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This paper considers a point process model with a monotonically decreasing or increasing ROCOF and the underlying distributions from the location-scale family. In terms of repairable system reliability analysis, the process is capable of modeling various restoration types including "better–than–new", i.e., the one not covered by the popular G-Renewal model (Kijima & Sumita, 1986). The characteristic property of the proposed process is that the times between successive events are obtained from the underlying distributions as the scale parameter of each is monotonically decreasing or increasing. This is similar to the scale parameter transformation in the Accelerated Life Model (Cox & Oakes, 1984). The paper discusses properties and statistical estimation of the proposed model for the case of the Exponential and Weibull underlying distributions.

The paper presents a new heuristic algorithm for determining optimal redundancy allocation of complex networks. The present algorithm is an iterative method; all the paths of the network are first arranged in decreasing order of their priority determined using a path sensitivity factor, the highest priority path is optimized first by adding redundant components for subsystems of the path iteratively based on proposed subsystem selection factor. In case of availability of any residual resources next lower priority paths are considered for redundancy allocation. The proposed algorithm not only demonstrates improved performance in comparison with most of the existing heuristic algorithms but also leaves minimum slack of components without any further possibility of redundancy.

In this paper we consider some algorithmic problems which appear in a calculation of a ruin probability in discrete time risk models with an interest force which creates stationary and reversible Markov chain. These problems are connected as with a generation of the Markov chain by its stationary distribution so with a calculation of the ruin probability.

It is displayed, that the specific number and average duration of joint estates of power-generating units of the state district power stations calculated analytically on indexes of reliability of power-generating units is essential exceed direct experimental data. Principal causes of a divergence are suppositions about equal reliability and undercount of run-down states of power-generating units. The pointed divergence leads to magnifying of an emergency standby of power.

In this article discrete time risk model with heavy tailed losses distribution and dependence between financial and insurance risks is considered. It is shown that known asymptotic formulas work with good accuracy for sufficiently large arguments. Direct methods based on calculation of ruin probability by solution of appropriate integral equations demand large volumes of calculations and so work for sufficiently small arguments. Fast and accuracy algorithms, based on approximation of loss distribution by mixture of exponential ones, to calculate ruin probability in this interval are developed. This approximation of considered model is based on continuity theorems and analog of Bernstein theorem in L_1 metrics.

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G1-RENEWAL PROCESS AS REPAIRABLE SYSTEM MODEL

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ABSTRACT

This paper considers a point process model with a monotonically decreasing or increasing ROCOF and the underlying distributions from the location-scale family. In terms of repairable system reliability analysis, the process is capable of modeling various restoration types including “better–than–new”, i.e., the one not covered by the popular G-Renewal model (Kijima & Sumita, 1986). The characteristic property of the proposed process is that the times between successive events are obtained from the underlying distributions as the scale parameter of each is monotonically decreasing or increasing. This is similar to the scale parameter transformation in the Accelerated Life Model (Cox & Oakes, 1984). The paper discusses properties and statistical estimation of the proposed model for the case of the Exponential and Weibull underlying distributions.

1. INTRODUCTION

In repairable system reliability analysis, if upon a failure, a system is restored to as "good-as-new" condition and the time between failures can be treated as an independent and identically distributed (IID) random variable, then the failure occurrence can be modeled by the *Ordinary Renewal Process* (ORP).

If upon a failure the system is restored to the "same-as-old" condition, then the appropriate model to describe the failure occurrence is the *Non-Homogeneous Poisson Process* (NHPP). The time between consecutive failures, in this case, is not an IID random variable. In a sense, the NHPP can be viewed as a renewal process with the "same-as-old" repair assumption. An important particular case of both ORP and NHPP is the *Homogeneous Poisson Process* (HPP), whose underlying failure times are distributed exponentially.

It is clear that even though attractive mathematically, the "good-as-new" and "same-as-old" repair assumptions are often exceptions rather than the rule, from the standpoint of practical reliability engineering. Generally, they could be treated as the "limiting" conditions to which a system could be restored. In reality, after the repair, the system is likely to find itself *between* the two conditions. Of great interest, therefore, is modeling other repair assumptions such as the intermediate "better-than-old-but-worse-than-new".

An early approach to cover more than one repair assumption within the same probabilistic model is discussed in (Brown and Proschan, 1982). This method assumes that upon a failure, a repair action restores the system to the "good-as-new" condition with probability of $p(t)$, or the "same-as-old" condition with probability of $1-p(t)$, where t is the age of the system at failure.

A more general model is the so-called *G-Renewal* process (Kijima M and Sumita, 1986), which treats ORP and NHPP as special cases. The GRP is introduced using the notion of *virtual age*:

$$A_n = qS_n ,$$

where A_n and S_n is the system's *virtual age* before and after the n-th repair, respectively, and q is the *restoration* (or *repair effectiveness*) *factor*.

It is clear that for $q = 0$, the age of the system after the repair is "re-set" to zero, which corresponds to the "good-as new" repair assumption and represents the ORP. With $q = 1$, the system is restored to the "same-as-old" condition, which is the case of the NHPP. The case of $0 < q < 1$ corresponds to the intermediate "better-than-old-but-worse-than-new" repair assumption. Finally, with $q > 1$, the virtual age is $A_n > S_n$, so that the repair damages the system to a higher degree than it was just before the respective failure, which corresponds to the "worse-than-old" repair assumption.

One limitation of the GRP model is its inability to model a "better than new" restoration, for which the need arises in some practical applications, e.g. *reliability growth modeling* (Crow, 1982). The proposed below G1-Renewal process overcomes this particular drawback.

The so-called *piecewise exponential model* (Sen and Bhattarcharyya, 1993; Sen, 1998) is the closest, in spirit, to the model proposed in this paper. In the framework of the piecewise exponential model, it is assumed that the times between successive failures $X_1, X_2, \ldots X_i, \ldots X_n$ are independent exponentially distributed random variables with the scale parameters $\alpha_i = \frac{\delta}{\mu} i^{\delta-1}$, $i = 1, 2, \ldots n$.

If $\delta = 1$, the model coincides with the HPP. If $\delta >$ (<) 1, the model reveals reliability improvement (deterioration).

2. G1-RENEWAL PROCESS: PROBABILISTIC MODEL

The location-scale family of underlying distributions is considered. After each i-th failure ($i = 1, 2, \ldots$), the system is restored (damaged) in such a way that its scale parameter α is changed to $\alpha(1+q)^{i-1}$, where q is the restoration (damage) parameter, $-1 < q < \infty$, so that for the time to the first failure $i = 1$, for the time between first and second failure $i = 2$, and so on. This transformation of the scale parameter is similar to the one used in the well-known accelerated life time model (Cox & Oaks, 1984; Nelson, 1990). To an extent, the suggested model makes more physical (reliability) sense than the respective NHPP model in terms of restoration assumption (i.e., "same-as-old" assumption). If $q = 0$, the process coincides with the ordinary Renewal process. If $q > 0$, the introduced process is obviously an improving one, and if $q < 0$, the process is aging (deteriorating). Table 1 shows multiplier $(1+q)^{i-1}$ to the scale parameter of the underlying distributions of the times between consecutive events for some values of q.

Table 1. *Multiplier* $(1+q)^{i-1}$ *to the scale parameter of the underlying distributions of the times between successive events for some values of q.*

Event, *i*	*q = 0.1*	*q = -0.1*	*q =0.2*	*q = -0.2*
1	1.000	1.000	1.000	1.000
2	1.100	0.900	1.200	0.800
3	1.210	0.810	1.440	0.640
4	1.331	0.729	1.728	0.512
5	1.464	0.656	2.074	0.410
6	1.611	0.590	2.488	0.328
7	1.772	0.531	2.986	0.262
8	1.949	0.478	3.583	0.210
9	2.144	0.430	4.300	0.168
10	2.358	0.387	5.160	0.134

We suggest calling the above introduced point process as the *G1-Renewal Process* due to a certain similarity to the G-Renewal Process introduced by Kijima and Sumita (1986). Again, by analogy with G-Renewal Equation, the equation for the cumulative intensity function (CIF) of the G1-Renewal Process will be correspondingly called the *G1-Renewal equation*.

2.1 G1-Renewal Equation

The location-scale distribution for a continuous random variable (r. v.) t is defined as having the cumulative distribution function (CDF) in the following form:

$$F(t) = F\left(\frac{t-u}{\alpha}\right) \tag{1}$$

The respective probability density function is

$$f(t) = \frac{1}{\alpha}\left(\frac{t-u}{\alpha}\right) \tag{2}$$

The time to the nth failure T_n is given by

$$T_n = X_1 + X_2 + \ldots + \ldots X_n = \sum_{i=1}^{n} X_i \tag{3}$$

where X_i ($i = 1, 2, \ldots, n$) are independent r.v., which, in the framework of the G1-Renewal Process, are distributed according to the following cumulative distribution function (CDF)

$$F_i(X_i) = F\left(\frac{X_i - u}{\alpha(1+q)^{i-1}}\right),\ i = 1, 2, \ldots, n \tag{4}$$

The distribution of the time to the nth failure T_n is difficult to find as a closed-form expression, even in the case of the ordinary renewal process, i.e. when $q = 0$ (except for the exponential and Gamma distribution among the popular lifetime distributions). Note that in the process considered, contrary to the ordinary renewal process, the X_i 's are not identically distributed.

The equation for the *cumulative intensity function* (CIF), also known as the *g-renewal equation* of the process can be found as

$$\mathrm{W(t)} = \sum_{k=1}^{\infty} F^{(k)}(t) \tag{5}$$

where $F^{(k)}(t)$ is k-fold convolution of the cumulative distribution functions (4). Note that $F^{(k)}(t) = Pr(T_k < t)$. The respective *rate of occurrence of failures* (ROCOF) can be found using its definition as

$$\mathrm{w(t)} = \frac{\mathrm{dW(t)}}{\mathrm{dt}} = \sum_{k=1}^{\infty} f^{(k)}(t) \tag{6}$$

2.2 G1-Renewal Process with Exponential Underlying Distribution

The process with exponential underlying distribution is considered. The time to the first failure has the exponential distribution with PDF

$$f_1(t) = \frac{1}{\alpha}\exp\left(-\frac{t}{\alpha}\right) \tag{7}$$

According to (4), the time between the first and the second failures has the following PDF

$$f_2(t) = \frac{1}{\alpha(1+q)}\exp\left(-\frac{t}{\alpha(1+q)}\right), \tag{8}$$

Correspondingly, the time between the (i-1)th and the i-th failures has the following PDF

$$f_i(t) = \frac{1}{\alpha(1+q)^{i-1}} exp\left(-\frac{t}{\alpha(1+q)^{i-1}}\right) \tag{9}$$

The convolution of $f_1(t)$ and $f_2(t)$, i.e., $f_1(t)*f_2(t)$ can be found as

$$\begin{aligned} f_1(t)*f_2(t) &\equiv f^{(2)}(t) = \int_0^t f_1(t-x)f_2(x)dx = \int_0^t f_2(t-x)f_1(x)dx \\ &= \frac{1}{\alpha^2(1+q)}\int_0^t exp\left(-\frac{t-x}{\alpha}\right)exp\left(-\frac{x}{\alpha(1+q)}\right)dx \\ &= \frac{1}{\alpha^2(1+q)} exp\left(-\frac{t}{a}\right)\int_0^t exp\left(\frac{xq}{\alpha(1+q)}\right)dx \\ &= \frac{1}{aq} exp\left(-\frac{t}{a}\right)\left[exp\left(\frac{tq}{a(1+q)}\right)-1\right] \end{aligned} \tag{10}$$

It can be shown that the Laplace transform of the PDF of time to the i^{th} failure $f_i(t)$ is given by

$$f_i^*(s) = \frac{1}{sa(1+q)^{i-1}+1} \tag{11}$$

Based on (11), the Laplace transform of the convolution $f^{(k)}(t)$ can be found as

$$f^{*(k)}(s) = \prod_{i=1}^{k} \frac{1}{sa(1+q)^{i-1}+1} \tag{12}$$

The inverse of (12) is not available in a closed form, which is why we default to obtaining the CIF for G1-Renewal Process via Monte Carlo (MC) simulation – similar to the solution of the G-Renewal Equation we suggested in (Kaminskiy & Krivtsov, 1998).

Figures 1 and 2 show the CIF's of the G1-Renewal Process with underlying exponential distribution. It is interesting to note that in the context of the G1-Renewal, the underlying exponential distribution provides a high flexibility in modeling both *improving* and *deteriorating* processes – contrary to the HPP.

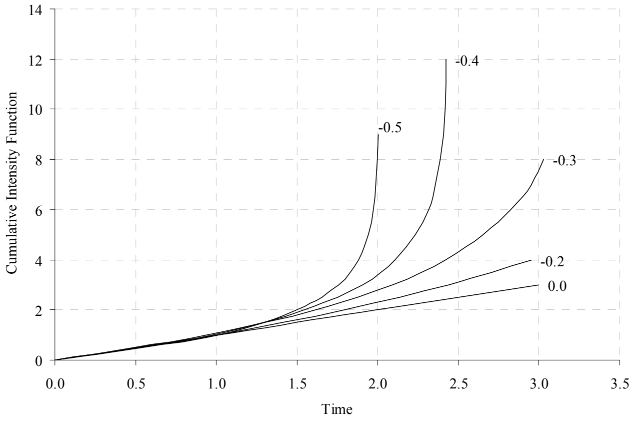

Figure 1. CIF of the G1-Renewal Process with underlying exponential distribution, scale parameter of 1 and various negative values of q.

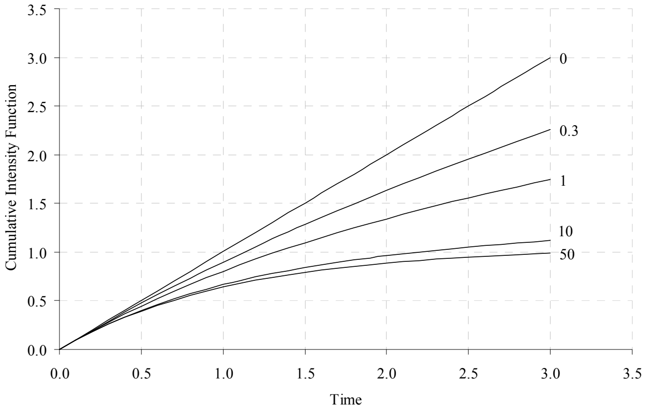

Figure 2. CIF of the G1-Renewal Process with underlying exponential distribution, scale parameter of 1 and various positive values of q.

2.3 G1-Renewal Process with Weibull Underlying Distribution

Figures 3 and 4 show the CIF's of the G1-Renewal Process with the positive restoration parameter and the underlying Weibull distribution with the scale parameter of 1 and the increasing and decreasing hazard functions, respectively.

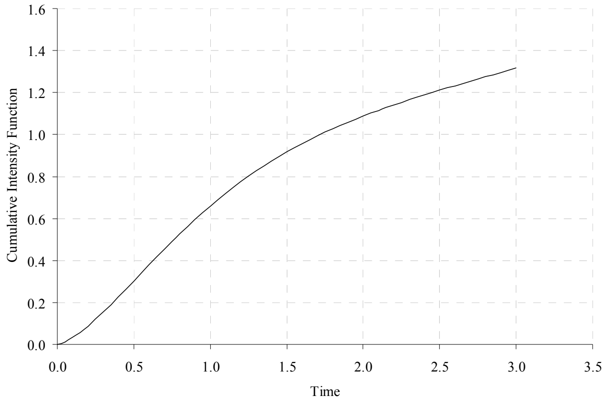

Figure 3. CIF of the G1-Renewal Process with underlying Weibull distribution, scale parameter of 1, shape parameter of 1.5 and restoration parameter of 3.

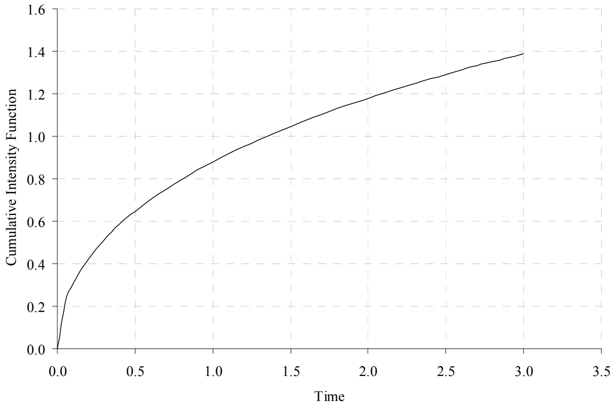

Figure 4. CIF of the G1-Renewal Process with underlying Weibull distribution, scale parameter of 1, shape parameter of 0.5 and restoration parameter of 3.

The concavity of the CIF for $t < \sim 0.7$ in Figure 3 might be related to the increasing hazard function of the underlying distribution. The subsequent convexity of the CIF for $t > 0.7$ might be explained by the positive restoration parameter, which corresponds to the improving G1R process. The overall convexity of the CIF in Figure 4 might be explained by the decreasing hazard function of the underlying distribution and the positive restoration parameter, which corresponds to the improving G1R process.

The concavity of the CIF in Figure 5 might be explained by the increasing hazard function of the time-to-first-failure distribution and a negative restoration parameter, which corresponds to the deteriorating G1R process. The relative "linearity" of the CIF in Figure 6 might be explained by the decreasing hazard function of the underlying distribution, which is partially "compensated" by the negative restoration parameter of the G1R process.

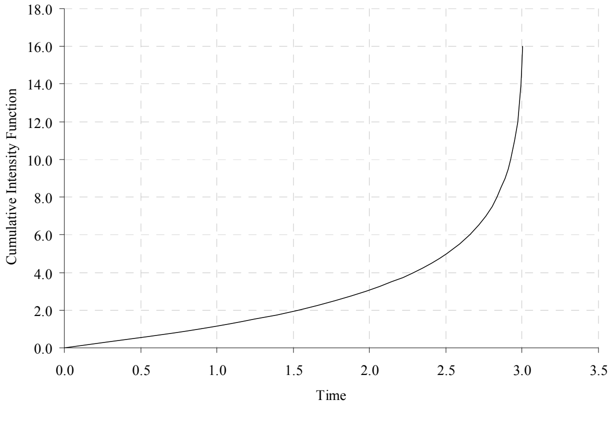

Figure 5. CIF of the G1-Renewal Process with underlying Weibull distribution, scale parameter of 1, shape parameter of 1.5 and restoration parameter of -0.3.

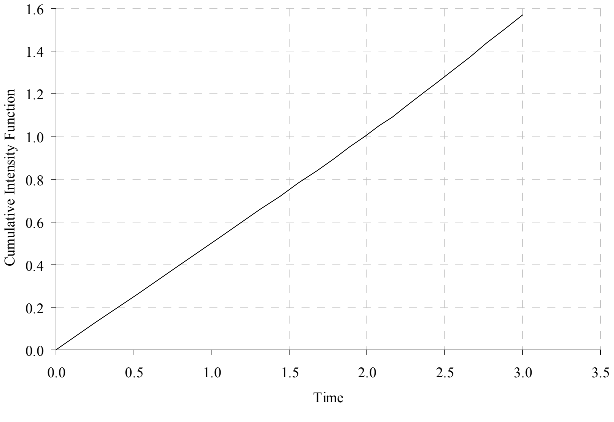

Figure 6. CIF of the G1-Renewal Process with underlying Weibull distribution, scale parameter of 1, shape parameter of 0.5 and restoration parameter of -0.3.

3. G1-RENEWAL PROCESS: MAXIMUM LIKELIHOOD ESTIMATION

3.1 Data

Let t_1 be time to the first failure, t_2 be the time between the first failure and the second failure, so that t_n is the time between the $(n$-1)th failure and the last nth failure. The test (observation) is terminated at the time $t = t_n$.

3.2 G1-Renewal Equation with Exponential Underlying Distribution

For the underlying distribution (7) the likelihood function can be written as follows:

$$L(a,q)=\left[\frac{1}{a}exp\left(\frac{-t_1}{a}\right)\right]\left[\frac{1}{a(1+q)}exp\left(\frac{-t_2}{a(1+q)}\right)\right]\left[\frac{1}{a(1+q)^2}exp\left(\frac{-t_2}{a(1+q)^2}\right)\right]\ldots$$
$$\left[\frac{1}{a(1+q)^{n-1}}exp\left(\frac{-t_2}{a(1+q)^{n-1}}\right)\right]$$

Taking logarithms of the function and differentiating with respect to a and q one gets

$$\left\{\begin{aligned}\frac{d\left(ln\left(L(a,q)\right)\right)}{da}&=\frac{1}{a}\sum_{i=1}^{n}\frac{t_i}{(1+q)^{i-1}}-n=0\\ \frac{d\left(ln\left(L(a,q)\right)\right)}{dq}&=\frac{1}{1+q}+\frac{n(1-n)}{2}-\frac{1}{a}\sum_{i=1}^{n}\frac{(1-i)t_i}{(1+q)^{i}}=0\end{aligned}\right\}\tag{13}$$

System of equations (13) can be solved numerically.

