics for the stochastic present value of aggregate claims in the renewal risk model. 2010. *Insurance: Mathematics and Economics.* Vol. 46, pp. 362-370.
4. Embrechts P., Kluppelberg C., Mikosch T. 1997. *Modelling Extremal Events in Finance and Insurance.* Springer.
5. Norberg R. 1999. Ruin problems with assets and liabilities of diffusion type. *Stochastic Process. Appl.* Vol. 81. No. 2.

RT&A # 02 (17)
(Vol.1) 2010, June

QUANTITATIVE SAFETY GOALS AND CRITERIA AS A BASIS FOR DECISION MAKING

Heinz-Peter Berg.
•
Bundesamt für Strahlenschutz, Salzgitter, Germany

e-mail: hberg@bfs.de

ABSTRACT

Internationally, probabilistic safety analyses represent the state of the art in the licensing process for new industrial facilities, but increasingly also for evaluating the safety level of older industrial plants, e. g. as part of periodic safety reviews of nuclear power plants. Quantitative safety goals have not yet reached the same level of acceptance. However, this depends on the type of industry. Most of the countries consider those criteria as safety targets rather than as sharply defined boundary values. The Netherlands and the United Kingdom are exceptions, they require demonstration of compliance with legally binding safety goals in the licensing procedure.

1 INTRODUCTION

1.1 General

Originated for applications in the nuclear industry, quantified risks and hazard analysis techniques are emerging as powerful tools for the safety management of hazardous plants in the process industry (chemical, petrochemical, petroleum and related industries).

Although the concept remains similar, i.e. is a probabilistic approach to risk quantification, there are apparent variations in methodological practices and particularly in the range of applications, focus and emphasis in the implementation of these tools for the different industries. This probably stems from the fundamental difference between the nuclear industry, essentially a one process industry, and the process industry which is characterized by a multitude of interdependent processes where raw materials undergo physical and chemical changes.

The more apparent variation between quantified risk and hazard analysis in the process industry and probabilistic safety assessment (PSA) in the nuclear industry lies in the relatively narrower range of applications of these tools in the process industry when compared to the more extensive use made use made by the nuclear industry in implementing PSA at the design and operational stages of nuclear power plants including plant changes (Cepin 2004, 2007).

There is much debate about the concept of acceptable risk. The question what level of risk should be tolerated and who determines acceptability is still controversial in the area of safety management. The importance of communicating is illustrated by the differential in willingness to tolerate risks from different sources, independent from benefit considerations, and the differential in willingness to accept types of risks between different groups of individuals.

The concept that some level of risk is tolerable is fundamental to risk assessment and risk management (Kumamoto 2007). Without the definition of such a tolerable risk criterion, risk assessment may be hampered in terms of decision making and formulation of risk management strategies. The setting of and adherence to precise and rigid criteria, however, does not acknowledge the limitation in accuracy of methodologies, nor does it allow for appropriate consideration of the benefits against which the acceptability of the risk may be assessed in each case. Furthermore, the extent of compliance with any risk criteria should not be the sole basis for evaluating the success of risk management measures. Other criteria include: the extent of risk and risk reductions achieved, the cost of risk reductions in social, economic and environmental terms, and the cost effectiveness of control measures.

As such, while debate will probably continue on the appropriateness of quantitative risk criteria as a measure of tolerability, future applications of quantitative risk assessment will greatly benefit from focusing more on the assessment process itself and the interpretation of such criteria as a target guideline.

As demonstrated by the wide spectrum of applications, the fact is illustrated that in the nuclear industry uses of PSA for other than compliance with formal criteria dominate. Some countries which operate nuclear power plants apply numerical safety objectives / criteria / rules / goals. The role and interpretation of such quantitative guidelines vary from country to country. A dominant opinion is that “the safety goals should not be used within a regulatory framework of strict acceptance or non-acceptance criteria but should be considered as one factor in arriving at regulatory judgement”.

1.2 Scope and purpose of the paper

The probabilistic safety analysis as already explained is the most powerful approach to quantification of risk and safety where risk is a combination of probability of harm and severity of that harm, while safety is freedom from unacceptable risk (Kumamoto 2007).

Basically, any plant, activity or item should be designed and operated in such a way as to satisfy a given set of safety goals. This is a goal-oriented approach where goals are first specified, and then the plant, activity or item is designed, created, operated and maintained accordingly. However, two problems must be answered for the goal-oriented approach:

1. How safe is safe enough? This requires a set of safety goals to be satisfied.
2. How to deal with uncertainties? The current risk quantification involves significant uncertainties.

The target discussed in this paper is mainly focused on a nuclear power plant. However, the implications can certainly be translated into other fields including process, aerospace, machinery, and automobile industries. Prevention of core damage in a nuclear power plant corresponds in general to prevention of vehicle collision (active safety), and accident mitigation by a containment structure corresponds to collision mitigation by an air bag (passive safety). However, one has to have in mind the more catastrophic consequence of a core damage compared with a vehicle accident. Prevention coupled with mitigation is an indispensable element of the defense-in-depth philosophy to cope with the uncertainty of current risk quantification.

Although the use of quantitative safety goals is sometimes questioned see for example (Aven & Vinnem 2005) and (Hokstad et al. 2004), many industries and countries have introduced such goals or criteria. This is due to the fact that probabilistic safety analysis is part of safety assessment to be submitted to the competent authority or licensing institution. This immediately rises the question how to assess the results, even in case – as in Germany – where no quantitative safety goals are determined.

The main underlying problem is that the quantitative results had to be evaluated together with the content, assumptions, models and data used which normally does not allow an easy comparison with the result of another plant or activity. Therefore, people performing a probabilistic safety analysis have to be aware to provide a very carefully elaborated analysis with high quality because the results may lead to costly technical changes or the shutdown of the respective plant. On the other hand, it is the responsibility of those performing risk assessment not to tailor the numbers used in the analysis to ensure that the results do not exceed the given goals.

2 PROBABILISTIC SAFETY ANALYSIS AND PROBABILISTIC SAFETY CRITERIA FOR THE NUCLEAR INDUSTRY

2.1 Methods and results of PSA

For historical reasons, the safety philosophy in the nuclear industry is mainly based on deterministic principles such as

– A multi-level safety concept ("defense in depth") with engineered safety systems to prevent or control anomalous events,

– 'Conservative' design, i. e. preference for proven technology and ample design margins

– Multiple barriers against the release of radioactivity,

– Redundant and diverse safety systems of high reliability.

The safety-related requirements which are the base of the plant safety design, are derived from events which are defined so that they represent in each case a whole class of similar events in an enveloping way.

In contrast, it is the essential task of the PSA also to determine the probability of occurrence of event sequences that are not covered by the design base and consequently cannot be assumed to be controlled by the engineered safety systems (Berg 1995). This goal is achieved by means of the accident sequence development analysis, an analysis tool which contains the following essential elements:

– Initiating event analysis,

– Definition of the event sequences and supporting analyses (e.g. thermo-hydraulic model calculations, success criteria analysis),

– Quantification of the probabilities of occurrence of the various event sequences with the aid of fault tree analyses.

Depending upon the nature of the initiating event and the plant status at the time of its occurrence, those functions of the operating and safety systems as well as the manual actions have to be determined that are planned for the control of the event sequence and are required. Inputs are the initiation or trip criteria for the safety systems; manual control actions of the operating staff can be also considered. The different sequences, that result depending on the availability (function or non-function) of these systems are to be represented in the form of event tree diagrams. The availability of a system function is derived quantitatively either from the operational experience or from fault tree analyses, by which the availability of a system is calculated from its logical structure and availability data based on operating experience for sub-units or components, respectively.

It is usual to distinguish three levels of PSA:

– Level 1: The analysis focuses on the responses from operating and safety systems to different initiating events. The end point of the analysis is either the occurrence of a core damage state or the stabilization of the plant state such that a core damage state is prevented.

– Level 2. Starting from the results of level 1 the physical processes of the accident progression post core damage are analyzed. Probabilities are determined for timing and mode of containment failure as well as for the release of radioactivity within the plant as well as for the source term for a release into the environment, determination of the time frame when a certain release is to be expected is of particular importance in this case.

– Level 3: Starting from the probabilities for releases as determined in level 2, the probabilities and extent of damages in the environment of the plant are determined as individual risk, dependent on the distance from the plant, as a complementary frequency distribution of individual risks (counting early fatalities only or including somatic late damages) or as a complementary frequency distribution of the collective dose.

2.2 Quantitative probabilistic safety criteria

In a PSA, safety relevant event sequences and the interaction of the safety systems are modelled for an entire plant. Accordingly, bottom line results do refer to the behaviour of the entire plant, not only to the reliability of single engineered safety systems or components. Those may be subject to design rules or the nuclear safety standards or other technical regulations containing quantitative reliability requirements. The most important of these integral results were:

- Total frequency of core damage states (Core Damage Frequency, CDF),
- Frequency of activity releases due to accidents, most important the Large Release Frequency (LRF). The latter may be defined in different ways as an activity release that requires immediate mitigation measures outside the plant (one typical definition: release of > 0,1 % of the core fission product inventory),
- Frequency of accident-caused damages and/or exposures.

These PSA result formats allow the evaluation of plant safety. They may be used together with results from the deterministic part of a Periodic Safety Review (PSR). Different strategies are followed in different countries (Görtz et al. 2001). The following basic strategies can be distinguished:

1. The PSA results can be used as additional information, without any change of the existing design rules and the regulatory framework which are the base for regulatory decisions in each individual case.
2. In addition to the existing design rules and the code of safety standards, requirements regarding quantitative PSA results are set (Example: The CDF is not to exceed a set limit).
3. Some of the existing design rules and/or safety standards are replaced by requirements that refer to quantitative PSA results.

Implementing these basic modes in national regulatory practice, numerous variants are possible, in particular with regard to the relative weight PSA results are accorded in the safety review process, the specific requirements regarding certification of safety in the review process and the applications of the results. In the chronological course transitions are conceivable between the basic modes. Typical pitfalls encountered when quantitative PSA results are added onto existing deterministic safety standards are shown in Figure 1.

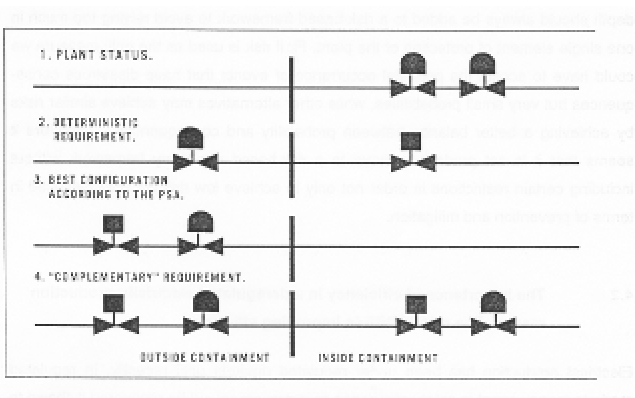


Figure 1. Possible pitfalls of "complementary" use of PSA-results (Villadóniga 2001).

2.3 International recommendations

2.3.1 IAEA

Quantitative probabilistic criteria were included in (INSAG 1999) and complemented with the annotation that for future plants 'another objective ... is the practical elimination of accident sequences that could lead to large early releases ... '. The recommendations of INSAG were adopted into IAEA Safety Guide No. NS-G-1.2 (IAEA 2001) in a more explicit form:

– 'Core damage frequency: For this, INSAG ... has proposed the following objectives:
–10^{-4} per reactor-year for existing plants,
–10^{-5} per reactor-year for future plants. '

– '... Large radioactive release. The following objectives are given:
–10^{-5} per reactor-year for existing plants,
–10^{-6} per reactor-year for future plants. '

Furthermore, the IAEA recommended in (IAEA 1992) to distinguish three regions as shown in Figure 2.

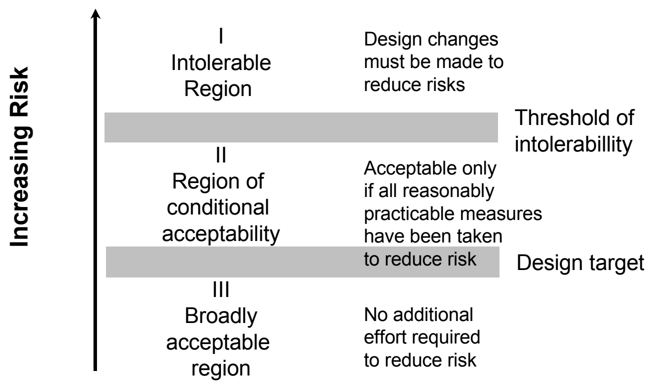


Figure 2. IAEA guidance on acceptance criteria.

This general scheme described has been implemented in various countries (see 2.4).

Figure 3 shows Decision Regions for strategic risk-informed decision making (RIDM) according to (IAEA 2007) The axis is CDFBL-A; it accounts for anticipated routine configurations for activities during the year. The ordinate is ΔCDF, accounting for a change in the CDF, the annual average CDF above CDFBL-0 as a result of a specific activity or situation being evaluated.

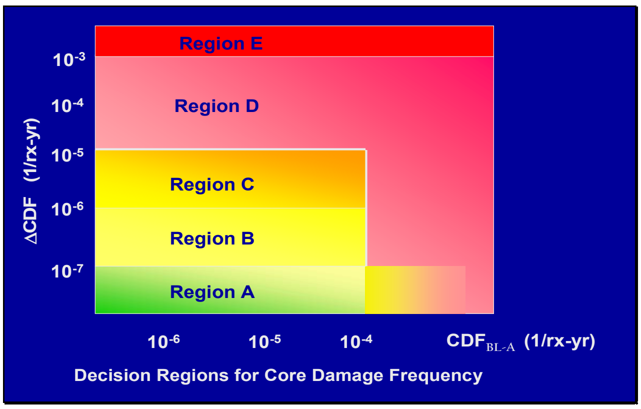


Figure 3. Strategic RIDM Decision Regions for CDF.

There are five decision regions, A to E in Figure 3:

- Results clearly inside Region A are considered to be normal operation and would be within the purview of licensed operators or equivalent.
- Results in Region B would be within the purview of facility management with possibly some regulatory approval depending upon the application, the facility license, and the regulatory structure.
- Results in Region C would require regulatory approval or licensed control depending upon the regulatory structure.
- Results in Region D would not normally be permitted and would always require regulatory approval. The regulatory authority would not normally permit operation in Region D.
- Results in Region E would not be permitted. Immediate action must be taken to remove operation from Region E, or the facility must be immediately shutdown in an orderly manner.

2.3.2 OECD/NEA

In (NEA 2007 a) it is underlined that regulatory bodies have the legal duty and authority to make final safety judgments on all nuclear activities under their responsibility. In a practical sense a nuclear activity is deemed to be safe if the perceived risks are judged to be acceptable. But the regulator can never have a certain quantitative assessment of the risk involved. Therefore, in arriving at its safety judgements, the regulatory body must be guided by the basic safety criteria embedded in its national laws, regulations and policies. One of these criteria is the level of safety protection required by the regulator. There are various statements of the basic level of safety required by OECD/NEA countries, but they all acknowledge that it is not possible to achieve absolute safety (i.e., zero risk) in nuclear activities. Some of these criteria are (see NEA 2007 a):

- no unreasonable risk,
- adequate protection of public health and safety,
- risk as low as reasonably practicable,
- safety as high as reasonably achievable,
- limit risk by use of best technologies at acceptable economic costs.

A related safety criterion is the degree of assurance needed by the regulator that the basic level of safety protection is being met. Here again, there are various formulations of this criterion among OECD/NEA countries.

In 2007 OECD/NEA has published a very exhaustive report on 'The Use, and Development of Probabilistic Safety Assessment (NEA 2007 b), compiled by the Working Group on Risk Assessment (WGRisk) of the Committee on the Safety of Nuclear Installations (CSNI). This report describes the current status of PSA programs in the member states, including general background information, rules and guidelines, different uses of PSA, essential results of recent analyses, brief descriptions of retrofits of plants initiated by PSA results and current topics from R & D in the field. The report is meant as a description of the current state of the art in the member states. A separate chapter, is dealing with quantitative safety criteria. The main statements are summarized in the following.

There are differences in the status of the numerical safety criteria that have been defined in different countries. Some have been defined in law and are mandatory, some have been defined by the regulatory authority (which is the case in the majority of countries where numerical safety criteria have been defined), some have been defined by an authoritative body such as a Presidents Commission and some have been defined by plant operators or designers. Hence there is a difference in the status of the numerical safety criteria which range from mandatory requirements that need to be addressed in law to informal criteria that have been proposed by plant operators or designers for guidance only.

There are a variety of reasons for defining the criteria which includes:

– a change in the law to introduce risk management into the environmental policy,

– the need to define an acceptable level of safety for nuclear power plants following an accident,

– a recommendation from a public enquiry to build a new plant,

– the need for guidance for improving old plants or designing new ones.

In some countries, high level qualitative and quantitative guidance has been defined and the has been used to derive lower level or surrogate criteria than are easier to address and are sufficient to demonstrate that the higher level criteria are met.

In some countries, criteria have been defined for existing plants and for new plants. In general, the expectation is that the target/ objective for the level of risk from a new plant should be about an order of magnitude lower than for existing plants.

In a number of countries no numerical safety criteria have been defined. However, there is a general requirement that the level of risk should be comparable to (or lower than) the risk from existing plants for which a PSA is available.

In most of the countries in which numerical safety criteria have been defined they have been defined as a "target", an "objective" or a "goal" where the recommendation is that the risk should be lower than the prescribed value with no guidance given on what action needs to be taken if it is exceeded.

The way that the safety criteria have been defined ranges from high level qualitative and quantitative requirements relating to individual and societal risk for members of the public to lower level criteria relating to core damage, a large release or a large early release of radioactivity to the environment, and radiation doses to an individual living near the plant.

The high level qualitative criteria state that the additional health effects to the public from operation of the nuclear power plant should not lead to a significant increase in the risk of death of members of the public. The high level quantitative goals state that the level of increase should be less than about 0.1% of the existing risks.

In some countries the risk criteria are defined for individual members of the public and for societal risks involving 10 or 100 members of the public. The societal risks are sometimes defined as acute fatalities that occur in a short time after the accident or in the longer term.

The most common metrics used are core damage frequency (CDF) and large release frequency (LRF) or large early release frequency (LERF). In some cases these criteria have been defined as surrogates for higher level metrics and some cases they have been defined in their own right.

2.4 Examples of national approaches

2.4.1 Quantitative safety criteria on level 3

The Netherlands

In the Netherlands risk based criteria were formulated to judge the safety and environmental effects of industrial plants with great hazard potential, nuclear power plants obviously belonging to these. One of these criteria refers to the individual risk, the other one limits the collective risk ('societal risk').

The maximally permissible individual risk, which means the risk of premature death as a result of the plant operation, is 10^{-6} / a. The individual risk is to be calculated according to a rather restrictive rule which postulates that a child one year old at the time of the accident will spend further seventy years at the location of the accident (Eendebak 1995).

According to Figure 4, societal risk is limited in such a way that the probability for ten fatalities is less than 10^{-5} per operating year, for a hundred fatalities less than 10^{-7} per year and so forth. Societal risk refers only to early radiation-induced fatalities, often designated as deterministic radiation-induced damages. In the calculations, accident mitigation measures are not taken into account.

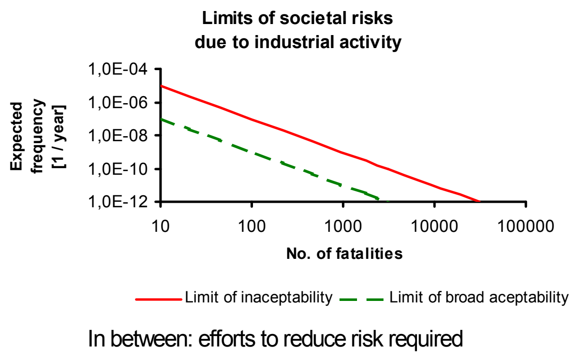


Figure 4. Limit of societal risk for any industrial plant in the Netherlands (Directorade 1989).

Although in the Netherlands nuclear power is of minor importance, there being a single NPP in operation providing less than 5 % of the electric power production, PSA is used to a considerable extent including PSA of level 3.

For new plants - NPPs or other nuclear installations - a PSA of level 3 is required in the licensing procedure. There is an official detailed guideline for performing PSAs, also describing the stipulation of specific atmospheric dispersion models and/or programs to be used. (JCSS 2008) gives an overview of the utilization of probabilistic acceptance criteria and the structure of the relevant code of standards, focussing mainly on the chemical industry.

For periodic safety reviews of NPPs, secondary safety criteria for evaluation of PSA results were derived from the above-mentioned societal risk limits. This means that for CDF a probability of < 10^{-4} per year is to be proved, the frequency of large early releases must not exceed 10^{-6} per year.

(Eendebak 1995) states that the PSAs carried out both for Borssele NPP and Doodeward NPP (meanwhile shutdown) show that the associated societal risks are small compared to those of other technical activities and that the Dutch acceptance criteria are unambiguously fulfilled. (Van der Borst & Versteeg 1996) shows this for Borssele NPP and points out the risk reduction effect of retrofitting measures that were initiated based on PSA insights (see Figure 5).

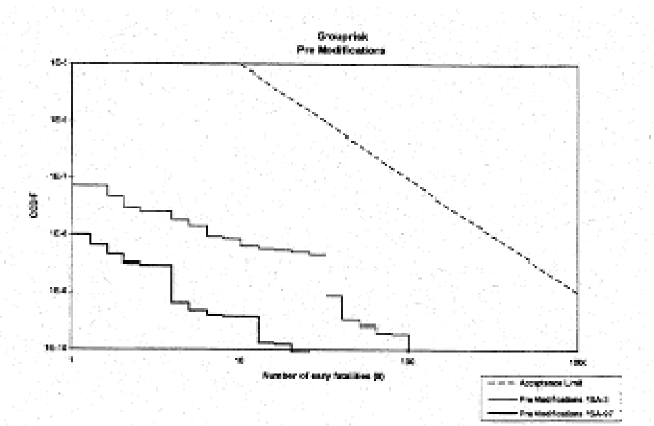


Figure 5. Societal risk of Borssele NPP, before and after AM measures and modifications.

Dutch regulations treat risks from nuclear installations in the same manner as those from non-nuclear installations, e. g. from chemical plants. Thus, an objective evaluation of diverse technical risks is achieved.

In (Vrijling et al. 1996, 2004) a possible extension of the Dutch concept of individual and societal risk is discussed. Application to airports, road traffic and to the transport of dangerous goods frequently shows surprisingly high risk figures compared with nuclear activities.

United Kingdom

The U.K Health and Safety Executive as the British regulatory authority, issued the paper "Tolerability of Risk from Nuclear Power Stations" (HSE 1988) as 'draft for comment'. The proposals contained in this paper became compulsory and were published as "Safety Assessment Principles for Nuclear Power Plants" in 1992. These safety assessment principles have been currently updated (see HSE 2006 a, b).

It must be emphasized that the Nuclear Installation Inspectorate (NII) in its 'Safety Assessment Principles' has a number of different quantitative safety goals. Like in the approach of the IAEA (cf. Fig. 6), there is between the 'broadly acceptable' region (below the Basic Safety Objective, BSO) and the 'unacceptable' region (above the Basic Safety Limit, BSL) an intermediate field in which risk optimization is to be carried out. It should be pointed out that in principle this criterion does not only apply to NPPs, but also to other nuclear installations.

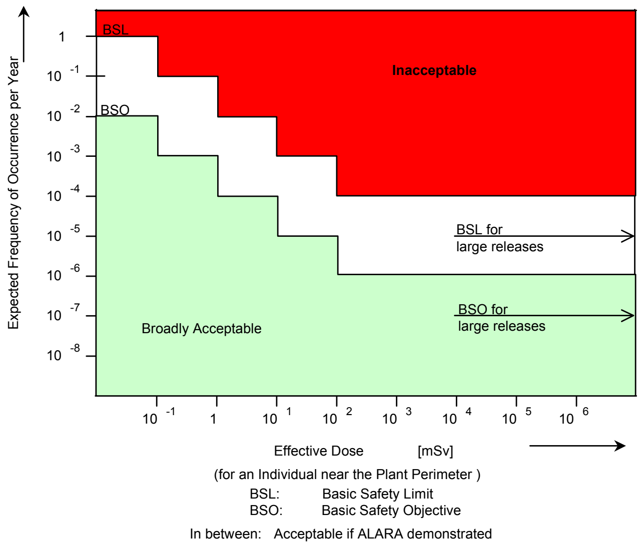


Figure 6. Limits to radiological effects vs. their expected frequencies of occurrence (acc. to HSE 1992). Doses are calculated for a person living approx. 1 km downwind from the plant.

2.4.2 Quantitative safety criteria on level 2

The Argentine code of regulations basically does not distinguish between NPPs on the one hand and 'other nuclear installations' on the other; rather does it only distinguish between relevant and non-relevant installations based on their associated radiological hazard (Berg et al. 2003). To the first category belong, besides NPPs, also larger test reactors and plants of the fuel cycle, e. g. fuel factories. There exist two criteria: one applicable to the general population near the plant and a second one applicable to the work force (cf. Fig. 7).

The criterion which links the effective dose with the expected frequency of occurrence of the event causing the exposure to a person of the general public outside of the plant boundary (Fig. 7) is defined so that no conceivable accident sequence will give rise to a risk greater than 10^{-7} per year. Together with the further criterion which limits the total plant hazard - the sum over all conceivable accident sequences - to 10^{-6} per year, this provides - at least implicitly - a quantitative criterion indicating whether the plant safety concept is well-balanced.

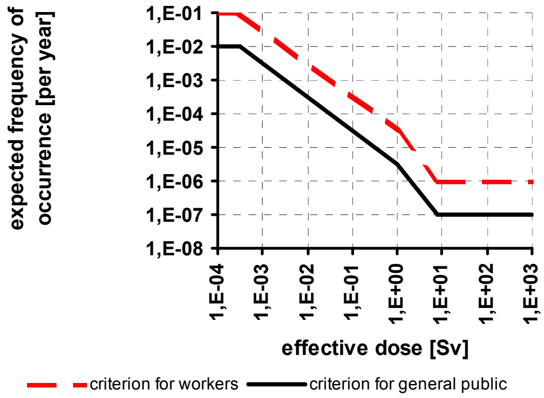


Figure 7. Boundary curves for the work force and for the general public in Argentina. Values to the right and above the curves are not acceptable.

Interpretation of the boundary curve in Figure 7: for effective doses less than 1 Sv, which are expected to yield only stochastic effects, a dose risk rate of 10^{-2} / Sv was used to build the curve. Effective doses larger than 1 Sv will yield non-stochastic effects and higher dose risk rates leading to an increased slope of the curves. Ultimately, effective doses larger than some 7 to 8 Sv (which correspond to the lethal dose in 30 days) may not occur with a probability larger than 10^{-7} per year for the general public (or 10^{-6} per year for workers).

In spite of the definition of the criteria in the form of a dose/frequency curve these are really criteria of level 2 since in the immediate vicinity of the plant the effective dose for the general public (or inside the plant for employees) is simply linearly dependent on the amount of released activity. Far field diffusion and accumulation effects do not play a significant role here, in contrast to criteria regulating collective doses in large areas.

2.4.3 Quantitative safety criteria on level 1

Quantitative safety criteria that are defined on level 1 – e. g. in the format of maximal allowed CDF values – are found less frequently in statutory or regulatory provisions. Some countries, despite having and applying quantitative safety criteria defined on a higher level, also have an explicit statutory or regulatory limitation for CDF.

As a typical example, Finland may be taken: besides limiting the expected frequency of occurrence for large off-site releases, the guideline (STUK 1996) sets a limit which restricts CDF for new plant to less than 10^{-5} per year (the value is designated a design objective).

Furthermore, countries actively promoting the expansion of their nuclear power plant park and the development of advanced NPP designs are known to apply design objectives for CDF like e. g. Canada does with the advanced heavy water moderated reactor type CANDU 9 (IAEA 2002).

In the safety review and for the evaluation of the necessity of back fits for the Ignalina NPP in Lithuania, quantitative probabilistic acceptance criteria were used in one application and more qualitative, quasi-probabilistic criteria in another one.

Given an initiating event (under the assumption that no safety device cuts in to control the event), that scheme combines in a single matrix the scale of possible accident consequences and the number and quality of available safety systems that are available for the control of the considered event sequence according to plant design. In this evaluation, safety systems (with conservative design, nuclear class quality, operational monitoring, single failure tolerance) and other, non safety-grade systems (with lower reliability, e. g. balance-of-plant systems) are distinguished; (Holloway & Butcher 1995) use the terms 'strong' and. 'weak lines of defense' respectively for these.

The former are attributed a failure probability of better than 10^{-2} per challenge, the latter one of between 10^{-2} and 10^{-1} per challenge.

With these roughly estimated values and the accident consequences sorted into four categories according to expected severity, an evaluation diagram is derived that points out broadly acceptable areas, those with long term tolerable safety weaknesses, those with only temporarily tolerable shortcomings safety-wise and, lastly, those areas with safety deficits which are not acceptable, even for limited time periods (Fig. 8).

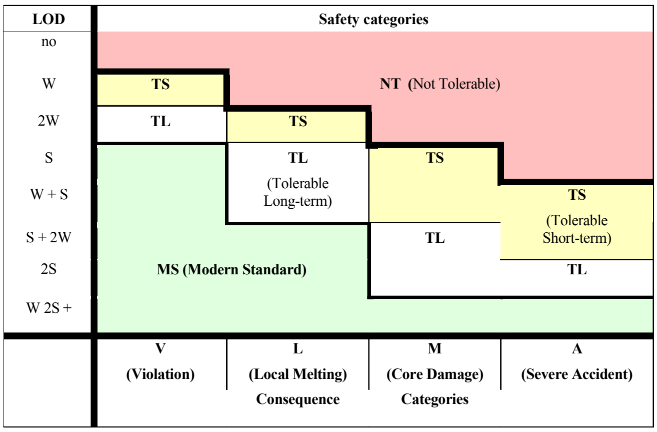


Figure 8. Scheme of the quasi-probabilistic LOD procedure for the evaluation of safety upgrade requirements for NPP Ignalina (acc. to Rimkevicius et al. 2002)
W: weak line of defense; S: strong line of defense

2.5 Discussion and evaluation

The variability of the examples presented in 2.4.1 to 2.4.3 demonstrates how many possibilities exist for the formulation of probabilistic safety criteria. Nevertheless the safety level described by these criteria - expressed either as core damage or large release frequencies - is largely comparable. The yardstick to compare the criteria are accidents leading to large releases. For their investigation, a PSA of at least level 2 is necessary, in the case of the Netherlands and the United Kingdom a PSA of level 3 is required to calculate the accident-caused individual or collective doses. The different criteria can be reformulated directly or implicitly into requirements on the expected frequency of large releases.

As conclusions three fundamental dose limits ca be defined, for additive annual doses from normal operation, for non-fatal health detriments (from a single brief exposure, i. e. accident-caused) and for acute fatalities due to large accident releases, determined the corresponding acceptable expected frequencies and thus derived a near-linear complementary cumulative distribution function (CCDF) for a Basic Safety Goal. This CCDF he compared with the BSO- and BSL-curves of the British HSE, the 'Safety Goal' of the USNRC, a safety design criterion for PWR of the ANS, and an ICRP-recommendation (which refers to radioactive waste repositories rather than to NPPs). Figure 9 shows a quite reasonable agreement between these rather differently formulated quantitative criteria (Hakata 2003).

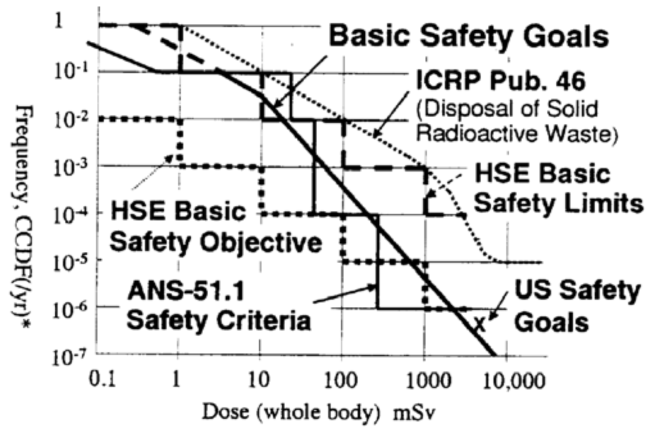


Figure 9. Comparison of different Safety Goals

HSE criteria are CCDF in each decade of doses; ANS and ICRP criteria are frequencies [per year]

3 RISK CRITERIA FOR OTHER INDUSTRIES

In the frame of the EU-project "Safety and Reliability of Industrial Products, Systems and Structures" (SAFERELNET), risk criteria used in the EU for population living in vicinity of hazardous facilities have been investigated. It can be seen from Table 1 that individual risk of 10^{-5} per year represents the upper limit in Europe for existing installations, while in the UK the intolerable limit is 10^{-4} but ALARP is strictly imposed, meaning that in reality the risk is well below the limit. The upper limit for individual risk for new installations in Czech Republic and in the Netherlands after 2010 is 10^{-6} per year. The quoted value for the Netherlands (10^{-5} and 10^{-6}) represent so-called location risk (risk contour), or the individual risk to a person who is permanently at the particular location. In addition, in the case of the Netherlands, the risk value corresponds to one establishment (facility), and the cumulative risks from several establishments are not taken into account.

The negligible risk levels specified in the UK as 10^{-7} per year and in the Netherlands as 10^{-8} per year are not questionable and it will be assumed that 10^{-8} can be a value accepted across the EU for the time being.

Table 1. Comparison of individual risk criteria

IRPA	UK	The Netherlands	Hungary	Czech Republic
10^{-4}	Intolerable limit for members of the public			
10^{-5}	Risk has to be reduced to the level As Low As Reasonably Practicable (ALARP)	Limit for existing installations. ALARA principle applies	Upper limit	Limit for existing installations. Risk reduction must be carried out
3×10^{-6}	LUP limit of acceptability (converted from risk of dangerous dose of 3×10^{-7})			
10^{-6}	Broadly acceptable level of risk	Limit for the new installations and general limit after 2010. ALARA applies	Lower limit	Limit for the new installations
10^{-7}	Negligible level of risk			
10^{-8}		Negligible level of risk		

In the Norwegian offshore petroleum industry, risk analysis are used for more than decades. These analysis have been closely linked to the use of risk acceptance criteria (see Aven &Vinnem 2005, Aven et al. 2006, Hokstad et al. 2004) as upper limits of acceptable risks.

In order to fulfil the requirements and acceptance criteria for major accidents the NORSOK Z-013 standard is usually applied.

In (Maharik & Vrijling 2002) is explained "If average fatality risk or average individual risk is used in the formulation of risk acceptance criteria, also criteria for areas or groups within the platform personnel shall be formulated. It is not sufficient just to have a platform average value as criterion. The risk estimates shall be considered on a "best estimate" basis, when considered in relation to the risk acceptance criteria, rather than on an optimistic or pessimistic (worst case) basis. The approach towards the best estimate shall, however, be from the conservative side, in particular when the data basis is scarce."

The standard (NORSOK 2001) does not prescribe explicit criteria; however, annex A provides some examples of typical risk acceptance criteria to be used, such as

– The fatality accident rate should be less than 10 for all personnel on the installation, where the fatality accident rate value is defined as the expected number of fatalities per 100 million exposed hours.

– The individual probability that a person is killed in an accident during one year should not exceed 0,1%.

In the railway sector, the European Railway Agency has got in December 2005 the mandate from the European Commission (2005) to develop a first set of common safety targets (CST):

"The CSTs shall define the safety levels that must at least be reached by different parts of the railway system and by the system as a whole in each Member State, expressed in risk acceptance criteria…"

Recommendations of this first set of CST will be available in September 2008 at the earliest.

For the signal technique for railways, safety standards are elaborated as EN 50129 (CENELEC 2003). A complete analysis of the possible hazards is not performed; instead only the hazard H="failure of level crossing to protect public from train" is considered. It is interpreted as covering all situations in which the level crossing should warn the public (of approaching trains), but fails to do so. The objective is now to determine the hazard rate HR [1/time] for H which is acceptable according to certain risk acceptance criteria.

The tolerable hazard rate of 10^{-9} per hour is in the railway area proposed as a target for all safety-critical functions (see Braband 2005). This approach is similar to that in civil aviation. It has been shown from operational experience with large aircraft fleets that the overall level has actually been met in practice. Tolerable hazard rates are correlated here to safety integrity levels (SIL) as shown in Table 2.

Table 2. Definition of safety integrity levels (SILs)

Tolerable Hazard Rate THR per hour and per functions	**Safety Integrity Level**
$10^{-9} \le THR < 10^{-8}$	4
$10^{-8} \le THR < 10^{-7}$	3
$10^{-7} \le THR < 10^{-6}$	2
$10^{-6} \le THR < 10^{-5}$	1

SIL is defined as the reliability to perform the required safety functions under all stated conditions within a stated operational environment and within a stated period of time.

According to the British Rail Safety and Standards Board, railway companies are required to make safety decisions to reduce risk to a level that is as is as low as is reasonably practicable (Dennis 2006). That is their legal duty. What is reasonably practicable must reflect their social duty to delver a railway that society demands and pays for through public subsidy and their commercial duty to shareholders and customers. The ALARP approach is, e.g., applied for risks of train passengers and workers (see Fig. 10).

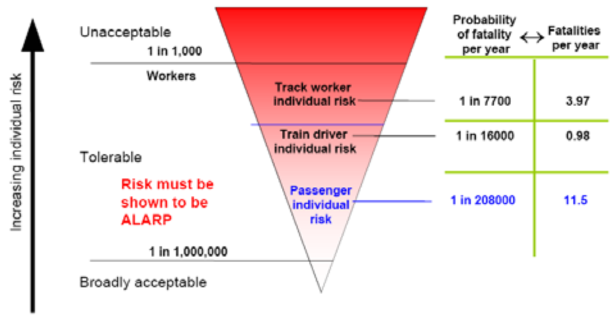


Figure 10. ALARP for risks of workers and train passengers.

In the maritime sector, international organisations have traditionally been capturing experience and knowledge into prescriptive legislation, thereby endeavouring to prevent past accidents from reoccurring. The current level of safety seems tolerable to the sector, however, the set of rules and regulations is extensive and it is not verified whether individual requirements are in balance with each other.

Thus, the Maritime Safety Committee - senior technical body on safety-related matters of the International Maritime Organisation (IMO) - agreed to further develop goal-based standards using a safety level approach; the task has a five-tier structure: goals (Tier I), functional requirements (Tier II), verification of compliance criteria (Tier III), technical procedures and guidelines, classification rules and industry standards (Tier IV) and codes of practice and safety and quality systems for shipbuilding, ship operation, maintenance, training, manning, etc. (Tier V).

Some reasons for the application of goal-based standards in shipping are seen by the Maritime Safety Committee:

- to assure a uniform minimum acceptable safety level across the merchant fleet;
- to facilitate the comparison between alternative risk control options,
- to facilitate the comparison of accident rates and risk acceptance criteria within the fleet and to other sectors such as aviation or offshore,
- to improve the transparency of the system by the incorporation of rationales; and
- to balance individual requirements with each other.

These goal based standards may use risk criteria as the ‘top’ goal which forms the ultimate goal to be achieved by subsequent IMO rules such as regulations for fire safety, navigation, life saving appliances as well as class society rules and regulations for structures, machinery etc.

4 UNCERTAINTIES IN RISK ASSESSMENT RESULTS

As large-scale accidents occur infrequently and are typically the result of some unique combination of human and system failure, there is inevitably a degree of imprecision or ambiguity associated with the predicted probability of occurrence of such accidents and uncertainty concerning the consequences, should such an accident happen. Procedures for tackling uncertainties when assessing risks are described in (HSE 2001).

These uncertainties may be linked to the relevance of the data basis, the models used in the estimation, the assumptions, simplifications or expert judgements that are made. This shall be reflected when quantitative safety goals are used to judge the results of a probabilistic safety assessment. The requirement may be satisfied in different ways:

- apply more conservation in the risk analysis.
- make sure that probabilistic safety assessment are satisfied with some margin.

Another way to capture uncertainties about a particular risk resulting from a plant, activity or item is to construct an exceedance probability (EP) curve. An EP curve specifies the probabilities that a certain level of losses will be exceeded. The losses can be measured in terms of technical damages, fatalities, financial consequences or some other relevant unit of the respective analysis.

By its nature, the EP curve inherently incorporates uncertainty associated with the probability of an event occurring and the magnitude of losses. This uncertainty is reflected in the 5 % and 95 % confidence interval curves in the EP curve. When determining quantitative safety goals, e.g., the competent regulatory body or institution has to provide guidance how to compare results from probabilistic safety assessments with these goals and how to deal with the uncertainties in the assessment taking into account that the degree of uncertainty in risk analysis increases at lower probabilities, which adds another dimension to the evaluation of potentially disastrous hazards and resulting consequences.

5 CONCLUDING REMARKS

Risk management and safety management, based on the results of risk analysis, support the process of decision making both for the industries and the respective regulatory bodies.

Whenever, on the basis of risk assessments, decision alternatives have been identified and ranked by comparing the expected value of benefits or losses, the risks must be considered in regard to their acceptability. It is suggested to differentiate between tangible and intangible risk, i.e. risks which may be easily expressed in monetary risks and others. Which intangible values should be considered in a given case has to be checked by the risk identification.

Therefore, the need for the development of risk criteria, which would support risk informed decision-making, is expressed worldwide. However, risk acceptance is also correlated to the cultural context, even if, e.g., the European Commission is acting in determining or harmonizing quantitative safety goals.

One way of determining quantitative risk criteria is to consider probabilistic safety assessment. Ideally, such quantitative safety goals are not limited to one type of plants but to any large industrial plant or any industrial activity that requires safety-related systems to ensure safety of aviation, aeronautical (Filip 2007), or railway.

REFERENCES

[1] Aven, T. & Vinnem, J.E. 2005. On the use of risk acceptance criteria in the offshore oil and gas industry. *Reliability Engineering and System Safety* 90: 15 – 24.

[2] Aven, T., Vinnem, J.E. & Røed, W. 2006. On the use of goals, quantitative criteria and requirements in safety management. Risk Management: an International Journal 8: 118 – 132.

[3] Berg, H. P. 1995. On the potential of probabilistic safety assessment. Kerntechnik 60: 71

[4] Berg, H. P., Görtz, R. & Schimetschka, E. 2003. Quantitative Probabilistic Safety Criteria for Licensing and Operation of Nuclear Plants, Comparison of the International Status and Development. BfS-Report, BfS-SK-03/03, Bundesamt für Strahlenschutz, Salzgitter, November 2003.

[5] Van der Borst & M. Versteeg, M. F. 1996. PSA supported severe accident management strategies for the Borssele NPP. Proc. International Topical Meeting on Probabilistic Safety Assessment PSA`96. Vol. 3. Park City, Utah, USA, September 29th – October 3rd, 1996

[6] Braband, J. 2005. Risikoanalysen in der Eisenbahn-Automatisierung. Eurailpress, Hamburg 2005.

[7] CENELEC. (2003). Railway applications – Communications, signalling and processing systems – Safety related electronic systems for signalling. EN 50129, February 2003.

[8] Cepin, M. 2004. Development of criteria for risk – informed decision – making. Proceedings International Conference Nuclear Energy for New Europe 2004, Nuclear Society of Slovenia, paper 501.

[9] Cepin, M. 2007. The risk criteria for assessment of temporary changes in a nuclear power plant. Risk Analysis 27, No 4: 991 – 998.

[10] Dennis, C. 2006. The use of risk assessment to inform safety decision making. Safety Risk Management as an Input to Business Strategy, Institute of Risk Management, 5th October 2006.

[11] Directorate General for Environmental Protection at the Ministry of Housing, Physical Planning and Environment 1989. Premises for Risk Management – Risk Limits in the Content of Environmental Policy.

[12] Eendebak, B. 1995. The use of probabilistic safety assessment for operating nuclear power plants. Fourth annual Two-Day Conference on PSA on the Nuclear Industry, London, November 29th – 30th, 1995.

[13] European Commission. Mandate to the European Railway Agency. December 16th, 2005.

[14] Filip, Ales 2007. Safety aspects of GNSS based train position determination for railway signalling. UIC GALILEO for Rail Symposium, Paris, October 18th – 19th, 2007.

[15] Görtz, R., Berg, H.P. & Schimetschka, E. 2001. Risk targets and reliability goals. Probabilistic Safety Assessment and Risk-informed Decision Making PSARID EUROCOURSE, Garching, March 2001.

[16] Hakata, T. 2003. Basic considerations on defining safety goals. Nuclear Technology 142: 243 – 249.

[17] Hokstad, P., Vatn, J., Aven, T. & Sorum, M. 2004. Use of risk acceptance criteria in Norwegian off-shore industry: dilemmas and challenges. Risk, Decision and Policy, 9: 193 – 206.

[18] Holloway, N. & J. Butcher, P. 1995. Ignalina RSR Task 10: a defense in depth approach to safety assessment for RMBK reactors. Serco Assurance Ltd., UK, December 1995, private communication by S. Simkevicius sigis@isag.lei.lt.

[19] Health & Safety Executive (HSE) 1988. The Tolerability of Risk from Nuclear Power Stations. HMSO, London 1988.

[20] Health & Safety Executive (HSE) 1992. Safety Assessment Principles for Nuclear Plants. HMSO, London, 1992.

[21] Health & Safety Executive (HSE) 2001. Reducing Risks, Protecting People. HMSO, Colegate, Norwich.

[22] Health & Safety Executive (HSE) 2006 a. Safety Assessment Principles for Nuclear Facilities. Edition 2006, HSE, Redgrave Court.

[23] Health & Safety Executive (HSE) 2006 b. Numerical Targets and Legal Limits in Safety Assessment Principles for Nuclear Facilities, An Explanatory Note. HSE, Redgrave Court, December 2006.

[24] International Atomic Energy Agency (IAEA) 1992. The Role of Probabilistic Safety Assessment and Probabilistic Safety Criteria in Nuclear Power Plant Safety. IAEA Safety Series No. 106, Vienna.

[25] International Atomic Energy Agency (IAEA) 2001. Safety Assessment and Verification for Nuclear Power Plants. Safety Guide No. NS-G-1.2, Vienna.

[26] International Atomic Energy Agency (IAEA) 2002. Heavy Water Reactors: Status and Projected Development. IAEA Technical Report Series No. 407, Vienna.

[27] International Atomic Energy Agency (IAEA) 2007. Risk-Informed Decision-Making. Draft B, Safety Guide, Vienna.

[28] International Nuclear Safety Advisory Group (INSAG) 1999. Basic Safety Principles for Nuclear Power Plants. 75-INSAG-3 Rev. 1, INSAG-12, IAEA, Vienna.

[29] Joint Committee of Structural Safety (JCSS) 2008. Risk Assessment in Engineering, Principles, System Representation & Risk Criteria. JCSS, June 2008.

[30] Kumamoto, H. 2007. Satisfying Safety Goals by Probabilistic Risk Assessment. Springer Series in Reliability Engineering, Springer-Verlag.

[31] Maharik, H. & Vrijling, J. K. 2002. Lessons from quantitative risk assessment in the Netherlands: national and international perspective. 6th International Conference on Probabilistic Safety Assessment and Management, San Juan, Puerto Rico, USA, June 23rd – 28th, 2002.

[32] NORSOK 2001. Risk and Emergency Preparedness Analysis. Standard Z-013.

[33] Nuclear Energy Agency/OECD 2007 b. Use and Development of Probabilistic Safety Assessment. NEA/CSNI/R (2007) 12, November 2007.

[34] Nuclear Energy Agency/OECD 2007 a. The Regulatory Goal of Assuring Nuclear Safety. Paris.

[35] Rimkevicius, S., Senkin, V. & Brandisauskas, D. 2002. Presentation at IAEA Technical Meeting on Analysis Results and Methods for PSRs of NPPs, Ljubljana, Slovenia, December 2nd – 6th, 2002.

[36] Radiation and Nuclear Safety Authority (STUK) 1996. Probabilistic Safety Analysis (PSA). Guide No. YVL 2.8, issued December 20th, 1996, updated version May 2003.

[37] Villadóniga, J. 2001. Towards the risk-informed approach. Probabilistic safety assessment and risk-informed decision making. PSARID EUROCOURSE, Garching, March 2001.

[38] Vrijling, J. K., van Hengel, W. & van Maanen, S. E. 1996. The application of the concept of societal risk to various activities in the Netherlands. Proceedings of the International Conference Probabilistic Safety Assessment and Management, ESREL 1996 – PSAM III, Springer-Verlag, Volume 2.

[39] Vrijling, J.K. et al. 2004. A framework for risk criteria for critical infrastructures : fundamentals and case studies in the Netherlands, Journal of Risk Research 7 (6): 569 – 579.

RISK MANAGEMENT: PROCEDURES, METHODS AND EXPERIENCES

Heinz-Peter Berg
•
Bundesamt für Strahlenschutz, Salzgitter, Germany

e-mail: hberg@bfs.de

ABSTRACT

Risk management is an activity which integrates recognition of risk, risk assessment, developing strategies to manage it, and mitigation of risk using managerial resources. Some traditional risk managements are focused on risks stemming from physical or legal causes (e.g. natural disasters or fires, accidents, death). Financial risk management, on the other hand, focuses on risks that can be managed using traded financial instruments. Objective of risk management is to reduce different risks related to a pre-selected domain to an acceptable. It may refer to numerous types of threats caused by environment, technology, humans, organizations and politics. The paper describes the different steps in the risk management process which methods are used in the different steps, and provides some examples for risk and safety management.

1 INTRODUCTION

1.1 Risk

Risk is unavoidable and present in every human situation. It is present in daily lives, public and private sector organizations. Depending on the context (insurance, stakeholder, technical causes), there are many accepted definitions of risk in use.

The common concept in all definitions is uncertainty of outcomes. Where they differ is in how they characterize outcomes. Some describe risk as having only adverse consequences, while others are neutral.

One description of risk is the following: risk refers to the uncertainty that surrounds future events and outcomes. It is the expression of the likelihood and impact of an event with the potential to influence the achievement of an organization's objectives.

The phrase "the expression of the likelihood and impact of an event" implies that, as a minimum, some form of quantitative or qualitative analysis is required for making decisions concerning major risks or threats to the achievement of an organization's objectives. For each risk, two calculations are required: its likelihood or probability; and the extent of the impact or consequences.

Finally, it is recognized that for some organizations, risk management is applied to issues predetermined to result in adverse or unwanted consequences. For these organizations, the definition of risk which refers to risk as "a function of the probability (chance, likelihood) of an adverse or unwanted event, and the severity or magnitude of the consequences of that event" will be more relevant to their particular public decision-making contexts.

1.2 Risk Management

Two different safety management principles are possible: consequence based safety management will claim that the worst conceivable events at an installation should not have consequences outside certain boundaries, and will thus design safety systems to assure this. Risk based safety management (usually called risk management) maintains that the residual risk should be analysed both with respect to the probabilistic and the nature of hazard, and hence give information for further risk mitigation. This implies that very unlikely events might, but not necessarily will, be tolerated.

Risk management is not new tool and a lot of standards and guidance documents are available (ACT 2004, AZ/NZS 2004, Committee 2004, DGQ 2007, FAA 2007, HB 2004, IEC 2008, ON 2008, Rio Tinto 2007, Treasury Board of Canada 2001). It is an integral component of good management and decision-making at all levels of an organization. All departments in an organization manage risk continuously whether they realize it or not, sometimes more rigorously and systematically, sometimes less. More rigorous risk management occurs most visibly in those departments whose core mandate is to protect the environment and public health and safety. At present, a further generic standard on risk management is in preparation as a common ISO/IEC standard (IEC 2007) describing a systemic top down as well as a functional bottom up approach (see Fig. 1) This standard is intended to support existing industry or sector specific standards.

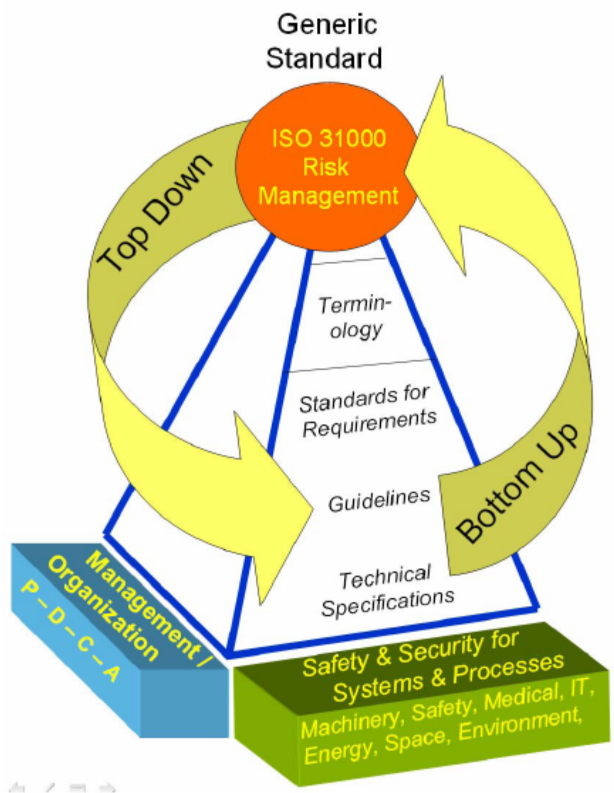


Figure 1. Approach of the planned generic standard on risk management.

As with the definition of risk, there are equally many accepted definitions of risk management in use. Some describe risk management as the decision-making process, excluding the identification and assessment of risk, whereas others describe risk management as the complete process, including risk identification, assessment and decisions around risk issues.

One well accepted description of risk management is the following: risk management is a systematic approach to setting the best course of action under uncertainty by identifying, assessing, understanding, acting on and communicating risk issues.

In order to apply risk management effectively, it is vital that a risk management culture be developed. The risk management culture supports the overall vision, mission and objectives of an organization. Limits and boundaries are established and communicated concerning what are acceptable risk practices and outcomes.

Since risk management is directed at uncertainty related to future events and outcomes, it is implied that all planning exercises encompass some form of risk management. There is also a clear implication that risk management is everyone's business, since people at all levels can provide some insight into the nature, likelihood and impacts of risk.

Risk management is about making decisions that contribute to the achievement of an organization's objectives by applying it both at the individual activity level and in functional areas. It assists with decisions such as the reconciliation of science-based evidence and other factors; costs with benefits and expectations in investing limited public resources; and the governance and control structures needed to support due diligence, responsible risk-taking, innovation and accountability.

A typical decision support for risk and safety management at strategic, normative and operational level is provided in (JCSS 2008).

1.3 Integrated Risk Management

The current operating environment is demanding a more integrated risk management approach (see Bolvin et al. 2007 and Treasury Board of Canada 2001). It is no longer sufficient to manage risk at the individual activity level or in functional silos. Organizations around the world are benefiting from a more comprehensive approach to dealing with all their risks.

Today, organizations are faced with many different types of risk (e.g., policy, program, operational, project, financial, human resources, technological, health, safety, political). Risks that present themselves on a number of fronts as well as high level, high -impact risks demand a co-ordinated, systematic corporate response.

Thus, integrated risk management is defined as a continuous, proactive and systematic process to understand, manage and communicate risk from an organization-wide perspective. It is about making strategic decisions that contribute to the achievement of an organization's overall corporate objectives.

Integrated risk management requires an ongoing assessment of potential risks for an organization at every level and then aggregating the results at the corporate level to facilitate priority setting and improved decision-making. Integrated risk management should become embedded in the organization's corporate strategy and shape the organization's risk management culture. The identification, assessment and management of risk across an organization helps reveal the importance of the whole, the sum of the risks and the interdependence of the parts.

Integrated risk management does not focus only on the minimization or mitigation of risks, but also supports activities that foster innovation, so that the greatest returns can be achieved with acceptable results, costs and risks.

From a decision-making perspective, integrated risk management typically involves the establishment of hierarchical limit systems and risk management committees to help to determine the setting and allocation of limits. Integrated risk management strives for the optimal balance at the corporate level. However, companies still vary considerably in the practical extent to which important risk management decisions are centralised (Basel Committee on Banking Supervision 2003).

1.4 Safety management

Apart from reliable technologies, the operational management of a industrial plant with high risk potential is also a highly important factor to ensure safe operation. Owing to the liberalisation of the markets and resulting cost pressure to the industries, the importance of operational management is growing since cost savings in the areas of personnel and organization result in reducing the number of personnel together with changes in the organizational structure and tighter working processes.

For small- and medium-sized companies, specific support is necessary and provided in (Rheinland-Pfalz 2008).

Experience with accidents in different branches of industry shows the importance of safe operational management. Today, effective safety management is seen as one crucial element of safe operational management (Hess & Gaertner 2006).

The term safety management subsumes the entirety of all activities relating to the planning, organization, management and supervision of individuals and work activities with a view to the efficient achievement of a high degree of safety performance, i.e. the achievement of a high quality of all activities that are important to safety, and to the promotion of a highly developed safety culture. Safety management is not limited to certain organization units but comprises the entire safety-related organization of the company. Safety management is the responsibility of the management level of a company.

For example in case of nuclear power plant in Germany (see ICBMU 2004), the licensee is according to the Atomic Energy Act responsible for the safety of the plant he operates. To fulfil the conditions associated with this responsibility, he has to implement an effective safety management system that complies with the requirements of the current regulations and with international standards. Typical management systems in nuclear power plants are described in (GRS 2007).

Sometimes risk management and safety management are seen as the same type of management, but in practice safety management is a main and important part of the risk management which also covers, e.g. financial risks.

2 RISK MANAGEMENT STEPS AND TOOLS

The risk management steps (see Fig. 2) are:

1. Establishing goals and context (i.e. the risk environment),
2. Identifying risks,
3. Analysing the identified risks,
4. Assessing or evaluating the risks,
5. Treating or managing the risks,
6. Monitoring and reviewing the risks and the risk environment regularly, and
7. Continuously communicating, consulting with stakeholders and reporting.

Some of the risk management tools are described in (IEC 2008) and (Oehmen 2005).

2.1 Establish goals and context

The purpose of this stage of planning enables to understand the environment in which the respective organization operates, that means to thoroughly understand the external environment and the internal culture of the organization. The analysis is undertaken through:

– establishing the strategic, organizational and risk management context of the organization, and

– identifying the constraints and opportunities of the operating environment.

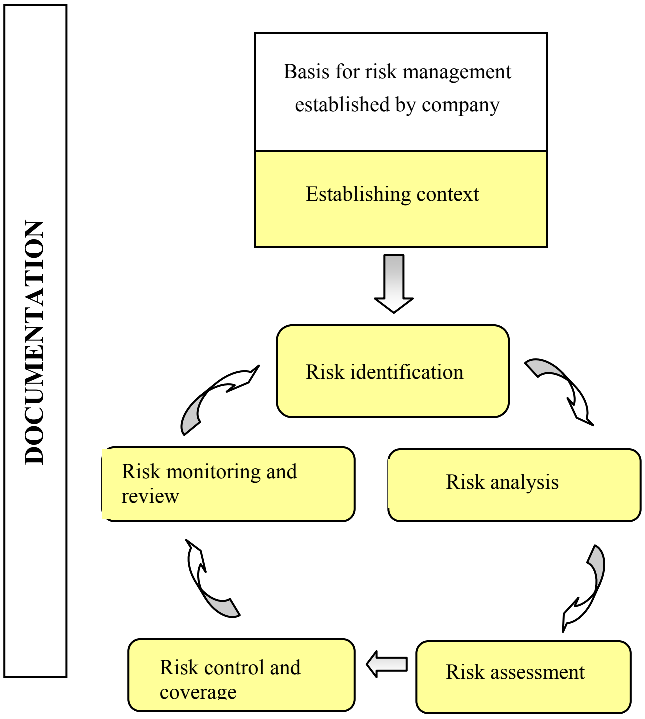


Figure 2. Risk management process.

The establishment of the context and culture is undertaken through a number of environmental analyses that include, e.g., a review of the regulatory requirements, codes and standards, industry guidelines as well as the relevant corporate documents and the previous year's risk management and business plans.

Part of this step is also to develop risk criteria. The criteria should reflect the context defined, often depending on an internal policies, goals and objectives of the organization and the interests of stakeholders. Criteria may be affected by the perceptions of stakeholders and by legal or regulatory requirements. It is important that appropriate criteria be determined at the outset.

Although the broad criteria for making decisions are initially developed as part of establishing the risk management context, they may be further developed and refined subsequently as particular risks are identified and risk analysis techniques are chosen. The risk criteria must correspond to the type of risks and the way in which risk levels are expressed.

Methods to assess the environmental analysis are SWOT (Strength, Weaknesses, Opportunities and Threats) and PEST (Political, Economic, Societal and Technological) frameworks, typically shown as tables.

2.2 Identify the risks

Using the information gained from the context, particularly as categorised by the SWOT and PEST frameworks, the next step is to identify the risks that are likely to affect the achievement of the goals of the organization, activity or initiative. It should be underlined that a risk can be an opportunity or strength that has not been realised.

Key questions that may assist your identification of risks include:

– For us to achieve our goals, when, where, why, and how are risks likely to occur?
– What are the risks associated with achieving each of our priorities?
– What are the risks of not achieving these priorities?
– Who might be involved (for example, suppliers, contractors, stakeholders)?

The appropriate risk identification method will depend on the application area (i.e. nature of activities and the hazard groups), the nature of the project, the project phase, resources available, regulatory requirements and client requirements as to objectives, desired outcome and the required level of detail.

The use of the following tools and techniques may further assist the identification of risks:

- Examples of possible risk sources,
- Checklist of possible business risks and fraud risks,
- Typical risks in stages of the procurement process,
- Scenario planning as a risk assessment tool ,
- Process mapping, and
- Documentation, relevant audit reports, program evaluations and / or research reports.

Specific lists, e.g. from standards, and organizational experience support the identification of internal risks. To collect experience available in the organization regarding internal risks, people with appropriate knowledge from the different parts of the organization should be involved in identifying risks. Creativity tools support this group process (see Fig. 3).

The identification of the sources of the risk is the most critical stage in the risk assessment process. The sources are needed to be managed for pro-active risk management. The better the understanding of the sources, the better the outcomes of the risk assessment process and the more meaningful and effective will be the management of risks.

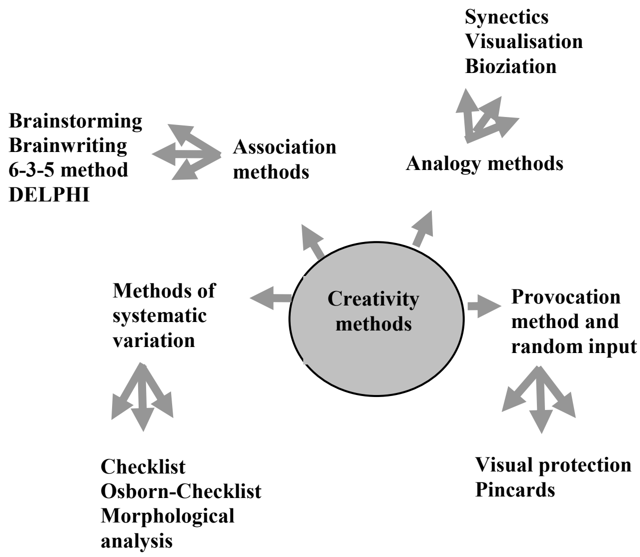


Figure 3. Creativity tools.

Key questions to ask at this stage of the risk assessment process to identify the impact of the risk are:

- Why is this event a risk?
- What happens if the risk eventuates?
- How can it impact on achieving the objectives/outcomes?

Risk identification of a particular system, facility or activity may yield a very large number of potential accidental events and it may not always be feasible to subject each one to detailed quantitative analysis. In practice, risk identification is a screening process where events with low or trivial risk are dropped from further consideration.

However, the justification for the events not studied in detail should be given. Quantification is then concentrated on the events which will give rise to higher levels of risk. Fundamental methods such as Hazard and Operability (HAZOP) studies, fault trees, event tree logic diagrams and Failure Mode and Effect Analysis (FMEA) are tools which can be used to identify the risks and assess the criticality of possible outcomes.

An example of a systematic method for identifying technical risks of a plant is the elaboration of a risk register where different types of risks and damage classes are correlated to local areas of a plant (cf. Fig. 4).

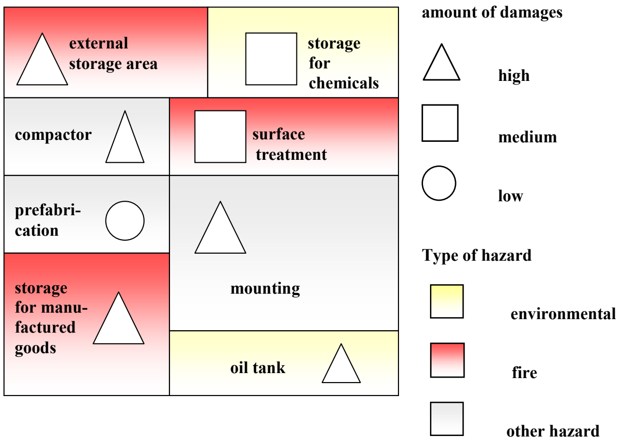


Figure 4. Example of a risk register.

2.3 Analyse the risk

Risk analysis involves the consideration of the source of risk, the consequence and likelihood to estimate the inherent or unprotected risk without controls in place. It also involves identification of the controls, an estimation of their effectiveness and the resultant level of risk with controls in place (the protected, residual or controlled risk). Qualitative, semi-quantitative and quantitative techniques are all acceptable analysis techniques depending on the risk, the purpose of the analysis and the information and data available.

Often qualitative or semi-quantitative techniques can be used for screening risks whereas higher risks are being subjected to more expensive quantitative techniques as required. Risks can be estimated qualitatively and semi-quantitatively using tools such as hazard matrices, risk graphs, risk matrices or monographs but noting that the risk matrix is the most common.

Applying the risk matrix, it is required to define for each risk its profile using likelihood and consequences criteria. Typical definitions of the likelihood and consequence are contained in the risk matrix (cf. Table 1).

Using the consequence criteria provided in the risk matrix, one has to determine the consequences of the event occurring (with current controls in place).

To determine the likelihood of the risk occurring, one can apply the likelihood criteria (again contained in the risk matrix). As before, the assessment is undertaken with reference to the effectiveness of the current control activities.

To determine the level of each risk, one can again refer to the risk matrix. The risk level is identified by intersecting the likelihood and consequence levels on the risk matrix.

Complex risks may involve a more sophisticated methodology. For example, a different approach may be required for assessing the risks associated with a significantly large procurement.

Table 1. Example of a risk matrix

Significance			Consequence				
			1 Insignificant Impact	2 Minor Impact to Small Population	3 Moderate- Minor Impact to Large Population	4 Major Impact to Small Population	5 Catastrophic – Major Impact to Large Population
Likelihood	1	Rare	Low	Low	Moderate	High	High
	2	Unlikely	Low	Low	Moderate	High	Very High
	3	Moderate / Possible	Low	Moderate	High	Very High	Very High
	4	Likely	Moderate	High	High	Very High	Extreme
	5	Almost Certain	Moderate	High	Very High	Extreme	Extreme

Special approaches exist to analyse major risk in complex projects, e. .g. described in (Cagno et al. 2007).

2.4 Evaluate the risk

Once the risks have been analysed they can be compared against the previously documented and approved tolerable risk criteria. When using risk matrices this tolerable risk is generally documented with the risk matrix. Should the protected risk be greater than the tolerable risk then the specific risk needs additional control measures or improvements in the effectiveness of the existing controls.

The decision of whether a risk is acceptable or not acceptable is taken by the relevant manager. A risk may be considered acceptable if for example:

- The risk is sufficiently low that treatment is not considered cost effective, or
- A treatment is not available, e.g. a project terminated by a change of government, or
- A sufficient opportunity exists that outweighs the perceived level of threat.

If the manager determines the level of risk to be acceptable, the risk may be accepted with no further treatment beyond the current controls. Acceptable risks should be monitored and periodically reviewed to ensure they remain acceptable. The level of acceptability can be organizational criteria or safety goals set by the authorities.

2.5 Treat the risk

An unacceptable risk requires treatment. The objective of this stage of the risk assessment process is to develop cost effective options for treating the risks. Treatment options (cf. Fig. 5), which are not necessarily mutually exclusive or appropriate in all circumstances, are driven by outcomes that include:

- Avoiding the risk,
- Reducing (mitigating) the risk,
- Transferring (sharing) the risk, and
- Retaining (accepting) the risk.

Avoiding the risk - not undertaking the activity that is likely to trigger the risk.

Reducing the risk - controlling the likelihood of the risk occurring, or controlling the impact of the consequences if the risk occurs.

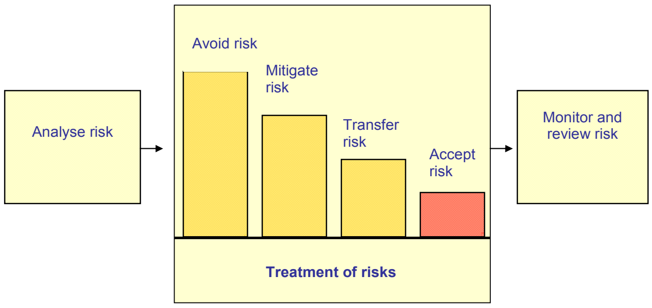


Figure 5. Treatment of risks

Factors to consider for this risk treatment strategy include:

– Can the likelihood of the risk occurring be reduced? (through preventative maintenance, or quality assurance and management, change in business systems and processes), or

– Can the consequences of the event be reduced? (through contingency planning, minimizing exposure to sources of risk or separation/relocation of an activity and resources).

Examples for the mitigation activity effectiveness are described in (Wirthin 2006).

Transferring the risk totally or in part - This strategy may be achievable through moving the responsibility to another party or sharing the risk through a contract, insurance, or partnership/joint venture. However, one should be aware that a new risk arises in that the party to whom the risk is transferred may not adequately manage the risk!

Retaining the risk and managing it - Resource requirements feature heavily in this strategy.

The next step is to determine the target level of risk resulting from the successful implementation of the preferred treatments and current control activities.

The intention of a risk treatment is to reduce the expected level of an unacceptable risk. Using the risk matrix one can determine the consequence and likelihood of the risk and identify the expected target risk level.

2.6 Monitoring the risk

It is important to understand that the concept of risk is dynamic and needs periodic and formal review.

The currency of identified risks needs to be regularly monitored. New risks and their impact on the organization may to be taken into account.

This step requires the description of how the outcomes of the treatment will be measured. Milestones or benchmarks for success and warning signs for failure need to be identified.

The review period is determined by the operating environment (including legislation), but as a general rule a comprehensive review every five years is an accepted industry norm. This is on the basis that all plant changes are subject to an appropriate change process including risk assessment.

The review needs to validate that the risk management process and the documentation is still valid. The review also needs to consider the current regulatory environment and industry practices which may have changed significantly in the intervening period.

The organisation, competencies and effectiveness of the safety management system should also be covered. The plant management systems should have captured these changes and the review should be seen as a ‘back stop’.

The assumptions made in the previous risk assessment (hazards, likelihood and consequence), the effectiveness of controls and the associated management system as well as people need to be monitored on an on-going basis to ensure risk are in fact controlled to the underlying criteria.

For an efficient risk control the analysis of risk interactions is necessary.

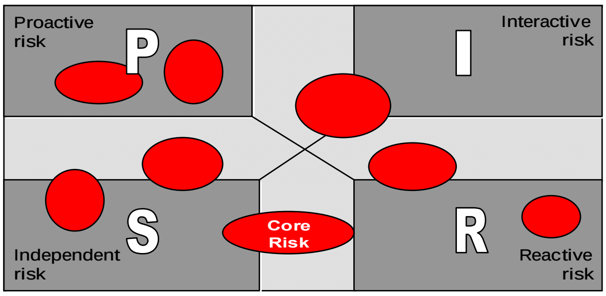


Figure 6. Results of a cross impact analysis.

This ensures that the influences of one risk to another is identified and assessed. Usual method for that purpose are a cross impact analysis (cf. Fig. 6), Petri nets or simulation tools.

A framework needs to be in place that enables responsible officers to report on the following aspects of risk and its impact on organizations´ operations:

– What are the key risks?
– How are they being managed?
– Are the treatment strategies effective? – If not, what else must be undertaken?
– Are there any new risks and what are the implications for the organization?

2.7 Communication and reporting

Clear communication is essential for the risk management process, i.e. clear communication of the objectives, the risk management process and its elements, as well as the findings and required actions as a result of the output.

Risk management is an integral element of organization´s management. However, for its successful adoption it is important that in its initial stages, the reporting on risk management is visible through the framework. The requirements on the reporting have to be fixed in a qualified and documented procedure, e. g., in a management handbook. The content of such a handbook is shown in Figure 7.

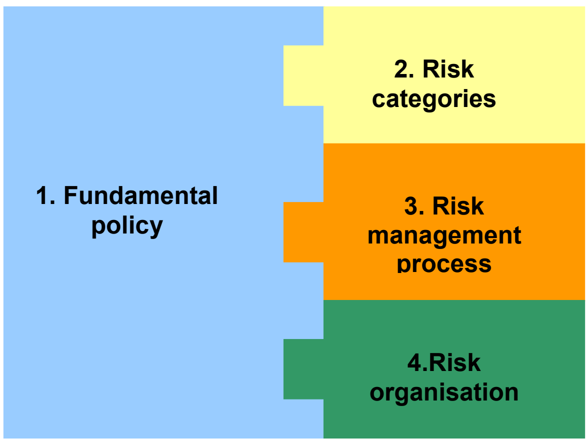


Figure 7. Structure of a risk management handbook.

Documentation is essential to demonstrate that the process has been systematic, the methods and scope identified, the process conducted correctly and that it is fully auditable. Documentation provides a rational basis for management consideration, approval and implementation including an appropriate management system.

A documented output from the above sections (risk identification, analysis, evaluation and controls) is a risk register for the site, plant, equipment or activity under consideration. This document is essential for the on-going safe management of the plant and as a basis for communication throughout the client organisation and for the on-going monitor and review processes. It can also be used with other supporting documents to demonstrate regulatory compliance.

3 EXAMPLES

3.1 NASA risk management to the SOFIA programm

NASA and DLR (German Aerospace Center) have been working together to create the Stratospheric Observatory For Infrared Astronomy (SOFIA). SOFIA is a Boing 747SP (Special Performance) aircraft, extensively modified to accommodate a 2.5 meter reflecting telescope and airborne mission control system. In (Datta 2007) it is shown how the SOFIA program handled one safety issue through appropriate use of NASA`s Risk Management Process based on (NASA 2002).

3.1.1 Risk identification

The safety issue was identified while reviewing the Probabilistic Risk Assessment of a depressurization scenario in the telescope cavity. The failure scenario itself was previously known where a leak in the telescope cavity door seal sucks air out from the telescope cavity creating a negative pressure differential between the telescope cavity and the aft cavity. Two negative pressure relief valves were designed to handle this and other cavity negative pressure scenarios. However, the proposed new scenario had a leak area that was beyond the original design basis. Nevertheless, this failure scenario was deemed credible but with a lower probability of occurrence.

3.1.2 Risk analysis

After identification of the safety issue, both the risk management and the engineering processes required an analysis of this depressurization scenario. Multiple models of the depressurization scenarios were created and analyzed at peak dynamic pressures. The results revealed that under some failure scenarios the relief valves might not be redundant. Both valves need to function for adequate pressure equalization without exceeding structural design loads. These conditions created a program risk state that needed to be mitigated.

All considerations within the risk analysis were based on prescribed project risk definitions.

3.1.3 Risk control

As a result, the program started a risk mitigation plan where a test will be performed to characterize the seal failure scenario by intentionally deflating the seal at lower dynamic pressure.

This risk continues to reside in the SOFIA program risk list so as to ensure that the risk mitigation plan is carried out in the future. The risk list is the listing of all identified risks in priority order from highest to lowest risk, together with the information that is needed to manage each risk and document its evolution over the course of the program. The highest risks are extracted from the list. The negative pressure relief valve risk has not yet reached among the top fifteen list of risks (see Datta 2007).

3.2 Construction of a nuclear power plant

Risk identification and risk analysis can not only be performed on component or system level, but also for a comprehensive technical project such as a (nuclear) power plant.

3.2.1 Risk context

Since many years, no new nuclear power plant has been constructed in USA. However, in near future, decisions have to be made which types of power plants will reset the nuclear power plants which have to be shut down in the next ten years. Thus, for a new project the resulting risks have to be evaluated.

The risk context is determined by the electricity market, the license, the technical aspects of the design, the construction of the plant, the operation of the new plant as well as the financing of the project.

3.2.2 Risk identification

On the background of this context, a potential operator has to take into account the following risks:

– Licensing risks: will the plant be licensed in a predictable time schedule or will this be a longer procedure, which strongly influences the start of the commercial operation.

– Design risks: is the plant completely designed before construction or are surprises to be expected which lead to cost- intensive changes of the plant and delay of the construction period.

– Technical risks: will the plant behave as planned or will unknown technical problems lead to shut down and thus fail the projected goals.

– Cost risks: will the plant to more expensive as planned and the chances in the free electricity market reduced.

– Time schedule risks: will the plant start the production at the scheduled time or have delays to be expected.

– Finance risks: which possible uncertainties have to be taken into account by investors with respect to the new project, e.g., how is the public acceptance of a new nuclear power plant.

3.2.3 Risk analysis

In a specific case, General Electric has analysed the risk of constructing a new plant in the following manner:

– License risks: the new reactor type has been developed in accordance with current nuclear safety standards and is already certified site-independently by the US licensing authority. Moreover, this type of reactor has already been licensed in Japan, where two plants are running successfully since five years.

– Design risks: the reactor type is completely planned with all necessary drawings. Material and costs are exactly known.

– Technical risks: the plants constructed in Japan have a total operating time of ten years with a high availability.

– Finance risks: main problem is the financing of a new nuclear power plant project because of experiences in the eighties with construction times up to 15 years.

3.2.4 Risk evaluation

Following this risk analysis, an evaluation of the risks has been the next step:

– License risk: the experiences listed in the risk analysis lead to the expectations that the licensing process should not last more than one year.

– Design risk: due to the completely available design documentation no larger deviations are expected that result in expensive delays.

– Technical risk: the risk evaluation of the potential operator and the investors will not only be based on the expected high availability, but also on the occurrence frequency of an accident and the acceptance by the public in comparison with other energy producing systems.

3.2.5 Risk treatment

General Electric has chosen from the different alternatives to treat risks as described in 2.5 to retain and accept the risks for costs and time schedule by offering a fixed price and a construction time which will be determined in the contract.

3.3 National foresight program "Poland 2020"

Totally different and more global types of risk management are so-called foresight programs. Foresight means a systematic method of building a medium and long-term vision of development of the scientific and technical policy, its directions and priorities, used as a tool for making on-going decisions and mobilizing joint efforts. The aim of foresight is to indicate future needs, opportunities and threats associated with the social and economic growth and to plan appropriate measures in the field of science and technology.

The scope of realization of the National Foresight Programme “Poland 2020” (see Fig. 8) covers the three research areas “sustainable development of Poland”, “information and telecommunications technologies” and “security”.

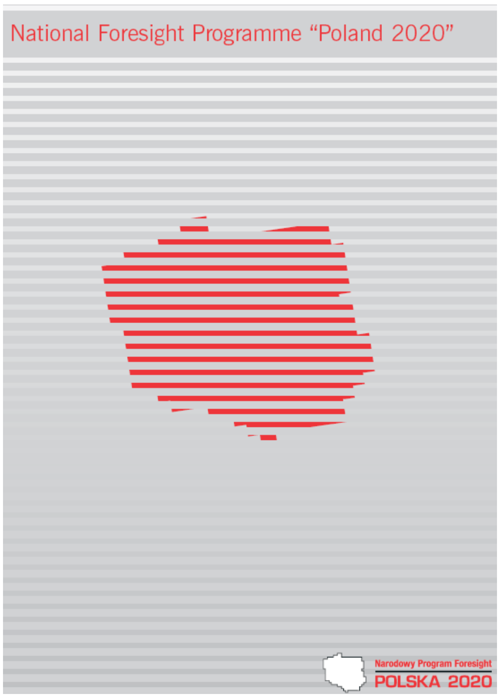


Figure 8. Cover of the brochure describing the Polish foresight program.

The aim of the National Foresight Programme "Poland 2020" is to:

– lay out the development vision of Poland until the year 2020,

– set out – through a consensus with the main beneficiaries – the priority paths of scientific research and development which will, in the long run, have an impact on the acceleration of the social and economic growth,

– put the research results into practice and create preferences for them when it comes to allotting funds from the budget,

– adjust the Polish scientific policy to the requirements of the European Union,

– shape the scientific and innovative police towards knowledge-based economy.

For the purpose of foresight, different methods can be applied to prepare long-term development scenarios (see Table 2).

Foresight can never be completely dominated by quantitative methods: the appropriate mix of methods depends on access to relevant expertise and the nature of the issues.

Various foresight methods are planned to be used in the National Foresight Programme "Poland 2020", among which the following methods will be the leading ones:

– Expert panels,
– SWOT analysis,
– Delphi analysis,
– PEST analysis,
– Cross-impact analysis,
– Scenarios of development.

Table 2. Methods typically used for foresight programs

Categories by Criteria	Methods
Quantitative methods (use of statistics and other data) to elaborate future trends and impacts	– Trend extrapolation – Simulation modelling – Cross impact analysis – System dynamics
Qualitative methods (drawing on expert knowledge) to develop long term strategies	– Delphi method – Experts panels – Brainstorming – Mindmapping – Scenario analysis workshops – SWOT analysis
Methods to identify key points of action to determine planning strategies	– Critical/ key technologies – Relevance trees – Morphological analysis

3.4 Risk management in the sector of banks and insurance companies

Basel II and the Capital Requirements Directive (Committee for 2005) are especially important for banks and small and medium sized companies. Rules on capital requirements are designed to protect savers and investors from the risk of the failure or bankruptcy of banks. They ensure that these institutions hold a minimum amount of capital. The Capital Requirements Directive was adopted on 14 June 2006 and comes into force January 2007 with full implementation by 2008. Capital adequacy rules set down the amount of capital a bank or credit institution must hold. This amount is based on risk.

Therefore, it is expected that this rules will have an important influence on the establishment of a risk management system.

Three main issues of the Capital Requirements Directive are:

– the new directive is more risk sensitive,

– costs to smaller banks and consequently to small-company growth, where the EU lags other regions, and

– moral hazard concerns in that risks are partly passed to insurers and banks, unlike insurers have potential last resort support from central banks.

Some commentators argue that strengthening the capital base of banks and encouraging the management of risk does not reduce the risk but merely passes it on elsewhere. Credit risk in particular is being passed on to insurance companies and funds, which are in turn passing it on to householders, i. e., one can ask the question whether ultimately, it may be the consumer who stands to lose if things go wrong.

Comparable to Basel II for the banks and investment institutions will Solvency II fundamentally change and support risk management of the insurance companies. The requirements on the capital equipment will then depend on the risk profile of the insurance company. Besides the quantitative determination of the capital equipment it is part of Solvency II to determine the internal risk management.

Basis in economics and finance is the so-called value at risk (VaR) method. VaR is the maximum loss, not exceeded with a given probability defined as the confidence level, over a given period of time. Although VaR is a very general concept that has broad applications, it is most commonly used by security firms or investment banks to measure the market risk of their asset portfolios (market value at risk). VaR is widely applied in finance for quantitative risk management for many types of risk. VaR does not give any information about the severity of loss by which it is exceeded.

A variety of models exist for estimating VaR. Each model has its own set of assumptions, but the most common assumption is that historical market data is the best estimator for future changes. Common models include:

– variance-covariance, assuming that risk factor returns are always (jointly) normally distributed and that the change in portfolio value is linearly dependent on all risk factor returns,

– the historical simulation, assuming that asset returns in the future will have the same distribution as they had in the past (historical market data),

– Monte Carlo simulation, where future asset returns are more or less randomly simulated.

In (Taleb 2007 a, b), VaR is seen as a dangerously misleading tool. Two issues are mentioned with regard to conventional calculation and usage of VaR:

– Measuring probabilities of rare events requires study of vast amounts of data. For example, the probability of an event that occurs once a year can be studied by taking 4-5 years of data. But high risk-low probability events like natural calamities, epidemics and economic disasters (like the bank crash of 1929) are once a century events which require at least 2-3 centuries of data for validating hypotheses. Since such data does not exist in the first place, it is argued, calculating risk with any accuracy is not possible.

– In the derivation of VaR normal distributions are assumed wherever the frequency of events is uncertain.

Although many problems are similar for the banking and insurance sector respectively, there are some distinctions between these two kinds of companies. Banks mainly deal with bounded risks, e. g., facing credit risks. On the other hand, insurance companies often have to consider unbounded risks, e. g., when heavy-tailed distributed financial positions are present. To address both situations, one always treats integrable but not necessarily bounded risks in this work. Furthermore, a main issue will be to develop risk management tools for dynamic models. These naturally occur when considering portfolio optimisation problems or in the context of developing reasonable risk measures for final payments or even stochastic processes. One considers only models in discrete time and denotes these approaches with dynamic risk management. In dynamic economic models one often faces a Markov structure of the underlying stochastic processes (Mundt 2008).

Systemic financial risk is the most immediate and the most severe. With so many potential consequences of the 2007 liquidity crunch unresolved, the outlook for the future is uncertain (WEF 2008).

The crisis of Société Générale in connection with the real estate credits in the US in 2007/2008 and the breakdown of further US banks in September 2008 might be a symptom for the fact that banks are underestimating the risks or do not apply the risk management tools in an appropriate manner.

4 CONCLUDING REMARKS

Risk management is, at present, implemented in many large as well as small and medium sized industries. In (Gustavsson 2006) it is outlined how a large company can handle its risks in practice and contains a computer based method for risk analysis that can generate basic data for decision-making in the present context. In that study, Trelleborg AB has been chosen as an example to illustrate the difficulties that can be encountered concerning risk management in a large company with different business areas. One typical difficulty is reaching the personnel. Another typical weakness is a missing system for controlling and following up on the results of the risk analysis that has been performed.

However, not only industries but also governmental organizations, research institutes and hospitals are now introducing risk management to some extent.

In case of hospitals. patient safety is endangered, e. g., by adverse events during medical treatment. Patient safety can be increased through risk management which reduces errors through error prevention. This presupposes the recognition of causes for errors and near misses which can be achieved through a critical incident reporting system (CIRS) with a detailed incident reporting form. CIRS is seen as an important instrument in the process of risk management and is, at present, of increasing importance and Switzerland and Germany.

Why is it important to have risk management in mind when performing risk assessment? The different tools support the answer to the following questions:

- risk analysis – how safe is the system, process or item to be investigated,
- risk evaluation – how safe is safe enough, e.g. by comparing the results of the risk analysis with prescribed safety criteria,
- risk management – how to achieve and ensure an adequate level of safety.

Thus, the results of technical risk assessments are one (often very important) part of an overall risk or safety assessment of an organization.

A further step is to couple knowledge management with risk management systems to capture and preserve lessons learned as described in (NASA 2007).

REFERENCES

[40] ACT Insurance Authority 2004. *Risk Management Toolkit.* February 2004.

[41] AZ/NZS 4360 2004. Risk Management. Standards Australia International Ltd, Sydney.

[42] Basel Committee on Banking Supervision 2003. Trends in Risk Integration and Aggregation, Basel, August 2003.

[43] Bolvin C., Farret, R., Salvi, O. 2007. Convergence towards integrated risk management: results from the European SHAPE-RISK project and other initiatives. Proc. ESREL 2007: 1683 – 1687.

[44] Cagno, E., Caron, F., Mancini, M. 2007. A multi-dimensional analysis of major risks in complex projects. Risk Management: 1–18.

[45] Committee for European Banking Supervisors 2005. Consultation Paper on the Supervisory Review Process under Pillar II of the Revised Basel Accord, Basel II), June 2005.

[46] Committee of Sponsoring Organizations of the Treadway Commission (ed.) 2004. Enterprise Risk Management – Integrated Framework – Application Techniques. September 2004.

[47] Datta, K. 2007. The application of the NASA risk management to the SOFIA program. Proc. Reliability and Maintainability Symposium 2007, Orlando, January 2007, 410 – 413.

[48] Deutsche Gesellschaft für Qualität e.V. 2007. Risk Management. DGQ 12 – 41, Beuth-Verlag, Berlin, April 2007 (in German).

[49] Federal Aviation Administration 2007. Safety Risk Management Guidance for System Organization. SRMGSA-Final Version 1.4a, February 2007.

[50] Federal Ministry for the Environment, Nature Conservation and Nuclear Safety (ICBNU) 2004. Fundamentals of Safety Management Systems in Nuclear Power Plants. June 2004.

[51] Federation of European Risk Management Associations 2003. A Risk Management Standard.

[52] Gesellschaft für Anlagen- und Reaktorsicherheit mbH (GRS) 2007. Management Systems in Nuclear Power Plants. GRS-229, Cologne, August 2007 (in German).

[53] Gustavsson, H. 2006. A Risk Management Framework Designed for Trelleborg AB. Report 5195.

[54] HB 436 2004. Handbook Risk Management Guidelines. Standards Australia International Ltd., Sydney 2004.

[55] Hess, S.M., Gaertner, J. P. 2006. Application of risk management as a cornerstone in ensuring nuclear plant safety. Proc. of the 8th International Conference on Probabilistic Safety Assessment and Management, May, 14 – 18, 2006, New Orleans, paper PSAM-0477.

[56] International Electrotechnical Commission (IEC) 2008. Draft IEC 31010 Ed. 1.0, Risk Management – Risk Assessment Techniques. May 2008.

[57] International Standardization Organization 2007. Draft ISO 31000, Risk Management Guidelines on Principles and Implementation of Risk Management. Final version to be issued in 2009.

[58] Joint Committee of Structural Safety (JCSS) 2008. Risk Assessment in Engineering, Principles, System Representation and Risk Criteria. JCSS, June 2008.

[59] Mundt, A.P. 2008. Dynamic risk management with Markov decision process. Universitätsverlag Karlsruhe, 2008.

[60] NASA 2002. Risk Management Procedural Requirements. NPR 8000.4, April 2002.

[61] NASA 2007. Exploration Systems, Risk Management Plan. August 2007.

[62] Oehmen, J. 2005. Approaches to Crisis Prevention in Lean Product Development by High Performance Teams and through Risk Management. Munich, September 2005.

[63] Oesterreichisches Normungsinstitut 2008. ONR 49000 Risikomanagement für Organisationen und Systeme. (in German).

[64] Rheinland-Pfalz 2008. SGU-Leitfaden. (in German).

[65] Rio Tinto 2007. Risk Policy and Standard. August 2007.

[66] Taleb, N. 2007 a. The Black Swan: The Impact of the Highly Improbable. Penguin, London.

[67] Taleb, N. 2007 b. Epistemology and risk management. Risk & Regulation Magazine, Summer 2007.

[68] Treasury Board of Canada 2001. Integrated Risk Management Framework. April 2001.

[69] Wirthin, R. 2006. Managing Risk and Uncertainty: Traditional Methods and the Lean Enterprise. MIT/LAI, Presentation April 18, 2006.

World Economic Forum (WEF) 2008. Global Risks 2008, A Global Risk Network Report. Cologny/Geneva, January 2008.

RELIABILITY AND AVAILABILITY OF A SHIPYARD SHIP-ROPE ELEVATOR IN VARIABLE OPERATION CONDITIONS

A. Blokus-Roszkowska
K. Kołowrocki
•
Gdynia Maritime University, Gdynia, Poland

e-mail: ablokus@am.gdynia.pl

ABSTRACT

In the paper the environment and infrastructure influence of the ship-rope elevator operating in Naval Shipyard in Gdynia on its operation processes is considered. The results are presented on the basis of a general model of technical systems operation processes related to their environment and infrastructure. The elevator operation process is described and its statistical identification is given. Next, the elevator is considered in varying in time operation conditions with different its components' reliability functions in different operation states. Finally, the reliability, risk and availability evaluation of the elevator in variable operation conditions is presented.

1 DESCRIPTION OF THE SHIP-ROPE ELEVATOR IN NAVAL SHIPYARD IN GDYNIA

Ship-rope elevators are used to dock and undock ships coming to shipyards for repairs. The elevator utilized in the Naval Shipyard in Gdynia, with the scheme presented in Figure 4, is composed of a steel platform-carriage placed in its syncline (hutch). The platform is moved vertically with 10 rope-hoisting winches fed by separate electric motors. Motors are equipped in ropes "Bridon" with the diameter 47 mm each rope having a maximum load of 300 tonnes. During ship docking the platform, with the ship settled in special supporting carriages on the platform, is raised to the wharf level (upper position). During undocking, the operation is reversed. While the ship is moving into or out of the syncline and while stopped in the upper position the platform is held on hooks and the loads in the ropes are relieved. Since the platform-carriage and electric motors are highly reliable in comparison to the ropes, which work in extremely aggressive conditions, in our further analysis we will discuss the reliability of the rope system only.

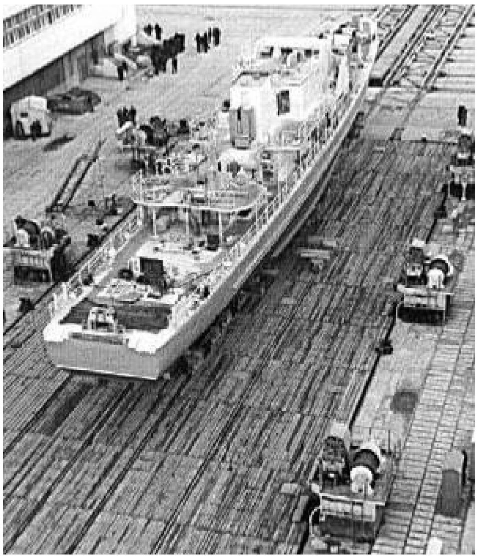


Figure 1. The ship-rope transportation system (upper position).

The system under consideration is composed of 10 ropes linked in series. Each of the ropes is composed of 22 strands: 10 outer and 12 inner.

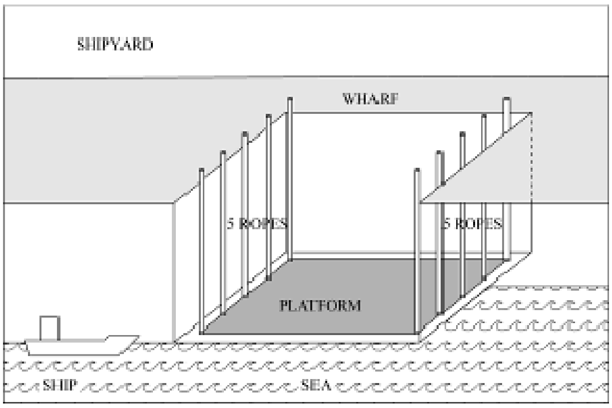


Figure 2. The scheme of the ship-rope elevator.

The assumption that ropes satisfy the technical conditions when at least one of its strands satisfies these conditions is not always true. In reality it is said that a rope is failed after some number of strands use. Therefore better, closer in reality approach to the system reliability evaluation is assumption that the ship-rope transportation system is "m out of l_n"-series system. Further we assume that m = 5.

2 OPERATION PROCESS AND ITS STATISTICAL IDENTIFICATION

Considering the tonnage of the docked and undocked ships by the rope elevator in Naval Shipyard in Gdynia we can divide the system's load, similarly as in the previous ships' transportation system, into six groups and due to fact that the rope elevator system depends mainly on the tonnage of docking ships we can distinguish the following (v = 6) operation states of the rope elevator system operation process:

- an operation state z_1 – without loading (the system is not working),
- an operation state z_2 – loading over 0 up to 500 tonnes,
- an operation state z_3 – loading over 500 up to 1000 tonnes,
- an operation state z_4 – loading over 1000 up to 1500 tonnes,
- an operation state z_5 – loading over 1500 up to 2000 tonnes,
- an operation state z_6 – loading over 2000 up to 2500 tonnes.

In all six operational states system has the same structure. There are 10 rope-hoisting winches equipped in identical ropes and each of the ropes is composed of 22 strands. We assume that the rope is "5 out of 22" system, so we consider the ship-rope elevator as a regular "5 out of 22"-series system.

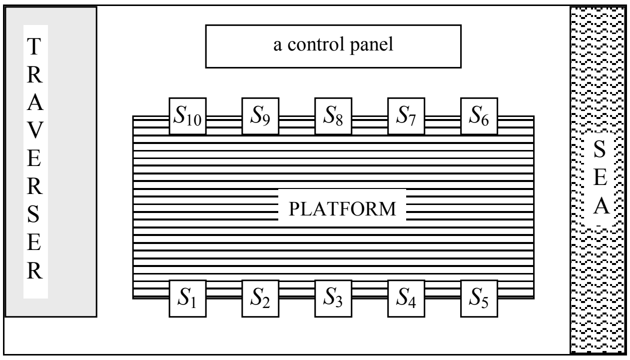


Figure 3. The scheme of the rope-hoisting winches placing.

On the basis of the statistical data coming from experts using the shipyard ship-rope elevator in Naval Shipyard in Gdynia (Blokus-Roszkowska et al. 2009) the transition probabilities p_{bl} from the operation state z_b into the operation state z_l, $b,l = 1,...,6, b \neq l,$ were evaluated. Their approximate evaluations are given in the matrix below.

$$[p_{bl}] = \begin{bmatrix} 0 & 0.2931 & 0.2931 & 0.2414 & 0.1207 & 0.0517 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

On the basis of the realizations of the operation process $Z(t)$ conditional sojourn times θ_{bl}, $b,l = 1,2,...,6,\ b \neq l,$ in the state z_b while the next transition is to the state z_l, given in (Blokus-Roszkowska et al. 2009), there were formulated hypotheses about the distributions of the conditional sojourn times θ_{bl}. These hypotheses allows us to estimate the conditional mean values $M_{bl} = E[\theta_{bl}],\ b,l = 1,2,...,6,\ b \neq l,$ of the lifetimes in the particular operation states:

$$M_{12} = 3057.06,\ M_{13} = 3319.12,\ M_{14} = 10406.07,\ M_{15} = 4687.86,\ M_{16} = 5540.00,$$
$$M_{21} = 58.00,\ M_{31} = 37.18,\ M_{41} = 183.21,\ M_{51} = 124.50,\ M_{61} = 270.00.$$

Hence, by (Kołowrocki & Soszyńska 2008), the unconditional mean sojourn times in the particular operation states are determined from the formula

$$M_b = E[\theta_b] = \sum_{l=1}^{6} p_{bl} M_{bl},\ b = 1,...,6,$$

and takes values:

$$M_1 \cong 5233.13,\ M_2 \cong 58.00,\ M_3 \cong 37.18,\ M_4 \cong 183.21,\ M_5 \cong 124.50,\ M_6 \cong 270.00.$$

Since from the system of equations below (Kołowrocki & Soszyńska 2008, Soszyńska 2006)

$$\begin{cases} [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6] = [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6][p_{bl}] \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 = 1, \end{cases}$$

we get

$$\pi_1 = 0.5,\ \pi_2 = 0.14655,\ \pi_3 = 0.14655,\ \pi_4 = 0.1207,\ \pi_5 = 0.06035,\ \pi_6 = 0.02585.$$

Then the limit values of the transient probabilities $p_b(t)$ at the operational states z_b, according to results given in (Blokus-Roszkowska et al. 2008b, Grabski 2002), are equal to:

$$p_1 = 0.9810,\ p_2 = 0.0032,\ p_3 = 0.0021,\ p_4 = 0.0083,\ p_5 = 0.0028,\ p_6 = 0.0026. \quad (1)$$

3 RELIABILITY OF THE SHIPYARD SHIP-ROPE ELEVATOR

According to rope reliability data given in their technical certificates and experts' opinions based on the nature of strand failures the following reliability states have been distinguished:

– a reliability state 3 – a strand is new, without any defects,

– a reliability state 2 – the number of broken wires in the strand is greater than 0% and less than 25% of all its wires, or corrosion of wires is greater than 0% and less than 25%,

– a reliability state 1 – the number of broken wires in the strand is greater than or equal to 25% and less than 50% of all its wires, or corrosion of wires is greater than or equal to 25% and less than 50%,

– a reliability state 0 – otherwise (a strand is failed).

We consider the strands as basic components of the system. The system of ropes is in the reliability state subset $\{1,2,3\}, \{2,3\}, \{3\}$, when all of its ropes are in this state subset and each of the ropes is in the reliability state subset $\{1,2,3\}, \{2,3\}, \{3\}$, if at least 5 of 22 strands are in this state subset. Thus, we conclude that the ship-rope elevator is a regular 4-states “5 out of 22”-series system composed of k_n = 10 series-linked subsystems (ropes) with l_n = 22 parallel-linked components (strands).

Then, taking into account above remarks, we obtain the reliability function of the considered ship-rope elevator given by the vector

$$\overline{\boldsymbol{R}}(t,\cdot) = [1, \overline{\boldsymbol{R}}(t,1), \overline{\boldsymbol{R}}(t,2), \overline{\boldsymbol{R}}(t,3)] = [1, \overline{\boldsymbol{R}}^{(5)}_{10,22}(t,1), \overline{\boldsymbol{R}}^{(5)}_{10,22}(t,2), \overline{\boldsymbol{R}}^{(5)}_{10,22}(t,3)],\ t \in <0,\infty). \quad (2)$$

We assume strands as a basic components of a system with the reliability functions given by the vector

$$R(t,\cdot) = [R(t,0), R(t,1), R(t,2), R(t,3)],\ t \in <0,\infty),$$

with the co-ordinates

$$R(t,u) = P(S(t) \ge u \mid S(0) = 3) = P(T(u) > t),\ t \in <0,\infty),\ u = 0,1,2,3,\ R(t,0) = 1.$$

$T(u)$ is independent random variable representing the lifetime of system components in the reliability state subset $\{u, u+1, ..., 3\}$, while they were at the reliability state 3 at the moment $t = 0$ and $S(t)$ are components’ reliability states at the moment t, $t \in <0,\infty)$.

Moreover we assume that the components of the ship-rope elevator i.e. strands have multi-state reliability functions

$$R^{(b)}(t,\cdot) = [1, R^{(b)}(t,1), R^{(b)}(t,2), R^{(b)}(t,3)],$$

with exponential co-ordinates $R^{(b)}(t,1)$, $R^{(b)}(t,2)$ and $R^{(b)}(t,3)$ different in various operation states z_b, $b = 1,2,...,6$.

At the system operational state z_1 the strands in the ropes have following conditional reliability functions co-ordinates:

$$R^{(1)}(t,1) = \exp[-0.1613t],\ R^{(1)}(t,2) = \exp[-0.2041t],\ R^{(1)}(t,3) = \exp[-0.2326t],\ t \ge 0.$$

Thus the conditional multi-state reliability function of the ship-rope elevator at the operational state z_1 is given by:

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(1)} = [1, [\overline{\boldsymbol{R}}(t,1)]^{(1)}, [\overline{\boldsymbol{R}}(t,2)]^{(1)}, [\overline{\boldsymbol{R}}(t,3)]^{(1)}],$$

where

$$[\overline{\boldsymbol{R}}(t,1)]^{(1)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,1)]^{(1)} = [1 - \sum_{i=1}^{4}\binom{22}{i}\exp[-0.1613it][1-\exp[-0.1613t]]^{22-i}]^{10}, \quad (3)$$

$$[\overline{\boldsymbol{R}}(t,2)]^{(1)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,2)]^{(1)} = [1 - \sum_{i=1}^{4}\binom{22}{i}\exp[-0.2041it][1-\exp[-0.2041t]]^{22-i}]^{10}, \quad (4)$$

$$[\overline{\boldsymbol{R}}(t,3)]^{(1)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,3)]^{(1)} = [1 - \sum_{i=1}^{4}\binom{22}{i}\exp[-0.2326it][1-\exp[-0.2326t]]^{22-i}]^{10}, \quad (5)$$

for $t \ge 0$.

The expected values and standard deviations of the ship-rope elevator conditional lifetimes in the reliability state subsets calculated from the above result given by (3)-(5), according to (Blokus-Roszkowska et al. 2008a, Kołowrocki 2004), at the operation state z_1, in years, are respectively given by:

$$\mu_1(1) \cong 6.4415, \mu_1(2) \cong 5.0907, \mu_1(3) \cong 4.4669, \tag{6}$$

$$\sigma_1(1) \cong 1.0563, \sigma_1(2) \cong 0.8345, \sigma_1(3) \cong 0.7323, \tag{7}$$

and further, using (6), from (Kołowrocki 2004) it follows that the conditional lifetimes in the particular reliability states at the operation state z_1, in years, are:

$$\bar{\mu}_1(1) \cong 1.3508,\ \bar{\mu}_1(2) \cong 0.6239,\ \bar{\mu}_1(3) \cong 4.4669.$$

At the system operational state z_2 the strands in the ropes have following conditional reliability functions co-ordinates:

$$R^{(2)}(t,1) = \exp[-0.2041t],\ R^{(2)}(t,2) = \exp[-0.2564t],\ R^{(2)}(t,3) = \exp[-0.2941t],\ t \geq 0.$$

Thus the conditional multi-state reliability function of the ship-rope elevator at the operational state z_2 is given by:

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(2)} = [1, [\overline{\boldsymbol{R}}(t,1)]^{(2)}, [\overline{\boldsymbol{R}}(t,2)]^{(2)}, [\overline{\boldsymbol{R}}(t,3)]^{(2)}],$$

where

$$[\overline{\boldsymbol{R}}(t,1)]^{(2)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,1)]^{(2)} = [1 - \sum_{i=1}^{4}\binom{22}{i}\exp[-0.2041it][1-\exp[-0.2041t]]^{22-i}]^{10}, \tag{8}$$

$$[\overline{\boldsymbol{R}}(t,2)]^{(2)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,2)]^{(2)} = [1 - \sum_{i=1}^{4}\binom{22}{i}\exp[-0.2564it][1-\exp[-0.2564t]]^{22-i}]^{10}, \tag{9}$$

$$[\overline{\boldsymbol{R}}(t,3)]^{(2)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,3)]^{(2)} = [1 - \sum_{i=1}^{4}\binom{22}{i}\exp[-0.2941it][1-\exp[-0.2941t]]^{22-i}]^{10}, \tag{10}$$

for $t \geq 0$.

The expected values and standard deviations of the ship-rope elevator conditional lifetimes in the reliability state subsets calculated from the above result given by (8)-(10), according to (Kołowrocki 2004) at the operation state z_2 are respectively given by:

$$\mu_2(1) \cong 5.0907, \mu_2(2) \cong 4.0523, \mu_2(3) \cong 3.5335, \tag{11}$$

$$\sigma_2(1) \cong 0.8345, \sigma_2(2) \cong 0.6639, \sigma_2(3) \cong 0.5744, \tag{12}$$

and further, using (11), from (Kołowrocki 2004) it follows that the conditional lifetimes in the particular reliability states at the operation state z_2 are:

$$\bar{\mu}_2(1) \cong 1.0384, \bar{\mu}_2(2) \cong 0.5188, \bar{\mu}_2(3) \cong 3.5335.$$

At the system operational state z_3 the strands in the ropes have following conditional reliability functions co-ordinates:

$$R^{(3)}(t,1) = \exp[-0.2222t],\ R^{(3)}(t,2) = \exp[-0.2857t],\ R^{(3)}(t,3) = \exp[-0.3226t],\ t \geq 0.$$

Thus the conditional multi-state reliability function of the ship-rope elevator at the operational state z_3 is given by:

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(3)} = [1, [\overline{\boldsymbol{R}}(t,1)]^{(3)}, [\overline{\boldsymbol{R}}(t,2)]^{(3)}, [\overline{\boldsymbol{R}}(t,3)]^{(3)}],$$

where

$$[\overline{\boldsymbol{R}}(t,1)]^{(3)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,1)]^{(3)} = [1 - \sum_{i=1}^{4}\binom{22}{i}\exp[-0.2222it][1-\exp[-0.2222t]]^{22-i}]^{10}, \tag{13}$$

$$[\overline{\boldsymbol{R}}(t,2)]^{(3)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,2)]^{(3)} = [1 - \sum_{i=1}^{4}\binom{22}{i}\exp[-0.2857it][1-\exp[-0.2857t]]^{22-i}]^{10}, \tag{14}$$

$$[\overline{\boldsymbol{R}}(t,3)]^{(3)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,3)]^{(3)} = [1 - \sum_{i=1}^{4}\binom{22}{i}\exp[-0.3226it][1-\exp[-0.3226t]]^{22-i}]^{10}, \tag{15}$$

for $t \geq 0$.

The expected values and standard deviations of the ship-rope elevator conditional lifetimes in the reliability state subsets calculated from the above result given by (13)-(15), according to results given in (Kołowrocki 2004), at the operation state z_3, in years are equal to:

$$\mu_3(1) \cong 4.6760, \mu_3(2) \cong 3.6367, \mu_3(3) \cong 3.2207, \tag{16}$$

$$\sigma_3(1) \cong 0.7665, \sigma_3(2) \cong 0.5956, \sigma_3(3) \cong 0.5273, \tag{17}$$

and further, from (16) and (Kołowrocki 2004) it follows that the conditional lifetimes in the particular reliability states at the operation state z_3 are:

$$\bar{\mu}_3(1) \cong 1.0393, \bar{\mu}_3(2) \cong 0.4160, \bar{\mu}_3(3) \cong 3.2207.$$

At the system operational state z_4 the strands in the ropes have following conditional reliability functions co-ordinates:

$$R^{(4)}(t,1) = \exp[-0.2702t],\ R^{(4)}(t,2) = \exp[-0.3508t],\ R^{(4)}(t,3) = \exp[-0.4167t],\ t \geq 0.$$

Thus the conditional multi-state reliability function of the ship-rope elevator at the operational state z_4 is given by:

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(4)} = [1, [\overline{\boldsymbol{R}}(t,1)]^{(4)},\ [\overline{\boldsymbol{R}}(t,2)]^{(4)}, [\overline{\boldsymbol{R}}(t,3)]^{(4)}],$$

where

$$[\overline{\boldsymbol{R}}(t,1)]^{(4)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,1)]^{(4)} = [1 - \sum_{i=1}^{4} \binom{22}{i} \exp[-0.2702it][1-\exp[-0.2702t]]^{22-i}]^{10}, \quad (18)$$

$$[\overline{\boldsymbol{R}}(t,2)]^{(4)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,2)]^{(4)} = [1 - \sum_{i=1}^{4} \binom{22}{i} \exp[-0.3508it][1-\exp[-0.3508t]]^{22-i}]^{10}, \quad (19)$$

$$[\overline{\boldsymbol{R}}(t,3)]^{(4)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,3)]^{(4)} = [1 - \sum_{i=1}^{4} \binom{22}{i} \exp[-0.4167it][1-\exp[-0.4167t]]^{22-i}]^{10}, \quad (20)$$

for $t \geq 0$.

The expected values and standard deviations of the ship-rope elevator conditional lifetimes in the reliability state subsets calculated from the above result given by (18)-(20), according to results in (Kołowrocki 2004), at the operation state z_4 are respectively given by:

$$\mu_4(1) \cong 3.8453, \mu_4(2) \cong 2.9618, \mu_4(3) \cong 2.4934, \quad (21)$$

$$\sigma_4(1) \cong 0.6301, \sigma_4(2) \cong 0.4846, \sigma_4(3) \cong 0.4074, \quad (22)$$

and further, using (21), from (Kołowrocki 2004) it follows that the conditional lifetimes in the particular reliability states at the operation state z_4 are:

$$\bar{\mu}_4(1) \cong 0.8835, \bar{\mu}_4(2) \cong 0.4684, \bar{\mu}_4(3) \cong 2.4934.$$

At the system operational state z_5 the strands in the ropes have following conditional reliability functions co-ordinates:

$$R^{(5)}(t,1) = \exp[-0.3333t],\ R^{(5)}(t,2) = \exp[-0.4762t],\ R^{(5)}(t,3) = \exp[-0.5882t],\ t \geq 0.$$

Thus the conditional multi-state reliability function of the ship-rope elevator at the operational state z_5 is given by:

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(5)} = [1, [\overline{\boldsymbol{R}}(t,1)]^{(5)},\ [\overline{\boldsymbol{R}}(t,2)]^{(5)}, [\overline{\boldsymbol{R}}(t,3)]^{(5)}],$$

where

$$[\overline{\boldsymbol{R}}(t,1)]^{(5)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,1)]^{(5)} = [1 - \sum_{i=1}^{4} \binom{22}{i} \exp[-0.3333it][1-\exp[-0.3333t]]^{22-i}]^{10}, \quad (23)$$

$$[\overline{\boldsymbol{R}}(t,2)]^{(5)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,2)]^{(5)} = [1 - \sum_{i=1}^{4} \binom{22}{i} \exp[-0.4762it][1-\exp[-0.4762t]]^{22-i}]^{10}, \quad (24)$$

$$[\overline{\boldsymbol{R}}(t,3)]^{(5)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,3)]^{(5)} = [1 - \sum_{i=1}^{4} \binom{22}{i} \exp[-0.5882it][1-\exp[-0.5882t]]^{22-i}]^{10}, \quad (25)$$

for $t \geq 0$.

The expected values and standard deviations of the ship-rope elevator conditional lifetimes in the reliability state subsets from the above result given by (31)-(33), and from (Kołowrocki 2004) at the operation state z_5 are respectively given in years by:

$$\mu_5(1) \cong 3.1173, \mu_5(2) \cong 2.1819, \mu_5(3) \cong 1.7664, \quad (26)$$

$$\sigma_5(1) \cong 0.5103, \sigma_5(2) \cong 0.3574, \sigma_5(3) \cong 0.2894, \quad (27)$$

and further, using (26), from (Kołowrocki 2004) it follows that the conditional lifetimes in the particular reliability states at the operation state z_5 are:

$$\bar{\mu}_5(1) \cong 0.9354, \bar{\mu}_5(2) \cong 0.4155, \bar{\mu}_5(3) \cong 1.7664.$$

At the system operational state z_6 the strands in the ropes have following conditional reliability functions co-ordinates:

$$R^{(6)}(t,1) = \exp[-0.4348t],\ R^{(6)}(t,2) = \exp[-0.7143t],\ R^{(6)}(t,3) = \exp[-0.9091t],\ \ t \ge 0.$$

Thus the conditional multi-state reliability function of the ship-rope elevator at the operational state z_6 is given by:

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(6)} = [1, [\overline{\boldsymbol{R}}(t,1)]^{(6)},\ [\overline{\boldsymbol{R}}(t,2)]^{(6)}, [\overline{\boldsymbol{R}}(t,3)]^{(6)}],$$

where

$$[\overline{\boldsymbol{R}}(t,1)]^{(6)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,1)]^{(6)} = [1 - \sum_{i=1}^{4}\binom{22}{i}\exp[-0.4348it][1-\exp[-0.4348t]]^{22-i}]^{10}, \tag{28}$$

$$[\overline{\boldsymbol{R}}(t,2)]^{(6)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,2)]^{(6)} = [1 - \sum_{i=1}^{4}\binom{22}{i}\exp[-0.7143it][1-\exp[-0.7143t]]^{22-i}]^{10}, \tag{29}$$

$$[\overline{\boldsymbol{R}}(t,3)]^{(6)} = [\overline{\boldsymbol{R}}^{(5)}_{10,22}(t,3)]^{(6)} = [1 - \sum_{i=1}^{4}\binom{22}{i}\exp[-0.9091it][1-\exp[-0.9091t]]^{22-i}]^{10}, \tag{30}$$

for $t \ge 0$.

The expected values and standard deviations of the ship-rope elevator conditional lifetimes in the reliability state subsets calculated from the above result given by (28)-(30), and from (Kołowrocki 2004) at the operation state z_6 are respectively given in years by:

$$\mu_6(1) \cong 2.3896, \mu_6(2) \cong 1.4546, \mu_6(3) \cong 1.1429, \tag{31}$$

$$\sigma_6(1) \cong 0.3918, \sigma_6(2) \cong 0.2378, \sigma_6(3) \cong 0.1865, \tag{32}$$

and further, from (31) and (Kołowrocki 2004) it follows that the conditional lifetimes in the particular reliability states at the operation state z_6 in years are equal to:

$$\bar{\mu}_6(1) \cong 0.9350, \bar{\mu}_6(2) \cong 0.3117, \bar{\mu}_6(3) \cong 1.1429.$$

In the case when the operation time is large enough its unconditional multi-state reliability function of the ship-rope elevator is given by the vector

$$\overline{\boldsymbol{R}}(t,\cdot) = [1, \overline{\boldsymbol{R}}(t,1), \overline{\boldsymbol{R}}(t,2),\ \overline{\boldsymbol{R}}(t,3)],\ \ t \in < 0, \infty),$$

where according to (Blokus-Roszkowska et al. 2008b, Soszyńska 2006), the vector co-ordinates are given respectively by:

$$\overline{\boldsymbol{R}}(t,u) = \sum_{i=1}^{6} p_i [\overline{\boldsymbol{R}}(t,u)]^{(i)},\ t \ge 0,\ u = 1,2,3, \tag{33}$$

where $[\overline{\boldsymbol{R}}(t,u)]^{(i)}$, $i = 1,\ldots,6$, are given by (3)-(5), (8)-(10), (13)-(15), (18)-(20), (23)-(25), (28)-(30).

The mean values and the standard deviations of the ship-rope elevator unconditional lifetimes in the reliability state subsets, according to (Kołowrocki & Soszyńska 2008, Soszyńska 2006) and after considering (6)-(7), (11)-(12), (16)-(17), (21)-(22), (26)-(27), (31)-(32) and (1), respectively are:

$$\mu(1) = \sum_{i=1}^{6} p_i\,\mu_i\,(1) \cong 6.3887,\ \sigma(1) \cong 1.1336, \tag{34}$$

$$\mu(2) = \sum_{i=1}^{6} p_i\,\mu_i\,(2) \cong 5.0463,\ \sigma(2) \cong 0.9041, \tag{35}$$

$$\mu(3) = \sum_{i=1}^{6} p_i\,\mu_i\,(3) \cong 4.4266,\ \sigma(3) \cong 0.7964\,. \tag{36}$$

Next, the unconditional mean values of the ship-rope elevator lifetimes in the particular reliability states, by (Kołowrocki 2004) and considering (34)-(36), in years are:

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 1.3424,\ \bar{\mu}(2) = \mu(2) - \mu(3) = 0.6197,\ \bar{\mu}(3) = \mu(3) = 4.4266.$$

If the critical reliability state is r = 2, then according to (Blokus-Roszkowska et al. 2008a), the system risk function takes the form

$$\boldsymbol{r}(t) = 1 - \overline{\boldsymbol{R}}(t,2) = 1 - \sum_{i=1}^{6} p_i [\overline{\boldsymbol{R}}(t,2)]^{(i)},\ t \ge 0,$$

where $\overline{\boldsymbol{R}}(t,2)$ is the unconditional reliability function of the ship-rope elevator at the critical state and $[\overline{\boldsymbol{R}}(t,2)]^{(i)}$, $i=1,\ldots,6$, are given by (4), (9), (14), (19), (24), (29).
Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (Blokus-Roszkowska et al. 2008a), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 3.577 \text{ years} \cong 3 \text{ years 205 days.}$$

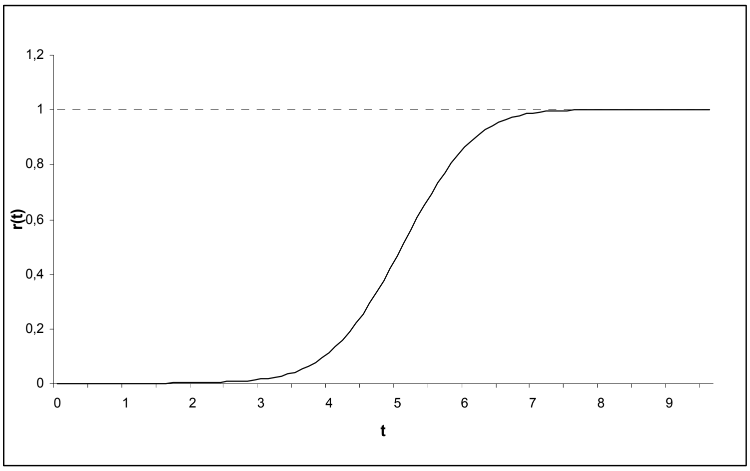


Figure 4. The graph of the ship-rope elevator risk function $r(t)$.

4 AVAILABILITY OF THE SHIPYARD SHIP-ROPE ELEVATOR

In this point the asymptotic evaluation of the basic reliability and availability characteristics of renewal systems with non-ignored time of renovation are determined in an example of the shipyard ship-rope elevator. The theoretical results of multi-state systems availability analysis can be found in (Blokus 2006, Blokus-Roszkowska et al. 2008a).

Assuming that the ship-rope elevator is repaired after its failure and that the time of the system renovation is not ignored and it has the mean value $\mu_0(2) = 0.0014 \cong 12$ hours and the standard deviation $\sigma_0(2) = 0.0002 \cong 2$ hours, applying results from (Blokus-Roszkowska et al. 2008a), we obtain the following results:

– the distribution function of the time $\overline{\overline{S}}_N(2)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution $N(5.0477N, 0.9041\sqrt{N})$, i.e.,

$$\overline{\overline{F}}^{(N)}(t,2) = P(\overline{\overline{S}}_N(2) < t) \cong F_{N(0,1)}\left(\frac{t - 5.0477N}{0.9041\sqrt{N}}\right), \quad t \in (-\infty,\infty), \quad N = 1,2,\ldots,$$

– the expected value and the variance of the time $\overline{\overline{S}}_N(2)$ until the *Nth* system's renovation take respectively forms

$$E[\overline{\overline{S}}_N(2)] \cong 5.0477N, \quad D[\overline{\overline{S}}_N(2)] \cong 0.8174N,$$

– the distribution function of the time $\overline{S}_N(2)$ until the *Nth* exceeding the reliability critical state 2 of this system takes form

$$\overline{F}^{(N)}(t,2) = P(\overline{S}_N(2) < t) = F_{N(0,1)}\left(\frac{t - 5.0477N + 0.0014}{0.9041\sqrt{N}}\right), \quad t \in (-\infty,\infty), \quad N = 1,2,\ldots,$$

– the expected value and the variance of the time $\overline{S}_N(2)$ until the *Nth* exceeding the reliability critical state 2 of this system take respectively forms

$$E[\overline{S}_N(2)] \cong 5.0463N + 0.0014(N-1), \quad D[\overline{S}_N(2)] \cong 0.8174N$$

– the distribution of the number $\overline{\overline{N}}(t,2)$ of system's renovations up to the moment $t, t \geq 0,$ is of the form

$$P(\overline{\overline{N}}(t,2) = N) \cong F_{N(0,1)}(\frac{5.0477N - t}{0.4024\sqrt{t}}) - F_{N(0,1)}(\frac{5.0477(N+1) - t}{0.4024\sqrt{t}}),\ N = 1,2,...\ ,$$

– the expected value and the variance of the number $\overline{\overline{N}}(t,2)$ of system's renovations up to the moment $t, t \geq 0,$ take respectively forms

$$\overline{\overline{H}}(t,2) \cong 0.1981t,\quad \overline{\overline{D}}(t,2) \cong 0.0064t,$$

– the distribution of the number $\overline{N}(t,2)$ of exceeding the reliability critical state 2 of this system up to the moment $t, t \geq 0,$ is of the form

$$P(\overline{N}(t,2) = N) \cong F_{N(0,1)}(\frac{5.0477N - t - 0.0014}{0.4024\sqrt{t + 0.0014}}) - F_{N(0,1)}(\frac{5.0477(N+1) - t - 00014}{0.4024\sqrt{t + 0.0014}}),\ N = 1,2,...\ ,$$

– the expected value and the variance of the number $\overline{N}(t,2)$ of exceeding the reliability critical state 2 of this system up to the moment $t, t \geq 0,$ are respectively given by

$$\overline{H}(t,2) \cong 0.1981t + 0.0003,\quad \overline{D}(t,1) \cong 0.0064(t + 0.0014),$$

– the availability coefficient of the system at the moment t is given by the formula

$$K(t,2) \cong 0.9997\ ,\ t \geq 0,$$

– the availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ is given by the formula

$$K(t,\tau,2) \cong 0.1981\int_{\tau}^{\infty}\overline{\boldsymbol{R}}(t,2)dt,\ t \geq 0,\ \tau > 0,$$

where the reliability function of a system at the critical state $\overline{\boldsymbol{R}}(t,2)$ is given by the formula (33).

5 CONCLUSIONS

In the paper an analytical model of port transportation systems environment and infrastructure influence on their operation process is presented. The theoretical results of reliability, risk and availability evaluation of industrial systems in variable operation conditions are applied to the shipyard ship-rope elevator in Naval Shipyard in Gdynia. These results may be considered as an illustration of the proposed methods possibilities of application in rope transportation systems reliability analysis. Other technical systems reliability evaluation related to their operation process are presented for example in (Blokus et al. 2005, Soszyńska 2006). The obtained evaluations may be discussed as an example in transportation systems reliability characteristics evaluation, especially during the design and while planning and improving its operation process safety and effectiveness.

ACKNOWLEDGEMENTS

The paper describes the work in the Poland-Singapore Joint Research Project titled "Safety and Reliability of Complex Industrial Systems and Processes" supported by grants from the Poland's Ministry of Science and Higher Education (MSHE grant No. 63/N-Singapore/2007/0) and the Agency for Science, Technology and Research of Singapore (A*STAR SERC grant No. 072 1340050).

REFERENCES

Blokus, A. 2006. Reliability and availability evaluation of large renewal systems. *International Journal of Materials & Structural Reliability* 4(1): 39-52.

Blokus, A., Kołowrocki, K. & Kwiatuszewska-Sarnecka, B. 2005. Reliability and availability of large "m out of n"-series systems. *Proceedings Qualita 2005 Conference*: 445-453.

Blokus, A., Kołowrocki, K. & Soszyńska, J. 2005. Reliability of port transportation systems related to their operation processes. *Proceedings 12-th International Congress of the International Maritime Association of the Mediterranean - IMAM 2005. Maritime Transportation and Exploitation of Ocean and Coastal Resources*: 1487-1495.

Blokus-Roszkowska, A, Guze, S., Kołowrocki, K., Kwiatuszewska-Sarnecka, B., Soszyńska, J. 2008a. General methods for safety, reliability and availability evaluation of technical systems. WP 3 – Task 3.1 – English – 31.05.2008. *Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.*

Blokus-Roszkowska, A., Guze, S., Kołowrocki, K., Kwiatuszewska-Sarnecka, B., Soszyńska, J. 2008b. Models of safety, reliability and availability evaluation of complex technical systems related to their operation processes. WP 4 – Task 4.1 – English – 31.05.2008. *Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.*

Blokus-Roszkowska, A., Kołowrocki, K., Namysłowski, M. 2009. Data mining for identification and prediction of safety and reliability characteristics of complex industrial systems and processes. WP 6 – Task 6.2 – English – 31.01.2009. Sub-Task 6.2.3 – Appendix 3A. Preliminary statistical data collection of a shipyard ship-rope elevator operation process and its preliminary statistical identification. *Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.*

Grabski, F. 2002. *Semi-Markov Models of Systems Reliability and Operations*. Warsaw: Systems Research Institute, Polish Academy of Sciences.

Kołowrocki, K. 2004. *Reliability of Large Systems*. Amsterdam - Boston - Heidelberg - London - New York - Oxford - Paris - San Diego - San Francisco - Singapore - Sydney – Tokyo: Elsevier.

Kołowrocki, K. & Soszyńska, J. 2006. Reliability and availability of complex systems. *Quality and Reliability Engineering International* 22(1): 79-99.

Kołowrocki, K. & Soszyńska, J. 2008. A general model of technical systems operation processes related to their environment and infrastructure. WP 2 – Task 2.1 – English – 31.05.2008. *Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.*

Soszyńska, J. 2006. Reliability evaluation of a port oil transportation system in variable operation conditions. *International Journal of Pressure Vessels and Piping* 83(4): 304-310.

A MONTE CARLO-BASED TECHNIQUE FOR ESTIMATING THE OPERATION MODES OF HYBRID DYNAMIC SYSTEMS

F. Cadini, D. Avram, E. Zio.

•

Politecnico di Milano – Dipartimento di Energia
Milan, Italy

e-mail: francesco.cadini@polimi.it

ABSTRACT

Many real systems are characterized by a hybrid dynamics of transitions among discrete modes of operation, each one giving rise to a specific continuous dynamics of evolution. The estimation of the state of these hybrid dynamic systems is difficult because it requires keeping track of the transitions among the multiple modes of system dynamics corresponding to the different modes of operation. A Monte Carlo-based estimation method is here illustrated through an application to a case study of literature.

1 INTRODUCTION

Diagnosis and prognosis of system faults rely on the knowledge or anticipation of the system state to provide advanced warning and lead time for preparing the necessary corrective actions to maintain the system in safe operation.

The related state estimation task becomes quite challenging for systems with a hybrid dynamic behavior characterized by continuous states and discrete modes. Sudden transitions of the discrete modes, often autonomously triggered by the continuous dynamics, affect the system evolution and a large computational effort is required to keep track of the multiple models of the discrete system modes and the autonomous transitions between them (Koutsoukos et al. 2002). Since the dynamic states cannot be directly observed, the problem becomes that of inferring the system state from related measured parameters.

The soundest model-based approaches to the estimation of the state of a dynamic system or component build a posterior probability distribution of the unknown states by combining the probability distribution assigned a priori to the possible states with the likelihood of the observations of the measurements actually collected (Doucet 1998, Doucet et al. 2001). In this Bayesian setting, the estimation method most frequently used in practice is the Kalman filter, which is optimal for linear state space models and independent, additive Gaussian noises. In practice, however, the dynamic evolution of many systems and components is non-linear and the associated noises are non-Gaussian (Kitagawa 1987). In these cases, one may resort to Monte Carlo sampling methods also known as particle filtering methods, which are capable of approximating the continuous distributions of interest by a discrete set of weighed ‘particles’ representing random trajectories of system evolution in the state space and whose weights are estimates of the probabilities of the trajectories (Doucet et al. 2000, Djuric et al. 2003, Cadini et al. 2009a, b).

In this paper, particle filtering is applied for the estimation of the state of a hybrid system of literature often taken as a benchmark for dynamic reliability estimation and fault diagnosis/prognosis methods (Aldemir et al. 1994, Marseguerra et al. 1996, Labeau et al. 1998, Wang et al. 2002). The system consists of a tank filled with a liquid whose level is autonomously maintained between two thresholds by actuators driving three filling and emptying flows triggered by the actual liquid level. The actuators discrete mode is estimated by the particle filter on the basis of noisy level and temperature measurements.

2 PARTICLE FILTERING FOR OPERATION MODE ESTIMATION

2.1 General model-based framework for state estimation

Let us consider a continuous system whose evolution can be described by:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \boldsymbol{\omega}) \tag{1}$$

where $\mathbf{x}$ is the system state vector, $\mathbf{f} : R^{n_x} \times R^{n_\omega} \to R^{n_x}$ is possibly non-linear and $\boldsymbol{\omega}$ is an independent identically distributed (i.i.d.) state noise vector of known distribution.

The state $\mathbf{x}$ cannot in general be directly observed; rather, information about $\mathbf{x}$ can be inferred from the observation of a related variable $\mathbf{z}$ whose relation to the state $\mathbf{x}$ is described in general terms by the equation:

$$\mathbf{z} = \mathbf{h}(\mathbf{x}, \boldsymbol{\upsilon}) \tag{2}$$

where $\mathbf{h} : R^{n_x} \times R^{n_\omega} \to R^{n_x}$ is possibly non-linear and $\boldsymbol{\upsilon}$ is an i.i.d. measurement noise vector sequence of known distribution. The measurements $\mathbf{z}$ are, thus, assumed to be conditionally independent given the state process $\mathbf{x}$.

The practical implementation of computational tools for state estimation requires that the continuous system dynamics be discretized appropriately. Regardless of the discretisation method adopted, the system state dynamics can be represented by an unobserved (hidden) Markov process of order one:

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \boldsymbol{\omega}_{k-1}) \tag{3}$$

where $\mathbf{f}_k : R^{n_x} \times R^{n_\omega} \to R^{n_x}$ is possibly non-linear and $\{\boldsymbol{\omega}_k,\ k \in \mathrm{N}\}$ is an independent identically distributed (i.i.d.) state noise vector sequence of known distribution.

The transition probability distribution $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ is defined by the system Equation (3) and the known distribution of the noise vector $\boldsymbol{\omega}_k$. The initial distribution of the system state $p(\mathbf{x}_0)$ is assumed known.

A sequence of measurements $\{\mathbf{z}_k,\ k \in \mathrm{N}\}$ is assumed to be collected at the successive time steps t_k. The sequence of measurement values is described by the measurement (observation) equation:

$$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \boldsymbol{\upsilon}_k) \tag{4}$$

where $\mathbf{h}_k : R^{n_x} \times R^{n_\omega} \to R^{n_x}$ is possibly non-linear and $\{\boldsymbol{\upsilon}_k,\ k \in \mathrm{N}\}$ is an i.i.d. measurement noise vector sequence of known distribution. The measurements $\{\mathbf{z}_k,\ k \in \mathrm{N}\}$ are, thus, assumed to be conditionally independent given the state process $\{\mathbf{x}_k,\ k \in \mathrm{N}\}$.

Within a Bayesian framework, the filtered posterior distribution p($\mathbf{x}_k|\mathbf{z}_{0:k}$) can be recursively computed in two stages: prediction and update (Doucet 1998, Arulampalam et al. 2002). Given the probability distribution $p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1})$ at time k–1, the prediction stage involves using the system model (3) to obtain the prior probability distribution of the system state $\mathbf{x}_k$ at time k via the Chapman-Kolmogorov equation:

$$p(\mathbf{x}_k|\mathbf{z}_{0:k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1})p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1})d\mathbf{x}_{k-1} = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1})d\mathbf{x}_{k-1} \tag{5}$$

where the Markovian assumption underpinning the system model (3) has been used.

At time k, a new measurement $\mathbf{z}_k$ is collected and used to update the prior distribution via Bayes rule, so as to obtain the required posterior distribution of the current state $\mathbf{x}_k$ (Arulampalam et al. 2002):

$$p(\mathbf{x}_k|\mathbf{z}_{0:k}) = \frac{p(\mathbf{x}_k|\mathbf{z}_{0:k-1})p(\mathbf{z}_k|\mathbf{x}_k)}{p(\mathbf{z}_k|\mathbf{z}_{0:k-1})} \tag{6}$$

where the normalizing constant is

$$p(\mathbf{z}_k|\mathbf{z}_{0:k-1}) = \int p(\mathbf{x}_k|\mathbf{z}_{0:k-1})p(\mathbf{z}_k|\mathbf{x}_k)d\mathbf{x}_k \tag{7}$$

The recurrence relations (5) and (6) form the basis for the exact Bayesian solution. Unfortunately, except for a few cases, including linear Gaussian state space models (Kalman filter) and hidden finite-state space Markov chains (Wohnam filter), it is not possible to evaluate analytically these distributions, since they require the evaluation of complex high-dimensional integrals.