3.3 G1-Renewal Equation with Weibull Underlying Distribution

As in the previous case, the same type of failure-terminated data are considered. The PDF of the underlying (time to the first failure) Weibull distribution is

$$f_1(t)=\frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right]\qquad \alpha,\beta>0,\ t\geq 0\tag{14}$$

For the above underlying distribution the likelihood function is

$$\begin{aligned}L(\alpha,\beta,q)=&\frac{\beta}{\alpha}\left(\frac{t_1}{\alpha}\right)^{\beta-1}exp\left[-\left(\frac{t_1}{\alpha}\right)^{\beta}\right]X\frac{\beta}{\alpha(1+q)}\left(\frac{t_2}{\alpha(1+q)}\right)^{\beta-1}exp\left[-\left(\frac{t_2}{\alpha(1+q)}\right)^{\beta}\right]\bullet\\ &\frac{\beta}{\alpha(1+q)^2}\left(\frac{t_3}{\alpha(1+q)^2}\right)^{\beta-1}exp\left[-\left(\frac{t_3}{\alpha(1+q)^2}\right)^{\beta}\right]\ldots\frac{\beta}{\alpha(1+q)^{n-1}}\left(\frac{t_n}{\alpha(1+q)^{n-1}}\right)^{\beta-1}exp\left[-\left(\frac{t_n}{\alpha(1+q)^{n-1}}\right)^{\beta}\right].\end{aligned}\tag{15}$$

Taking the logarithm of this likelihood function one gets:

$$\begin{aligned}ln\left(L(\alpha,\beta,q)\right)=&ln\left(\frac{\beta}{\alpha}\right)+(\beta-1)ln\left(\frac{t_1}{\alpha}\right)-\left(\frac{t_1}{\alpha}\right)^{\beta}+ln\left(\frac{\beta}{\alpha(1+q)}\right)+(\beta-1)ln\left(\frac{t_2}{\alpha(1+q)}\right)-\left(\frac{t_2}{\alpha(1+q)}\right)^{\beta}+\\ &+ln\left(\frac{\beta}{\alpha(1+q)^2}\right)+(\beta-1)ln\left(\frac{t_3}{\alpha(1+q)^2}\right)-\left(\frac{t_3}{\alpha(1+q)^2}\right)^{\beta}+\ldots+ln\left(\frac{\beta}{\alpha(1+q)^{n-1}}\right)+(\beta-1)ln\left(\frac{t_n}{\alpha(1+q)^{n-1}}\right)-\left(\frac{t_n}{\alpha(1+q)^{n-1}}\right)^{\beta}\end{aligned}\tag{16}$$

Differentiating this function with respect to α, β and q, and equating the derivatives to zero one gets:

$$\frac{d\ln\big(L(\alpha,\beta,q)\big)}{d\alpha}=-\frac{\beta}{\alpha}+\frac{\beta}{\alpha}\left(\frac{t_1}{\alpha}\right)^{\beta}-\frac{\beta}{\alpha}+\frac{\beta}{\alpha}\left(\frac{t_2}{\alpha(1+q)}\right)^{\beta}-\frac{\beta}{\alpha}+\frac{\beta}{\alpha}\left(\frac{t_3}{\alpha(1+q)^2}\right)^{\beta}\ldots-\frac{\beta}{\alpha}+\frac{\beta}{\alpha}\left(\frac{t_n}{\alpha(1+q)^{n-1}}\right)^{\beta}$$

$$=-\frac{n\beta}{\alpha}+\frac{\beta}{\alpha}\sum_{i=1}^{n}\left(\frac{t_i}{\alpha(1+q)^{i-1}}\right)^{\beta}=-n+\alpha^{-\beta}\sum_{i=1}^{n}\left(\frac{t_i}{(1+q)^{i-1}}\right)^{\beta}=0$$

Thus, the first equation is

$$\alpha=\left(\frac{\sum_{i=1}^{n}\left(\frac{t_i}{(1+q)^{i-1}}\right)^{\beta}}{n}\right)^{\frac{1}{\beta}} \tag{17-1}$$

Taking the derivative with respect to β one gets

$$\frac{d\,ln\big(L(\alpha,\beta,q)\big)}{d\beta}=\frac{1}{\beta}+ln\left(\frac{t_1}{\alpha}\right)-\left(\frac{t_1}{\alpha}\right)^{\beta}ln\left(\frac{t_1}{\alpha}\right)+\frac{1}{\beta}+ln\left(\frac{t_2}{\alpha(1+q)}\right)-\left(\frac{t_2}{\alpha(1+q)}\right)^{\beta}ln\left(\frac{t_2}{\alpha(1+q)}\right)+\frac{1}{\beta}+ln\left(\frac{t_3}{\alpha(1+q)^2}\right)-$$

$$-\left(\frac{t_3}{\alpha(1+q)^2}\right)^{\beta}ln\left(\frac{t_3}{\alpha(1+q)^2}\right)+\ldots+\frac{1}{\beta}+ln\left(\frac{t_n}{\alpha(1+q)^{n-1}}\right)-\left(\frac{t_n}{\alpha(1+q)^{n-1}}\right)^{\beta}ln\left(\frac{t_n}{\alpha(1+q)^{n-1}}\right)$$

$$=\frac{n}{\beta}+\sum_{i=1}^{n}ln\left(\frac{t_i}{a(1+q)^{i-1}}\right)\left[1-\left(\frac{t_i}{a(1+q)^{i-1}}\right)^{\beta}\right]=0$$

Accordingly, the second equation is

$$\beta=\frac{n}{\sum_{i=1}^{n}ln\left(\frac{t_i}{a(1+q)^{i-1}}\right)\left[\left(\frac{t_i}{a(1+q)^{i-1}}\right)^{\beta}-1\right]} \tag{17-2}$$

And taking the derivative with respect to q one gets the third equation

$$\frac{d\,ln\big(L(\alpha,\beta,q)\big)}{dq}=0\,,$$

which is

$$\beta\sum_{i=1}^{n}\frac{i-1}{1+q}\left[\left(\frac{t_i}{a}\right)^{\beta}(1+q)^{\beta(1-i)}-1\right]=0\,. \tag{17-3}$$

Again, Equations (17.1–3) can be solved numerically to obtain MLE estimates of the G1R Process with the underlying Weibull distribution.

3.4 Numeric Example

Consider failure times between 12 consecutive failures discussed by Basu & Rigdon (2000): {3, 6, 11, 5, 16, 9, 19, 22, 37, 23, 31, 45}. The data are of the failure-terminated type. The G1-Renewal process with the underlying exponential distribution is assumed as a probabilistic model. Figure 7 shows MLE of the CIF obtained by solving System (15).

It is interesting to note that the CIF exhibits pronounced convexity, contrary to linearity, which might be intuitively expected from a point process with the underlying exponential distribution. The exponential distribution parameter is estimated to be 4.781 and G1-R restoration parameter as 0.232.

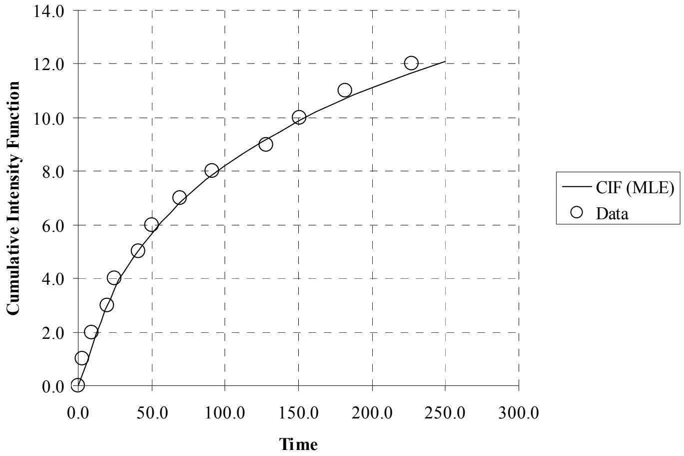

Figure 7. G1-Renewal with Exponential Underlying distribution as a Model to Data Set of Basu & Rigdon (2000).

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AN EFFICIENT HEURISTIC ALGORITHM FOR DETERMINING OPTIMAL REDUNDANCY ALLOCATION OF COMPLEX NETWORKS

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ABSTRACT

The paper presents a new heuristic algorithm for determining optimal redundancy allocation of complex networks. The present algorithm is an iterative method; all the paths of the network are first arranged in decreasing order of their priority determined using a path sensitivity factor, the highest priority path is optimized first by adding redundant components for subsystems of the path iteratively based on proposed subsystem selection factor. In case of availability of any residual resources next lower priority paths are considered for redundancy allocation. The proposed algorithm not only demonstrates improved performance in comparison with most of the existing heuristic algorithms but also leaves minimum slack of components without any further possibility of redundancy.

Keywords: constrained redundancy optimization; complex networks; heuristic algorithm.

1. INTRODUCTION

The problem of redundancy allocation generally has been solved as a single objective optimization problem to maximize system reliability subject to several constraints such as cost, weight, volume, etc. The solutions of such optimization problems have been obtained using mathematical models like dynamic programming [1-3], heuristic methods [4-23] and meta-heuristics such as genetic algorithms [24-26], tabu search [27], ant colony optimization [28], etc.

In recent works, major focus is on the development of heuristic and meta-heuristic algorithms for redundancy allocation problems to improve system reliability [4, 29]. Many heuristic algorithms have been proposed in the literature for solving redundancy reliability optimization problems which search for the solutions not only in feasible regions but also do excursion in infeasible/bounded infeasible regions for finding possibly improved solutions [30-

32]. For example, Shi method [19] searches solution only in feasible region and shows good temporal efficiency. The Nakagawa –Nakashima (N-N) method [15] also search solution in the feasible region and is superior to other heuristics in terms of optimality rates and relative errors, but have poor temporal efficiency [4,29]. The Agarwal - Gupta (A-G) algorithm [32] is the one of the recent methods for solving redundancy optimization of complex combinatorial problems and allows the search for optimal solution not only in the feasible region but also into the bounded infeasible region. Recently, Kumar *et al.* [22,23] presented heuristic algorithms which shows better performance.

The purpose of this paper is to present an efficient heuristic algorithm for determining optimal redundancy allocation of complex systems. The proposed algorithm [P-Alg] consists of: arranging all the path sets of the network in the decreasing order of path sensitivity factor value and then selecting the highest priority path set for redundancy allocation. A redundant parallel subsystem is added to the unsaturated subsystem of the chosen path set having maximum value of subsystem selection factor, if no constraints are violated. In case of violation of any constraint, the subsystem is excluded from further consideration and the next subsystem of the path set having highest value of subsystem selection factor is considered for redundancy allocation. The path is removed only after exhausting all the subsystems of the path set. The proposed algorithm not only demonstrates improved performance in comparison with most of the existing heuristic algorithms but it also leaves minimum slack of components without any further possibility of redundancy. The P-Alg not only utilizes a different formulation for subsystem selection factor but it also differs significantly from Kumar [22] the way the path sets are removed from further consideration in case of violation of any constraints.

The computational experiments are conducted on 4-, 5-, 7-, 10- and 15-unit complex systems with linear constraints. The numerical results obtained with P-Alg, Kumar [22], Shi [19], N-N [15] and A-G [31] methods are compared in terms of performance measures like average relative error (*A*), maximum relative error (*M*), optimality rate (*O*) and average execution time (*T*) [22, 30,31].

2. PROBLEM FORMULATION

2.1 Assumptions:

1. There are n subsystems in the system.
2. The system and subsystems are coherent. The subsystem structure is not restricted.
3. Subsystem states are mutually and statistically independent.
4. Constraints are separable and additive among components. Each constraint is an increasing function of x_i for the subsystem.
5. Redundant components can not cross subsystem boundaries.

2.2 Problem Definition:

A complex system consists of several components connected to each other neither in series nor in parallel. Figs. 1-5 show 5 such complex systems for 4-, 5-, 7-, 10- & 15- unit networks respectively. The problem of constrained redundancy optimization can be reduced to the following integer programming problem:

$$\text{Maximize: } R_s(x) \tag{1}$$

$$\text{Subject to: } \sum_{i=1}^{n} g_i^j(x_i) \le C_j, \quad j = 1,2,\dots,k$$

$$x_i \geq 1, \text{ for } i = 1, 2, \ldots, \text{n}.$$

3. PROBLEM FORMULATION

3.1 Algorithm

Assuming that the system reliability expression $R_s(x)$ is known (Shi [**1**]), the proposed algorithm uses following steps for finding the solution.

First of all, unsaturated minimal path sets of the system are arranged in their decreasing order of priority using path sensitivity factor $a_l(x)$

$$a_l(X) = \frac{\prod_{i\in P_l} R_i(x_i)}{\sum_{i\in P_l}\sum_{j=1}^{k}(g_i^j(x_i)/k\,C_j)}, \qquad l = 1,2,..,m \tag{2}$$

From the above ordered set of minimal path sets, path set having highest sensitivity factor- $a_l(X)$ is considered for optimization and subsystem selection factor $b_i(x_i)$ of all the unsaturated subsystem is found using

$$b_i(x_i) = \frac{\Delta R_i}{\sum_{j=1}^{k}(g_i^j(x_i)/k\,C_j)}, \quad \text{for each } i \in P_l \tag{3}$$

where $\Delta R_i = R_i(x_i) - R_i(x_i - 1)$

ΔR_i is termed as increment in subsystem reliability when a unit is added in parallel to the subsystem. After finding the subsystem having highest value of subsystem selection factor $b_i(x_i)$ of the chosen path set, a redundant parallel component is added to the unsaturated subsystem if no constraint is violated. In case of violation of any constraint the subsystem is removed from further consideration for redundancy and the next subsystem of the path set having highest value of subsystem selection factor $b_i(x_i)$ is considered for redundancy allocation. The iteration continues either till all the subsystems are removed from further consideration or all the resources are consumed. If all the resources are exactly consumed the iteration stops giving the optimal solution. But in case of all the subsystems of the chosen path set are removed from further consideration and there are still some resources available, the minimal path set having next highest value of sensitivity factor $a_l(X)$ is considered for optimization and then the steps are repeated till optimal solution is reached.

3.2 Steps of the Proposed Algorithm

Step1: Find all minimal path sets P_l (for all $l = 1,2,\ldots,m.$) of the system using any method.

Step2: Let $x_i = 1$ for all i; $i = 1, 2,\ldots, n$ such that $X = (1, 1,\ldots, 1)$

Step3: All unsaturated minimal path sets P_l (for all $l = 1,2,\ldots,m.$) of the system are arranged in their decreasing order of priority Ps using sensitivity factor $a_l(X)$.

Step4: The path set $[P_{Ps(Q)}]_{(Q = 1)}$ is selected for optimization.

Step5: For the above chosen path set, calculate $b_i(x_i)$ for all the subsystems of the path set and find i^* such that $b_{i^*}(x_i) = max\,[b_i(x_i)]$.

Step6: Check, by adding one redundant unit to unsaturated subsystem i^*:

i. if no constraints are violated, add one redundant subsystem to unsaturated subsystem i^* by replacing x_{i^*} with $x_{i^*}+1$ and go to step 5.

ii. if at least one constraint is exactly satisfied and others are not violated, then add one redundant subsystem to unsaturated subsystem i^* by replacing x_{i^*} with $x_{i^*}+1$.

The $x^* = x$ is the optimal solution. Go to step 7.

iii. if at least one constraint is violated then exclude the subsystem i^* from further consideration and the next subsystem i^* of the same path set having highest value $b_{i*}(x_i)$ is considered for redundancy allocation, go to step 6.

iv. if at least one constraint is violated and all other subsystems of the path $[P_{Ps(Q)}]$ are excluded; exclude the path $[P_{Ps(Q)}]$ from further consideration; consider the next path $[P_{Ps(Q+1)}]$ having next highest value of $a_l(X)$ and go to step 5.

v. if all the subsystems and/or all the minimal path sets are excluded from further consideration, than $x^* = X$ is the optimal solution; go to step 7.

Step7: Calculate the system reliability, $R_s(x^*)$.

4. COMPUTATION AND RESULT

The redundancy allocation problem for complex systems is formulated with the objective of maximization of the system reliability under constraint environment. In this paper, the test problems of computational experiments are generated for 4-, 5-, 7-, 10- and 15- unit complex networks shown in Figs. 1-5 respectively. As an illustration of P-Alg method two sets of bench mark examples consisting of 4- unit composite network (Fig. 1) and 5-unit bridge network (Fig. 2) with linear constraints are considered. The same examples are also solved by Kumar, Shi, N-N and A-G methods for comparison.

***Example 1**:* 4-unit composite network (Fig. 1) with linear constraints (n = 4 and k = 2). The constraint redundancy optimization problem is expressed as the following integer programming problem:

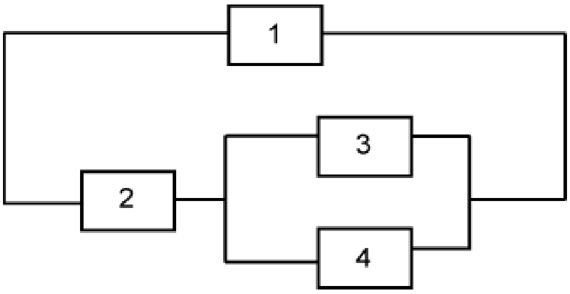

Figure 1. 4-Unit Composite Network

$$\text{Max. } R_s(x) = \begin{array}{l} R_1(x_1) + Q_1(x_1)R_2(x_2)R_4(x_4) \\ + Q_1(x_1)R_2(x_2)R_3(x_3)Q_4(x_4) \end{array} \tag{4}$$

$$\text{Subject to: } \sum_{i=1}^{4} c_{1i}(x_i) \le 132$$

$$\sum_{i=1}^{4} c_{2i}(x_i) \le 341 \tag{5}$$

$$x_i \ge 1, \text{ for i = 1, 2, 3, 4.}$$

$$L(x) = (1, 1, 1, 1), \quad U(x) = (6, 1, 13, 4)$$

The problem is solved for P-Alg, Kumar, Shi, N-N and A-G methods with the randomly generated values of parameters r_i, c_{1i}, c_{2i}, C_1 and C_2 shown in Table 1. It is interesting to note that all the methods yield same solution x^* = (3, 1, 2, 1) for $R_s(x^*)$ = 0.989612, which is also the global optima. The time taken by P-Alg (0.00413 sec.) is comparable with Kumar (0.00342 sec.) and Shi (0.00264 sec.) methods and is much smaller than N-N (0.01963 sec.) and A-G (0.90486 sec.) methods.

Table 1. Data for Figure 1

I	1	2	3	4
r_i	0.6984	0.625	0.8464	0.7536
c_{1i}	2	64	3	4
c_{2i}	48	74	23	74
C_1			132	
C_2			341	

***Example 2**:* 5-unit complex network (Fig. 2) with linear constraint (n = 5 and k = 1). The problem is defined as:

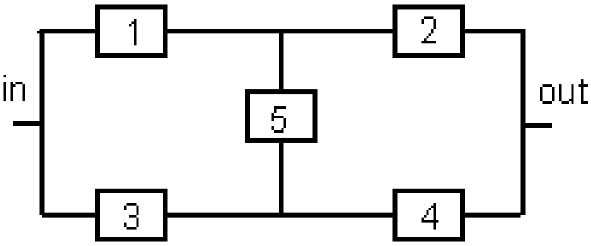

Figure 2. 5-Unit Bridge Network

$$\text{Maximise } R_s(X) = \begin{aligned} & R_1(x_1)R_2(x_2) + R_3(x_3)R_4(x_4)(Q_1(x_1) \\ & + R_1(x_1)Q_2(x_2)) \\ & + R_1(x_1)Q_2(x_2)Q_3(x_3)R_4(x_4)R_5(x_5) \\ & + Q_1(x_1)R_2(x_2)R_3(x_3)Q_4(x_4)R_5(x_5) \end{aligned} \tag{6}$$

Subject to:
$$\sum_{i=1}^{5} c_{1i}(x_i) \le 290 \tag{7}$$

$$x_i \ge 1, \quad \text{for } i = 1, 2, 3, 4, 5.$$
$$L(x) = (1, 1, 1, 1, 1),\ U(x) = (4, 146, 19, 3, 5)$$

The problem is solved for all the above methods with randomly generated values of parameters given in Table 2.

Table 2. Data for Figure 2

i	1	2	3	4	5
r_i	0.8106	0.6940	0.6974	0.8068	0.633 1
c_{1i}	45	1	8	56	35
C_1			290		

The results obtained for $x^*, g_1(x^*)$, $R_s(x)$ and T with various methods are compared in Table 3. From the table it is evident that P-Alg and A-G methods obtain best optimal solution x^* = (4, 11, 1, 1, 1) for $R_s(x)$ = 0.999546 without any slack of components, but the execution time of P-Alg is much lower than that of A-G. The A-G method takes more time by an order of 25 in comparison with P-Alg method. Though both Shi and N-N methods also do not leave any slack of components but their solution quality is inferior in comparison with P-Alg and A-G methods. Although, Kumar method takes minimum execution time but the quality of solution is poorest of all and it leaves maximum slack of components. To illustrate the working of different iterations of the algorithm, an example of 7- unit bridge network (Fig. 3(a)) has been solved for random generated data set in Appendix B.

Table 3. Comparison of Figure 2 Complex Network

Methods	x^*	$g_1(x^*)$	$R_s(x^*)$	T (sec.)
P-Alg	(4,11,1,1,1)	290	0.999546	0.04992
Kumar	(3,7,3,1,1)	257	0.999293	0.00326
Shi	(2,61,6,1,1)	290	0.997432	0.24205
N-N	(3,16,6,1,1)	290	0.999514	0.05007
A-G	(4,11,1,1,1)	290	0.999546	1.26784

5. PERFORMANCE MEASURES

In addition to the above, each of the n = 4- and 5- unit (Fig. 1 and 2 respectively) complex networks is further solved for 9 additional sets of randomly generated parameters r_i , c_{ji} and C_j for the comparison of performance measures of different methods.

The performance in terms of computational efficiency and solution quality of P-Alg, Kumar, Shi, N-N and A-G methods, defined as u =1, 2, 3, 4, 5 respectively, is illustrated with number of examples. The test problems of computational experiments are generated for 4-, 5-, 7-, 10-, and 15- unit examples of complex systems (shown in Figs. 1- 5) used by Kumar [22, 23] with linear constraints. The algorithms of different methods for the test problems are compared through performance measures such as average relative error (A_u), maximum relative error (M_u), optimality rate (O_u) and average execution time (T) for ten randomly generated initial data (v = 1, 2, …., 10), defined as:

Average relative error for method u,

$$A_u = \frac{1}{10} \sum_{v=1}^{10} (R_v^* - R_{uv}) / R_v^* \tag{8}$$

where

R_{uv} is system reliability obtained by method u for test problem v; and R_v^* is the best system reliability obtained by any of the four methods or by complete enumeration for the test problem v.

Maximum Relative Error for method u,

$$M_u = \max_v \{(R_v^* - R_{uv}) / R_v^*\} \tag{9}$$

Optimality rate for method u, O_u = number of times (out of 10 problems) method u yields the best system reliability.

T is average execution time of 10 test problems (sec.)