One way to overcome this problem is to resort to Monte Carlo sampling or PF methods (Pulkkinen 1991, Doucet et al. 2000, Doucet et al. 2001, Seong et al. 2002). Assuming that a set of random samples (particles) $\mathbf{x}^i_{0:k}$, i = 1, 2,…, N_s, of the system state at the time k-1 is available as a realization of the posterior probability $p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1})$, the predicting step at time k is accomplished by sampling from the probability distribution of the system noise $\boldsymbol{\omega}_{k-1}$ and simulating the system dynamics (3) to generate a new set of samples $\mathbf{x}^i_k$ which are realizations of the predicted probability distribution $p(\mathbf{x}_k|\mathbf{z}_{0:k-1})$.

In the update step, based on the likelihoods of the observations $\mathbf{z}_k$ collected at time k, each sampled particle $\mathbf{x}^i_{k-1}$ is assigned a weight:

$$w^i_k = \frac{p(\mathbf{z}_k|\mathbf{x}^i_k)}{\sum_{j=1}^{N} p(\mathbf{z}_k|\mathbf{x}^j_k)} \tag{8}$$

An approximation of the posterior distribution $p(\mathbf{x}_k|\mathbf{z}_{0:k})$ can be obtained in terms of the weighted samples ($\mathbf{x}^i_k$, w^i_k), i = 1, 2,…, N_s (Doucet et al. 2001).

One difficulty that arises in the implementation of PF is the degeneracy problem: as the algorithm evolves in time, the weight variance increases (Doucet 1998) and the importance weight distribution becomes progressively skewed, until (after a few iterations) all but one particle have negligible weights (Arulampalam et al. 2000, Doucet et al. 2000, Andrieu et al. 2001). As a result, the approximation of the target distribution $p(\mathbf{x}_k|\mathbf{z}_{0:k})$ becomes very poor and significant computational resources are spent trying to update particles with minimum relevance. A possible solution to this problem is to proceed to a resampling of a new swarm of realizations $\mathbf{x}^i_k$ from the approximate posterior distribution, constructed on the weighted samples previously drawn; all particles thereby generated are assigned equal weights, $w^i_k = 1/N_s$ (Doucet et al. 2001).

As the final step, one has to resample from the posterior distribution a new swarm of points $\mathbf{x}^i_k$. The prediction, update and resample steps form a single iteration, recursively applied at each time k.

2.2 Hybrid system model

Let us consider a hybrid system whose dynamic evolution can be described by:

$$\begin{cases} \mathbf{x}_k = \mathbf{f}_{\beta_k}\left(\mathbf{x}_{k-1}, \boldsymbol{\omega}_{k-1}\right) \\ \beta_k = \mathbf{g}_k\left(\beta_{k-1}, \mathbf{x}_{k-1}\right) \end{cases} \tag{9}$$

$\beta_k = \{1,2,3,\ldots,M\}$ is the discrete state which indicates the mode in which the system is evolving at time k, $\mathbf{f}_{\beta k}$ is the non-linear function describing the (discretized) continuous evolution of system state x when the system is in mode β_k at time k, $\mathbf{g}_k$ is the discrete mode transition function. In what follows, we shall consider only autonomous transitions between the system modes, i.e. those triggered by the control of the continuous state $\mathbf{x}$ which demands transitions among the system modes when reaching specified thresholds.

Let $s^i_k = (\beta^i_k, \mathbf{x}^i_k)$ indicate the i^{th} sample of the extended hybrid system state, where $\mathbf{x}^i_k$ is the random sample drawn from the importance function $p(\mathbf{x}_k|\mathbf{x}^i_{k-1})$ and β^i_k is the corresponding discrete mode of system behavior. Then, the posterior probability density of the continuous and discrete states can be represented by the random measure $\{s^i_k, w_k^i, i=1...N_s\}$, where w_k^i is the particle weight of the i^{th} sample of the hybrid state at time k after resampling.

The estimation of system mode of operation as the most likely one is given by:

$$\hat{\beta}_k = \arg\max \sum_{i \in \hat{G}_j} w_k^i \tag{10}$$

where $\hat{G}_j = \left\{ i \middle| \beta_k^i = j \right\}$. Whereas, the posterior estimate mean of the continuous state $\mathbf{x}_k$ and its variance $\hat{\sigma}_k^2$ are given by:

$$\hat{\mathbf{x}}_k = \frac{\sum_{i\in\hat{G}} w_k^i \mathbf{x}_k^i}{\sum_{i\in\hat{G}} w_k^i} \tag{11}$$

$$\hat{\sigma}_k^2 = \frac{\sum_{i\in\hat{G}}^{N_s} w_k^i \left(\mathbf{x}_k^i - \hat{\mathbf{x}}_k\right)}{\sum_{i\in\hat{G}} w_k^i} \tag{12}$$

where only the particles belonging to the most likely mode $\hat{\beta}_k^i$ are considered $\hat{G}_j = \left\{ i \middle| \beta_k^i = \hat{\beta}_k^i \right\}$.

3 APPLICATION TO A TANK CONTROL SYSTEM

The particle filter estimation algorithm is applied to a hybrid system of literature (Aldemir et al. 1994, Marseguerra et al. 1996, Labeau et al. 1998, Wang et al. 2002). The system consists of a tank containing a fluid whose level is controlled by three control units which open or close depending on the fluid level crossing of predefined thresholds (*HLV* and *HLP*) (Figure 1). The fluid in the tank is uniformly heated, under adiabatic conditions, by a thermal power source W.

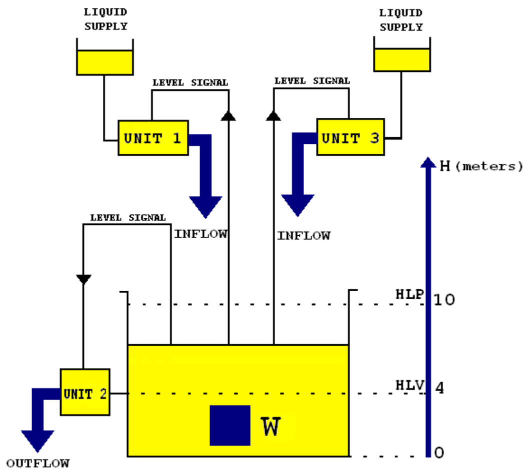


Figure 1. Tank control system (Aldemir et al. 1994, Marseguerra et al. 1996, Labeau et al. 1998, Wang et al. 2002).

The control aims at maintaining the fluid level x_1 in the range ($x_{1,\min}$ = *HLV*, $x_{1,\max}$ = *HLP*), while also monitoring the fluid temperature x_2 which may become relevant from a safety point of view.

The operational states of the control units at time *k* are described by the Boolean indicator $\alpha_{l,k}$, l = 1,2,3, where $\alpha_{l,k}$ assumes the value 1 or 0 according to whether the unit is on ($\alpha_{l,k}$ = 1) or off ($\alpha_{l,k}$ = 0). The autonomous control actions modify the states $\alpha_{l,k}$ of the units according to the following rules:

$$\alpha_{1,k} = \begin{cases} 1 & if\ x_1 < HLV \\ 0 & if\ x_1 > HLP \\ 0\,or\,1 & dependingon\ previousswitching \end{cases}$$

$$\alpha_{2,k} = \begin{cases} 1 & if\ x_1 > HLV \\ 0 & otherwise \end{cases} \tag{13}$$

$$\alpha_{1,k} = \begin{cases} 1 & if\ x_1 < HLV \\ 0 & if\ x_1 > HLP \\ 0\,or\,1 & dependingon\ previousswitching \end{cases}$$

Thus, the following four modes of system dynamic evolution may be identified:

$$\beta_k = \begin{cases} 1 & if\ x_{1,k} < HLV \\ 2 & if\ HLV < x_{1,k} < HLP \text{ and } \alpha_{1,k-1} = \alpha_{3,k-1} = 1 \\ 3 & if\ HLV < x_{1,k} < HLP \text{ and } \alpha_{1,k-1} = \alpha_{3,k-1} = 0 \\ 4 & if\ x_{1,k} > HLP \end{cases} \tag{14}$$

With the additional simplifying physical assumptions that the fluid input in the tank by units 1 and 3 mixes instantaneously and the flow rate through the outlet unit 2 is independent of the fluid level, and the discretisation of the system dynamics, the time evolution of the state $x_{1,k}$ and $x_{2,k}$ can be described by two first-order, decoupled, non-linear difference equations determined by the mass and energy conservation laws (Wang et al. 2002).

The aim of the analysis is that of estimating the discrete mode of the system, i.e. the operational states of the three control units on the basis of N_s trajectories drawn from the system model and a sequence of noisy measurements of the level $x_{1,k}$ and the temperature $x_{2,k}$:

$$\begin{aligned} z_{1,k} &= x_{1,k} + \upsilon_{1,k} \\ z_{2,k} &= x_{2,k} + \upsilon_{2,k} \end{aligned} \tag{15}$$

where $\upsilon_{1,k}$ and $\upsilon_{2,k}$ are the measurement noises. Knowledge of the system mode of operation allows the proper control and maintenance of its components.

Let us suppose that the control system starts from $x_{1,0}$ =6m and $x_{2,0}$ =10m. The time horizon considered for the evolution of the system dynamics is N_t = 40h, with level and temperature observations at discrete time steps of Δt = 30min (N_k = 80). As in the application of reference (Wang et al. 2002), the inlet fluid temperature is ϑ_m = 15ºC, the level thresholds are set at HLV = 4m and HLP = 10m and the fluid flow rates are Q_1 = 1m/h, Q_2 = 4m/h and Q_3 = 4.5m/h. A zero – mean Gaussian noise with variance σ^2_Q = 0.0025 is added to the flow rates, for closer adherence to reality. The process and the measurement noises are assumed Gaussian with zero mean and variances σ^2_ω = [0.02 0.01] and σ^2_υ = [0.16 0.05] respectively.

Assuming independence of the level and temperature measurements, the observation likelihood in (8) can be written as:

$$p\left(\mathbf{z}_k \middle| \mathbf{x}_k^i\right) = \prod_{h=1,2} p\left(z_{h,k} \middle| \mathbf{x}_k^i\right) = \frac{1}{\sqrt{2\pi}\sigma_{\upsilon_1}} e^{-\frac{1}{2}\left(\frac{z_{1,k}-\mu_{1,k}}{\sigma_{\upsilon_1}}\right)^2} \frac{1}{\sqrt{2\pi}\sigma_{\upsilon_1}} e^{-\frac{1}{2}\left(\frac{z_{2,k}-\mu_{2,k}}{\sigma_{\upsilon_2}}\right)^2} \tag{16}$$

First, a crude, measurement-based, empirical algorithm is proposed for the estimation of the mode β_k at time k:

$$\hat{\beta}_k = \begin{cases} 1 & if\ z_{1,k} < HLV \\ 2 & if\ HLV < z_{1,k} < HLP \text{ and } \alpha_{1,k-1} = \alpha_{3,k-1} = 1 \\ 3 & if\ HLV < z_{1,k} < HLP \text{ and } \alpha_{1,k-1} = \alpha_{3,k-1} = 0 \\ 4 & if\ z_{1,k} > HLP \end{cases} \tag{17}$$

where $z_{1,k}$ is the level measurement at time k.

Figure 2 shows the estimated mode $\hat{\beta}$ (dot-dashed line) and the model simulated one β (solid line). The performance is not satisfactory because the noise υ_1 generates spurious oscillations in the level measurement z_1 with respect to the model-simulated x_1 actually driving the mode transitions.

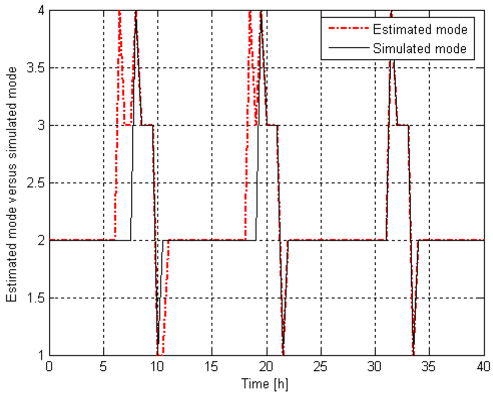


Figure 2. Measurement-based estimated system modes (dotted line) and model-based simulated system modes (solid line).

To overcome this problem, the particle filter method is implemented with a number of particles N_s = 1000 (Figure 3). Figure 4 shows the particle filter-estimated mode $\hat{\beta}$ (dot-dashed line) and the model simulated one β (solid line). The agreement is satisfactory, with the only exception in correspondence of the first time when the system enters mode $\beta = 4$, i.e. the fluid level is higher than *HLP*. This is due to the fact that the first few observations of the fluid level higher than *HLP* do not provide the filter with enough information for properly performing the mode estimation. This is confirmed in Figure 5, where the estimated level $\hat{x}_1$ (dotted) is affected by a larger uncertainty $\hat{\sigma}_{1,k}$ when approaching the threshold *HLP* for the first time.

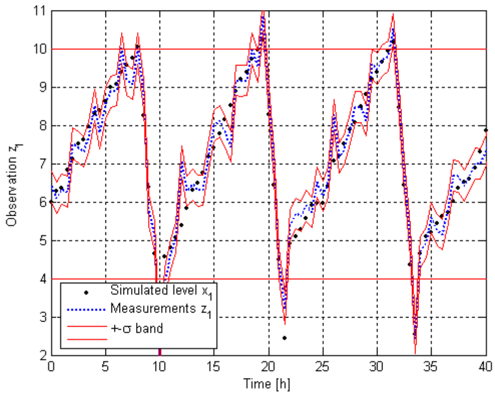


Figure 3. Fluid level measurements (dotted line), with measurement noise uncertainty $\pm 1\sigma_{\upsilon_1}$ bands (solid line); model-simulated fluid level (dots).

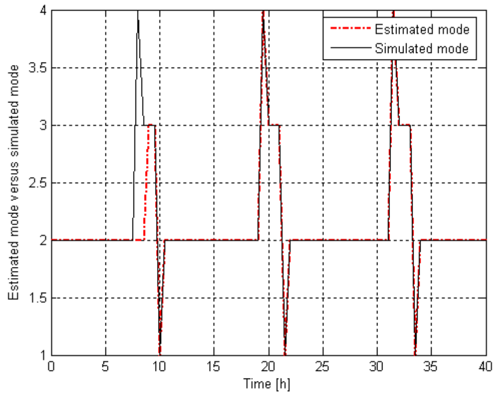


Figure 4. Particle filter-estimated (dotted line) and model-simulated (solid line) modes.

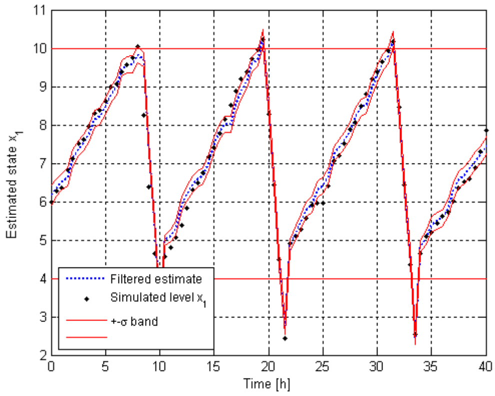


Figure 5. Particle filter-estimated mean fluid level (dotted line), with $\pm 1\sigma_{\upsilon_1}$ uncertainty bands (solid line) and model-simulated fluid level (dots).

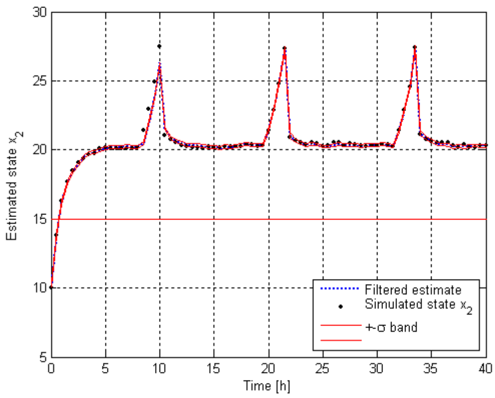


Figure 6. Particle filter-estimated mean of the fluid temperature (dotted line), with $\pm 1\sigma_{\upsilon_1}$ uncertainty bands (solid line) and simulated fluid temperature (dots).

Similar satisfactory results (not reported for brevity's sake) have been obtained in the estimation of the temperature state variable, x_2.

4 CONCLUSIONS

In this paper, a Monte Carlo-based filter has been devised for estimating both the continuous states and the discrete modes of a controlled system, whose transitions between the discrete modes are autonomously triggered by the continuous states. Comparison with a crude algorithm which bases its estimates directly on the observed measurements, shows the higher performance of the particle filter on a wider range of measurements noises, thus counterbalancing the larger computational effort required.

5 PREFERENCES

Aldemir, T., Siu, N., Mosleh, A., Cacciabue, P.C. & Goktepe, B.G. (1994). Eds.: *Reliability and Safety Assessment of Dynamic Process Systems*. NATO-ASI Series F. 120, Berlin: Springer-Verlag.

Andrieu, C., Doucet, A., & Punskaya, E. (2001) .*Sequential Monte Carlo Methods for Optimal Filtering*. Sequential Monte Carlo Methods in Practice, Doucet, A., de Freitas, N., and Gordon, N., Eds. New York: Springer-Verlag.

Arulampalam, M.S., Maskell, S., Gordon, N. & Clapp, T. (2002). A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking. *IEEE Trans. On Signal Processing*. 50, 2, 174-188.

Cadini, F., Zio, E. & Avram, D. (2009a). Model-based Monte Carlo state estimation for condition-based component replacement. *Reliab. Eng. and Sys. Saf*. 94, 752-758.

Cadini, F., Zio, E. & Avram, D. (2009b). Monte Carlo-based filtering for fatigue crack growth estimation. *Probabilistic Engineering Mechanics*. 24, 367-373.

Djuric, P.M., Kotecha, J.H., Zhang, J., Huang, Y., Ghirmai, T., Bugallo, M. F. & Miguez, J. (2003). Particle Filtering, *IEEE Signal Processing Magazine*. 19-37.

Doucet, A. (1998). On Sequential Simulation-Based Methods for Bayesian Filtering. Technical Report, University of Cambridge, Dept. of Engineering, CUED-F-ENG-TR310.

Doucet, A., Godsill, S. & Andreu, C. (2000). On Sequential Monte Carlo Sampling Methods for Bayesian Filtering. *Statistics and Computing*. 10, 197-208.

Doucet, A., de Freitas, J.F.G. & Gordon, N.J. (2001). *An Introduction to Sequential Monte Carlo Methods, in Sequential Monte Carlo in Practice*. A. Doucet, J.F.G. de Freitas and N.J. Gordon, Eds., New York: Springer-Verlag,.

Kitagawa, G. (1987). Non-Gaussian State-Space Modeling of Nonstationary Time Series. *Journal of the American Statistical Association*. 82, 1032-1063.

Koutsoukos, X., Kurien, J. & Zhao, F. (2002). Monitoring and diagnosis of hybrid systems using particle filtering methods. *Proceedings of the 15th International Symposium on the Mathematical Theory of Networks and Systems* (MTNS).

Labeau, P.E. & Zio, E. (1998). The Cell-to-Boundary Method in the Frame of Memorization-Based Monte Carlo Algorithms. A New Computational Improvement in Dynamic Reliability. *Mathematics and Computers in Simulation*. 47, 2-5, 329-347.

Marseguerra, M. & Zio, E. (1996). Monte Carlo approach to PSA for dynamic process systems. *Reliab. Eng. and Sys. Saf.* 52, 227-241.

Pulkkinen, U. (1991). A Stochastic Model for Wear Prediction through Condition Monitoring. K. Holmberg & A. Folkeson Eds.. *Operational Reliability and Systematic Maintenance*. London/New York: Elsevier. 223-243.

Seong, S.-H., Park, H.-Y., Kim, D.-H., Suh, Y.-S., Hur, S., Koo, I.-S., Lee, U.-C.,. Jang, J.-W & Shin, Y.-C. (2002). Development of Fast Running Simulation Methodology Using Neural Networks for Load Follow Operation. *Nuclear Science and Engineering*. 141, 66-77.

Wang, P., Chen X.M. and Aldemir T. (2002). DSD: a Generic Software Package for Model-Based Fault Diagnosis in Dynamic Systems. *Reliab. Eng. and Sys. Saf*. 75, 31-39.

BOOTSTRAP METHODS FOR THE CENSORED DATA IN EMPIRICAL BAYES ESTIMATION OF THE RELIABILITY PARAMETERS

F. Grabski, A. Załęska-Fornal

•

Department of Mathematics, Naval University, Gdynia, Poland

e-mail: *f.grabski@amw.gdynia.pl* ; *a.fornal@amw.gdynia.pl*

ABSTRACT

Bootstrap and resampling methods are the computer methods used in applied statistics. They are types of the Monte Carlo method based on the observed data. Bradley Efron described the bootstrap method in 1979 and he has written a lot about it and its generalizations since then. Here we apply these methods in an empirical Bayes estimation using bootstrap copies of the censored data to obtain an empirical prior distribution.

1 INTRODUCTION

The bootstrap is a computer-based method used in applied statistics. It is a databased method of simulation for assessing statistical accuracy. The term bootstrap derives from the phrase *'to pull oneself up by one's bootstrap'* which can be found in the eighteenth century Adventures of Baron Munchausen by Rudolf Erich Raspe. The method was proposed by Bradley Efron in 1979 as a method to estimate the standard error of a parameter. The main goal of the bootstrap method is a computer-based fulfilling of basic statistical ideas. The recent environment applications of bootstrap can be found in toxicology, fisheries survey, ground water and air pollution modeling, hydrology etc. Bootstrapping is a methodology whose implementation involves a powerful principle: creating many repeated data samples from a single one we have and making inference from those samples.
We apply bootstrap in empirical estimation using the so-called bootstrap copies of the censored data to obtain an empirical distribution.

2 BOOTSTRAP AND RESAMPLING COPIES OF THE CENSORED

The random variable X denotes time to failure of an element. The probability distribution of the time to failure is defined by the cumulative distribution function (*cdf*)

$$F_\theta(x) = P(X \le x) \tag{1}$$

where $\theta \in \Theta$ is true but unknown parameter. To assess this distribution we test n identical elements $e_1, e_2, ..., e_n$ through the times $y_1, y_2, ..., y_n$ correspondingly. Suppose, that the numbers $x_1, x_2, ..., x_n$ are the times to failures of the elements mentioned above. A vector $\mathbf{x}_n = (x_1, x_2, ..., x_n)$ of the data is assumed to be the value of the random vector $\mathbf{X}_n = (X_1, X_2, ..., X_n)$, where random variables $X_1, X_2, ..., X_n$ are mutually independent and identically distributed (i.i.d.). That random vector is a sample from the distribution $F_\theta(\cdot)$. A vector $y_n = (y_1, y_2, ..., y_n)$ of the testing times of elements (times of the observations, censoring points) we can treat as the value of the random vector $\mathbf{Y}_n = (Y_1, Y_2, ..., Y_n)$. We assume that $Y_1, Y_2, ..., Y_n$ are mutually independent random variables and they are also independent of X's. Probability distributions of the random variables $Y_1, Y_2, ..., Y_n$ are defined by *cdf*

$$G_i(y_i) = P(Y_i \le y), \quad i = 1,2,...,n \tag{2}$$

Those functions do not depend on the parameter $\theta \in \Theta$. In many cases those functions are defined as

$$G_i(y) = \begin{cases} 0 & \text{for} \quad y < y_i \\ 1 & \text{for} \quad y \ge y_i \end{cases} \qquad y_i \in [0, \infty] \ .$$

It means that the quantities of $Y_1, Y_2, ..., Y_n$ are determined.
The observations are described by the random variables

$$U_j = \min(X_j, Y_j),\ j = 1,...,n \tag{3}$$

$$\Delta_j = \begin{cases} 1 \text{ for } X_j \le Y_j \\ 0 \text{ for } X_j > Y_j \end{cases} \ . \tag{4}$$

The sufficient statistic describing observations can be written as the vector $\mathbf{Z}_n = ((U_1, \Delta_1), ..., (U_n, \Delta_n))$. The value of that random vector is $\mathbf{z}_n = ((u_1, \delta_1), ..., (u_n, \delta_n))$, which allows to obtain the vector $\mathbf{z}_{(n)} = (z_{(1)}, z_{(2)}, ..z_{(k)}, z_{(k+1)}, ..., z_{(n)})$, where $z_{(1)}, z_{(2)}, ..., z_{(k)}$ are the instants of the elements failure and $z_{(k+1)}, z_{(k+2)}, ..., z_{(n)}$ are the times observations of the working elements. Suppose that we are able to estimate a parameter $\theta \in \Theta$ by using estimator $\bar{\boldsymbol{\theta}}_n = T(\mathbf{Z}_n)$ (or $\bar{\boldsymbol{\theta}}_n = \tilde{T}(\mathbf{Z}_{(n)})$). The numbers $\bar{\theta}_n = T(\mathbf{z}_n)$ (or $\bar{\theta}_n = \tilde{T}(\mathbf{z}_{(n)})$) are their values. After that we can use the distribution $F_{\bar{\theta}_n}(\cdot)$ to simulate so-called *bootstrap copies*

$$\mathbf{z}_{(n)}^{*(b)} = (z_{(1)}^{*(b)}, z_{(2)}^{*(b)}, ..., z_{(n)}^{*(b)}), \ \ b = 1,2,....,B$$

of data $\mathbf{z}_{(n)} = (z_{(1)}, z_{(2)}, ..., z_{(n)})$. The bootstrap copies of data are the values of the random vectors

$$\mathbf{Z}_{(n)}^{*(b)} = (Z_{(1)}^{*(b)}, Z_{(2)}^{*(b)}, ..., Z_{(n)}^{*(b)}), \ \ b = 1,2,....,B,$$

that are called the *bootstrap samples.* The function $F_{\bar{\theta}_n^b}(\cdot)$ is a cumulative probability distribution of the independent random variables $Z_1^{*(b)}, Z_2^{*(b)}, ..., Z_n^{*(b)}$.
If we have a vector of observation $\mathbf{z}_{(n)} = (z_{(1)}, z_{(2)}, ..., z_{(n)})$ of size n, we can define the empirical cumulative distribution function $\bar{F}$ as

$$\bar{F}(z; \mathbf{z}_{(n)}) = \frac{\#\{z_{(i)} : z_{(i)} \le z\}}{n}$$

that is equivalent to the discrete distribution

$$\bar{p}_k = \frac{n_k}{n}, \quad k = 1,2,...,l\,,$$

where $n_k = \#\{i : z_{(i)} = z_{(k)}\}$.
This distribution can be expressed as a vector of frequencies $\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2, ..., \bar{p}_l)$.
Vectors of the data

$$\mathbf{z}_n^{\circ(r)} = (z_1^{\circ(r)}, z_2^{\circ(r)}, ..., z_n^{\circ(r)}), \ \ r = 1,2,....,R$$

coming from distribution $\bar{F}(z; \mathbf{z}_{(n)})$ are said to be *resampling copies* of the data

$$\mathbf{z}_{(n)} = (z_{(1)}, z_{(2)}, ..., z_{(n)})\,.$$

In other words a resampling copy of the data $\mathbf{z}_n^{\circ(r)} = (z_1^{\circ(r)}, z_2^{\circ(r)}, ..., z_n^{\circ(r)})$ is generated by randomly sampling $n-$times with replacement from the original data points $\mathbf{z}_{(n)} = (z_{(1)}, z_{(2)}, ..., z_{(n)})$. The randomly sampling means the random choice of an element from among $z_{(1)}, z_{(2)}, ..., z_{(n)}$ in each of n drawings. The resampling copy of the data is composed of the elements of the original sample, some of them can be taken zero times, some of them can be taken ones or twice etc. Notice that in $\mathbf{z}_n^{\circ(r)} = (z_1^{\circ(r)}, z_2^{\circ(r)}, ..., z_n^{\circ(r)})$ the resampling copy, the elements are repeated as a rule.
The typical number of the bootstrap B or resampling copies of the data range from 50 to 1000.

3 BOOTSTRAP ESTIMATORS

Let $\mathbf{Z}_n^* = (Z_1^*, Z_2^*, ..., Z_n^*)$ be a bootstrap sample for the given vector of data $\mathbf{z}_n = (z_1, z_2, ..., z_n)$. A random variable $\mathbf{\theta}_n^* = T(\mathbf{Z}_n^*)$ is said to be a bootstrap estimator of the parameter θ.

The distribution of the statistics $\mathbf{\theta}_n^* - \overline{\mathbf{\theta}}_n$ for the bootstrap sample with the fixed values data is close to the distribution of the statistics $\overline{\mathbf{\theta}}_n - \theta$.

From that rule it follows that the shapes of the distributions of the statistics $\mathbf{\theta}_n^*$, $\overline{\mathbf{\theta}}_n$ are similar .To obtain empirical distribution of the random variable $\mathbf{\theta}_n^*$ we have to simulate bootstrap copies

$$\mathbf{z}_n^{*(b)} = (z_1^{*(b)}, z_2^{*(b)}, ..., z_n^{*(b)}), \; b = 1,2,....,B$$

of data $\mathbf{z}_n = (z_1, z_2, ..., z_n)$. After that we calculate the values of statistics

$$\theta_n^{*(b)} = T(\mathbf{z}_n^{*(b)}), \; b = 1,2,....,B .$$

We can use a nonparametric kernel estimator to obtain the estimate of probability density of the bootstrap estimator $\mathbf{\theta}_n^*$. The value of this estimator with Gaussian kernel is given by

$$\overline{g}(\vartheta) = \frac{1}{B\,h} \sum_{b=1}^{B} K\left(\frac{\vartheta - \theta_n^{*(b)}}{h} \right)$$

where $K(\vartheta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\vartheta^2}{2}}, \; \vartheta \in (-\infty, \infty)$,

and $h = 1.06\, s\, B^{-0.2}$, s - standard deviation of $\theta_n^{*(b)}$, $b = 1,2,....,B$.

4 THE BOOTSTRAP ESTIMATE OF STANDARD ERROR

$\mathbf{z}_n^{*(b)} = (z_1^{*(b)}, z_2^{*(b)}, ..., z_n^{*(b)}), \; b = 1,2,....,B$

are the bootstrap replications of the statistics values

$$\theta_n^{*(b)} = T(z_n^{*(b)}), \; b = 1,2,....,B \tag{5}$$

and they correspond to the bootstrap censoring data.

The bootstrap estimate of the standard error of $\overline{\theta}$ is defined by the following formula

$$se_{\overline{\theta}^*} = \sqrt{\frac{\sum_{b=1}^{B} \left(\overline{\theta}^{*(b)} - \breve{\theta}^* \right)^2}{B-1}}, \tag{6}$$

where $\breve{\theta}^* = \frac{\sum_{b=1}^{B} \overline{\theta}^{*(i)}}{B}$.

The bootstrap algorithm for estimating standard errors is as follows:

- Get B independent bootstrap samples $\mathbf{z}_n^{*(b)} = (z_1^{*(b)}, z_2^{*(b)}, ..., z_n^{*(b)}),\ b = 1,2,....,B$ (for estimating a standard error, the number of B should be in the range 30-200).
- Compute the bootstrap replication correspond each bootstrap sample, $\theta_n^{*(b)} = T(\mathbf{z}_n^{*(b)}),\ b = 1,2,....,B$.
- Compute the standard error $se_{\overline{\theta}^*}$ by the sample standard deviation of B replications according to (6).

5 EMPIRICAL BAYES ESTIMATION

The recent work deal with empirical Bayes estimation has been stimulated by the work of Robbins (1955). It is well known that the value of Bayes estimator $\overline{\theta}_B$ of parameter θ under the squared-loss function is an expectation in posterior distribution. If $\overline{\theta}$ is a value of sufficient statistics for parameter θ, than the value of Bayes estimator $\overline{\theta}_B$ of the parameter θ is

$$\overline{\theta}_B = E(\theta \mid \overline{\theta}) = \frac{\int_{\Theta} \theta \tilde{\mathbf{f}}(\overline{\theta} \mid \theta) g(\theta) d\nu(\theta)}{\int_{\Theta} \tilde{\mathbf{f}}(\overline{\theta} \mid \theta) g(\theta) d\nu(\theta)} \tag{7}$$

where ν denotes a discrete counting measure or Lebesgue measure and $g(\theta)$ is a prior density function of the parameter θ with respect to measure ν.

We suppose that a prior density of mentioned above parameter is unknown. In classical empirical Bayesian procedure a prior distribution is assessed from the *past data*. Very often the only data we have is the small sample $\mathbf{z} = (z_1, z_2, ..., z_n)$. In those cases instead of past data, we can use the vectors $\mathbf{z}_n^{*(b)} = (z_1^{*(b)}, z_2^{*(b)}, ..., z_n^{*(b)}),\ b = 1,2,....,B$, that are values of the *bootstrap samples* corresponding to an unknown distribution $F_\theta(\cdot)$ of a random variable X denoting (for example) time to failure. The bootstrap copies for censored data are generated from the distribution $F_{\overline{\theta}}(\cdot)$, where $\overline{\theta} = T(\mathbf{z}_{(n)})$. To estimate the unknown parameter θ we have to calculate values of the bootstrap statistics $\theta^{*(b)} = T(z_n^{*(b)}),\ b = 1,2,....,B$ of that one. As a prior density we propose discrete density function

$$g(\theta) = \frac{m_i}{m} \delta(\theta, \theta^{*(i)}), \quad i \in \{j_1, j_2, ..., j_w\} \subseteq \{1, ..., B\} \tag{8}$$

where $m_i = \#\{k: \ \theta^{*(k)} = \theta^{*(i)}\}$ denotes number observations equal to $\theta^{*(i)}$,

$$\delta(\theta, \theta^{*(i)}) = \begin{cases} 1 & \text{for} \quad \theta = \theta^{*(i)} \\ 0 & \text{for} \quad \theta \neq \theta^{*(i)} \end{cases}$$

and $m = \sum_{i=1}^{w} m_{j_i} = B$.

From (7), for the counting measure ν and the density function defined by (8) we obtain

$$\overline{\theta}_B = E(\theta \mid \overline{\theta}) = \frac{\sum_{i=1}^{w} m_i \theta^{*(i)} \tilde{f}(\overline{\theta} \mid \theta^{*(i)})}{\sum_{i=1}^{w} m_i \tilde{f}(\overline{\theta} \mid \theta^{*(i)})} = \frac{\sum_{i=1}^{B} \theta^{*(i)} \tilde{f}(\overline{\theta} \mid \theta^{*(i)})}{\sum_{i=1}^{B} \tilde{f}(\overline{\theta} \mid \theta^{*(i)})}. \tag{9}$$

Let $\mathbf{f}_\theta(\mathbf{z}^{*(b)}_{(n)}) = l(\mathbf{z}^{*(b)}_{(n)};\theta)$ be a likelihood function for the bootstrap sample

$$\mathbf{z}^{*(b)}_{(n)} = (z^{*(b)}_{(1)}, z^{*(b)}_{(2)}, \dots, z^{*(b)}_{(n)})$$

with unknown parameter $\theta \in \Theta$. The function is defined by the formula

$$l(\mathbf{z}^{*(b)}_{(n)}, \theta) = \prod_{i=1}^{k_b} f_\theta(z^{*(b)}_{(i)}) \prod_{i=k_b+1}^{n} [1 - F_\theta(z^{*(b)}_{(i)})]. \tag{10}$$

Notice that a prior distribution is constructed on the basis on the bootstrap samples. Since, a value of bootstrap empirical Bayes estimator has the form of (9).

6 EXAMPLES

Example 1.

Suppose that we wish to estimate a failure rate $\theta = \lambda$ in the exponential distribution given by *pdf*

$$f_\theta(x) = \lambda e^{-\lambda x}, \quad x \ge 0, \quad \lambda > 0 . \tag{11}$$

Assume that we have data, which is the vector

$$\mathbf{z}_{(n)} = (z_{(1)}, z_{(2)}, ..z_{(k)}, z_{(k+1)}, \dots, z_{(n)}),$$

where $z_{(1)}, z_{(2)}, \dots, z_{(k)}$ are times to failure of the tested elements and $z_{(k+1)}, z_{(k+2)}, \dots, z_{(n)}$ are times of the working elements observations. In that case a likelihood function is

$$l(\mathbf{z}_{(n)}, \lambda) = \prod_{i=1}^{k} f_\theta(z_{(i)}) \prod_{i=k+1}^{n} [1 - F_\theta(z_{(i)})] =$$

$$= \prod_{i=1}^{k} \lambda e^{-\lambda\, z_{(i)}} \prod_{i=k+1}^{n} [e^{-\lambda\, z_{(i)}}] = \lambda^k e^{-\lambda \sum_{i=1}^{n} z_{(i)}}. \tag{12}$$

The number

$$\tau = \sum_{i=1}^{n} z_{(i)} \tag{13}$$

is the value of some sufficient statistics for the unknown parameter λ. By substitution we obtain the likelihood function

$$l(\tau, \lambda) = \lambda^k e^{-\lambda \tau},$$

which depends on τ. To find the value of the maximum likelihood estimator we have to solve an equation

$$\frac{\partial \ln l(\tau, \lambda)}{\partial \lambda} = 0.$$

The solution of it is

$$\overline{\lambda} = \frac{k}{\tau} = \frac{k}{\sum_{i=1}^{n} z_{(i)}}. \tag{14}$$

The same way, using formula (7) for the bootstrap samples $\mathbf{z}^{*(b)}_{(n)} = (z^{*(b)}_{(1)}, z^{*(b)}_{(2)}, \dots, z^{*(b)}_{(n)}),\ b = 1,2,\dots,B$ we obtain the values of the maximum likelihood estimator of λ

$$\lambda^{*(b)} = \frac{k^{*(b)}}{\tau^{*(b)}} = \frac{k^{*(b)}}{\sum_{i=1}^{n} z_i^{*(b)}}, \quad b = 1,2,\dots,B$$

The function (9) in this case is given by the formula

$$\overline{\lambda}_B = E(\lambda \mid \overline{\lambda}) = \frac{\sum_{i=1}^{w} m_i \lambda^{*(i)} \widetilde{f}(\overline{\lambda} \mid \lambda^{*(i)})}{\sum_{i=1}^{w} m_i \widetilde{f}(\overline{\lambda} \mid \lambda^{*(i)})},$$

where $\widetilde{f}(\overline{\lambda} \mid \lambda^{*(i)}) = (\lambda^{*(i)})^k e^{-\frac{k\lambda^{*(i)}}{\overline{\lambda}}}$.

Finally we obtain

$$\overline{\lambda}_B = \frac{\sum_{i=1}^{w} m_i \lambda^{*(i)} (\lambda^{*(i)})^k e^{-\frac{k\lambda^{*(i)}}{\overline{\lambda}}}}{\sum_{i=1}^{w} m_i (\lambda^{*(i)})^k e^{-\frac{k\lambda^{*(i)}}{\overline{\lambda}}}} = \frac{\sum_{j=1}^{B} (\lambda^{*(j)})^{k+1} e^{-\frac{k\lambda^{*(j)}}{\overline{\lambda}}}}{\sum_{j=1}^{B} (\lambda^{*(j)})^k e^{-\frac{k\lambda^{*(j)}}{\overline{\lambda}}}},$$

where $\overline{\lambda} = \frac{k}{\sum_{i=1}^{n} z_{(i)}}$, $\lambda^{*(b)} = \frac{k^{*(b)}}{\sum_{i=1}^{n} z_i^{*(b)}}$, $b = 1,2,....,B.$

By repetition we can obtain a sequence of values of a Bayes estimator that we can use to construct its empirical distribution.

Example 2.

We wish to estimate a value of an exponential reliability function

$$R_\theta(x) = e^{-\lambda x},\ x \ge 0,\ \lambda > 0,\ \theta = \lambda. \tag{15}$$

At a fixed moment x_0 the number

$$r = R_\theta(x_0) = e^{-\lambda x_0}$$

is a value of the reliability function. Hence

$$\lambda = \frac{\ln r}{x_0}. \tag{16}$$

There is a given vector

$$\mathbf{z}_{(n)} = (z_{(1)}, z_{(2)}, .. z_{(k)}, z_{(k+1)}, ..., z_{(n)})$$

the coordinates of which have the same meaning as in Example 1. Let τ be described by (13).
A likelihood function of the parameter λ for $\mathbf{z}_n$ is

$$l(\tau, \lambda) = \lambda^k e^{-\lambda \tau}.$$

Substituting the value of λ and $r = e^{\ln r}$ we get the form of the likelihood function

$$l(\tau, \lambda) = \mathbf{f}(\tau \mid r) = \left(-\frac{\ln r}{x_0}\right)^k e^{\left(\frac{\ln r}{x_0}\right)\tau} = \left(-\frac{\ln r}{x_0}\right) r^{\frac{\tau}{x_0}}. \tag{17}$$

The likelihood equation

$$\frac{\partial \ln l(\tau, \lambda)}{\partial \lambda} = 0$$

is carried out to the following form $\frac{k}{r \ln r} + \frac{\tau}{r x_0} = 0$.

A root of the equation is a value of the maximum likelihood estimate of r and it has a form of

$$\bar{r} = e^{-\left(\frac{kx_0}{\tau}\right)} . \qquad (18)$$

Using the bootstrap samples

$$\mathbf{z}_{(n)}^{*(b)} = (z_{(1)}^{*(b)}, z_{(2)}^{*(b)}, ..., z_{(n)}^{*(b)}), \; b = 1,2,....,B$$

we obtain the values of the maximum likelihood estimator of r and it is defined by

$$r^{*(b)} = e^{\left(-\frac{k^{*(b)}x_0}{\sum_{i=1}^{n} z_i^{*(b)}}\right)}, \quad b = 1,2,...,B .$$

As

$$\tilde{f}(\bar{r} \mid r^{*(i)}) = \ln(r^{*(i)}, \tau) = \left(-\frac{\ln r^{*(i)}}{x_0}\right)^k (r^{*(i)})^{\frac{\tau}{x_0}},$$

then the value of the Bayes empirical estimate of r computed on the basis on

$$\bar{r}_B = E(r \mid \bar{r}) = \frac{\sum_{i=1}^{w} m_i r^{*(i)} \tilde{f}(\bar{r} \mid r^{*(i)})}{\sum_{i=1}^{w} m_i \tilde{f}(\bar{r} \mid r^{*(i)})}$$

has the following form $\bar{r}_B = \dfrac{\sum_{i=1}^{w} m_i r^{*(i)} \left(-\frac{\ln r^{*(i)}}{x_0}\right)^k (r^{*(i)})^{\frac{\tau}{x_0}}}{\sum_{i=1}^{w} m_i \left(-\frac{\ln r^{*(i)}}{x_0}\right)^k (r \;\;)^{\frac{\tau}{x_0}}}$.

7 CONCLUSIONS

In that paper we present the possibility of applying the bootstrap and resampling methods in empirical Bayes estimation. The bootstrap and resampling copies of the given data are used to construct an empirical prior distribution.

REFERENCES

1. Belyaev Yu. K.: *Resampling and bootstrap methods in analysis of reliability data*. Safety & Reliability. ESREL 2001, p.1877-1882.
2. Grabski F. Jaźwiński J.: *Metody bayesowskie w niezawodności I diagnostyce*. WKŁ, Warszawa 2001.
3. Efron B., Tibshirani R. J.: *An introduction to the Bootstrap*, Chapman & Hall, New York, London 1993.
4. Koronacki J. Mielniczuk J.: *Statystyka dla studentów kierunków technicznych i przyrodniczych*, Wydawnictwa Naukowo-Techniczne, Warszawa 2001.
5. Savĉuk W.P. *Bayesian methods of the statistical assessment*. Nauka, Moskva 1989 (Russian).

RANDOM FUZZY CONTINUOUS-TIME MARKOV JUMP PROCESSES

R. Guo
University of Cape Town, Cape Town, South Africa
Renkuan.Guo@uct.ac.za
•
D. Guo
South African National Biodiversity Institute, Cape Town, South Africa
Guo@sanbi.org
•
T. Dunne
University of Cape Town, Cape Town, South Africa
Tim.Dunne@uct.ac.za

ABSTRACT

Continuous-time Markov chains are an important subclass in stochastic processes, which have facilitated many applications in business decisions, investment risk analysis, insurance policy making and reliability modeling. One should be fully aware that the existing continuous-time Markov chains theory is merely a framework under which the random uncertainty governs the phenomena. However, the real world phenomena often reveal a reality in which randomness and vagueness co-exist, and thus probabilistic continuous-time Markov chains modeling practices may be not wholly adequate. In this paper, we define random fuzzy continuous-time Markov chains, explore the related average chance distributions, and propose both a scheme for parameter estimation and a simulation scheme. It is expected that a foundational base can be established for reliability modeling and risk analysis, particularly, repairable system modeling.

1 INTRODUCTION

One should be fully aware that vagueness is an intrinsic feature of today's diversified business environments. As Carvalho and Machado (2006) commented, "In a global market, companies must deal with a high rate of changes in business environment. … The parameters, variables and restrictions of the production system are inherently vagueness." Therefore a co-existence of random uncertainty and fuzzy uncertainty is inevitable aspect of safety and reliability analysis and modelling.

It is obvious that probabilistic modeling is a good approximation to real world problem only when random uncertainty governs the phenomenon. Philosophically, if fuzziness and randomness both appear then probabilistic modeling alone may be questionable or inadequate. Therefore, it is logical to develop appropriate models for co-existent fuzziness and randomness.

Markov processes have been applied to large and complex system modeling and analysis in the reliability literature, for example, in recent work of Kolowrocki (2007), Love et al. (2000), Soszynska (2007), and Tamura (2004), etc.

We may also note that in recent year researchers in repairable system modeling, particularly in Asian reliability communities, proposed repair impact scenario models, which assume that the repair impacts to a repairable system may be classified into several states: no improvement, minor improvement, medium improvement, and major improvement. Hence one may utilize Kijima's age models (Kijima, 1989) to estimate those repair effects on the system repair states for optimal maintenance policy decision making, see Chan and Shaw (1993), Dohi et al. (2006), Lim and Lie (2000), Love et al. (2000), Wang, H. and Pham, H. (1996), Sheu et al. (2004), and Zhang (2002). However, less attention has been paid to the repair effect estimation, except for a few authors, Guo and Love (1992, 2004), Lim and Lie (2000), Yun et al. (2004), etc.

In this paper, we will give a systematic treatment for random fuzzy continuous-time Markov chains not only in the mathematical sense (building models based on postulates and definitions), but also in the statistical sense (estimation and hypothesis testing based on sample data).

2 PROBABILISTIC CONTINUOUS-TIME MARKOV JUMP PROCESSES

Grimmett and Stirzaker (1992) and also Guo (2009) describe continuous-time Markov jump processes by focusing the stochastic semigroup and the rate matrix.

Let $X=\{X_t,t\ge 0\}$ be a Markov chain with state space $\mathbb{S}=\{0,1,2,\cdots,N-1\}$. Further, let

$$p_{ij}(s,t)=\Pr\{X_t=j\mid X_s=i\} \tag{1}$$

be the transition probabilities. For the stationary Markov chain

$$p_{ij}(0,t-s)=p_{ij}(s,t),\ \forall s<t \tag{2}$$

Definition 1: (Grimmett and Stirzaker (1992)) A stochastic semigroup $\mathrm{P}=\{\mathrm{P}_t,t\ge 0\}$, with $\mathrm{P}_t=\left(p_{ij}(t)\right)_{N\times N}$ satisfies the following properties:

(a) $\mathrm{P}_0=\mathrm{I}$, an $N\times N$ identity matrix;

(b) For $\forall t$, $0\le p_{ij}(t)\le 1$, $\sum_j p_{ij}(t)=1$;

(c) The Chapman-Kolmogorov equations, for any $s,t>0$, $\mathrm{P}_{t+s}=\mathrm{P}_t\mathrm{P}_s$.

A stochastic semigroup $\mathrm{P}=\{\mathrm{P}_t,t\ge 0\}$ is standard if $\lim_{t\downarrow 0}\mathrm{P}_t=\mathrm{I}$. The characterization of a stochastic semigroup $\mathrm{P}=\{\mathrm{P}_t,t\ge 0\}$ can be stated as a theorem.

Theorem 1: For a standard stochastic semigroup $\mathrm{P}=\{\mathrm{P}_t,t\ge 0\}$, the limit

$$\lim_{h\downarrow 0}\frac{p_{ii}(h)-1}{h}=-q_i\left(=q_{ii}\right) \tag{3}$$

exists (maybe $-\infty$), while the limit

$$\lim_{h\downarrow 0}\frac{p_{ij}(h)}{h}=q_{ij} \tag{4}$$

exists and is finite. Guo (2009) detailed the proof of Theorem 1.

Definition 2: The matrix Q

$$\mathrm{Q}=\begin{bmatrix} -q_0 & q_{01} & \cdots & q_{0,N-1} \\ q_{10} & -q_{11} & \cdots & q_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ q_{N-1,1} & q_{N-1,2} & \cdots & -q_{N-1,N-1} \end{bmatrix} \tag{5}$$

where

$$\lim_{h\downarrow 0}\frac{p_{ij}(h)-\delta_{ij}}{h}=q_{ij} \tag{6}$$

with $\delta_{ij}=1,\ i=j$, *0* otherwise.

Lemma 1: In the rate matrix Q,

$$q_i = \sum_{j=0,\, j\neq i}^{N-1} q_{ij},\ i, j = 0,1,2,\cdots, N-1 \tag{7}$$

The rate matrix Q characterizes the movements of the continuous-time Markov chain $X = \{X_t, t \geq 0\}$. The following theorem reveals that fundamental fact.

Theorem 2: If the process $X = \{X_t, t \geq 0\}$ is currently halted at state i, it halts in state i during a time exponentially distributed with parameter q_i, independently of how the process reached state i and of how long it takes to get there. Furthermore, The process $X = \{X_t, t \geq 0\}$ leaves state i, and moves to state j with probability $q_{ij}/q_i \ (i \neq j)$.

Theorem 3. A standard stochastic semigroup $\mathrm{P} = \{\mathrm{P}_t, t \geq 0\}$ satisfies Kolmogorov equations:

$$\begin{aligned} \frac{d}{dt}\mathrm{P}_t &= \mathrm{P}_t\mathrm{Q} \text{ (Forward)} \\ \frac{d}{dt}\mathrm{P}_t &= \mathrm{Q}\mathrm{P}_t \text{ (Backward)} \end{aligned} \tag{8}$$

Corollary 1. A standard stochastic semigroup $\mathrm{P} = \{\mathrm{P}_t, t \geq 0\}$ satisfies

$$\mathrm{P}_t = e^{\mathrm{Q}t} \tag{9}$$

where matrix

$$e^{\mathrm{Q}t} = \sum_{i=0}^{\infty} \frac{1}{i!}(\mathrm{Q}t)^i \tag{10}$$

It is well-established fact that every entry of P_t, say $p_{ij}(t)$, can be expressed by a linear combination of $e^{\rho_l t}$ with appropriate coefficient $c(l)$, where ρ_l is the l^{th} eigenvalue of Q or of an appropriate minor matrix of Q, i.e.,

$$p_{ij}(t) = \sum_{l=0}^{N-1} c(l) e^{\rho_l t} \tag{11}$$

Example 1: Two-state continuous-time Markov chain. Let the rate matrix

$$\mathrm{Q} = \begin{bmatrix} -\nu & \nu \\ \lambda & -\lambda \end{bmatrix} \tag{12}$$

The eigenvalues are $(\rho_1, \rho_2) = (0, -(\nu + \lambda))$, thus

$$\mathrm{P}_t = \begin{bmatrix} \dfrac{\lambda}{\lambda+\nu} + \dfrac{\nu}{\lambda+\nu} e^{-(\lambda+\nu)t} & \dfrac{\nu}{\lambda+\nu} - \dfrac{\nu}{\lambda+\nu} e^{-(\lambda+\nu)t} \\ \dfrac{\lambda}{\lambda+\nu} - \dfrac{\lambda}{\lambda+\nu} e^{-(\lambda+\nu)t} & \dfrac{\nu}{\lambda+\nu} + \dfrac{\lambda}{\lambda+\nu} e^{-(\lambda+\nu)t} \end{bmatrix} \tag{13}$$

which confirms the formal result Equation (11).

3 FOUNDATION OF RANDOM FUZZY PROCESSES

Without a solid understanding of the intrinsic feature of random fuzzy processes, there is no base for exploring the modelling of random fuzzy continuous-time Markov chains. Liu’s (2004, 2007) hybrid variable theory established on the axiomatic credibility measure and probability measure foundations provides the mathematical foundation.

Guo et al. (2009) gave a systematic review on random fuzzy variable theory. In order to shorten the current paper, we keep only contents necessary for notational clarity, for details, see Guo and Guo (2009), or directly Liu's books (2004, 2007).

First let us review the credibilistic fuzzy variable theory. Let Θ be a nonempty set, and $\mathfrak{P}(\Theta)$ the power set on Θ.

Definition 3: Any set function $\mathrm{Cr}:\mathfrak{P}(\Theta)\to[0,1]$ which satisfies Liu's four Axioms (2004, 2007) is called a credibility measure. The triple $(\Theta,\mathfrak{P}(\Theta),\mathrm{Cr})$ is called the credibility measure space.

Definition 4: A fuzzy variable ξ is a measurable mapping, i.e., $\xi:(\Theta,\mathfrak{P}(\Theta))\to(\mathbb{R},\mathfrak{B}(\mathbb{R}))$.

A fuzzy variable is not a fuzzy set in the sense of Zadeh's fuzzy theory (1965, 1978), in which a fuzzy set is defined by a membership function.

Definition 5: (Liu (2004, 2007)) The credibility distribution $\Lambda:\mathbb{R}\to[0,1]$ of a fuzzy variable ξ on $(\Theta,\mathfrak{P}(\Theta),\mathrm{Cr})$ is

$$\Lambda(x)=\mathrm{Cr}\left\{\theta\in\Theta\left|\xi(\theta)\le x\right.\right\} \tag{14}$$

Liu (2004, 2007) defines a random fuzzy variable as a mapping from the credibility space $(\Theta,2^{\Theta},\mathrm{Cr})$ to a set of random variables.

Definition 6: (Guo et al, (2007)) A random fuzzy variable, denoted as $\xi=\left\{X_{\beta(\theta)},\theta\in\Theta\right\}$, is a set of random variables X_{β} defined on the common probability space $(\Omega,\mathfrak{A},\mathrm{Pr})$ and indexed by a fuzzy variable $\beta(\theta)$ defined on the credibility space $(\Theta,2^{\Theta},\mathrm{Cr})$.

Definition 7: (Liu (2004, 2007)) Let ξ be a random fuzzy variable, then the average chance measure denoted by $\mathrm{ch}\{\cdot\}$, of a random fuzzy event $\{\xi\le x\}$, is

$$\mathrm{ch}\{\xi\le x\}=\int_0^1\mathrm{Cr}\left\{\theta\in\Theta|\mathrm{Pr}\left\{\xi(\theta)\le x\right\}\ge\alpha\right\}\mathrm{d}\alpha \tag{15}$$

Then function $\Psi(\cdot)$ is called as average chance distribution if and only if

$$\Psi(x)=\mathrm{ch}\{\xi\le x\} \tag{16}$$

Definition 8: A random fuzzy process is a family of random fuzzy variables defined on the common Product measure space $(\Theta,2^{\Theta},\mathrm{Cr})\times(\Omega,\mathfrak{A},\mathrm{Pr})$, denoted by $\xi=\{\xi_t,t\in\mathbb{T}\}$, where $\mathbb{T}$ is an index set.

Theorem 4: Let ζ be a fuzzy variable defined on the credibility space $(\Theta,\mathfrak{P}(\Theta),\mathrm{Cr})$ and τ be a random variable defined on the probability space $(\Omega,\mathfrak{A}(\Omega),P)$, then

(1) Let $\oplus$ be an arithmetic operator, which can be "$+$", "$-$", "$\times$" or "$\div$" operations, such that $\zeta\oplus\tau$ maps from $(\Theta,\mathfrak{P}(\Theta),\mathrm{Cr})$ to a collection of random variables on $(\Omega,\mathfrak{A}(\Omega),P)$, denoted by ξ. Then ξ is a random fuzzy variable defined on hybrid product space $(\Theta,\mathfrak{P}(\Theta),\mathrm{Cr})\times(\Omega,\mathfrak{A}(\Omega),P)$.

(2) Let $f:\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ be a continuous mapping, such that $f(\zeta,\tau)$ maps from $(\Theta,\mathfrak{P}(\Theta),\mathrm{Cr})$ to a collection of random variables on $(\Omega,\mathfrak{A}(\Omega),P)$, denoted by ξ. Then $\xi=f(\zeta,\tau)$ is a random fuzzy variable defined on hybrid product space $(\Theta,\mathfrak{P}(\Theta),\mathrm{Cr})\times(\Omega,\mathfrak{A}(\Omega),P)$.

(3) Let $F(x;\theta)$ be the probability distribution of random variable τ with parameter θ (possibly vector-valued), then $F(x;\zeta)$ defines a random fuzzy variable ξ on the hybrid product space $(\Theta,\mathfrak{P}(\Theta),\mathrm{Cr})\times(\Omega,\mathfrak{A}(\Omega),P)$.

Note that Theorem 4 merely repeats facts stated in Liu's books, (2004, 2007).

4 STATIONARY RANDOM FUZZY CONTINUOUS-TIME MARKOV CHAIN

Let $X=\{X_t,t\geq 0\}$ be a Markov process with a standard stochastic semigroup $\mathrm{P}=\{\mathrm{P}_t,t\geq 0\}$ having a fuzzy rate matrix Q defined on credibility space $(\Theta,\mathfrak{P}(\Theta),\mathrm{Cr})$ with credibility distribution function matrix $\Lambda=(\Lambda_{ij})_{N\times N}$. Then by a direct application of Theorem 4, Item (3), a random fuzzy continuous-time Markov chain can be obtained.

Definition 9: A process is called a random fuzzy continuous-time Markov chain $\xi=\{\xi_t,t\geq 0\}$ taking values in set $\mathbb{S}=\{0,1,2,\cdots,N-1\}$, if

(a) $\xi=\{\xi_t,t\geq 0\}$ satisfies a Markov property:

$$\begin{aligned}&\Pr\left\{\xi_t=j\,|\,\xi_{t_1}=i_1,\xi_{t_2}=i_2,\cdots,\xi_s=i\right\}\\&=\Pr\left\{\xi_t=j\,|\,\xi_s=i\right\}\end{aligned}\tag{17}$$

for all $t_1<t_2<\cdots<s<t$ and any $i_1,i_2,\cdots,i,j\in\mathbb{S}$.

(b) the stochastic semigroup $\mathrm{P}=\{\mathrm{P}_t,t\geq 0\}$ is standard;

(c) and the fuzzy rate matrix

$$\mathrm{Q}=(q_{ij})_{N\times N}=\lim_{t\downarrow 0}\frac{\mathrm{P}_t-\mathrm{I}}{t}\tag{18}$$

is defined on credibility space $(\Theta,\mathfrak{P}(\Theta),\mathrm{Cr})$ with credibility distribution function matrix $\Lambda=(\Lambda_{ij})_{N\times N}$.

It is obvious that in Definition 9 for a given value of matrix $\mathrm{Q}=\mathrm{Q}_0$, $\xi=\{\xi_t,t\geq 0\}$ is a probabilistic continuous-time Markov chain. However, if Q is a fuzzy matrix, then for any given time t, the count ξ_t is a random fuzzy variable according to Theorem 5. Therefore, Definition 9 defines a stationary random fuzzy Poisson process.

Theorem 5: If the process $\xi=\{\xi_t,t\geq 0\}$ is currently halted at state i, it halts in state i during a a time interval which is exponentially distributed with fuzzy parameter q_i, independently of how and when the process reached state i and of how long it has been there. Furthermore, The process $\xi=\{\xi_t,t\geq 0\}$ leaves state i, and moves to state j with a fuzzy probability q_{ij}/q_i $(i\neq j)$.

Proof: A straightforward application of Definition 9 and Theorem 2.

Corollary 2: If q_{ij} $(i\neq j)$, $i,j=0,1,\cdots,N-1$, follow piecewise linear credibility distributions

$$\Lambda_{ij}(x)=\begin{cases} 0 & x<a_{ij} \\ \dfrac{x-a_{ij}}{2\left(b_i-a_{ij}\right)} & a_{ij}\le x<b_{ij} \\ \dfrac{x+c_{ij}-2b_{ij}}{2\left(c_{ij}-b_{ij}\right)} & b_{ij}\le x<c_{ij} \\ 1 & x\ge c_{ij} \end{cases}, \ i\ne j \tag{19}$$

The halting times, denoted by T_i, $i=0,1,\cdots,N-1$, are independent random fuzzy exponential variables with fuzzy parameter $q_i=\sum_j q_{ij}$ following a piecewise linear credibility distribution

$$\Lambda_i(x)=\begin{cases} 0 & x<a_i \\ \dfrac{x-a_i}{2\left(b_i-a_i\right)} & a_i\le x<b_i \\ \dfrac{x+c_i-2b_i}{2\left(c_i-b_i\right)} & b_i\le x<c_i \\ 1 & x\ge c_i \end{cases}, \tag{20}$$

where

$$\begin{cases} a_i=\sum_{j=1,j\ne i}^{N-1} a_{ij} \\ b_i=\sum_{j=1,j\ne i}^{N-1} b_{ij} \\ c_i=\sum_{j=1,j\ne i}^{N-1} c_{ij} \end{cases} \tag{21}$$

Thus the average chance distributions (for holding times) are

$$\begin{aligned}\Psi_i(t)&=\int_0^1 \mathrm{Cr}\left\{\theta: q_i(\theta)\ge -\ln(1-\alpha)/t\right\}\mathrm{d}\alpha \\ &=1+\frac{e^{-b_i t}-e^{-a_i t}}{2\left(b_i-a_i\right)t}+\frac{e^{-c_i t}-e^{-b_i t}}{2\left(b_i-c_i\right)t}\end{aligned} \tag{22}$$

Proof: Note that

$$\Pr\left\{T\left(q_i\right)\le t\right\}=1-e^{-q_i t} \tag{23}$$

Therefore the event $\left\{\theta:\Pr\left\{T\left(q_i(\theta)\right)\le t\right\}\ge\alpha\right\}$ is a fuzzy event and is equivalent to the fuzzy event $\left\{\theta: q_i(\theta)\ge -\ln(1-\alpha)/t\right\}$. As a critical part of the derivation of the average chance distribution, it is necessary to calculate the credibility measure for fuzzy event $\left\{\theta: q_i(\theta)\ge -\ln(1-\alpha)/t\right\}$, i.e., to obtain the expression for

$$\mathrm{Cr}\left\{\theta: q_i(\theta)\ge -\ln(1-\alpha)/t\right\} \tag{24}$$

Recall that for the credibilistic fuzzy variable, $q_i=\sum_{j\ne i} q_{ij}$, the credibility measure takes the form

$$\mathrm{Cr}\left\{\theta : q_i\left(\theta\right) > x\right\} = \begin{cases} 1 & x < a_i \\ \dfrac{2b_i - a_i - x}{2\left(b_i - a_i\right)} & a_i \le x < b_i \\ \dfrac{c_i - x}{2\left(c_i - b_i\right)} & b_i \le x < c_i \\ 0 & x \ge c_i \end{cases}, \tag{25}$$

Accordingly, the range for integration over α can be determined as shown in Table 1. Recall that the expression of $x = -\ln\left(1-\alpha\right)/t$ appears in Equation (25), which constitutes the link between intermediate variable α and average chance measure.

The average chance distribution for the exponentially distributed random fuzzy lifetime is then derived by splitting the integration into five terms according to the range of α and the corresponding mathematical expression for the credibility measure $\mathrm{Cr}\left\{\theta : q_i\left(\theta\right) \ge -\ln\left(1-\alpha\right)/t\right\}$, which is detailed in the following table.

Table 1. Range analysis for α

x	α and credibility measure expression	
$-\infty < x < a$	Range for α	$0 \le \alpha \le 1 - e^{-at}$
	$\mathrm{Cr}\{\lambda(\theta) \ge -\ln(1-\alpha)/t\}$	1
$a \le x < b$	Range for α	$1 - e^{-at} < \alpha \le 1 - e^{-bt}$
	$\mathrm{Cr}\{\lambda(\theta) \ge -\ln(1-\alpha)/t\}$	$1 - (x-a)/(2(b-a))$
$b \le x < c$	Range for α	$1 - e^{-bt} < \alpha \le 1 - e^{-ct}$
	$\mathrm{Cr}\{\lambda(\theta) \ge -\ln(1-\alpha)/t\}$	$(c-x)/2(c-b)$
$c \le x < +\infty$	Range for α	$1 - e^{-ct} < \alpha \le 1$
	$\mathrm{Cr}\{\lambda(\theta) \ge -\ln(1-\alpha)/t\}$	0

Then the exponential random fuzzy lifetime has an average chance distribution function:

$$\begin{aligned} \Psi\left(t\right) &= \int_0^1 \mathrm{Cr}\left\{\theta : \lambda\left(\theta\right) \ge -\ln\left(1-\alpha\right)/t\right\} \mathrm{d}\alpha \\ &= 1 + \frac{e^{-bt} - e^{-at}}{2\left(b-a\right)t} + \frac{e^{-bt} - e^{-ct}}{2\left(c-b\right)t} \end{aligned} \tag{26}$$

and the average chance density is

$$\begin{aligned} \psi(t) &= \frac{e^{-at} - e^{-bt}}{2\left(b-a\right)t^2} + \frac{be^{-bt} - ae^{-at}}{2\left(b-a\right)t} \\ &+ \frac{e^{-bt} - e^{-bt}}{2\left(c-b\right)t^2} + \frac{ce^{-ct} - be^{-bt}}{2\left(c-b\right)t} \end{aligned} \tag{27}$$

This expression concludes the proof.

Similarly to the probabilistic reliability theory, we define a reliability function or survival function for a random fuzzy lifetime and accordingly name it as the average chance reliability function, which is defined accordingly as

$$\bar{\Psi}\left(t\right) = 1 - \Psi\left(t\right) \tag{28}$$

Then, for an exponential random fuzzy lifetime, the average chance reliability function is

$$\overline{\Psi}(t)=\frac{e^{-at}-e^{-bt}}{2(b-a)t}+\frac{e^{-bt}-e^{-ct}}{2(c-b)t} \tag{29}$$

Remark 1: The average chance distributions of jump probabilities q_{ij}/q_j do not have closed forms, which require the application of Zadeh's extension theorem (1978). However, the values of fuzzy probability q_{ij}/q_j fall in intervals

$$\left[\min\left(\frac{a_{ij}}{\sum_{j\neq i}a_{ij}},\frac{c_{ij}}{\sum_{j\neq i}c_{ij}}\right),\max\left(\frac{a_{ij}}{\sum_{j\neq i}a_{ij}},\frac{c_{ij}}{\sum_{j\neq i}c_{ij}}\right)\right] \tag{30}$$

which will inform the explorations of the process $\xi=\{\xi_t,t\geq 0\}$.

5 NON-STATIONARY RANDOM FUZZY CONTINUOUS-TIME MARKOV CHAIN

The probabilistic non-stationary continuous-time Markov chain is an extension to the stationary case except that the rate matrix is function of time, i.e., time-dependent. Hence, a non-stationary random fuzzy continuous-time Markov chain can be defined as follows.

Definition 10: A process is called as random fuzzy continuous-time non-stationary Markov chain $\xi=\{\xi_t,t\geq 0\}$ taking values in state space $\mathbb{S}=\{0,1,2,\cdots,N-1\}$, if:

(a) $\xi=\{\xi_t,t\geq 0\}$ satisfies Markov property:

$$\begin{aligned}&\Pr\left\{\xi_t=j\mid\xi_{t_1}=i_1,\xi_{t_2}=i_2,\cdots,\xi_s=i\right\}\\&=\Pr\left\{\xi_t=j\mid\xi_s=i\right\}\end{aligned} \tag{31}$$

for all $t_1<t_2<\cdots<s<t$ and any $i_1,i_2,\cdots,i,j\in\mathbb{S}$.

(b) for $\forall s<t$, $p_{ij}(s,t)=\Pr\{\xi_t=j\mid\xi_s=i\}$, the transitional probabilities satisfy:

(i) for a small time-increment *h*, $\xi=\{\xi_t,t\geq 0\}$ moves from state i to state j with (fuzzy) probability:

$$p_{ij}(t,t+h)=q_{ij}(t)h+o(h)\ h\downarrow 0(i\neq j) \tag{32}$$

(ii) for a small time-increment *h*, $\xi=\{\xi_t,t\geq 0\}$ remaining in state i with (fuzzy) probability:

$$p_{ii}(t,t+h)=1-q_i(t)h+o(h)\ h\downarrow 0 \tag{33}$$

where the rate functions are given by

$$q_i(t)=\sum_{j=0,j\neq i}^{N-1}q_{ij}(t),\ i=0,1,\cdots,N-1 \tag{34}$$

(c) The parameters of rate functions, i.e., the entries of the fuzzy rate matrix $\mathrm{Q}(t)=\left(q_{ij}(t)\right)_{N\times N}$ are credibilistic fuzzy variables defined on the common credibility measure space $(\Theta,\mathfrak{P}(\Theta),\mathrm{Cr})$.

Theorem 6: If the process $\xi=\{\xi_t,t\geq 0\}$ is currently halted at state i, it halts in state i during a time interval that is exponentially distributed with fuzzy parameter $q_i(t)$, independently of how and when the process reached state i and of how long it has been there. Furthermore, the process $\xi=\{\xi_t,t\geq 0\}$ leaves state i, and moves to state j with a fuzzy probability $q_{ij}(t)/q_i(t)\ (i\neq j)$.

Corollary 3: The probability distribution of halting times given the current state $\xi_{w_{l-1}}=x_{l-1}\in\mathbb{S}$, is

$$\begin{aligned}&\Pr\left\{W_l - w_{l-1} > t, \xi_{w_{l-1}} = x_{l-1}\right\} = \\ &\exp\left(-\left(m_{x_{l-1}}\left(w_{l-1}+t\right)\right) - m_{x_{l-1}}\left(w_{l-1}+t\right)\right)\end{aligned} \tag{35}$$

where

$$m_i\left(t\right) = \int_0^t q_i\left(u\right) du \tag{36}$$

is called the i^{th} integrated rate function.

Example 2: Assume a linear rate function:

$$q_{ij}\left(t\right) = \beta_{0,ij} + \beta_{1,ij} t,\ \left(j \neq i\right), \beta_{0,ij} > 0,\ \beta_{1,ij} > 0 \tag{37}$$

Further, we assume that β_0 and β_1 both have piecewise linear credibility distribution:

$$\Lambda_{ij}^{(k)}\left(x\right) = \begin{cases} 0 & x < a_{ij}^{(k)} \\ \dfrac{x - a_{ij}^{(k)}}{2\left(b_{ij}^{(k)} - a_{ij}^{(k)}\right)} & a_{ij}^{(k)} \le x < b_{ij}^{(k)} \\ \dfrac{x + c_{ij}^{(k)} - 2b_{ij}^{(k)}}{2\left(c_{ij}^{(k)} - b_{ij}^{(k)}\right)} & b_{ij}^{(k)} \le x < c_{ij}^{(k)} \\ 1 & x \ge c_{ij}^{(k)} \end{cases},\ k = 0,1 \tag{38}$$

Then the diagonal entries $q_i\left(t\right)$, $i = 0,1,\cdots,N-1$, have credibility distributions

$$\Lambda_i\left(x\right) = \begin{cases} 0 & x < a_i \\ \dfrac{x - a_i}{2\left(b_i - a_i\right)} & a_i \le x < b_i \\ \dfrac{x + c_i - 2b_i}{2\left(c_i - b_i\right)} & b_i \le x < c_i \\ 1 & x \ge c_i \end{cases},\ i = 0,1,\cdots,N-1 \tag{40}$$

where

$$\begin{cases} a_i = \sum_{j=0}^{N-1}\left(a_{ij}^{(0)} + a_{ij}^{(1)} t\right) \\ b_i = \sum_{j=0}^{N-1}\left(b_{ij}^{(0)} + b_{ij}^{(1)} t\right) \\ c_i = \sum_{j=0}^{N-1}\left(c_{ij}^{(0)} + c_{ij}^{(1)} t\right) \end{cases} \tag{41}$$

The integrated diagonal entries of $\mathrm{Q}\left(t\right)$:

$$m_i\left(t\right) = \beta_{0,i} t + \beta_{1,i} t^2 \tag{42}$$

will have credibility distributions:

$$\Lambda_{m_i(t)}(y) = \begin{cases} 0 & y < A_i \\ \dfrac{y - A_i}{2(B_i - A_i)} & A_i \le y < B_i \\ \dfrac{y + C_i - 2B_i}{2(C_i - B_i)} & B_i \le y < C_i \\ 1 & y \ge C_i \end{cases} \tag{43}$$

where

$$\begin{cases} A_i = \sum_{j=0}^{N-1} \left(a_{ij}^{(0)} t + a_{ij}^{(1)} t^2 \right) \\ B_i = \sum_{j=0}^{N-1} \left(b_{ij}^{(0)} t + b_{ij}^{(1)} t^2 \right) \\ C_i = \sum_{j=0}^{N-1} \left(c_{ij}^{(0)} t + c_{ij}^{(1)} t^2 \right) \end{cases} \tag{44}$$

In general, to obtain the credibility distribution denoted as $\Lambda_{m_i(t)}$, of the integrated intensity function $m(t)$, it is necessary to apply Zadeh's extension principle (1978), but for the piecewise linear credibility distribution case, the mathematical arguments are relatively simple.
Now let us derive the average chance distribution for the first halting times at i^{th} state (the initial state).

$$\Psi_T(t) = \int_0^1 \mathrm{Cr}\left(\theta : \Pr\left\{T_1(\theta) \le t\right\} \ge \alpha\right) d\alpha \tag{45}$$

Note that for the first arrival time,

$$\begin{aligned} &\left\{\theta : \Pr\left\{T_1(\theta) \le t\right\} \ge \alpha\right\} \\ &= \left\{\theta : 1 - \exp\left(-\int_0^t (\beta_0 + \beta_1 u)\, du \right) \ge \alpha \right\} \\ &= \left\{\theta : 1 - e^{-m(t)} \ge \alpha \right\} \\ &= \left\{\theta : m(t) \ge -\ln(1-\alpha) \right\} \end{aligned} \tag{46}$$

Therefore, the average chance distribution for T_1, the first halting at state *i*, is

$$\begin{aligned} &\Psi_{T_i}(t) \\ &= \int_0^1 \mathrm{Cr}\left(\theta : \Pr\left\{T_1(\theta) \le t\right\} \ge \alpha\right) d\alpha \\ &= \int_0^1 \mathrm{Cr}\left(\theta : m(t) \ge -\ln(1-\alpha)\right) d\alpha \end{aligned} \tag{47}$$

We observe that $y = -\ln(1-\alpha)$, therefore,

$$\operatorname{Cr}\left\{m(t)>y\right\}=\begin{cases}1 & y<A_i\\ \dfrac{2B_i-2-y}{2\left(B_i-A_i\right)} & A_i\le y<B_i\\ \dfrac{C_i-y}{2\left(C_i-B_i\right)} & B_i\le y<C_i\\ 0 & y\ge C_i\end{cases} \tag{48}$$

Hence,

$$\begin{aligned}&\Psi_{T_i}(t)=\\&1-e^{-m(A_i)}+\frac{2B_i-A_i-1}{2\left(B_i-A_i\right)}\left(e^{-m(A_i)}-e^{-m(B_i)}\right)\\&+\frac{1}{2(B_i-A_i)}\left(-m\left(B_i\right)e^{-m(B_i)}+m\left(A_i\right)e^{-m(A_i)}\right)\\&+\frac{C_i-1}{2\left(C_i-B_i\right)}\left(e^{-m(B_i)}-e^{-m(C_i)}\right)\\&+\frac{1}{2(C_i-B_i)}\left(-m\left(C_i\right)e^{-m(C_i)}+m\left(B_i\right)e^{-m(B_i)}\right)\end{aligned} \tag{49}$$

6 A PARAMETER ESTIMATION SCHEME

Here parameter estimation is essentially a problem of estimation of credibility distributions from fuzzy observations. Guo and Guo (2009) recently proposed a maximally compatible random variable to a credibilistic fuzzy variable and thus the fuzzy estimation problem is converted into estimating the distribution function of the maximally compatible random variable. The following scheme is for estimating a piecewise linear credibility distribution.

Definition 11: Let X be a random variable defined in $\left(\mathbb{R},\mathfrak{B}(\mathbb{R})\right)$ such that

$$\mu^c=\operatorname{Cr}\circ\xi^{-1}=\mu=P\circ X^{-1} \tag{50}$$

Then X is called a maximally compatible to fuzzy variable ξ.

In other words, a random variable X can take all the possible real-values the fuzzy variable ξ may take and the distribution of X , $F_X(r)$ equals the credibility distribution of ξ, $\Lambda_\xi(r)$ for all $r\in\mathbb{R}$.

It is observed that the induced measure $\mu^c=\operatorname{Cr}\circ\xi^{-1}$ and measure $\mu=P\circ X^{-1}$ are defined on the same measurable space $\left(\mathbb{R},\mathfrak{B}(\mathbb{R})\right)$. Furthermore, we note that the pre-image $\xi^{-1}(B)\in\mathfrak{P}(\Theta)$, but the pre-image $X^{-1}(B)\in\mathfrak{A}(\Theta)\subset\mathfrak{P}(\Theta)$, which implies that for the same Borel set $B\in\mathfrak{B}(\mathbb{R})$, the pre-images under fuzzy variable ξ and random variable X are not the same. It is expected that

$$\left\{\theta\in\Theta:X(\theta)\le r\right\}\subseteq\left\{\theta\in\Theta:\xi(\theta)\le r\right\} \tag{51}$$

but

$$\Pr\left\{\theta\in\Theta:X(\theta)\le r\right\}=\operatorname{Cr}\left\{\theta\in\Theta:\xi(\theta)\le r\right\} \tag{52}$$

The statistical estimation scheme for parameters (a,b,c) of the credibility distribution based on fuzzy observations $\{x_1,x_2,\cdots,x_n\}$ can be stated as:

Estimation Scheme 1:

Step 1: Rank fuzzy observations $\{x_1, x_2, \cdots, x_n\}$ to obtain "order" statistics $\{x_{(1)}, x_{(2)}, \cdots, x_{(n)}\}$ in ascending order;

Step 2: Set $\hat{a} = x_{(1)}$ and $\hat{c} = x_{(n)}$;

Step 3: Set a tentative estimator for b,

$$\hat{b}_e = \frac{4\overline{x}_n - x_{(1)} - x_{(n)}}{2} \tag{53}$$

where

$$\overline{x}_n = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{54}$$

Step 4: Identify $x_{(i_0)}$ from $\{x_{(1)}, x_{(2)}, \cdots, x_{(n)}\}$ such that $x_{(i_0)} \le \hat{b}_e < x_{(i_1)}$ and $1 < i_0 < i_1$, then we may see $\{x_{(1)}, x_{(2)}, \cdots, x_{(i_0)}\}$ as a set of order statistics from uniform [*a*,*b*]. Hence the "sufficient" statistic for parameter b is $x_{(i_0)}$.

Then $(\hat{a}, \hat{b}, \hat{c}) = (x_{(1)}, x_{(i_0)}, x_{(n)})$ is the parameter estimator for the piecewise linear credibility distribution.

$$\hat{\Lambda}(x) = \begin{cases} 0 & x < \hat{a} \\ \dfrac{x - \hat{a}}{2(\hat{b} - \hat{a})} & \hat{a} \le x < \hat{b} \\ \dfrac{x + \hat{c} - 2\hat{b}}{2(\hat{c} - \hat{b})} & \hat{b} \le x < \hat{c} \\ 1 & x \ge \hat{c} \end{cases} \tag{55}$$

The next issue is how to extract the information on matrix rate Q in the stationary random fuzzy continuous-time Markov chain. Basawa and Prakasa Rao (1980) developed a maximum likehood procedure for estimating the entries q_{ij} in Q.

It is noted that for a given random fuzzy continuous-time Markov chain $\xi = \{\xi_t, t \ge t\}$, if we fix the fuzzy rate matrix at a given value Q_0, then $\xi = \{\xi_t, t \ge t\}$ becomes a probabilistic continuous Markov chain, Obtain the sample of the process: $K_\tau = \{N_\tau, X(0), W_1, X(W_1+), W_2, \cdots, W_{N_\tau} X(W_{N_\tau}+)\}$, which is sufficient. Then an MLE estimator for Q_0, denoted as $\hat{\mathrm{Q}}_0$ is obtained. Repeat the sampling procedure from the random fuzzy continuous-time Markov chain as many times as possible, say, m times, then the fuzzy rate matrix "observation" sequence is

$$\{\hat{\mathrm{Q}}_1, \hat{\mathrm{Q}}_2, \cdots, \hat{\mathrm{Q}}_m\} = \{(\hat{q}_{ij}^{(1)}), (\hat{q}_{ij}^{(2)}), \cdots, (\hat{q}_{ij}^{(m)})\} \tag{56}$$

Apply the Estimation Scheme 1 to the estimated observations at $(i,j)^{th}$ entry of rate matrix Q $\{\hat{q}_{ij}^{(1)}, q_{ij}^{(2)}, \cdots, \hat{q}_{ij}^{(m)}\}$, then the piecewise linear credibility distribution shown in Equation (55) for q_{ij}.

For the non-stationary random fuzzy continuous-time Markov chain, the parameters specifying the rate matrix $\mathrm{Q}(t;\underline{\beta})$, we may use a maximum likelihood procedure for estimating the parameters that define fuzzy parameters $\underline{\beta}$. Therefore the idea is similar to that of stationary case but the credibility distribution treatments involved may be very complicated, since Zadeh's extension principle (1978) must be applied. And mean measure involves two linear piecewise credibility distributions for fuzzy parameters β_0 and β_1 respectively.

7 A SIMULATION SCHEME

Simulation of a random fuzzy continuous-time Markov chain is intrinsically two-stage procedure: a fuzzy parameter simulation for generating realizations $\left\{\left(q_{ij}^{(1)}\right),\left(q_{ij}^{(2)}\right),\cdots,\left(q_{ij}^{(m)}\right)\right\}$ from a matrix of credibility distribution functions $\left(\Lambda_{ij}\right)$ and then for each realization of $\left(q_{ij}\right)$, a probabilistic continuous-time Markov chain is simulated. Repeat this procedure until all the $\left(q_{ij}\right)$ realizations are complete.

As to the fuzzy parameter simulation, following Guo and Guo (2009), we utilize the concept of a maximally compatible random variable to a fuzzy variable and the inverse transformation of the probability distribution function approach to generate fuzzy variable realizations. An algorithm is stated as follows:

Simulation scheme 1:

Step 1: Simulate a uniform random variable U[0,1], and denote the simple random sample as $\{u_1,u_2,\cdots,u_n\}$;

Step 2: Set $\Lambda(x_i)=u_i,\ (k=1,2,\cdots,n)$;

Step 3: Set $x_i,\ (i=1,2,\cdots,n)$:

$$x_i=\begin{cases} a+2(b-a)u_i & \text{if } 0\le u_i\le 0.5 \\ 2b-c+2(c-b)u_i & \text{if } 0.5\le u_i\le 1 \end{cases} \tag{57}$$

Then $\{x_1,x_2,\cdots,x_n\}$ is a sample from the fuzzy variable ξ with a piecewise linear credibility distribution Λ.

Step 4: Repeat Step 1 to Step 3, until m realizations of fuzzy rate matrix $\{Q_1,Q_2,\cdots,Q_m\}$ are obtained.

Step 5: For each rate matrix, say, Q_i, simulate a probabilistic continuous-time Markov chain, until m set of realizations of random fuzzy continuous-time Markov chain are obtained.

It should be mentioned that simulating a probabilistic continuous-time Markov chain is well-established in the literature.

8 CONCLUSION

In this paper, we give a systematic treatment of random fuzzy continuous-time Markov chains not only for the stationary one, and then for the non-stationary case, but also propose a parameter estimation scheme and a simulation scheme. In this way, the foundation is provided for the random fuzzy continuous-time Markov chains, although in its early stage. The applications to reliability engineering fields and the risk analysis now can extend from case with only random uncertainty to case with both co-existing randomness and fuzziness. It is expected that this development will assist reliability and risk analysis researchers as well as reliability analysts and engineers.

9 REFERENCES

Basawa, I.V. and Prakasa Rao, B. L.S. 1980. *Statistical Inference for Stochastic Processes*. London: Academic Press.

Carvalho, H. and Machado, V.C. 2006. Fuzzy set theory to establish resilient production systems. *Proceedings of IIE Annual Conference and Exhibition 2006*.

Chan, J.K. and Shaw, L. 1993. Modelling Repairable systems with failure rates that depend on age & maintenance. *IEEE Transactions on Reliability*, 42(4): 566-571.

Dohi, T., Aoki, T., Kaio, N., and Osaki, S. 2006. Statistical estimation algorithms for some repair-limit replacement scheduling problems under earning rate criteria, *Computers & Math. Applic.*, 51, 345-356.

Grimmett, G.R. and Stirzaker, D.R. 1992. *Probability and Random Processes*. Second Edition. Clarendon Press, Oxford, London.

Guo, R. and Love, C.E. 1992. Statistical Analysis of An Age Model for Imperfectly Repaired System. *Quality and Reliability Engineering International*, 8: 133-146.

Guo, R., and Love, C.E. 2004. Fuzzy Covariate Modelling an Imperfectly Repaired System. *Quality Assurance*, 10(37), 7-15, 2004.

Guo, R., Zhao, R.Q., Guo, D. and Dunne, T. 2007. Random Fuzzy Variable Modeling on Repairable System. *Journal of Uncertain Systems*, 1(3): 222-234, August, 2007.

Guo, R. 2009. Stochastic Processes. Unpublished Lecture Notes.

Guo, R. and Guo, D. 2009. Statistical Simulating Fuzzy Variable, Proceedings of the Eighth International Conference on Information and Management Sciences, Kunming & Banna, China, July 20-28, 2009, pp. 775-778. Series of Information & Management Sciences, Series Editor: Eldon Y. Li, Cal Poly State University.

Kijima, M. 1989. Some Results for Repairable Systems with General Repair. *Journal of Applied Probability*, 26: 89-102.

Lim, T.J. and Lie, C.H. 2000. Analysis of system reliability with dependent repair models. *IEEE Transactions on Reliability*, 49: 153-162.

Kolowrocki, K. 2007. Reliability modelling of complex system – part 1. *Proceedings of the First Summer Safety and Reliability Seminars*, July 22-29, 2007, Gdańsk-Sopot, Poland, 219-230.

Kolowrocki, K. 2007. Reliability modelling of complex system – part 2. *Proceedings of the First Summer Safety and Reliability Seminars*, July 22-29, 2007, Gdańsk-Sopot, Poland, 231-242.

B. D. Liu. 2004. *Uncertainty Theory: An Introduction to Its Axiomatic Foundations*. Berlin: Springer-Verlag Heidelberg.

B. D. Liu. 2007. *Uncertainty Theory: An Introduction to Its Axiomatic Foundations*. 2nd Edition. Berlin: Springer-Verlag Heidelberg.

Love, C.E., Zhang, Z.G., Zitron, M.A., and Guo, R. 2000. A Discrete Semi-Markov Decision Model to Determine the Optimal Repair/Replacement Policy under General Repairs. *European Journal of Operational Research*, 125: 398-409.

Wang, H. and Pham, H. 1996. Optimal age-dependent preventive maintenance policies with imperfect maintenance. *International Journal of Reliability, Quality and SafetyEngineering*, 3: 119-135.

Sheu, S.H., Lin, Y. B., and Liao, G. L. 2004. Optimum Policies with Imperfect Maintenance. *Proceedings of the 2004 Asian International Workshop*, *Advanced Reliability Modeling*, Dohi, T. and Yun W.Y. (Eds), 459-466.

Soszynska, J. 2007. Systems reliability analysis in variable operation conditions. *Proceedings of the First Summer Safety and Reliability Seminars*, July 22-29, 2007, Gdańsk-Sopot, Poland, 319-330.

Tamura, N. 2004. On a Markovian Deteriorating System with uncertain repair and replacement. *Proceedings of the 2004 Asian International Workshop*, *Advanced Reliability Modeling*, Dohi, T. and Yun W.Y. (Eds), 523-530.

Yun, W.Y., Lee, K.K., Cho, S.H. and Nam, K.H. 2004. Estimating Parameters of Failure Model for Repairable Systems with Different Maintenance Effects. *Proceedings of the 2004 Asian International Workshop*, *Advanced Reliability Modeling*, Dohi, T. and Yun W.Y. (Eds), 609-616.

Zadeh, L.A. 1965. Fuzzy sets. *Information and Control*, 8: 338-353.

Zadeh, L.A. 1978. Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems*, 1: 3-28.

Zhang, Y.L. Yam, R.C.M., and Zuo, M.J. 2002. Optimal replacement policy for a multistate repairable system. *Journal of the Operational Research Society*, 53: 336-341.

THE COMPUTER PROGRAM TO VERIFY THE HYPOTHESIZES AND TO PREDICT OF THE PARAMETERS FOR OPERATIONAL PROCESS

S. Guze
B, Kwiatuszewska-Sarnecka
J. Soszyńska
•
Maritime University, Gdynia, Poland

e-mail: sambor@am.gdynia.pl

ABSTRACT

The theoretical background and technical information for the program are presented. Further, the components of the program are described and user manual is given.

1 INTRODUCTION

The computers programs are the tool to make life easier. Especially it is important if we have to perform a lot of complex and laborious calculations.

It is usually when we are a contactors in research projects. Since 2007 team of Department of Mathematics in Gdynia Maritime University is working on Poland – Singapore Joint Research Project entitled “Safety and Reliability of Complex Industrial Systems and Processes”. The described computer program is one of the tools for this Project.

2 THEORETICAL BACKGROUND AND TECHNICAL INFORAMTION

The computer program is written in Java with using SSJ V2.1.3. The SSJ is a Java library for stochastic simulation, developed in the Département d'Informatique et de Recherche Opérationnelle (DIRO), at the Université de Montréal.
The computer program implements the results from WP6 Poland – Singapore Joint Research Project.
Its first part is verifying the hypothesizes about the conditional distribution functions $H_{bl}(t)$ of the system operation process $Z(t)$ sojourn times θ_{bl}, $b, l = 1,2,...,\nu$, $b \neq l$, in the state z_b while the next transition is the state z_l on the base of their realizations θ_{bl}^k, $k = 1,2,...,n_{bl}$ during the experiment time Θ. We assume that the typical distributions to describe these sojourn times are:

- the uniform distribution;
- the triangle distribution;
- the double trapezium distribution;
- the quasi-trapezium distribution;
- the exponential distribution;
- the Weibull distribution;
- the normal distribution;
- the chimney distribution.

The computer program uses to verify the hypothesizes a non-parametric chi-square goodness-of-fit test.
Second aim of the program is to estimate the unknown parameters of the system operation process.

It estimates the following parameters:
- the matrix of probabilities of the system operation process $Z(t)$ transitions between the operation states $[p_{bl}]$
- the mean values $M_b = E[\theta_b]$ of the unconditional sojourn times θ_b, $b = 1,2,...,v$,
- the steady probabilities π_b, $b = 1,2,...,v$,

the limit values of the transient probabilities at the particular operation states p_b.

3 COMPONENTS OF THE COMPUTER PROGRAM

There are two main tabs in computer application. One of them gives possibility to verifying hypotheses about distribution function of sojourn times in particular operational states. Second one gives the predicts of parameters of operational process. (see Figure 1 and Figure 2).

Figure 1. Main window for verifying the hypothesizes

As it is shown in Figure 1 the window of the section for verifying hypothesizes has the two parts: INPUT and OUTPUT.
The INPUT is composed by the following components:
- button to choose the file with probe;
- text field with the path to chosen file;
- text field to set a factor α - the level of significance for investigated hypothesizes,
- button to start of the verification of the hypothesizes.

In OUTPUT it is shown the following results in particular text fields:
- size of probe – n_{bl}
- number of subintervals – r;
- length of subintervals – d;
- the begin of the interval and the end of the one (x_{bl}, y_{bl}),
- mean value from probe,
- values qbl, wbl and in case of quasi-trapezium distribution: zbl1, zbl2,
- values of theoretical statistics,
- values of statistics u_n,

- mean value of the conditional sojourn times θ_{bl} of the system operations process at the operations state $H_{bl}(t)$ when the next transition is to the operation state θ_{bl} – Mbl.

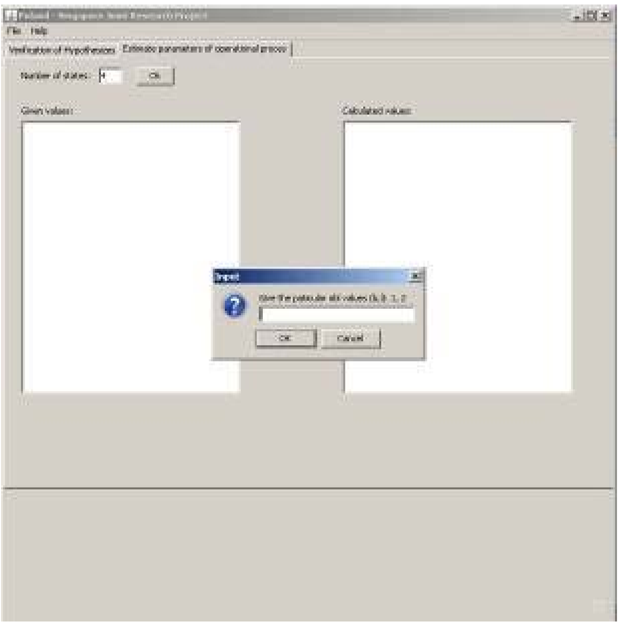


Figure 2. Main window for estimation the parameters of the operational process

In the text area it is shown:
- name of the validate distribution
- density function for this distribution.

In the case of using the computer program to estimate the unknown parameters of operational process, the main window has following components:
a) text field to set a number of operational states,
b) button to accept setting parameter,
c) text area to present the given data
d) text area to present the determined values.

After pressing the button "OK":
a) the program allows to set the following values:
   - the matrix of the realizations of the numbers of the transients of the system operation process between the operation states,
   - the matrix of the realizations of the mean values M_{bl} of the conditional sojourn times θ_{bl} of the system operations process at the operations state $H_{bl}(t)$ when the next transition is to the operation state θ_{bl} ;
b) the program determines:
   - the matrix of the realizations of the probabilities p_{bl} , $b, l = 1,2,..., v$, of the system operations process transitions from the operations state z_b to the operations state z_l during the experiment time Θ,
   - the vector of the mean values $M_b = E[\theta_b]$ of the unconditional sojourn times θ_b, $b = 1,2,..., v$,
   - the vector of the probabilities π_b of the vector $[\pi_b]_{1xv}$, $b = 1,2,..., v$,

the vector of the limit values of the transient probabilities at the particular operation states.

4 INSTRUCTIONS FOR USERS

Now, we present the steps how fluently using the particular sections of the computer program.

4.1 Verifying the hypothesizes

Our work with program we start from preparing the data file. This text file should include a data set in one column as below example shows. (see *Example 1*)

Example 1. Correct form of the text file with data.

...
34.6
31.0
56.9
60.4
...

When we have the text file in correct form we can use the program with the following instruction of use:

In the section “INPUT”:
Step 1. Press the button “Read TXT file with data” to choose the file with the data set,

Step 2. Set the level of significant alfa;

Step 3. Press the button “TESTING”.

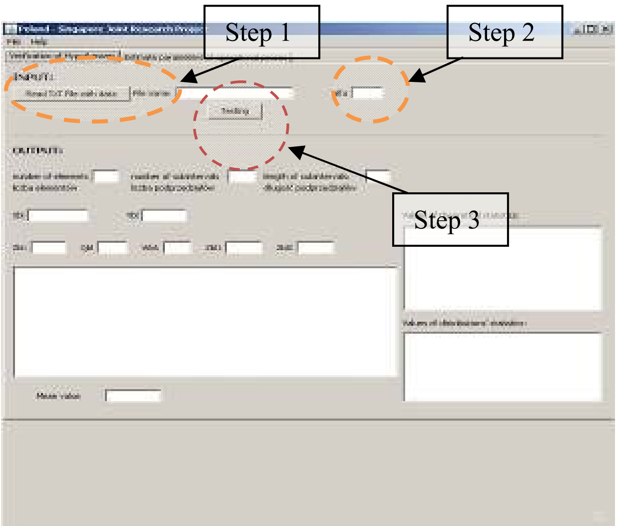


Figure 3. Instructions for users

The computer program fits the correct distribution function for the included file and shows results in section “OUTPUT” as it has been described before.

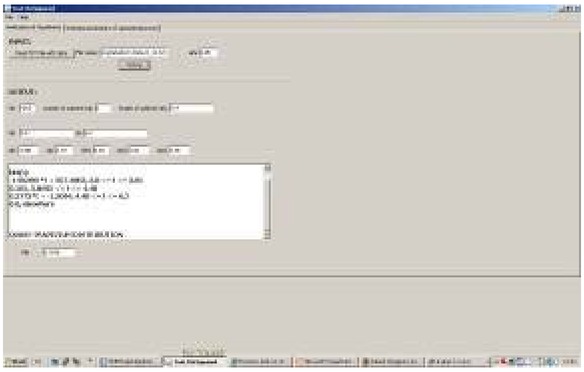


Figure 4. Exemplary results of the computer program

4.2 Predicts of operational process parameters

The instruction of use is as follows:

Step 1. Set the number of states.

Step 2. Press the "OK" button.

Step 3. Set the realizations of the numbers of the transients of the system operation process between the operation states,

Step 4. Set the matrix of the realizations of the mean values M_{bl} of the conditional sojourn times θ_{bl} of the system operations process at the operations state $H_{bl}(t)$ when the next transition is to the operation state θ_{bl} ;

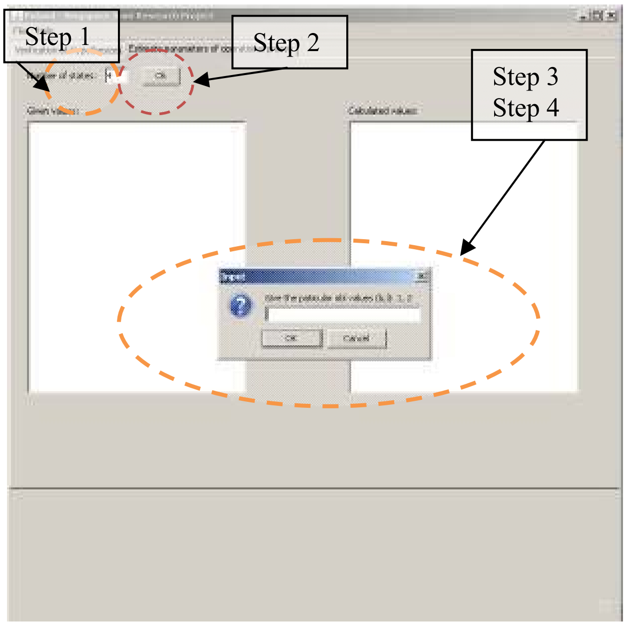


Figure 5. Instruction of use for prediction parameters

The computer program based on these values calculates:

a) the matrix of the realizations of the probabilities p_{bl} , $b, l = 1,2,...,\nu$, of the system operations process transitions from the operations state z_b to the operations state z_l during the experiment time Θ,

b) the vector of the mean values $M_b = E[\theta_b]$ of the unconditional sojourn times θ_b, $b = 1,2,...,v$,

c) the vector of the probabilities π_b of the vector $[\pi_b]_{1xv}$, $b = 1,2,...,v$,

d) the vector of the limit values of the transient probabilities at the particular operation states.

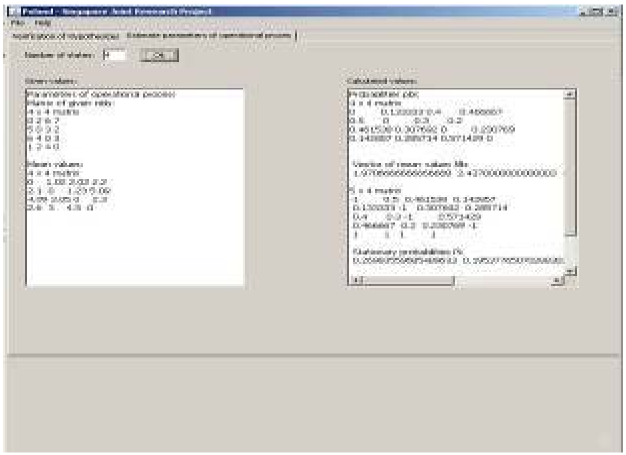
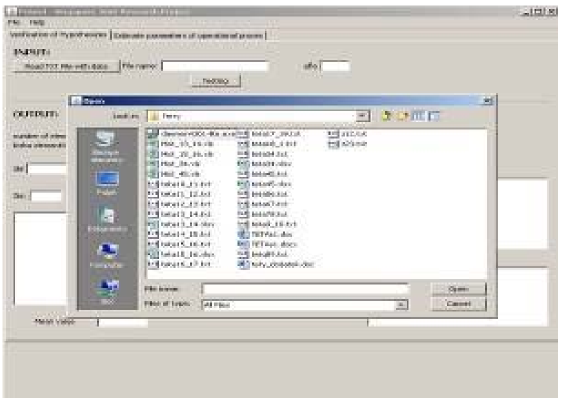
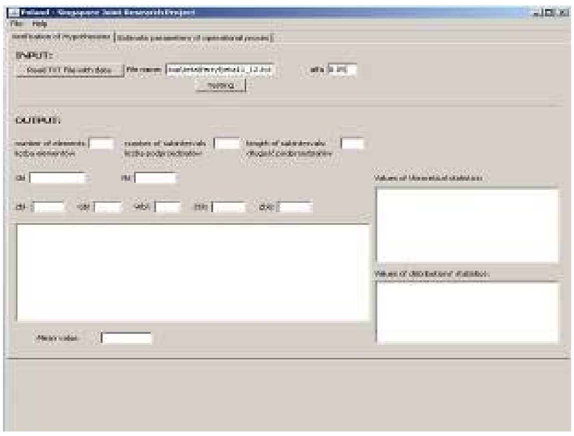


Figure 4. Exemplary results for estimating parameters

5 APPLICATIONS

Example 2. There are shown the consecutive steps of using the computer programme for hypothesizes verification.





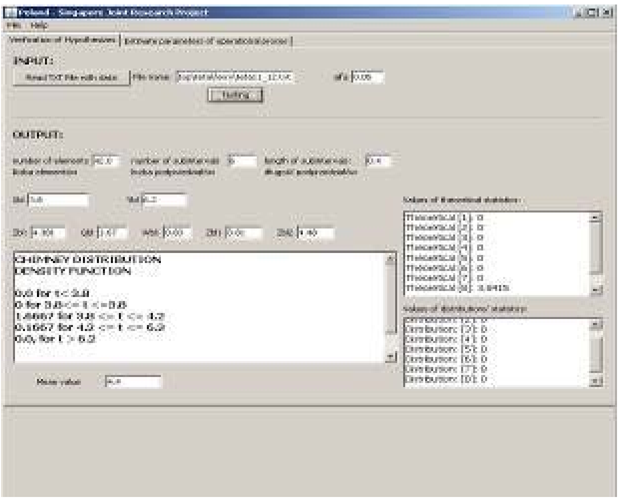
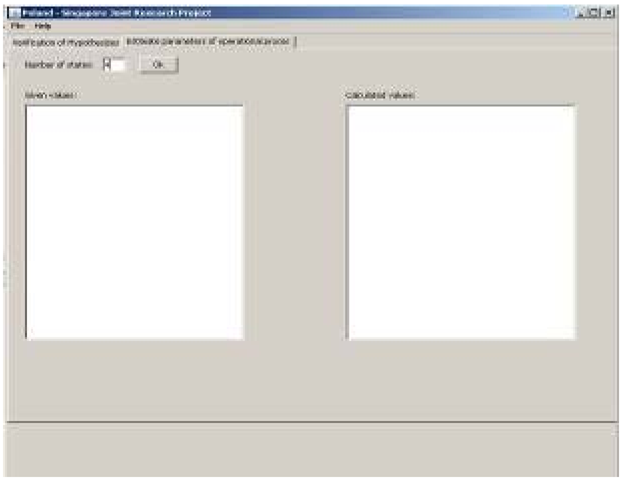
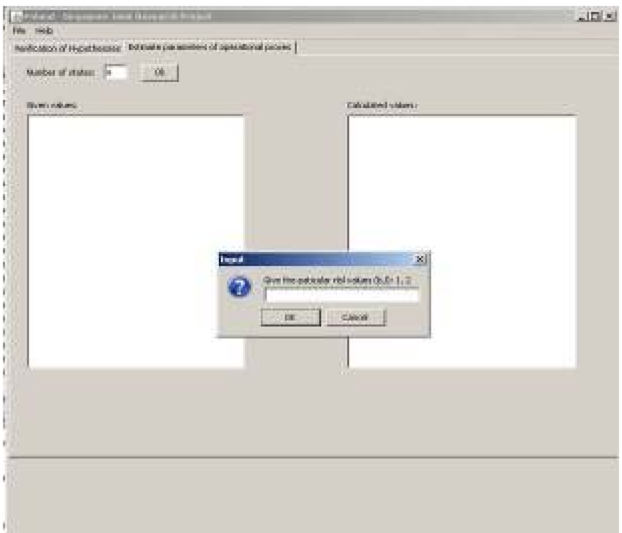


Figure 5. The steps of using computer programme for the verify the hypothesizes

Example 3. It is shown how to use the computer programme for validation of parameters of operational processes.





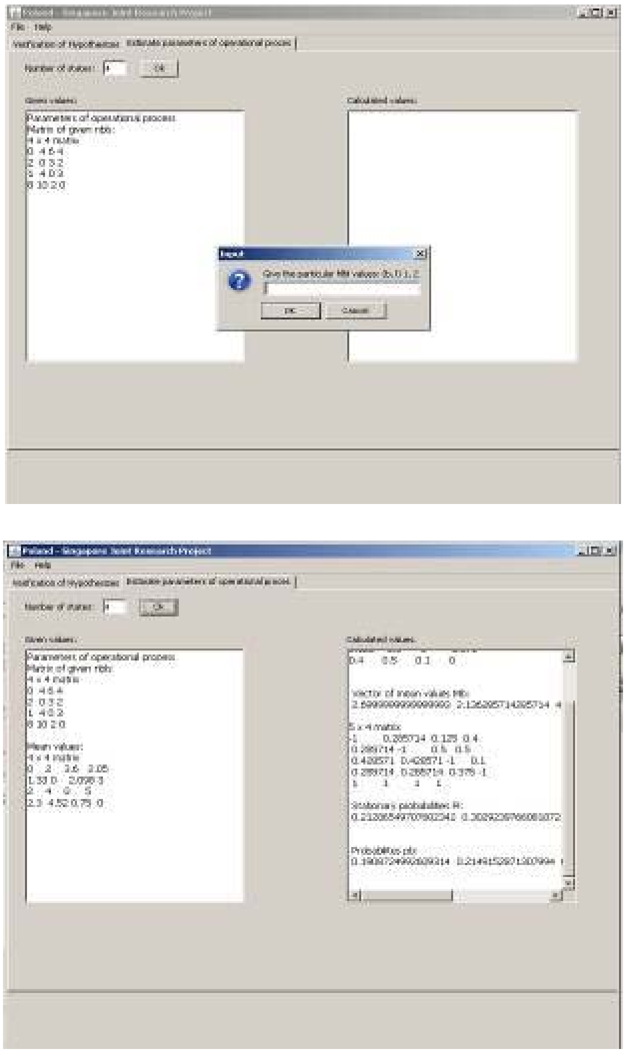


Figure 6. The steps of using the computer programme for prediction of parameters of operational process

6 CONCLUSIONS

The paper has described the computer program for Poland - Singapore Joint Research Project. The theoretical backgrounds and the technical information have been presented. Further, the short introduction about components of the program have been discussed and the manual for users has been given. The computer program can be used to verification and prediction for every operational process.

Acknowledgements

The paper describes the work in the Poland-Singapore Joint Research Project titled “Safety and Reliability of Complex Industrial Systems and Processes” supported by grants from the Poland’s Ministry of Science and Higher Education (MSHE grant No. 63/N-Singapore/2007/0) and the Agency for Science, Technology and Research of Singapore (A*STAR SERC grant No. 072 1340050).

REFERENCES

Barlow, R. E. & Proschan, F. 1975. *Statistical Theory of Reliability and Life Testing. Probability Models*. Holt Rinehart and Winston, Inc., New York.

Eckel, B. 2005. *Thinking in Java, 4th Edition*. Prentice Hall.

Kołowrocki, K. Soszyńska, J. *A general model of technical systems operation processes related to their environment and infrastructure. Task 2.1*, Poland - Singapore Joint Research Project, internal papers.

Kołowrocki, K. Kwiatuszewska – Sarnecka, B. *Methods – Algorithms for evaluating unknown parameters of operation processes. Task 6.1*, Poland – Singapore Joint research Project, Internal papers.

On-line documentation of SSJ. http://www.iro.umontreal.ca/~simardr/ssj/indexe.html

CONFIDENCE BOUNDS FOR THE RELIABILITY OF A SYSTEM FROM SUBSYSTEM DATA

O. Hryniewicz

Systems Research Institute, Warsaw, Poland

e-mail: Olgierd.Hryniewicz@ibspan.waw.pl

ABSTRACT

The paper is concerned with the construction of lower bounds for the reliability of a system when statistical data come from independent tests of its elements. The overview of results known from literature and obtained under the assumption that elements in a system are independent is given. It has been demonstrated using a Monte Carlo experiment that in case when these elements are dependent and when their dependence is described by Clayton and Gumbel copulas these confidence bounds are not satisfactory. New simple bounds have been proposed which in some practical cases perform better than the classical ones.

1 INTRODUCTION

Reliability indices of complex systems can be estimated from the results of lifetime tests. When a system is treated as a one entity we can distinguish two different types of reliability tests. In the first one, we observe consecutive failures of a system, and after each of them a failed system is completely renewed. In such a case, random times between consecutive failures are described by random variables having independent and identical probability distributions. If this assumption is true, we can estimate a required reliability characteristic using a sample of observed lifetimes. In the second case, we have to observe several identical systems working in the same conditions. Times to first failures of these systems constitute a sample which may be used for the estimation of the considered reliability characteristic. In both cases, however, we need to have either sufficiently long time of test or sufficiently large number of observed systems. Both these requirements are seldom met in practice. Thus, this method of the reliability estimation is rarely used in practice despite the fact that from a statistical point of view the required estimators are obtained in the simplest possible way. Moreover, in such a case we do not profit from the information about the structure of the considered system, and from the knowledge of times to failure of its elements.

In practice we are frequently faced with a different problem: how to evaluate reliability characteristics of a system on its design stage. There exist many methods for the prediction of reliability using available statistical data. In this paper we consider the simplest one, when we can utilize the results of reliability tests of system's elements performed in presumably the same conditions as the conditions of work of the designed complex system.

Research studies on statistical methods aimed at the estimation of system's reliability using the results of reliability tests of its elements were initiated independently in the 1950s in the United States and the Soviet Union, where they were performed by prominent mathematicians and statisticians. Some strong mathematical results were obtained, and these results can be used for both point and interval estimation of system's reliability using the data obtained for its elements or subsystems. In this paper we will focus our attention on the interval estimation. The reason for the importance of the results of this type stems from practice. Usually we can use scarce reliability data, and thus the obtained point estimators are not very precise. Therefore, we need to know some lower bounds for the predicted reliability characteristics.

Preliminary analysis of the theoretical results shows undoubtedly that even in the cases of simple systems exact analytical methods require utilization of complex mathematical tools such as nonlinear mathematical programming. On the other hand, interesting approximate results, obtained mainly by American researchers, can be used in practice when a sufficiently large number of failures have been observed. For this reasons already in the 1980s the reliability theoreticians lost their interest in further research on those problems. However, the problem is still interesting for practitioners who need approximate, or even heuristic, methods which may be used for the prediction of reliability using existing statistical data.

The paper is partly based on the lecture for young scientists working in the area of reliability. Therefore the purpose of this paper is two-fold. In first six sections we give a short overview of different methods for the construction of confidence intervals for the reliability of systems. In all these methods it has been assumed that the elements of a system are independent. In the last section of the paper we present new heuristically designed bounds which seem to be robust to deviations from this assumption in certain practical cases.

2 GENERAL METHODOLOGY FOR THE EVALUATION OF SYSTEM'S RELIABILITY

Evaluation of reliability of complex systems became the subject of intensive theoretical investigation in the beginning of 1960s. Fundamental results were summarized in the famous book (Barlow & Proschan 1965). In the developed mathematical models we assume that both the system as a whole, and system's elements at any time instant $t>0$ are either in the state of functioning (or failure-free state), when the random variable $X(t)$ describing the reliability state adopts the value 1, or in the state of failure, when this random variable adopts the value 0. When the considered system consists of m elements, then its reliability state is described by the random vector $\mathbf{X}=(X_1,X_2,\ldots,X_m)$, and the probability of the observation of any reliability state is given by

$$P(\mathbf{X})=\prod_{i=1}^{m} p_i^{X_i}\left(1-p_i\right)^{1-X_i} \tag{1}$$

where

$$p_i = P(X_i = 1) = E(X_i),\ i = 1,\ldots,n\,. \tag{2}$$

In the above formulae we have omitted time t assuming that in case of specific calculations it adopts the same value for all components of the random vector.

Reliability state of the whole system depends on the states of all individual system's elements. Denote by Ω the set of all 2^m possible states of system's elements. We can divide this set into two exclusive subsets: the subset of all functioning states of the system G, and the system of all failure states of this system $\overline{G}$ ($G \cup \overline{G} = \Omega$). The function

$$\Psi(\mathbf{X})=\begin{cases}1 & \mathbf{X}\in G\\ 0 & \mathbf{X}\in \overline{G}\end{cases} \tag{3}$$

is called the *structure function*, and it describes the relation between reliability state of the whole system and reliability states of its elements. The effective construction of this function is the subject of numerous research works. Particular results may be found in all classical textbooks on reliability such as (Barlow & Proschan 1965, 1975).

Probability that the considered system is in the failure-free state depends on the vector $\mathbf{p}=(p_1,p_2,\ldots,p_m)$ that describes the probabilities of failure-free functioning of system's elements, and system's reliability structure function. It is given by the function called the reliability function which is given by the following formula

$$R(\mathbf{p}) = P(\mathbf{X} \in G) = E[\Psi(\mathbf{X})] = \sum_{\mathbf{X} \in G} \Psi(\mathbf{X}) \prod_{i=1}^{m} p_i^{X_i} (1-p_i)^{1-X_i} . \tag{4}$$

Below, we present the respective formulae for the reliability structures which are most frequently met in practice.

a) In case of a system with series reliability structures which consists of m groups of identical n_i, $i=1,\ldots,m$ elements we have:

$$R(\mathbf{p}) = \prod_{i=1}^{m} p_i^{n_i} \tag{5}$$

b) For the system with a parallel reliability structure which consists of m elements the respective formula is given by

$$R(\mathbf{p}) = 1 - \prod_{i=1}^{m} (1-p_i) = 1 - \prod_{i=1}^{m} q_i . \tag{6}$$

c) In case of a series-parallel reliability system which consists of m connected in series groups, where each of these groups consists of n_i connected in parallel identical elements, the reliability function is given by the formula:

$$R(\mathbf{p}) = \prod_{i=1}^{m} \left[1 - (1-p_i)^{n_i}\right]. \tag{7}$$

d) For a parallel-series system consisting of m connected in parallel groups, where each of these groups consists of n_i identical elements connected in series, the reliability function is given by the formula:

$$R(\mathbf{p}) = 1 - \prod_{i=1}^{m} \left[1 - \prod_{j=1}^{n_i} p_{ij}\right]. \tag{8}$$

In formulae (5) – (8) p_{ij} denotes the probability that the j-th element in the i-th subsystem is in a failure-free state.

The systems with structures described above belong to a more general class of systems called coherent systems, or systems with monotonic structure. The system has monotonic structure if

$$\Psi(\mathbf{X}) \geq \Psi(\mathbf{Y}) \tag{9}$$

holds when $X_i \geq Y_i$, $i=1,\ldots,m$, and when

$$\Psi(\mathbf{0}) = 0 ,\ldots \Psi(\mathbf{1}) = 1 , \tag{10}$$

with $\mathbf{0}=(0,\ldots,0)$ and $\mathbf{1}=(1,\ldots,1)$. For systems with a monotonic structure the reliability function can be always computed. However, for large and complex systems this can be a hard computational task.

In order to compute the probability that the system is in the failure-free state we need to know the estimates of the elements of the vector $\mathbf{p}$. These estimates can be obtained from the results of reliability tests. We assume that for each of system's elements we have the results of *independent* reliability tests. From these tests we obtain the vector of estimates $\mathbf{p}^*=(p_1^*, p_1^*,\ldots, p_m^*)$. The estimators p_i^* are unbiased estimators of unknown probabilities p_i only in certain particular cases.

However, in the majority of practical cases, when we apply the maximum likelihood method of estimation, these estimators are asymptotically unbiased, but in practice the conditions of asymptotics usually do not hold due to the limited number of the pertinent statistical data. The knowledge of estimates $\mathbf{p}^*=(p_1^*, p_1^*,\ldots, p_m^*)$ allows for simple estimation of the reliability $R(\mathbf{p})$. In such a case we apply the method of substitution. We substitute in (4) unknown probabilities $\mathbf{p}$ with their estimates $\mathbf{p}^*$. The estimator of the reliability of the whole system $R(\mathbf{p}^*)$ is unbiased only in a particular case of systems with a series reliability structure and unbiased estimators of p_i. In all other cases $R(\mathbf{p}^*)$ is biased or at best asymptotically unbiased. Therefore, in practical situations the estimates of the system's reliability are very uncertain, and we need to have methods for the computation of lower bounds for its possible value. Such bounds may be obtained by the calculation of confidence intervals for $R(\mathbf{p})$.

Let us now consider a system consisting of element s of m different types. Suppose that the reliability of the element of the i-th type, $i=1,\ldots,m,$ is a certain function of a parameter θ_i whose value is unknown. Thus, we may assume that the reliability of the whole system is described by a function $R(\boldsymbol{\theta})$ which depends on the vector $\boldsymbol{\theta}=(\theta_1, \theta_2,..., \theta_m)$ of parameters describing the reliability of system's elements. Moreover, we assume that the information from reliability tests of system's elements is denoted by x_i, $i=1,\ldots,m$. Thus, the results of the tests are described by a vector $\mathbf{x}=(x_1, x_2,\ldots, x_m)$. We have to note that the values of θ_i and x_i only in special cases are represented by single numbers. In a general case they are represented by vectors of numbers. The interval $(\underline{R},\overline{R})$, where $\underline{R}=\underline{R}(\mathbf{x})$ and $\overline{R}=\overline{R}(\mathbf{x})$ is the two-sided confidence interval, calculated on the confidence level γ, for the unknown value of $R(\boldsymbol{\theta})$ if the following condition is fulfilled

$$P_{\boldsymbol{\theta}}\left(\underline{R}\le R(\boldsymbol{\theta})\le \overline{R}\right)\ge\gamma\,. \tag{11}$$

In an analogical way we can define one-sided lower and upper confidence intervals for the reliability function $R(\boldsymbol{\theta})$. In the sections which follow we present methods for the calculation of such confidence intervals. In this presentation we use notation given in the book (Gnedenko *et al.* 1999).

3 CONFIDENCE INTERVALS FOR SYSTEM'S RELIABILITY IN THE CASE OF DISCRETE RELIABILITY DATA

Let us consider the problem of reliability estimation when the results of reliability tests of system's elements are available in a discrete form. Let us assume that the elements of all types are independently tested in exactly the same conditions as the work conditions of the considered system. In the simplest case we test samples of size N_i, $i=1,\ldots,m,$ for all m types of elements, and the duration of all tests is the same, and is equal to t. In this simplest case we assume that we know the reliability state of each tested element at the end of the test. Thus, we assume that we know the numbers of elements d_i, $i=1,\ldots,m,$ which failed during the test. The test result is described, therefore, by pairs of integer numbers (d_i,N_i), $i=1,\ldots,M.$ In such a case we say that the reliability tests, also known as pass-fail tests, are performed according to a *binomial scheme*. In this simple case there exists an unbiased estimator of the reliability of a tested element given by a simple formula

$$\hat{p}_i=1-\frac{d_i}{N_i},\ i=1,\ldots,m \tag{12}$$

The random number of the observed failures is thus described by the binomial distribution

$$P\left(d_i=d_i^*\right)=\binom{N_i}{d_i^*}(1-p_i)^{d_i^*}p_i^{N_i-d_i^*},\ i=1,\ldots,m \tag{13}$$

Calculation of the confidence interval for the reliability p_i is not simple. For a given confidence level γ one can calculate the confidence interval using a so called fiducial approach. The respective formulae are known as the Clopper-Pearson formulae, and in the considered case of reliability estimation they have the form given in (Gnedenko *et al.* 1999). The lower bound $\underline{p}$ of the one-sided confidence interval for the reliability p is given as the solution of the following equation

$$\sum_{k=0}^{d}\binom{N}{k}(1-\underline{p})^k(\underline{p})^{N-k}=1-\gamma\,, \tag{14}$$

and the upper bound $\overline{p}$ of the one-sided confidence interval for the reliability p is given as the solution of the equation

$$\sum_{k=0}^{N-d}\binom{N}{k}\overline{p}^k(1-\overline{p})^{N-k}=1-\gamma\,. \tag{15}$$

In case of $d=N$ we have $\overline{p}=1$, and when $d=0$ we have $\underline{p}=0$. It is worth noticing that if we replace $1-\gamma$ in (14) – (15) with $0{,}5<\alpha<1$ and $0{,}5<\beta<1$, respectively, we can use these formulae for the calculation of a two-sided confidence interval for the reliability p on the confidence level equal to $1-\alpha-\beta$.

When the probability of a failure is low, or when reliability is high, i.e. when the strong inequality $q_i=1-p_i<<1$, $i=1,\ldots,m$ holds, and when the number of tested elements N_i, $i=1,\ldots,m$ is large, the probability distribution of the number of failed elements d_i, $i=1,\ldots,m$ can be approximated by the Poisson distribution with the parameter $\Lambda_i=q_iN_i$, and the probability mass function given by the formula

$$P(d_i=d_i^*)=\frac{\Lambda_i^{d_i^*}}{d_i^*!}e^{-\Lambda_i}\,,\ i=1,\ldots,m \tag{16}$$

This approximation is valid when in case of $q\to 0$ and $N\to\infty$ the condition Nq=const holds.

One-sided confidence intervals for the parameter Λ of the Poisson distribution can be found by solving the following equations

$$e^{-\underline{\Lambda}}\sum_{j=0}^{d-1}\frac{\underline{\Lambda}^j}{j!}=1-\beta \tag{17}$$

$$e^{-\overline{\Lambda}}\sum_{j=0}^{d}\frac{\overline{\Lambda}^j}{j!}=\alpha \tag{18}$$

When $d=0$ we have $\underline{\Lambda}=0$. For further calculation we can use the connection between the Poisson distribution and the special case of the gamma distribution, namely the chi-square distribution. The confidence intervals can be thus calculated from the formulae:

$$\underline{\Lambda}=\frac{1}{2}\chi_{\beta}^2(2d), \tag{19}$$

$$\overline{\Lambda}=\frac{1}{2}\chi_{1-\alpha}^2(2d+2), \tag{20}$$

where $\chi_{\gamma}^2(n)$ is the quantile of order γ of the chi-square distribution with n degrees of freedom. Similarly, as in the case of the binomial distribution, for $0{,}5<\alpha<1$ and $0{,}5<\beta<1$ we can use (19) – (20) for the calculation of the two-sided confidence interval for the parameter Λ on the confidence level $1-\alpha-\beta$.

The Poisson distribution can be also used when the times to failure are described by the exponential distribution. When all elements failed during the test are replaced by new ones, and the duration of the test is equal to T, the observed number of failures is described by the Poisson distribution with the parameter $\Lambda=\lambda NT$, where λ is the failure (hazard) rate in the exponential distribution, and N is the number of simultaneously tested elements. Confidence intervals for the parameter Λ (and for the failure rate λ) are in this case calculated from the formulae (19) – (20).

4 CONFIDENCE INTERVALS IN THE ABSENCE OF OBSERVED FAILURES

Contemporary technical systems are built of very reliable elements. For such elements we usually do not observe failures during reliability tests. In such a case, the point estimation of system's reliability is trivial, and is equal to 1. However, we are interested in the lower bound for this characteristic which may be interpreted as kind of guaranteed reliability. Suppose, that for each of the m types of elements the system is built of we test N_i, i=1,…,m, elements, and in every case the number of observed failures is d_i = 0, i=1,…,m. For such results tests the upper bound for the confidence interval is always equal to $\overline{R} = 1$. On the other hand, it is possible to calculate the lower bound $\underline{R}$ of the confidence interval for the reliability of the considered system. In the book (Gnedenko *et al.* 1999), where results of many works were summarized, it has been shown that the computation of this bound is equivalent to solving the following optimization problem

$$\underline{R} = \min_{\mathbf{p} \in H_0} R(\mathbf{p}), \tag{21}$$

where the set H_0 contains all values of the vector $\mathbf{p}=(p_1, p_2, ..., p_m)$ such that

$$\prod_{i=1}^{m} p_i^{N_i} \geq 1 - \gamma \tag{22}$$

and

$$0 \leq p_i \leq 1,\ i = 1, \ldots, m\,. \tag{23}$$

In many interesting cases there exist closed solutions to this optimization problem. In case of a series system such solution was given in (Mirnyi & Solovev 1964). They showed that the lower bound of the confidence interval for system's reliability is given by a simple formula

$$\underline{R} = \min_i \underline{p}_i \tag{24}$$

where $\underline{p}_i$ is the lower bund of the one-sided confidence interval, calculated according to the Clopper-Pearson method (14). It is easy to show that this bound can be calculated from an equivalent formula

$$\underline{R} = (1 - \gamma)^{1/N^*} \tag{25}$$

where

$$N^* = \min_i N_i\,. \tag{26}$$

For systems with a more complicated structure very strong theoretical results were obtained in (Pavlov 1982) who considered systems with a convex cumulative risk function defined as follows

$$R(t) = e^{-H(t)}\,. \tag{27}$$

He has shown that for such systems

$$\underline{R} = \min_i R(1, \ldots, 1, \underline{p}_i, 1, \ldots, 1) \tag{28}$$

where

$$\underline{p}_i = (1 - \gamma)^{1/N_i},\ i = 1, \ldots, m\,. \tag{29}$$

The solutions of this problem for parallel, series-parallel, parallel-series, and k-out-of-n systems have been presented in the book (Gnedenko *et al.* 1999). For example, in the case of a system with a parallel reliability structure, consisting of n different elements the lower bound of the one-sided confidence interval for system's reliability is given by:

$$\underline{R} = 1 - \prod_{j=1}^{n} \frac{t}{t + N_j} \tag{30}$$

where t is the solution of the following equation

$$\sum_{j=1}^{n} N_j \, ln\left(1+\frac{t}{N_j}\right) = -ln(1-\gamma) \tag{31}$$

In a particular case, when $N_1=\ldots=N_n=N$ Tyoskin and Kurskiy obtained a simple analytic solution (see (Gnedenko *et al.* 1999))

$$\underline{R} = 1-\left[1-(1-\gamma)^{1/nN}\right]^n . \tag{32}$$

For systems with a more general coherent structure such simple solutions do not exist. However, in the book (Gnedenko *et al.* 1999) two boundaries for the lower bound of the confidence interval have been proposed. Consider the set of all minimal cuts of the system, and assume that the minimal cut with the smallest number of elements consists of b elements. Then, consider the set of all possible minimal paths. For this set consider its all possible subsets consisting of independent, i.e. having no common elements, paths. Let a be the number of such paths in the subset with the largest number of independent paths. Assume additionally, that for each type of system elements exactly N elements have been tested. The boundaries for the lower bound for the system's reliability are the given by

$$1-\left[1-(1-\gamma)^{1/Na}\right]^a \le \underline{R} \le 1-\left[1-(1-\gamma)^{1/Nb}\right]^b \tag{33}$$

In a particular case of $a = b$ we have

$$\underline{R} = 1-\left[1-(1-\gamma)^{1/Nb}\right]^b \tag{34}$$

The authors of (Gnedenko *et al.* 1999) notice, that this case is typical for many reliability structures such as lattice or radial structures which are typical for large network systems.

Another very interesting method for the calculation of the lower bound of the confidence interval for system's reliability was presented in (Gnedenko *et al.* 1999). Let us assume that the same vector of reliabilities $\mathbf{p}=(p_1,p_2,\ldots,p_m)$ is used for the calculation of reliability of two systems: the reliability $R(\mathbf{p})$ of the considered complex system, and the reliability $R^{'}(\mathbf{p})$ of a simple (e.g. series) auxillary system. For this auxillary system we must know the lower bound of the respective confidence interval $\underline{R}^{'}(\mathbf{p})$. In order to find the lower bound of the confidence interval for the reliability of the considered system we have to solve the following optimization problem:

$$\underline{R} = \min_{\mathbf{p}} R(\mathbf{p}) \tag{35}$$

where the element of the vector **p** must fulfill the following constraint

$$\prod_{i=1}^{m} p_i \ge \underline{R}^{'} ,\, 0 \le p_i \le 1,\, i = 1,\ldots,m . \tag{36}$$

The lower bound calculated in this way fulfills all the requirements for a lower bound of a confidence interval, but the length of such interval is usually not the shortest possible.

5 CONFIDENCE INTERVALS IN THE PRESENCE OF OBSERVED FAILURES

When failures are observed during reliability tests of system's elements the problem of building confidence intervals for the reliability of the whole system becomes much more complicated. Comprehensive information about available methods can be found in the fundamental book (Gnedenko *et al.* 1999). Below, we present only some basic results considered in this book and related literature.

Let us assume that the considered system consists of elements of m different types. For each of these types we test a sample of N_i elements, and for each sample we observe $d_i \ge 0$, $i=1,\ldots,m$ failures. Let

$$S = R\left(\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_m\right) \tag{37}$$

be the point estimator of system's reliability, where $\hat{p}_i, i = 1, \ldots, m$ are the estimators of the reliability of systems elements calculated according to (12). Now, denote by $\mathbf{d}^* = (d_1^*, d_2^*, \ldots, d_m^*)$ the vector of numbers of observed failures. Moreover, denote by $S^* = S(\mathbf{d}^*)$ the observed value of the estimator of system's reliability presented as the function of the vector $\mathbf{d}^*$. The lower bound of the confidence interval for the system's reliability is now calculated from the formula

$$\max_{\mathbf{p} \in A_R} \sum_{S(\mathbf{d}) \le S(\mathbf{d}^*)} \prod_{i=1}^{m} \binom{N_i}{d_i} p_i^{N_i - d_i} (1 - p_i)^{d_i} = 1 - \gamma \,, \tag{38}$$

where maximum is calculated over the set A_R of vectors $(p_1, p_2, \ldots, p_m)$, such that

$$R(p_1, p_2, \ldots, p_m) = R,\ 0 \le p_i \le 1, 1 = 1, \ldots, m \,. \tag{39}$$

The sum in (38) is calculated over all possible values of the vector $\mathbf{d}^* = (d_1^*, d_2^*, \ldots, d_m^*)$ that fulfill the condition given for this sum in (38). In certain cases other formulation of this optimization problem is more suitable for computations. According to this formulation we denote by $n(\mathbf{d}) = n(d_1, d_2, \ldots, d_m)$ a non-decreasing, with respect to all components, series of vectors. The first element of this series is the vector (0,0,…,0), and then we have the vectors of the type (0,..,0,1,0,…,0), etc. The lower bound of the confidence interval for system's reliability can be calculated from

$$\underline{R} = \min R(p_1, p_2, \ldots, p_m), \tag{40}$$

where minimum is taken over the set of all values of the vector $(p_1, p_2, \ldots, p_m)$ such that

$$\sum_{n(\mathbf{d}) \le n(\mathbf{d}^*)} \prod_{i=1}^{m} \binom{N_i}{d_i} p_i^{N_i - d_i} (1 - p_i)^{d_i} \ge 1 - \gamma, \quad 0 \le p_i \le 1, i = 1, \ldots, m \tag{41}$$

The optimization problem given by (40) – (41) was formulated first time in (Buehler 1957) where a system consisted of two elements was considered. This was the first result of the calculation of the confidence interval for system's reliability.

Let us now consider the series system consisted of m different elements. The optimization problem is now the following:

$$\underline{R} = \min \prod_{i=1}^{m} p_i \,, \tag{42}$$

where minimum is taken over all vectors $(p_1, p_2, \ldots, p_m)$ such that

$$\sum_{R(\mathbf{d}) \ge R(\mathbf{d}^*)} \prod_{i=1}^{m} \binom{N_i}{d_i} p_i^{N_i - d_i} (1 - p_i)^{d_i} \ge 1 - \gamma,\ 0 \le p_i \le 1, i = 1, \ldots, m \tag{43}$$

The calculation of the lower bound of the confidence interval for system's reliability $\underline{R}$ can be simplified when the probabilities of failures are small, i.e. when the inequality $q_i = 1 - p_i \ll 1$, $i = 1, \ldots, m$ holds. In such a case we can assume that the number of failures is described by the Poisson distribution with the parameter $\Lambda_i = q_i N_i$, $i = 1, \ldots, m$. It has been shown in (Gnedenko *et al.* 1999) that in this case we have

$$\underline{R} = e^{-\bar{f}} \tag{44}$$

where

$$\bar{f} = \max\left(\sum_{i=1}^{m} \frac{\Lambda_i}{N_i}\right), \tag{45}$$

and the maximum in (45) is taken over all vectors $\boldsymbol{\Lambda} = (\Lambda_1, \Lambda_2, ..., \Lambda_m)$ such that

$$\sum_{R(\mathbf{d}) \geq R(\mathbf{d}^*)} \prod_{i=1}^{m} e^{-\Lambda_i} \left(\frac{\Lambda_i^{d_i}}{d_i!} \right) \geq 1-\gamma,\ \lambda_i \geq 0,\ i = 1,...,m \tag{46}$$

This practical result was obtained first time in (Bol'shev & Loginov 1966) for the case of equal values of N_i, and, independently, in (Pavlov 1973) and (Sudakov 1974) for any values of these numbers.