Following section illustrate the solution of 3 complex networks (Figs. 3-5). For n = 7 (Fig. 3(a)) network, 4 problems are formed by taking two constraints k = 1, 5 and two different values of parameter C_j as '*small*' and '*large*' defined as

$$\{ C_j \} = w_j * \sum_{i=1}^{n} g_i^j \tag{10}$$

here w_j denotes random uniform deviates from 1.5 to 2.5 for 'small', and from 2.5 to 3.5 for 'large'. Data for the parameters of the problems are generated randomly by taking $\{ g_i^j(x_i) \}$ a random uniform deviates from 0 to 100 and random uniform deviates $\{r_i\}$ from 0.6 to- 0.85. Another $n = 7$ (Fig. 3(b)), 10- (Figs 4) and 15- (Figs 5) unit networks are solved for two different data sets of parameters $k = 5$, C_j = *'small'* and $k = 5$, C_j = *'large'*. Thus, in total 12 test problems are solved for 6 different networks (Figs. 1-5) with various methods using MATLAB on a Pentium(R) D, 3.4 GHz CPU based computer. Each of the test problems is then solved for 10 randomly generated data sets for r_i , c_{ji} and C_j.

To obtain optimal solutions, P-Alg, Kumar and Shi methods use single initial solution (1, 1,…, 1) whereas A-G method uses 10 initial solutions generated randomly by a 2-phase procedure of Kim and Yum [30]. For N-N method, each problem is solved by taking initial solution (1, 1, …, 1) and 10 values of the balancing coefficient α as 0.1, 0.2,…., 1.0. Thus, out of 10 such solutions obtained by N-N and A-G methods best solution is selected for comparison with other methods. Thus in total 1320 test problems are solved (120 test problems by P-Alg, Kumar and Shi methods and 1200 test problems by N-N and A-G methods).

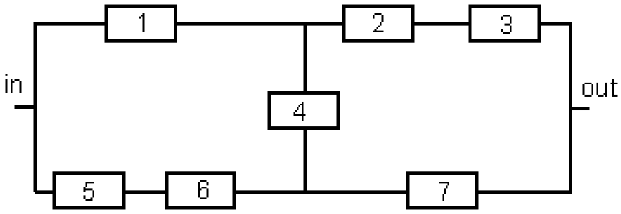

Figure 3(a). 7-Unit Complex System ($n = 7$)

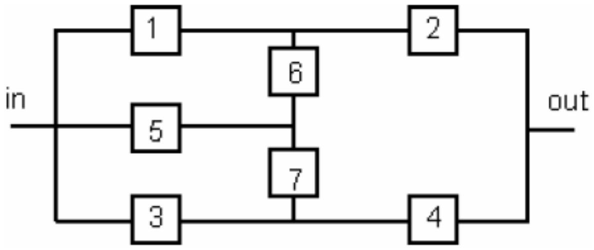

Figure 3(b). 7-Unit Complex System ($n = 7$)

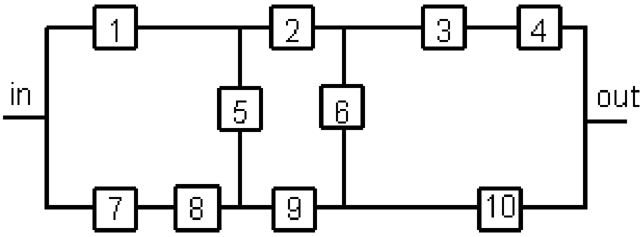

Figure 4. 10-Unit Complex System ($n = 10$)

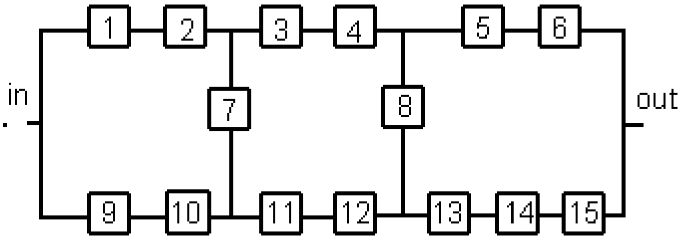

Figure 5. 15-Unit Complex System ($n = 15$)

The results of performance measures, i.e., average relative error (*A*), maximum relative error (*M*), optimality rate (*O*) and average execution time (*T*) obtained with P-Alg, Kumar, Shi, N-N, and A-G methods, are compared in Table 4. The performance measures *A, M* and *O* of P-Alg are consistently better than all other methods, except in case of 4,7x5-'large*'* and 8,10x5-*'small'* network examples where N-N method shows better performance. The average execution time (*T*) of P-Alg is also better or comparable with Kumar in most of the cases.

Table 4. Comparison of Performance Measures
for 4-, 5-, 7-, 10-, 15-Unit Networks (Fig. 1-5 respectively)

Example (m, *n* x *k*)	Methods	Performance Measures			
		A	M	O	T (sec.)
3, 4 x 2 Figure 1	P-Alg	0	0	10	0.06020
	Kumar	0.000353	0.00186	5	0.053242
	Shi	0.000344	0.00186	6	0.27828
	N-N	9.5E-06	9.5E-05	9	0.02385
	A-G	0.000296	0.00173	1	12.38190
4, 5 x 1 Figure 2	P-Alg	4E-07	4E-06	9	0.04992
	Kumar	0.001382	0.003364	1	0.003262
	Shi	0.003845	0.007585	0	0.24205
	N-N	3.6E-06	3.6E-05	9	0.05007
	A-G	0.000593	0.002059	2	1.26784
4, 7 x 1 'small' Figure 3(a)	P-Alg	7.95E-05	0.000795	9	0.05076
	Kumar	0.001152	0.004727	2	0.05165
	Shi	0.048417	0.182504	0	0.98615
	N-N	0.000486	0.003217	8	0.08194
	A-G	0.003395	0.010776	0	33.03050
4, 7 x 5 'small' Figure 3(a)	P-Alg	0	0	10	0.05164
	Kumar	0.003966	0.010224	4	0.050893
	Shi	0.011981	0.06485	3	0.61064
	N-N	0.000895	0.005394	8	0.05421
	A-G	0.001741	0.009337	1	7.80162
4, 7 x 1 'large' Figure 3(a)	P-Alg	0	0	10	0.04938
	Kumar	0.000217	0.000444	0	0.05124
	Shi	0.005614	0.025473	0	1.41909
	N-N	2.6E-06	1.4E-05	8	0.14222
	A-G	0.000112	0.000215	0	14.51660
4, 7 x 5 'large' Figure 3(a)	P-Alg	0.008177	0.021486	4	0.04226
	Kumar	0.001184	0.004195	0	0.05186
	Shi	0.005277	0.019515	0	1.07131
	N-N	0.000274	0.000788	5	0.09980
	A-G	0.000436	0.001459	1	8.62698
6, 7 x 5 'small' Figure 3(b)	P-Alg	0	0	10	0.05200
	Kumar	0.006507	0.015626	0	0.03064
	Shi	0.012012	0.040473	0	0.66712
	N-N	0.005372	0.008265	0	0.05283
	A-G	0.004888	0.00917	0	12.40320
6, 7 x 5 'large' Figure 3(b)	P-Alg	0.001255	0.010007	8	0.05080
	Kumar	0.000235	0.001405	0	0.05107
	Shi	0.000231	0.001419	0	1.16433
	N-N	3.22E-05	0.000131	2	0.09677
	A-G	9.84E-05	0.000724	0	13.93550
8, 10 x 5 'small' Figure 4	P-Alg	0.01371	0.058709	3	0.04383
	Kumar	0.010458	0.058121	5	0.05611
	Shi	0.043408	0.080705	0	1.95416
	N-N	0.001176	0.010479	7	0.15770
	A-G	0.005039	0.013028	0	3.62803

Example (m, *n* x *k*)	Methods	Performance Measures			
		A	M	O	T (sec.)
8, 10 x 5 'large' Figure 4	P-Alg	0.003071	0.02704	8	0.04433
	Kumar	0.02487	0.041681	1	0.05511
	Shi	0.044704	0.080265	0	1.95288
	N-N	0.015437	0.038307	0	0.15536
	A-G	0.014913	0.038306	1	14.09680
8, 15 x 5 'small' Figure 5	P-Alg	0	0	10	0.04407
	Kumar	0.041613	0.064621	0	0.08765
	Shi	0.088131	0.174326	0	3.70830
	N-N	0.033755	0.059137	0	1.08656
	A-G	0.037845	0.059135	0	15.34900
8, 15 x 5 'large' Figure 5	P-Alg	0	0	10	0.05097
	Kumar	0.067226	0.091703	0	0.08684
	Shi	0.112797	0.178473	0	3.65437
	N-N	0.059578	0.07733	0	1.07747
	A-G	0.063573	0.077328	0	62.50630

CONCLUSIONS

In this paper, an efficient heuristic algorithm for determining optimal redundancy allocation of complex systems has been proposed. It has been shown that quality of solution in P-Alg is better than all other methods in most of the cases. The computational time of P-Alg is either better or comparable with Kumar and Shi methods. The method also leaves minimum slack of components without any further possibility of redundancy. Therefore, the P-Alg method finds its greater utility for solving redundancy allocation problems where both the solution quality and computational time are of prime importance.

APPENDIX A

Notation

$a_l(X)$	Sensitivity factor of l^{th} minimal path set
$b_i(x_i)$	Subsystem selection factor for i^{th} subsystem with x_i components
c_{ji}	Cost of subsystem *i* for k^{th} constraint.
C_j	Total amount of resource *j* available
$g_i^j(x_i)$	Amount of resources-*j* consumed in subsystem-*i* with x_i components
k	Number of constraints, $j = 1, 2, \ldots, k$
$L(x)$	$(L_{x_1}, L_{x_2}, \ldots, L_{x_n})$, Lower limit of each of subsystem *i,*
m	Number of minimal path sets, $l = 1, 2,., m$
n	Number of subsystems, $i = 1, 2, \ldots, n$
P_l	l^{th} minimal path set of the system
Ps	$(l^1, l^2, \ldots, l^{min})$: priority vector s.t. l^1 and l^{min} are the number of minimal path sets respectively having maximum and minimum value of path selection parameter $a_l(X)$.
$Q_i(x_i)$	Unreliability of subsystem *i* with x_i components.
r_i	Reliability of a component at subsystem *i*.
$R_i(x_i)$	Reliability of subsystem *i* with x_i components.
R_r	Residual resources $C_j - \sum c_{ij} x_i$
$R_s(x)$	System reliability
T	Average execution time of 10 test problems (sec.)
$U(x)$	$(U_{x_1}, U_{x_2}, \ldots, U_{x_n})$, Upper limit of each of subsystem *i,*
x^0	Initial feasible solution
x^*	Optimal solution
x_i	Number of components in subsystem *i;* $i = 1,2,\ldots.n$
X	$(x_1, \ldots\ldots x_n)$
α	Balancing coefficient for N-N method

APPENDIX B

This section presents how the P-Alg is applied for constraint redundancy reliability optimization problem of a 7- unit bridge network with $n = 7$, $k = 1$ (Fig. 3(a)) for data given in Table 5 and the procedure is illustrated below:

Table 5. Data for Figure 3(a)

i	1	2	3	4	5	6	7
r_i	0.7321	0.6109	0.7963	0.7013	0.6247	0.7104	0.6631
c_{1i}	54	48	95	24	68	9	99
C_1				794			

Illustration

Step1: Initialize $i = 7$, $m = 4$, $k=1$, r_i = [0.7321, 0.6109, 0.7963, 0.7013, 0.6247, 0.7104, 0.6631], c_{1i} = [54, 48, 95, 24, 68, 9, 99], $R_r = C_j = 794$, and P_l (for l = 1, 2, 3, 4.) of the system are P_1 = [1, 2, 3], P_2 = [1, 4, 7], P_3 = [5, 6, 7] and P_4 = [2, 3, 4, 5, 6] .

Step2: Let $x_i = 1$ for all i; $i = 1, 2,\ldots,7$ *i.e* $X = (1,1,1,1,1,1,1)$ and for this value of X, $R_r = 397$.

Step3: The sensitivity factor $a_l(X)$ i.e. $[a_1, a_2, a_3, a_4]$ = [1.435, 1.527, 1.328 0.493], hence, Ps = [2, 1, 3, 4].

Step4: The path set $[P_{Ps(Q)}]_{(Q = 1)} = [P_{Ps(1)}] = P_2$ is selected for optimization.

Step5: For P_2 all values of $b_i(x_i)$ for each $i \in P_2$ = [10.765, 23.201, 5.318] $_{(i=1,4,7)}$, $i^* = 4$ as $b_4(x_4) = 23.201$ is the maximum.

Step6(i): By incrementing $x_4 = x_4 + 1$, $X = (1,1,1,2,1,1,1)$ and $R_r = 373$ (i.e. a +ve number), no constraints is violated.

Step5: For P_2, $b_i(x_i)$ = [10.765, 6.930, 5.318] $_{(i=1,4,7)}$, $i^* = 1$ as $b_1(x_1) = 10.765$,

Step6(i): Increment $x_4 = x_4 + 1$, $X = (2,1,1,2,1,1,1)$ and $R_r = 319$, no constraint is violated. Repeat the Step5 and Step6(i) until $X = (3,1,1,4,1,1,3)$ and $R_r = 19$, $b_i(x_i)$ = [0.773, 0.618, 0.604] $_{(i=1,4,7)}$, $i^* = 1$ as $b_1(x_1) = 0.773$ is the maximum.

Step6(ii): Increment $x_1 = x_1 + 1$, $X = (4,1,1,4,1,1,3)$ and $R_r = -35$, constraint is violated, X is reinstated at its previous step value i.e. (3,1,1,4,1,1,3), subsystem $i^* = 1$ of P_2 is removed from further consideration, and $b_i(x_i)$ = [--, 0.618, 0.604] $_{(i=1,4,7)}$, next highest value of $b_i(x_i)$ is $b_4(x_4) = 0.618$ for $i^* = 4$.

Step6(ii): Increment $x_4 = x_4 + 1$, $X = (3,1,1,4,1,1,3)$ and $R_r = -5$, constraint is violated, X is reinstated at its previous step value i.e. (3,1,1,4,1,1,3), subsystem $i^* = 4$ of P_2 is removed from further consideration, and $b_i(x_i)$ = [--, --, 0.604] $_{(i=1,4,7)}$, next highest value of $b_i(x_i)$ is $b_7(x_7) = 0.604$ $i^* = 7$. Repeat the Step6 until all the subsystems of path P_2 are removed from further consideration and X = (3, 1, 1, 4, 1, 1, 3), $b_i(x_i)$ = [--, --, --] $_{(i=1,4,7)}$, $a_l(X)$ i.e. $[a_1, a_2, a_3, a_4]$ = [1.435, ++, 1.328 0.493].

Step6(iii) $Q = Q + 1$, $P_{Ps(Q)}]_{(Q = 2)} = [P_{Ps(2)}] = P_1$, for P_1 all values of $b_i(x_i)$ for each $i \in P_1$ = [--, 10.105, 6.555]$_{(i=1,2,3)}$, as subsystem $i = 1$ has already been optimized hence not considered further.

Step5: The next highest value of $b_i(x_i)$ is $b_2(x_2) = 10.105$ for $i^* = 2$.

Step6: By repetitively checking the various conditions for the sub steps in similar manners as described above, it is found that there is no possible of redundancy for any of the subsystem of the path set P_1, hence all the subsystems the path set are removed from further consideration, hence the path set is removed. $X = (3,1,1,4,1,1,3)$, $b_i(x_i)$ = [--, --, --] $_{(i=1,2,3)}$, $a_l(X)$ i.e. $[a_1, a_2, a_3, a_4]$ = [++, ++, 1.328 0.493].

Step6(iii) $Q = Q + 1$, $P_{Ps(Q)}]_{(Q = 3)} = [P_{Ps(3)}] = P_3$, for P_3 all values of $b_i\,(x_i)$ for each $i \in P_3 =$ [7.294, 62.673, --]$_{(i = 5,6,7)}$, as subsystem $i = 7$ has already been optimized hence not considered further.

This way, the above Step5 and Step6 are again repeated for the selected path set P_3 until all the subsystems and/or all the minimal path sets are excluded from further consideration, till optimal solution $x^* = X = (3,1,1,4,1,3,3)$ is obtained for $R_r = 1$. All the intermediate steps and the values of different parameter during different steps are given in Table 6.

Step7: System reliability, $R_s\,(x^*) = 0.971495$ is determined for the optimal solution $x^* = X = (3, 1, 1, 4, 1, 3, 3)$ and $T = 0.05076$ sec.

Table 6. Procedure of P-Alg for Figure 3 Complex Network

Allocation	Residual resources	Minimal path set sensitivity factor				Components selection factor						
$(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$	$C_j-\sum c_{ij}x_i$	$(a_1$	a_2	a_3	$a_4)$	$(b_1$	b_2	b_3	b_4	b_5	b_6	$b_7)$
1,1,1,1,1,1,1	397	1.435	1.527*	1.328	0.493	10.765			23.201$^{\#}$			5.318
1,1,1,2,1,1,1	373					10.765$^{\#}$			6.930			5.318
2,1,1,2,1,1,1	319					2.884			6.930$^{\#}$			5.318
2,1,1,3,1,1,1	295					2.884			2.070			5.318$^{\#}$
2,1,1,3,1,1,2	196					2.884$^{\#}$			2.070			1.792
3,1,1,3,1,1,2	142					0.773			2.070$^{\#}$			1.792
3,1,1,4,1,1,2	118					0.773			0.618			1.792$^{\#}$
3,1,1,4,1,1,3	19					0.773$^{\#}$			0.618			0.604
4,1,1,4,1,1,3	-35$^{\$}$					--			0.618$^{\#}$			0.604
3,1,1,5,1,1,3	-5$^{\$}$					--			--			0.604$^{\#}$
3,1,1,4,1,1,4	-80$^{\$}$					--			--			--
3,1,1,4,1,1,3	19	1.435*	++	1.328	0.493	--	10.105$^{\#}$	6.655	--			--
3,2,1,4,1,1,3	-29$^{\$}$					--	--	6.655$^{\#}$	--			--
3,1,2,4,1,1,3	-76$^{\$}$					--	--	--	--			--
3,1,1,4,1,1,3	-19$^{\$}$	++	++	1.328*	0.493	--	--	--	--	7.294	62.673$^{\#}$	--
3,1,1,4,1,2,3	10					--	--	--		7.294	18.150$^{\#}$	--
3,1,1,4,1,3,3	1					--	--	--	--	7.294$^{\#}$	5.256	--
3,1,1,4,2,3,3	-67$^{\$}$					--	--	--		--	5.256$^{\#}$	--
3,1,1,4,1,3,3	1	++	++	++	0.493$^{\%}$	--	--	--	--	--	--	--

* This minimal path set has the highest value of the sensitivity factor.
$^{\#}$ A redundant component is to be added to this subsystem for highest selection factor.
$^{\$}$ Constraint violation.
-- Subsystem is removed from further consideration.
++ Minimal path set removed from further consideration.
$^{\%}$ All the subsystems of the path have already been optimized, hence no further possibility of redundancy.

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ALGORITHMIC PROBLEMS IN DISCRETE TIME RISK MODEL

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ABSTRACT

In this paper we consider some algorithmic problems which appear in a calculation of a ruin probability in discrete time risk models with an interest force which creates stationary and reversible Markov chain. These problems are connected as with a generation of the Markov chain by its stationary distribution so with a calculation of the ruin probability.

Keywords: ruin probability, transportation problem, asymptotic formula, enumeration problem

1. INTRODUCTION

In this paper we consider some algorithmic problems which occur in a calculation of a ruin probability in discrete time risk model with an interest force which creates stationary and reversible Markov chain. Such model of the interest force is suggested by A.A. Novikov. Algorithmic problems are connected as with accuracy calculation of the ruin probability so when we deal with its asymptotic analysis. First numerical experiments show that without a solution of these algorithmic problems it is impossible to construct programs of numerical calculation of the ruin probability.

Markov chain generation reduces to a definition of permissible solutions of appropriate transportation problems. The ruin probability calculation is connected as with a definition of special sums of exponents so with a calculation of ruin probability asymptotic. These procedures also need to solve some auxiliary algorithmic problems: of convenient designations and symbolic calculations, some enumeration problem and so on. A specific of these problems is that primitive variants of their solution have very high complexity. Moreover some calculation procedures can not be realized without a solution of these problems.