6 APPROXIMATE CONFIDENCE INTERVALS FOR SYSTEM'S RELIABILITY

Computation of exact bounds of confidence intervals for system's reliability requires, with only few exceptions, solving difficult optimization problems. Therefore, its practical applicability is somewhat limited unless specialized software is available. For this reason several authors, mainly American, have tried to obtain approximate, but relatively easy for computation, solutions. Different approximate solutions have been proposed in the papers like (Madansky 1965, Myhre & Saunders 1968, Easterling 1972, Mann 1974 a, b, Mann & Grubbs 1972, 1974). Comprehensive review of such results can be found in a well known book (Mann, Shaefer & Singpurwalla 1974). However, probably the most interesting from a practical point of view result was presented in one of the first textbooks on reliability (Lloyd & Lipow 1962). These authors presented a heuristic method, attributed to Lindstrom and Madden, for the calculation of the approximate confidence interval for the system with a series reliability structure. This method utilizes the concept of so called equivalent tests. To present this method we consider, following the book (Gnedenko *et al.* 1999), a system with a series-parallel structure which has the same elements in its parallel subsystems. Let R^* be the estimated value of the reliability function for the considered system, and N_i, $i=1,\ldots,m$ be the number of tested items for the element of the i-th type. The equivalent number of failures D_i^* for the element of this type is then calculated from the equation

$$R\left(1,\ldots,1,1-D_i^*/N_i,1,\ldots,1\right) = R^* \tag{47}$$

At the next stage of the computation procedure, for each equivalent test (N_i, D_i^*) we calculate the lower bound of the confidence interval $\underline{P}_i\left(N_i, D_i^*\right)$ by solving the equation

$$B_p\left(N_i - D_i^*, D_i^* + 1\right) = 1-\gamma, \tag{48}$$

where

$$B_p(a,b) = \frac{\int_0^p x^{a-1}(1-x)^{b-1}dx}{\int_0^1 x^{a-1}(1-x)^{b-1}dx} \tag{49}$$

is the incomplete beta function whose values can be computed using an available numerical procedure. The lower bound of the confidence interval is now calculated from a simple formula

$$\underline{R} = \min_{1\leq i\leq m} R\left(1,\ldots,1,\underline{P}_i\left(N_i, D_i^*\right),1,\ldots,1\right). \tag{50}$$

The Lindstrom-Madden method was proposed as an approximate heuristic method. However, it has been proved (see (Gnedenko *et al.* 1999) for additional information) that for many simple reliability structures it produces exact confidence intervals.

Another method which uses the concept of equivalent tests, and which can be used for the analysis of complex systems consisted of many simple subsystems, was proposed in (Martz & Duran 1985). In this method it is assumed that for each simple subsystem we are able to calculate the value of its reliability estimator R_i, and the lower bound for the respective confidence interval $\underline{R}_i$. Next, from a set of equations

$$1 - \frac{r_i}{M_i} = R_i \tag{51}$$

and

$$\underline{R}_i = \underline{P}_i(M_i, r_i) \tag{52}$$

we calculate the parameters (M_i, r_i) of the equivalent binomial reliability tests. In further analysis the considered subsystem is treated as a single element described by the equivalent test. Note, that for the application of this method it is not important how we have found the values of R_i and $\underline{R}_i$.

7 SOME REMARKS ABOUT OTHER METHODS FOR THE CALCULATION OF CONFIDENCE INTERVALS FOR SYSTEM'S RELIABILITY

In the previous sections we have presented methods for the calculation of confidence intervals for system's reliability for the case of discrete reliability data from tests, i.e. when the numbers of tested elements and the numbers of observed failures are known. It is a well known fact that the knowledge of lifetime distributions combined with the knowledge of observed times to failures may increase the accuracy of reliability estimation. Moreover, this knowledge may be sufficient for the prediction of reliability at time instants other than the times of the performed reliability tests. Unfortunately, even in the simplest case of the exponential distribution of lifetimes the exact and practically applicable solutions are known only in few cases when lifetime tests are performed according to the type-II censoring scheme (a fixed number of observed failures). For example, (Lentner & Buehler 1963) considered the case of a series system with only two elements. Their result was generalized in an unpublished PhD thesis (El Mawaziny 1965) who proposed an iterative method for the calculation of the lower bound of the confidence interval for reliability of a series system consisted of m elements. Because of its complicated nature this algorithm has not been described in reliability textbooks. However, there exists a good approximation proposed in (Mann & Grubbs 1972), and in a simplified version in (Mann 1974b).

Consider the case when the lifetimes are exponentially distributed, and reliability tests provide type-II censored data. For each type of system elements we test a sample of n_i items, and observe times $t_{i,j}$ of the first $r_i>0$, $i=1,\ldots,m$ failures. The respective value of the total time on test z_i, is given by

$$z_i = \sum_{j=1}^{r_i} t_{i,j} + (n_i - r_i) t_{i,r_i}, \quad i = 1, \ldots, m \tag{53}$$

Denote by $z_{(1)}$ the minimal value of z_i, $i=1,\ldots,m$. (Mann 1974b) has shown that the estimator of the hazard rate of the series system has approximately the expected value given by

$$\mu = \sum_{i=1}^{k} \frac{r_i - 1}{z_i} + \frac{1}{z_{(1)}}, \tag{54}$$

and the variance given by

$$v = \sum_{i=1}^{k} \frac{r_i - 1}{z_i^2} + \frac{1}{z_{(1)}^2}. \tag{55}$$

To approximate the optimum lower bound on series system reliability $\underline{R}_s(t)$ at confidence level β, using the Wilson-Hilferty transformation, one calculates

$$\underline{R}_s(t) = \exp\left[-t\mu\left(1 - \frac{\nu}{9\mu^2} + \frac{y_\gamma \nu^{1/2}}{3\mu}\right)^3\right], \tag{56}$$

where y_γ is the quanitile from the standardized normal distribution. For systems with more complex structures an interesting approach has been proposed in (Gnedenko *et al.* 1999). According to this approach first we have to calculate upper bounds for the hazard rates of system's elements using the following simple formula

$$\overline{\lambda}_i = \frac{\chi_\gamma^2(2r_i)}{2S_i}, i = 1,\ldots,m, \tag{57}$$

where $\chi_\gamma^2(2r)$ is the quantile of the γ order from the chi-square distribution with $2r$ degrees of freedom. When we insert these lower bounds into a formula for the calculation of the system's reliability function instead of respective hazard rates, i.e. if we calculate

$$\underline{R} = R(\overline{\lambda}_1.\overline{\lambda}_2,\ldots,\overline{\lambda}_m), \tag{58}$$

the obtained value usually fulfills the requirements for a confidence interval. (Pavlov 1982) has shown that in case of γ>0,778 this approach allows to calculate confidence intervals for a broad class of reliability structures for lifetime distributions having non-decreasing (in time) hazard rates (i.e. for elements with the ageing property).

The general methodology for the calculation of confidence intervals for system's reliability was proposed in (Belyaev 1966, 1968). Other, but completely equivalent general method, was proposed in (Bol'shev & Loginov 1966). Below, we present the main results of Belyaev.

Suppose that we know the statistic S which can be used as a point estimator of system's reliability, i.e. $S = \hat{R}$. Moreover, we assume that this statistic is a function of a vector of parameters $\boldsymbol{\theta}$ describing probability distributions of lifetimes of system's elements. Additionally, we assume that the probability distribution of this statistic *is known*, i.e. we know

$$F(t,\boldsymbol{\theta}) = P_{\boldsymbol{\theta}}(S \le t). \tag{59}$$

For a given value of the vector $\boldsymbol{\theta}$ we can now introduce two functions $t_1(\boldsymbol{\theta})$ and $t_2(\boldsymbol{\theta})$, such that

$$F(t_1,\boldsymbol{\theta}) = \alpha \tag{60}$$

and

$$F(t_2,\boldsymbol{\theta}) = 1 - \beta. \tag{61}$$

Now, let's denote by

$$A_R = \{\boldsymbol{\theta} : R(\boldsymbol{\theta}) = R\} \tag{62}$$

the set of all values of the vector $\boldsymbol{\theta}$ for which the reliability function adopts a given value R. Next, introduce two functions

$$K_1(R) = \min_{\boldsymbol{\theta}\in A_R} t_1(\boldsymbol{\theta}) \tag{63}$$

and

$$K_2(R) = \min_{\boldsymbol{\theta}\in A_R} t_2(\boldsymbol{\theta}). \tag{64}$$

The lower and upper bounds of the confidence interval on the confidence level $1-\alpha-\beta$ for system's reliability can be found by solving equations

$$K_1(\underline{R}) = S^* \tag{65}$$

and

$$K_2(\overline{R}) = S^*, \tag{66}$$

where S^* is the observed value of the statistic S.

The described general methodology is based on the original methodology for the construction of confidence sets proposed in (Neyman 1935), and is valid for any type of reliability data, and any type of reliability structure. However, its practical applicability is limited only to rather simple cases.

8 APPROXIMATE LOWER BOUNDS FOR SYSTEM'S RELIABILITY BASED ON MINIMUM VALUES OF THE RELIABILITY OF SYSTEM'S ELEMENTS

Computation of optimal (i.e. the shortest) and exact confidence intervals is, with a few exceptions, a very difficult task. Moreover, in all published results it is assumed that the elements in a system are mutually independent. Additional problems arise from a fact that confidence intervals used for the description of test results may be conservative, as in the case of intervals based on the Clopper-Pearson formula. In this section we present approximate bounds for system's reliability which, under certain conditions, may replace lower bounds of confidence intervals.

In order to investigate the robustness of the confidence intervals for system's reliability against the departure from the assumption of independence of system's elements let us introduce the notion of a *copula*. According to a famous theorem of Sklar (see e.g. (Nelsen 2006)) any two-dimensional probability distribution function $H(x,y)$ with marginals $F(x)$ and $G(y)$ is represented using a function C called a *copula* in the following way:

$$H(x,y) = C(F(x), G(y)) \tag{67}$$

for all $x,y \in R$. Conversely, for any distribution functions F and G and any copula C, the function H defined by (67) is a two-dimensional distribution function with marginals F and G. Moreover, if F and G are continuous, then the copula C is unique. In our investigation we will consider three types of copulas:

a) Clayton copula defined as

$$H(x,y) = \left[F^{-\theta}(x) + G^{-\theta} - 1\right]^{-1/\theta}, \theta > 0 \tag{68}$$

b) Gumbel copula defined as

$$H(x,y) = exp\left(-\left[(-ln\,F(x))^{\theta} + (-ln\,G(y))^{\theta}\right]^{1/\theta}\right), \theta > 0, \tag{69}$$

c) Fairlie-Gumbel-Morgenstern (FGM) copula defined as

$$H(x,y) = F(x)G(y)(1 + \theta(1 - F(x))(1 - G(y))), -1 \le \theta \le 1 \tag{70}$$

The Clayton and Gumbel copulas can be used for modeling a positive stochastic dependence. The FGM copula can be used for modeling both negative (θ<1) and positive (θ>1) dependence. The Clayton copula is especially interesting in reliability applications as it describes stronger dependence for smaller lifetimes than for larger ones. If this type of dependence exists the reliability of a series system with dependent elements is greater than in the case of independence. On the other hand, for a parallel system the reliability of a system with dependent elements is smaller.

In the majority practical cases the reliability of tested elements is high, and even for moderate sample sizes the number of observed failures is small. This suggests utilization of the result obtained for the case of zero-failure tests for the calculation of the lower bounds for reliability of a series system given by the expression (24). To analyze the properties of this approximation let us consider a two-element series system whose elements are equally reliable. We also assume that the sample sizes for both elements are the same. On Figure 1 we present the comparison of the values of our simple approximate bound with the bounds calculated for this system using a substitution method. For obtaining the presented results we performed a Monte Carlo simulation experiments, and in each of them we generated 500 000 test cases, Our approximate bound, plotted against the expected number of observed failures in a sample (for a probability of failure equal to 0,01), is represented by a continuous upper curve. The middle curve represents the bound calculated by the insertion into (5) the respective lower bound of the confidence intervals for the reliability of elements, calculated for the same confidence level (γ=0,9).The lower curve is a similar to the previous one, but calculated for the confidence level equal to $\sqrt{\gamma}$, as it is suggested in statistical literature.

Then, we calculated the coverage probability of the considered confidence intervals. The results of the comparison are presented on Figure 2 for our approximate bound, and the bound represented by a middle curve on Figure 1.

As we can see, our simple bound fulfills requirements for a confidence interval not only for zero-failure reliability tests, but for all tests with the expected number of failures not greater than 1,95. The classical and much wider confidence intervals have the probability of coverage close to 1, i.e. much greater than the designed value of 0,9.

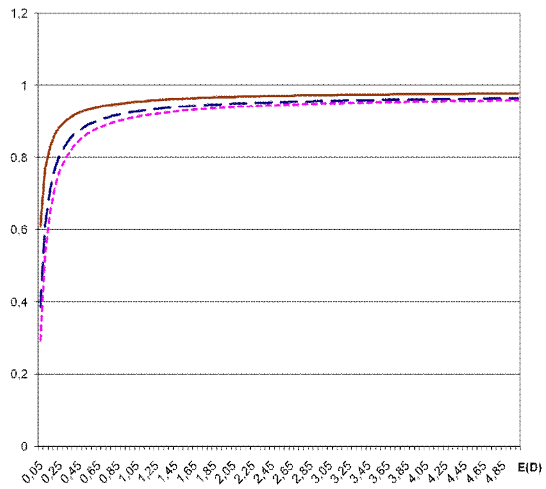


Figure 1. Lower bounds for a series system

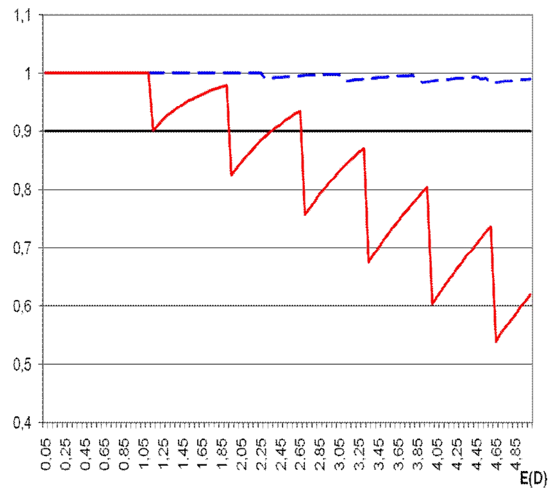


Figure 2. Coverage probabilities for a series system in case of independence

Now, let us consider the case when the elements of the system are dependent. On Figure 3 we show the coverage probability when this dependence is described by the Clayton copula with dependence parameter θ=2, and the Gumbel copula, with dependence parameter θ=2. For this value of the parameter the Kendall measure of dependence τ for both copulas is equal to 0,5. It means that the dependence is positive and fairly strong.

The coverage probability in the case of the Clayton copula (solid line) is greater than the designed value for tests with the expected value of observed failures greater than 5. However, in the case of the dependence described by the Gumbel copula (dashed line) this feature is guaranteed only for this value not greater than 2. It shows, how the type of dependence influences the results despite the fact that the popular measure of dependence, such as Kendall τ in both cases gives exactly the same value.

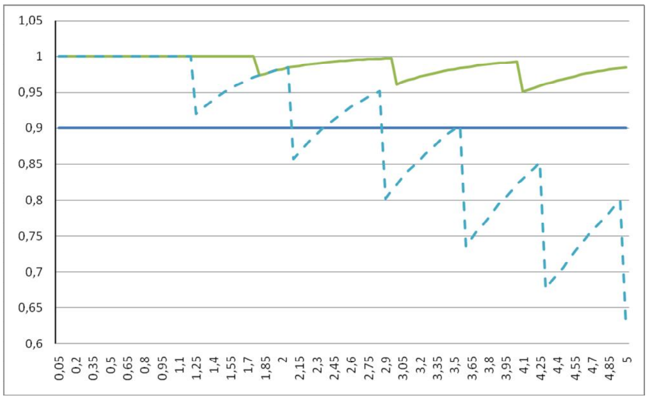


Figure 3. Coverage probabilities for a series system in case of dependence

Now, let us consider the case of the system with elements connected in parallel. For such systems a simple for computation bound which is similar to that for a series system does not exist. Instead we propose the following approximation

$$\underline{R} = 1 - \min_i \overline{q}_i \,, \tag{71}$$

where $\overline{q}_i$ is the upper bound of the confidence interval for the probability of failure. The lower bound calculated according to (71) is *always* smaller than the bound obtained by substitution of the probabilities of failures q_i with their respective upper bounds $\overline{q}_i$. Thus, the coverage probability in case of independent elements of the system, calculated according to (71), is always greater than the respective confidence level. It can be seen at Figure 4, where this probability is always equal to 1. (Note that the coverage probability in case of the bound obtained by substitution is also much greater than the confidence level which is equal to 0,9).

The situation changes dramatically when the elements of the system are dependent, and when their dependence is described either by the Clayton copula or by the Gumbel copula. On Figure 5 we present the coverage probabilities in such cases when the confidence intervals are calculated using the substitution method.

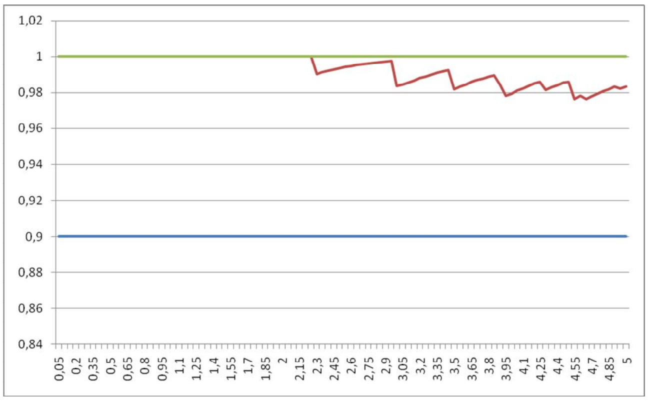


Figure 4. Coverage probabilities for a parallel system in case of independence

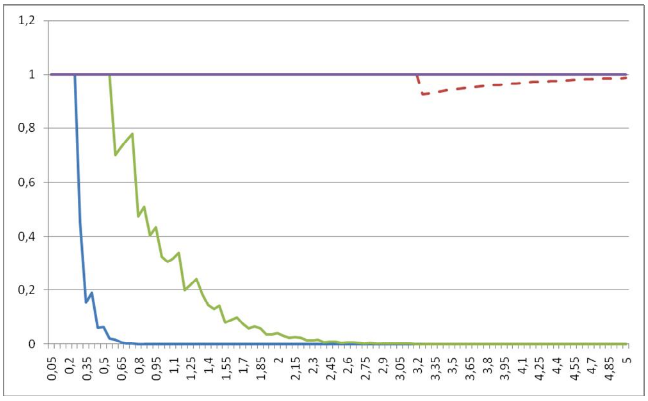


Figure 5. Coverage probabilities for a parallel system in case of dependence

The coverage probabilities (the left-most curve for the Clayton copula, and the curve next to it for the Gumbel copula) show dramatically that the confidence intervals obtained by substitution under the assumption of independence are too narrow. On the other hand, the interval calculated according to (71) has the coverage probability (depicted by a dashed curve for the Clayton copula, and equal to one for the Gumbel copula) greater than the confidence level.

9 CONCLUSIONS

Many prominent authors, mainly from USA and the Soviet Union, contributed to the problem of computing the lower confidence bounds for system's reliability using the data from tests of separate elements or subsystems. The proposed exact bounds are usually difficult to compute. Good approximations exist, but they are usually obtained under the assumption that failures of all elements or subsystems are observed during the tests. In the paper we have shown using Monte Carlo simulation that in case when elements working together in a system are dependent these bounds are inaccurate or even useless, as it is the case of parallel (redundant) systems. In the paper, we have proposed very simple bounds characterized by satisfactory performance, at least for highly reliable system elements, which are robust against the presence of positive dependence of the elements of a system.

REFERENCES

Barlow, R. E. & Proschan, F. (1965). *Mathematical Theory of Reliability*. John Wiley, New York.

Barlow, R. E. & Proschan, F. (1975). *Statistical Theory of Reliability and Life Testing. Probability Models*. Holt Rinehart and Winston, Inc., New York.

Belyaev, Y. K. (1966). Confidence intervals for the functions of many unknown parameters. *Proc. of the Sov. Acad. Sci,* 169: 755-758 (in Russian)

Belyaev, Y. K. (1968). On simple methods of Confidence Limits Construction. *Eng. Cybernetics*, No.5: 96-101 (in Russian, exists English translation of this journal)

Bol'shev, L.N. Loginov E.A. (1966). Interval estimates under noisy parameters. *Theory of Prob. and its Appl.* v.11, No.1: 94-107 (in Russian, exists English translation of this journal)

Buehler R.J. (1957), Confidence Intervals for the Product of two Binomial Parameters. *Journal of the American Statistical Association*, 52: 482-493.

Easterling R.G. (1972), Approximate Confidence Limits for the System Reliability. *Journal of the American Statistical Association*, 67: 220-222.

El Mawaziny, A.H. (1965), *Chi-square distribution theory with applications to reliability problems*. PhD Thesis, Iowa State Univ., Ames, Iowa.

Gnedenko B., Pavlov I., Ushakov I. & Chakravarty S. (1999), *Statistical Reliability Engineering*. John Wiley, New York.

Lentner, M.M. & Buehler, R.J. (1963), Some inferences about gamma parameters with an application to a reliability problem. *Journal of the Amer. Stat. Assoc.* 58: 670-677.

Lloyd D.K. & Lipow M. (1962), *Reliability Management. Methods and Mathematics*. Prentice-Hall, Englewood Cliffs, NJ.

Madansky A. (1965), Approximate Confidence Limits for the Reliability of Series and Parallel Systems. *Technometrics*, 7: 493-503.

Mann N.R. (1974a), Approximate Optimum Confidence Bounds on Series and Series-Parallel System Reliability for Systems with Binomial Subsystem Data. *IEEE Transactions on Reliab*ility, R-23: 295-304.

Mann N.R. (1974b), simplified expressions for obtaining approximately optimum system-reliability confidence bounds from exponential subsystem data. *Journal of the Amer. Stat. Assoc*., 69: 492-495

Mann N.R. & Grubbs F.E. (1972), Approximately Optimum Confidence Bounds on Series-System Reliability for Exponential Time to Fail Data. *Biometrika*, 50: 191-204.

Mann N.R. & Grubbs F.E. (1974), Approximately Optimum Confidence Bounds for System Reliability Based on Component Test Plan. *Technometrics*, 16: 335-347

Mann N.R., Schaefer R.E. & Singpurwalla N.D. (1974), *Methods of Statistical Analysis of Reliability and Life Data*. John Wiley, New York.

Martz H.F. & Duran I.S. (1985), A Comparison of Three Methods for Calculating Lower Confidence Limits on System Reliability Using Binomial Component Data. *IEEE Transactions on Reliability*, R-34: 113-120.

Mirnyi, R.A. & Solovyev, A.D. (1964), Estimation of system reliability on the basis on its units tests. In: *Cybernetics in Service for Communism*, vol.2, Energiya, Moscow: 213-218.

Myhre J. & Saunders S.C. (1968), Comparison of Two Methods of Obtaining Approximate Confidence Intervals for System Reliability. *Technometrics*, 10: 37-49.

Nelsen, R.B. (2006) *An Introduction to Copulas* (2nd edition), Springer, New York.

Neyman J. (1935), On the Problem of Confidence Intervals. *Annals of Mathematical Statistics*, 6: 111-116.

Pavlov, I.V. (1973), Confidence limits for system reliability on the basis on its components testing. *Eng. Cybernetics*, No.1: 52-61 (in Russian, exists English translation of this journal).

Pavlov, I.V. (1982), *Statistical methods of reliability estimation by tests results*. Radio i Svyaz, Moscow (in Russian).

Sudakov, R.S. (1974), About interval estimation of reliability of series system. *Eng. Cybernetics*, No.3: 86-94 (in Russian, exists English translation of this journal).

TIME DIFFERENCES IN PERATION STATES OF STENA BALTICA FERRY DURING THE OPEN WATER AREAS PASSAGE

M. Jurdziński[a]
S. Guze[a]
P. Kamiński[b]
•
[a] Maritime University, Gdynia Poland
[b] Stena Line, Poland

e-mail: sambor@am.gdynia.pl

ABSTRACT

The paper deals with analysis of ships operation stages in open water areas effected by environmental constraints influencing on ship sea keeping parameters in application to ferry "Stena Baltica" operated in the Baltic Sea between Gdynia and Karlskrona harbors.

1 INTRODUCTION

Sea effects the ship during open water passage induces ship responses as motion, slamming, rolling etc. as the main constraints the whole system. To describe the ship system as a simply model in a seaway, it is necessary to introduce dynamic responses of a ship during passage (see *Figure 1*).

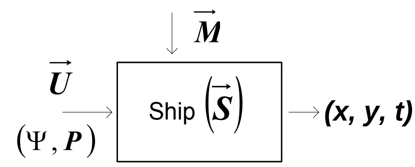


Figure 1. Model of a ships in a seaway

The state of a ship is describe by control vector $\vec{U}$ expressed by heading (Ψ) and power output ($\boldsymbol{P}$). The ship motion is depicted by sea keeping constraints vector ($\vec{M}$). The ship position is expressed by $\boldsymbol{x}$ (lat) $\boldsymbol{y}$ (long) and time ($\boldsymbol{t}$) determines the actual ship's position in a seaway.
Vector ($\vec{S}$) specifying the ship's geometrical parameters (Chen). Based on the type of ship and her operation criteria of the vessel and a set of operating criteria is recommended for each trip. The main available sea keeping criteria recommended for passenger ferry operation should be her lack of casual contents to ship, less fatigue to passenger (crew injure) and to cargo damages.
Judgment of degree of danger to the ship is dependent on control vector: speed and course. Control of the ship movement in open sea is based on recommendation for reducing speed and or course changes in proper time.
There are the main factors that should be taken into account to establish state of ship control vector:

- Observation of waves;
- Encounter degrees of waves;
- Observation of wind;
- Main engine revolution;
- Propeller slip;
- Shipping seas on deck the bow;

- Degree of rolling;
- Degree of yawing;
- Degree of slamming;
- Other general observation of the ship behavior.

Additionally it is recommended to reduce fuel consumption, expected time to arrival (ETA) and in conclusion to care of ship safety.

The analysis of environmental effects on ship movements during sea passage must be considered taking into account the following aspects (IMO 2002):

- Ship category (type of cargo, age, geometrical parameters etc);
- Ship systems or functions (layout, type of propulsion);
- Ship operation (voyage duration, areas);
- External influences (weather, season, navigational infrastructure, shore based systems);
- Risk associated with consequences (damage to ship or fatalities to passengers or crew);
- Accident category.

Every master of a ship is obliged to receive an accurate description of the sea environmental condition before departure and during sea passage.

There are emergency states in which the ship can be found during her operation as damage by waves, taking water, collision, fire, grounding, oil spill, the crew or passenger sickness, or total loss. The main forecast environmental data is given in Table 1.

Table 1. Forecast environmental data

Kind of data	Units	Remarks
Wind	[m/s, [^{o}]	Speed, direction
Sea	[m], [^{o}], [s]	Height, direction, period
Swell	[m], [^{o}], [s]	Height, direction, period
Currents	[m/s], [o]	Speed, direction
Dangers	-	Ice, Fog etc.

It is important to every master the knowledge of the ship's responses as waves and winds components that may met the ship during her sea passage.

Information on surface currents are important specially during navigation in restricted water areas.

Ships sailing in rough seas are subject to motions and in consequences are loosing their speed.

In Figure 2 there has been shown the environmental effects on speed loss by ship in rough seas.

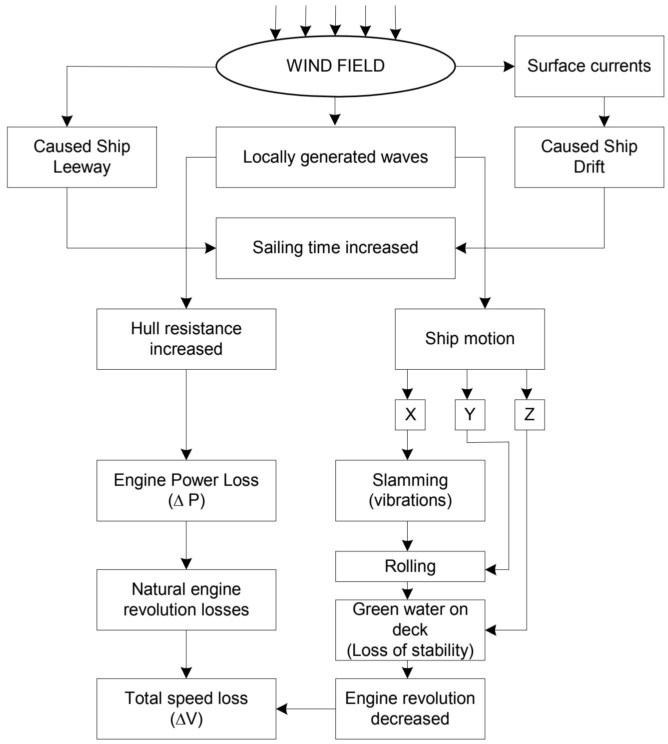


Figure 2. Environmental effects on speed loss during sea passage in rough seas (Jurdziński 1989)

The speed depends on the hull form, draft of the ship, depth of water, environmental condition and state of the engine power, or propeller setting. In Figure 3 there have been shown two different ship's hull reaction on environmental condition in different phase of navigation.

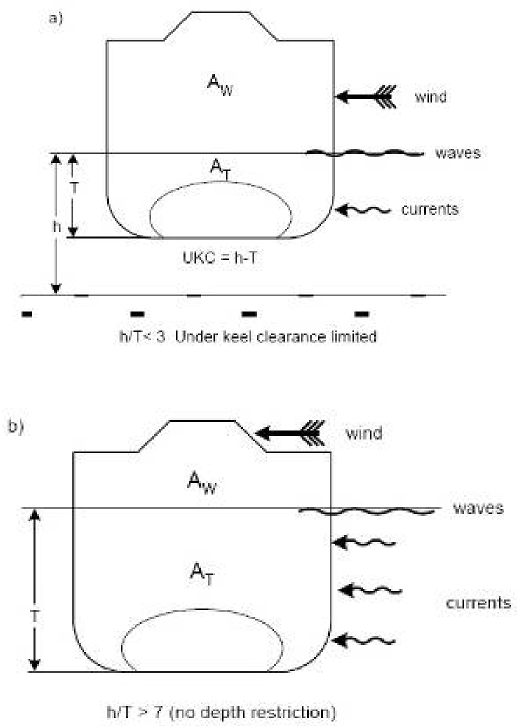


Figure 3. Different ship's curves reaction on environmental conditions: a. navigation in restricted water areas (Ferry); b. navigation in open water areas (Bulk Carriers) (Jurdziński 2003)

For practical application the ship speed loss curves are used. Environmental parameters such as waves, swell, wind and current are used in calculation. A ship being influence by many factors which interact in a complex manner. Relation between the ship speed and total hull resistant will clarify the action of particular forces on hull during ship movement in different environmental condition.

2 ENVIRONMENTAL CONDITIONS AND THE SHIP SPEED LOSS

The thrust that is gain by the propeller effects is equal to the sum of calm water resistance, environmental effects as wave, wind, currents and shallow water resistances. (See Figure 4.)

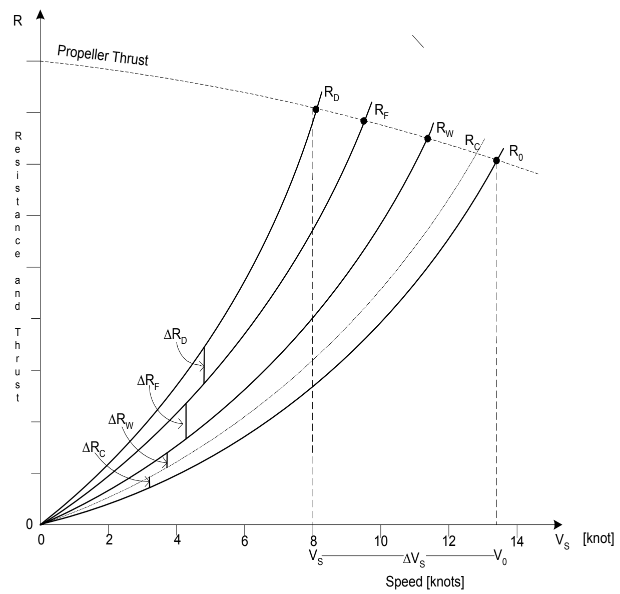


Figure 4. The ship speed changes due to environmental conditions

Total resistance of the ship during her movements is given by:

$$R_T = R_O + R_C + R_W + R_F + R_D \qquad [kN] \qquad (1)$$

Where:
R_T – total hull resistance in difficult environmental conditions;
R_O – resistance in calm water;
R_W – additional resistance in waves;
R_F – additional resistance in wind;
R_D –additional resistance in shallow waters;
R_C –additional resistance in currents.

In the same way is determined the speed loss of the ship during her moving:

$$V_S = V_0 - \Delta V_S,$$

$$\Delta V_S = \Delta V_C + \Delta V_W + \Delta V_F + \Delta V_D \qquad [knot] \qquad (2)$$

Where
V_0 – speed in calm water;
V_S – speed in different environmental constraints;

ΔV_S – total speed loss due to environmental conditions;
ΔV_C – speed loss in currents;
ΔV_W – speed loss in waves;
ΔV_F – speed loss in wind;
ΔV_D – speed loss in shallow water.

Ferry make vessel especially susceptible to wind due to her large windage of super structure A_W but the external forces of waves seems to be small (see *Figure 3*).
Dynamic characteristic of ship motion is important to predict ship responses in term of wave spectra and ship geometry during her sea passage.
Speed is the main ship performance characteristic. The actual ship speed can be expressed in functional form as:

$$V_S = V_O - \Delta V_W \quad \text{[knot]} \qquad (3)$$

$$V_0 = F(n) \quad \text{[knot]} \qquad (4)$$

Where:
$a_1 b_1$ – coefficients obtained by experimental method;
F{n} - propeller revolution function [r p m];

Loss of speed in waves during passage in open waters conditions is given by formula:[5]

$$\Delta V_W = aH + bH^2 + cH\cos q_w \quad \text{[knot]} \qquad (5)$$

where:
a,b, c – coefficients obtained by experimental method;
H - significant wave heights [m];
q_w – wave to ship track angle [^{0}].

Prediction of engine power in the open sea phase of navigation is given by formula:[11]

$$P = P_O - \Delta P \quad \text{[kW]} \qquad (6)$$

$$P_O = a_1 n^3 \quad \text{[kW]} \qquad (7)$$

$$\Delta P = b_1 \Delta V_W + c_1 \Delta V_{W2} \quad \text{[kW]} \qquad (8)$$

where:
a_1, b_1, c_1 – coefficient;
n – propeller revolutions [rpm];
ΔV_w – speed loss due to waves [knot].

Speed loss presented in graphical form is given in *Figure 5*.

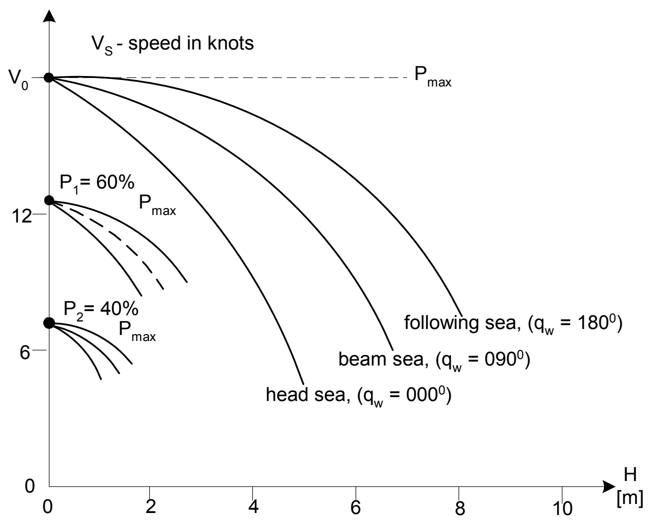


Figure 5. Speed curves for various engine power setting. It is possible to establish the functional relation between ship, wave period and wave to ship track in relation to available power output

Such function is given as (Chan):

$$V_S = f(P, H^2, q_w, T_w) \qquad \text{[knots]} \qquad (9)$$

where:
P – power output [kW];
H – significant wave height [m];
T_W – predominant wave period [s];
q_W– wave to ship track [^{0}].

For safety reason an approach to model of the ship speed function on a seaway should be prepared for every ship. Information recorded from ship's logbook as speed against wave height or wind speed will make possible construct speed curves. The speed function to established is effective and useful in navigation passage planning.

3 SHIP SPEED IN SHALLOW WATERS

The shallow water influence the ship speed. There have been given information in reference (Barrass 2004) on depth influence the ship speed. The formula is as fallow:

$$h = k \cdot T \qquad \text{[m]} \qquad (10)$$

where:
h – depth of water influencing on ship speed[m];
k – coefficient equals to: $\frac{4.44}{C_B^{1.3}}$;
C_B – ship block coefficient;
T – ship draft [m].

The amounts that the ship is reducing her speed will depend on the following elements (Barrass 2004):

- Type of ship;
- Proportion of water depth (h) to static mean draft of the ship (T), (i.e. the h/T value);
- Ship block coefficient (C_B).

Loss in speed in shallow water is given in Figure 6.

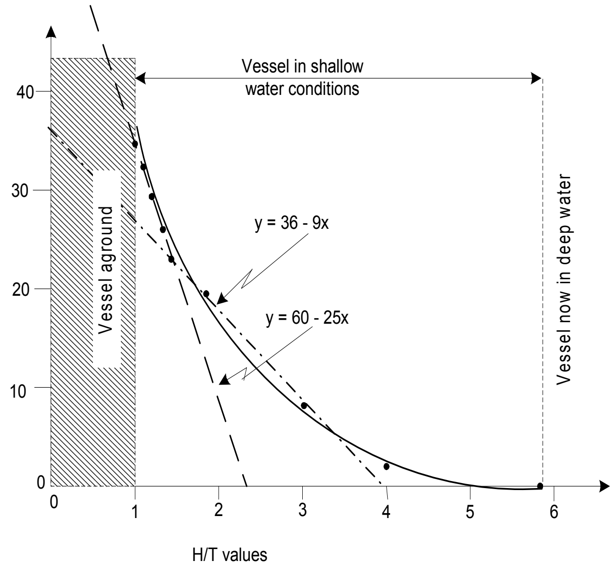


Figure 6. Loss in speed in shallow water (Barrass 2004)

The loss of speed in % equals to:

$$\Delta V_D = 60 - (25 \cdot h/T) \quad , \quad [\%] \qquad (11)$$

for an h/T of 1.10 – 1.40

$$\Delta V_D = 36 - (9 \cdot h/T), \quad [\%] \qquad (12)$$

for an h/T of 1.5 – 3.0
The formula (11) and (12) shows the percentage of loss speed relative to full service speed in deep water (h/T > 7).
In conclusion the ship speed can decrease by about 30% when h/T is 1.10 – 1.40.
Propeller rpm can decrease by about 15% when h/T is 1.10 – 1.40 (Barrass 2004).
According to above the times of each ships operation stage during sea passage is different in each trip. Ship liner as ferry is covering in calm water the same distances from A to B ports. Weather the times of the time of sea passage during each voyage is changing due to degree of environmental constraints. Distances to cover may change in order to alternatives for course diversion. This make increasing in fuel consumption.
The fuel consumption during sea passage depends on the following factors:

- Ship parameters such as from of hull, weight type of main engines, propellers, etc.;
- Number of engaged main engines;
- Ship speed relative to ground;
- Water depth;
- Weather, current, wind, waves;
- Ship draft.

A set of collected statistic on time difference in which the ship is operated in different states there will developed the realistically ship operation criteria to establish save speed during passages in open sea phase.

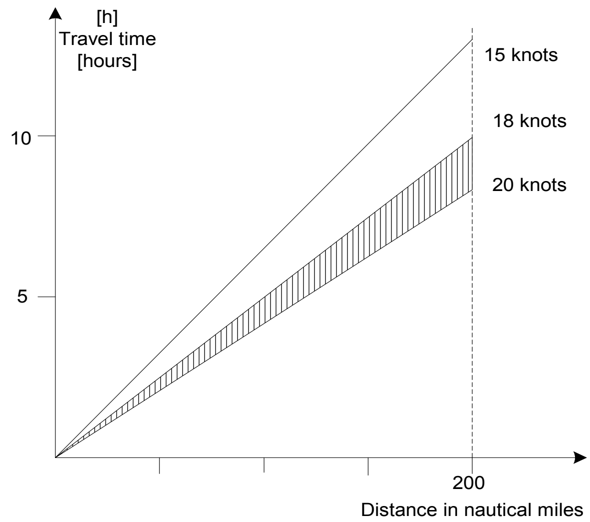


Figure 7. Heuristically defined feasible state space as a function of voyage distance and max/min ship speed in open sea phase (Chen).

4 THE SHIP SPEED LOSS OF FERRY "STENA BALTICA"

The speed characteristics of ferry were estimated in empirical way. Number of observations collected in 2008 (winter time) was limited to 319.

This has given us a rough estimation the speed loss mainly in bad weather condition. (See Figure 8).The ship speed over the ground was measured against wind speed in Beaufort scale using GPS navigator and ECDiS systems.

The ferry has large superstructure in the transverse projection area above waterline so the ship is very susceptible to wind, less to waves.

The ratio of superstructure area to transverse projected area below waterline (draft of the ship) AT/Aw equals to 7.8. She moves at sea as a sailing vessel. The high speed loss in the head winds is suspected to be cause by the fact that the forward ship superstructure amounts to 573 m^2. The side superstructure area equals to 4200 m^2. In this case the speed loss characteristics have been constructed against wind speed.

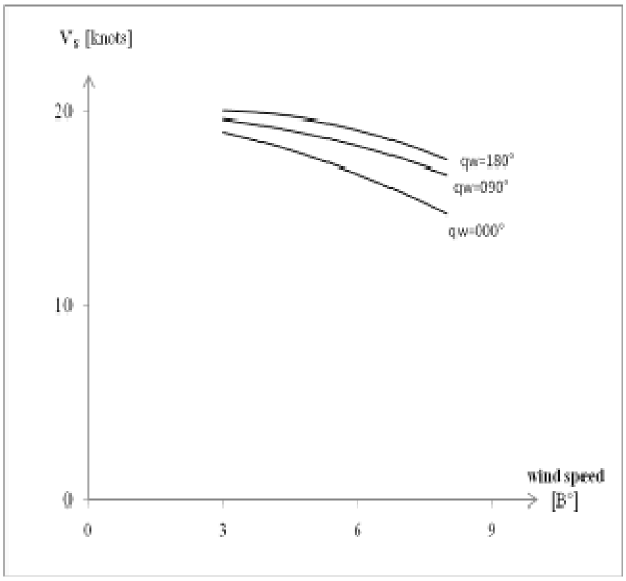


Figure 8. The ship speed in knot against the wind speed in Beaufort scale.

To establish the ferry speed characteristic the polynomial regression function have been used. The output shows the results a second order polynomial model to describe the relationship between ΔV_s and B. (See equations 13-16). The equations to fitted model is:

$$\Delta V_{Si} = a_1 B + b_1 B^2 + r_1, \quad (13)$$

for an $q_w = 000^0$,

$$\Delta V_{Si} = a_2 B + b_2 B^2 + r_2, \quad (14)$$

for an $q_w = 090^0$,

$$\Delta V_{Si} = a_3 B + b_3 B^2 + r_3, \quad (15)$$

for an $q_w = 180^0$,

then

$$V_{Si} = V_o - \Delta V_{Si}, \quad (16)$$

where
$a_1 = +0.14958$, $b_1 = + 0.63520$, $r_1 = - 0.00121$,

$a_2 = -0.04758$, $b_2 = + 0.056061$, $r_2 = - 0.00667$,

$a_3 = - 0.50050$, $b_3 = + 0.91883$, $r_3 = - 0.59286$.

The actual speed curves for different q_w have been shown in Figure 8.

5 THE STATISTICS OF TIME DIFFERENCES IN OPEN SEA OPERATION STATES OF "STENA BALTICA"

Taking into account the operation process of the considered ferry we distinguish the following as its eighteen operation states (Jurdizński et. al 2008):

- an operation state z_1 – loading at Gdynia Port,
- an operation state z_2 – unmooring operations at Gdynia Port,
- an operation state z_3 – leaving Gdynia Port and navigation to "GD" buoy,
- an operation state z_4 – navigation at restricted waters from "GD" buoy to the end of Traffic Separation Scheme,
- an operation state z_5 – navigation at open waters from the end of Traffic Separation Scheme to "Angoring" buoy,
- an operation state z_6 – navigation at restricted waters from "Angoring" buoy to "Verko" Berth at Karlskrona,
- an operation state z_7 – mooring operations at Karlskrona Port,
- an operation state z_8 – unloading at Karlskrona Port,
- an operation state z_9 – loading at Karlskrona Port,
- an operation state z_{10} – unmooring operations at Karlskrona Port,
- an operation state z_{11} – ship turning at Karlskrona Port,
- an operation state z_{12} – leaving Karlskrona Port and navigation at restricted waters to "Angoring" buoy,
- an operation state z_{13} - navigation at open waters from "Angoring" buoy to the entering Traffic Separation Scheme,
- an operation state z_{14} – navigation at restricted waters from the entering Traffic Separation Scheme to "GD" buoy,
- an operation state z_{15} – navigation from "GD" buoy to turning area,
- an operation state z_{16} – ship turning at Gdynia Port,
- an operation state z_{17} – mooring operations at Gdynia Port,
- an operation state z_{18} – unloading at Gdynia Port.

To identify all parameters of "Stena Baltica" ferry operation process the statistical data about this process, have been collected during 42 round trip. (Soszyńska et. al.)

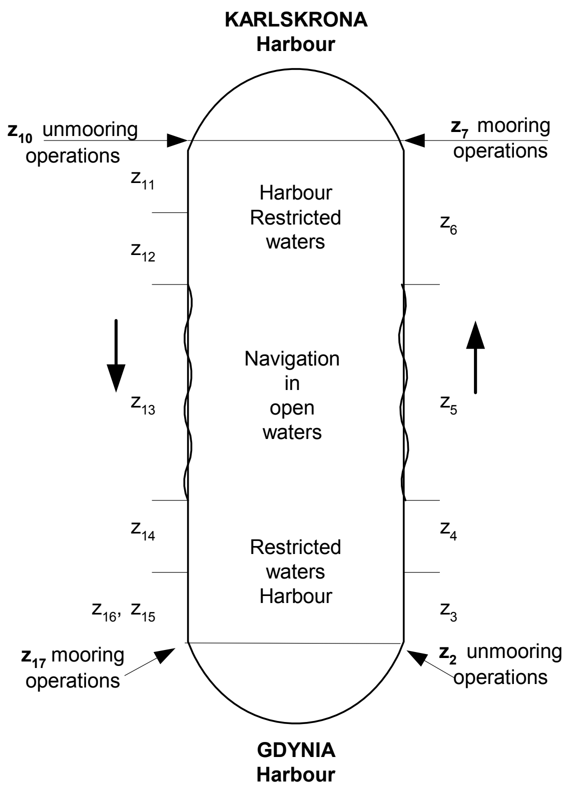


Figure 9. The "Stena Baltica" round trip operation process at sea

In *Figure 9* there have been distinguished only the states of the ferry in her round trip when she was in the navigation process.
In *Table 2* there have been collected the statistical data time differences in sea navigation states.

Table 2. The time difference of states during navigation passage

State	Phase of navigation	Time of operation		Remarks
		$t_{min\ [h]}$	$t_{max\ [h]}$	
z_3	Harbour. Restricted waters	0.50	0.75	Harbour, regulation speed limitation
z_4	Restricted waters	0.75	1.50	Weather restrictions
z_5	Open waters	7.75	12.00	Environmental constraints
z_6	Restricted waters. Harbour	0.50	0.67	Different weather condition
z_{11}	Harbour. Restricted waters	0.05	0.10	Ships turning abilities
z_{12}	Restricted waters	0.35	0.67	Weather and ships traffic condition
z_{13}	Open waters	7.75	<12.00	Environmental constraints
z_{14}	Restricted waters	0.70	1.15	VTS operation and harbour regulations
z_{15} z_{16}	Harbour	0.77	0.77	Due to dense traffic in harbour, speed limitation

These experimental data have shown that the main constrains in ferry operation states in open sea is the speed loss due to bad weather condition.

6 CONCLUSION

1. The ferry operation states z_5 and z_{13} are the longest time differences occurred in open water navigation due to speed loss during unexpected environmental constraints.
2. The major uncertainties involved in the present analysis of the speed loss characteristic are introduced by calculation using small amount of information (319 observations).

3. The required ship speed loss appeared not to exceed 25 percent of full speed in calm water in forward direction of the wind speed below 8-9 Beaufort scale.
4. To say more about the ferry sea keeping characteristic that the route optimization especially in winter season is expected to improve the economics.
5. In commercial applications the most important objective function for ship operation problem is the minimize the voyage cost.
In modern ship operation the following criteria are commonly used;

a. Ship safety;
b. Prevention of ship damage;
c. Maintenance of time schedule;
d. Passenger / crew comfort;
e. Economy of navigation;
f. Minimize the voyage costs (mainly fuel costs).

6. The recorded date from the ship's logbook the wave height, speed and power output, from the past voyages, will help to further development in establish the ship sea keeping characteristic.
7. Information on the actual speed of the ship in different phase of navigation and in different environmental constraints will help the navigator to establish ETA (Estimated Time of Arrival) with good approximation to every position of the ship destination.

REFERENCES

International Maritime Organization, MSC/Circ.1023, MEPC/Circ. 392, Guidelines for Formal Safety Assessment (FSA/for use in the IMO Rule – Making Process. 5 April 2002.

Barrass C.B., 2004. Ship Design and Performance, *Elsevier Butterworth Heinemann, Oxford*.

Boun, R., 2005. Admitance Policy Tidal Bound Ships. *AVV Transport Research Centre*.

Chen H., Weather Routeing a new Approach, www.ocean-system.com

Chen H., 1978. A Dynamic Program for Minimum Cost Ship Routeing Under Uncertainty, *PhD Thesis, M.I.T*.

Hagiwara, H., 2007 Study on Fuel Saving by Ship Weather Routeing, *Tokyo University of Marine Science and Technology.*

Jurdziński, M., 1989. Navigational Passage Planning. *Wydawnictwo Morskie*. Gdańsk (in Polish).

Jurdziński, M., 2003. Navigational Passage Planning in restricted waters. *Fundacja Rozwoju Akademii Morskiej w Gdyni*, (in Polish).

Jurdziński, M. Kołowrocki, K. Soszyńska, J., 2008. An approach to ship operation analysis with primary application to „Stena Baltica" ferry operating at Baltic Sea. *Proc. 2nd Summer Safety and Reliability Seminars – SSARS 2008*, Gdańsk-Sopot, Vol. 2, p. 197-204.

Mizikami, S.at al., Sep. 2007. *Monitoring of Service Performance of a ROPAX Ferry*. *Technical Review*, Vol. 44 No3.

Soszyńska, J. Kołowrocki, K. Kamiński, P. Jurdziński, M. Guze, S. Milczek, B. Golik, P. Preliminary safety and risk analysis and evaluation of „Stena Baltica" ferry in constant and variable operation conditions. *Sub-Task 4.2.5 Safety and Reliability of Complex Industrial Systems and Processes. Poland-Singapore Joint Project 01.12.2007-30.11.2010.*

Stoler, P.H., 2009. Ship Weather Routeing, The Meteorologist's Job? *Meteo Consultant B.V. Wageningen.* www.google.com.

http://research.germanlloyd.de/Projects/DEXTREMEL.

www.bmtsseatech.co.uk *Ship Performance*.

PRELIMINARY RELIABILITY, RISK AND AVAILABILITY ANALYSIS AND EVALUATION OF BULK CARGO TRANSPORTATION SYSTEM IN VARIABLE OPERATION CONDITIONS

K. Kołowrocki, B. Kwiatuszewska-Sarnecka, J. Soszyńska

•
Gdynia Maritime University, Gdynia, Poland

e-mail: kwiatek@am.gdynia.pl

ABSTRACT

In the paper, definitions and theoretical results on system operations process, multi-state system reliability, risk and availability modelling are illustrated by the example of their application to a bulk cargo transportation system operating in Gdynia Port Bulk Cargo Terminal. The bulk cargo transportation system is considered in varying in time operation conditions. The system reliability structure and its components reliability functions are changing in variable operation conditions. The system reliability structures are fixed with a high accuracy. Whereas, the input reliability characteristics of the bulk cargo transportation system components and the system operation process characteristics are not sufficiently exact because of the lack of statistical data. Anyway, the obtained evaluation may be a very useful example in simple and quick systems reliability characteristics evaluation, especially during the design and improving the transportation systems operating in ports.

1 INTRODUCTION

Taking into account the importance of the reliability and operating process effectiveness of technical systems it seems reasonable to expand the two-state approach to multi-state approach in their reliability analysis. The assumption that the systems are composed of multi-state components with reliability states degrading in time gives the possibility for more precise analysis and diagnosis of their reliability and operational processes' effectiveness. This assumption allows us to distinguish a system reliability critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operational process effectiveness. Then, an important system reliability characteristic is the time to the moment of exceeding the system reliability critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state reliability function that is a basic characteristic of the multi-state system. Determining the multi-state reliability function, the risk function and the availability of systems on the base of their components' reliability functions is then the main research problem. Modeling complicated systems operations' processes is difficult mainly because of large number of operations states and impossibility of precise describing of changes between these states. One of the useful approaches in modeling these complicated processes is applying the semi-markov processes Modeling of multi-state real technical systems' reliability and linking it with semi-markov model of these systems' operation processes is the main and practically important research problem of this paper. The paper is devoted to this research problem with reference to basic reliability structures of technical systems and particularly to reliability analysis of a port bulk cargo transportation system in variable operation conditions. This approach to system reliability investigation is based on the multi-state system reliability analysis and on semi-markov processes modeling.

2 THE BULK CARGO TRANSPORTATION SYSTEM DESCRIPTION

The Baltic Bulk Terminal Ltd. In Gdynia is designated for storage and reloading of bulk cargo such as different kinds of fertilisers i.e.: ammonium sulphate, but its main area of activity is to load bulk cargo on board the ships for export. The BBT is not equipped with any devices to enable the discharge of vessels.

There are two independent shipment systems:
1. The system of reloading rail wagons.
2. The system of loading vessels.

Cargo is brought to BBT by trains consisting of self discharging wagons which are discharged to a hopper and then by means of conveyors are transported into the one of four storage tanks (silos). Loading of fertilizers from storage tanks on board the ship is done by means of special reloading system which consists of several belt conveyors and one bucket conveyor which allows the transfer of bulk cargo in a vertical direction. Researched system is a system of belt conveyors, called later on the transport system.
In the conveyor reloading system we distinguish the following transportation subsystems:
S_1, S_2, S_3 – the belt conveyors.
In the conveyor loading system we distinguish the following transportation subsystems:
S_4 – the dosage conveyor,
S_5 – the horizontal conveyor,
S_6 – the horizontal conveyor,
S_7 – the sloping conveyors,
S_8 – the dosage conveyor with buffer,
S_9 – the loading system.
The scheme of this system is presented in **Figure 1**.

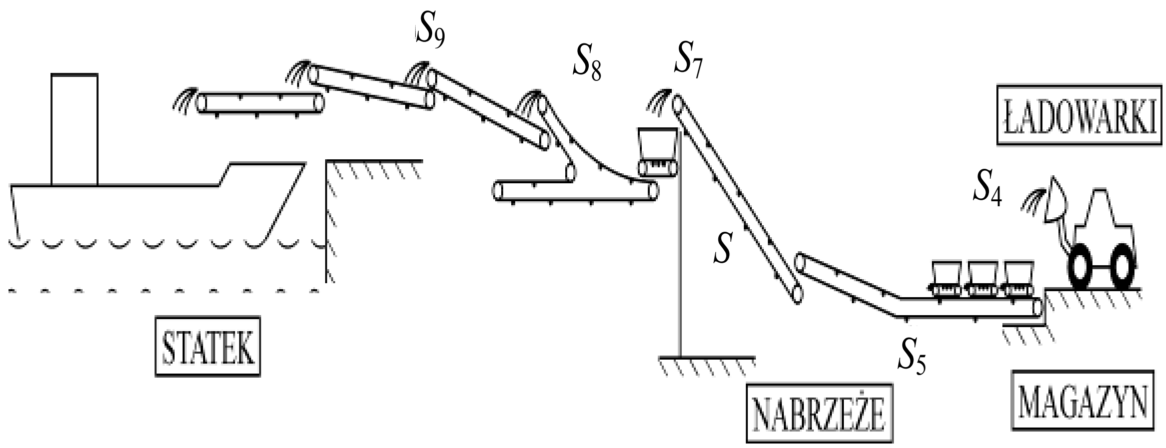


Figure 1. The scheme of port bulk cargo transportation system

After discussion with experts, taking into account the reliability of the operation of the system, we distinguish the following four reliability states of its components:
- a reliability state 3 – ensuring the highest efficiency of the conveyor,
- a reliability state 2 – ensuring less efficient of the working conveyor by spilling cargo out of the belt caused by partial damage to some of the rollers or misalignment of the belt,
- a reliability state 1 – ensuring less efficiency of the working conveyor controlled directly by operator caused by i.e.: stretched or slightly damaged belt,
- a reliability state 0 – the conveyor unable to work which my be caused by i.e.: breakage of the belt, failure of rollers or elongated belt beyond adjustment range.

We preliminarily assume that the bulk cargo transportation system is composed of nine subsystems S_1, S_2, S_3 and S_4, S_5, S_6, S_7, S_8, S_9, having an essential influence on its reliability.

We mark the reliability functions of these subsystems respectively by the vectors

$$R_i(t,\cdot) = [R_i(t,0), R_i(t,1), R_i(t,2), R_i(t,3)], \quad t \in <0,\infty), \ i = 1,2,3, \ldots,9,$$

with the co-ordinates

$$R_i(t,u) = P(S_i(t) \geq u \mid S_i(0) = z) = P(T_i(u) > t),$$

defined for $t \in <0,\infty)$, $u = 0,1,2,3$, $i = 1,2,3, \ldots,9$, where $T_i(u)$, $i = 1,2,3,\ldots,9$, are independent random variables representing the lifetimes of subsystems S_i in the reliability state subset $\{u,u+1,...,3\}$.

Further, assuming that the system is in the reliability state subset $\{u,u+1,...,3\}$ if all its subsystems are in this subset of reliability states, we conclude that the system is a series system of subsystems S_1, S_2, S_3 and S_4, S_5, S_6, S_7, S_8, S_9 with a general scheme presented in **Figure 2**.

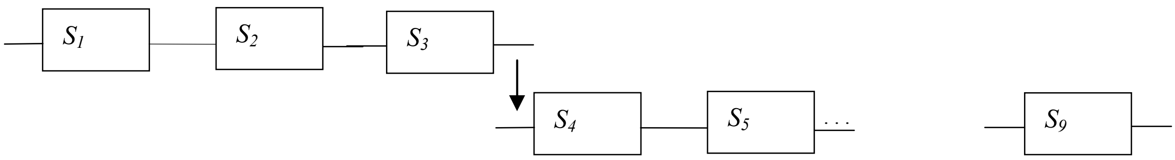


Figure 2. General scheme of transportation system structure

The bulk cargo transportation system consists nine subsystems S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9:

- the subsystem S_1 composed of 1 rubber belt, 2 drums, set of 121 bow rollers, set of 23 belt supporting rollers,
- the subsystem S_2 composed of 1 rubber belt, 2 drums, set of 44 bow rollers, set of 14 belt supporting rollers,
- the subsystem S_3 composed of 1 rubber belt, 2 drums, set of 185 bow rollers, set of 60 belt supporting rollers,
- the subsystem S_4 composed of three identical belt conveyors, each composed of 1 rubber belt, 2 drums, set of 12 bow rollers, set of 3 belt supporting rollers,
- the subsystem S_5 composed of 1 rubber belt, 2 drums, set of 125 bow rollers, set of 45 belt supporting rollers,
- the subsystem S_6 composed of 1 rubber belt, 2 drums, set of 65 bow rollers, set of 20 belt supporting rollers,
- the subsystem S_7 composed of 1 rubber belt, 2 drums, set of 12 bow rollers, set of 3 belt supporting rollers,
- the subsystem S_8 composed of 1 rubber belt, 2 drums, set of 162 bow rollers, set of 53 belt supporting rollers,
- the subsystem S_9 composed of 3 rubber belts, 6 drums, set of 64 bow rollers, set of 20 belt supporting rollers.

3 THE BULK CARGO TRANSPORTATION SYSTEM OPERATION PROCESS CHARACTERISTICS EVALUATION

Technical subsystems $S_1,\ S_2,\ldots,\ S_9$, indicated in **Figure 1** are forming a general bulk cargo transportation system reliability structure presented in **Figure 2**. However, the bulk cargo transportation system reliability structure and the subsystems reliability depend on its changing in time operation states. Taking into account the operation process of the considered system we distinguish the following as its three operation states:

- an operation state $z_1 -$ the discharging rail wagons to storage tanks or hall when subsystems S_1, S_2, S_3, are used.
- an operation state $z_2 -$ the loading of fertilizers from storage tanks or hall on board the ship is done by using S_4, S_5, S_6, S_7, S_8 , S_9, subsystems.
- an operation state $z_3 -$ the loading of fertilizers from rail wagons on board the ship is done by using S_1, S_2, S_3, S_6, S_7, S_8 , S_9 subsystems.

According to expert opinions in the operation process, $Z(t),\ \ t \ge 0$, we distinguished three operation states: z_1, z_2, z_3. On the basis of data coming from experts, the probabilities of transitions between the operation states are approximately given by

$$[p_{bl}]_{3x3} = \begin{bmatrix} 0 & 0.37 & 0.63 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Using the results given above and when the mean values of the conditional sojourn times M_{bl}, $b,l = 1,2,3$, $b \ne l$, are given, we can find the mean values of the unconditional sojourn times M_b, $b = 1,2,3$, in particular operation states z_b. Next, knowing the mean values of the unconditional sojourn times in particular operation states z_b and solving the system of equations

$$\begin{cases} [\pi_1, \pi_2, \pi_3] = [\pi_1, \pi_2, \pi_3][p_{bl}]_{3x3} \\ \pi_1 + \pi_2 + \pi_3 = 1, \end{cases}$$

$$\pi_1 = 0.5,\ \pi_2 = 0.185,\ \pi_3 = 0.315,$$

we obtain the limit values of the transient probabilities $p_b(t)$ at the operational states z_b,

$$p_1 = 0.6679, \quad p_2 = 0.0945, \quad p_3 = 0.2376. \tag{1}$$

4 THE BULK CARGO TRANSPORTATION SYSTEM IN VARIABLE OPERATION CONDITIONS RELIABILITY, RISK AND AVAILABILITY EVALUATION

We assume as earlier that the bulk cargo transportation system is composed of $n = 9$ subsystems S_i, $i = 1,2,3,...,9$ and that the changes of the process $Z(t)$ of the bulk cargo transportation system operation states have an influence on the system subsystems S_i reliability and on the reliability structure as well. Thus, we denote the conditional reliability function of the bulk cargo transportation system subsystem S_i while the system is at the operational state z_b, $b = 1,2,3$, by

$$R_i^{(b)}(t,\cdot) = [1,\ R_i^{(b)}(t,1),\ R_i^{(b)}(t,2),\ \ldots,\ R_i^{(b)}(t,z)],$$

where for $t \in < 0,\infty)$, $b = 1,2,3$, $u = 1,2,\ldots,z$,

$$R_i^{(b)}(t,u) = P(T_i^{(b)}(u) > t \mid Z(t) = z_b),$$

and the conditional reliability function of the bulk cargo transportation system while the system is at the operational state z_b, $b = 1,2,3,$ by

$$\boldsymbol{R}_{n_b}^{(b)}(t,\cdot) = [1,\ \boldsymbol{R}_{n_b}^{(b)}(t,1),\ \boldsymbol{R}_{n_b}^{(b)}(t,2), .., \boldsymbol{R}_{n_b}^{(b)}(t,z)\ ],$$

where for $t \in <0,\infty)$, $b = 1,2,3,$ $n_b \in \{1,2,...,9\}$, $u = 1,2,..., z,$

$$\boldsymbol{R}_{n_b}^{(b)}(t,u) = P(T^{(b)}(u) > t | Z(t) = z_b).$$

We assume that the bulk cargo transportation system subsystems S_i, $i = 1,2,3,...,9,$ are its three-state components, i.e. $z = 3$, with the multi-state reliability functions

$$R_{ij}^{(b)}(t,\cdot) = [1,\ R_{ij}^{(b)}(t,1),\ R_{ij}^{(b)}(t,2),\ R_{ij}^{(b)}(t,3)\ ],$$

with exponential co-ordinates different in various operation states z_b, $b = 1,2,3.$ From Kołowrocki K. & Kwiatuszewska-Sarnecka B. (2008), we have following results.

At system operational state z_1, system is composed of three series subsystems S_1, S_2, S_3.

The subsystem S_1 is a multi-state series system consists of n = 147 components and its reliability function is given by

$$[\,\overline{\boldsymbol{R}}_{147}(t,\cdot)\,]^{(1)} = [1, [\,\overline{\boldsymbol{R}}_{147}(t,1)\,]^{(1)}, [\,\overline{\boldsymbol{R}}_{147}(t,2)\,]^{(1)}, [\,\overline{\boldsymbol{R}}_{147}(t,3)\,]^{(1)}],\ \ t \in <0,\infty), \tag{2}$$

where

$$[\,\overline{\boldsymbol{R}}_{147}(t,1)\,]^{(1)} = \exp[-13.132t], \tag{3}$$

$$[\,\overline{\boldsymbol{R}}_{147}(t,2)\,]^{(1)} = \exp[-16.569t], \tag{4}$$

$$[\,\overline{\boldsymbol{R}}_{147}(t,3)\,]^{(1)} = \exp[-21.642t]. \tag{5}$$

The subsystem S_2 is a multi-state series system, consists of n = 61 components, its reliability function is given by

$$[\,\overline{\boldsymbol{R}}_{61}(t,\cdot)\,]^{(1)} = [1, [\,\overline{\boldsymbol{R}}_{61}(t,1)\,]^{(1)}, [\,\overline{\boldsymbol{R}}_{61}(t,2)\,]^{(1)}, [\,\overline{\boldsymbol{R}}_{61}(t,3)\,]^{(1)}],\ \ t \in <0,\infty), \tag{6}$$

where

$$[\,\overline{\boldsymbol{R}}_{61}(t,1)\,]^{(1)} = \exp[-5.204\text{t}], \tag{7}$$

$$[\,\overline{\boldsymbol{R}}_{61}(t,2)\,]^{(1)} = \exp[-6.517\text{t}], \tag{8}$$

$$[\,\overline{\boldsymbol{R}}_{61}(t,3)\,]^{(1)} = \exp[-8.406\text{t}]. \tag{9}$$

The subsystem S_3 is a multi-state series system, consists of n = 248 components, its reliability function is given by

$$[\,\overline{\boldsymbol{R}}_{248}(t,\cdot)\,]^{(1)} = [1, [\,\overline{\boldsymbol{R}}_{248}(t,1)\,]^{(1)}, [\,\overline{\boldsymbol{R}}_{248}(t,2)\,]^{(1)}, [\,\overline{\boldsymbol{R}}_{248}(t,3)\,]^{(1)}],\ \ t \in <0,\infty), \tag{10}$$

where

$$[\,\overline{\boldsymbol{R}}_{248}(t,1)\,]^{(1)} = \exp[-21.227t], \tag{11}$$

$$[\,\overline{\boldsymbol{R}}_{248}(t,2)\,]^{(1)} = \exp[-26.577\text{t}], \tag{12}$$

$$[\,\overline{\boldsymbol{R}}_{248}(t,3)\,]^{(1)} = \exp[-34.232\text{t}]. \tag{13}$$

The reliability function of the bulk cargo transportation system, at the operational state z_1, is given by

$$[\,\overline{\boldsymbol{R}}(t,\cdot)]^{(1)} = [1, [\,\overline{\boldsymbol{R}}(t;1)\,]^{(1)}, [\,\overline{\boldsymbol{R}}(t;2)\,]^{(1)},\ \overline{\boldsymbol{R}}(t;3)\,]^{(1)}], \tag{14}$$

where

$$[\,\overline{\boldsymbol{R}}(t;1)\,]^{(1)} = \exp[-39.563t], \tag{15}$$

$$[\,\overline{\boldsymbol{R}}(t;2)\,]^{(1)} = \exp[-49.663t], \tag{16}$$

$$[\,\overline{\boldsymbol{R}}(t;3)\,]^{(1)} = \exp[-64.280t]. \tag{17}$$

The expected values and standard deviations of the bulk cargo transportation system conditional lifetimes in the reliability state subsets calculated from the above result given by (15)-(17), at the operational state z_1, are:

$$m^{(1)}(1) \cong 0.025,\ m^{(1)}(2) \cong 0.020,\ m^{(1)}(3) \cong 0.016 \text{ y}, \tag{18}$$

$$\sigma^{(1)}(1) \cong 0.025,\ \sigma^{(1)}(2) \cong 0.020,\ \sigma^{(1)}(3) \cong 0.016, \tag{19}$$

from (18)-(19) it follows the conditional lifetimes in the particular reliability states at the operational state z_1 are:

$$\overline{m}^{(1)}(1) \cong 0.005,\ \overline{m}^{(1)}(2) \cong 0.004,\ \overline{m}^{(1)}(3) \cong 0.016.$$

At system operational state z_2, system is composed of one series-parallel subsystem S_4 and five series subsystems S_5, S_6, S_7, S_8, S_9 .

The subsystem S_4 consists of k = 3 identical belt conveyors, each composed of l = 18 components. Thus the subsystem S_4 is a multi-state series-parallel system, its reliability function is given by

$$[\boldsymbol{R}_{3,18}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}_{3,18}(t,1)]^{(2)}, [\boldsymbol{R}_{3,18}(t,2)]^{(2)}, [\boldsymbol{R}_{3,18}(t,3)]^{(2)}],\ t \in <0,\infty), \tag{20}$$

where

$$[\boldsymbol{R}_{3,18}(t,1)]^{(2)} = 1-[1-\exp[-2.751t]]^3 = \exp[-8.253t]-3\exp[-5.502t]+3\exp[-2.751t], \tag{21}$$

$$[\boldsymbol{R}_{3,18}(t,2)]^{(2)} = 1-[1-\exp[-2.956t]]^3 = \exp[-8.868t]-3\exp[-5.912t]+3\exp[-2.956t], \tag{22}$$

$$[\boldsymbol{R}_{3,18}(t,3)]^{(2)} = 1-[1-\exp[-3.276t]]^3 = \exp[-9.828t]-3\exp[-6.552t]+3\exp[-3.276t]], \tag{23}$$

Thus the subsystem S_5 is a multi-state series system, its reliability function, is given by

$$[\overline{\boldsymbol{R}}_{173}(t,\cdot)]^{(2)} = [1, [\overline{\boldsymbol{R}}_{173}(t,1)]^{(2)}, [\overline{\boldsymbol{R}}_{173}(t,2)]^{(2)}, [\overline{\boldsymbol{R}}_{173}(t,3)]^{(2)}],\ t \in <0,\infty), \tag{24}$$

where

$$[\overline{\boldsymbol{R}}_{173}(t,1)]^{(2)} = \exp[-14.642t], \tag{25}$$

$$[\overline{\boldsymbol{R}}_{173}(t,2)]^{(2)} = \exp[-18.297t], \tag{26}$$

$$[\overline{\boldsymbol{R}}_{173}(t,3)]^{(2)} = \exp[-23.662t]. \tag{27}$$

Thus the subsystem S_6 is a multi-state series system, consists of n = 88 components, its reliability function, is given by

$$[\overline{\boldsymbol{R}}_{88}(t,\cdot)]^{(2)} = [1, [\overline{\boldsymbol{R}}_{88}(t,1)]^{(2)}, [\overline{\boldsymbol{R}}_{88}(t,2)]^{(2)}, [\overline{\boldsymbol{R}}_{88}(t,3)]^{(2)}],\ t \in <0,\infty), \tag{28}$$

where

$$[\overline{\boldsymbol{R}}_{88}(t,1)]^{(2)} = \exp[-7.547t], \tag{29}$$

$$[\overline{\boldsymbol{R}}_{88}(t,2)]^{(2)} = \exp[-9.457t], \tag{30}$$

$$[\overline{\boldsymbol{R}}_{88}(t,3)]^{(2)} = \exp[-12.272t]. \tag{31}$$

Thus the subsystem S_7 is a multi-state series system, consists of n = 18 components, its reliability function, is given by

$$[\overline{\boldsymbol{R}}_{18}(t,\cdot)]^{(2)} = [1, [\overline{\boldsymbol{R}}_{18}(t,1)]^{(2)}, [\overline{\boldsymbol{R}}_{18}(t,2)]^{(2)}, [\overline{\boldsymbol{R}}_{18}(t,3)]^{(2)}],\ t \in <0,\infty), \tag{32}$$

where

$$[\overline{\boldsymbol{R}}_{18}(t,1)]^{(2)} = \exp[-2.751t], \tag{33}$$

$$[\overline{\boldsymbol{R}}_{18}(t,2)]^{(2)} = \exp[-2.956t], \tag{34}$$

$$[\overline{\boldsymbol{R}}_{18}(t,3)]^{(2)} = \exp[-3.276t]. \tag{35}$$

Thus the subsystem S_8 is a multi-state series system, consists of n = 218 components, its reliability function, is given by

$$[\overline{\boldsymbol{R}}_{218}(t,\cdot)]^{(2)} = [1, [\overline{\boldsymbol{R}}_{218}(t,1)]^{(2)}, [\overline{\boldsymbol{R}}_{218}(t,2)]^{(2)}, [\overline{\boldsymbol{R}}_{218}(t,3)]^{(2)}],\ t \in <0,\infty), \tag{36}$$

where

$$[\overline{\boldsymbol{R}}_{218}(t,1)]^{(2)} = \exp[-18.639t], \quad (37)$$

$$[\overline{\boldsymbol{R}}_{218}(t,2)]^{(2)} = \exp[-23.333t], \quad (38)$$

$$[\overline{\boldsymbol{R}}_{218}(t,3)]^{(2)} = \exp[-30.226t]. \quad (39)$$

Thus the subsystem S_9 is a multi-state series system, consists of $n = 93$ components, its reliability function, is given by

$$[\overline{\boldsymbol{R}}_{93}(t,\cdot)]^{(2)} = [1, [\overline{\boldsymbol{R}}_{93}(t,1)]^{(2)}, [\overline{\boldsymbol{R}}_{93}(t,2)]^{(2)}, [\overline{\boldsymbol{R}}_{93}(t,3)]^{(2)}], \ t \in <0,\infty), \quad (40)$$

where

$$[\overline{\boldsymbol{R}}_{93}(t,1)]^{(2)} = \exp[-5.926t], \quad (41)$$

$$[\overline{\boldsymbol{R}}_{93}(t,2)]^{(2)} = \exp[-8.063t], \quad (42)$$

$$[\overline{\boldsymbol{R}}_{93}(t,3)]^{(2)} = \exp[-10.152t]. \quad (43)$$

The reliability function of the bulk cargo transportation system, at the operational state z_2, is given by

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(2)} = [1, [\overline{\boldsymbol{R}}(t;1)]^{(2)}, [\overline{\boldsymbol{R}}(t;2)]^{(2)}, [\overline{\boldsymbol{R}}(t;3)]^{(2)}], \ t \geq 0, \quad (44)$$

where

$$[\overline{\boldsymbol{R}}(t;1)]^{(2)} = \exp[-57.758t]-3\exp[-55.007t]+3\exp[-52.256t], \quad (45)$$

$$[\overline{\boldsymbol{R}}(t;2)]^{(2)} = \exp[-70.974t]-3\exp[-68.018t]+3\exp[-65.062t], \quad (46)$$

$$[\overline{\boldsymbol{R}}(t;3)]^{(2)} = \exp[-89.416t]-3\exp[-86.140t]+3\exp[-82.864t], \quad (47)$$

The expected values and standard deviations of the bulk cargo transportation system lifetimes at the operational state z_2, in the safety state subsets calculated from the above result, according to (45)-(47), are:

$$m^{(2)}(1) \cong 0.020, \ m^{(2)}(2) \cong 0.016, \ m^{(2)}(3) \cong 0.013, \quad (48)$$

$$\sigma^{(2)}(1) \cong 0.020, \ \sigma^{(2)}(2) \cong 0.016, \ \sigma^{(2)}(3) \cong 0.013, \quad (49)$$

and further, using (48), it follows the conditional lifetimes in the particular reliability states at the operational state z_2, are:

$$\overline{m}^{(2)}(1) \cong 0.004, \ \overline{m}^{(2)}(2) \cong 0.003, \ \overline{m}^{(2)}(3) \cong 0.013.$$

At system operational state z_3, system is composed of seven non-homogenous series subsystems S_1, S_2, S_3, S_6, S_7, S_8, S_9.

The reliability function of the bulk cargo transportation system, at the operational state z_3, is given by

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(3)} = [1, [\overline{\boldsymbol{R}}(t;1)]^{(3)}, [\overline{\boldsymbol{R}}(t;2)]^{(3)}, [\overline{\boldsymbol{R}}(t;3)]^{(3)}], \ t \geq 0, \quad (50)$$

where

$$[\overline{\boldsymbol{R}}(t;1)]^{(3)} = \exp[-74.426t], \quad (51)$$

$$[\overline{\boldsymbol{R}}(t;2)]^{(3)} = \exp[-93.472t], \quad (52)$$

$$[\overline{\boldsymbol{R}}(t;3)]^{(3)} = \exp[-150.206t]. \quad (53)$$

The expected values and standard deviations of the bulk cargo transportation system lifetimes at the operational state z_3, in the safety state subsets calculated from the above result, according to (51)-(53), are:

$$m^{(3)}(1) \cong 0.013, \ m^{(3)}(2) \cong 0.011, \ m^{(3)}(3) \cong 0.007, \quad (54)$$

$$\sigma^{(3)}(1) \cong 0.013, \ \sigma^{(3)}(2) \cong 0.011, \sigma^{(3)}(3) \cong 0.007, \quad (55)$$

and further, using (54), it follows the conditional lifetimes in the particular reliability states at the operational state z_3, are:

$$\overline{m}^{(3)}(1) \cong 0.002,\ \overline{m}^{(2)}(2) \cong 0.004,\ \overline{m}^{(3)}(3) \cong 0.007.$$

In the case when the system operation time is large enough, the unconditional reliability function of the bulk cargo transportation system is given by the vector

$$\boldsymbol{R_9}(t,\cdot) = [1,\ \boldsymbol{R_9}(t,1),\ \boldsymbol{R}_9(t,2),\ \boldsymbol{R_9}(t,3)],\ t \geq 0, \tag{56}$$

the vector co-ordinates are given respectively by

$$\begin{aligned}\boldsymbol{R}_9(t,1) &= p_1[\overline{\boldsymbol{R}}(t,1)]^{(1)} + p_2[\overline{\boldsymbol{R}}(t,1)]^{(2)} + p_3[\overline{\boldsymbol{R}}(t,1)]^{(3)}\\ &= 0.6679\exp[-39.563t] + 0.0945\,(\exp[-57.758t] - 3\exp[-55.007t] + 3\exp[-52.256t])\\ &\quad + 0.2376\exp[-74.426t] \ \text{ for } t \geq 0,\end{aligned} \tag{57}$$

$$\begin{aligned}\boldsymbol{R}_9(t,2) &= p_1[\overline{\boldsymbol{R}}(t,2)]^{(1)} + p_2[\overline{\boldsymbol{R}}(t,2)]^{(2)} + p_3[\overline{\boldsymbol{R}}(t,2)]^{(3)}\\ &= 0.6679\exp[-49.663t] + 0.0945\,(\exp[-70.974t] - 3\exp[-68.018t] + 3\exp[-65.062t])\\ &\quad + 0.2376\exp[-93.472t] \text{ for } t \geq 0,\end{aligned} \tag{58}$$

$$\begin{aligned}\boldsymbol{R}_9(t,3) &= p_1[\overline{\boldsymbol{R}}(t,3)]^{(1)} + p_2[\overline{\boldsymbol{R}}(t,3)]^{(2)} + p_3[\overline{\boldsymbol{R}}(t,3)]^{(3)}\\ &= 0.6679\exp[-64.280t] + 0.0945\,(\exp[-89.416t] - 3\exp[-86.140t] + 3\exp[-82.864t])\\ &\quad + 0.2376\exp[-150.206t] \text{ for } t \geq 0,\end{aligned} \tag{59}$$

where $[\boldsymbol{R}(t,1)]^{(1)}$, $[\boldsymbol{R}(t,1)]^{(2)}$, $[\overline{\boldsymbol{R}}(t,1)]^{(3)}$ are given by (15), (45), (51) and $[\boldsymbol{R}(t,2)]^{(1)}$, $[\boldsymbol{R}(t,2)]^{(2)}$, $[\overline{\boldsymbol{R}}(t,2)]^{(3)}$ are given by (16), (46), (52) and $[\boldsymbol{R}(t,3)]^{(1)}$, $[\boldsymbol{R}(t,3)]^{(2)}$, $[\overline{\boldsymbol{R}}(t,3)]^{(3)}$ are given by (17), (47), (53).

The mean values of the system unconditional lifetimes in the reliability state subsets, after considering (17), (47), (81), and (57)-(59), respectively are

$$m(1) = 0.022,\ m(2) = 0.017,\ m(3) = 0.014.$$

The mean values of the system lifetimes in the particular reliability states are

$$\overline{m}(1) = 0.005,\ \overline{m}(2) = 0.003,\ \overline{m}(3) = 0.014.$$

If the critical safety state is $r = 2$, then the system risk function, is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}_9(t,2) \quad t \geq 0,$$

where

$$\begin{aligned}\boldsymbol{r}(t) &= 1 - [0.6679\exp[-49.663t] + 0.0945\,(\exp[-70.974t] - 3\exp[-68.018t]\\ &\quad + 3\exp[-65.062t]) + 0.2376\exp[-93.472t]],\ t \geq 0.\end{aligned}$$

Hence, the moment when the system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.00084 \text{ year}.$$

If $\delta = 0.1$ then $\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.00173$ year.

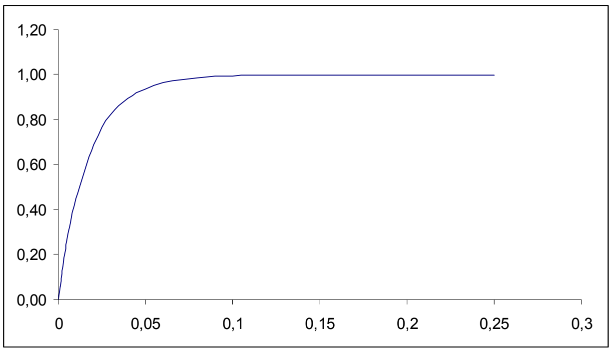


Figure 3. The graph of the port bulk cargo transportation system risk function $r(t)$

Further, assuming that the bulk cargo transportation system is repaired after its failure and that the time of the system renovation is ignored, we obtain the following results:

i) the distribution of the time $S_N(2)$ until the *Nth* exceeding of reliability critical state 2 of this system, for sufficiently large *N*, has approximately normal distribution $N(0.017N, 0.017\sqrt{N})$, i.e.,

$$F^{(N)}(t,2) = P(S_N(2) < t) \cong F_{N(0,1)}(\frac{0.017N - t}{0.017\sqrt{N}}),\ t \in (-\infty, \infty),$$

ii) the expected value and the variance of the time $S_N(2)$ until the *Nth* exceeding the reliability critical state 1 of this system take respectively forms

$$E[S_N(2)] = 0.017N,\ D[S_N(2)] = 0.000289N,$$

iii) the distribution of the number $N(t,2)$ of exceeding the reliability critical state 2 of this system up to the moment t, $t \geq 0$, for sufficiently large *t*, is approximately of the form

$$P(N(t,2) = N) \cong F_{N(0,1)}(\frac{0.017N - t}{0.00222\sqrt{t}}) - F_{N(0,1)}(\frac{0.017(N+1) - t}{0.00222\sqrt{t}}),\ N = 0,1,2,...,$$

iv) the expected value and the variance of the number $N(t,2)$ of exceeding the reliability critical state 2 of this system at the moment t, $t \geq 0$, for sufficiently large *t*, approximately take respectively forms

$$H(t,1) = 58.824t,\ D(t,1) = 58.824t.$$

Further, assuming that the bulk cargo transportation system is repaired after its failure and that the time of the system renovation is not ignored and it has the mean value $\mu_0(2) = 0.001$, and the standard deviation $\sigma_0(2) = 0.001$, we obtain the following results:

i) the distribution function of the time $\overline{\overline{S}}_N(2)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution $N(0.018N, 0.01703\sqrt{N})$, i.e.,

$$\overline{\overline{F}}^{(N)}(t,2) = P(\overline{\overline{S}}_N(2) < t) \cong F_{N(0,1)}(\frac{t - 0.018N)}{0.01703\sqrt{N}}),\ t \in (-\infty, \infty),\ N = 1,2,...,$$

ii) the expected value and the variance of the time $\overline{\overline{S}}_N(2)$ until the *Nth* system's renovation take respectively forms

$$E[\overline{\overline{S}}_N(2)] \cong 0.018N,\ D[\overline{\overline{S}}_N(2)] \cong 0.00029N,$$

iii) the distribution function of the time $\overline{S}_N(2)$ until the *Nth* exceeding the reliability critical state 1 of this system takes form

$$\overline{F}^{(N)}(t,2) = P(\overline{S}_N(2) < t) = F_{N(0,1)}(\frac{t - 0.018N + 0.001}{\sqrt{0.00029N - 0.000001}}),\ t \in (-\infty, \infty),\ N = 1,2,...,$$

iv) the expected value and the variance of the time $\overline{S}_N(2)$ until the *Nth* exceeding the reliability critical state 2 of this system take respectively forms

$$E[\overline{S}_N(2)] \cong 0.017N + 0.001(N-1),\ D[\overline{S}_N(2)] \cong 0.000289N + 0.000001(N-1),$$

v) the distribution of the number $\overline{\overline{N}}(t,2)$ of system's renovations up to the moment t, $t \geq 0$, is of the form

$$P(\overline{\overline{N}}(t,2) = N) \cong F_{N(0,1)}(\frac{0.018N - t)}{0.1269\sqrt{N}}) -_{N(0,1)} (\frac{0.018(N+1) - t)}{0.1269\sqrt{N}}),\quad N = 1,2,...,$$

vi) the expected value and the variance of the number $\overline{\overline{N}}(t,2)$ of system's renovations up to the moment t, $t \geq 0$, take respectively forms

$$\overline{\overline{H}}(t,2) \cong 55.556t,\ \overline{\overline{D}}(t,2) \cong 49.743t,$$

vii) the distribution of the number $\overline{N}(t,2)$ of exceeding the reliability critical state 2 of this system up to the moment $t, t \geq 0,$ is of the form

$$P(\bar{N}(t,2) = N) \cong F_{N(0,1)}(\frac{0.018N - t - 0.001}{0.1269\sqrt{t + 0.001}}) - F_{N(0,1)}(\frac{0.018(N+1) - t - 0001}{0.1269\sqrt{t - 0.001}}),\ N = 1,2,...,$$

viii) the expected value and the variance of the number $\overline{N}(t,2)$ of exceeding the reliability critical state 2 of this system up to the moment $t, t \geq 0,$ are respectively given by

$$\bar{H}(t,2) \cong \frac{t + 0.001}{0.018},\ \ \bar{D}(t,1) \cong 49.743(t + 0.001),$$

ix) the availability coefficient of the system at the moment t is given by the formula

$$K(t,2) \cong 0.9444\ ,\ \ t \geq 0,$$

x) the availability coefficient of the system in the time interval $< t, t+\tau),\ \ \tau > 0,$ is given by the formula

$$K(t,\tau,2) \cong 55.556 \int_{\tau}^{\infty} \boldsymbol{R}_3(t,2)dt,\ \ t \geq 0,\ \ \tau > 0.$$

5 CONCLUSION

In the paper the multi-state approach to the reliability, risk and availability analysis and evaluation of complex technical systems operating in variable operation conditions has been practically applied. Theoretical definitions and results have been illustrated by the example of their application in the reliability, risk and availability evaluation of a bulk cargo transportation system.

6 PREFERENCES

- Barlow, R. E. & Proschan, F. (1975). *Statistical Theory of Reliability and Life Testing. Probability Models*. Holt Rinehart and Winston, Inc., New York.
- Kwiatuszewska-Sarnecka, B. (2002). Reliability improvement of a port bulk cargo transport system. *Proc. 3-rd International Congress on Maritime Technological Innovations and Research*, Bilbao, 591-599.
- Kołowrocki K. & Kwiatuszewska-Sarnecka B. (2005). Ocena niezawodności i gotowości portowego systemu transportu towarów sypkich. *Zeszyty Naukowe Akademii Morskiej w Szczecinie* 6, *Materiały XI Międzynarodowej Konferencji Naukowo-Technicznej „Inżynieria Ruchu Morskiego – IMR 2005*, Świnoujście, 199-210.
- Kołowrocki K., Kwiatuszewska-Sarnecka B. & Soszyńska J. (2008). Modelling of operational processes of port bulk cargo system. *Proc. 2nd Summer Safety and Reliability Seminars – SSARS 2008*, Gdańsk-Sopot, Vol. 2, 217-222.
- Kołowrocki K. & Kwiatuszewska-Sarnecka B. (2008). Reliability and risk analysis of large systems with ageing components. *Reliability Engineering & System Safety, Volume 93*, 1821-182.
- Kołowrocki K. & Kwiatuszewska-Sarnecka B. (2008). *Preliminary modelling, statistical identification and evaluation of reliability, risk, availability and safety of port, shipyard and ship technical systems in constant and variable operation conditions.* WP4 - Task 4.2. *Preliminary reliability, risk and availability analysis and evaluation of a port bulk cargo transportation system in constant and variable operation conditions.* WP4 - Sub-Task

4.2.2 – English – 30.11.2008. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.

- Kołowrocki K. & Kwiatuszewska-Sarnecka B. (2008). *Data Mining for Identification and Prediction of Safety and Reliability Characteristics of Complex Industrial Systems and Processes.* WP6 - Task 6.2. *Preliminary reliability, risk and availability analysis and evaluation of bulk cargo transportation system in variable operation conditions.* WP6 - Sub-Task 6.2.2 – English. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.

Acknowledgements
The paper describes part of the work in the Poland-Singapore Joint Research Project titled "Safety and Reliability of Complex Industrial Systems and Processes" supported by grants from the Poland's Ministry of Science and Higher Education (MSHE grant No. 63/N-Singapore/2007/0) and the Agency for Science, Technology and Research of Singapore (A*STAR SERC grant No. 072 1340050).

METHODS AND ALGORITHMS FOR EVALUATING UNKNOWN PARAMETERS OF OPERATION PROCESSES OF COMPLEX TECHNICAL SYSTEMS

K. Kolowrocki, J. Soszynska.

•

Gdynia Maritime University, Gdynia, Poland

e-mail: katmatkk@am.gdynia.pl, joannas@am.gdynia.pl

ABSTRACT

The paper objectives are to present the methods and tools useful in the statistical identifying unknown parameters of the operation models of complex technical systems and to apply them in the maritime industry. There are presented statistical methods of determining unknown parameters of the semi-markov model of the complex technical system operation processes. There is also presented the chi-square goodness-of-fit test applied to verifying the distributions of the system operation process conditional sojourn times in the particular operation states. Applications of these tools to identifying and predicting the operation characteristics of a ferry operating at the Baltic Sea waters are presented as well.

1 INTRODUCTION

Many real transportation systems belong to the class of complex systems. It is concerned with the large numbers of components and subsystems they are built and with their operating complexity. Modeling the complicated system operation processes, is difficult because of the large number of the operation states, impossibility of their precise defining and because of the impossibility of the exact describing the transitions between these states. The changes of the operation states of the system operations processes cause the changes of these systems reliability structures and also the changes of their components reliability functions (Blokus-Roszkowska et all. 2008b). The models of various multistate complex systems are considered in (Blokus-Roszkowska et all. 2008a). The general joint models linking these system reliability models with the models of their operation processes (Kolowrocki, Soszynska 2008), allowing us for the evaluation of the reliability and safety of the complex technical systems in variable operations conditions, are constructed in (Blokus-Roszkowska et all. 2008b).

In order to be able to apply these general models practically in the evaluation and prediction of the reliability of real complex technical it is necessary to elaborate the statistical methods concerned with determining the unknown parameters of the proposed models, namely the probabilities of the initials system operation states, the probabilities of transitions between the system operation states and the distributions of the sojourn times of the system operation process in the particular operation states and also the unknown parameters of the conditional multistate reliability functions of the system components in various operation states. It is also necessary the elaborating the methods of testing the hypotheses concerned with the conditional sojourn times of the system operations process in particular operations states and the hypotheses concerned with the conditional multistate reliability functions of the system components in the system various operation states. The model of the operation process of the complex technical system with the distinguished their operation states is proposed in (Kolowrocki, Soszynska 2008). The semi-markov process is used to construct a general probabilistic model of the considered complex industrial system operation process. To construct this model there were defined the vector of the probabilities of the system initial operation states, the matrix of the probabilities of transitions between the operation states, the matrix of the distribution functions and the matrix of the density functions of the conditional sojourn times in the particular operation states. To describe the system operation process conditional sojourn times in the particular operation states the uniform distribution, the triangular distribution, the double trapezium distribution, the quasi-trapezium distribution, the exponential distribution, the Weibull's distribution, the normal distribution and the chimney distribution are suggested in (Kolowrocki, Soszynska 2008). In this paper, the formulae estimating unknown parameters of these distributions are given and the chi-square test is applied to verifying the hypotheses about these distributions validity. Moreover these tools are applied to unknown parameters estimation and characteristics prediction of the Stena Baltica ferry operation process.

2 IDENTYFICATION OF THE OPERATION PROCESS OF THE COMPLEX TECHNICAL SYSTEM

2.1. Estimation of unknown parameters of the semi-markov model of the operation process

We assume, similarly as in (Blokus-Roszkowska et all 2008b, Kolowrocki, Soszynska 2008) that a system during its operation at the fixed moment t, $t \in <0, +\infty>$, may be in one of v, $v \in N$, different operations states z_b, b = 1,2, ..., v. Next, we mark by $Z(t)$, $t \in <0, +\infty>$, the system operation process, that is a function of a continuous variable t, taking discrete values in the set Z = $\{z_1, z_2, ... z_v\}$ of the operation states. We assume a semi-markov model (Blokus-Roszkowska et all 2008b, Kolowrocki, Soszynska 2008) of the system operation process $Z(t)$ and we mark by θ_{bl} its random conditional sojourn times at the operation states z_b when its next operation state is z_l, $b, l = 1,2,...,v,\ b \neq l.$
Under these assumption, the operation process may be described by the vector $[p_b(0)]_{1\times v}$ of probabilities of the system operation process staying in particular operations states at the initial moment t = 0, the matrix $[p_{bl}(t)]_{v\times v}$ of the probabilities of the system operation process transitions between the operation states and the matrix $[H_{bl}(t)]_{v\times v}$ of the distribution functions of the conditional sojourn times θ_{bl} of the system operation process at the operation states or equivalently by the matrix $[h_{bl}(t)]_{v\times v}$ of the distribution functions of the conditional sojourn times θ_{bl} of the system operation process at the operation states.
To estimate the unknown parameters of the system operations process, firstly during the experiment, we should collect necessary statistical data performing the following steps:
i) to analyze the system operation process and either to fix or to define its following general parameters:
- the number of the operation states o the system operation process v,
- the operation states o the system operation process z_1, z_2, …, z_v,
- the duration time of the experiment Θ;
ii) to fix and to collect the following statistical data necessary to evaluating the probabilities of the initial states of the system operations process:
- the number of the investigated (observed) realizations of the system operation process $n(0)$,
- the numbers of staying of the operation process respectively in the operations states z_1, z_2, …, z_v, at the initials moment $t = 0$ of all $n(0)$ observed realizations of the system operation process $n_1(0)$, $n_2(0)$, …, $n_v(0)$;
iii) to fix and to collect the following statistical data necessary to evaluating the probabilities of transitions between the system operation states:

- the numbers n_{bl}, $b, l = 1,2,...,v$, $b \neq l$, of the transitions of the system operation process from the operation state z_b to the operation state z_l during all observed realizations of the system operation process;
- the numbers n_b, $b = 1,2,...,v$, of departures of the system operation process from the operation states z_b;
iv) to fix and to collect the following statistical data necessary to evaluating the unknown parameters of the distributions of the conditional sojourn times of the system operation process in the particular operation states:
- the realizations θ_{bl}^k, $k = 1,2, \ldots, n_{bl}$, for each $b, l = 1,2,...,v$, $b \neq l$ of the conditional sojourn times θ_{bl} of the system operations process at the operation state z_b when the next transition is to the operation state z_l during the observation time;
After collecting the above statistical data it is possible to estimate the unknown parameters of the system operation process performing the following steps:
i) to determine the vector

$$[p(0)] = [p_1(0), p_2(0), \ldots, p_v(0)],$$

of the realizations of the probabilities $p_b(0)$, $b = 1,2,...,v$, of the initial states of the system operation process, according to the formula

$$p_b(0) = \frac{n_b(0)}{n(0)} \text{ for } b = 1,2,...,v,$$

where $n(0) = \sum_{b=1}^{v} n_b(0),$ is the number of the realizations of the system operation process starting at the initial moment $t = 0$;
ii) to determine the matrix

$$[p_{bl}] = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1v} \\ p_{21} & p_{22} & \cdots & p_{2v} \\ \cdots \\ p_{v1} & p_{v2} & \cdots & p_{vv} \end{bmatrix},$$

of the realizations of the probabilities p_{bl}, $b,l = 1,2,...,v$, of the system operations process transitions from the operations state z_b to the operations state z_l during the experiment time Θ, according to the formula

$$p_{bl} = \frac{n_{bl}}{n_b} \text{ for } b,l = 1,2,...,v, \ b \neq l, \ p_{bb} = 0 \ \text{ for } b = 1,2,...,v,$$

where $n_b = \sum_{b \neq l}^{v} n_{bl}$, $b = 1,2,...,v$, is the realization of the total number of the system operations process departures from the operations state z_b during the experiment time Θ;
iii) to determine the following empirical characteristics of the realizations of the conditional sojourn time of the system operation process in the particular operation states:

- the realizations of the mean values $\bar{\theta}_{bl}$ of the conditional sojourn times θ_{bl} of the system operations process at the operations state $H_{bl}(t)$ when the next transition is to the operation state θ_{bl} , according to the formula

$$\bar{\theta}_{bl} = \frac{1}{n_{bl}} \sum_{k=1}^{n_{bl}} \theta_{bl}^{k} \;\; b,l = 1,2,...,\nu, \; b \neq l,$$

- the number θ_{bl}^{k}, of the disjoint intervals $k = 1,2,...,n_{bl},$, θ_{bl}, that include the realizations θ_{bl}^{k}, $k = 1,2,...,n_{bl}$, of the conditional sojourn times θ_{bl} at the operation state z_b when the next transition is to the operation state z_l, according to the formula

$$\bar{r} \cong \sqrt{n_{bl}} \; ,$$

- the length d of the intervals $I_j =< a_{bl}^{j}, b_{bl}^{j})$, $j = 1,2,...,\bar{r}$, according to the formula

$$d = \frac{\bar{R}}{\bar{r} - 1}, \text{ where } \bar{R} = \max_{1 \leq k \leq n_{bl}} \theta_{bl}^{k} - \min_{1 \leq k \leq n_{bl}} \theta_{bl}^{k},$$

- the ends a_{bl}^{j}, b_{bl}^{j}, of the intervals $I_j =< a_{bl}^{j}, b_{bl}^{j})$, $j = 1,2,...,\bar{r}$, according to the formulae

$$a_{bl}^{1} = \min_{1 \leq k \leq n_{bl}} \theta_{bl}^{K} - \frac{d}{2}, \; b_{bl}^{j} = a_{bl}^{1} + jd, \; j = 1,2,...,\bar{r}, \;\; a_{bl}^{j} = b_{bl}^{j-1}, \;\; j = 2,3,...,\bar{r},$$

in the way such that

$$I_1 \cup I_2 \cup ... \cup I_{\bar{r}} =< a_{bl}^{1}, b_{bl}^{\bar{r}}),$$

and

$$I_i \cap I_j = \emptyset \text{ for all } i \neq j, \; i,j \in \{1,2,...,\bar{r}\},$$

- the numbers n_{bl}^{j} of the realizations θ_{bl}^{k} in particular intervals I_j, $j = 1,2,...,\bar{r}$, according to the formula

$$n_{bl}^{j} = \# \{k : \theta_{bl}^{k} \in I_j, k \in \{1,2,...,n_{bl}\}\}, \; j = 1,2,...,\bar{r},$$

where $\sum_{j=1}^{\bar{r}} n_{bl}^{j} = n_{bl}$, whereas the symbol # means the number of elements of the set;

iv) to estimate the parameters of the distributions of the conditional sojourn times of the system operation process in the particular operation states for the following distinguished distributions respectively in the following way:

- the uniform distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{1}{y_{bl} - x_{bl}}, & x_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where $0 \le x_{bl} < y_{bl} < +\infty$,
the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^{1}, \; y_{bl} = x_{bl} + \bar{r}d \; ;$$

- the triangular distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{2}{y_{bl} - x_{bl}} \dfrac{t - x_{bl}}{z_{bl} - x_{bl}}, & x_{bl} \le t \le z_{bl} \\ \dfrac{2}{y_{bl} - x_{bl}} \dfrac{y_{bl} - t}{y_{bl} - z_{bl}}, & z_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where $0 \le x_{bl} < z_{bl} < y_{bl} < +\infty$,
the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^{1}, \; y_{bl} = x_{bl} + \bar{r}d \; , \quad j = 1,2,...,\bar{\bar{r}};$$

- the double trapezium distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ q_{bl} + \dfrac{C_{bl} - q_{bl}}{z_{bl} - x_{bl}}(t - x_{bl}), & x_{bl} \le t \le z_{bl} \\ w_{bl} + \dfrac{C_{bl} - w_{bl}}{y_{bl} - z_{bl}}(y_{bl} - t), & z_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where

$$C_{bl} = \frac{2 - q_{bl}(z_{bl} - x_{bl}) - w_{bl}(y_{bl} - z_{bl})}{y_{bl} - x_{bl}}, \; 0 \le x_{bl} < z_{bl} < y_{bl} < +\infty, \;\; 0 \le q_{bl} < +\infty, \; 0 \le w_{bl} < +\infty,$$
$$0 \le q_{bl}(z_{bl} - x_{bl}) + w_{bl}(y_{bl} - z_{bl}) \le 2;$$

the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^{1}, \; y_{bl} = x_{bl} + \bar{r}d \; , \; q_{bl} = \frac{n_{bl}^{1}}{n_{bl}d}, w_{bl} = \frac{n_{bl}^{\bar{r}}}{n_{bl}d}, \; z_{bl} = \bar{\theta}_{bl},$$

- the quasi-trapezium distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ q_{bl} + \dfrac{A_{bl} - q_{bl}}{z_{bl}^1 - x_{bl}}(t - x_{bl}), & x_{bl} \le t \le z_{bl}^1 \\ A_{bl}, & z_{bl}^1 \le t \le z_{bl}^2 \\ w_{bl} + \dfrac{A_{bl} - w_{bl}}{y_{bl} - z_{bl}^2}(y_{bl} - t), & z_{bl}^2 \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where

$$A_{bl} = \frac{2 - q_{bl}(z_{bl}^1 - x_{bl}) - w_{bl}(y_{bl} - z_{bl}^2)}{z_{bl}^2 - z_{bl}^1 + y_{bl} - x_{bl}}, 0 \le x_{bl} < z_{bl}^1 \le z_{bl}^2 < y_{bl} < +\infty,\ 0 \le q_{bl} < +\infty,$$
$$0 \le w_{bl} < +\infty,\ 0 \le q_{bl}(z_{bl}^1 - x_{bl}) + w_{bl}(z_{bl}^2 - y_{bl}) \le 2,$$

the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^1,\ y_{bl} = x_{bl} + \bar{r}d,\ q_{bl} = \frac{n_{bl}^1}{n_{bl}d},\ w_{bl} = \frac{n_{bl}^{\bar{r}}}{n_{bl}d},\ z_{bl}^1 = \bar{\theta}_{bl}^1,\ z_{bl}^2 = \bar{\theta}_{bl}^2,$$

where

$$\bar{\theta}_{bl}^1 = \frac{1}{n_{(me)}} \sum_{j=1}^{n(me)} \theta_{bl}^j,\ \bar{\theta}_{bl}^2 = \frac{1}{n_{bl} - n_{(me)}} \sum_{j=n(me)+1}^{n_{bl}} \theta_{bl}^j,\ n_{(me)} = [\frac{n_{bl}+1}{2}];$$

- the exponential distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl}, x_{bl} \ge 0, \\ \alpha_{bl} \exp[-\alpha_{bl}(t - x_{bl})], & t \ge x_{bl}, \end{cases}$$

where $0 \le \alpha_{bl} < +\infty,\ 0 \le x_{bl} = a_{bl}^1,$,
the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^1,\ \alpha_{bl} = \frac{1}{\bar{\theta}_{bl} - x_{bl}},$$

- the Weibull's distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl}, x_{bl} \ge 0, \\ \alpha_{bl}\beta_{bl}(t - x_{bl})^{\beta_{bl}-1} \exp[-\alpha_{bl}(t - x_{bl})^{\beta_{bl}}], & t \ge x_{bl}, \end{cases}$$

where $0 \le \alpha_{bl} < +\infty,\ 0 \le \beta_{bl} < +\infty,\ 0 \le x_{bl} = a_{bl}^1,$,
the estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^1, \ \alpha_{bl} = \frac{n_{bl}}{\sum_{j=1}^{n_{bl}} (\theta_{bl}^j)^{\beta_{bl}}}, \ \alpha_{bl} = \frac{\frac{n_{bl}}{\beta_{bl}} + \sum_{j=1}^{n_{bl}} \ln(\theta_{bl}^j - x_{bl})}{\sum_{j=1}^{n_{bl}} (\theta_{bl}^j)^{\hat{\beta}_{bl}} \ln(\theta_{bl}^j - x_{bl})};$$

- the normal distribution with a density function

$$h_{bl}(t) = \frac{1}{\sigma_{bl}\sqrt{2\pi}} \exp[-\frac{(t - m_{bl})^2}{2\sigma_{bl}^{\ 2}}], \quad t \in (-\infty, \infty),$$

where $-\infty < m_{bl} < +\infty, \ 0 \le \sigma_{bl} < +\infty,$
the estimates of the unknown parameters of this distribution are:

$$m_{bl} = \bar{\theta}_{bl}, \ \sigma_{bl}^{\ 2} = \bar{\sigma}_{bl}^2 = \frac{1}{n_{bl}} \sum_{j=1}^{n_{bl}} (\theta_{bl}^j - m_{bl})^2,$$

- the chimney distribution with a density function

$$h_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \frac{a_{bl}}{z_{bl}^1 - x_{bl}}, & x_{bl} \le t \le z_{bl}^1 \\ \frac{c_{bl}}{z_{bl}^2 - z_{bl}^1}, & z_{bl}^1 \le t \le z_{bl}^2 \\ \frac{d_{bl}}{y_{bl} - z_{bl}^2}, & z_{bl}^2 \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where $0 \le x_{bl} \le z_{bl}^1 \le z_{bl}^2 \le y_{bl} < +\infty, \ 0 \le q_{bl} < +\infty, \ 0 \le w_{bl} < +\infty, \ a_{bl} \ge 0, \ c_{bl} \ge 0, \ d_{bl} \ge 0,$ $a_{bl} + c_{bl} + d_{bl} = 1.$

The estimates of the unknown parameters of this distribution are:

$$x_{bl} = a_{bl}^1, \ y_{bl} = x_{bl} + \bar{r}d,$$

and moreover, if

$$\hat{n}_{bl} = \max_{1 \le j \le \bar{r}} \{n_{bl}^j\}, \ i = j, \text{ where } j \in \{12,..., \bar{r}\},$$

is such a number of the interval for which $n_{bl}^j = \hat{n}_{bl}$, then:

for $i = 1$

$$z_{bl}^1 = x_{bl} + (i-1)d, \ z_{bl}^2 = x_{bl} + id, \ a_{bl} = 0, \ c_{bl} = \frac{n_{bl}^i}{n_{bl}}, \ d_{bl} = \frac{n_{bl}^{i+1} + ... + n_{bl}^{\bar{r}}}{n_{bl}},$$

when $n_{bl}^{i+1} = 0$ or $n_{bl}^{i+1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i+1}} \geq 3,$

$$z_{bl}^{1} = x_{bl} + (i-1)d,\ z_{bl}^{2} = x_{bl} + (i+1)d,\ a_{bl} = 0,\ c_{bl} = \frac{n_{bl}^{i} + n_{l}^{i+1}}{n_{bl}},\ d_{bl} = \frac{n_{bl}^{i+2} + \ldots + n_{bl}^{\bar{r}}}{n_{bl}},$$

when $n_{bl}^{i+1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i+1}} < 3,$

for $i = 2,3,\ldots,\bar{r}-1$

$$z_{bl}^{1} = x_{bl} + (i-1)d,\ z_{bl}^{2} = x_{bl} + id,\ a_{bl} = \frac{n_{bl}^{1} + \ldots + n_{bl}^{i-1}}{n_{bl}},\ c_{bl} = \frac{n_{bl}^{i}}{n_{bl}},\ d_{bl} = \frac{n_{bl}^{i+1} + \ldots + n_{bl}^{\bar{r}}}{n_{bl}},$$

when $n_{bl}^{i-1} = 0$ or $n_{bl}^{i-1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i-1}} \geq 3$ and

$$z_{bl}^{1} = x_{bl} + (i-1)d,\ z_{bl}^{2} = x_{bl} + (i+1)d,\ a_{bl} = \frac{n_{bl}^{1} + \ldots + n_{bl}^{i-1}}{n_{bl}},\ c_{bl} = \frac{n_{bl}^{i} + n_{l}^{i+1}}{n_{bl}},\ d_{bl} = \frac{n_{bl}^{i+2} + \ldots + n_{bl}^{\bar{r}}}{n_{bl}},$$

when $n_{bl}^{i-1} = 0$ or $n_{bl}^{i-1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i-1}} \geq 3$ and

$$z_{bl}^{1} = x_{bl} + (i-2)d,\ z_{bl}^{2} = x_{bl} + id,\ a_{bl} = \frac{n_{bl}^{1} + \ldots + n_{bl}^{i-2}}{n_{bl}},\ c_{bl} = \frac{n_{bl}^{i-1} + n_{l}^{i}}{n_{bl}},\ d_{bl} = \frac{n_{bl}^{i+1} + \ldots + n_{bl}^{\bar{r}}}{n_{bl}},$$

when $n_{bl}^{i-1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i-1}} < 3$ and when $n_{bl}^{i+1} = 0$ or $n_{bl}^{i+1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i+1}} \geq 3,$

$$z_{bl}^{1} = x_{bl} + (i-2)d,\ z_{bl}^{2} = x_{bl} + (i+1)d,\ a_{bl} = \frac{n_{bl}^{1} + \ldots + n_{bl}^{i-1}}{n_{bl}},\ c_{bl} = \frac{n_{bl}^{i-1} + n_{bl}^{i} + n_{l}^{i+1}}{n_{bl}},\ d_{bl} = \frac{n_{bl}^{i+2} + \ldots + n_{bl}^{\bar{r}}}{n_{bl}},$$

when $n_{bl}^{i-1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i-1}} < 3$ and when $n_{bl}^{i+1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i+1}} < 3,$

for $i = \bar{r}$

$$z_{bl}^{1} = x_{bl} + (i-1)d,\ z_{bl}^{2} = x_{bl} + id,\ a_{bl} = \frac{n_{bl}^{1} + \ldots + n_{bl}^{i-1}}{n_{bl}},\ c_{bl} = \frac{n_{bl}^{i}}{n_{bl}},\ d_{bl} = 0,$$

when $n_{bl}^{i-1} = 0$ or $n_{bl}^{i-1} \neq 0$ and $\frac{n_{bl}^{i}}{n_{bl}^{i-1}} \geq 3$,

$$z_{bl}^1 = x_{bl} + (i-2)d,\ \ z_{bl}^2 = x_{bl} + id,\ \ a_{bl} = \frac{n_{bl}^1 + ... + n_{bl}^{i-2}}{n_{bl}},\ \ c_{bl} = \frac{n_{bl}^{i-1} + n_{bl}^i}{n_{bl}},\ \ d_{bl} = 0,$$

when $n_{bl}^{i-1} \neq 0$ and $\frac{n_{bl}^i}{n_{bl}^{i-1}} < 3$.

2.2. Identification of distributions of conditional sojourn times in operation states

To formulate and next to verify the non-parametric hypothesis concerning the form of the distribution function $H_{bl}(t)$ of the system conditional sojourn time θ_{bl} at the operation state z_b when the next transition is to the operation state z_l, on the basis of its realizations θ_{bl}^k, $k = 1,2,...,n_{bl}$, it is necessary to proceed according to the following scheme:

- to construct and to plot the realization of the histogram of the system conditional sojourn time θ_{bl} at the operation states, defined by the following formula

$$\bar{h}_{n_{bl}}(t) = \frac{n_{bl}^j}{n_{bl}} \ \text{for}\ t \in I_j,$$

- to analyze the realization of the histogram, comparing it with the graphs of the density functions $h_{bl}(t)$ of the previously distinguished distributions, to select one of them and to formulate the null hypothesis H_0 and the alternative hypothesis H_A, concerning the unknown form of the distribution function $H_{bl}(t)$ of the conditional sojourn time θ_{bl} in the following form:

H_0 : The system conditional sojourn time θ_{bl} at the operation state z_b when the next transition is to the operations state z_l, has the distribution function $H_{bl}(t)$,

H_A : The system conditional sojourn time θ_{bl} at the operation state z_b when the next transition is to the operations state z_l, has the distribution function different from $H_{bl}(t)$,

- to join each of the intervals I_j that has the number n_{bl}^j of realizations is less than 4 either with the neighbour interval I_{j+1} or with the neighbour interval I_{j-1} this way that the numbers of realizations in all intervals are not less than 4,
- to fix a new number of intervals $\bar{\bar{r}}$,
- to determine new intervals $\bar{I}_j = < \bar{a}_{bl}^j, \bar{b}_{bl}^j)$, $j = 1,2,..,\bar{\bar{r}}$,
- to fix the numbers $\bar{n}_{bl}^j$ of realizations in new intervals $\bar{I}_j$, $j = 1,2,..,\bar{\bar{r}}$,
- to calculate the hypothetical probabilities that the variable θ_{bl} takes values from the interval $\bar{I}_j$, under the assumption that the hypothesis H_0 is true, i.e. the probabilities

$$p_j = P(\theta_{bl} \in \bar{I}_j) = P(\bar{a}_{bl}^j \le \theta_{bl} < \bar{b}_{bl}^j) = H_{bl}(\bar{b}_{bl}^j) - H_{bl}(\bar{a}_{bl}^j),\ j = 1,2,...,\bar{\bar{r}},$$

where $H_{bl}(\bar{b}_{bl}^j)$ and $H_{bl}(\bar{a}_{bl}^j)$ are the values of the distribution function $H_{bl}(t)$ of the random variable θ_{bl} defined in the null hypothesis H_0,

- to calculate the realization of the χ^2(*chi-square*)-Pearson's statistics $U_{n_{bl}}$, according to the formula

$$u_{n_{bl}} = \sum_{j=1}^{\bar{\bar{r}}} \frac{(\bar{n}_{bl}^{j} - n_{bl} p_j)^2}{n_{bl} p_j},$$

- to assume the significance level α ($\alpha = 0.01$, $\alpha = 0.02$, $\alpha = 0.05$ or $\alpha = 0.10$) of the test,
- to fix the number $\bar{\bar{r}} - l - 1$ of degrees of freedom, substituting for l for the distinguished distributions respectively the following values: $l = 0$ for the uniform, triangular, double trapezium, quasi-trapezium and chimney distributions, $l = 1$ for the exponential distribution, $l = 2$ for the Weibull's and normal distributions,
- to read from the Tables of the χ^2 – Pearson's distribution the value u_α for the fixed values of the significance level α and the number of degrees of freedom $\bar{\bar{r}} - l - 1$ such that the following equality holds

$$P(U_{n_{bl}} > u_\alpha) = 1 - \alpha,$$

and next to determine the critical domain in the form of the interval $(u_\alpha, +\infty)$ and the acceptance domain in the form of the interval $< 0, u_\alpha >$,
- to compare the obtained value $u_{n_{bl}}$ of the realization of the statistics $U_{n_{bl}}$ with the red from the Tables critical value u_α of the chi-square random variable and to verify previously formulated the null hypothesis H_0 in the following way: if the value $u_{n_{bl}}$ does not belong to the critical domain, i.e. when $u_{n_{bl}} \le u_\alpha$, then we do not reject the hypothesis H_0, otherwise if the value $u_{n_{bl}}$ belongs to the critical domain, i.e. when $u_{n_{bl}} > u_\alpha$, then we reject the hypothesis H_0 in favor of the hypothesis H_A.

3 APPLICATION IN MARITIME TRANSPORT

3.1. The Stena Baltica ferry operation process and its statistical identification

Taking into account the operation process of the considered ferry we distinguish the following as its eighteen operation states:

- an operation state z_1 – loading at Gdynia Port,
- an operation state z_2 – unmooring operations at Gdynia Port,
- an operation state z_3 – leaving Gdynia Port and navigation to "GD" buoy,
- an operation state z_4 – navigation at restricted waters from "GD" buoy to the end of Traffic Separation Scheme,
- an operation state z_5 – navigation at open waters from the end of Traffic Separation Scheme to "Angoring" buoy,
- an operation state z_6 – navigation at restricted waters from "Angoring" buoy to "Verko" Berth at Karlskrona,
- an operation state z_7 – mooring operations at Karlskrona Port,
- an operation state z_8 – unloading at Karlskrona Port,
- an operation state z_9 – loading at Karlskrona Port,
- an operation state z_{10} – unmooring operations at Karlskrone Port,
- an operation state z_{11} – ship turning at Karlskrone Port,
- an operation state z_{12} – leaving Karlskrone Port and navigation at restricted waters to "Angoring" buoy,

- an operation state z_{13} – navigation at open waters from "Angoring" buoy to the entering Traffic Separation Scheme,
- an operation state z_{14} – navigation at restricted waters from the entering Traffic Separation Scheme to "GD" buoy,
- an operation state z_{15} – navigation from "GD" buoy to turning area,
- an operation state z_{16} – ship turning at Gdynia Port,
- an operation state z_{17} – mooring operations at Gdynia Port,
- an operation state z_{18} – unloading at Gdynia Port.

To identify all parameters of Stena Baltica ferry operation process the statistical data about this process is needed. The statistical data that has been collected up to now is given in *Tables 1-7* in (Soszynska et all 2009, Appendix 5A). In the *Tables 1-7* there are presented the realizations θ_{bl}^{k}, $k = 1,2,...,42$, for each $b = 1,2,...,17$, $l = b+1$ and $b = 18$, $l = 1$ of the ship operation process conditional sojourn times θ_{bl}, $b = 1,2,...,17$, $l = b+1$ and $b = 18$, $l = 1$ in the state z_b while the next transition is the state z_l during the experiment time $\Theta = 42$ days.

These statistical data allow us, applying the methods and procedures given in the section 2, to formulate and to verify the hypotheses about the conditional distribution functions $H_{bl}(t)$ of the Stena Baltica ferry operation process sojourn times θ_{bl}, $b = 1,2,...,17$, $l = b+1$ and $b = 18$, $l = 1$ in the state z_b while the next transition is to the state z_l on the base of their realizations θ_{bl}^{k}, $k = 1,2,...,42$.

On the basis of the statistical data, given in the Appendix 5A in [5], the vector of the probabilities of the system initial operation states was evaluated in the following form

$$[p_b(0)] = [1, 0, 0, \ldots, 0, 0].$$

The matrix of the probabilities p_{bl} of transitions from the operation state z_b into the operation state z_l were evaluated as well. Their evaluation are given in the matrix below

$$[p_{bl}]_{18x18} = \begin{bmatrix} 0\,1\,0\ldots 0\,0 \\ 0\,0\,1\ldots 0\,0 \\ \ldots \\ 0\,0\,0\ldots 0\,1 \\ 1\,0\,0\ldots 0\,0 \end{bmatrix},$$

Next the matrix $[h_{bl}(t)]_{18x18}$ of conditional density functions of the system operation process $Z(t)$ conditional sojourn times θ_{bl} (Soszynska et all 2009, Appendix 5A) were evaluated.

The results of the distributions unknown parameters estimation and the hypotheses testing are as follows:

- the conditional sojourn time θ_{12} have a triangular distribution with the density function

$$h_{12}(t) = \begin{cases} 0, & t < 7, \\ 0.00044t - 0.0031, & 7 \le t < 54, \\ 0.044 - 0.00043t; & 54 \le t < 103, \\ 0, & t \ge 103; \end{cases}$$

- the conditional sojourn time θ_{23} have an exponential distribution with the density function

$$h_{23}(t) = \begin{cases} 0, & t < 1.6 \\ 1.03\exp[-1.03(t-1.6)], & t \ge 1.6; \end{cases}$$

- the conditional sojourn time θ_{34} have a steep chimney distribution with the density function

$$h_{34}(t) = \begin{cases} 0, & t < 29, \\ 0.0278, & 29 \le t < 35, \\ 0.1984, & 35 \le t < 38, \\ 0.0266, & 38 \le t < 47, \\ 0, & t \ge 47; \end{cases}$$

- the conditional sojourn time θ_{45} have a chimney distribution with the density function

$$h_{45}(t) = \begin{cases} 0, & t < 41, \\ 0.0095, & 41 \le t < 46, \\ 0.0762, & 46 \le t < 56, \\ 0.0127, & 56 \le t < 71, \\ 0, & t \ge 71; \end{cases}$$

- the conditional sojourn time θ_{56} have a double trapezium distribution with the density function

$$h_{56}(t) = \begin{cases} 0, & t < 467.8, \\ -0.00004t + 0.0277, & 467.8 \le t \le 525.95, \\ -0.00006t + 0.0397, & 525.95 \le t \le 650.2, \\ 0, & t < 650.2; \end{cases}$$

- the conditional sojourn time θ_{67} have a double trapezium distribution with the density function

$$h_{67}(t) = \begin{cases} 0, & t < 31.9, \\ 0.0067t - 0.1747, & 31.9 \le t \le 37.17, \\ 0.0031t - 0.0395, & 37.7 \le t \le 45.1, \\ 0, & t > 45.1; \end{cases}$$

- the conditional sojourn time θ_{78} have a double trapezium distribution with the density function

$$h_{78}(t) = \begin{cases} 0, & t < 4.5, \\ -0.0183t + 0.2922, & 4.5 \le t \le 7.02, \\ -0.0069t + 0.2122, & 7.02 \le t \le 10.5, \\ 0, & t > 10.5; \end{cases}$$

- the conditional sojourn time θ_{89} have a triangular distribution with the density function

$$h_{89}(t) = \begin{cases} 0, & t < 0, \\ 0.0021t, & 0 \le t \le 21.4 \\ -0.002t + 0.087, & 21.4 \le t \le 44.4, \\ 0, & t > 44.4; \end{cases}$$

- the conditional sojourn time θ_{910} have a double trapezium distribution with the density function

$$h_{910}(t) = \begin{cases} 0, & t < 14.6, \\ 0.0001t + 0.0109, & 14.6 \le t \le 52.26, \\ 0.0062, & 52.2 \le t \le 127.4, \\ 0, & t > 127.4; \end{cases}$$

- the conditional sojourn time θ_{1011} have a double trapezium distribution with the density function

$$h_{1011}(t) = \begin{cases} 0, & t < 1.6, \\ -0.3071t + 1.0014, & 1.6 \le t \le 2.93, \\ 0.0398t - 0.0145, & 2.93 \le t \le 6.4, \\ 0, & t > 6.4; \end{cases}$$

- the conditional sojourn time θ_{1112} have a quasi-trapezium distribution with the density function

$$h_{1112}(t) = \begin{cases} 0, & t < 3.8, \\ -148.899t + 567.4863, & 3.8 \le t \le 3.81, \\ 0.181, & 3.81 \le t \le 4.48, \\ 0.3773t - 1.5094, & 4.48 \le t \le 6.2, \\ 0, & t > 6.2; \end{cases}$$

- the conditional sojourn time θ_{1213} have a triangular distribution with the density function

$$h_{1213}(t) = \begin{cases} 0, & t < 18.7, \\ 0.025t - 0.4675, & 18.7 \le t \le 23.86, \\ -0.012t + 0.4116, & 23.86 \le t \le 34.3, \\ 0, & t > 34.3; \end{cases}$$

- the conditional sojourn time θ_{1314} have a chimney distribution with the density function

$$h_{1314}(t) = \begin{cases} 0, & t < 410, \\ 0.0017, & 410 \le t < 478, \\ 0.0189, & 478 \le t < 512, \\ 0.0024, & 512 \le t < 614, \\ 0, & t \ge 614; \end{cases}$$

- the conditional sojourn time θ_{1415} have a double trapezium distribution with the density function

$$h_{1415}(t) = \begin{cases} 0, & t < 36.8, \\ -0.0006t + 0.0518, & 36.8 \le t \le 50.14, \\ 0.0003t + 0.0062, & 50.14 \le t \le 75.2, \\ 0, & t > 75.2; \end{cases}$$

- the conditional sojourn time θ_{1516} have a chimney distribution with the density function

$$h_{1516}(t) = \begin{cases} 0, & t < 30, \\ 0.0317, & 30 \le t < 33, \\ 0.2698, & 33 \le t < 36, \\ 0.0084, & 36 \le t < 48, \\ 0, & t \ge 48; \end{cases}$$

- the conditional sojourn time θ_{1617} have a triangular distribution with the density function

$$h_{1617}(t) = \begin{cases} 0, & t < 2.7, \\ 0.305t - 0.823, & 2.7 \le t \le 4.52, \\ -0.313t + 1.9719, & 4.52 \le t \le 6.3, \\ 0, & t > 6.3; \end{cases}$$

- the conditional sojourn time θ_{1718} have a double trapezium distribution with the density function

$$h_{1718}(t) = \begin{cases} 0, & t < 2.3, \\ -0.1134t + 0.6707, & 2.3 \le t \le 5.62, \\ 0.0071t - 0.0063, & 5.62 \le t \le 10.7, \\ 0, & t > 10.7; \end{cases}$$

- the conditional sojourn time θ_{181} have a triangular distribution with the density function

$$h_{181}(t) = \begin{cases} 0, & t < 0, \\ 0.0023t, & 0 \le t \le 18.74, \\ -0.0016t + 0.0729, & 18.74 \le t \le 45.59, \\ 0, & t > 45.59. \end{cases}$$

3.2. The Stena Baltica ferry operation process prediction

On the basis of the previous section, the mean values $M_{bl} = E[\theta_{bl}],\ b,l = 1,2,...,18,\ b \ne l$, (12) in (Kolowrocki, Soszynska 2008) of the system operation process $Z(t)$ conditional sojourn times in particular operation states were determined and there are given by:

$$M_{12} = 54.33,\ M_{23} = 2.57,\ M_{34} = 36.57,$$

$$M_{45} = 52.5,\ M_{56} = 525.95,\ M_{67} = 37.16,$$

$$M_{78} = 7.02,\ M_{89} = 21.43,\ M_{910} = 53.69,$$

$$M_{1011} = 2.93,\ M_{1112} = 4.38,\ M_{1213} = 23.86,$$

$$M_{1314} = 509.69,\ M_{1415} = 50.14,\ M_{1516} = 34.28,$$

$$M_{1617} = 4.52,\ \ M_{1718} = 5.62,\ \ M_{181} = 18.74.$$

Hence, by (21) in (Kolowrocki, Soszynska 2008), the unconditional mean sojourn time in the particular operation states are given by:

$$M_1 = E[\theta_1] = p_{12}M_{12} = 1 \cdot 54.33 = 54.33,$$

$$M_2 = E[\theta_2] = p_{23}M_{23} = 1 \cdot 2.57 = 2.57,$$

$$M_3 = E[\theta_3] = p_{34}M_{34} = 1 \cdot 36.57 = 36.57,$$

$$M_4 = E[\theta_4] = p_{45}M_{45} = 1 \cdot 52.5 = 52.5,$$

$$M_5 = E[\theta_5] = p_{56}M_{56} = 1 \cdot 525.95 = 525.95,$$

$$M_6 = E[\theta_6] = p_{67}M_{67} = 1 \cdot 37.16 = 37.16,$$

$$M_7 = E[\theta_7] = p_{78}M_{78} = 1 \cdot 7.02 = 7.02,$$

$$M_8 = E[\theta_8] = p_{89}M_{89} = 1 \cdot 21.43 = 21.43,$$

$$M_9 = E[\theta_9] = p_{910}M_{910} = 1 \cdot 53.69 = 53.69,$$

$$M_{10} = E[\theta_{10}] = p_{1011}M_{1011} = 1 \cdot 2.93 = 2.93,$$

$$M_{11} = E[\theta_{11}] = p_{1112}M_{1112} = 1 \cdot 4.38 = 4.38,$$

$$M_{12} = E[\theta_{12}] = p_{1213}M_{1213} = 1 \cdot 23.86 = 23.86,$$

$$M_{13} = E[\theta_{13}] = p_{1314}M_{1314} = 1 \cdot 509.69 = 509.69,$$

$$M_{14} = E[\theta_{14}] = p_{1415}M_{1415} = 1 \cdot 50.14 = 50.14,$$

$$M_{15} = E[\theta_{15}] = p_{1516}M_{1516} = 1 \cdot 34.28 = 34.28,$$

$$M_{16} = E[\theta_{16}] = p_{1617}M_{1617} = 1 \cdot 4.52 = 4.52,$$

$$M_{17} = E[\theta_{17}] = p_{1718}M_{1718} = 1 \cdot 5.62 = 5.62,$$

$$M_{18} = E[\theta_{18}] = p_{181}M_{181} = 1 \cdot 18.74 = 18.74.$$

Since from the system of equations below (23) in (Kolowrocki, Soszynska 2008) that takes the form

$$\begin{cases} [\pi_1, \pi_2, ..., \pi_{18}] = [\pi_1, \pi_2, ..., \pi_{18}][p_{bl}]_{18x18} \\ \pi_1 + \pi_2 + ... + \pi_{18} = 1, \end{cases}$$

we get

$$\pi_1 = \pi_2 = ... = \pi_{18} = 0.056,$$

then the limit values of the transient probabilities (the portions of time of a week, as the operation process is periodic) $p_b(t)$ at the operational states z_b, according to (22) in (Kolowrocki, Soszynska 2008), are given by

$$p_1 = 0.037, \quad p_2 = 0.002, \quad p_3 = 0.025,$$

$$p_4 = 0.036, \quad p_5 = 0.364, \quad p_6 = 0.025,$$

$$p_7 = 0.005, \quad p_8 = 0.014, \quad p_9 = 0.037,$$

$$p_{10} = 0.002, \quad p_{11} = 0.003, \quad p_{12} = 0.017,$$

$$p_{13} = 0.354, \quad p_{14} = 0.035, \quad p_{15} = 0.024,$$

$$p_{16} = 0.003, \quad p_{17} = 0.004, \quad p_{18} = 0.013.$$

Hence by (26) in (Kolowrocki, Soszynska 2008), the mean values of the system operation process total sojourn times $\bar{\theta}_b$ in the particular operation states z_b, for the operation time $\theta = 1$ month = 720 hours are approximately given by

$$E[\bar{\theta}_1] = p_1\theta = 26.64, \quad E[\bar{\theta}_2] = p_2\theta = 1.44,$$

$$E[\bar{\theta}_3] = p_3\theta = 18.00, \quad E[\bar{\theta}_4] = p_4\theta = 25.92,$$

$$E[\bar{\theta}_5] = p_5\theta = 262.08, \quad E[\bar{\theta}_6] = p_6\theta = 18.00,$$

$$E[\bar{\theta}_7] = p_7\theta = 3.6, \quad E[\bar{\theta}_8] = p_8\theta = 10.08,$$

$$E[\bar{\theta}_9] = p_9\theta = 26.64, \quad E[\bar{\theta}_{10}] = p_{10}\theta = 1.44,$$

$$E[\bar{\theta}_{11}] = p_{11}\theta = 2.16, \quad E[\bar{\theta}_{12}] = p_{12}\theta = 12.24,$$

$$E[\bar{\theta}_{13}] = p_{13}\theta = 25.88, \quad E[\bar{\theta}_{14}] = p_{14}\theta = 25.20,$$

$$E[\bar{\theta}_{15}] = p_{15}\theta = 17.28, \quad E[\bar{\theta}_{16}] = p_{16}\theta = 2.16,$$

$$E[\bar{\theta}_{17}] = p_{17}\theta = 2.88, \quad E[\bar{\theta}_{18}] = p_{18}\theta = 9.36.$$

4 CONCLUSION

The statistical methods and algorithms for the unknown parameters of the operation process of complex technical systems in variable operation conditions are proposed. Next, these methods are applied to estimating the operation process of Stena Baltica ferry operating between Gdynia Port in Poland and Karsklone Port in Sweden. The proposed methods other very wide applications to port and shipyard transportation systems operation processes characteristics evaluation are obvious. The results are expected to be the basis to the reliability and safety of complex technical systems optimization and their operation processes effectiveness and cost analysis

Acknowledgements

The paper describes part of the work in the Poland-Singapore Joint Research Project titled “Safety and Reliability of Complex Industrial Systems and Processes” supported by grants from the Poland’s Ministry of Science and Higher Education (MSHE grant No. 63/N-Singapore/2007/0) and the Agency for Science, Technology and Research of Singapore (A*STAR SERC grant No. 072 1340050).

5 REFERENCES

Blokus-Roszkowska, A., Guze, S., Kolowrocki, K., Kwiatuszewska-Sarnecka, B., Soszynska, J. 2008a. *Models of safety, reliability and availability evaluation of complex technical systems related to their operation processes*. WP 4 – Task 3.1 – 31.05.2008. Poland-Singapore Joint Research Project, 2007-2010.

Blokus-Roszkowska, A., Guze, S., Kolowrocki, K., Kwiatuszewska-Sarnecka, B., Soszynska, J. 2008b. *Models of safety, reliability and availability evaluation of complex technical systems related to their operation processes*. WP 4 – Task 4.1 – 31.05.2008. Poland-Singapore Joint Research Project, 2007-2010.

Blokus-Roszkowska, A. Guze, S. Kolowrocki, K., Kwiatuszewska-Sarnecka, B., Milczek B., Soszynska, J. 2009. *Data Mining for Identification and Prediction of Safety and Reliability Characteristics of Complex Industrial Systems and Processes.* WP6 – Task 6.1. Methods-algorithms for evaluating unknown parameters of system operation processes, reliability and safety models. WP6 - Task 6.1.2 – 16.03.2009. Poland-Singapore Joint Research Project, 2007-2010.