2. PRELIMINARIES

Consider recurrent discrete time risk model (with annual step) with initial capital x, $x \ge 0$ and nonnegative losses Z_n, $n = 1,2,...$, $P(Z_n < t) = F(t)$:

$$S_0 = x\,,\ S_n = B_n S_{n-1} + A_n\,,\ n = 1,2...\,, \tag{1}$$

Here annual income A_n, $n = 1,2,...$ to end of n-th year is defined as difference between unit premium sum and loss $A_n = 1 - Z_n$. Assume that $B_n > 1$ is inflation factor from $n-1$ to n year, $n = 1,2,...$. In [1] $X_n = -A_n$ is called insurance risk and $Y_n = B_n^{-1}$ is called financial risk. In this model with initial capital x ruin time is defined by formula

$$\tau(x) = \inf\{n = 1,2,...: S_n \le 0 | S_0 = x\}$$

and finite time ruin probability $\psi(x,n)$ - by formula

$$\psi(x,n)=P(\tau(x)\le n).$$

So the sum S_n money accumulated by insurance company to *n*- th year end satisfies recurrent formula

$$S_0=x\ ,S_n=x\prod_{j=1}^{n}B_j+\sum_{i=1}^{n}A_i\prod_{j=i+1}^{n}B_j\ ,\quad n=1,2...,\tag{2}$$

where $\prod_{j=n+1}^{n}=1$ by convention. According to the notation above, we can rewrite the discounted value of the surplus S_n in (2) as

$$\tilde{S}_0=x,\qquad \tilde{S}_n=S_n\prod_{j=1}^{n}Y_j=x-\sum_{i=1}^{n}X_i\prod_{j=1}^{i}Y_j=x-W_n\,.$$

Hence, we easily understand that, for each $n=0,1,...$,

$$\psi(x,n)=\mathrm{P}(U_n>x),\ U_n=\max\left\{0,\max_{1\le k\le n}W_k\right\},\ U_0=0\,.\tag{3}$$

Suppose that the sequence $\{Y_n,n\ge 0\}$ is stationary and reversible Markov chain with state set $\left\{r_q^{-1},q\in Q\right\}$, $Q=\{1,...,m\}$ consisting of different positive numbers and transition matrix $\left\|\pi_{q',q}\right\|_{q',q\in Q}$. It means that the following formulas are true

$$P\left(Y_n=r_q^{-1}\right)=p_q,\ 0<p_q,\ \sum_{q\in Q}p_q=1,\ p_{q'}\pi_{q'q}=p_q\pi_{qq'},\ q,q'\in Q,\ n\ge 0$$

and consequently [2,Theorem 2.4]

$$(Y_1,...,Y_n)\overset{(d)}{=}(Y_n,...,Y_1),\ n\ge 1\,.\tag{4}$$

Assume that the random sequence $\{\omega_n,n\ge 0\}$ consists of independent and identically distributed random variables (i.i.d.r.v.`s) with uniform distribution on interval [0,1]. Suppose that random sequences $\{Y_n,n\ge 0\},\{\omega_n,n\ge 0\}$ are independent. Introduce distribution functions (d.f.`s) $F_q,q\in Q$ and designate $F^{-1}(\omega)$, $0\le\omega\le 1$, inverse function to distribution function $F(t)$, $-\infty<t<\infty$. Denote $Z_n=F_{Y_n}^{-1}(\omega_n)$, $n\ge 0$, then from the formula (4) we obtain the formula

$$((X_1,Y_1),...,(X_n,Y_n))\overset{(d)}{=}((X_n,Y_n),...,(X_1,Y_1)),\ n\ge 1\,.\tag{5}$$

In such a way it is possible to introduce dependence between financial and insurance risks provided financial risks create stationary and reversible Markov chain.
Define another random sequence

$$V_0=0,\ V_n=Y_n\max(0,X_n+V_{n-1}),\ n=1,2,....\tag{6}$$

Using recurrent formula (6) we introduce Markov chain (Y_n,V_n), $n=1,2,...$ and designate

$$\psi_{n,q}(x)=P\left(Y_n=r_q^{-1},V_n>x\right),\ q\in Q,\ x\ge 0,\ n\ge 0\,.$$

Theorem 1. The formula

$$\psi_n(x)=\sum_{q\in}\psi_{n,q}(x),\ n=0,1,...,\ x\ge 0,\tag{7}$$

is true.

Proof . The result (7) is trivial for the case when $n=0$. Now we aim at (7) for each $n=1,2,\ldots$. Let $n\geq 1$ be fixed. In view of the equality (5) we replace X_i and Y_j in U_n respectively by X_{n+1-i} and Y_{n+1-j} in deriving the following relations:

$$U_n=\max\left\{0,\max_{1\le k\le n}\sum_{i=1}^{k}X_i\prod_{j=1}^{i}Y_i\right\}\overset{(d)}{=}\max\left\{0,\max_{1\le k\le n}\sum_{i=1}^{k}X_{n+1-i}\prod_{j=1}^{i}Y_{n+1-j}\right\}=$$

$$=\max\left\{0,\max_{1\le k\le n}\sum_{i^*=n+1-k}^{n}X_{i^*}\prod_{j^*=i^*}^{n}Y_{j^*}\right\}=\max\left\{0,\max_{1\le k^*\le n}\sum_{i^*=k^*}^{n}X_{i^*}\prod_{j^*=i^*}^{n}Y_{j^*}\right\}. \qquad (8)$$

If we write the right-hand side of (8) as $\tilde{V}_n$, which satisfies the recurrence equation

$$V_n^*=Y_n\max\left(0,X_n+V_{n-1}^*\right),\ n=1,\ldots,$$

which is just the same as (6). So we immediately conclude that $V_n^*=V_n$ for each $n=1,\ldots$. Finally, it follows from (8) that (7) holds for each $n=1,\ldots$. This ends the proof of Theorem 1.

3. RECURRENT ALGORITHMES OF RUIN PROBABILITY CALCULATIONS

Introduce m -dimensional vectors $1_q=\left(\delta_{1,q},\ldots,\delta_{m,q}\right)$ where $\delta_{i,j}$ is Kronecker symbol and

$$R=(r_1,\ldots,r_m),\ K=(k_1,\ldots,k_m),\ r_i>0,\ k_i\in\{0,1,\ldots\},\ i=1,\ldots,m,$$

and denote

$$R^K=\prod_{q\in Q}r_q^{k_q},\ |K|=\sum_{q\in Q}k_q .$$

Redefine the function $\exp(-t)$ so that for $t<0$ we have $\exp(-t)=1$ and for $t\geq 0$ this function is defined as usual. Introduce the function

$$E(t)=\begin{cases}1,t\le 0,\\0,t>0.\end{cases}$$

Suppose that

$$\overline{F}_q(t)=\sum_{i=1}^{l}a_{q,i}\exp(-\lambda_i t),\ n\ge 1,\ t\ge 0,$$

with

$$-\infty<a_{\ i}<\infty,\ i=1,\ldots,l,\ \sum_{i=1}^{l}a_{q,i}=1,\ q\in Q .$$

Theorem 2. Suppose that

$$R^K\lambda_i\ne\lambda_j,1\le i,\ j\le l,\ 1\le|K| . \qquad (9)$$

Then there are real numbers $B_{n,i,q}^K$, $i=1,\ldots,l$, $1\le|K|\le n$, which satisfy for $n\geq 1$, $i=1,\ldots,l,$ initial conditions

$$B_{1,i,q}^{1_q}=p_q a_{q,i\exp(-\lambda_i)},\ B_{1,i,q}^{1_{q'}}=0,q,q'\notin Q,\ q\ne q' . \qquad (10)$$

and recurrent formulas for $q\in Q$:

$$B_{n+1,i,q}^{1_q}=\sum_{q'\in Q}p_{q'}\pi_{q',q}\left[\sum_{1\le|K|\le n}\sum_{j=1}^{l}\frac{B_{n,j,q'}^K a_{q,i}}{R^K\lambda_j-\lambda_i}R^K\lambda_j\exp(-\lambda_i)+B_{n,q'}^0 a_{q,i}\exp(-\lambda_i)\right],\ q=q',$$

$$B_{n+1,i,q}^{1_{q'}} = 0\,,\ q \neq q'\,, \tag{11}$$

$$B_{n,j,q}^{K} = -\sum_{q' \in Q} p_{q'} \pi_{q',q} I\left(k_q > 0\right) \sum_{j=1}^{l} \frac{B_{n,j,q'}^{K-1_q} a_{q,j} \lambda_j}{R^{K-1_q}\lambda_i - \lambda_j} \exp\left(-R^{K-1_q}\lambda_i\right),\ 1 < |K| \leq n+1\,, \tag{12}$$

so that

$$\psi_{s,q(t)} = \sum_{1 \leq |K| \leq s} \sum_{i=1}^{l} B_{s,i,q}^{K} \exp\left(-R^{K}\lambda_i t\right) + B_{s,q}^{0} E(t)\,,\ s > 0\,, \tag{13}$$

where

$$B_{s,q}^{0} = p_q - \sum_{1 \leq |K| \leq s} \sum_{i=1}^{l} B_{s,i,q}^{K}\ . \tag{14}$$

Proof . If positive random variables ξ, η are independent and

$$P(\xi > t) = \exp(-\mu t),\ P(\eta > t) = \exp(-\lambda t)\,,\ \lambda, \mu > 0\,,\ \lambda \neq \mu\,,$$

then it is easy to obtain that

$$P(\xi + \eta > t) = \frac{\mu \exp(-\lambda t) - \lambda \exp(-\mu t)}{\mu - \lambda}\,. \tag{15}$$

Calculating for $t > 0$

$$P\left(Y_1 = r_q^{-1}, Y_1(Z_1 - 1) > t\right)| = p_q P\left(F_q^{-1}(\omega_1) - 1 > R^{1_q} t\right) = p_q P|\left(F_q^{-1}(\omega_1) >| R^{1_q} t + 1\right) =$$

$$p_q \sum_{i=1}^{l} a_{q,i} \exp\left(-\lambda_i\left(R^{1_q} t + 1\right)\right) = p_q \sum_{i=1}^{l} a_{q,i} \exp(-\lambda_i) \exp\left(-\lambda_i R^{1_q} t\right)$$

we obtain that

$$\psi_{1,q}(t) = p_q \sum_{i=1}^{l} a_{q,i} \exp(-\lambda_i) \exp\left(-\lambda_i R^{1_q} t\right) + B_{1,q}^{0} E(t) =$$

$$= \sum_{i=1}^{l} B_{1,i,q}^{1_q} \exp\left(-\lambda_i R^{1_q} t\right) + B_{1,q}^{0} E(t)\,.$$

So the formula (13) is true for $s = 1$ with the initial conditions (10) and the equality (14).

Suppose that the formula (13) takes place for $s = n$ and using the formula (15) calculate

$$\psi_{n+1,q}(t) = P\left(Y_{n+1} = r_q^{-1}, Y_{n+1}(V_n + Z_{n+1} - 1) > t\right) =$$

$$= P\left(Y_{n+1} = r_q^{-1}, V_n + F_q^{-1}(\omega_{n+1}) > r_q t + 1\right) = \sum_{q' \in Q} p_{q'} \pi_{q',q} P\left(Y_{n+1} = r_{q'}^{-1}, V_n + F_q^{-1}(\omega_{n+1}) > r_q t + 1\right).$$

As for $x > 0$

$$P\left(Y_n = r_{q'}^{-1}, V_n + F_q^{-1}(\omega_{n+1}) > x\right) =$$

$$= \sum_{1 \leq |K| \leq n} \sum_{i=1}^{l} \sum_{j=1}^{l} \frac{B_{n,i,q'}^{K} a_{q',j}}{R^{K}\lambda_i - \lambda_j} \left(R^{K}\lambda_i \exp\left(-|\lambda_j x\right) - \lambda_j \exp\left(-R^{K}\lambda_i x\right)\right) + B_{n,q'}^{0} \sum_{i=1}^{l} a_{q,i} \exp(-\lambda_i x)\,,$$

so for $t > 0$

$$P\left(Y_n = r_{q'}^{-1}, V_n + F_q^{-1}(\omega_{n+1}) > r_q t + 1\right) = \sum_{1 \leq |K| \leq n} \sum_{i=1}^{l} \sum_{j=1}^{l} \frac{B_{n,i,q'}^{K} a_{q,j}}{R^{K}\lambda_i - \lambda_j} A_{i,j,q,n}^{k}(t) +$$

$$+B^0_{n,q'}\sum_{i=1}^{l} a_{q,i}\exp(-\lambda_i x)\exp\left(-R^{1_q}\lambda_i t\right)$$

with

$$A^K_{i,j,q,n}(t)=R^K\lambda_i\exp\left(-\lambda_j\left(r_q t+1\right)\right)-\lambda_j\exp\left(-R^K\lambda_i\left(r_q t+1\right)\right)=$$

$$=R^K\lambda_i\exp\left(-\lambda_j\right)\exp\left(-\lambda_j r_q t\right)-\lambda_j\exp\left(-R^K\lambda_i\right)\exp\left(-R^K\lambda_i r_q t\right)=$$

$$=R^K\lambda_i\exp\left(-\lambda_j\right)\exp\left(-\lambda_j R^{1_q}t\right)-\lambda_j\exp\left(-R^K\lambda_i\right)\exp\left(-R^{K+1_q}\lambda_i t\right)=$$

Consequently we obtain for $t>0$

$$\psi_{n+1,q}(t)=\sum_{q'\in Q}\Big[p_{q'}\pi_{q',q}\sum_{1\le|K|\le n}\sum_{i=1}^{l}\sum_{j=1}^{l}\frac{B^K_{n,i,q'}a_{q,j}}{R^K\lambda_i-\lambda_j}\Big[R^K\lambda_i\exp\left(-\lambda_j\right)\exp\left(-\lambda_j R^{1_q}t\right)-$$

$$-\lambda_j\exp\left(-R^K\lambda_i\right)\exp\left(-R^{K+1_q}\lambda_i t\right)\Big]+B^0_{n,q'}\sum_{i=1}^{l}a_{q,i}\exp(-\lambda_i)\exp\left(-R^{1_q}\lambda_i t\right)\Big]=$$

$$=\sum_{q'\in Q}p_{q'}\pi_{q',q}\sum_{1\le|K|\le n}\sum_{i=1}^{l}\sum_{j=1}^{l}\frac{B^K_{n,i,q'}a_{q,i}}{R^K\lambda_j-\lambda_i}R^K\lambda_j\exp(-\lambda_i)\exp\left(-\lambda_i R^{1_q}t\right)-$$

$$-\sum_{q'\in Q}p_{q'}\pi_{q',q}\sum_{1\le|K|\le n}\sum_{i=1}^{l}\sum_{j=1}^{l}\frac{B^{K'}_{n,i,q'}a_{q,i}}{R^K\lambda_i-\lambda_j}\lambda_j\exp\left(-R^{K'}\lambda_i\right)\exp\left(-R^{K'+1_q}\lambda_i t\right)+$$

$$+\sum_{q'\in Q}p_{q'}\pi_{q',q}+B^0_{n,q'}\sum_{i=1}^{l}a_{q,i}\exp(-\lambda_i)\exp\left(-R^{1_q}\lambda_i t\right)\Big]=\sum_{1\le|K|\le n}\sum_{i=1}^{l}B^K_{n+1,i,q}\exp\left(-R^K\lambda_i t\right)=$$

$$=\sum_{q'\in Q}\sum_{i=1}^{l}B^{1_{q'}}_{n+1,i,q}\exp\left(-R^{1_{q'}}\lambda_i t\right)+\sum_{2\le|K|\le n+1}\sum_{i=1}^{l}B^K_{n+1,i,q}\exp\left(-R^K\lambda_i t\right).$$

So the formula (13) is true for $s=n+1$. Here for $i=1,...,l$, $1<|K|\le n+1$ we have the recurrent formulas (12) and for $i=1,...,l$, $|K|=1$ the recurrent formulas (11) and $B^0_{n+1,q}$ the equality (14).

4. ASYMPTOTIC FORMULAS

Using the complete probability formula we obtain

$$P(V_n>t)=\psi_n(t)=\sum_{q_1,\dots,q_n\in Q}\psi_{n,q_1,\dots,q_n}(t)\,p_{q_1}\pi_{q_1,q_2}\cdot\ldots\cdot\pi_{q_{n-1},q_n} \tag{16}$$

with

$$\psi_{n,q_1,\dots,q_n}(t)=P\left(V_n>t/Y_1=r^{-1}_{q_1},\dots,Y_n=r^{-1}_{q_n}\right).$$

(C) Suppose that $F_q(t)\in\mathcal{S}$, $q\in Q$, where $\mathcal{S}$ is the class of subexponential distributions. Assume that for any $q_1,q_2\in Q$, $q_1\ne q_2$ and for any positive a one of the following equalities is true

$$\overline{F}_{q_1}(t)=O\left(\overline{F}_{q_2}(at)\right)\text{ or }\overline{F}_{q_2}(at)=O\left(\overline{F}_{q_1}(t)\right),\ t>0. \tag{17}$$

Then using [3, Lemma 3.2] it is possible to obtain that

$$F_{q_1}(t)*F_{q_2}(at)\in\mathcal{S}\ ,\ \overline{F_{q_1}(t)*F_{q_2}(at)}\sim\overline{F}_{q_1}(t)+\overline{F}_{q_2}(at)\Big|,\ t\to\infty. \tag{18}$$

Here $F*G$ is a conjuncture of distributions F,G. Further we consider equivalences "~" only for $t\to\infty$.

Theorem 3. If the condition **(C)** is true then

$$\psi_n(t) \sim \sum_{k=1}^{n} \sum_{q_1,\dots,q_n \in Q} \overline{F}_{q_1}\left(t \prod_{i=1}^{n-k+1} r_{q_i}\right) p_{q_1} \pi_{q_1,q_2} \cdot \ldots \cdot \pi_{q_{n-k},q_{n-k+1}} . \tag{19}$$

Proof . It is obvious that

$$\psi_{1,q_1}(t) = P\left(Y_1 X_1 > t / Y_1 = r_{q_1}^{-1}\right) = P\left(X_1 > r_{q_1} t\right) = \overline{F}_{q_1}\left(r_{q_1} t\right).$$

Using the condition **(C)** and the formula (18) we obtain for $n > 1$ that

$$\psi_{n,q_1,\dots,q_n}(t) \sim P\left(V_{n-1} > r_{q_n} t / Y_1 = r_{q_1}^{-1},\dots,Y_n = r_{q_n}^{-1}\right) + P\left(X_n > r_{q_n} t / Y_1 = r_{q_1}^{-1},\dots,Y_n = r_{q_n}^{-1}\right) =$$

$$= \psi_{n-1,q_1,\dots,q_{n-1}}\left(r_{q_n} t\right) + \overline{F}_{q_n}\left(r_{q_n} t\right).$$

So an induction by n and the formula (16) give the equivalence

$$\psi_{n,q_1,\dots,q_n}(t) \sim \sum_{k=1}^{n} \overline{F}_{q_k}\left(t \prod_{i=k}^{n} r_{q_i}\right).$$

Consequently using the formula (16) it is easy to obtain the equivalence

$$\psi_n(t) \sim \sum_{q_1,\dots,q_n \in Q} p_{q_1} \pi_{q_1,q_2} \cdot \ldots \cdot \pi_{q_{n-1},q_n} \sum_{k=1}^{n} \overline{F}_{q_k}\left(t \prod_{i=k}^{n} r_{q_i}\right).$$

So the formula (19) is true.

Consider the following conditions.

1) There are positive numbers c_q, α_q, $q \in Q$, $\alpha_1 < \alpha_q$, $1 < q \le m$, so that $\overline{F_q}(t) \sim c_q t^{-\alpha_q}$.

2) There are positive numbers c_q, $q \in Q$, α, so that $\overline{F_q}(t) \sim c_q t^{-\alpha}$.

3) There are positive numbers c_q, β_q, $q \in Q$, $\beta_1 < \beta_q$, $1 < q \le m$, so that $\overline{F_q}(t) \sim \exp\left(-c_q t^{\beta_q}\right)$.

4) There are positive numbers c_q, $q \in Q$, β, so that $\overline{F_q}(t) \sim \exp\left(-c_q t^{\beta}\right)$ and $c_1 r_1^{\beta} < c_q r_q^{\beta}$, $q \in Q$, $q \ne 1$.

It is easy to prove that the family $F_q(t)$, $q \in Q$ under each of the conditions 1) - 4) satisfies the condition **(C)**.

Using Theorem 3 it is possible to obtain the following statements. If the condition 1) is true then

$$\psi_n(t) \sim c_1 p_1 (r_1 t)^{-\alpha_1} \sum_{k=1}^{n} S_{n-k+1},$$

with

$$S_1 = 1,\ S_2 = \sum_{q_2 \in Q} S_{2,q_2},\ S_{2,q_2} = \pi_{1,q_2} r_{q_2}^{-\alpha_1},$$

$$S_1 = \sum_{q_i \in Q} S_{i,q_i},\ S_{i,q_i} = \sum_{q_{i-1} \in Q} S_{i-1,q_{i-1}} \pi_{q_{i-1},q_i} r_{q_i}^{-\alpha_1},\ 2 < i\ . \tag{20}$$

If the condition 2) is true then

$$\psi_n(t) \sim t^{-\alpha} \sum_{k=1}^{n} T_{n-k+1}$$

with

$$T_1 = \sum_{q_1,q_1 \in Q} T_{1,q_1,q_1},\ T_{1,q_1,q_1} = c_{q_1} p_{q_1} r_{q_1}^{-\alpha},$$

$$T_i = \sum_{q_1,q_i \in Q} T_{i,q_1,q_i},\ T_{i,q_1,q_i} = \sum_{q_1,q_{i-1} \in Q} T_{i-1,q_1,q_{i-1}} \pi_{q_{i-1},q_i} r_{q_i}^{-\alpha},\ 1 < i\ . \tag{21}$$

The formulas (20), (21) show that to find asymptotic constants in the conditions 1), 2) it is necessary to use number of arithmetical operations proportional to n. If the condition 3) is true then

$$\psi_n(t) \sim p_1 \exp\left(-c_1 (r_1 t)^{-\beta_1}\right).$$

If the condition 4) is true then

$$\psi_n(t) \sim p_1 \exp\left(-c_1 (r_1 t)^{\beta}\right).$$

5. GENERATION OF TRANSITION MATRICES FOR STATIONARY AND REVERSIBLE MARKOV CHAINS

Consider stationary and reversible Markov chain Y_n, $n| \geq 0$, with stationary distribution p_q, $q \in Q$. Then its transition matrix $\left\|\pi_{i,q}\right\|_{i,q=1}^{m}$ satisfies the equalities:

$$A_{i,j} = A_{j,i} > 0\,,\ \sum_{j=1}^{m} A_{i,j} = p_i = \sum_{j=1}^{m} A_{j,i}\,,\ 1 \leq i, j \leq m\ . \tag{22}$$

where $A_{i,j} = p_i \pi_{i,j}$. So symmetric matrix $\left\|A_{i,j}\right\|_{i,j=1}^{m}$ with positive elements is a permissible solution of the transportation problem (22) with n sources and n consumers. If we have the problem (22) solution $\left\|A_{i,j}\right\|_{i,j=1}^{m}$ then it is possible to find the transition matrix $\left\|\pi_{i,j}\right\|_{i,j=1}^{m}$ using the formula $\pi_{i,j} = A_{i,j} / p_i$.

Each permissible solution of the transportation problem (2) may be found by the following sequence of algorithms.

The algorithm $\{p_1, ..., p_m\}$ generates $A_{1,1}, ..., A_{1,m}$ so that

$$0 < A_{1,1} < p_1, ..., 0 < A_{1,m} < p_m\,,\ \sum_{k=1}^{m} A_{1,k} = p_1$$

and put $A_{2,1} = A_{1,2}, ..., A_{m,1} = A_{1,m}$ and redefines $p_1 := p_1 - p_1 = 0$, $p_2 := p_2 - A_{1,2}, ..., p_m := p_m - A_{1,m}$.

As a result the transportation problem (2) with n sources and n consumers is transformed into the transportation problem

$$A_{i,j} = A_{j,i} > 0\,,\ \sum_{j=2}^{m} A_{i,j} = p_j\,,\ 2 \leq i, j \leq m\ . \tag{23}$$

with $n-1$ sources and $n-1$ consumers. So the algorithm $\{p_1, ..., p_m\}, \{p_2, ..., p_m\}, ..., \{p_{m-1}, ..., p_m\}$ generates arbitrary solution of the transportation problem (22).

The algorithm $\{p_1, ..., p_m\}$ consists of m steps.

Step 1. Define $A_{1,1}$ from the inequalities $0 < A_{1,1} < p_1$, $p_1 - A_{1,1} < p_2 + ... + p_m$ and put $p_1 := p_1 - A_{1,1}$.

Step 2. Define $A_{1,2}$ from the inequalities $0 < A_{1,2} < p_1$, $0 < A_{1,2} < p_2$, $p_1 - A_{1,2} < p_3 + ... + p_m$ and put $p_1 := p_1 - A_{1,2}$.

Step m-1. Define $A_{1,m-1}$ from the inequalities $0 < A_{1,m-1} < p_1$, $0 < A_{1,m-1} < p_{m-1}$, $p_1 - A_{1,m-1} < p_m$ and put $p_1 := p_1 - A_{1,m-1}$.

Step m. Define $A_{1,m-1} = p_1$

The algorithm $\{p_1,...,p_m\},\{p_2,...,p_m\},...,\{p_{m-1},...,p_m\}$ is a modification of the algorithm for constructing the routing matrix for an open network [4, p.177].

6. ENUMERATION PROBLEM

Assume that vector K consists of components 0,1,… and its dimension is dim $\dim K$. Introduce the sets of vectors $\mathrm{K}_i^j=\{K:\dim K=j,|K|=i\}$ and designate $|\mathrm{K}_i^j|$ number of vectors in the set $\mathrm{K}_i^j, i\ge 0, j\ge 1$. Our purpose is to enumerate all vectors K of the set

$$\{K:\dim K=m,1\le|K|\le n\}=\bigcup_{i=1}^{n}\mathrm{K}_i^m.$$

It is easy to construct algorithm to define the set K_i^m from the set K_{i-1}^m as follows: $\mathrm{K}_i^m=\{K+1_q: K\in\mathrm{K}_{i-1}^m, q=1,...,m\}$. But a complexity of this algorithm is proportional to m^n and it may generate coinciding vectors. So it is worthy to construct more efficient algorithm for example with power by n complexity.

It is obvious that K_i^{j+1} is a union of nonintersecting sets

$$\mathrm{K}_i^{j+1}=\bigcup_{k_{j+1}=0}^{i}\{(K,k_{j+1}):K\in\mathrm{K}_{i-k_{j+1}}^j\} \tag{24}$$

and consequently

$$|\mathrm{K}_i^{j+1}|=\sum_{t=0}^{i}|\mathrm{K}_{i-t}^j|=\sum_{t=0}^{i}|\mathrm{K}_t^j| \text{ where } j\ge 1, i\ge 0. \tag{25}$$

Here K_0^j consists of single j- dimensional vector with zero components, $|\mathrm{K}_0^j|=1$ and $\mathrm{K}_0^j\le\mathrm{K}_1^j\le...\le\mathrm{K}_i^j$. From the formula (25) we have by induction that

$$|\mathrm{K}_i^{j+1}|=\sum_{t=0}^{i}|\mathrm{K}_t^j|\le(i+1)\mathrm{K}_i^j\le(i+1)^{j+1}. \tag{26}$$

As K_i^1 consists of single one dimensional vector i so to find the set K_n^m using the formula (24) we construct the sequence of the sets

$$\mathrm{K}_0^1,\mathrm{K}_0^2,...,\mathrm{K}_0^m;$$
$$\mathrm{K}_1^1,\mathrm{K}_1^2,...,\mathrm{K}_1^m;$$
$$.....................;$$
$$\mathrm{K}_n^1,\mathrm{K}_n^2,...,\mathrm{K}_n^m.;$$

Consequently complexity of this algorithm is not larger than $(n+1)^{m+1}$.

7. SOLUTION OF SMALL DENOMINATORS PROBLEM

Consider how to find $s_1,...,s_m,\nu_1,...,\nu_l$ so that for any fixed $\varepsilon>0$ the inequalities

$$|s_1-r_1|<\varepsilon,...,|s_m-r_m|<\varepsilon,|\nu_1-\lambda_1|<\varepsilon,...,|\nu_l-\lambda_l|<\varepsilon \tag{27}$$

are true and for any K we have an analogy of the condition (9):

$$S^K\nu_i\ne\nu_j\ 1\le i,j\le l,1\le|K|, \tag{28}$$

where $S=(s_1,...,s_m)$. For this aim we take integer b so that $2^b<\varepsilon$ and choose fractions $s_i=\frac{v_i}{2^b}, i=1,...,m,\ \nu_j=\frac{u_j}{2^b}, j=1,...,l$, with odd numerators v_i so that the formula (27) is true. Then the formula (28) takes place.

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THE ANALYSIS OF JOINT CONDITIONS OF POWER BLOCK OF A STATE DISTRICT POWER STATION

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ABSTRACT

It is displayed, that the specific number and average duration of joint estates of power-generating units of the state district power stations calculated analytically on indexes of reliability of power-generating units is essential exceed direct experimental data. Principal causes of a divergence are suppositions about equal reliability and undercount of run-down states of power-generating units. The pointed divergence leads to magnifying of an emergency standby of power.

As a joint condition of power block (PB) a state district power station during the any moment of time t we shall understand realization of set of possible combinations of their workers (PB it is included in a network) and non-working (PB is disconnected) conditions. If to designate number PB through n_b, that j-th realization of this set, be presented as sequence of conditions of station numbers PB placed in ascending order $S_{1,j}, S_{2,j}, ... S_{n_b}$. Joint conditions PB are defined by process of change of a condition of everyone PB and interrelation of conditions PB

Non-working conditions are subdivided into following types:

- emergency idle time. This condition takes place at emergency switching-off PB for the reasons, not dependent on reliability of its equipment and devices (for example, at power interruptions, absence of fuel, short circuits on the switching centre where it is connected PB, etc.);
- sudden emergency repair. PB it is under abnormal condition disconnected owing to refusal of its equipment or devices. Switching-off occurs automatically or under abnormal condition manually and defines an emergency reserve of capacity. Sudden emergency switching-off can occur both at sudden refusals, and at the refusals caused by deterioration;
- emergency repair at switching-off PB under the emergency application. Consequences of switching-off PB in this condition, as a rule, it is essential below, than at sudden switching-off;
- emergency repair owing to refusal at start-up PB. Refusals at start-up are caused by poor-quality repair or lacks of a retentively of the equipment at idle time PB;
- emergency repair at repeated refusals. Occurs owing to poor-quality repair or the insufficient control of a technical condition;
- cold reserve.
- scheduled average repair;
- scheduled major overhaul.

Let's note one prominent feature of process of change of conditions PB. With a view of decrease number of start-up on power stations translation PB from one non-working condition in another practices. For example, PB from a cold reserve it is translated in emergency repair at detection as a result of test of the equipment and devices PB of implicit defects. From scheduled repair PB can be deactivated, etc.

Now the state district power stations have the unique statistical data describing changes previous the present moment of conditions PB. On the basis of these data can be calculated as the average parameters of reliability (PR) PB a state district power station, and parameters of individual reliability. These data allow define and PR, describing joint conditions PB. However PR joint conditions, owing to labors input of the manual account and absence of the corresponding software,

are estimated by settlement methods for which are characteristic a number of assumptions quite often far from the validity [2]. These PR have as independent value, and form a basis of specification of analytical methods of calculation, methods of imitating modeling of process of change of conditions PB at forecasting reliability of a state district power station, the analysis of dynamics of change of a necessary reserve of capacity. Great volumes of the information, bulkiness and labors input of the manual account, possible mistakes demand development of specialized algorithms and programs of the analysis of statistical data and estimation PR of joint conditions PB, as component of the automated system of the analysis of reliability of a state district power station [2].

Algorithm of recognition type of joint condition PB and calculation of parameters of reliability.

The integrated block diagram of algorithm is resulted on fig.1, and on fig.2 time diagrams of some variants of joint conditions PB are shown.

Block 1. The set version of sequence of joint conditions (SJC) gets out of the menu. The menu provides an opportunity of recognition and calculation:

- numbers and duration of a finding in the disconnected condition of several PB. In particular, specific number, total duration and average duration of joint conditions, relative size the electric power, operating ratio of working capacity. As it has noted been [1], these parameters are calculated for any way set interval of time;
- number and duration of a finding in the same non-working condition of several PB;
- numbers of simultaneous switching-off of several PB;
- laws of change in time PR of joint conditions.

Block 2. The algorithm provides input of following data:

- interval of time during which there is an analysis of joint conditions PB (ΔT). For example, an interval of the autumn-winter period;
- number PB (n_b);
- certain in ascending order the moments of change of conditions of everyone PB sequence of conditions (SC) all PB a state district power station. The information file of each condition includes: date and time of the beginning and the end of conditions, number of the block, type of a condition, a kind of switching-off, the name of the given up installation and a number of other data. This information is concentrated in empirical tables of a database [1].

Block 3. The current number PB, forming a joint condition is defined.

Block 4. From SC gets out next i- th a condition, where i=1, m_c, m_c-number of conditions

Block 5. If this first condition from SC, management transferred the *block 6*, otherwise-to the *block 21*.

Block 6. The moment of the beginning of the first condition $T_{H1} \geq T_H$ is registered.

Block 7. Duration of a joint condition, as a difference between the moments of occurrence (T_{Hi}) and end (T_{Kj}) this condition is defined.

Block 8. In the empirical table data on joint condition PB are brought. These data include: date and time of the beginning and the termination of a joint condition, its duration, number PB, being a non-working condition, station and a serial number disconnected PB, with the instruction of type of a condition and a kind of switching-off of everyone PB.

Block 9. The moment of the beginning of the new joint condition T_{Hj} equal to the moment of end of the preceded joint condition $T_{K,(j-1)}$ is fixed.

Block 10. If joint non-working conditions during the moment T_{Hj} are absent, management is transferred the block 11. Otherwise – to the *block 12*.

Block 11. In this block end of consideration of all non-working conditions SC is supervised. If $i \leq m_S$, management is transferred the *block 3*. Otherwise – to the *block 23*, where the analysis of statistical data SJC PB is carried out.

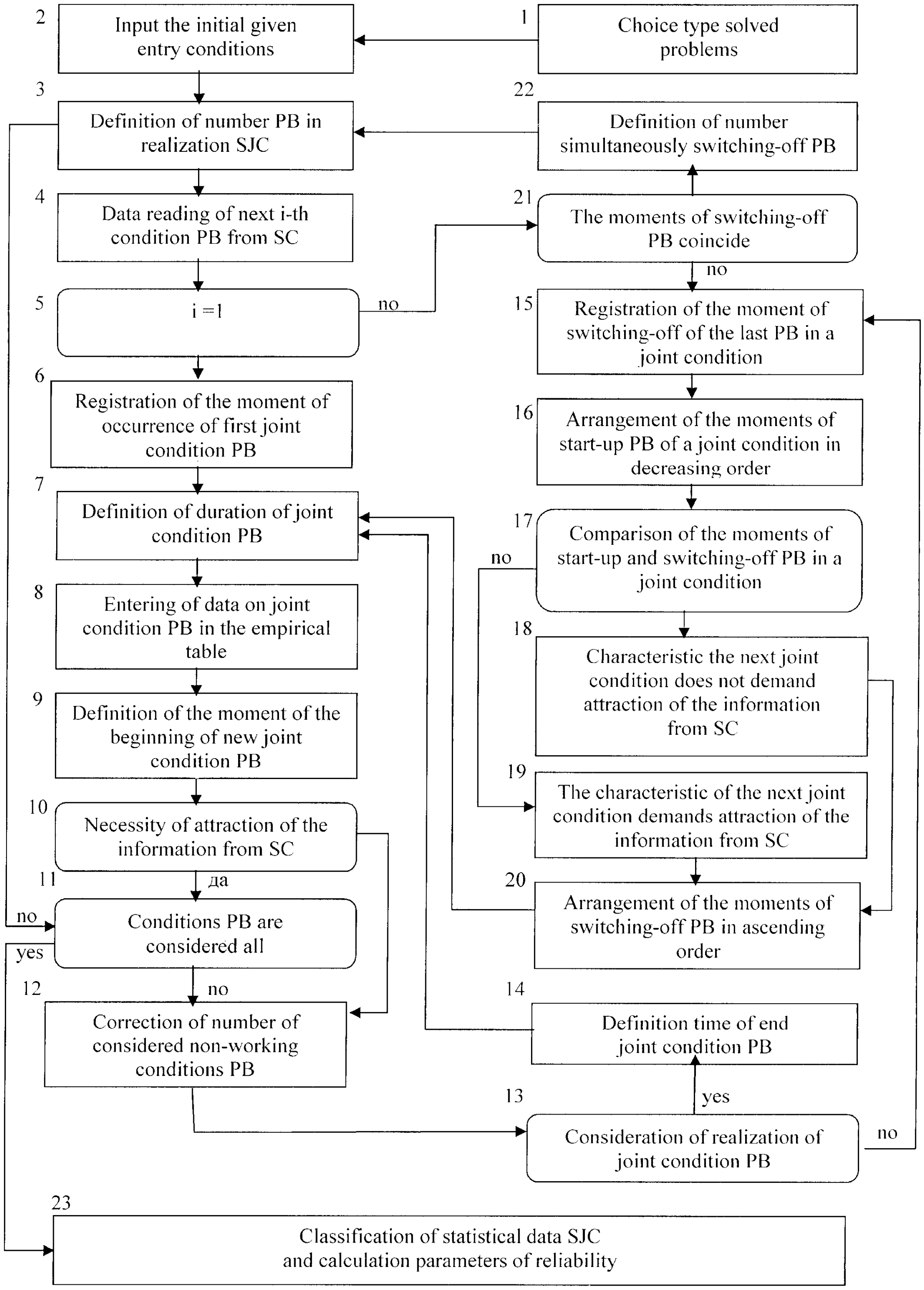


Fig. 1 Integrated block diagram of algorithm of recognition of type of joint condition PB

Block 12. Joint non-working conditions PB presented by set of consistently occurring various joint conditions PB. So, on the time diagram fig. 1в on an interval $(T_{K1} - T_{K5})$ three joint non-working conditions from which one includes three non-working conditions PB and two – two non-working conditions PB take place. And on the time diagram fig.2c took place two joint non-working conditions of two PB and one (on an interval $(T_{H4} - T_{K3})$) overlapping of three non-working conditions PB. In the *block 12* correction of number of the remained joint conditions made.

Block 13. If all joint conditions are considered, management is transferred *the block 14* for calculation of time of last joint condition with the subsequent transfer of management to the block 17. Otherwise - to the block 15. *Blocks 15-19* are intended for the characteristic of next joint condition ЭБ.

Block 15. The moment of last switching-off PB in a joint condition is defined.

Block 16. Accommodation of the moments of start-up PB of a joint condition in decreasing order is spent.

Block 17. Comparison of the moments of start-up $T_{K\xi}$ and switching-off PB in a joint condition is spent. If the moment of start-up does not exceed the moment of a stop corresponding PB management is transferred the *block 18* where duration of a condition is calculated, the moment of the termination of a condition is fixed and, the main thing, is registered absence of necessity of attraction of the new information from SC for the characteristic of the subsequent joint condition. Otherwise – to the *block 19*

Block 19. It is inherently similar to the *block 18* with that difference that in this block necessity of attraction of data from SC about next non-working condition PB is marked.

Block 20. Here it is restored initial SC the considered joint condition PB, changed in the *block 16.* Then management transferred the *block 17*.

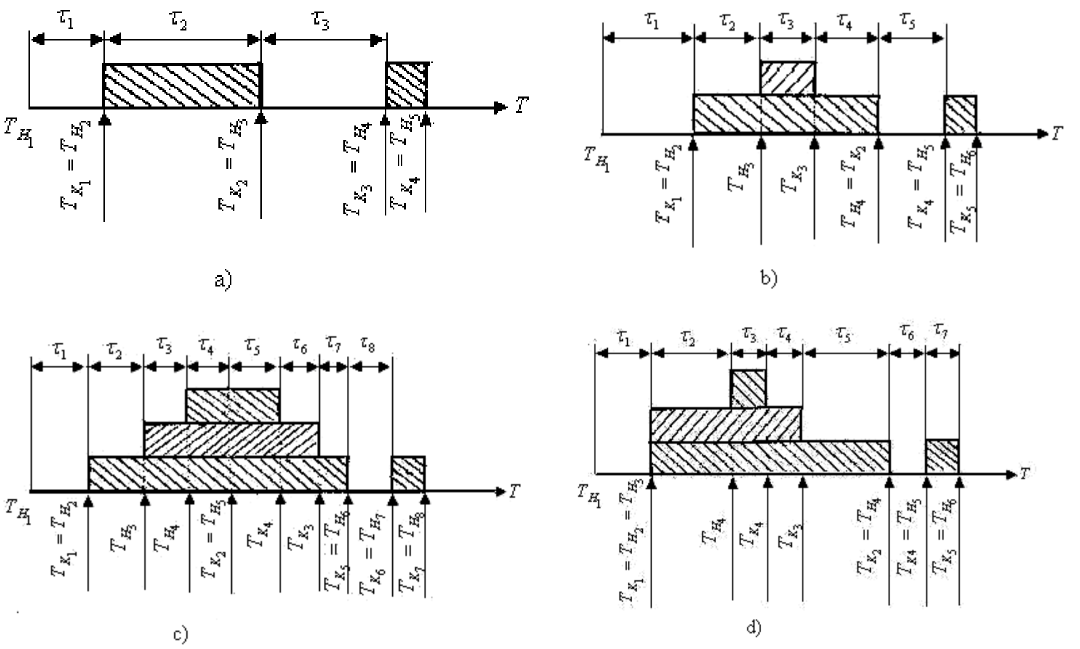


Fig.2. Time diagrams of joint conditions PB a state district power station.

Block 21. At power interruptions (increase of frequency) or at short circuits on the switching centre 110 kv. also simultaneous switching-off of some PB are above possible. The indicator of simultaneous switching-off of some PB is equality of the moments of switching-off PB. If for two adjacent conditions PB the moments of occurrence of non-working conditions coincide, the number simultaneously disconnected PB (reference value L=1) increases for unit (*the block 22*) and management is transferred to the *block 3* that allows to define the general number simultaneously disconnected PB. Otherwise, management transferred to the *block 15*.

Block 23. Classification of statistical data SJC spent according to the *block 1* where research problems formulated

To classification attributes concern:

1. Number PB (i), simultaneously being a non-working condition (i=0,n_b). The version of an attribute i=0 corresponds to a case when all PB are in working order.

2. Type of non-working condition PB by way of their stop. At coding versions of attributes the identifier excluding classification to this attribute is entered.

3. A kind of switching-off PB by way of their stop. We shall distinguish sudden switching-off (automatically or under abnormal condition manually), switching-off under the emergency application and scheduled switching-off. By analogy to versions of type of switching-off the identifier shunting this attribute is entered.

These attributes allow classifying SJC for an estimation of a real number of joint conditions under the list of solved problems. In particular, it is possible to estimate number of joint conditions i PB, deduced in emergency repair, or simultaneously disconnected at a power interruption, the number of emergency switching-off PB during a finding of one of PB in a reserve, etc. should be had in view of, that the number of joint non-working conditions PB of the set type insufficiently full characterizes reliability of work PB. If to address to fig.2b it is easy to notice, that on an interval $(T_{H2} - T_{K2})$ three joint conditions took place, in two of which one has been disconnected PB, and in the third – two PB. But from the point of view of switching-off PB one of n_b PB has originally been disconnected, and then, during emergency repair – one more was disconnected PB. I.e. differing joint conditions can arise both as a result of switching-off PB, and at start-up PB. From the point of view of duration of conditions and the general number of arising joint conditions spent by algorithm fig.1 classification is authentic. To define number of cases of occurrence of the joint conditions connected only with switching-off PB, it is necessary from the general number of joint conditions i PB to subtract the general number of cases of joint conditions (i+1) PB with $i \geq 2$ provided that the conditions caused by simultaneous switching-off of some PB are excluded from consideration, connected with failures in system. On the basis sample of the empirical table of a joint condition of j-th type assuming a finding in non-working condition $n_{b,j}$ PB, following PR are calculated:

- Estimation of specific number of joint conditions of j-th type under the formula

$$\lambda^*_{jc,j} = \frac{\sum\limits_{\nu=1}^{C_{n_b}^{n_{b,j}}} n_{jc,j,\nu}}{\sum\limits_{\nu=1}^{C_{n_b}^{n_{b,j}}} T_{w.\Sigma,\nu}} \qquad (1)$$

where $n_{jc,j,\nu}$ - number of joint conditions of j-th type and ν -th combination PB;

$T_{w.\Sigma,\nu}$ - total duration of working condition $n_{b,j}$ PB on the considered period of time (ΔT).

$$T_{w.\Sigma,\nu} = n_{b,\nu} \cdot T_{sr,\Sigma,\nu} - T_{cr,\Sigma,\nu} + \sum_{i=2}^{n_{b,j}} (i-1) \cdot T_{jc.\Sigma,\nu,i} \qquad (2)$$

$T_{sr.\Sigma,\nu}$ - total duration of scheduled repairs $n_{b,j}$ PB for ν -th combination; $T_{cr.\Sigma,\nu}$ - total duration of a condition of cold reserve $n_{b,j}$ PB for ν -th combination; $T_{jc.\Sigma,\nu,i}$ - total duration i joint conditions PB disconnected on scheduled repair or in a cold reserve for ν -th combination PB;.