Grabski, F. 2002. *Semi-Markov Models of Systems Reliability and Operations Analysis*. Warsaw: Systems Research Institute, Polish Academy of Science.

Kolowrocki, K., Soszynska, J. 2008. *A general model of technical systems operation processes related to their environment and infrastructure*. WP 2 – Task 2.1 – 31.05.2008. Poland-Singapore Joint Research Project, 2007-2010.

Soszynska, J., Kołowrocki, K., Kamiński, P., Jurdziński, M., Guze, S., Milczek, B., Golik, P. 2009. *Data Mining for Identification and Prediction of Safety and Reliability Characteristics of Complex Industrial Systems and Processes.* WP6– Task 6.2. Preliminary safety and risk analysis and evaluation of a Stena Baltica ferry in variable operation conditions. Sub-Task 6.2.5 - Sub-Task 6.2.5 – 16.03.2009. Poland-Singapore Joint Research Project, 2007-2010.

Kolowrocki, K., Kwiatuszewska- Sarnecka, B. 2009. *Data Mining for Identification and Prediction of Safety and Reliability Characteristics of Complex Industrial Systems and Processes.* WP6 – Task 6.1. Methods-algorithms for evaluating unknown parameters of system operation processes, reliability and safety models – a preliminary study. WP6 - Task 6.1.1 – 16.03.2009. Poland-Singapore Joint Research Project, 2007-2010.

METHODS AND ALGORITHMS FOR EVALUATING UNKNOWN PARAMETERS OF COMPONENTS RELIABILITY OF COMPLEX TECHNICAL SYSTEMS

K. Kolowrocki, J. Soszynska.

•

Gdynia Maritime University, Gdynia, Poland

e-mail: katmatkk@am.gdynia.pl, joannas@am.gdynia.pl

ABSTRACT

The paper objectives are to present the methods and tools useful in the statistical identifying the unknown parameters of the components reliability and safety of complex industrial systems and to apply them in the maritime industry. There are presented statistical methods of estimating the unknown intensities of departure from the reliability state subsets of the exponential distribution of the component lifetimes of the multistate systems operating in various operation states. The goodness-of-fit method applied to testing the hypotheses concerned with the exponential form of the multistate reliability function of the particular components of the complex technical system in variable operations conditions is suggested. An application of these tools to reliability characteristics of a ferry operating at the Baltic Sea waters is presented as well.

1 INTRODUCTION

Many real transportation systems belong to the class of complex systems. It is concerned with the large numbers of components and subsystems they are built and with their operating complexity. Modeling the complicated system operation processes, first of all, is difficult because of the large number of the operation states, impossibility of their precise defining and because of the impossibility of the exact describing the transitions between these states. The changes of the operation states of the system operations processes cause the changes of these systems reliability structures and also the changes of their components reliability functions (Blokus-Roszkowska et all 2008a). The models of various multistate complex systems are considered in (Blokus-Roszkowska et all 2008b). The general joint models linking these system reliability models with the models of their operation processes, allowing us for the evaluation of the reliability and safety of the complex technical systems in variable operations conditions, are constructed in (Kolowrocki, Soszynska 2008). In these general joint reliability and safety models of the complex systems it was assumed that the conditional multistate reliability functions of the considered systems components in the particular operations states are exponential.

In order to be able to apply these general models practically in the evaluation and prediction the reliability of real complex technical it is necessary to elaborate the statistical methods concerned with determining the unknown parameters of the proposed models. Namely, the probabilities of the initials system operation states, the probabilities of transitions between the system operation states and the distributions of the sojourn times of the system operation process in the particular operation states and also the unknown parameters of the conditional multistate reliability functions of the system components in various operation states. It is also necessary the elaborating the methods of testing the hypotheses concerned with the conditional sojourn times of the system operations process in particular operations states and the hypotheses concerned with the conditional multistate reliability functions of the system components in the system various operation states. In this paper, the methods for evaluating unknown parameters of the exponential reliability functions in various experimental cases with a special stress on small samples and unfinished investigations are defined and formulae for estimating the intensities of departure from the reliability state subsets in all cases are proposed. The common principle to formulate and to verify the hypotheses about the exponential distribution functions of the lifetimes in the reliability state subsets of the multistate system components by chi-square test is also discussed. These tools on exemplary application to estimating unknown intensity of departure on Stena Baltica ferry component is shown.

2 IDENTYFICATION OF CONDITIONAL MULTISTATE RELIABILITY FUNCTIONS OF THE SYSTEM COMPONENTS

2.1. Estimation of intensities of departure from the reliability state subsets

We assume as in (Blokus-Roszkowska et all 2008b) that the changes of operations states of the multistate system operations process $Z(t)$ have an influence on the reliability functions of the system components and we mark the conditional multistate reliability function of the system component when the system is in the operation state z_b, $b = 1,2,...,\nu$, by

$$[R(t,\cdot)]^{(b)} = [1,\ [R(t,1)]^{(b)}, ...,\ [R(t,z)]^{(b)}\ ], \tag{1}$$

where

$$[R(t,u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_b) \tag{2}$$

for $t \in <0,\infty)$, $u = 1,2,...,z$, $b = 1,2,...,\nu$,

is the conditional reliability function standing the probability that the conditional lifetime $T^{(b)}(u)$ of the system component in the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, is greater than t, while the system operation process $Z(t)$ is in the operation state z_b, $b = 1,2,...,\nu$.

Further, we assume that the coordinates of the vector of the conditional multistate reliability function (1) are exponential reliability functions of the form

$$R^{(b)}(t,u) = R(t,\lambda^{(b)}(u)) = \exp[-\lambda^{(b)}(u)t] \ \text{for}\ t \in <0,\infty), \quad u = 1,2,...,z,\ b = 1,2,...,\nu. \tag{3}$$

Te above assumptions mean that the density functions of the system component conditional life time $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,...z\}$, $u = 1,2,...,z$, at the operations state z_b, $b = 1,2,...,\nu$, are exponential of the form

$$f^{(b)}(t,u) = f(t,\lambda^{(b)}(u)) = \lambda^{(b)}(u)\exp[-\lambda^{(b)}(u)t] \ \text{for}\ t \in <0,\infty), \tag{4}$$

where $\lambda^{(b)}(u)$, $\lambda^{(b)}(u) \geq 0$, is an unknown intensity of departure from this subset of the reliability states.

We want to estimate the value of this unknown intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset $\{u, u+1,...z\}$, $u = 1,2,...,z$, on the basis empirical data. The estimators of the of the unknown intensity of departure $\lambda^{(b)}(u)$, i.e. the unknown failure rate $\lambda^{(b)}$, in the case of the two-state system reliability for various experimental conditions, are determined by maximum likelihood method in (Kolowrocki, Kwiatuszewska-Sarnecka 2009). The modified and transmitted to the multistate system reliability results obtained in (Soszynska et all 2009) are presented below.

Case 1.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Completed investigations, the same observation time on all experimental posts

We assume that during the time τ, $\tau > 0$, we are observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,...z\}$, $u = 1,2,...,z$, at the operation state z_b, $b = 1,2,...,\nu$, on n identical experimental posts. Moreover, we assume that during the fixed

observation time τ all components left the reliability states subset and we mark by $t_i^{(b)}(u)$, $i = 1,2,...,n$, the moment of departure from the reliability states subsets of the component on the $i-$th observational post, i.e. the realizations of the identical component lifetimes $T_i^{(b)}(u)$, $i = 1,2,...,n$, to the first departure from the reliability states subsets, that are the independent random variables with the exponential distribution defined by the density function (4).
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset is

$$[\overline{\lambda}(u)]^{(b)} = \frac{n^{(b)}}{\sum_{i=1}^{n^{(b)}} t_i^{(b)}(u)}, \quad u = 1,2,...,z \tag{5}$$

Case 2.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, the same observation time on all experimental posts
We assume that during the time τ, $\tau > 0$, we are observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,...z\}$, $u = 1,2,...,z$, at the operation state z_b, $b = 1,2,...,\nu$, on n identical experimental posts. Moreover, we assume that during the fixed observation time τ not all components left the reliability states subset and we mark by m_1, $m_1 < n$, the number of components that left the reliability states subset and by $t_i^{(b)}(u)$, $i = 1,2,...,m_1$, the moments of their departures from the reliability states subsets, i.e. the realizations of the identical component lifetimes $T_i^{(b)}(u)$, $i = 1,2,...,m_1$, to the first departure from the reliability states subsets, that are the independent random variables with the exponential distribution defined by the density function (4).
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset is

$$[\overline{\lambda}(u)]^{(b)} = \frac{m^{(b)}(u)}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \tau^{(b)}[n^{(b)} - m^{(b)}(u)]}, \; u = 1,2,...,z. \tag{6}$$

Assuming the observation time τ as the moment of departure from the reliability states subset of the components that have not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset of the form

$$[\overline{\lambda}(u)]^{(b)} = \frac{n^{(b)}}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \tau^{(b)}[n^{(b)} - m^{(b)}(u)]}, \; u = 1,2,...,z. \tag{6'}$$

Case 3.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, different observation times on particular experimental posts
We assume that we are observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,...z\}$, $u = 1,2,...,z$, at the operation state z_b, $b = 1,2,...,\nu$, on n identical experimental posts. We assume that the observation times on particular experimental posts are different and we mark by $\tau^{(i)}$, $\tau^{(i)} > 0$, $i = 1,2,...,n$, the observation time respectively on the *i*-th experimental post. Moreover, we assume that during the fixed observation times $\tau^{(i)}$ not all components left the reliability states subset and we mark by m_1, $m_1 < n$, the number of components

that left the reliability states subset and by $t_i^{(b)}(u)$, $i = 1,2,...,m_1$, the moments of their departures from the reliability states subsets, i.e. the realizations of the identical component lifetimes $T_i^{(b)}(u)$, $i = 1,2,...,m_1$, to the first departure from the reliability states subsets, that are the independent random variables with the exponential distribution defined by the density function (4).
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset is

$$[\overline{\lambda}(u)]^{(b)} = \frac{m^{(b)}(u)}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \sum_{i=m^{(b)}(u)+1}^{n^{(b)}} \tau_i^{(b)}}, \ u = 1,2,...,z. \quad (7)$$

Assuming the observation times $\tau^{(i)}$, $i = m_1$, $m_1+1,...n,$ as the moment of departure from the reliability states subset of the components that have not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset of the form

$$[\overline{\lambda}(u)]^{(b)} = \frac{n^{(b)}}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \sum_{i=m^{(b)}(u)+1}^{n^{(b)}} \tau_i^{(b)}}, \ u = 1,2,...,z. \quad (7')$$

Case 4.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flow (stream) on one experimental post

We assume that we are observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,...,z,$ at the operation state z_b, $b = 1,2,...,\nu$, on one experimental post. We assume that at the moment when the component is leaving the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,...,z,$ it is replaced at once by the same new component staying at the best reliability state z. Moreover, we assume that the renewal process of the components is continuing during the observation time $\tau^{(b)}$, $\tau^{(b)} > 0,$ and that during this time $m_1^{(b)}(u) = m_1$, $m_1^{(b)}(u) < n^{(b)}$, components have left the reliability states subset $\{u, u+1,..., z\}$ and we mark by $t_i^{(b)}(u) = t_i$, $i = 1,2,...,m_1^{(b)}(u),$ the moments of their departures from the reliability states subsets, i.e. the realizations of the identical component lifetimes $T_i^{(b)}(u)$, $i = 1,2,...,m_1^{(b)}(u),$ to the first departure from the reliability states subset $\{u, u+1,..., z\}$, that are the independent random variables with the exponential distribution defined by the density function (4).
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset is

$$[\overline{\lambda}(u)]^{(b)} = \frac{m^{(b)}(u)}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + d^{(b)}(u)}, \ u = 1,2,...,z, \quad (8)$$

where

$$d^{(b)}(u) = \begin{cases} \tau^{(b)} - \sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(1) \ if \ m^{(b)}(u) = m^{(b)} \\ 0 \qquad\qquad if \ m^{(b)}(u) = m^{(b)} + 1, \ u = 1,2,...,z. \end{cases}$$

In the case if $m^{(b)}(u) = m^{(b)}$, $u = 1,2,...,z,$ after assuming the observation time $\tau^{(b)}$ as the moment of departure from the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,...,z,$ of the last component that has not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z,$ of the form

$$[\bar{\lambda}(u)]^{(b)} = \frac{m^{(b)} + 1}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + d^{(b)}(u)}, \ u = 1,2,...,z. \quad (8')$$

Case 5.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flows (streams) on several experimental posts – The same observation time on all experimental posts

We assume that we are observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1, ...z\}$, $u = 1,2,...,z,$ at the operation state z_b, $b = 1,2,...,\nu$, on n experimental posts. We assume that at the moment when the component is leaving the reliability states subset $\{u, u+1, ...z\}$, $u = 1,2,...,z,$ it is replaced at once by the same new component staying at the best reliability state z. Moreover, we assume that the renewal process of the components is continuing at all experimental posts during the same observation time τ, $\tau > 0,$ and that during this time m_k, $k = 1,2,...,n,$ components at the k-th experimental post left the reliability states subset $\{u, u+1, ...z\}$ and we mark by $[t_i^{(b)}(u)]^{(k)}$, $i = 1,2,...,m_k$, the moments of their departures from the reliability states subsets, i.e. the realizations of the identical component lifetimes $[T_i^{(b)}(u)]^{(k)}$, $i = 1,2,...,m_k$, to the first departure from the reliability states subset $\{u, u+1, ...z\}$, that are the independent random variables with the exponential distribution defined by the density function (4).
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset is

$$[\bar{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)}(u)}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} d_j^{(b)}(u)}, \ u = 1,2,...,z, \quad (9)$$

where for $j = 1,2,...,n^{(b)}$

$$d_j^{(b)}(u) = \begin{cases} \tau^{(b)} - \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(1)]^{(j)} & \text{if } m_j^{(b)}(u) = m_j^{(b)} \\ 0 & \text{if } m_j^{(b)}(u) = m_j^{(b)} + 1, \ u = 1,2,...,z. \end{cases}$$

In the case if there exist j, $j \in \{1,2,...,n^{(b)}\}$, such that $m_j^{(b)}(u) = m_j^{(b)}$, $u = 1,2,...,z,$ assuming the observation time $\tau^{(b)}$ as the moment of departures from the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z,$ of the last components on all experimental posts that have not left this reliability states subset we get so called pessimistic evaluation of the intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z,$ of the form

$$[\bar{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)} + n^{(b)}}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} d_j^{(b)}(u)}, \ u = 1,2,...,z. \quad (9')$$

Case 6.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flows (streams) on several experimental posts – Different observation times on experimental posts

We assume that we are observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1, ...z\}$, $u = 1,2,...,z,$ at the operation state z_b, $b = 1,2,...,\nu$, on n experimental posts. We assume that at the moment when the component is leaving the reliability states subset $\{u, u+1, ...z\}$, $u = 1,2,...,z,$ it is replaced at once by the same new component staying at the best reliability state z. Moreover, we assume that the renewal process of the components is continuing at the k-th experimental post during the observation time $\tau^{(k)}$, $\tau^{(k)} > 0,$ and that during this time m_k, $k = 1,2,...,n,$ components at this experimental post left the reliability states subset $\{u, u+1, ...z\}$ and we mark by $[t_i^{(b)}(u)]^{(k)}$, $i = 1,2,...,m_k$, the moments of their departures from the reliability states subsets, i.e. the realizations of the identical component lifetimes $[T_i^{(b)}(u)]^{(k)}$, $i = 1,2,...,m_k$, to the first departure from the reliability states subset $\{u, u+1, ...z\}$, that are the independent random variables with the exponential distribution defined by the density function (4). In this case, the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset is

$$[\bar{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)}(u)}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} \bar{d}_j^{(b)}(u)}, \quad u = 1,2,...,z, \tag{10}$$

where for $j = 1,2,...,n^{(b)}$

$$\bar{d}_j^{(b)}(u) = \begin{cases} \tau_j^{(b)} - \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(1)]^{(j)} & if \ m_j^{(b)}(u) = m_j^{(b)} \\ 0 & if \ m_j^{(b)}(u) = m_j^{(b)} + 1, \ u = 1,2,...,z. \end{cases}$$

In the case if there exist j, $j \in \{1,2,...,n^{(b)}\}$, such that $m_j^{(b)}(u) = m_j^{(b)}$, $u = 1,2,...,z,$ assuming the observation times $\tau_j^{(b)}$, $j = 1,2,...,n^{(b)}$, as the moments of departures from the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z,$ of the last components on experimental posts that have not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z,$ of the form

$$[\bar{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)} + n^{(b)}}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} \bar{d}_j^{(b)}(u)}, \quad u = 1,2,...,z. \tag{10'}$$

2.2. Identification of distributions of conditional lifetimes of system components in reliability state subsets

To formulate and next to verify the non-parametric hypothesis concerning the exponential form of the coordinate

$$R^{(b)}(t,u) = R(t, \lambda^{(b)}(u)) = \exp[-\lambda^{(b)}(u)t] \ \text{for} \ t \in <0,\infty), \quad u = 1,2,...,z, \ b = 1,2,...,\nu. \tag{11}$$

of the vector

$$[R(t,\cdot)]^{(b)} = [1, \ [R(t,1)]^{(b)}, ..., \ [R(t,z)]^{(b)}], \tag{12}$$

of the conditional multistate reliability function of the system component when the system is at the operations state z_b, $b = 1,2,...,\nu,$ it is necessary to ct according to the scheme below:

- to fix the realizations $t_1^{(b)}(u),\ t_2^{(b)}(u),\ \ldots,\ t_n^{(b)}(u),\ u = 1,2,\ldots,z,$ of the system component conditional lifetimes $T^{(b)}(u)$, $b = 1,2,\ldots,\nu$, in the reliability states subsets $\{u, u+1,\ldots,z\},\ u = 1,2,\ldots,z,$
- to determine the number $\bar{r}$ of the disjoint intervals $I_j = < x_j, y_j)$, $j = 1,2,\ldots,\bar{r}$, that include the realizations $t_1^{(b)}(u),\ t_2^{(b)}(u),\ \ldots,\ t_n^{(b)}(u)$ of the system component conditional lifetimes $T^{(b)}(u)$ in the reliability states subset, according to the formula

$$\bar{r} \cong \sqrt{n},$$

- to determine the length d of the intervals $I_j = < x_j, y_j)$, $j = 1,2,\ldots,\bar{r}$, according to the formula

$$d = \frac{\bar{R}}{\bar{r} - 1},$$

where

$$\bar{R} = \max_{1 \le i \le n} t_i^{(b)} - \min_{1 \le i \le n} t_i^{(b)},$$

- to determine the ends x_j, y_j, of the intervals $I_j = < x_j, y_j)$, $j = 1,2,\ldots,\bar{r}$, according to the formulae

$$x_1 = \min_{1 \le i \le n} t_i^{(b)} - \frac{d}{2},\ y_j = x_1 + jd,\ \ j = 1,2,\ldots,\bar{r},\ x_j = y_{j-1},\ \ j = 2,3,\ldots,\bar{r},$$

in the way such that

$$I_1 \cup I_2 \cup \ldots \cup I_{\bar{r}} = < x_1, y_{\bar{r}}),$$

and

$$I_i \cap I_j = \emptyset \text{ for all } i \ne j,\ i, j \in \{1,2,\ldots,\bar{r}\},$$

- to determine the numbers of realizations n_j in particular intervals I_j, $j = 1,2,\ldots,\bar{r}$, according to the formula

$$n_j = \#\{i : t_i^{(b)} \in I_j, i \in \{1,2,\ldots,n\}\},\ \ j = 1,2,\ldots,\bar{r},$$

where

$$\sum_{j=1}^{\bar{r}} n_j = n,$$

whereas the symbols # means the number of elements of a set,
- to evaluate the value of the unknown intensity of the component departure $\lambda^{(b)}(u)$, from the reliability states subset, applying suitable formula from the section 3.1,
- to construct and to plot the realization of the histogram of the conditional system component lifetime $T^{(b)}(u),\ b = 1,2,\ldots,\nu,$ in the reliability states subset $\{u, u+1,\ldots,z\},\ u = 1,2,\ldots,z,$ at the system operation state z_b, $b = 1,2,\ldots,\nu,$

$$\bar{f}_n^{(b)}(t,u) = \frac{n_j}{n} \text{ for } t \in I_j,$$

- to analyze the realization of the histogram, comparing it with the graph of the exponential density function

$$f^{(b)}(t,u) = f(t, \lambda^{(b)}(u)) = \lambda^{(b)}(u) \exp[-\lambda^{(b)}(u) t] \text{ for } t \in < 0, \infty),$$

of the system component lifetime $T^{(b)}(u)$ in the reliability states subset $\{u, u+1, \ldots z\}$ at the operations state z_b, corresponding the reliability function coordinate

$$R^{(b)}(t,u) = R(t, \lambda^{(b)}(u)) = \exp[-\lambda^{(b)}(u) t] \text{ for } t \in < 0, \infty),$$

of the vector of the conditional multistate reliability function of the system component

$$[R(t,\cdot)]^{(b)} = [1,\ [R(t,1)]^{(b)}, \ldots,\ [R(t,z)]^{(b)}\ ],$$

and to formulate the null hypothesis H_0 and the alternative hypothesis H_A, concerned with the form of the component multistate reliability $[R(t,\cdot)]^{(b)}$ in the following form:

H_0 : The conditional multistate reliability function of the system component

$$[R(t,\cdot)]^{(b)} = [1,\ [R(t,1)]^{(b)}, ...,\ [R(t,z)]^{(b)}\ ],$$

has the exponential reliability functions coordinates of the form

$$R^{(b)}(t,u) = R(t,\lambda^{(b)}(u)) = \exp[-\lambda^{(b)}(u)t] \text{ for } t \in <0,\infty),$$

H_A : The conditional multistate reliability function of the system component has different from the exponential reliability functions coordinates,

- to join each of the intervals I_j, that has the number n_j of realizations is less than 4 either with the neighbor interval I_{j+1} or with the neighbor interval I_{j-1}, this way that the numbers of realizations in all intervals are not less than 4,
- to fix a new number of intervals
$\bar{\bar{r}}$,
- to determine new intervals
$\bar{I}_j =< \bar{x}_j, \bar{y}_j),\quad j = 1,2,..,\bar{\bar{r}},$
- to fix the numbers $\bar{n}_j$ of realizations in new intervals $\bar{I}_j$, $j = 1,2,..,\bar{\bar{r}}$,
- to calculate the hypothetical probabilities that the variable $T^{(b)}(u)$ takes values from the interval $\bar{I}_j$, under the assumption that the hypothesis H_0 is true, i.e. the probabilities

$$p_j = P(T^{(b)}(u) \in \bar{I}_j) = P(\bar{x}_j \le T^{(b)}(u) < \bar{y}_j) = R^{(b)}(\bar{x}_j,u) - R^{(b)}(\bar{y}_j,u),\ j = 1,2,...,\bar{\bar{r}},$$

where $R^{(b)}(\bar{x}_j,u)$ and $R^{(b)}(\bar{y}_j,u)$ are the values of the coordinate reliability function $R^{(b)}(t,u)$ of the multistate reliability function defined in the null hypothesis H_0.

- to calculate the realization of the χ^2 (*chi-square*)-Pearson's statistics U_n, according to the formula

$$u_n = \sum_{j=1}^{\bar{\bar{r}}} \frac{(\bar{n}_j - np_j)^2}{np_j},$$

- to assume the significance level α ($\alpha = 0.01,\ \alpha = 0.02,\ \alpha = 0.05$ or $\alpha = 0.10$) of the test,

- to fix the number $\bar{\bar{r}} - l - 1$ of degrees of freedom, substituting $l = 1$,

- to read from the Tables of the χ^2 – Pearson's distribution the value u_α for the fixed values of the significance level α and the number of degrees of freedom $\bar{\bar{r}} - l - 1$ such that the following equality holds

$$P(U_n > u_\alpha) = 1 - \alpha,$$

and next to determine the critical domain in the form of the interval $(u_\alpha, +\infty)$ and the acceptance domain in the form of the interval $< 0, u_\alpha >$,

- to compare the obtained value u_n of the realization of the statistics U_n with the red from the Tables critical value u_α of the chi-square random variable and to verify previously formulated the null hypothesis H_0 in the following way: if the value u_n does not belong to the critical domain, i.e. when $u_n \le u_\alpha$, then we do not reject the hypothesis H_0, otherwise if the value u_n belongs to the critical domain, i.e. when $u_n > u_\alpha$, then we reject the hypothesis H_0 in favor of the hypothesis H_A.

3 APPLICATION IN MARITIME TRANSPORT

3.1. The Stena Baltica ferry reliability characteristic statistical identification

The exact evaluation of the Stena Baltica ferry is not possible at the moment because of the complete lack of statistical data about the changes the reliability state subsets by the ferry components and subsystems. Currently, we have only one information about the change from the reliability state subset $\{1.2\}$ into the worst reliability state $z = 0$ (a failure) one of two stern loading platforms operating at the ferry main deck. This departure happened after its good working for around 22 years. The remaining components and subsystems of the ferry under considerations are high reliable and none of them failed during the observation time $\tau = 22.5$ years.
The estimation of this failed component intensity of departure from the reliability states subset $\{1.2\}$ can be done by the formula (6) derived in *Case 2*. Substituting in this formula $\tau = 22.5$, $u = 1$, $n = 2$, $m_1 = 1$ and $t_1^{(b)}(1) = 22$, we get the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(1)$ from the reliability states subset $\{1.2\}$ is

$$\overline{\lambda}^{(b)}(1) = \frac{1}{22 + 22.5(2-1)} \cong 0.0225.$$

The estimation of this failed component intensity of departure from the reliability states subset $\{1.2\}$ can also be done by the formula (9) derived in *Case 5*. Substituting in this formula $\tau = 22.5$, $u = 1$, $n = 2$, $m_1 = 1$ and $t_1^{(b)}(1) = 22$, we get the maximum likelihood evaluation of the unknown component intensity of departure $\lambda^{(b)}(1)$ from the reliability states subset $\{1.2\}$ is

$$\overline{\lambda}^{(b)}(1) = \frac{1+0}{2 \cdot 22.5} \cong 0.0222.$$

The unknown intensities of departures from the reliability state subsets for the components that have not failed during the observation time can be evaluated using so called pessimistic estimation (7')-(11'), derived in (Kolowrocki, Kwiatuszewska-Sarnecka 2009).

4 CONCLUSION

The statistical methods estimating the unknown intensities of the components' exponential reliability functions existing in the joint general model of complex technical systems reliability operating in variable operation conditions linking a semi-markov modeling of the system operation processes with a multi-state approach to system reliability and availability analysis are proposed. Next, these methods are applied to estimating the reliability characteristics of Stena Baltica ferry operating between Gdynia Port in Poland and Karsklone Port in Sweden. The proposed methods other very wide applications to port and shipyard transportation systems reliability and safety characteristics evaluation are obvious. The results are expected to be the basis to the reliability and safety of complex technical systems optimization and their operation processes effectiveness and cost analysis.

Acknowledgements

The paper describes part of the work in the Poland-Singapore Joint Research Project titled "Safety and Reliability of Complex Industrial Systems and Processes" supported by grants from the Poland's Ministry of Science and Higher Education (MSHE grant No. 63/N-Singapore/2007/0) and the Agency for Science, Technology and Research of Singapore (A*STAR SERC grant No. 072 1340050).

5 REFERENCES

Blokus-Roszkowska, A., Guze, S., Kolowrocki, K., Kwiatuszewska-Sarnecka, B., Soszynska, J. 2008a. *Models of safety, reliability and availability evaluation of complex technical systems related to their* *operation processes*. WP 4 – Task 3.1 – 31.05.2008. Poland-Singapore Joint Research Project, 2007-2010.

Blokus-Roszkowska, A., Guze, S., Kolowrocki, K., Kwiatuszewska-Sarnecka, B., Soszynska, J. 2008b. *Models of safety, reliability and availability evaluation of complex technical systems related to their operation processes*. WP 4 – Task 4.1 – 31.05.2008. Poland-Singapore Joint Research Project, 2007-2010.

Blokus-Roszkowska, A. Guze, S. Kolowrocki, K., Kwiatuszewska-Sarnecka, B., Milczek B., Soszynska, J. 2009. *Data Mining for Identification and Prediction of Safety and Reliability Characteristics of Complex Industrial Systems and Processes.* WP6 – Task 6.1. Methods-algorithms for evaluating unknown parameters of system operation processes, reliability and safety models. WP6 - Task 6.1.2 – 16.03.2009. Poland-Singapore Joint Research Project, 2007-2010.

Grabski, F. 2002. *Semi-Markov Models of Systems Reliability and Operations Analysis*. Warsaw: Systems Research Institute, Polish Academy of Science.

Kolowrocki, K., Soszynska, J. 2008. *A general model of technical systems operation processes related to their environment and infrastructure*. WP 2 – Task 2.1 – 31.05.2008. Poland-Singapore Joint Research Project, 2007-2010.

Soszynska, J., Kołowrocki, K., Kamiński, P., Jurdziński, M., Guze, S., Milczek, B., Golik, P. 2009. *Data Mining for Identification and Prediction of Safety and Reliability Characteristics of Complex Industrial Systems and Processes.* WP6– Task 6.2. Preliminary safety and risk analysis and evaluation of a Stena Baltica ferry in variable operation conditions. Sub-Task 6.2.5 - Sub-Task 6.2.5 – 16.03.2009. Poland-Singapore Joint Research Project, 2007-2010.

Kolowrocki, K., Kwiatuszewska- Sarnecka, B. 2009. *Data Mining for Identification and Prediction of Safety and Reliability Characteristics of Complex Industrial Systems and Processes.* WP6 – Task 6.1. Methods-algorithms for evaluating unknown parameters of system operation processes, reliability and safety models – a preliminary study. WP6 - Task 6.1.1 – 16.03.2009. Poland-Singapore Joint Research Project, 2007-2010.

MODELS OF RELIABILITY AND AVAILABILITY IMPROVEMENT OF SERIES AND PARALLEL SYSTEMS RELATED TO THEIR OPERATION PROCESSES

B. Kwiatuszewska-Sarnecka,

•

Gdynia Maritime University, Gdynia, Poland

e-mail: kwiatek@am.gdynia.pl

ABSTRACT

Integrated general models of approximate approaches of complex multi-state series and parallel systems, linking their reliability and availability improvement models and their operation processes models caused changing reliability and safety structures and components reliability characteristics in different operation states, are constructed. These joint models are applied to determining improved reliability and availability characteristics of the considered multi-state series and parallel systems related to their varying in time operation processes. The conditional reliability characteristics of the multi-state systems with hot, cold single reservation of component and the conditional reliability characteristics of the multi-state systems with reduced rate of departure by a factor of system components are defined.

1 INTRODUCTION

Taking Most real transportation systems are very complex. Large numbers of components and subsystems and their operating complexity cause that the evaluation and optimization of their reliability improvement and availability improvement is complicated. A convenient tool for solving this problem is semi-markov modeling of the systems operation processes combining with three methods of reliability and availability improvement proposed in this paper. Therefore, the common usage of the system's reliability and availability improvement models and the semi-markov model for the system's operation process modeling in order to construct the joint general system reliability and availability improvement model related to its operation process is proposed.

.

2 RELIABILITY IMPROVEMENT OF MULTI-STATE SYSTEM COMPONENT IN VARIABLE OPERATION CONDITIONS

We assume that the reliability of a single system component can be improved by using hot or cold reserve of this component or by replacing this component by an improved component with the reduced rates of departure from the reliability state subset $\{u, u+1, ...z\}$ by a factor $\rho(u)$, $0 < \rho(u) < 1$, $u = 1,2,\ldots,z$. Further, we assume that the basic and reserve component have the same multi-state exponential reliability function. If basic and reserve components of the multi-state system at the operational state z_b, $b = 1,2,...,\nu$, have the same exponential reliability functions given by

$$[R_i(t,\cdot)]^{(b)} = [1, [R_i(t,1)]^{(b)}, \ldots, [R_i(t,z)]^{(b)}], t \in (-\infty,\infty), \; b = 1,2,...,\nu, \quad (1)$$

where

$$[R_i(t,u)]^{(b)} = 1 \text{ for } t < 0, \; [R_i(t,u)]^{(b)} = \exp[-\lambda_i^{(b)}(u)t] \text{ for } t \geq 0, \; \lambda_i^{(b)}(u) > 0, \quad (2)$$

$i = 1,2,...,n,\ u = 1,2,\ldots,z,\ b = 1,2,...,\nu,$
then
(a) the reliability function of multi-state system component with a single hot reservation at the operational state z_b, $b = 1,2,...,\nu$, is respectively given by

$$[R_i^{(1)}(t,\cdot)]^{(b)} = [1,[R_i^{(1)}(t,1)]^{(b)},\ldots,[R_i^{(1)}(t,z)]^{(b)}],\ t \in (-\infty,\infty),\ b = 1,2,...,\nu,$$

(3)
where

$$[R_i^{(1)}(t,u)]^{(b)} = 1-[[F_i(t,u)]^{(b)}]^2 = 1,\ t < 0,$$

$$[R_i^{(1)}(t,u)]^{(b)} = 1-[[F_i(t,u)]^{(b)}]^2 = 2\exp[-\lambda_i^{(b)}(u)t] - \exp[-2\lambda_i^{(b)}(u)t],\ t \geq 0, \quad (4)$$

$\lambda_i^{(b)}(u) > 0,\ i = 1,2,...,n,\ u = 1,2,\ldots,z,\ b = 1,2,...,\nu,$
(b) the reliability function of multi-state system component with single cold reservation at the operational state z_b, $b = 1,2,...,\nu$, is respectively given by

$$[R_i^{(2)}(t,\cdot)]^{(b)} = [1,[R_i^{(2)}(t,1)]^{(b)},\ldots,[R_i^{(2)}(t,z)]^{(b)}],\ t \in (-\infty,\infty),\ b = 1,2,...,\nu, \quad (5)$$

where

$$[R_i^{(2)}(t,u)]^{(b)} = 1-[[F_i(t,u)]^{(b)} * [F_i(t,u)]^{(b)}] = 1,\ t < 0,$$

$$[R_i^{(2)}(t,u)]^{(b)} = 1-[[F_i(t,u)]^{(b)} * [F_i(t,u)]^{(b)}] = [1+\lambda_i^{(b)}(u)t]]\exp[-\lambda_i^{(b)}(u)t],\ t \geq 0, \quad (6)$$

(c) the exponential reliability function of multi-state system component with the reduced rate of departure by a factor $\rho(u)$, $u = 1,2,...,z$, at the operational state z_b, $b = 1,2,...,\nu$, is respectively given by

$$[R_i^{(3)}(t,\cdot)]^{(b)} = [1,[R_i^{(3)}(t,1)]^{(b)},\ldots,[R_i^{(3)}(t,z)]^{(b)}]\ t \in (-\infty,\infty),\ b = 1,2,...,\nu, \quad (7)$$

where

$$[R_i^{(3)}(t,u)]^{(b)} = 1 \text{ for } t < 0,\ [R_i^{(3)}(t,u)]^{(b)} = \exp[-\lambda_i^{(b)}(u)\rho(u)t] \text{ for } t \geq 0, \quad (8)$$

3 ASYMPTOTIC APPROACH TO EVALUATION OF RELIABILITY IMPROVEMENT OF LARGE MULTI-STATE SYSTEMS IN VARIABLE OPERATION CONDITIONS

Technical Main results concerning asymptotic approach to multi-state system reliability improvement with ageing components in fixed operation conditions are comprehensively detailed. In order to combine the results on the reliability improvement of multi-state systems related to their operation processes and the results concerning limit reliability functions of the multi-state systems, and to obtain results of asymptotic approach to evaluation of the multi-state system reliability improvement in variable operation conditions (Kołowrocki K. & Kwiatuszewska-Sarnecka B. (2008)), we use the following definition.
A reliability function

$$\boldsymbol{\mathfrak{R}}(t,\cdot) = [1, \boldsymbol{\mathfrak{R}}(t,1),..., \boldsymbol{\mathfrak{R}}(t,z)],\ t \in (-\infty,\infty),$$

where

$$\boldsymbol{\mathfrak{R}}(t,u) = \sum_{b=1}^{\nu} p_b[\boldsymbol{\mathfrak{R}}(t,u)]^{(b)},\quad u = 1,2,...,z,$$

is called a limit reliability function of a multi-state system in its operation process with reliability function

$$\boldsymbol{R}_n(t,\cdot) = [\boldsymbol{R}_n(t,0), \boldsymbol{R}_n(t,1),..., \boldsymbol{R}_n(t,z)],$$

where

$$\boldsymbol{R}_n(t,u) \cong \sum_{b=1}^{\nu} p_b[\boldsymbol{R}_n(t,u)]^{(b)},\ u = 1,2,...,z,$$

where

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l},\ b = 1,2,...,v,$$

are the limit values of the transient probabilities at the particular operation states

$$p_b(t) = P(Z(t) = z_b),\ t \in <0,+\infty),\ b = 1,2,...,v,$$

and the probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases}$$

if there exist normalising constants

$$a_n^{(b)}(u) > 0,\ b_n^{(b)}(u) \in (-\infty,\infty),\ u = 1,2,...,z,\ b = 1,2,...,v,$$

such that for $t \in C_{[\boldsymbol{\mathfrak{R}}(u)]^{(b)}}$, $u = 1,2,...,z,\ b = 1,2,...,v,$

$$\lim_{n\to\infty}[\boldsymbol{R}_n(a_n^{(b)}(u)t + b_n^{(b)}(u),u)]^{(b)} = [\boldsymbol{\mathfrak{R}}(t,u)]^{(b)}.$$

Hence, the following approximate formulae are valid

$$\boldsymbol{R}_n(t,\cdot) = [\boldsymbol{R}_n(t,0), \boldsymbol{R}_n(t,1),..., \boldsymbol{R}_n(t,z)],$$

where

$$\boldsymbol{R}_n(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{\mathfrak{R}}(\frac{t - b_n^{(b)}(u)}{a_n^{(b)}(u)},u)]^{(b)},\ t \in (-\infty,\infty),\ u = 1,2,...,z.$$

The following propositions are concerned with the homogeneous exponential systems i.e. the systems which components have at operational states z_b, $b = 1,2,...,v,$ exponential reliability functions.

Proposition 3.1

If components of the homogeneous multi-state series system at the operational state z_b have exponential reliability functions and the system have:

(a) a single hot reservation of system components and

$$a_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)\sqrt{n}},\ b_n^{(b)}(u) = 0,\ u = 12,...,z,\ b = 1,2,...,v,$$

then

$$\overline{\boldsymbol{\mathfrak{R}}}^{(1)}(t,\cdot) = [1, \overline{\boldsymbol{\mathfrak{R}}}^{(1)}(t,1),\ldots, \overline{\boldsymbol{\mathfrak{R}}}^{(1)}(t,z)],\ t \in (-\infty,\infty), \quad (9)$$

where

$$\overline{\boldsymbol{\mathfrak{R}}}^{(1)}(t,u) = 1 \text{ for } t < 0,\ \overline{\boldsymbol{\mathfrak{R}}}^{(1)}(t,u) = \sum_{b=1}^{v} p_b \exp[-t^2] \text{ for } t \ge 0, \quad (10)$$

is its unconditional multi-state limit reliability function. Hence, the following exact formulae is valid

$$\overline{\boldsymbol{R}}_n^{(1)}(t,\cdot) = [1, \overline{\boldsymbol{R}}_n^{(1)}(t,1),..., \overline{\boldsymbol{R}}_n^{(1)}(t,z)], \quad (11)$$

where

$$\overline{\boldsymbol{R}}_n^{(1)}(t,u) = 1 \text{ for } t < 0,\ \overline{\boldsymbol{R}}_n^{(1)}(t,u) = \sum_{b=1}^{v} p_b \exp[-(\lambda^{(b)}(u)\sqrt{n}t)^2],\ t \ge 0,\ u = 1,2,...,z, \quad (12)$$

(b) a single cold reservation of system components and

$$a_n^{(b)}(u) = \frac{\sqrt{2}}{\lambda^{(b)}(u)\sqrt{n}},\ b_n^{(b)}(u) = 0,\ u = 1,2,...,z,\ b = 1,2,...,v,$$

then

$$\overline{\boldsymbol{\mathfrak{R}}}^{(2)}(t,\cdot) = [1, \overline{\boldsymbol{\mathfrak{R}}}^{(2)}(t,1),\ldots, \overline{\boldsymbol{\mathfrak{R}}}^{(2)}(t,z)],\ t \in (-\infty,\infty), \quad (13)$$

where

$$\overline{\boldsymbol{\mathfrak{R}}}^{(2)}(t,u) = 1 \text{ for } t < 0, \quad \overline{\boldsymbol{\mathfrak{R}}}^{(2)}(t,u) = \sum_{b=1}^{v} p_b \exp[-t^2] \text{ for } t \geq 0,\ u = 1,2,...,z \tag{14}$$

is its unconditional multi-state limit reliability function. Hence the following approximate formulae is valid

$$\overline{\boldsymbol{R}}_n^{(2)}(t,\cdot) = [1, \overline{\boldsymbol{R}}_n^{(2)}(t,1), ..., \overline{\boldsymbol{R}}_n^{(2)}(t,z)], \tag{15}$$

where

$$\overline{\boldsymbol{R}}_n^{(2)}(t,u) = 1 \text{ for } t < 0,\ \overline{\boldsymbol{R}}_n^{(2)}(t,u) \cong \sum_{b=1}^{v} p_b \exp[-(\frac{\lambda^{(b)}(u)\sqrt{n}}{\sqrt{2}} t)^2],\ t \geq 0,\ u = 1,2,..., z, \tag{16}$$

(c) components with the reduced rate of departure by a factor $\rho(u)$, $u = 1,2,..., z$, and

$$a_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)\rho(u)n},\ b_n^{(b)}(u) = 0,\ u = 1,2,...,z,\ b = 1,2,..., v,$$

then

$$\overline{\boldsymbol{\mathfrak{R}}}^{(3)}(t,\cdot) = [1, \overline{\boldsymbol{\mathfrak{R}}}^{(3)}(t,1), \ldots, \overline{\boldsymbol{\mathfrak{R}}}^{(3)}(t,z)],\ t \in (-\infty,\infty), \tag{17}$$

where

$$\overline{\boldsymbol{\mathfrak{R}}}^{(3)}(t,u) = 1 \text{ for } t < 0,\ \overline{\boldsymbol{\mathfrak{R}}}^{(3)}(t,u) = \sum_{b=1}^{v} p_b \exp[-t],\ t \geq 0,\ u = 1,2,...,z, \tag{18}$$

is its unconditional multi-state limit reliability function. Hence the following exact formulae is valid

$$\overline{\boldsymbol{R}}_n^{(3)}(t,\cdot) = [1, \overline{\boldsymbol{R}}_n^{(3)}(t,1), ..., \overline{\boldsymbol{R}}_n^{(3)}(t,z)], \tag{19}$$

where

$$\overline{\boldsymbol{R}}_n^{(3)}(t,u) = 1 \text{ for } t < 0,\ \overline{\boldsymbol{R}}_n^{(3)}(t,u) = \sum_{b=1}^{v} p_b \exp[-\lambda^{(b)}(u)\rho^{(b)}(u)nt,\ t \geq 0,\ u = 1,2,..., z. \tag{20}$$

Proposition 3.2

If components of the homogeneous multi-state parallel system at the operational state z_b have exponential reliability functions and the system have

(a) a single hot reservation of system components and

$$a_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)},\ b_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)} \log 2n,\ u = 1,2,...,z,\ b = 1,2,..., v,$$

then

$$\boldsymbol{\mathfrak{R}}^{(1)}(t,\cdot) = [1, \boldsymbol{\mathfrak{R}}^{(1)}(t,1), \ldots, \boldsymbol{\mathfrak{R}}^{(1)}(t,z)],\ t \in (-\infty,\infty), \tag{21}$$

where

$$\boldsymbol{\mathfrak{R}}^{(1)}(t,u) = 1 - \sum_{b=1}^{v} p_b \exp[-\exp[-t]],\ t \in (-\infty,\infty),\ u = 1,2,...,z, \tag{22}$$

is its unconditional multi-state limit reliability function. Hence the following approximate formulae is valid

$$\boldsymbol{R}_n^{(1)}(t,\cdot) = [1, \boldsymbol{R}_n^{(1)}(t,1), ..., \boldsymbol{R}_n^{(1)}(t,z)], \tag{23}$$

where

$$\boldsymbol{R}_n^{(1)}(t,u) \cong 1 - \sum_{b=1}^{v} p_b \exp[-\exp[-(\lambda^{(b)}(u)t - \log 2n)]],\ t \in (-\infty,\infty),\ u = 1,2,...,z, \tag{24}$$

(b) a single cold reservation of system components and

$$a_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)},\ \frac{\exp[\lambda^{(b)}(u)b_n^{(b)}(u)]}{\lambda^{(b)}(u)b_n^{(b)}(u)} = n,\ u = 1,2,...,z,\ b = 1,2,..., v, \tag{25}$$

then

$$\boldsymbol{\mathfrak{R}}^{(2)}(t,\cdot) = [1, \boldsymbol{\mathfrak{R}}^{(2)}(t,1), \ldots, \boldsymbol{\mathfrak{R}}^{(2)}(t,z)],\ t \in (-\infty,\infty), \tag{26}$$

where

$$\mathfrak{R}^{(2)}(t,u) = 1 - \sum_{b=1}^{v} p_b \exp[-\exp[-t]],\ t \in (-\infty,\infty),\ u = 1,2,...,z, \tag{27}$$

is its unconditional multi-state limit reliability function, hence, the following approximate formulae is valid

$$\boldsymbol{R}_n^{(2)}(t,\cdot) = [1, \boldsymbol{R}_n^{(2)}(t,1),..., \boldsymbol{R}_n^{(2)}(t,z)], \tag{28}$$

where

$$\boldsymbol{R}_n^{(2)}(t,u) \cong 1 - \sum_{b=1}^{v} p_b \exp[-\exp[-(\lambda^{(b)}(u)t - \lambda^{(b)}(u)b_n^{(b)}(u))]],\ t \in (-\infty,\infty),\ u = 1,2,..., z, \tag{29}$$

(c) components with the reduced rate of departure by a factor $\rho(u)$, $u = 1,2,..., z$, and

$$a_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)\rho(u)},\ b_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)\rho(u)}\log n,\ u = 1,2,...,z,\ b = 1,2,..., v,$$

then

$$\mathfrak{R}^{(3)}(t,\cdot) = [1, \mathfrak{R}^{(3)}(t,1),\ldots, \mathfrak{R}^{(3)}(t,z)],\ t \in (-\infty,\infty), \tag{30}$$

where

$$\mathfrak{R}^{(3)}(t,u) = 1 - \sum_{b=1}^{v} p_b \exp[-\exp[-t]],\ t \in (-\infty,\infty),\ u = 1,2,...,z, \tag{31}$$

is its unconditional multi-state limit reliability function hence, the following approximate formulae is valid

$$\boldsymbol{R}_n^{(3)}(t,\cdot) = [1, \boldsymbol{R}_n^{(3)}(t,1),..., \boldsymbol{R}_n^{(3)}(t,z)], \tag{32}$$

where

$$\boldsymbol{R}_n^{(3)}(t,u) \cong 1 - \sum_{b=1}^{v} p_b \exp[-\exp[-(\lambda^{(b)}(u)\rho^{(b)}(u)t - \log n)]],\ t \in (-\infty,\infty),\ u = 1,2,..., z. \tag{33}$$

4 AVAILABILITY OF MULTI-STATE SERIES AND PARALLEL SYSTEMS IN VARIABLE OPERATION CONDITIONS

There is presented a combination of reliability, availability improvement models of multi-state renewal systems only with non-ignored time of renovation in a model of variable in time operation processes. On the basis of those joined models, with assumption, that systems' improved conditional reliability functions dependent on variable in time operation states are the same as improved limit reliability functions of the exponential non-renewal multi-state series and parallel systems, improved availability characteristics of the systems are determined.

4.1 Multi-state systems with non-ignored time of renovation in variable operation conditions

We assume similarly as in non renewal systems considered in Point 3 that the changes of the process *Z*(*t*) states have an influence on the multi-state system reliability structure. The main characteristics of multi-state renewal system with hot, cold single reservation of system components and system with improved component's reliability related to their operation process can be approximately determined by taking account described system's operation process properties.

Proposition 4.1

If components of the multi-state renewal system with non-ignored time of renovation at the operational states z_b, $b = 1,2,..., v$, have exponential reliability functions and the time of the system renovation has the mean value $[\mu_o(r)]^{(b)}$ and the standard deviation $[\sigma_o^2(r)]^{(b)}$, then:

i) the distribution function of the time $\overline{\overline{S}}_N^{(k)}(r)$, $k = 1,2,3,$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution

$$N(N(\mu^{(k)}(r) + \mu_o(r)), \sqrt{N(\sigma^{(k)2}(r) + \sigma_o^2(r))})$$

i.e.,

$$\overline{\overline{F}}^{(N)(k)}(t,r) = P(\overline{\overline{S}}_N^{(k)}(r) < t) \cong F_{N(0,1)}{}^{(k)}\left(\frac{t - N(\mu^{(k)}(r) + \mu_o(r))}{\sqrt{N(\sigma^{(k)2}(r) + \sigma_o^2(r))}}\right),$$

$t \in (-\infty,\infty)$, $N = 1,2,...$, $r \in \{1,2,..., z\}$,

ii) the expected value and the variance of the time $\overline{\overline{S}}_N^{(k)}(r)$, $k = 1,2,3,$ until the *Nth* system's renovation take respectively forms

$$E[\overline{\overline{S}}_N^{(k)}(r)] \cong N(\mu^{(k)}(r) + \mu_o(r)),\ D[\overline{\overline{S}}_N^{(k)}(r)] \cong N(\sigma^{(k)2}(r) + \sigma_o^2(r)),\ r \in \{1,2,..., z\},$$

iii) the distribution function of the time $\overline{S}_N^{(k)}(r)$, $k = 1,2,3,$ until the *Nth* exceeding the reliability critical state *r* of this system takes form

$$\overline{F}^{(N)(k)}(t,r) = P(\overline{S}_N^{(k)}(r) < t) \cong F_{N(0,1)}{}^{(k)}\left(\frac{t - N(\mu^{(k)}(r) + \mu_o(r)) + \mu_o(r)}{\sqrt{N(\sigma^{(k)2}(r) + \sigma_o^2(r)) - \sigma_o^2(r)}}\right),$$

$t \in (-\infty,\infty)$, $N = 1,2,...$, $r \in \{1,2,...,z\}$,

iv) the expected value and the variance of the time $\overline{S}_N^{(k)}(r)$, $k = 1,2,3,$ until the *Nth* exceeding the reliability critical state *r* of this system take respectively forms

$$E[\overline{S}_N^{(k)}(r)] \cong N\mu^{(k)}(r) + (N-1)\mu_o(r),\ D[\overline{S}_N^{(k)}(r)] \cong N\sigma^{(k)2}(r) + (N-1)\sigma_o^2(r),\ r \in \{1,2,..., z\},$$

v) the distribution of the number $\overline{\overline{N}}^{(k)}(t,r)$, $k = 1,2,3,$ of system's renovations up to the moment t, $t \geq 0,$ is of the form

$$P(\overline{\overline{N}}^{(k)}(t,r) = N) \cong F_{N(0,1)}{}^{(k)}\left(\frac{N(\mu^{(k)}(r) + \mu_o(r)) - t}{\sqrt{\frac{t}{\mu^{(k)}(r) + \mu_0(r)}(\sigma^{(k)2}(r) + \sigma_o^2(r))}}\right)$$

$$- F_{N(0,1)}{}^{(k)}\left(\frac{(N+1)(\mu^{(k)}(r) + \mu_o(r)) - t}{\sqrt{\frac{t}{\mu^{(k)}(r) + \mu_0(r)}(\sigma^{(k)2}(r) + \sigma_o^2(r))}}\right),\quad N = 1,2,...,\ r \in \{1,2,...,z\},$$

vi) the expected value and the variance of the number $\overline{\overline{N}}^{(k)}(t,r)$, $k = 1,2,3,$ of system's renovations up to the moment t, $t \geq 0,$ take respectively forms

$$\overline{H}^{(k)}(t,r) \cong \frac{t}{\mu^{(k)}(r) + \mu_o(r)},\quad \overline{D}^{(k)}(t,r) \cong \frac{t}{(\mu^{(k)}(r) + \mu_o(r))^3}(\sigma^{(k)2}(r) + \sigma_o{}^2(r)),\ r \in \{1,2,..., z\},$$

vii) the distribution of the number $\overline{N}^{(k)}(t,r)$, $k = 1,2,3,$ of exceeding the reliability critical state *r* of this system up to the moment t, $t \geq 0,$ is of the form

$$P(\overline{N}^{(k)}(t,r) = N) \cong F_{N(0,1)}{}^{(k)}\left(\frac{N(\mu^{(k)}(r) + \mu_o(r)) - t - \mu_0(r)}{\sqrt{\frac{t + \mu_0(r)}{\mu^{(k)}(r) + \mu_0(r)}(\sigma^{(k)2}(r) + \sigma_o^2(r))}}\right)$$

$$-F_{N(0,1)}{}^{(k)}\left(\frac{(N+1)(\mu^{(k)}(r)+\mu_o(r))-t-\mu_0(r)}{\sqrt{\frac{t+\mu_0(r)}{\mu^{(k)}(r)+\mu_0(r)}(\sigma^{(k)^2}(r)+\sigma_o^2(r))}}\right),\ N=1,2,\ldots,\ r\in\{1,2,\ldots,z\},$$

viii) the expected value and the variance of the number $\overline{N}^{(k)}(t,r)$, $k=1,2,3,$ of exceeding the reliability critical state r of this system up to the moment $t, t\geq 0,$ for sufficiently large t, are approximately respectively given by

$$\bar{H}^{(k)}(t,r)\cong\frac{t+\mu_0(r)}{\mu^{(k)}(r)+\mu_o(r)},\ \bar{D}^{(k)}(t,r)\ \cong\frac{t+\mu_0(r)}{(\mu^{(k)}(r)+\mu_o(r))^3}(\sigma^{(k)^2}(r)+\sigma_o{}^2(r)),\ r\in\{1,2,\ldots,z\},$$

ix) the availability coefficient of the system at the moment t is given by the formula

$$K^{(k)}(t,r)\cong\frac{\mu^{(k)}(r)}{\mu^{(k)}(r)+\mu_o(r)},\ t\geq 0,\ r\in\{1,2,\ldots,z\},$$

x) the availability coefficient of the system in the time interval $<t,t+\tau), \tau>0,$ is given by the formula

$$K^{(k)}(t,\tau,r)\cong\frac{1}{\mu^{(k)}(r)+\mu_o(r)}\int_\tau^\infty \boldsymbol{R}_n{}^{(k)}(t,r)dt,\ t\geq 0,\ \tau>0,\ r\in\{1,2,\ldots,z\},$$

where for $u=r,$ and $\mu^{(k)}(r)$ and $\sigma^{(k)}(r)$, $k=1,2,3,$ are given by:
- for a homogeneous series system
(a) with a single hot reservation of system components

$$\mu^{(1)}(r)\cong\sum_{b=1}^{v}p_b\frac{\sqrt{\pi}}{2\lambda^{(b)}(r)n},\ r\in\{1,2,\ldots,z\},\tag{34}$$

$$[\sigma^{(1)}(r)]^2=\int_{-\infty}^{+\infty}t^2d\overline{\boldsymbol{F}}_n{}^{(1)}(t,r)-[\mu^{(1)}(r)]^2,\ r\in\{1,2,\ldots,z\},$$

where $\mu^{(1)}(r)$ is given by the formula (34)

$$\overline{\boldsymbol{F}}_n{}^{(1)}(t,r)=1-\overline{\boldsymbol{R}}_n{}^{(1)}(t,r),\ t\in(-\infty,\infty),$$

and $\overline{\boldsymbol{R}}_n{}^{(1)}(t,r)$ is given by the formulae (11)-(12) for $u=r,\ r\in\{1,2,\ldots,z\}$,
(b) with a single cold reservation of system components

$$\mu^{(2)}(r)\cong\sum_{b=1}^{v}p_b\frac{\sqrt{\pi}}{\lambda^{(b)}(r)\sqrt{2n}},\ r\in\{1,2,\ldots,z\},\tag{35}$$

$$[\sigma^{(2)}(r)]^2=\int_{-\infty}^{+\infty}t^2d\overline{\boldsymbol{F}}_n{}^{(2)}(t,r)-[\mu^{(2)}(r)]^2,\ r\in\{1,2,\ldots,z\},$$

where $\mu^{(2)}(r)$ is given by the formula (35)

$$\overline{\boldsymbol{F}}_n{}^{(2)}(t,r)=1-\overline{\boldsymbol{R}}_n{}^{(2)}(t,r),\ t\in(-\infty,\infty),$$

and $\overline{\boldsymbol{R}}_n{}^{(2)}(t,r)$ is given by the formulae (15)-(16) for $u=r,\ r\in\{1,2,\ldots,z\}$,
(c) with the reduced rate of departure by a factor $\rho(u),\ u=1,2,\ldots,z,$ of its components

$$\mu^{(3)}(r)\cong\sum_{b=1}^{v}p_b\frac{1}{\lambda^{(b)}(r)\rho(r)n},\ r\in\{1,2,\ldots,z\},\tag{36}$$

$$[\sigma^{(3)}(r)]^2=\int_{-\infty}^{+\infty}t^2d\overline{\boldsymbol{F}}_n{}^{(3)}(t,r)-[\mu^{(3)}(r)]^2,\ r\in\{1,2,\ldots,z\},$$

where $\mu^{(3)}(r)$ is given by the formula (36)

$$\overline{\boldsymbol{F}}_n{}^{(3)}(t,r)=1-\overline{\boldsymbol{R}}_n{}^{(3)}(t,r),\ t\in(-\infty,\infty),$$

and $\overline{\boldsymbol{R}}_n^{(3)}(t,r)$ is given by the formulae (19)-(20) for $u = r,\ r \in \{1,2,...,z\}$,

- for a homogeneous parallel system

(a) with a single hot reservation of system components

$$\mu^{(1)}(r) \cong \sum_{b=1}^{v} p_b (C/\lambda^{(b)}(r) + \log 2n/\lambda^{(b)}(r)),\ \ r \in \{1,2,...,z\}, \tag{37}$$

where $C \cong 0.5772$ is Euler's constant

$$[\sigma^{(1)}(r)]^2 = \int_{-\infty}^{+\infty} t^2 d\overline{\boldsymbol{F}}_n^{(1)}(t,r) - [\mu^{(1)}(r)]^2,\ \ r \in \{1,2,...,z\},$$

where $\mu^{(1)}(r)$ is given by the formula (37)

$$\boldsymbol{F}_n^{(1)}(t,r) = 1 - \boldsymbol{R}_n^{(1)}(t,r),\ \ t \in (-\infty,\infty),$$

and $\boldsymbol{R}_n^{(1)}(t,r)$ is given by the formulae (23)-(24) for $u = r,\ r \in \{1,2,...,z\}$,

(b) with a single cold reservation of system components

$$\mu^{(2)}(r) \cong \sum_{b=1}^{v} p_b (C/\lambda^{(b)}(r) + \log(n\lambda^{(b)}(r) b_n^{(b)}(r)),\ r \in \{1,2,...,z\}, \tag{38}$$

where $C \cong 0.5772$ is Euler's constant and

$$[\sigma(r)]^2 = \int_{-\infty}^{+\infty} t^2 d\overline{\boldsymbol{F}}_n\ (t,r) - [\mu(r)]^2,\ \ r \in \{1,2,...,z\},$$

where $\mu^{(2)}(r)$ is given by the formula (38)

$$\boldsymbol{F}_n^{(2)}(t,r) = 1 - \boldsymbol{R}_n^{(2)}(t,r),\ \ t \in (-\infty,\infty),$$

and $\boldsymbol{R}_n^{(2)}(t,r)$ is given by the formulae (25)-(26) for $u = r,\ r \in \{1,2,...,z\}$,

(c) with the reduced rate of departure by a factor $\rho(u),\ u = 1,2,...,z,$ of its components

$$\mu^{(3)}(r) \cong \sum_{b=1}^{v} p_b (C/(\lambda^{(b)}(r)\rho(r)) + \log n/(\lambda^{(b)}(r)\rho(r))), r \in \{1,2,...,z\}, \tag{39}$$

where $C \cong 0.5772$ is Euler's constant

$$[\sigma^{(3)}(r)]^2 = \int_{-\infty}^{+\infty} t^2 d\overline{\boldsymbol{F}}_n^{(3)}(t,r) - [\mu^{(3)}(r)]^2,\ \ r \in \{1,2,...,z\},$$

where $\mu^{(3)}(r)$ is given by the formula (39)

$$\boldsymbol{F}_n^{(3)}(t,r) = 1 - \boldsymbol{R}_n^{(3)}(t,r),\ \ t \in (-\infty,\infty),$$

and $\boldsymbol{R}_n^{(3)}(t,r)$ is given by the formulae (32)-(33) for $u = r,\ r \in \{1,2,...,z\}$.

5 EFFECTS OF COMPARISON ON RELIABILITY AND AVAILABILITY IMPROVEMENT

In order to determine the value of the coefficient $\rho(r)$, $r \in \{1,2,...,z\}$, by which it is necessary to reduce (multiply by) the failure rates of departure of the reliability states subsets of basic components of the non renewal system or renewal with non-ignored time of renovation in order to receive the system having the mean value of the number of exceeding the critical reliability state during the time t as the mean value of the number of exceeding the critical reliability state during the time t of the system with either hot or cold single reserve of basic components we can solve either the equation

$$\mu^{(1)}(r) = \mu^{(3)}(r),\ \ t \geq 0,\ \ r \in \{1,2,...,z\}, \tag{40}$$

or respectively the equation

$$\mu^{(2)}(r) = \mu^{(3)}(r),\ \ t \geq 0,\ \ r \in \{1,2,...,z\}, \tag{41}$$

and if the system is the system with not ignored time of renovation in order to receive the system having the same availability as the availability of the system with either hot or cold single reserve of basic components we can solve either the equation

$$K^{(1)}(r) = K^{(3)}(r), \quad t \ge 0, \; r \in \{1,2,..., z\}, \tag{42}$$

or respectively the equation

$$K^{(2)}(r) = K^{(3)}(r), \quad t \ge 0, \; r \in \{1,2,..., z\}. \tag{43}$$

Proposition 5.1

If components of the multi-state non renewal system or renewal system with non ignored time of renovation at the operational states z_b, $b = 1,2,..., v$, have exponential reliability functions then the coefficient $\rho(r)$, $r \in \{1,2,..., z\}$, is in the form

- for a homogeneous series system

in the first case by (40) , after considering (34) and (36), we get

$$\rho(r) = \frac{2}{\sqrt{\pi n}}, \; r \in \{1,2,..., z\},$$

in the second case by (41), after considering (35) and (36), we get

$$\rho(r) = \frac{\sqrt{2}}{\sqrt{\pi n}}, \; r \in \{1,2,..., z\},$$

- for a homogeneous parallel system

in the first case by (40) , after considering (36) and (38), we get

$$\rho(r) = \frac{C + \log n}{C + \log 2n}, \; r \in \{1,2,..., z\},$$

in the second case by (41), after considering (37) and (38), we get

$$\rho(r) = \frac{C + \log n}{C + \log \lambda^{(b)}(r) b_n^{(b)}(r) n}, \; r \in \{1,2,..., z\},$$

where $C \cong 0.5772$ is Euler's constant and $\dfrac{\exp[\lambda^{(b)}(r) b_n^{(b)}(r)]}{\lambda^{(b)}(r) b_n^{(b)}(r)} = n$, $r \in \{1,2,..., z\}$.

6 CONCLUSION

In the paper the multi-state approach to the improvement of systems' reliability, and availability for series and parallel systems has been presented. Constructed in this paper the final integrated, general analytical models of complex systems reliability and availability improvement related to their operation processes are very important for the further optimization of system operation costs.

6 PREFERENCES

- Barlow, R. E. & Proschan, F. (1975). *Statistical Theory of Reliability and Life Testing. Probability Models*. Holt Rinehart and Winston, Inc., New York.
- Gniedenko, B. W. (1943). Sur la distribution limite du terme maximum d'une serie aleatoire. *Ann. of Math.* 44, 432–453.
- Kołowrocki, K. (2004). *Reliability of Large Systems*. Elsevier, Amsterdam - Boston - Heidelberg - London - New York - Oxford - Paris - San Diego - San Francisco - Singapore - Sydney - Tokyo.
- Kołowrocki K. & Kwiatuszewska-Sarnecka B. (2008). *Safety and Reliability Optimization of Complex Industrial Systems and Processes.* WP5 - Task 5.1. *Models of safety, reliability*

and availability improvement and optimization of complex industrial systems related to their operation processes. – English – 31.05.2009. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.

- Soszyńska, J. (2007). *Systems reliability analysis In variable operation conditions.* PhD Thesis, Gdynia Maritime University-System Research Institute Warsaw, *(in Polish).*

Acknowledgements

The paper describes part of the work in the Poland-Singapore Joint Research Project titled "Safety and Reliability of Complex Industrial Systems and Processes" supported by grants from the Poland's Ministry of Science and Higher Education (MSHE grant No. 63/N-Singapore/2007/0) and the Agency for Science, Technology and Research of Singapore (A*STAR SERC grant No. 072 1340050)

CORRELATION AND REGRESSION ANALYSIS OF SPRING STATISTICAL DATA OF MARITIME FERRY OPERATION PROCESS

M.S. Habibullah, Fu Xiuju

•
Institute of High Performance Computing, Singapore

e-mail: mohdsh@ihpc.a-star.edu.sg

K. Kolowrocki, J. Soszynska
•
Gdynia Maritime University, Gdynia, Poland

e-mail: katmatkk@am.gdynia.pl, joannas@am.gdynia.pl

ABSTRACT

These are presented statistical methods of correlation and regression analysis of the operation processes of complex technical systems. The collected statistical data from the Stena Baltica ferry operation process are analysed and used for determining correlation coefficients and linear and multiple regression equations, expressing the influence of the operation process conditional sojourn times in particular operation states on the ferry operation process total conditional sojourn time.

1 INTRODUCTION

Many real transportation systems belong to the class of complex systems. First, and foremost, these systems are concerned with the large numbers of components and subsystems they are built and with their operating complexities. Modeling of these complicated system operation processes is, first of all, difficult because of the large number of the operation states, impossibility of their precise definition as well as the impossibility of the exact description of the transitions between these states. Generally, the change of the operation states of the system operations processes causes the changes of these systems reliability structures and their components reliability functions. Therefore, the system operation process and its operation states proper definition and accurate identification of the interactions between the particular operation states and their influence on the entire system operation process is very important.

The model of the operation processes of the complex technical systems (Blokus et al. 2008) with distinguishes their operation states is proposed in (Kolowrocki & Soszynska 2008). The semi-markov process (Grabski 2002) is used to construct a general probabilistic model of the considered complex industrial system operation process. To apply this model in practice its unknown parameters have to be identified. Namely, the vector of the probabilities of the system initial operation states, the matrix of the probabilities of transitions between the operation states and the matrix of the distribution functions or equivalently the matrix of the density functions of the conditional sojourn times in the particular operation states, needs to be estimated on the basis of the statistical data. The methods of these unknown parameters evaluation are developed and presented in details in (Kolowrocki & Soszynska 2009A-B). In addition to these methods the simple data mining techniques such as correlation coefficient, linear and multiple regression as well as root mean square error can be used on the statistical data samples to perform the analyses. The results of that analysis as well as relevant conclusions that can be reached from the studies may give practically important information in the operation processes of the complex technical systems investigation.