- average duration of a joint condition of j-th type - under the formula

$$M^*(\tau_{jc,j}) = \frac{\sum\limits_{\nu=1}^{C_{n_b}^{n_{b,j}}} \sum\limits_{i=1}^{n_{jc,j}} \tau_{jc,j,i,\nu}}{\sum\limits_{\nu=1}^{C_{n_b}^{n_{b,j}}} n_{jc,j,\nu}} \qquad (3)$$

where $\tau_{jc,j,i,\nu}$ - realization of duration of a joint condition of j-th type

- an estimation of probability of occurrence of a joint condition of j-th type – under the formula

$$K^*_{jc,j} = \frac{\sum_{\nu=1}^{C^{n_b,j}_{n_b}} \sum_{i=1}^{n_{ss,j}} \tau_{jc,j,i,\nu}}{\sum_{\nu=1}^{C^{n_b,j}_{n_b}} T_{w,\Sigma,\nu} + \sum_{\nu=1}^{C^{n_b,j}_{n_b}} \sum_{i=1}^{n_{jc,j,\nu}} \tau_{jc,j,i,\nu}} \qquad (4)$$

The size $K^*_{\Gamma,jc,j} = 1 - K^*_{jc,j}$, as a matter of fact, represents analogue of a known parameter in practice – factor of readiness, with that difference that event of refusal of one PB, and event of occurrence of a joint condition of j-th type (an estimation of probability of is considered not that on the set interval of time simultaneous emergency repair $n_{b,j}$ PB will not occur).

We shall consider sequence of work of algorithm for each of presented on fig.2 variants of joint conditions PB. With a view of decrease in the bulkiness, carried out calculations we shall present sequence of numbers of blocks of algorithm.

Example 1 (fig.2a). The simple case – switching-off of one PB during the moment T_{H2} with duration of idle time τ_2 is considered most.

$\tau_1 \rightarrow$ 1,2,3,4,5,6,7,8,9,10,11,3
$\tau_2 \rightarrow$ 3,4,5,21,15,16,17,18,20,7,8,9,10,12
$\tau_3 \rightarrow$ 12,13,14,7,8,9,10,3

Example 2 (fig.2b). In this example the case when during idle time of one PB another is disconnected PB is considered. And duration of its idle time does not exceed residual duration of idle time before disconnected PB

$\tau_1 \rightarrow$ (look τ_1. an example 1)
$\tau_2 \rightarrow$ 3,4,5,21,15,16,17,19,7,8,9,10,11
$\tau_3 \rightarrow$ 12,13,14,7,8,9,10,12
τ_4 and $\tau_5 \rightarrow$ (look τ_3 an example 1)

Example 3 (fig.2c). In this example number PB, being simultaneously in a non-working condition to equally three. Besides first of disconnected PB (for example, in a reserve) during the moment T_{H5} has been translated in other type of a condition (for example, at its start-up there was a refusal of one of elements and PB has been deduced in emergency repair). The algorithm of calculation τ_1 also τ_2 is similar to calculation of intervals τ_1 and τ_2 an example 1, the algorithm of calculation $\tau_4 \div \tau_5$ is similar to calculation τ_3 of an example 2, and the algorithm of calculation $\tau_6 \div \tau_8$ is similar to calculation τ_3, an example 1.

Example 4 (fig. 2d). In this example during the moment T_{H2} there is a simultaneous switching-off of two PB and further in an interval of their idle time there is a switching-off of the third PB. The sequence of calculation of duration of joint conditions PB τ_2 looks like:

$\tau_2 \rightarrow$ 3,4,5,21,22,3

Calculation of duration of conditions $\tau_3 \div \tau_6$ is similar to calculation, accordingly, conditions τ_4, $\tau_6 \div \tau_8$ fig.2c.

Thus, despite of essential distinction of variants of the joint conditions represented on fig. 2, sequence of calculation of duration of set of joint conditions it is same and, in essence, are reduced to calculation of duration of joint conditions, when:

- all PB are in work;
- the number simultaneously being in due course increases in disconnected condition PB;
- the same, but decreases;
- simultaneous switching-off of some PB takes place.

SOME RESULTS OF CALCULATIONS.

The purpose of these calculations are, first of all, quantitative characteristics of joint conditions and revealing of features which should be considered at imitating modeling process of change SJC with the purpose of forecasting of reliability of a state district power station. In table 1 the average parameters of joint non-working conditions PB for a number of years of supervision, and in table 2 similar parameters for one year of operation are resulted. Data of these tables confirm intuitively clear values of number of joint conditions PB. Even for one year of operation their number is estimated in hundreds.

Is of interest laws of change specific number of joint non-working conditions and average duration of these conditions depending on number simultaneously disconnected PB. It would seem, the number simultaneously being disconnected condition PB is more, the specific number of such joint conditions will be less. The same assumption can be made and concerning average duration of joint conditions. However, results of the analysis (see tab.1 and tab.2) testify to an inaccuracy of these assumptions. As the number of joint conditions is great enough, these features are observed both in tab.1, and in tab.2 they obviously cannot be explained only casual character of estimations of these parameters. As to relative duration of joint conditions here our representations about its monotonous decrease with growth of number of simultaneously disconnected conditions PB prove to be true.

Table 1

Characteristic of number and duration of a finding in disconnected condition of several power block

Number of simultan. switch-off PB	Total duration of condition, h	Number of condition	Average of conditions (con.year)	Average duration of a condition,h	Relative duration of a condition,h	Relative size not developed the electric power, %
0	5918,8	147	21	40,3	9,64	0
1	18765,6	336	48	55,9	30,58	3,82
2	13603,6	381	54	35,7	22,17	5,54
3	13094,3	330	47	39,7	21,34	8
4	8205,5	182	26	45,1	13,37	6,69
5	1395,5	49	7	28,5	2,27	1,42
6	213,8	10	1	21,4	0,35	0,26
7	14,5	1	0	14,5	0,02	0,02
8	0,4	1	0	0,4	0,001	0,001
Operating ratio of working capacity of station – 74,2%						

Table 2

Characteristic of number and duration of a finding in disconnected condition of several power block

Number of simultan. switch-off PB	Total duration of condition, h	Number of condition	Average of conditions (con.year)	Average duration of a condition,h	Relative duration of a condition,h	Relative size not developed the electric power, %
0	451,3	15	15	30,1	5,15	0
1	3040,7	29	29	104,9	34,71	4,34
2	1730,2	54	51	33,9	19,72	4,94
3	2811,2	64	64	43,9	32,09	12,03
4	1032,1	29	29	35,6	11,78	5,89
5	8,9	3	3	3	0,0	0,06
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
Operating ratio of working capacity of station – 72,7%						

Is of interest also laws of change of relative size not developed the electric power during the periods of switching-off of several PB. Here there are the laws which are distinct from laws of change of specific number and average duration of joint conditions PB. If we shall subtract from unit total value of relative size not developed the electric power on all joint conditions we shall receive a component of operating ratio of the established generating capacity (K_R) on a state district power station caused by switching-off PB. Results of calculations show, that this size according to tab.1 makes 74,2%, and according to tab.2 – 72,7%. If to calculate K_R as the attitude of the electric power developed for the considered period to product of duration of the settlement period and total capacity PB we shall receive accordingly K_R=75,1% and K_R=74,7%. Hence, K_R approximately on 95% depends on joint non-working conditions PB. Thus, increase КИ can be reached, first of all, by decrease in specific number and average duration unforeseen by a production schedule of non-working conditions PB.

Feature tab.1 and tab.2 is independence of a joint condition of type of switching-off PB

Considering casual character of occurrence of refusals, joint emergency repair of some PB can arise in current of all period of supervision. However the probability of a joint finding in emergency repair PB depends as on number disconnected PB in a cold reserve and on scheduled repair, and from results of restoration of deterioration.

One of versions of joint conditions is simultaneous switching-off PB at the power interruptions connected with infringement of parallel work of adjacent power supply systems and automatic switching-off of significant loading, and at short circuits on trunks 110 kv. and above switching centers of a state district power station. The number of such switching-off changes from $(1 \div n_b)$, however the probability of occurrence of the set number simultaneously disconnected PB, as one would expect, is various.

Let us consider known methodology of analytical calculation of number and duration of a finding in a condition of emergency repair of several PB and it is comparable these estimations to real data. According to one year of operation 39 emergency switching-off of eight PB with the average duration of restoration equal 67h were observed. In seven cases overlapping emergency repairs of two PB took place. Hence, on the average occurred 39/8=4,9 emergency switching-off of everyone PB in a year (the first assumption: parameters of reliability of all PB the same). Total duration of idle time of one PB is equal emergency repair 4,9·67=326,6h. On this interval remained seven PB could give up 326,3·7·4,9/8760=1,28 sw./year (the second assumption: all PB, except for given up, are in work). For eight PB the general number совмещений conditions of repairs of two PB will be equal 1,28·8=10,2 sw./year. The valid size is equal 7sw./year. Average duration of a condition of simultaneous finding PB on statistical data is equal emergency repair 22h., and the settlement size makes 67/2=33,5h. If to consider the second assumption noted divergence of estimations decreases.

Let's lead similar calculations for a joint condition in which two PB are deduced in a reserve. For the same year of operation it was observed 51 switching-off PB in a reserve with average duration of a condition 176h. On the average, each of blocks was disconnected in a reserve 51/8=6,4h. Total duration of idle time PB is equal a cold reserve of one 6,4·176=1126,4h. On this interval remained seven PB could be disconnected (1126,4·7·6,4)/8760=5,8h. For eight PB the general number of the combined conditions of a reserve of two PB is equal 5,8·8≈46h. The real number of joint conditions of a cold reserve of two PB for the considered period equally 37. Average duration of a simultaneous finding of two PB in a condition of a cold reserve equally 66h., and settlement value is equal 176/2=88h. Here, as well as in preceded calculation it is supposed, that during switching-off PB in a cold reserve all PB are in work. Besides non-uniformity of switching-off PB in a cold reserve within a year is not considered. In particular, the analysis of time diagrams of switching-off PB in a reserve has shown a state district power station that the specific number of switching-off of two PB in a reserve during march-september increases, and cases of switching-off in a reserve of three PB occurs only during the period specified above. This process has mainly determined character with that difference, that in a cold reserve the least reliable are disconnected and the least economic PB. For this reason switching-off concrete PB in a cold reserve is event casual, depending from technical condition PB. The condition of a cold reserve is often used for carrying out of operating repair, elimination of defects which liquidation demands small time, carrying out of preventive tests of the equipment and devices.

Let's consider one more characteristic example of calculation. We shall assume, that one of PB is deduced in a cold reserve and during a finding of it PB in a cold reserve there is a switching-off of one of remained PB in emergency repair. It is necessary to estimate parameters of reliability of this condition and it is comparable them to statistical data. This example is interesting to that allows to check up the assumption of casual character of refusals PB. We shall take advantage of data of previous calculations. Total duration of idle time of one PB is equal a cold reserve 1126,4h. On the average, 4,9 emergency switching-off of everyone PB were observed. Hence, number of emergency switching-off PB during a finding of one of PB in a cold reserve equally 1126,4*4,9*7*8/8760=35,3 sw./year. According to operation it was observed 23 sw./year. Average duration of this condition pays off as an average geometrical длительностей conditions of emergency repair and a condition of a cold reserve, i.e. 176*67/(176+67)=48,5h. Statistical data testify that average duration of a considered condition is equal 30h.

Comparison of settlement and experimental estimations of parameters of joint conditions of eight power block 300 Mwt is resulted by capacity in tab.3, These data as a whole testify that the error of calculation makes tens percent, and settlement estimations of parameters of reliability are exceeded by operational data. operational settlement

Table 3.

Comparison of settlement and experimental estimations of parameters of joint conditions of power block

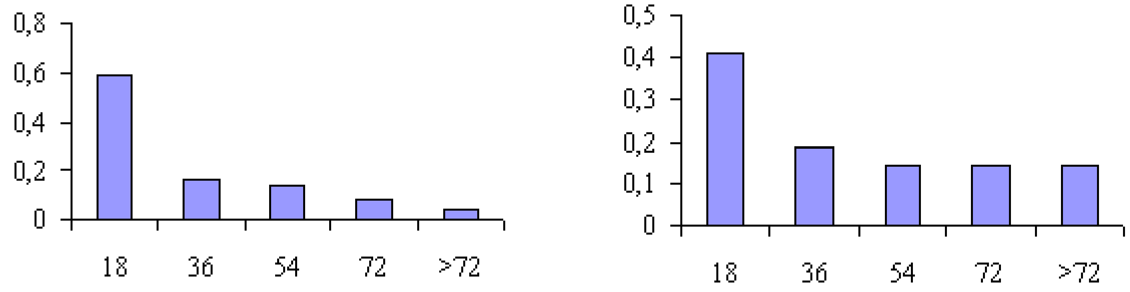
Condition PB	Parameter of a condition			
	Specific number		Duration of a condition	
	operation	settlement	operation	settlement
Emergency repair of one PB	4,9	-	67,0	-
Cold reserve of one PB	6,4	-	176,0	-
Overlapping of emergency repair of two PB	7,0	10,2	22,0	33,5
Overlapping of a cold reserve of two PB	37,0	46,0	66,0	88,0
Overlapping of cold reserve PB with emergency repair of another PB	23,0	35,3	30,0	48,5

The observable divergence of results of calculation is caused both functional and statistical by components. The importance of a functional component is characterized by a degree of distinction of real process of display of a joint condition of the set type and used model, distinction of parameters of individual reliability PB. Really, the distinction of the same parameters of reliability PB is more, the less estimation of the parameters describing joint condition PB. The statistical component is most full shown by consideration of improbable events. For example, calculation average duration of a joint condition of emergency repair three and greater number PB. Influence of a statistical component can be lowered according to SJC for some years of supervision.

However the increase in the period of supervision is not always expedient, since thus the new assumption of constant reliability of everyone PB in this period is entered. In conditions when service life exceeds settlement, this assumption, as a rule, is unacceptable. Change of parameters of reliability before capital and average scheduled repairs is not considered also.

Fluctuations of distinction of compared estimations PR can be lowered if even to consider distinction PR PB and finding PB in a non-working condition when the analyzed joint condition appears impossible. At the same time, the account of these and of some other features leads to bulkiness, labors input an analytical method, including «green light» for development and uses of imitating model.

Alongside with dot estimations of parameters of number and duration of joint non-working conditions and, in particular, conditions of emergency repair are of interest their functions of distribution of duration and law of change of number of joint conditions in time. On fig.3a,b histograms of change of duration of a joint finding of two PB in emergency repair and in a cold reserve are shown. Asymmetry of distribution of duration of emergency repair of two PB (fig.3a) speaks aspiration of operation personnel as much as possible to accelerate restoration and more quickly to put into operation one of PB. During the years period at presence of a cold reserve instead of damaged PB, as a rule, it is entered into work PB, being in a cold reserve. And after restoration of damage under abnormal condition disconnected PB, as a rule, it is translated in a condition of a cold reserve. Distribution of duration of a joint finding of two PB in a condition of a cold reserve also is asymmetric (see fig.3b). However, asymmetry of distribution in this case speaks dependence of duration of a joint condition on a season. The greatest values of duration it is observed the years period, and the least - in the spring and autumn periods. The analysis of data SJC testifies that the probability of occurrence of simultaneous emergency repair of two PB does not depend on a season, however changes essentially on years. It appears appreciable at end капительных repairs PB and especially appreciable after modernization PB.



a) b)

Fig. 3 Histograms of change of duration of a joint finding of two PB in emergency repair (a) and in a cold reserve (b).

CONCLUSION

1. The algorithm and the program of the analysis of joint conditions PB according to about conditions of everyone PB are developed. The program is a component of the automated system of the analysis of technical condition PB a state district power station [2].
2. Comparison of operational data and results of analytical calculation testifies to their essential divergence. A principal cause to that is not the account distinction of parameters of reliability PB, opportunities of finding PB in scheduled repair and a cold reserve, interrelation of conditions PB on years, etc. Such divergence is natural non-uniformity of occurrence of joint conditions, as the account of these features much more increases bulkiness of analytical formulas, labors input of calculation, probability of occurrence of mistakes.
3. Direct comparison of experimental and settlement data, fluctuation of their parity in various intervals of time demands, applications of the special approach considering casual character of estimations of parameters of reliability
4. Overcoming of methodical and information difficulties can be reached on the basis of the imitating model reflecting features of change of conditions PB [3].

Discussion of the imitating model developed by authors is beyond present clause. It is possible to note only, that the developed algorithm has allowed estimating parameters of reliability and their accuracy for various number PB disconnected in emergency repair and by that to solve a problem of an objective estimation of distribution of emergency decrease in capacity of a state district power station.

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ACCURACY FORMULAS OF RUIN PROBABILITY CALCULATIONS FOR DISCRETE TIME RISK MODEL WITH DEPENDENCE OF FINANCIAL AND INSURANCE RISKS

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ABSTRACT

In this article discrete time risk model with heavy tailed losses distribution and dependence between financial and insurance risks is considered. It is shown that known asymptotic formulas work with good accuracy for sufficiently large arguments. Direct methods based on calculation of ruin probability by solution of appropriate integral equations demand large volumes of calculations and so work for sufficiently small arguments. Fast and accuracy algorithms, based on approximation of loss distribution by mixture of exponential ones, to calculate ruin probability in this interval are developed. This approximation of considered model is based on continuity theorems and analog of Bernstein theorem in L_1 metrics.

INTRODUCTION

Discrete time risk model with dependence between financial and insurance risks is considered. In modern period of strong economical crisis such dependence may be recognized easily in different large anthropogenic catastrophes. So a problem to analyze asymptotically this dependence influence on the ruin probability is actual now.

This problem is discussed in risk theory and in queueing theory (Asmussen & Bladt 1996, Asmussen 2000, Feldmann & Whitt 1998, Dufresne 2005, Albrecher, Teugels & Tichy 2001). For heavy tailed distributions of losses it is shown that known asymptotic formulas (Embrechts & Veraverbeke 1982, Embrechts, Kluppelberg & Mikosch 1997, Tang 2004) work with good accuracy for sufficiently large arguments (Asmussen 2000, Kalashnikov 1997, Kalashnikov 1999). Direct methods based on calculation of ruin probability by solution of appropriate integral equations demand large volumes of calculations (Skvarnik 2004) and so work for sufficiently small arguments. As a result an interval of mean arguments appears. This interval is interesting practically and in it asymptotic formulas still do not work and direct methods already do not work. So it is interesting to process sufficiently fast and accuracy algorithms to calculate ruin probability in this interval.

To solve this problem an analogy with the queueing theory with an approximation of heavy tailed distribution by a mixture of exponential ones is used. It is known that waiting times in an one server queueing system which creates the Lindley chain coincide by a distribution with maximums of sequential sums and are continuous for fluctuations of distributions (Borovkov 1971, Zolotarev 1976). Analogously in discrete time risk model with dependent financial and insurance risks a finite interval ruin probability coincides with tail of distribution of some Markov chain.

In this paper finite interval ruin probability is represented by sum of exponents with unknown coefficients. To find these coefficients some recurrent procedure is suggested. It allows to consider risk model with exponential, Pareto, Weibull and some other loss distributions. We consider special model of insurance and financial risks dependence based on suggestion that a financial risk has a finite number of meanings and for each meaning an insurance risk has its own distribution.

Asymptotic formulas for the ruin probability in a case of independent financial and insurance risks have been obtained in (Tang & Tsitsiashvili 2003 (Stochastic Processes Applied)). More complicated cases with special restrictions on insurance risks dependence are considered for example in (Tang & Tsitsiashvili 2003 (Extremes), Tang & Wang 2010). First asymptotic formulas for dependent financial and insurance risks are obtained in (Tsitsiashvili 2010).

1 PRELIMINARIES

Consider recurrent discrete time risk model (with annual step) with initial capital $x, x \geq 0,$ and nonnegative losses $Z_n, n = 1,2..., P(Z_n < t) = F(t)$:

$$S_0 = x,\ S_n = B_n\ S_{n-1} + A_n,\ n = 1,2,... \tag{1}$$

Here income $A_n, n = 1,2,...,$ to end of *n*-th year is defined as difference between unit premium sum and loss $A_n = 1 - Z_n$. Assume that $B_n > 1$ is inflation factor from $n-1$ to n year, $n = 1,2,...$ In (Norberg 1999) $X_n = -A_n$ is called insurance risk and $Y_n = B_n^{-1}$ is called financial risk. Suppose that $\{(A_n, B_n),\ n \geq 1\}$ is sequence of independent and identically distributed random vectors (i.i.d.r.v.`s). In this model with initial capital x ruin time is defined by formula

$$\tau(x) = \inf\{n = 1,2,...:\ S_n \leq 0 \mid S_0 = x\}$$

and finite time ruin probability by formula

$$\psi_n(x) = P(\tau(x) \leq n).$$

So the sum S_n money accumulated by insurance company to *n*-th year end satisfies recurrent formula

$$S_0 = x,\ S_n = x\prod_{j=1}^{n} B_j + \sum_{i=1}^{n} A_i \prod_{j=i+1}^{n} B_j,\ n = 1,2,..., \tag{2}$$

where $\prod_{j=n+1}^{n} B_j = 1$ by convention. According to the notation above we can rewrite the discounted value of the surplus S_n in (2) as

$$\tilde{S}_0 = x,\ \tilde{S}_n = S_n \prod_{j=1}^{n} Y_j = x - \sum_{i=1}^{n} X_i \prod_{j=1}^{i} Y_j = x - W_n.$$

Hence we easily understand that for each *n*=0,1,...

$$\psi_n(x) = P(U_n > x),\ U_n = \max\left\{0, \max_{1\leq k\leq n} W_k\right\},\ U_0 = 0. \tag{3}$$

Define another Markov chain as

$$V_0 = 0,\ V_n = Y_n \max(0, X_n + V_{n-1}),\ n = 1,2,... \tag{4}$$

In (Tang & Tsitsiashvili 2003 (Stochastic Processes Applied), Tsitsiashvili 2010) the following statement is proved.

Theorem 1. The formula $\psi_n(x) = P(V_n > x),\ n = 1,2,...$ is true.

Suppose that $Q = \{1,...,m\}$ and introduce *m*-dimensional vectors $1_q = \{\delta_{1q},...,\delta_{mq}\}$ where δ_{ij} is Kroneker symbol and

$$R = (r_1,...,r_m),\ K = (k_1,...,k_m),\ r_i > 0,\ k_i \in \{0,1,...\},\ i = 1,...,m,$$

and denote

$$R^K = \prod_{q\in Q} r_q^{k_q},\ |K| = \sum_{q\in Q} k_q.$$

Redefine the function e^{-t} so that for $t<0$ we have $e^{-t}=1$ and for $t\ge 0$ this function is defined as usual. Introduce the function $E(t)=\begin{cases}0, & t>0,\\ 1, & t\le 0.\end{cases}$ Suppose that i.i.d.r.vectors $(Y_n, Z_n),\ n\ge 1,$ have following distributions

$$(Y_n^{-1}=r_q, Z_n=Z_n^q)=p_q,\ P(Z_n^q>t)=\overline{F}_q(t),\ n\ge 1,\ q\in Q.$$

Consider disturbed Markov chain $\tilde{V}_n,\ n\ge 0,$ so that

$$\tilde{V}_0=0,\ \tilde{V}_n=Y_n\max(0, \tilde{X}_n+\tilde{V}_{n-1}),\ n=1,2,... \tag{5}$$

have following distributions

$$(Y_n^{-1}=r_q, \tilde{Z}_n=\tilde{Z}_n^q)=p_q,\ P(\tilde{Z}_n^q>t)=\overline{G}_q(t),\ n\ge 1,\ q\in Q.$$

Denote $\varphi_n(x)=P(\tilde{V}_n>x).$

2 RECURRENT ALGORITHMS OF RUIN PROBABILITY CALCULATIONS

Theorem 2. Suppose that there are real numbers $a_{qi},\ q\in Q,\ i=1,...,l,$ and $p_q>0,\ q\in Q,$ $\sum_{q\in Q}p_q=1,$ so that $\overline{G}_q(t)=\sum_{i=1}^{l}a_{qi}\exp(-\lambda_i t),\ q\in Q, n\ge 1,$ and

$$R^K\lambda_i\ne\lambda_j,\ 1\le i,j\le l,\ |K|\ge 1. \tag{6}$$

Then there are real numbers $B_{n,i}^K,\ i=1,...,l,\ 1\le|K|\le n,$ which satisfy for $n\ge 1, i=1,...,l$ initial conditions

$$B_{1,i}^{1_q}=p_q\ a_{qi}\exp(-\lambda_i),\ q\in Q, \tag{7}$$

and recurrent formulas

$$B_{n+1,i}^{1_q}=p_q\sum_{1\le|K|\le n}\sum_{j=1}^{l}\frac{B_{n,j}^K\,a_{qi}\,R^K\lambda_j}{R^K\lambda_j-\lambda_i}\ \exp(-\lambda_i)+p_qB_n^0a_{qi}\exp(-\lambda_i),\ q\in Q, \tag{8}$$

$$B_{n+1,i}^K=-\sum_{q\in Q}I(k_q>0)p_q\sum_{j=1}^{l}\frac{B_{n,i}^{K-1_q}\ a_{qj}\ \lambda_j}{R^{K-1_q}\lambda_i-\lambda_j}\ \exp(-R^{K-1_q}\lambda_i),\ 1<|K|\le n+1, \tag{9}$$

so that

$$\varphi_s(t)=\sum_{1\le|K|\le s}\sum_{i=1}^{l}B_{s,i}^K\exp(-R^K\lambda_i\,t)\ +B_s^0E(t),\ s>0, \tag{10}$$

where

$$B_s^0=1-\sum_{1\le|K|\le s}\sum_{i=1}^{l}B_{s,i}^K\ . \tag{11}$$

Proof. If random variables ξ,η are independent and

$$P(\xi>t)=\backslash\exp(-\mu t), P(\eta>t)=\backslash\exp(-\lambda t),\ \lambda,\mu>0,\ \lambda\ne\mu,$$

then it is easy to obtain that

$$P(\xi+\eta>t)=\frac{\mu\exp(-\lambda t)-\lambda\exp(-\mu t)}{\mu-\lambda}. \tag{12}$$

Calculating

$$P(Y_1(\tilde{Z}_1-1)>t)=\sum_{q\in Q}p_q\ P(\tilde{Z}_1^q-1>R^{1_q}t)=\sum_{q\in Q}p_q\ P(\tilde{Z}_1^q>R^{1_q}t+1)=$$

$$=\sum_{q\in Q} p_q \sum_{i=1}^{l} a_{qi} \exp(-\lambda_i(R^{1_q}t+1)) = \sum_{q\in Q} p_q \sum_{i=1}^{l} a_{qi} \exp(-\lambda_i)\exp(-\lambda_i R^{1_q}t)$$

we obtain that

$$\varphi_1(t) = \sum_{q\in Q} p_q \sum_{i=1}^{l} a_{qi} \exp(-\lambda_i)\exp(-\lambda_i R^{1_q}t) + B_1^0 E(t) = \sum_{q\in Q}\sum_{i=1}^{l} B_{1,i}^{1_q} \exp(-\lambda_i R^{1_q}t) + B_1^0 E(t).$$

So the formula (10) is true for s=1 with the initial conditions (7) and the equality (11).

Suppose that the formula (10) takes place for s=n and using the formula (12) calculate $P(Y_{n+1}(\tilde{V}_n + \tilde{Z}_{n+1} \text{-}1) > t) = \sum_{q\in Q} p_q P(\tilde{V}_n + \tilde{Z}_{n+1}^q > r_q t + 1).$ As

$$P(\tilde{V}_n + \tilde{Z}_{n+1}^q > x) = \sum_{1\le|K|\le n}\sum_{i=1}^{l}\sum_{j=1}^{l} \frac{B_{n,i}^K a_{qj}}{R^K\lambda_i \text{-} \lambda_j}(R^K\lambda_i \exp(\text{-}\lambda_j x) - \lambda_j \exp(\text{-}R^K\lambda_i x)) + B_n^0 \sum_{i=1}^{l} a_{qi}\exp(\text{-}\lambda_i x)$$

so

$$P(\tilde{V}_n + \tilde{Z}_{n+1}^q > r_q t + 1) = \sum_{1\le|K|\le n}\sum_{i=1}^{l}\sum_{j=1}^{l} \frac{B_{n,i}^K a_{qj}}{R^K\lambda_i \text{-} \lambda_j}[R^K\lambda_i \exp(\text{-}\lambda_j)\exp(\text{-}\lambda_j R^{1_q}t) - \lambda_j \exp(\text{-}R^K\lambda_i)\exp(\text{-}R^{K+1_q}\lambda_i t)] +$$

$$+B_n^0 \sum_{i=1}^{l} a_{qi}\exp(\text{-}\lambda_i)\exp(\text{-}\lambda_i R^{1_q}t).$$

Consequently we obtain

$$\varphi_{n+1}(t) = \sum_{q\in Q} p_q \sum_{1\le|K|\le n}\sum_{i=1}^{l}\sum_{j=1}^{l} \frac{B_{n,i}^K a_{qj}}{R^K\lambda_i \text{-} \lambda_j}[R^K\lambda_i \exp(\text{-}\lambda_j)\exp(\text{-}\lambda_j R^{1_q}t) - \lambda_j \exp(\text{-}R^K\lambda_i)\exp(\text{-}R^{K+1_q}\lambda_i t)] +$$

$$+\sum_{q\in Q} p_q B_n^0 \sum_{i=1}^{l} a_{qi}\exp(\text{-}\lambda_i)\exp(\text{-}\lambda_i R^{1_q}t) + B_{n+1}^0 E(t) =$$

$$= \sum_{q\in Q} p_q \sum_{i=1}^{l}\sum_{1\le|K|\le n}\sum_{j=1}^{l} \frac{B_{n,j}^K a_{qi}}{R^K\lambda_j \text{-} \lambda_i} R^K\lambda_j \exp(\text{-}\lambda_i)\exp(\text{-}\lambda_i R^{1_q}t) -$$

$$-\sum_{q\in Q} p_q \sum_{i=1}^{l}\sum_{1\le|K'|\le n}\sum_{j=1}^{l} \frac{B_{n,i}^{K'} a_{qj}}{R^{K'}\lambda_i \text{-} \lambda_j} \lambda_j \exp(\text{-}R^{K'}\lambda_i)\exp(\text{-}R^{K'+1_q}\lambda_i t)] +$$

$$+\sum_{q\in Q} p_q B_n^0 \sum_{i=1}^{l} a_{qi}\exp(\text{-}\lambda_i)\exp(\text{-}\lambda_i R^{1_q}t) + B_{n+1}^0 E(t) = \sum_{1\le|K|\le n+1}\sum_{i=1}^{l} B_{n+1,i}^K \exp(\text{-}R^K\lambda_i t) + B_{n+1}^0 E(t) =$$

$$= \sum_{q\in Q}\sum_{i=1}^{l} B_{n+1,i}^{1_q}\exp(\text{-}R^{1_q}\lambda_i t) + \sum_{2\le|K|\le n+1}\sum_{i=1}^{l} B_{n+1,i}^K \exp(\text{-}R^K\lambda_i t) + B_{n+1}^0 E(t).$$

So the formula (10) is true for s=n+1. Here for $i=1,...,l,\ 1<|K|\le n+1$ we have the recurrent formulas (9) and for $i=1,...,l,\ |K|=1$ the recurrent formulas (8) and for B_{n+1}^0 the equality (11). The theorem is proved.

3 CONTINUITY OF RISK MODEL IN L$_1$ METRICS

In the sequel we assume that

$$(Y_n, Z_n) = \sum_{q=1}^{m} I(i_n = q)(r_q^{-1},\ F_q^{-1}(\omega_n))\ ,\ (Y_n, \tilde{Z}_n) = \sum_{q=1}^{m} I(i_n = q)(r_q^{-1},\ G_q^{-1}(\omega_n)). \qquad (13)$$

Here i.i.d.r.v\`s $\omega_n, n\ge 1,$ are uniformly distributed on interval [0, 1], $F^{-1}(\omega), 0\le\omega\le 1,$ is inverse to d.f. *F(t)* function. Then using uniform metrics

$$\rho(F,G) = \sup_{x\ge 0} |F(x) \text{-} G(x)|$$

and results on stability of queueing systems (Zolotarev 1976) it is simple to obtain following statement.

Theorem 3. For fixed $n \geq 0$ inequality

$$\rho(\psi_n, \varphi_n) \leq n\rho(F, G) \tag{14}$$

is true.

Say that distribution density $f(t)$ concentrated on $[0, \infty)$, is absolutely monotone if it has derivatives of all orders and $(-1)^k f^{(k)}(t) \geq 0$ for all $t > 0$ and $k \geq 1$. Example of such distribution is Pareto distribution satisfying equality $\overline{F}(x) = (1+bx)^{-\alpha},\ x > 0$. From Bernstein theorem (Feldmann 998) it is known that for d.f. F with absolutely monotone density there is sequence of d.f.`s represented as sums of exponents

$$F_s(x) = \sum_{i=1}^{l_s} p_{si}(1 - \exp(-\lambda_{si} x))\ ,\ x \geq 0,\ s > 0,$$

where $0 < \lambda_{si},\ p_{si} < \infty,\ p_{s1} + ... + p_{sl_s} = 1$ and $\rho(F, F_s) \to 0,\ s \to \infty.$

Theorem 3 and Bernstein theorem allow to construct approximative algorithm for a calculation of ruin probability. But linear by n upper bound in (14) is not convenient for this aim. So we begin to reformulate these results in terms of L_1 metrics. Denote $EY_n = a^{-1}$ and introduce the metrics $L_1(F, G)$ between d.f.`s *F, G* as follows

$$L_1(F, G) = \int_{-\infty}^{\infty} | F(t) - G(t) |\, dt. \tag{15}$$

Theorem 4. If $\delta = \max_{q \in Q} L_1(F_q, G_q)$ and *a* >1 then

$$L_1(\varphi_n, \psi_n) \leq \frac{\delta}{a - 1}. \tag{16}$$

Proof. From the formulas (4), (5) we have that $E | V_0 - \tilde{V}_0 | = 0$ and

$$E | V_n - \tilde{V}_n | = E\sum_{q \in Q} I(i_n = q) r_q^{-1} | \max(0,\ V_{n-1} + F_q^{-1}(\omega_n) - 1) - \max(0,\ \tilde{V}_{n-1} + G_q^{-1}(\omega_n) - 1) | \leq$$

$$\leq E\sum_{q \in Q} I(i_n = q) r_q^{-1}(| V_{n-1} - \tilde{V}_{n-1} | + | F_q^{-1}(\omega_n) - G_q^{-1}(\omega_n) |) =$$

$$= \sum_{q \in Q} r_q^{-1} p_q (E | V_{n-1} - \tilde{V}_{n-1} | + E | F_q^{-1}(\omega_n) - G_q^{-1}(\omega_n) |) = \frac{E | V_{n-1} - \tilde{V}_{n-1} | + \delta}{a},\ n \leq 1.$$

Consequently an induction by *n* gives the formula

$$E | V_n - \tilde{V}_n | \leq \delta \sum_{k=1}^{n} a^{-k} \leq \frac{\delta}{a - 1}.$$

As the minimum of the complex probability metrics $E | V_n - \tilde{V}_n |$ by all joint distributions which conserve marginal distributions of r.v.`s $V_n, \tilde{V}_n$ is $L_1(\varphi_n, \psi_n)$ (Zolotarev 1976) so from Theorem 1 we obtain the inequality (16). The theorem is proved.

It is easy to obtain from (Tsitsiashvili 2004, Kalashnikov & Rachev 1988) that in conditions

$$a > 1,\ \max_{g \in Q} \int_0^{\infty} \overline{F}_q(t)\,dt = C < \infty \tag{17}$$

there is nonincreasing function $\psi(t)$ so that $\psi(0) = 1,\ \psi(t) \to 0, t \to \infty$ and

$$\lim_{n \to \infty} \psi_n(t) = \psi(t),\ t \geq 0.$$

Indeed from Theorem 1 and the formula (3) the sequence $\psi_n(t),\ n \geq 0,$ satisfies the inequalities $\psi_{n+1}(t) \geq \psi_n(t),\ n \geq 0,$ and so it has the limit $\psi(t)$. Choosing r.v. V_∞ so that $P(V_\infty > t) = \psi(t),$

$t \geq 0,$ and applying Theorem 4 proof to the sequence $EV_n,\ n > 0,$ it is possible to obtain the inequality

$$EV_\infty \leq \frac{C}{a-1} < \infty$$

and consequently $\psi(t) \to 0, t \to \infty.$

Theorem 5. If the conditions (17) are true then

$$L_1(\psi, \psi_n) \leq \frac{C}{(a-1)a^{n-1}},\ n > 0. \tag{18}$$

Proof. For n=1 the formula (18) is true. Prove the formula (18) using an induction by n. Suppose that (18) takes place for some $n > 0$. Introduce the following joint distribution of r.v.`s $V_n, V_\infty$ which conserves their marginal distributions $V_n = \psi_n^{-1}(\omega), V_\infty = \psi^{-1}(\omega).$ Here r.v $\omega$ is independent on r.v.`s V_n, V_∞ and has uniform distribution on the interval [0, 1] so $E\,|\,V_n - V_\infty\,| = L_1(\psi_n, \psi).$ Then for r.v.`s $Z_{n+1}, Y_{n+1}$ independent on r.v.`s V_n, V_∞ we have the equalities

$$Y_{n+1}\max(0, V_n + Z_{n+1} - 1) = V_{n+1},\ Y_{n+1}\max(0, V_\infty + Z_{n+1} - 1) \overset{(d)}{=} V_\infty.$$

So from minimal property of metrics L_1 we obtain using mathematical induction by n that

$$L_1(\psi_{n+1}, \psi) \leq E\,|\,V_{n+1} - V_\infty\,| = E\,|\,Y_{n+1}\max(0, V_n + Z_{n+1} - 1) - Y_{n+1}\max(0, V_\infty + Z_{n+1} - 1)\,| \leq$$
$$\leq EY_{n+1}E\,|\,V_n - V_\infty\,| = a^{-1}E\,|\,V_n - V_\infty\,| \leq a^{-n}E\,|\,V_1 - V_\infty\,| \leq \frac{EV_\infty}{a^n}.$$

The formula (18) is true. The theorem is proved.

Denote $n(\varepsilon) = \inf(n : L_1(\psi, \psi_n) < \varepsilon)$ then from Theorem 5 we have the inequality

$$n(\varepsilon) \leq \inf(n : R^{-n+1}L_1(\psi, \psi_1) < \varepsilon)$$

and so

$$n(\varepsilon) \leq 2 + \frac{\ln L_1(\psi, \psi_1) - \ln \varepsilon}{\ln R} = n_1(\varepsilon). \tag{19}$$

Remark 1. The formula (19) allows to establish that if $L_1(\psi, \psi_{n(\varepsilon)}) < \varepsilon$ then it is enough to find $\psi_n,\ 1 \leq n \leq n_1(\varepsilon).$ From Theorem 2 we obtain that to calculate $\varphi_n(t)$ it is necessary $O(n^{m+1})$ arithmetical operations for $n \to \infty$. So to find $\varphi_{n(\varepsilon)}(t)$ we need $O(|\ln \varepsilon|^{m+1})$ arithmetical operations for $\varepsilon \to 0.$

Suppose that the condition (6) of Theorem 2 is not true then it is possible to approximate $G_q(t), q \in Q,$ in metrics L_1 so that the condition becomes true. We formulate this statement in the following way.

4 SMALL DENOMINATORS PROBLEM

Suppose that the condition (6) of Theorem 2 is not true and so we deal with zero denominators in recurrent formulas (8), (9). Then it is possible to approximate $G_q(t),\ q \in Q,$ in metrics L_1 so that the condition becomes true. We formulate this statement in the following way.

Theorem 6. Assume that for some $\delta > 0$ positive numbers $\lambda_1, ..., \lambda_l$ satisfy the condition

$$|\,\lambda_i - \lambda_j\,| > 3\delta,\ 1 \leq i \neq j \leq l.$$

Suppose that $r_q = t_q / T$ where t_q, T are coprimes and T is prime, $q \in Q$. Then for any $\varepsilon > 0$ there are positive and rational numbers $\tilde{\lambda}_1, ..., \tilde{\lambda}_l$ so that

$$|\lambda_i - \tilde{\lambda}_i| < \varepsilon,\ \tilde{\lambda} \neq R^K \tilde{\lambda}_j,\ 1 \le i \neq j \le l, |K| > 0.$$

Proof. Fix $\varepsilon,\ 0 < \varepsilon < \delta.$ There are integers $N,\ s_1, \ldots, s_l$ so that

$$\frac{1}{NT} < \frac{\varepsilon}{2},\ \left|\lambda_i - \frac{s_i}{N}\right| < \frac{\varepsilon}{2}.$$

Choose $\tilde{\lambda}_i = (s_i T + 1)/NT$ then $|\lambda_i - \tilde{\lambda}_i| < \varepsilon$ and $s_i T + 1, T$ are coprimes, $1 \le i \le l$, so rational number

$$\prod_{q \in Q} t_q^{k_q} \frac{s_i T + 1}{T^{|K|}}$$

can not be integer and consequently

$$s_i T + 1 \neq \prod_{q \in Q} t_q^{k_q} \frac{s_i T + 1}{T^{|K|}},\ 1 \le i \neq j \le l, |K| > 0, K = (k_1, \ldots, k_m).$$

The theorem is proved.

Remark 2. Fix $\varepsilon > 0$. If $r_q > 0$ and r_q is noninteger then there is prime T and rational noninteger number $r_q^* = t_q / T$ so that $|r_q - r_q^*| < \varepsilon, q \in Q.$

Introduce Markov chain $V_0^* = 0,\ V_n^* = Y_n^* \max(0, \tilde{X}_n + V_{n-1}^*),\ \ n = 1, 2, \ldots,\ \varphi_n^*(x) = P(V_n^* > x).$

Theorem 7. Suppose that $|r_q - r_q^*| < \varepsilon, L_1(F_q, G_q) < \varepsilon, q \in Q.$ If the condition (17) is true and $1/a < d < 1,\ 0 < \varepsilon < d - 1/a$ then

$$L_1(\varphi_n, \varphi_n^*) \le \frac{\varepsilon D}{1-d}, n > 0, D = \frac{aC + ad - 1}{a - 1} < \infty. \tag{20}$$

Proof. From Theorem 7 condition we have that $E\tilde{Z}_n \le C + d - 1/a$ and from Theorem 5 proof we obtain $E\tilde{V}_\infty \le (C + d - 1/a)/(a-1).$ Assume that $P(Y_n = r_q, Y_n^* = r_q^*) = p_q,\ q \in Q,$ then

$$E|\tilde{V}_{n+1} - V_{n+1}^*| \le EY_{n+1} E|\tilde{V}_n - V_n^*| + E|Y_{n+1} - Y_{n+1}^*|(V_n^* + \tilde{Z}_{n+1}) \le \frac{E|\tilde{V}_n - V_n^*|}{a} +$$
$$+ \varepsilon(C + d - 1/a + E\tilde{V}_n + E|\tilde{V}_n - V_n^*|) \le E|\tilde{V}_n - V_n^*| d + \varepsilon(C + d - 1/a + E\tilde{V}_\infty) \le E|\tilde{V}_n - V_n^*| d + \varepsilon D.$$

Using mathematical induction by *n* and minimal property of metrics L_1 we obtain the formula (20).