The aim of this report is to use these techniques in studying the patterns that can be derived, from realizations of the conditional sojourn times, obtained from the Stena Baltica ferry operation process, for the early spring data (Kolowrocki et al. 2009A-B).

The report is organized is the following way. In Section 1, some general comments on complex technical systems operation processes modeling are given and the problem considered in this report is defined. In Section 2, the general assumptions on the complex system operation process are presented. In Section 3, the Stena Baltica ferry operation process is described. In Section 4, the formulae for the total conditional sojourn time its mean and standard deviation are presented and applied to the spring statistical data of the Stena Baltica ferry operation process. This is then followed by determining the correlation coefficient, linear and multiple regression and root mean square error for the ferry operation process spring data. In Section 5, the report summary is given.

2 SYSTEM OPERATION PROCESS

We assume, similarly as in (Blokus et al. 2008, Kolowrocki & Soszynska 2008), that a system during its operation at the fixed moment t, $t \in <0,+\infty>$, may be in one of v, $v \in N$, different operations states z_b, $b = 1,2, ..., v$. Next, we mark by $Z(t)$, $t \in <0,+\infty>$, the system operation process, that is a function of a continuous variable t, taking discrete values in the set $Z = \{z_1, z_2, ..., z_v\}$ of the operation states. We assume a semi-markov model (Blokus et al. 2008, Grabski 2002, Kolowrocki & Soszynska 2008) of the system operation process $Z(t)$ and we mark by θ_{bl} its random conditional sojourn times at the operation states z_b, when its next operation state is z_l, $b, l = 1,2,...,v$, $b \neq l$.

Under these assumptions, the operation process may be described by the vector $[p_b(0)]_{1 \times v}$ of probabilities of the system operation process staying in particular operations states at the initial moment $t = 0$, the matrix $[p_{bl}(t)]_{v \times v}$ of the probabilities of the system operation process transitions between the operation states and the matrix $[H_{bl}(t)]_{v \times v}$ of the distribution functions of the conditional sojourn times θ_{bl} of the system operation process at the operation states or equivalently by the matrix $[h_{bl}(t)]_{v \times v}$ of the density functions of the conditional sojourn times θ_{bl}, $b, l = 1,2,...,v$, $b \neq l$, of the system operation process at the operation states.

To estimate the unknown parameters of the system operations process, the first phase in the experiment, is to collect necessary statistical data. This is performed in the following steps (Kolowrocki et al. 2009A-B):

i) To analyze the system operation process and either to fix or to define the following general parameters:
   - the number of the operation states of the system operation process v;
   - the operation states of the system operation process z_1, z_2, …, z_v;

ii) To fix and collect the following statistical data necessary in evaluating the probabilities of the initial states of the system operations process:
   - the duration time of the experiment Θ;
   - the number of the investigated (observed) realizations of the system operation process $n(0)$;
   - the numbers of staying operation process respectively in the operations states z_1, z_2, …, z_v, at the initial moment $t = 0$ of all $n(0)$ observed realizations of the system operation process $n_1(0)$, $n_2(0)$, …, $n_v(0)$, where $n_1(0) + n_2(0) + n_v(0) = n(0)$;

iii) To fix and collect the following statistical data necessary to evaluating the transient probabilities between the system operation states:

- the numbers n_{bl}, $b, l = 1,2,...,v$, $b \neq l$, of the transitions of the system operation process from the operation state z_b to the operation state z_l during all observed realizations of the system operation process;
- the numbers n_b, $b = 1,2,...,v$, of departures of the system operation process from the operation states z_b, where $n_b = \sum_{l=1}^{v} n_{bl}$;

iv) To fix and collect the following statistical data necessary in evaluating the unknown parameters of the distributions of the conditional sojourn times of the system operation process in the particular operation states:

- the realizations θ_{bl}^k, $k = 1,2, \ldots, n_{bl}$, $b, l = 1,2,...,v$, $b \neq l$, of the conditional sojourn times θ_{bl} of the system operations process at the operation state z_b when the next transition is to the operation state z_l during the observation time;

After collecting the above statistical data it is possible to estimate the unknown parameters of the system operation process (Kolowrocki & Soszynska 2009A-B). It is also possible to analyze rather accurately the system operation process sojourn times in the particular operation states and their influence on the entire system operation process total sojourn time (Kolowrocki et al. 2009B).

3 STENA BALTICA FERRY OPERATION PROCESS

The problem considered in this report is based on real maritime statistical data, obtained from Stena Baltica ferry operation process, whereby the ferry performs continuous journeys from Gdynia in Poland to Kalskrona in Sweden. Table 1 show the operation states that the Stena Baltica ferry undertakes, beginning with loading at Gdynia then passing through the Traffic Separation Scheme to Karlskrona for unloading/loading and back to Gdynia for unloading/loading. This operation process is repeated continuously and it is assumed that one voyage from Gdynia to Kalskrona and back to Gdynia is a single realization of its operation process. For the voyage described, time-series data were collected for the realization of the conditional sojourn times θ_{bl} of the system operations process at the operation state z_b when the next transition is to the operation state z_l for spring conditions. These data are shown in the Appendix in Tables A1-A4 coming from (Kolowrocki et al. 2009B).

Table 1. Stena Baltica ferry operation states

Operation state	Description	Operation State	Description
z_1	Gdynia: Loading	z_{10}	Karlskrona: Unmooring
z_2	Gdynia: Unmooring	z_{11}	Karlskrona: Turning
z_3	Gdynia: Navigating to GD buoy	z_{12}	Karlskrona: Navigating to Angoring buoy
z_4	Gdynia: Navigating to TSS	z_{13}	Karlskrona: Navigating to TSS
z_5	Gdynia: Navigating to Angoring buoy	z_{14}	Karlskrona: Navigating to GD buoy
z_6	Karlskrona: Navigating to Verko berth	z_{15}	Karlskrona: Navigating to Turning Area
z_7	Karlskrona: Mooring	z_{16}	Gdynia: Ferry Turning

Operation state	Description	Operation State	Description
z_8	Karlskrona: Unloading	z_{17}	Gdynia: Mooring
z_9	Karlskrona: Loading	z_{18}	Gdynia: Unloading

It is also important to note that the operation process is very regular and cyclic, in the sense that the operation states changes from the particular state z_b, where $b = 1,2....17$ to the neighbouring state z_{b+1}, where $b = 1,2....17$ only and from z_{18} to z_1. Therefore, based on this definition the spring realization of the ferry conditional sojourn times $\theta^k_{b\,b+1}$, where $b = 1,2....17$ and $\theta^k_{18\;1}$ for $k = 1,2,...,n_{bl}$, where $n_{bl} = 42$, are given in Tables A1-A4. Also included in Tables A1-A4 are the values of the total conditional sojourn times for each realization, θ^k_T, for $k = 1,2,...,n_{bl}$, where $n_{bl} = 42$. In our analysis the values of θ^k_T are important in analyzing the behaviour of the Stena Baltic ferry operation process.

4 DATA ANALYSIS ON STENA BALTICA OPERATION PROCESS

In this section, the use of several data mining techniques on the system total conditional sojourn time is described. The techniques adopted are namely, correlation coefficient, linear and multiple regression and root mean square error. These techniques are applied on the early spring data from the Stena Baltica ferry operation process.

4.1 Total conditional sojourn time

As discussed above, the Stena Baltica ferry operation process data for spring is shown in the Appendix in Tables A1-A4 for spring. In analyzing the behavior of the data patterns, this report examines the ferry total conditional sojourn time (the time length of one ferry voyage) θ_T by analyzing its successive realizations θ^k_T, defined as

$$\theta^k_T = \sum_{b=1}^{17} \theta^k_{b\,b+1} + \theta^k_{18\;1} \tag{1}$$

for $k = 1,2,...,n_{bl}$, where $n_{bl} = 42$ for spring data. Using equation (1), the total conditional sojourn times were then calculated for both spring with the values shown in Tables A1-A4. These values form the basis of our conjecture in this paper.

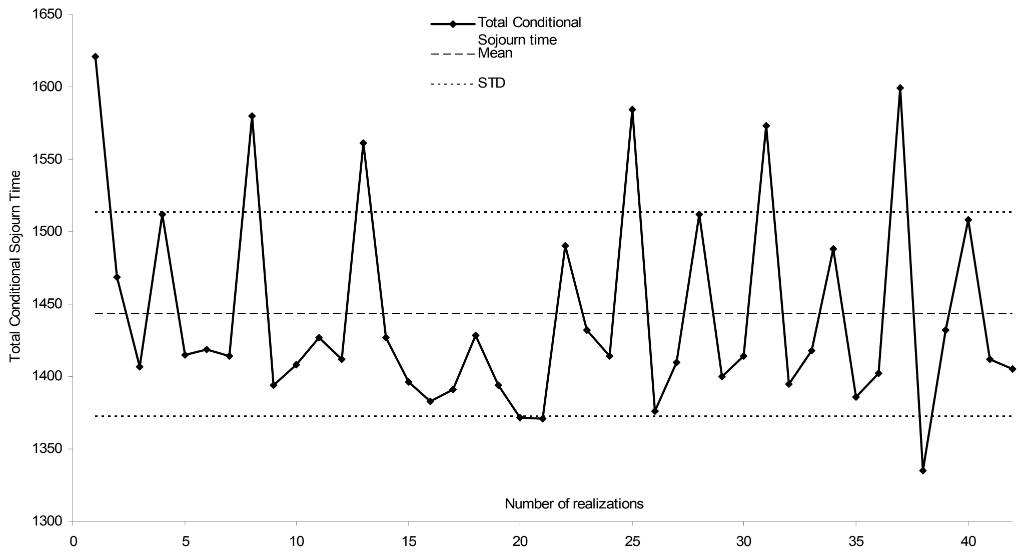


Figure 1. Plot of realizations θ_T^k of total conditional sojourn time θ_T for spring data

Figure 1 shows the plot of the realizations θ_T^k of the ferry total conditional sojourn time θ_T against the realization number k for spring data. In the picture, by STD there are marked 1-sigma lower $\overline{\theta}_T - \overline{\sigma}_T$ and upper $\overline{\theta}_T + \overline{\sigma}_T$ bounds for the ferry total conditional sojourn time θ_T.
Although the ferry operation process is regular and cyclic, i.e. the operation states follows the process in Table 1, it can be observed that the values of θ_T are not constant. Furthermore, by using the mean total conditional sojourn time $\overline{\theta}_T$, evaluated from the following equation

$$\overline{\theta}_T = \frac{1}{n_{bl}} \sum_{k=1}^{n_{bl}} \theta_T^k \tag{2}$$

and the standard deviation defined as

$$\overline{\sigma}_T = \sqrt{\frac{1}{n_{bl}} \sum_{k=1}^{n_{bl}} (\overline{\theta}_T^k - \overline{\theta}_T)^2} \tag{3}$$

it was found that nearly 26% of the θ_T^k values fall outside of the interval $< \overline{\theta}_T - \overline{\sigma}_T, \overline{\theta}_T + \overline{\sigma}_T >$.
The results in Figures 1 seem to indicate a pattern whereby in each realization the contribution of the ferry conditional sojourn time $\theta_{b\,l}^k$ for some operation states towards θ_T^k is more for some than that for others. Thus, identifying the conditional sojourn time for such operation states, which has major effect on the ferry total operation process times enable the total conditional sojourn time for the operation process to be studied, analysed and predicted. These are discussed in the following sections where the use of data mining techniques to understand the behaviour of θ_T^k is presented.

4.2 Correlation

Correlation analysis is a method commonly used to establish, with certain degree of probability, whether a linear relationship exists between two measured quantities. This means that when there is correlation it implies that there is a tendency for the values of the two quantities to effect one another. Vice-versa also holds true if there is no correlation which implies no effect on each other. Furthermore, using the values of the correlation coefficient, a positive or negative relationship can also be identified. If the coefficient values are close to 1, it implies positive linear relationship, whilst values close to 0 imply no linear relationship. Thus, based on the values of the correlation coefficient, the relationship between two measured quantities can be determined. The adopted formula for evaluating the correlation coefficient r_{bl} between the ferry conditional sojourn time θ_{bl} in particular operation states and the ferry total conditional sojourn time θ_T is given by

$$r_{bl} = \frac{\sum_{k=1}^{n_{bl}} (\theta_{bl}^k - \overline{\theta}_{bl})(\theta_T^k - \overline{\theta}_T)}{\sqrt{\sum_{k=1}^{n_{bl}} (\theta_{bl}^k - \overline{\theta}_{bl})^2} \sqrt{\sum_{k=1}^{n_{bl}} (\theta_T^k - \overline{\theta}_T)^2}}, \tag{4}$$

for $b = 1,2,...,17$, $l = b+1$ and $b = 18$, where $n_{bl} = 42$ is the number of realizations, θ_{bl}^k is the k-th realization of the conditional sojourn time θ_{bl}, θ_T^k is the k-th realization of the total conditional sojourn time θ_T evaluated from (1), $\overline{\theta}_T$ is the mean total conditional sojourn time evaluated from the equation (2) and $\overline{\theta}_{bl}$ is the mean conditional sojourn time obtained from

$$\overline{\theta}_{bl} = \frac{1}{n_{bl}} \sum_{k=1}^{n_{bl}} \theta_{bl}^k. \tag{5}$$

Thus, using the values from Tables A1-A4, the correlation coefficient, r_{bl}, were then evaluated using equation (4). Table 2 shows the values of $r_{b\,l}$ for the spring data.

Table 2. Correlation coefficient $r_{b\,l}$ values for spring data

Operation State	**Correlation coefficient**	**Operation state**	**Correlation coefficient**
z_1	0.221169	z_{10}	0.401463
z_2	0.298071	z_{11}	0.324054
z_3	-0.13934	z_{12}	0.306238
z_4	0.642635	z_{13}	0.640848
z_5	0.738339	z_{14}	0.365648
z_6	0.020627	z_{15}	0.099242
z_7	-0.04948	z_{16}	0.142937
z_8	0.2035	z_{17}	0.149159
z_9	0.1559	z_{18}	0.057029

Figure 2 shows the plot of the correlation coefficient r_{bl} against the number b of the operation state z_b. It can be seen that $\theta_{4\,5}$, $\theta_{5\,6}$ and $\theta_{13\,14}$ has the strongest positive linear relationship, as compared to the conditional sojourn times in the remaining operation states, where $\theta_{5\,6}$ and $\theta_{13\,14}$ coincides with the longest parts of the voyage. This implies that any variations in the conditional sojourn times $\theta_{b\,b+1}$ associated with these 3 operation states, namely z_4, z_5 and z_{13}, will significantly effect the total conditional sojourn time θ_T.

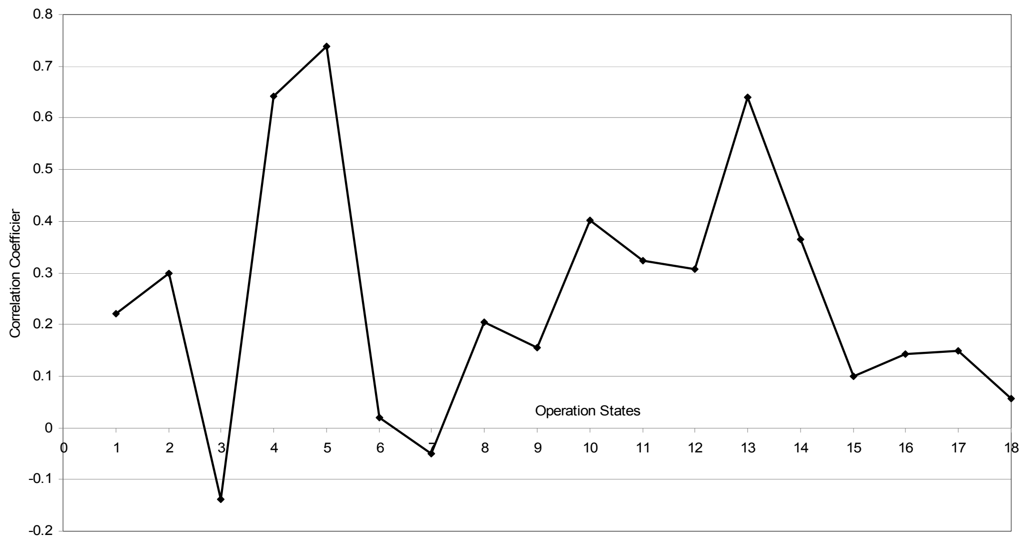


Figure 2. Plot of correlation coefficient r_{bl} between conditional sojourn time and total conditional sojourn time for spring data

The plots given in Figure 2 also shows that most of the r_{bl} values are more than 0, which seems to indicate a positive linear relationship, albeit weak linear relationship for some. Thus, from the correlation coefficient values, it can be deduced that the values of the total conditional sojourn time θ_T is strongly dependent on the conditional sojourn times $\theta_{b\,l}$ for some operation states. In the following section, this understanding of the data behaviour will be used in the regression model to predict the values of the total conditional sojourn time θ_T.

4.3 Regression

Regression analysis is a data mining technique used in modeling, analyzing and predicting numerical data. In linear regression, input statistical data are necessary, whereby the data is modeled as a function, in coming out with the model parameters. These parameters are then estimated so as to give a "best fit" of the data, which are then used to predict future data behaviour. Multiple regression is another type of regression model. It is similar to linear regression but in this model the interest is on examining more than one predictor variables. In this technique the aim is to determine whether the inclusion of additional predictor variables leads to increased prediction of the outcome. Here, the use of both linear and multiple regression models on the spring data are described.

From the above discussions, it can be seen that the aim of using the linear regression technique is to use initial sample data of the conditional sojourn times θ_b to predict subsequent behavior of the total conditional sojourn time θ_T. In the paper the equation adopted is given by

$$\theta_T = \alpha_b + \beta_b \theta_{b\,l} + \varepsilon_b \tag{6}$$

for $b = 1,2,...,17,\ l = b+1$ and $b = 18,\ l = 1,$ where $\alpha_b,\ \beta_b$ are the unknown regression coefficients and ε_b is the random noise.

Before predicting the subsequent behavior, the values of α_b and β_b based on varying realizations of the operation process need to be evaluated. Here, the unknown regression coefficients α_b and β_b are evaluated by minimizing the functions

$$\Delta(\alpha_b, \beta_b) = \sum_{k=1}^{N} [\theta_T^k - (\alpha_b + \beta_b \theta_{b\,l}^k)]^2 \tag{7}$$

for $b = 1,2,...,17,\ l = b+1$ and $b = 18,$ defined as the measure of divergences between the empirical values θ_T^k and defined by (6) the predicted values $\theta_T(\theta_{bl}^k) = \alpha_b + \beta_b \theta_{b\,l}^k$ of the total conditional sojourn time θ_T .

From the necessary condition, i.e. after finding the first partial derivatives of $\Delta(\alpha_b, \beta_b)$ with respect to α_b and β_b and putting them equal to zero, we get the system of equalities involving the realizations θ_T^k of the total conditional sojourn time θ_T and the realizations θ_{bl}^k of the conditional sojourn times $\theta_{b\,l}$ defined as follows

$$\begin{aligned} N\alpha_b + \sum_{k=1}^{N} \theta_{b\,l}^k \beta_b &= \sum_{k=1}^{N} \theta_T^k \\ \sum_{k=1}^{N} \theta_{bl}^k \alpha_b + \sum_{k=1}^{N} (\theta_{b\,l}^k)^2 \beta_b &= \sum_{k=1}^{N} \theta_{bl}^k \theta_T^k \end{aligned} \tag{8}$$

for $b = 1,2,...,17,\ l = b+1$ and $b = 18,\ l = 1$ and $N = 1,2,...,n_{bl}$.

The remaining question that needs to be addressed is how many realizations marked by N does it take to obtain a reasonable representation of α_b and β_b. By using Matlab and putting the values from Tables A1-A4 into the system of equations (6), the varying α_b and β_b values were calculated for $N = 1,2,...,n_{bl}$.

Figure 3 shows the plot of the regression coefficient β_b against N , for the operation states of z_5 and z_{13}. From the discussions in Section 4.2, these 2 operation states represents among the longest part of the voyage and has major influence on the total conditional sojourn time. From the plot, it can be observed that other than the initial instability for low values of N , the values of β_b seems to stabilize for larger N . In our analyses, it was discovered that the value of β_b stabilizes at $N = 30$. Although not shown in the paper this behavior also holds true for all the other operation states.

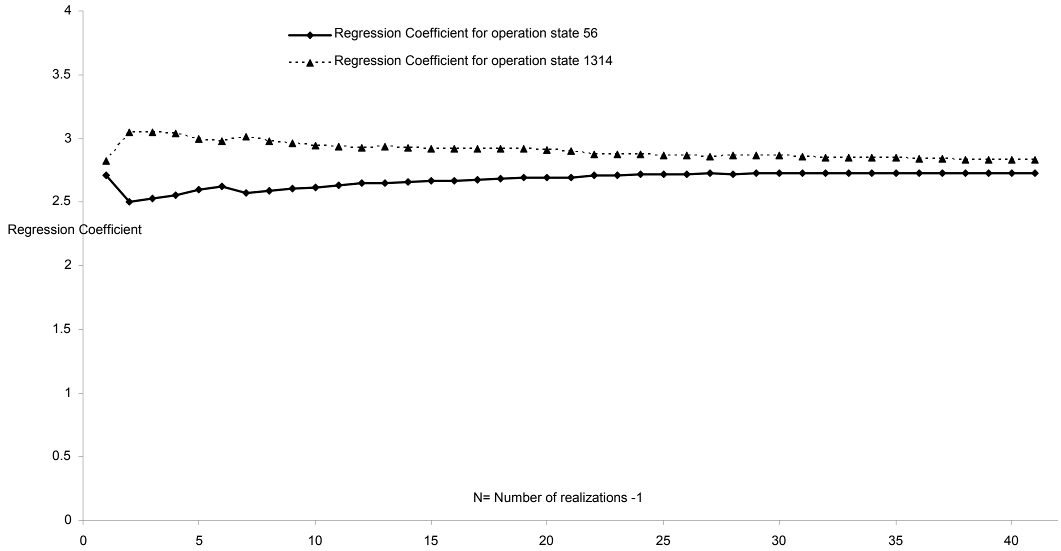


Figure 3. Plot of regression coefficient β_b for spring data

Thus, based on the above observations, the predicted total conditional sojourn time θ_T^* can then be evaluated using β_b values at $N = 30$. In evaluating θ_T, the formulations in the system of equations (8), lead to

$$\theta_T^* = \alpha_b^* + \beta_b^* \theta_{b\,l} \tag{9}$$

for $b = 1,2,...,17$, $l = b + 1$ and $b = 18$, $l = 1$, where α_b^* and β_b^* are respectively the value of α_b and β_b at $N = 30$.

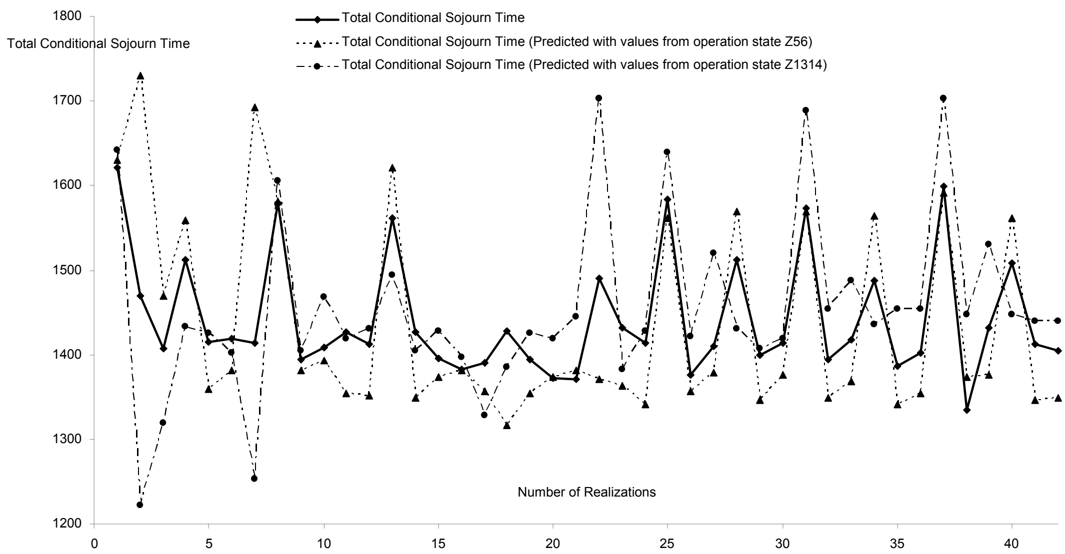


Figure 4. Plots of empirical realizations and predicted from linear regression values of total conditional sojourn time for spring data

Figure 4 shows the comparison plots of the values of the empirical realizations θ_T^k of the total conditional sojourn time θ_T and the predicted values θ_{T*}^k of the total conditional sojourn time θ_T^* defined by the equation (9) against the number of realizations k for summer data. It can be observed that for both the operation states of z_5 and z_{13}, the predicted θ_{T*}^k values are not close to the empirical θ_T^k values. Similar pattern of behaviour were also seen when the values of θ_{T*}^k for other operation states, were considered. These results seem to indicate that linear regression does not provide an accurate means of predicting the behaviour of the Stena Baltica ferry operation process.

Since linear regression does not provide an accurate prediction of the total conditional sojourn time the multiple regression technique is explored instead. As described earlier, the difference in the multiple regressions technique is that in this method, more than one predictor variables are considered. It is envisaged that the inclusion of additional predictor variables will lead to increased prediction of the total conditional sojourn time. Thus, for multiple regressions, the equation adopted is given by

$$\theta_T = \alpha_B + \sum_{b=1}^{B} \beta_b \theta_{b\,l} + \varepsilon_b \tag{10}$$

for $b = 1,2,...,17$, $l = b+1$ and $b = 18$, $l = 1$ and $B = 1,2,...,\nu$, $\nu = 18$, where α_B, β_1, β_2, …, β_B are the unknown regression coefficients and ε_b is the random noise.

Before predicting the subsequent behaviour of α_B, β_1, β_2, …, β_B values based on varying realizations of the operation process need to be evaluated. The unknown regression coefficients α_B, β_1, β_2, …, β_B are obtained by minimizing the functions,

$$\Delta(\alpha_B, \beta_1, \beta_2, ..., \beta_B) = \sum_{k=1}^{N} [\theta_T^k - (\alpha_B + \sum_{b=1}^{B} \beta_b \theta_{b\,l}^k)]^2 \tag{11}$$

for $b = 1,2,...,17$, $l = b+1$ and $b = 18$, $l = 1$ and $B = 1,2,...,\nu$, $\nu = 18$, that is the measure of divergences between the empirical values θ_T^k and predicted values $\theta_T(\theta_{1\,l}^k, \theta_{2\,l}^k, ..., \theta_{B\,l}^k) = \alpha_B + \sum_{b=1}^{B} \beta_b \theta_{b\,l}^k$ of the total conditional sojourn time θ_T defined by (8).

From the necessary condition, *i.e.* after finding the first partial derivatives of $\Delta(\alpha_B, \beta_1, \beta_2, ..., \beta_B)$ with respect to α_B, β_1, β_2, …, β_B and putting them equal to zero, we get the system of equalities involving the realizations θ_T^k of the total conditional sojourn time θ_T and the realizations θ_{bl}^k of the conditional sojourn times $\theta_{b\,l}$ defined as follows,

$$\begin{aligned} & N\alpha_B + \sum_{b=1}^{B}\sum_{k=1}^{N} \theta_{b\,l}^k \beta_b = \sum_{k=1}^{N} \theta_T^k \\ & \sum_{k=1}^{N} \theta_{1\,l}^k \alpha_B + \sum_{b=1}^{B}\sum_{k=1}^{N} \theta_{1\,l}^k \theta_{b\,l}^k \beta_b = \sum_{k=1}^{N} \theta_{1\,l}^k \theta_T^k \\ & \ldots\ldots\ldots \end{aligned} \tag{12}$$

$$\sum_{k=1}^{N}\theta_{B\,l}^{k}\alpha_{B}+\sum_{b=1}^{B}\sum_{k=1}^{N}\theta_{B\,l}^{k}\theta_{b\,l}^{k}\beta_{b}=\sum_{k=1}^{N}\theta_{B\,l}^{k}\theta_{T}^{k}$$

for $b=1,2,...,17$, $l=b+1$ and $b=18$, $l=1$ and $B=1,2,...,\nu$, $\nu=18$ and $N=1,2,...,n_{bl}$. The remaining question that needs to be addressed here is that how many realizations marked by N in (12) does it take to obtain a reasonable representation of α_B, β_1, β_2, …, β_B. Thus, by using Matlab and putting the values from Tables A1-A4 into the system of equations (10) for $N=1,2,...,n_{bl}$, the varying α_B, β_1, β_2, …, β_B values were calculated.

In our analyses on the values of α_B, β_1, β_2, …, β_B, the observation is that the values of α_B, β_1, β_2, …, β_B stabilizes at $N=30$. It was also observed that α_B, β_1, β_2, …, β_B vary with respect to the number B, $B=1,2,...,\nu$, $\nu=18$, of predictor variables considered changing 1 to 18. The argument for this method is that by using more than one predictor variables, better results will be obtained. The aim is also to use as minimal number of predictor variables to generate accurate results, within as short period of time. Thus, based on the above observations, the predicted total conditional sojourn time, θ_T, can then be evaluated using α_B, β_1, β_2, …, β_B values at $N=30$. In evaluating θ_T, the formulations in the system of equations (10) lead to

$$\theta_T^{*}=\alpha_B^{*}+\sum_{b=1}^{B}\beta_b^{*}\theta_{b\,l} \quad (13)$$

for $b=1,2,...,17$, $l=b+1$ and $b=18$, $l=1$ and $B=1,2,...,\nu$, $\nu=18$, where α_B^{*}, β_1^{*}, β_2^{*}, ..., β_B^{*} are respectively the value of α_B, β_1, β_2, …, β_B at $N=30$.

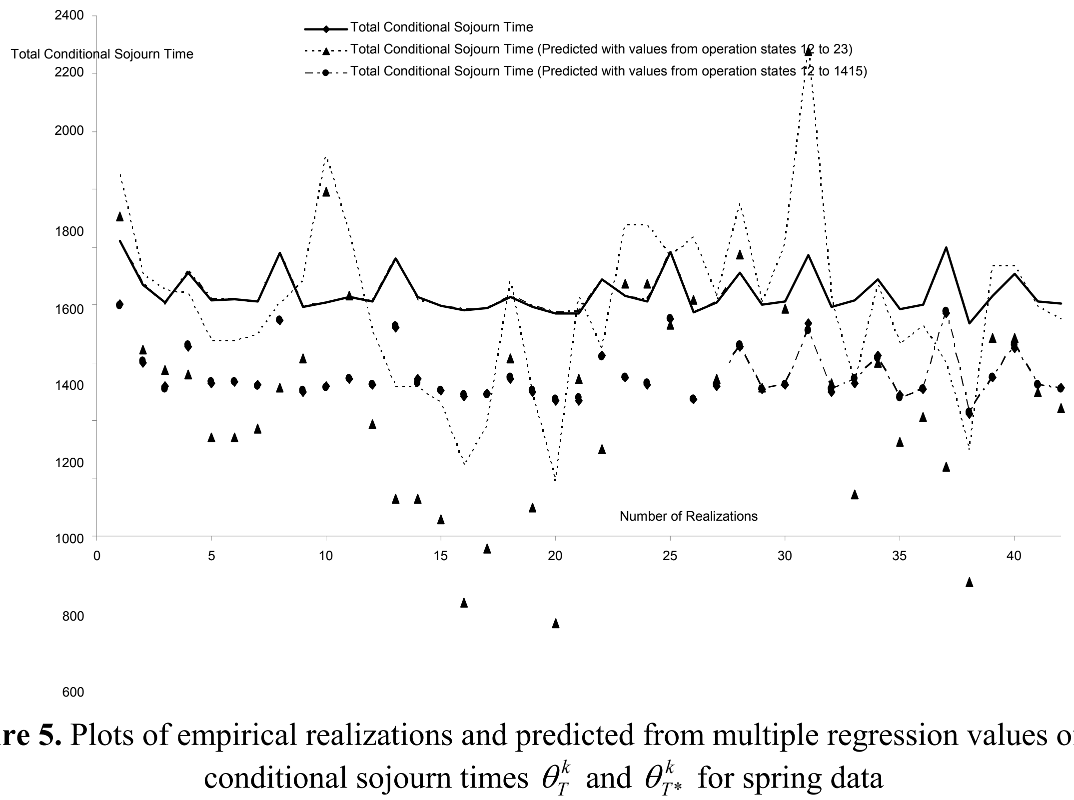


Figure 5. Plots of empirical realizations and predicted from multiple regression values of total conditional sojourn times θ_T^k and θ_{T*}^k for spring data

Figure 5 shows the comparison plots of the values of the empirical realizations θ_T^k of the total conditional sojourn time θ_T and the predicted values θ_{T*}^k of the total conditional sojourn time θ_T^* defined by the equation (11) against the number of realizations k for summer data. It can be seen that if only 2 predictor variables $\theta_{1\,2}$ and $\theta_{2\,3}$ $(B = 2)$ are used in the equation (11), then the predicted values differ much from the empirical values θ_{T*}^k and are not accurate at all. It was discovered that as we increased the number of predictor variables, the accuracy improves, leading to the best accuracy at $B = 14$ predictor variables $\theta_{1\,2}$, $\theta_{2\,3}$, …, $\theta_{14\,15}$. It was also observed that if more than 14 predictor variables were used, the results doesn't change much, indicating that 14 predictors variables provides a good representation of the prediction. The analyses also show that multiple regression is a better method of predicting the behaviour of the Stena Baltica ferry data than the linear regression.

4.4 Accuracy

To further access the accuracy of the predicted data the root mean square error ε is applied. The root mean square error is commonly used to calculate the error and is often used to measure the success of numerical prediction. If the value of ε is 0 it simply means that there is no error to the prediction and the prediction is accurate. The greater values of ε mean that the more inaccurate is the prediction. Here, the values of the root mean square errors for both the linear and multiple regressions are calculated. The adopted for the root mean square error equation is given by

$$\varepsilon = \sqrt{\frac{1}{n_{bl}} \sum_{k=1}^{n_{bl}} (\theta_T^{*k} - \theta_T^k)^2}, \qquad (14)$$

where $\theta_T^{*k} = \theta_T^*(\theta_{b\,l}^k)$ for linear regression, $\theta_T^{*k} = \theta_T^*(\theta_{1\,l}^k, \theta_{2\,l}^k, ..., \theta_{B\,l}^k)$ for multiple regression and $n_{bl} = 42$ in the case of spring data. By using the predicted values θ_T^{*k} for both linear and multiple regressions and the empirical value of θ_T^k from the spring data the values of ε were calculated. It was found for spring data that for instance for linear regression with one predictor variable $\theta_{5\,6}$ that $\varepsilon \cong 77.9$ and for multiple regression with 14 predictor variables $\theta_{1\,2}$, $\theta_{2\,3}$, …, $\theta_{14\,15}$ this value was $\varepsilon \cong 5.3$. These values of the the root mean square errors validate the results obtained from the regression analyses, indicating the accuracy of multiple regressions as compared to linear regression.

5 SUMMARY

This report has described the use of simple data mining techniques on the Stena Baltica ferry operation process statistical data given in Tables A1-A4. The aim is to observe the behaviour of the ferry operation process total conditional sojourn time and use it to predict future behaviours. In our analyses, we applied the correlation coefficient, linear and multiple regressions and root mean square error on spring data. From the results, it can be concluded that multiple regressions technique provides an accurate of predicting the ferry total conditional sojourn time.

8 REFERENCES

Blokus-Roszkowska, A., Guze, S., Kołowrocki, K., Kwiatuszewska-Sarnecka, B., Soszyńska, J. 2008. *Models of safety, reliability and availability evaluation of complex technical systems related to their operation processes.* WP 4 – Task 4.1 – English – 31.05.2008. Poland-Singapore Joint Project.

Grabski, F. 2002. *Semi-Markov Models of Systems Reliability and Operations*. Monograph. Analysis. Monograph. System Research Institute, Polish Academy of Science, (in Polish).

Kołowrocki, K., Soszyńska, J. 2008. *A general model of technical systems operation processes related to their environment and infrastructure.* WP 2 – Task 2.1 – English – 31.05.2008. Poland-Singapore Joint Project.

Kołowrocki, K., Soszyńska, J. 2009A. *Methods and algorithms for evaluating unknown parameters of operation processes of complex systems*. Proc. Summer Safety and Reliability Seminars - SSARS *2009*, Vol. 2, 211-221.

Kołowrocki, K., Soszyńska, J. 2009B. *Data mining for identification and prediction of safety and reliability characteristics of complex systems and processes.* Proc. European Safety and Reliability Conference – ESREL 2009, Vol. 2, 853-863.

Kołowrocki, K., Soszyńska, J., Kamiński, P., Jurdziński, M., Guze, S., Milczek, B., Golik, P. 2009A. Data mining for identification and prediction of safety and reliability characteristics of complex industrial systems and processes.WP6 - Task 6.2. Preliminary statistical data collection of the Stena Baltica ferry operation process and its preliminary statistical identification. WP6 - Sub-Task 6.2.5 – Appendix 5A - English – 31.10.2009. Poland-Singapore Joint Project.

Kołowrocki, K., Soszyńska, J., Salahuddin Habibullah, M., Xiuju, F. 2009B. *Data mining for identification and prediction of safety and reliability characteristics of complex industrial systems and processes.*WP6 - Task 6.1.3. Experimental statistical data correlation and regression analysis - Correlation and regression analysis of experimental statistical data of the operation process of the Stena Baltica ferry. Task 6.1.3 – Section 4 and Section 5.5.4 - English – 31.08.2009. Poland-Singapore Joint Project.

Appendix

Statistical summer data collection of the Stena Baltica ferry operation process

In the *Tables A1-A4* there are given realizations of the conditional sojourn times in particular operation states on the basis of a sample composed of $n = 42$ realizations of the Stena Baltica ferry operation process. It is assumed that one voyage from Gdynia to Kalskrone and back to Gdynia of the ferry is a single realization of its operation process. The conditional sojourn times in particular operation states of each single realization of the ferry operation process are given in separate columns. The operation process is very regular in the sense that the operation state changes are from the particular state z_b, $b = 1,2,...,17,$ to the neighboring state z_{b+1}, $b = 1,2,...,17,$ only and from z_{18} to z_1. Therefore the realizations of the conditional sojourn times θ^j_{bb+1}, $b = 1,2,...,17,$ $j = 1,2,...,42,$ are given in the Tables b-th row and the realizations of the conditional sojourn time $\theta^j_{18\,1}$, $b = 1,2,...,17,$ are given in the Tables 18-th row.

Appendix 5A

5A. 1. Statistical summer data collection of the Stena Baltica ferry operation process

Date/2008	24/25 Jan	26/27 Jan	27/28 Jan	11/12 Feb	12/13 Feb	26/27 Feb	27/28 Feb	28/01 Mar	01/02 Mar	02/03 Mar	11/12 Mar	12/13 Mar

In the *Tables A1-A4* there are given realizations of the conditional sojourn times in particular operation states on the basis of a sample composed of $n = 42$ realizations of the Stena Baltica ferry operation process. It is assumed that one voyage from Gdynia to Kalskrone and back to Gdynia of the ferry is a single realization of its operation process. The conditional sojourn times in particular operation states of each single realization of the ferry operation process are given in separate columns. The operation process is very regular in the sense that the operation state changes are from the particular state z_b, $b = 1,2,...,17,$ to the neighboring state z_{b+1}, $b = 1,2,...,17,$ only and from z_{18} to z_1. Therefore the realizations of the conditional sojourn times θ^j_{bb+1}, $b = 1,2,...,17,$ $j = 1,2,...,42,$ are given in the Tables b-th row and the realizations of the conditional sojourn time $\theta^j_{18\,1}$, $b = 1,2,...,17,$ are given in the Tables 18-th row.

Table A1: Realization of conditional sojourn times in operations states (early spring)

Realization number k	1	2	3	4	5	6	7	8	9	10	11	12
	Realization of conditional sojourn times in operations states (in minutes)											
Operation state z_b	θ^1_{bb+1}	θ^2_{bb+1}	θ^3_{bb+1}	θ^4_{bb+1}	θ^5_{bb+1}	θ^6_{bb+1}	θ^7_{bb+1}	θ^8_{bb+1}	θ^9_{bb+1}	θ^{10}_{bb+1}	θ^{11}_{bb+1}	θ^{12}_{bb+1}
z_1	55	52	47	75	60	60	62	43	50	61	65	63
z_2	4	3	3	2	2	2	2	3	3	4	3	2
z_3	28	31	32	35	37	48	33	38	39	43	40	42
z_4	52	46	48	65	53	47	49	62	45	46	51	47
z_5	598	635	539	572	499	507	621	580	507	511	497	496
z_6	35	42	42	44	35	37	34	40	36	33	38	38
z_7	7	9	8	7	7	5	5	5	5	5	8	7
z_8	25	20	23	27	20	31	15	17	16	21	33	34
z_9	75	59	56	40	66	47	26	60	65	25	55	40
z_{10}	5	3	2	3	2	3	5	6	3	4	4	2
z_{11}	6	5	4	5	4	5	4	4	4	6	4	5
z_{12}	25	22	25	25	23	25	20	33	24	24	22	22
z_{13}	574	427	461	501	498	490	438	561	491	513	496	500
z_{14}	61	43	43	46	49	52	42	63	46	60	50	50
z_{15}	33	32	33	36	35	33	35	34	31	33	34	36
z_{16}	4	4	5	4	4	4	3	4	4	4	4	4
z_{17}	8	10	6	5	5	6	4	5	8	7	6	7
z_{18}	26	26	30	20	16	17	16	22	17	8	17	17
Total θ^k_T	1621	1469	1407	1512	1415	1419	1414	1580	1394	1408	1427	1412

Table A2: Realization of conditional sojourn times in operations states (early spring)

Date/2008	13/15 Mar	15/16 Mar	16/17 Mar	17/18 Mar	18/19 Mar	19/20 Mar	20/21 Mar	21/22 Mar	22/23 Mar	23/24 Mar	08/09 Apr	09/10 Apr
Realization number k	13	14	15	16	17	18	19	20	21	22	23	24
	Realization of conditional sojourn times in operations states (in minutes)											
Operation state z_b	θ_{bb+1}^{13}	θ_{bb+1}^{14}	θ_{bb+1}^{15}	θ_{bb+1}^{16}	θ_{bb+1}^{17}	θ_{bb+1}^{18}	θ_{bb+1}^{19}	θ_{bb+1}^{20}	θ_{bb+1}^{21}	θ_{bb+1}^{22}	θ_{bb+1}^{23}	θ_{bb+1}^{24}
z_1	45	45	40	20	33	50	43	15	45	57	97	68
z_2	2	2	2	2	2	3	2	2	3	2	2	3
z_3	35	36	36	36	37	35	34	34	36	36	39	36
z_4	51	51	51	49	53	44	51	52	50	53	53	54
z_5	595	495	504	507	498	483	497	504	507	503	500	492
z_6	34	39	38	39	38	35	37	36	37	34	38	40
z_7	7	8	7	10	8	8	7	8	8	8	7	9
z_8	18	16	13	3	15	6	9	25	19	31	30	35
z_9	75	77	60	73	82	118	71	55	30	24	34	41
z_{10}	5	2	2	2	3	4	2	2	3	3	2	5
z_{11}	4	4	4	4	4	4	4	4	4	4	4	4
z_{12}	24	24	25	24	23	22	23	22	22	22	26	22
z_{13}	522	491	499	488	464	484	498	496	505	595	483	499
z_{14}	72	50	48	50	48	52	47	53	51	61	61	48
z_{15}	34	35	35	34	35	34	31	32	33	46	34	34
z_{16}	5	5	5	4	4	4	5	5	3	4	6	6
z_{17}	7	7	6	4	4	7	5	5	7	5	4	5
z_{18}	26	40	21	34	40	35	28	22	8	2	12	13
Total θ_T^k	1561	1427	1396	1383	1391	1428	1394	1372	1371	1490	1432	1414

Table A3: Realization of conditional sojourn times in operations states (early spring)

Date/2008	10/12 Apr	12/13 Apr	13/14 Apr	14/15 Apr	15/16 Apr	16/17 Apr	18/19 Apr	19/20 Apr	20/21 Apr	05/06 May	06/07 May	07/08 May
Realization numbr k	25	26	27	28	29	30	31	32	33	34	35	36
	Realization of conditional sojourn times in operations states (in minutes)											
Operation state z_b	θ_{bb+1}^{25}	θ_{bb+1}^{26}	θ_{bb+1}^{27}	θ_{bb+1}^{28}	θ_{bb+1}^{29}	θ_{bb+1}^{30}	θ_{bb+1}^{31}	θ_{bb+1}^{32}	θ_{bb+1}^{33}	θ_{bb+1}^{34}	θ_{bb+1}^{35}	θ_{bb+1}^{36}
z_1	58	35	45	75	72	62	37	44	46	78	59	65
z_2	3	4	3	3	2	3	6	3	2	2	2	2
z_3	37	36	35	39	37	36	37	36	36	37	36	36
z_4	67	51	50	62	49	48	64	51	53	63	55	53
z_5	573	498	506	576	494	505	576	495	502	574	492	497
z_6	36	37	35	38	38	36	35	39	37	36	38	37
z_7	8	7	5	7	10	9	10	6	7	7	6	6
z_8	25	11	17	31	23	25	23	15	18	19	18	24
z_9	55	55	43	45	52	48	50	58	53	30	30	45
z_{10}	3	3	3	3	2	3	2	2	3	3	2	2
z_{11}	4	4	5	5	4	5	4	5	4	5	4	4
z_{12}	23	22	23	26	23	23	24	23	24	23	28	24
z_{13}	573	497	531	500	492	496	590	508	520	502	508	508
z_{14}	58	51	54	47	40	51	47	47	56	47	46	42
z_{15}	34	35	33	35	35	34	33	34	35	36	35	35
z_{16}	5	5	6	5	4	6	5	5	4	4	5	4
z_{17}	4	5	5	5	7	6	5	6	6	10	5	4
z_{18}	18	20	11	10	16	18	25	18	12	12	17	14
Total θ_T^k	1584	1376	1410	1512	1400	1414	1573	1395	1418	1488	1386	1402

Table A4: Realization of conditional sojourn times in operations states (early spring)

Date/2008	08/09 May	10/11 May	11/12 May	12/13 May	13/14 May	14/15 May						
Realization number k	37	38	39	40	41	42						
	Realization of conditional sojourn times in operations states (in minutes)											
Operation state z_b	θ_{bb+1}^{37}	θ_{bb+1}^{38}	θ_{bb+1}^{39}	θ_{bb+1}^{40}	θ_{bb+1}^{41}	θ_{bb+1}^{42}						
z_1	53	25	55	84	71	67						
z_2	2	2	3	2	2	2						
z_3	38	37	40	36	37	34						
z_4	60	49	46	57	53	51						
z_5	584	504	505	573	494	495						
z_6	38	35	36	39	36	36						
z_7	5	7	5	5	6	6						
z_8	15	6	40	28	32	28						
z_9	70	35	35	47	40	50						
z_{10}	2	2	3	3	3	2						
z_{11}	5	4	5	5	4	4						
z_{12}	25	25	24	23	26	24						
z_{13}	595	506	535	506	503	503						
z_{14}	42	45	47	46	51	43						
z_{15}	34	35	34	34	33	33						
z_{16}	6	4	4	5	5	4						
z_{17}	5	3	4	5	3	5						
z_{18}	20	11	11	10	13	18						
Total θ_T^k	1599	1335	1432	1508	1412	1405						

CONTRIBUTION TO RELIABILITY ANALYSIS OF HIGHLY RELIABLE ITEMS

D. Valis, Z. Vintr
•
University of Defence, Brno, Czech Republic

e-mail: david.valis@unob.cz, zdenek.vintr@unob.cz

M. Koucky
•
Technical University of Liberec, Liberec, Czech Republic

e-mail: miroslav.koucky@tul.cz

ABSTRACT

In recent years the intensive efforts in developing and producing electronic devices have more and more critical inference in many areas of human activity. Engineering is one of the areas which have been also importantly affected. The paper deals with dependability namely reliability analysis procedure of a highly reliable item. The data on manufacturing and operating of a few hundred thousands pieces of electronic item are available and they are statistically a very important collection/set. However, concerning some items the manufacturing procedure was not checked and controlled accurately. The procedure described in the paper is based on the thorough data analysis aiming at the operating and manufacturing of these electronic elements. The results indicate some behaviour differences between correctly and incorrectly made elements. It was proved by the analysis that dependability and safety of these elements was affected to a certain degree. Although there is a quite big set of data the issue regarding the statistical comparability is very important.

1 INTRODUCTION

The application of electronic elements introduces a number of advantages as well as disadvantages. Let us start with operating process itself – the operating is more ecological, smoother and cheaper. Also the area of safety, both passive and active, is optimised. On the other hand the complexity of a system is getting higher as well as its sensitivity to previously not perceived factors. The electronic elements are also applied into so called service and comfort systems. However new the technology would be, all the elements are subject to certain factors set by a design, manufacturing, operating and environment in which they are used. Besides performance and utility properties we are supposed to follow dependability as well. Regarding electronic elements they are highly reliable and in terms of dependability measures they are at the highest level. If the elements are well manufactured and their construction and software equipment meets the required dependability level, we are usually satisfied and there is no reason to act otherwise. If occasional fluctuations in the dependability level do not limit the function or safety of a system or its operating, the problem of unreliability of electronic elements in systems is not so serious. The real problem is not meeting the requirements and errors.

In the paper we are going to address reliability assessment of a highly reliable electronic item. In this paper the evaluated application is perceived as an item produced for systems´ specific use/utilization. Item is implemented in a system in order to control one of the step functions of the system. The manufacturer has had long term experience of item manufacturing. This item is also widely introduced into the market where it successfully meets the parameters within technical applications. The introduced item has been applied in the systems´ environment many times and no major problems have been detected regarding its function.

As we know from previous publications the item is initialised by start power. Unfortunately non-intentional causes resulted in non-compliance with the manufacturing process during development and manufacturing a new item. While manufacturing the item a relatively minor shortening of program protocol took place, thereby shortening the initialisation time. This situation resulted in the production of many tens of thousands of incorrectly manufactured items where the initialisation time was shortened by the program. The non-compliance with the manufacturing process was detected only by accident and that was after some time. However, most of the items manufactured this way have been mounted in systems and they have been in operation.

The non-compliance with the manufacturing process itself, thereby shortening the programming time might not be a serious problem. More related circumstances might be the real problem. The first one is the fact that the items have been mounted in systems and they have been in operation. Another quite serious problem is the fact that a item function failure can result in failure occurrence on the device which is supposed to perform a system's step function. If a system step function is just being used, its interruption-failure might lead to a critical accident with serious consequences. In case this type failure occurs, it affects significantly system's dependability. Moreover, it breaks the confidence in the step function which leads to the lack of confidence in a system as a whole.

Resulting from the arguments mentioned above the producer decided to solve the problem immediately. The producer wanted to find out if the errors occurring when manufacturing items have a possible effect upon operational dependability – reliability. Basically a few solutions could have been taken into account at that moment. Finally two of the solutions were chosen to be accomplished.

One of the options is to carry out a one-side interval calculation of a item reliability measure at a required confidence level. This intention is easy to be fulfilled since the data on the item operation were carefully and systematically collected. The aim of the paper is to describe an estimation procedure of a reliability measure and assess the validation of the statistical hypothesis testing based on the available data.

Suggesting and carrying out an accelerated reliability test of item is another option. However, this method is not included in this paper and represents a separate methodology. All terms mentioned here are in accordance with the (IEC 60050/191).

2 FIELD DATA ASSESSMENT PROCEDURE

The procedure follows widely known and basic approaches and terminology (IEC 60050/191). The producer provided data on the item operation over a complete period. Regarding the nature of the analysis the following facts were agreed on:

1) The aim of the analysis was to calculate the one-side item reliability interval. The item "programmed incorrectly" was assessed first, and the item "programmed correctly" was assessed as the second. The calculation of a reliability one-side interval determined for each set separately was the outcome of the analysis.
2) The next step was to compare both items sets and decide whether the „incorrect programming" can/cannot affect the item's reliability. A one-side interval was determined at a required confidence level and it specifies a minimal reliability level of a item set obtained by a calculation.
3) The operation time of the item started the moment a production range was produced plus two weeks (the assumption that it will be delivered to the customer, mounting into the system, and physical start of the operation).
4) The real operation time equivalent was determined by recommending the standards and is based on a calendar time (GS 95003-1, GS 95003, GS 95003-4). The real operation time is believed to start at the moment as stated in point 3). The transforming coefficient value following the sources/standards mentioned above is: dormant time versus operation time ≈ 24,836 : 1.

5) The standard IEC 60605-4 “Equipment reliability testing - Part 4: Statistical procedures for exponential distribution - Point estimates, confidence intervals, prediction intervals and tolerance intervals” has been used for calculating the reliability measure one-side interval at a required confidence level.
6) The reliability confidence interval was set according to common roles. One of the very accurate levels which were decided to be used is 95%. This level was used for following calculations.
7) End of observation, censoring by time is given by the date of 31st December 2008. This was negotiated with the item producer.
8) The hour [h] is a reliability measure unit.

Since the standard IEC 60605-4 deals with a few possible types of the assessed sets, it is necessary to determine what type it is referred to. The operation profile and the agreement that the analysis assessment will be finished on a certain day indicate that this is a case of a specific field test finished by time without replacing the item. This assumption resulted in the following solution taking into account the standard mentioned above (IEC 60605-4).
Following the standard (IEC 60605-4) recommendation a lower limit of mean time to failure at the required confidence level was calculated. In order to estimate one-side interval of a lower level of mean time to failure we used the following equation (see also Holub 1992, Lipson & Sheth 1973, Neson 1982, Kapur & Lambertson 1977):

$$m_{l\,F/C} = \frac{2.T^{*F/C}}{\chi^2_{\alpha,\nu}} \qquad (1)$$

where $m_{l\,F/C}$ - is a lower limit of mean time to failure of either „*F*“ – „incorrectly“ programmed sets or „*C*“ – „correctly“ programmed sets; $T^{*F/C}$ - is accumulated operation time of all items sets (either „*F*“ – „incorrectly“ programmed or „*C*“ – „correctly“ programmed) observed in the operation during an evaluation period. It is calculated using the equation $T^{*F/C} = \sum_{t=i}^{n} t_i^{F/C}$ („t_i“ = accumulated real operation time of all items of *i*-th production range of either „*F*“ – „incorrectly“ programmed sets or „*C*“ – „correctly“ programmed sets, where the *n* is number of the production ranges. The interval is the period in which they are put into operation which lasts up to the day when the temporary observation is finished; $\chi^2_{\alpha,\nu}$ -chi square for a given number of degrees of freedom ν; „α“ – confidence level agreed on 95%.

Since it is a one side censored set (it is censored by the agreed date when the observation is to be finished; this date is the last possible day when the operation record is to be made), the number of degrees of freedom ν to determine chi square is going to be calculated using the standard recommendation**Ошибка! Источник ссылки не найден.** following the formula:

$$\nu = 2r^{F/C} + 1 \qquad (2)$$

where: *r* is a number of events (failures) in a given group of sets.

Based on the assumptions and the calculation which have been made before, the reliability measure values for correctly and incorrectly programmed items were found. These values were calculated at the required confidence level. By comparing these values we were able to determine whether the error affects the item reliability during a manufacturing process.
However, concerning the field data we face a theoretical problem. The data set is apparently different concerning a digit place in terms of the operation time of the item sets. It means that correctly manufactured items obviously operate for a shorter time than the ones manufactured

incorrectly. This situation can affect a calculation procedure as well as a comparison of the results. Taking into account this situation it is necessary to test the field data using the statistical test which is supposed to prove their comparability. The procedures proving the statistical equivalence of the evaluated sets is part of another contribution. The objective of the statistical analyses is to compare two sets of data both of which have non-similar size.

2.1 Example of the application of above mentioned procedure

Here will be presented restricted part of the above mentioned procedure. The procedure given in this example is the same as used in the whole analysis. The difference is that no information about portion of data or other relevant indicators will be provided.

Data were provided in following form:

Number of production range:	1.
Number of items produced in this range:	4 200
Date of production:	16.1. 2006
Number of failed items in this range:	1
Date of failure:	12.10. 2006

Ad section 2, point 3), 4), 7), 8)

Number of days in operation:	43 days
Number of hours in operation for 4 199 items:	1030 h
For 1 item:	238 h
Total hours in operation for all items from this range:	4 325 710 h

Following the calculation (1) and (2) it is:

$\nu = 2r^{F/C} + 1 = 2.1 + 1 = 3$

and

$$m_l = \frac{2.T^*}{\chi^2_{\alpha,\nu}} = \frac{2\,.\,4\,325\,710}{7{,}8} \cong 1\,109\,156 \cong 1.10^6\,h$$

It means that lover limit of one side confidence interval for MTTF of the item is approximately $1.10^6\,h$.

The assessment of the other sets which represent the electronic items made (both correctly and incorrectly manufactured) is carried out in the same way. Finally the decision about the failure rate comparability is performed. From the reason of keeping the industrial confidence of the data and their assessment we can not present full range of the calculations made. We can only present that the difference between correctly and incorrectly manufactured items is noticeable.

3 RISK ANALYSIS RESULTING FROM THE FAILURE OCCURRENCE

In this phase of observing the object we are talking about partially predictive risk assessment. We could choose fully theoretical way of assessment usually made during design. However we do have the field data are available so we may also use the process approach. Following one of the approaches we would focus on individual risk contributors which would be thoroughly examined. The classic probability methods might be used for determining the event occurrence probability. The expert assessment based on the defined scales would be used for analysing the consequences. Next issue which might be used is recommendation of the standards dealing with such kind of items. One very suitable method is mentioned in the standard SAE J 1739:2002.

Usually we do not count on other factors when dealing with theoretical risk analysis. However, some special characteristics still exist and that is the reason why one of the possible approaches where another factor occurs is described below. However, further verification and validation of the obtained result will pose a problem while assessing the risk theoretically. In our case, when undesired event occurrence probability might be recorded when observing the field data, the result will be more realistic and consequent verification of the result will be also possible. Such event occurrence information is not a prediction then, but it is estimation based on the real information. Consequence decisions resulting from the occurred event might be regarded as a prediction in this case. Consequences description options are stated below.
Using either fully standardised approach, namely industrial standards or software support can be another option when analysing the risk. An event occurrence rate or its criticality may be obtained using well known dependability analysis methods, e.g. FMECA, PHA or OSHA. The total risk is usually based on these two contributors we often work with in industry practice. Concerning software support when analysing the risk it is possible to use widely available tools, e.g. Risk Spectrum based on the FTA method supported by the ETA method, or the tools by Relia Soft or Item Software – Item QRAS which uses both methods individually but basically leads to the same result.
Using so called soft methods when analysing the risk and dependability is another possibility. It is namely about non-stochastic methods which are based mostly on the deterministic approach and iteration principles. Also the probability plays an important role but most approaches of these methods are based just on empiricism and practice. The methods would be used namely for analysing the event consequences and also the event occurrence but on a limited scale. The determination is not often unambiguous and also it is not easy to decide what defined scale the consequences belong to. We would highly recommend fuzzy logic which allows us to work very well with qualitative characteristics of some events, and which is able to quantify them. If we were to define individual process states in system operation and they would represent the periods in which the system is run, we would be able to determine to what extent the event belongs to a defined state while an event occurs. That is how we would cover the failure criticality level regarding the defined states set and the time vector in which a system might occur during its operation/technical life. Unfortunately, in this paper there is no space for presentation and development of this approach.
Generally speaking we can use standardized criteria by which every failure is evaluated following the previously defined scales. Using the point estimations the Risk Priority Number is added to each failure mode. The RPN is then used for downward arrangement of the assessed failures. The failures with a risk number going above the defined scale undergo the corrective actions which are supposed to reduce the risk number sufficiently.

3.1 Evaluated factors

The existing model described in standards (e.g. IEC 60812:2006 and SAE J 1739) considers two evaluated factors, Probability – P and Severity – S, or three evaluated factors, Probability, Detection and Failure Consequences. These factors result from a fully quantitative assessment where the risk is expressed by a conjunction of probability and consequences

$$R = P * S \tag{3}$$

The Detection Factor – D in a full quantitative assessment would decrease the probability that a failure will not be detected during design/manufacturing process (see e.g. 0), thus

$$R = P * D * S \tag{4}$$

whereas its value would belong to the interval <0;1> (or <0;100%>).

As we deal with an electronic item which might be installed inside systems, the SAE standard is very suitable to be applied.

3.2 Scales for assessment

In the standards (e.g. IEC 60812:2006 or SAE J 1739) for example there are scales for assessment for all three criteria which are used in industry. The scales are put in the form of tables with verbal explanation of every level at the scale. These are severity, occurrence probability and detection scales. Sometimes a consequence scale in relation either to the customer or manufacturing process or operation is completed. These scales are going to be used in the next procedure. Other existing and used scales are for example those which are applied in a part of software, Item Toolkit or Reliasoft XFMEA.

3.3 Risk Priority Number *RPN*

The Risk Priority Number is a crucial criterion for detecting weak points in a system, and corrective actions which decrease the risk resulting from the device failure are convenient to be applied to these weak points. The magnitude of the Risk Priority Number *RPN* is given by conjunction of point estimations of probability, detection and consequences. Since the Risk Priority Number is given by conjunction of point estimations, it is a case of a dimensionless quantity as in equation (4). $RPN = P * D * S$

The values interval depends on the selection of assessment scales. Concerning the scales put in the (see IEC 60605-4 or MIL-STD-1629a) the range of the Risk Priority Number is 1 up to 1000 (=10^3) (EN 60812:2006, MIL-STD-1629a, SAE J 1739). The application of corrective actions involves all the events of the Risk Priority Number value exceeding 125. In our case we can talk namely about:

The effort to minimize an event occurrence - this was achieved especially by detecting the manufacturing disagreement and correcting it. This act should provide reliable item operation at a higher level;

The effort to minimize consequences severity of a failure which might occur – this is provided by using standard security measures which are not expected to modify;

The effort to improve detection of a possible failure – this is provided by a sufficient quality manufacturing.

The consequence of an event occurrence is in the range “9” to “10” according to the standard. The frequency according to the same standard is “low” and detection is “moderate” at maximum using the same source. Therefore we need to carry out some design change to improve the item´s *RPN*. From this point of view is the service of such item very dangerous and may cause inadvertent situation with very sad consequences.

Example of the assessment:

Using the approaches above and the recommendations in the standards we may get following values for the *RPN* calculation.

The occurrence might be “2” at minimum according to the rating.

The severity might be “9” at minimum according to the rating.

The detection ability might be “5” at minimum according to the rating.

Therefore, the calculation of the *RPN* is:

$RPN = P * D * S = 2 * 9 * 5 = 90$

This is the lowest level of the *RPN* which might be got.

As said before, as we see that one of the values is “9”, we have to apply countermeasures.

3.4 Criticality matrix

In some applications where no detection is assessed apart from failure probability and its consequences it is possible to use a so-called criticality matrix (sometimes it is designated as a risk matrix). The measures used in the matrix correspond with those ones which have been discussed above.

Contrary to an exact value calculation as it takes place when assessing by the *RPN*, an event positioning in the matrix is a crucial one. The example of criticality matrix which could be used for risk assessment is taken from the standard (EN 60812:2006) and is put in Table 1.

To place a failure mode into a certain matrix field, the scales categories for consequence assessment are to be defined (in Table it is put as Severity Levels and Occurrence Frequency of Failure Effect). The weak point of such scales is the fact that they can be different considering more application fields, and they are defined mostly by an analyst/decision maker. The following scale used for assessing the probability put in the standard serves as an example.

Criticality number 1 or E, Improbable, probability of occurrence: $0 \le P_i < 0{,}001$;

Criticality number 2 or D, Remote, probability of occurrence: $0{,}001 \le P_i < 0{,}01$;

Criticality number 3 or C, Occasional, probability of occurrence: $0{,}01 \le P_i < 0{,}1$;

Criticality number 4 or B, Probable, probability of occurrence: $0{,}1 \le P_i < 0{,}2$;

Criticality number 5 or A, Frequent, probability of occurrence: $P_i \ge 0{,}2$.

Regarding the criteria described above we can talk about the following intervals distribution of *RPN* components:

The Severity Component could range over the values 5 – 10;

The Probability Occurrence Component could range over the values 1 – 2;

The Detection Component could range over the values 1 – 3.

Adequate corrective measures for decreasing all the values of the obtained *RPN* components were taken.

Table 1. The example of a risk criticality matrix

Frequency of occurrence of failure effect	Severity levels			
	1 Insignificant	2 Marginal	3 Critical	4 Catastrophic
5. Frequent	Undesirable	Intolerable	Intolerable	Intolerable
4. Probable	Tolerable	Undesirable	Intolerable	Intolerable
3. Occasional	Tolerable	Undesirable	Undesirable	Intolerable
2. Remote	Negligible	Tolerable	Undesirable	Undesirable
1. Improbable	Negligible	Negligible	Tolerable	Tolerable

4 CONCLUCION

The procedure as described above was used to calculate reliability of the single sets which served as correctly and incorrectly programmed items. Following the obtained results a possible effect of a manufacturing error upon the items reliability was estimated. Following the results it is obvious that manufacturing error could affect items reliability in some way. Both sets are from the statistical point of view slightly different, which is an essential piece of information. This fact should be referred to when carrying out statistical data evaluation using the introduced tools.

5 ACKNOWLEDGEMENTS

This paper was supported by the GA Czech Republic project number 101/08/P020 „Contribution to Risk Analysis of Technical Sets and Equipment", and by the Ministry of

Education, Czech Republic project number 1M06047 „The Centre for Production Quality and Dependability“.

REFERENCES

BMW Group Standard; GS 95003-1 *Electrical/Electronic Assemblies in Motor Vehicles – General Information*.
BMW Group Standard GS 95003 *(Supplement 1) Electrical/Electronic Assemblies in Motor Vehicles – Tests*.
BMW Group Standard GS 95003-4 *Electrical/Electronic Assemblies in Motor Vehicles - Climatic Requirements*.
IEC 600 50 (191) (IEV) 1990. *Dependability and quality of services*.
IEC 60605-4 2004. *Equipment reliability testing - Part 4: Statistical procedures for exponential distribution - Point estimates, confidence intervals, prediction intervals and tolerance intervals*.
EN 60812 2006. *Analysis techniques for system reliability - Procedure for failure mode and effects analysis (FMEA)*.
MIL-STD-1629a 1998. *Procedures for performing a failure mode, effects and criticality analysis*.
SAE J 1739 2006. *Potential Failure Mode and Effects Analysis in Design, Manufacturing and Assembly and for Machinery (Design FMEA, Process FMEA and Machinery FMEA)*.
Holub, R. 1992. *Dependability tests (stochastic methods)*. Brno: Military Academy, 1992.
Lipson, CH., SHETH, N.J. 1973. *Statistical Design and Analysis of Engineering Experiments*; New York: Mc Graw Hill.
Neson, V. 1982. *Applied Life Date Analysis*, New York: John Wiley and Sons.
Kapur, K.C.; Lamberson, L.R. 1977. *Reliability in Engineering Design*; New York: John Wiley & Sons.

ISSN 1932-2321