5 BERSTEIN THEOREM IN L_1 METRICS

Bernstein theorem allows for any $q \in Q$ to approximate d.f. $F_q(t)$ by a mixture of exponential distributions in uniform metrics. But we need analogous approximation in L_1 metrics. Suppose that d.f. $F(t), \bar{F}(t) = 1 - F(t)$ concentrated on $[0, \infty)$ has mean

$$M = \int_0^\infty \bar{F}(t)dt < \infty \tag{21}$$

and continuous positive density $f(t)$ so that for any $T > 0$

$$\inf_{0 \le t \le T} f(t) = \frac{1}{A(t)} > 0. \tag{22}$$

Lemma 1. If d.f. *F* satisfies the conditions (21), (22) then for any $\varepsilon > 0$ it is possible to choose discrete d.f. G_n with finite number *n* of positive atoms so that $L_1(F, G_n) < 2\varepsilon.$

Proof. Fix positive ε and using the condition (21) find T_ε so that

$$\int_{T_\varepsilon}^{\infty} \overline{F}(t)dt < \varepsilon. \tag{23}$$

Using the condition (22) define integer *n* so that

$$\frac{A(T_\varepsilon)}{n} < \varepsilon \tag{24}$$

and put $\nu = \overline{F}(T_\varepsilon)$. Define $t_i, 1 \le i < n,$ from the equalities

$$F(t_i) = \frac{i(1-\nu)}{n}, 1 \le i < n,\ F(t_n) = F(T_\varepsilon) = 1-\nu.$$

Suppose that discrete d.f. G_n satisfies equalities

$$G_n(t) = \begin{cases} 0,\ 0 \le t < t_1, \\ F(t_2), t_1 \le t < t_2, \\ F(t_3), t_2 \le t < t_3, \\ \ldots \\ F(t_n), t_{n-1} \le t < T_\varepsilon, \\ 1, T_\varepsilon \le t < \infty. \end{cases} \tag{25}$$

Using the formulas (15), (23) - (25) it is easy to obtain the inequality $L_1(F, G_n) < 2\varepsilon$. The lemma is proved.

Theorem 8. If d.f. *F* satisfies the conditions (21), (22) then for any $\varepsilon > 0$ it is possible to choose d.f. $R_n(t)$ concentrated on $[0, \infty)$ with tail

$$\overline{F}(t) = \sum_{i=1}^{r} a_i \exp(-b_i t),\ t > 0,\ -\infty < a_i < \infty,\ 0 < b_i < \infty, \tag{26}$$

so that $L_1(F, R_n) < 4\varepsilon$.

Proof. From Lemma 1 it is easy to obtain that $G_n(t)$ is probability mixture of point distributions

$$G_n(t) = \frac{1-\nu}{n}\left[2\,\mathbf{1}(t - t_1) + \sum_{i=2}^{n-1} \mathbf{1}(t - t_i)\right] + \nu\,\mathbf{1}(t - T_\varepsilon)$$

where $\mathbf{1}(t - u)$ is d.f. concentrated in real point *u*.

Fix $\varepsilon > 0$ and natural *k*. Following (Dufresne 2005, Ko & Ng 2007) let $Erl(m, \lambda)$ denote the Erlang distribution with density function

$$\frac{\lambda^m t^{m-1} \exp(-\lambda t)}{(m-1)!}, t > 0,$$

which describes the sum of *m* independent r.v.`s with exponential distribution with parameter λ. Then $Erl(m, \lambda)$ has mean m/λ and variance m/λ^2. To approximate a degenerate distribution at $k > 0$ we consider $Erl(m, m/k)$ and let *m* tend to infinity. So we may choose m_ε so that variance of d.f. $H_k = Erl(m, m/k)$ is smaller than ε^2 and consequently $L_1(\mathbf{1}(t - k), H_k(t)) \le \varepsilon$. If Erlang d.f.`s $H_1(t), \ldots, H_n(t)$ satisfy the inequalities

$$L_1(\mathbf{1}(t - t_1), H_1(t)) < \varepsilon, \ldots, L_1(\mathbf{1}(t - t_n), H_n(t)) < \varepsilon$$

then $L_1(G_n, Q_n) < \varepsilon$ where

$$Q_n(t) = \frac{1-\nu}{n}\left[2\,H_1(t) + \sum_{i=2}^{n-1} H_i(t)\right] + \nu\,H_n(t).$$

Following (Ko & Ng 2007) let $\eta_1,...,\eta_m$ are i.i.d. r.v.`s with exponential d.f. and with parameter $\lambda$. Then random sum $\sum_{i=1}^{m}\eta_i$ has d.f. $Erl(m,\lambda)$. Suppose that independent r.v.`s $\xi_1,...,\xi_m$ with exponential distributions and with parameters $\lambda_1,...,\lambda_m$ so that

$$\sum_{i=1}^{m}\left|\frac{1}{\lambda}-\frac{1}{\lambda_i}\right|<\varepsilon,\ \lambda_i\neq\lambda_j,\ 1\le i\neq j\le m,$$

and there are i.i.d. r.v.`s $\omega_n, n\ge 1,$ uniformly distributed on interval [0,1] so that

$$\eta_i=-\frac{\ln\omega_i}{\lambda},\ \xi_i=-\frac{\ln\omega_i}{\lambda_i},\ 1\le i\le m.$$

Denote $S(t)=P\left(\sum_{i=1}^{m}\xi_i<t\right)$. Then we have the inequality

$$E\left|\sum_{i=1}^{m}\eta_i-\sum_{i=1}^{m}\xi_i\right|\le\sum_{i=1}^{m}E\,|\,\eta_i-\xi_i\,|\le\varepsilon.$$

If we replace in last inequality complex probability metric $E\,|\,X-Y\,|$ by its minimum $L_1(P(X<t),P(Y<t))$ among all joint distributions which conserve marginal distributions of $P(X<t),P(Y<t)$ then we obtain $L_1(Erl(m,\lambda),S)<\varepsilon.$ Using the formula (12) it is easy to represent d.f. $S(t)$ in the form (26).

So it is possible to find d.f. S concentrated on $[0,\infty)$ with the tail $\overline{S}(t)$ which has the form (26) so that $L_1(Erl(m,\lambda),S)<\varepsilon.$ If d.f.`s $S_1(t),...,S_n(t)$ which has the form (26) satisfy the inequalities

$$L_1(H_1,S_1)<\varepsilon,...,L_1(H_n,S_n)<\varepsilon.$$

Then $L_1(Q_n,R_n)<\varepsilon$ where

$$R_n(t)=\frac{1-\nu}{n}\left[2\,S_1(t)+\sum_{i=2}^{n-1}S_i(t)\right]+\nu\,S_n(t)$$

and d.f. $R_n(t)$ satisfies the condition (26) also.

Consequently from Lemma 1 we obtain the inequalities

$$L_1(F,R_n)\le L_1(F,G_n)+L_1(G_n,Q_n)+L_1(Q_n,R_n)<4\varepsilon.$$

The theorem is proved.

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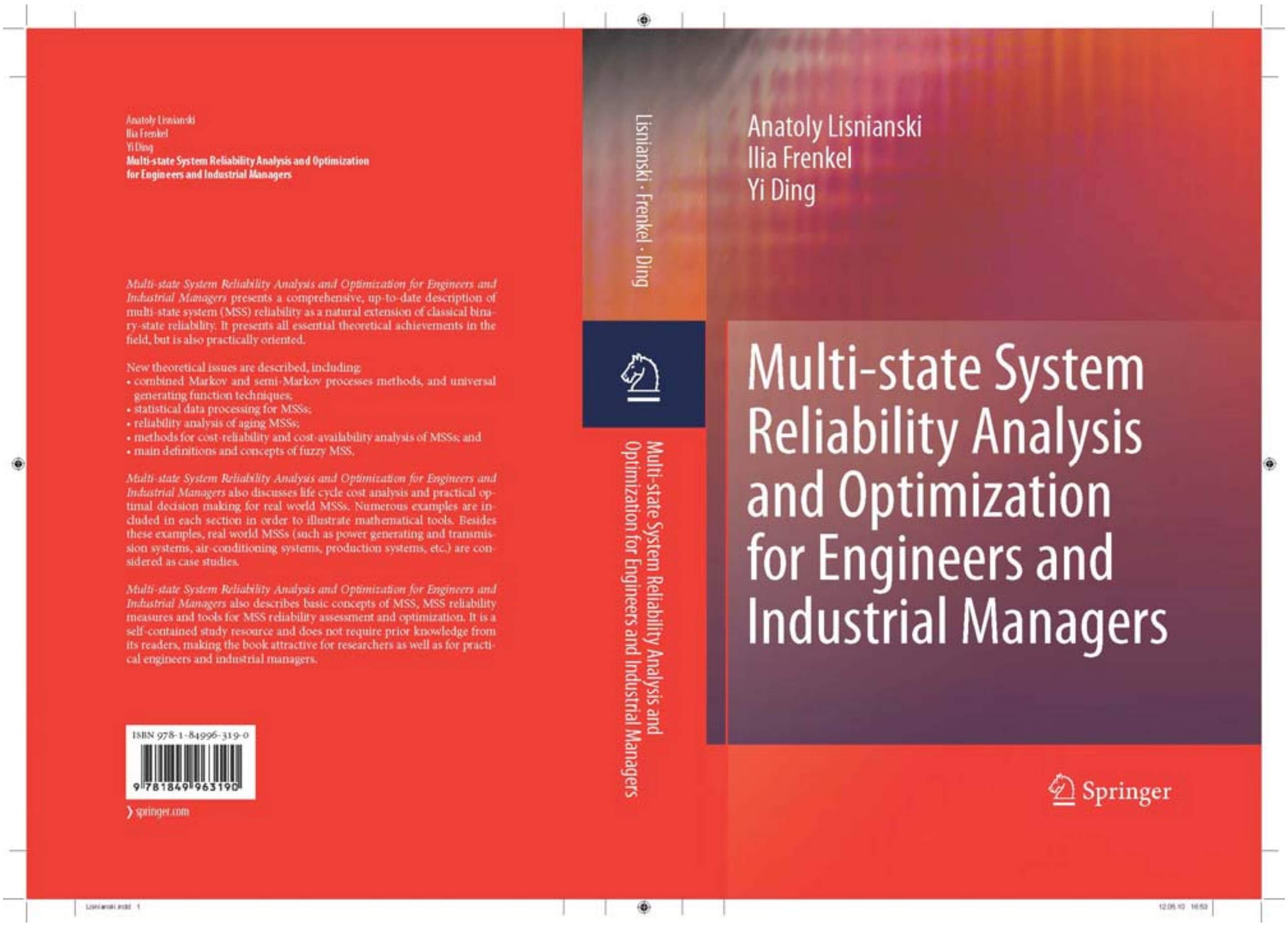

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Yi Ding
Multi-state System Reliability Analysis and Optimization for Engineers and Industrial Managers

Multi-state System Reliability Analysis and Optimization for Engineers and Industrial Managers presents a comprehensive, up-to-date description of multi-state system (MSS) reliability as a natural extension of classical binary-state reliability. It presents all essential theoretical achievements in the field, but is also practically oriented.

New theoretical issues are described, including:
- combined Markov and semi-Markov processes methods, and universal generating function techniques;
- statistical data processing for MSSs;
- reliability analysis of aging MSSs;
- methods for cost-reliability and cost-availability analysis of MSSs; and
- main definitions and concepts of fuzzy MSS.

Multi-state System Reliability Analysis and Optimization for Engineers and Industrial Managers also discusses life cycle cost analysis and practical optimal decision making for real world MSSs. Numerous examples are included in each section in order to illustrate mathematical tools. Besides these examples, real world MSSs (such as power generating and transmission systems, air-conditioning systems, production systems, etc.) are considered as case studies.

Multi-state System Reliability Analysis and Optimization for Engineers and Industrial Managers also describes basic concepts of MSS, MSS reliability measures and tools for MSS reliability assessment and optimization. It is a self-contained study resource and does not require prior knowledge from its readers, making the book attractive for researchers as well as for practical engineers and industrial managers.

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Yi Ding

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Springer

Multi-state System Reliability Analysis and Optimization for Engineers and Industrial Managers

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Thomas L. Saaty
CONFLICTS RESOLUTION AS A GAME WITH PRIORITIES: MULTIDIMENSIONAL CARDINAL PAYOFFS, PART 1
There are two ways to consider increasing the effectiveness of the theory of games in applications. The first is to derive priorities for the payoffs using a cardinal absolute relative scale instead of an ordinal or interval scale to do equilibrium analysis. Our approach using cardinal payoffs is illustrated with one example in an application to OPEC strategies that the author published in the International Journal of Game Theory.

Vyacheslav Abramov
FURTHER ANALYSIS OF CONFIDENCE INTERVALS FOR LARGE CLIENT/SERVER COMPUTER NETWORKS
In the recent paper [Abramov, RTA, 2 (2007), pp. 34-42], confidence intervals have been derived for symmetric large client/server computer networks with client servers, which are subject to breakdowns. The present paper mainly discusses the case of asymmetric network and provides another representation of confidence intervals.

Boyan Dimitrov, George Hayrapetyan, Peter Stanchev, Zohel Khalil

AGING AND LONGEVITY CONTROL OF BIOLOGICAL SYSTEMS VIA DRUGS - A RELIABILITY MODEL

The treatments in bio-systems correspond to respective repairs known in reliability. Some treatments may make the biological objects younger; others may make them older, or not deteriorate their current age. Such kind of "maintenance" has some analogous failure/repair models in reliability. We use it to incorporate some results of reliability and bio modeling for the quantitative studies of the aging and resistance of bio-systems to environmental stress factors. We call "calendar age" the age of a bio-object which does not use treatments, or uses it without age improvement, or deterioration. All bio-objects, which are using treatments of same strength and direction of effect, have "virtual age". We explain here what the virtual age is, and how is it related to age correcting factors. We illustrate our common results about the virtual ages on the example of the Gompertz-Makenham law of mortality, and discuss the relations of the longevity, mechanism of aging and age affecting control. As a consequence, a concept of age determination is proposed. Numeric and graphical examples are provided.

Yakov Genis

RELIABILITY AND RISK ASSESSMENT OF SYSTEMS OF PROTECTION AND BLOCKING WITH FAST RESTORATION

There is examined a system with fast restoration which should be operational beginning from some moments of time. If beginning from these moments of time the system is defective during the time more than the assigned random time interval it is considered failed. Such system includes the models of systems with the protection and blocking and systems with the discrete periodic functions. The estimations of indices of failure-free performance and maintainability of these systems and the estimation of indices of risk and losses, connected with the failure (accident) of the system with protection are obtained. This material was presented in the Mathematical Methods in Reliability 2007 Conference in Glasgow, UK.

Gurami Tsitsiashvili, A. Losev

AST ALGORITHMS OF ASYMPTOTIC ANALYSIS OF NETWORKS WITH UNRELIABE ARCS

A problem of a reliability in networks with unreliable elements naturally origin in technical applications. But a direct calculation of the reliability demands a number of operations which increases geometrically dependently on a number of arcs. So it is necessary to use approximate methods and particularly asymptotic one. In other work asymptotic reliability is calculated in analogous asymptotic suggestions on the network arcs. Main parameters in these asymptotic are a shortest way length and a maximal flow in a network. In this paper different partial classes of networks are considered and effective algorithms of their parameters calculations are suggested. These networks are networks originated by dynamic systems, networks with integer-valued lengths of arcs, superposition of networks and bridge schemes.

Gurami Tsitsiashvili

BOTTLENECKS IN GENERAL TYPE LOGICAL SISTEMS WITH UNRELIABE ELEMENTS

In this paper a model of general type logical system with unreliable elements is considered. An asymptotic analysis of its work (failure) probability is made in appropriate conditions on work (failure) probabilities of the system elements. A concept of bottlenecks of this system is constructed on a suggestion that an increase (a decrease) of elements reliabilities lead to an increase (a decrease) of the system reliability. A construction of general type logical system is founded on concepts of disjunctive and conjunctive normal forms (DNF and CNF) of a logical function.

Mark Kaminskiy, Vasili Krivtsov
AN INTEGRAL MEASURE OF AGING/REJUVENATION FOR REPAIRABLE AND NON-REPAIRABLE SYSTEMS
This paper introduces a simple index that helps to assess the degree of aging or rejuvenation of a non-repairable system. The index ranges from -1 to 1 and is negative for the class of decreasing failure rate distributions (or deteriorating point processes) and is positive for the increasing failure rate distributions (or improving point processes). The introduced index is distribution free.

Revaz Kakubava
ANALYSIS OF ALTERNATING RENEWAL PROCESSES WITH DEPENDED COMPONENTS
In the terms of operational calculus the probability characteristics of direct and reverse residual renewal time of alternating renewal process, where renewal time depends on life-time, are found.

Edward Korczak
COMPUTATION OF FAILURE/REPAIR FREQUENCY OF MULTI-STATE MONOTONE SYSTEMS
The paper deals with calculation methods for failure and repair frequencies of multi-state monotone systems, both for the instantaneous and steady state cases. Being based on the binary representation of multi-state structure, new general formula for the failure/repair frequency is derived. This formula is used to obtain simple rules for the calculation of failure/repair frequency. In particular, the use of the algebra of dual numbers is presented.

Mark Bebbington, Chin-Diew Lai, Ricardas Zitikis
LIFETIME ANALYSIS OF INCANDESCENT LAMPS: THE MENON-AGRAWAL MODEL REVISITED
The use of the Weibull distribution to model lifetimes of incandescent lamps was originally suggested by Leff (1990). Following this suggestion, Agrawal and Menon have offered and investigated, in a series of papers, an improved model constructed from physical considerations and laws of mathematical statistics. In the present paper we offer supplementary thoughts concerning the Agrawal-Menon model and its several modifications. In addition, we discuss the use of Pinelis's l'Hospital-type calculus rules in the analysis of ageing properties of lifetime distributions.

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Zio Enrico, Sansavini Giovanni, Maja Roberto, Marchionni Giovanna
ANALYSIS OF THE SAFETY EFFICIENCY OF A ROAD NETWORK: A REAL CASE STUDY

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Yu. M. Paramonov
BAYES-FIDUCIAL APPROUCH FOR AIRCRAFT SPECIFIED LIFE NOMINATION
The problem of nomination of Retirement or Specified Life (SL) of aircraft on the base of full-scale fatigue test result processing is considered. SL can be defined (1) by requirement of fatigue failure probability limitation or (2) by economics reasons. For optimization problem the Bayes-fiducial (BF) approach is offered. BF decision is always a function of sufficient statistics and, by contrast with maximum likelihood method, it is based on the use of specific loss function. For the problem of failure probability limitation in case when sufficient statistics coincides with the sample itself (for example, for Weibull distribution) usually the Monte Carlo method is used but in this paper for the distributions with location and scale parameters an analytical solution is offered.
Some numerical examples for lognormal, Weibull distributions are given.

O.V.Abramov, Y.V.Katueva and D.A.Nazarov
CONSTRUCTION OF ACCEPTABILITY REGIONS FOR PARAMETRIC RELIABILITY OPTIMIZATION
The problem of representation and analysis of analog technical devices and systems acceptable regions is introduced. This problem occurs during designing and controlling in view of parametric dithering. The algorithms of constructing circumscribed parallelepiped, representation of acceptable region as a set of non-overlapping parallelepipeds are offered. The algorithm of acceptable region centre of mass computation is offered as the example of utilizing the acceptable region representation.

Farhadzadeh E.M., Muradaliyev A.Z., Farzaliyev Y.Z.
MATCHING OF CRITERIA THE DISCERNMENT OF THE FUNCTIONAL CHARACTERISTICS OF INDEXES OF RELIABILITY OF PLANTS EES
References on variation of reliability on the curves received at analysis of statistical data can appear erratic if not to consider a random in character of assessments of indexes of reliability. The comparison method of criteria of a discernment of the functional characteristics indexes of reliability reduced at ordinal and nominal dials of variation of argument.

Maxim Finkelstein
ON ENGINEERING RELIABILITY CONCEPTS AND BIOLOGICAL AGING
In this study, various stochastic approaches to biological aging modeling are discussed. We assume that an organism acquires a random resource at birth. Death occurs when the accumulated damage (wear) exceeds this initial value. Another source of death of an organism is also taken into account, when it occurs as a consequence of a shock or of a demand for energy, which is a generalization of the Strehler-Mildvan's model. Biological age, based on the observed degradation, is also defined. Finally, aging properties of imperfectly repaired systems are discussed. We show that aging slows down with age in this case. This presents another possible explanation for human mortality rate plateaus.

G. F. Kovalev

"SAFETY" AS A CHARACTERISTIC OF ONE OF THE SINGLE RELIABILITY PROPERTIES

The paper deals with the problems of interrelation between two most important properties of technical (production) systems: safety and reliability that were considered historically separately. However, recently both properties have proven to be increasingly more interrelated, which makes their joint study topical. And the safety may be treated as the most important reliability property, determining to a great extent all the remaining single reliability properties. The relation between the notions of "technical safety" and "energy safety" is also studied.

B. A. Kulik

N-TUPLE ALGEBRA-BASED PROBABILISTIC LOGIC

The concept of "probabilistic logic" known in artificial intelligence needs a more thorough substantiation. A new approach to constructing probabilistic logic based on the N-tuple algebra developed by the author is proposed. A brief introduction is given to the N-tuple algebra and its properties that provide efficient paralleling of algorithms for solving problems of logical analysis of systems in computer implementation are generalized. Methods for solving direct and inverse problems of probabilistic simulation of logical systems are considered.

Nicholas A. NECHVAL, Konstantin N. NECHVAL

TECHNIQUE FOR FINDING SAMPLING DISTRIBUTIONS FOR TRUNCATED LAWS WITH SOME APPLICATIONS TO RELIABILITY ESTIMATION

In this paper, the problem of finding sampling distributions for truncated laws is considered. This problem concerns the very important area of information processing in Industrial Engineering. It remains today perhaps the most difficult and important of all the problems of mathematical statistics that require considerable efforts and great skill for investigation. The technique discussed here is based on use of the unbiasedness equivalence principle, the idea of which belongs to the authors, and often provides a neat method for finding sampling distributions. It avoids explicit integration over the sample space and the attendant Jacobian but at the expense of verifying completeness of the recognized family of densities. Fortunately, general results on completeness obviate the need for this verification in many problems involving exponential families. The proposed technique allows one to obtain results for truncated laws via the results obtained for non-truncated laws. It is much simpler than the known techniques. The examples are given to illustrate that in many situations this technique allows one to find the results for truncated laws and to estimate system reliability in a simple way.

G. Albeanu & H. Madsen, B. Burtschy, Fl. Popentiu-Vl?dicescu, Manuela Ghica

BOOTSTRAPPING TIME SERIES WITH APPLICATION TO RISK MANAGEMENT

The bootstrap method is an extensive computational approach, based on Monte Carlo simulation, useful for understanding random samples and time series. It is a powerful tool, especially when only a small data set is used to predict the behaviour of systems or processes. This paper presents the results of an investigation on using bootstrap resampling (different types: uniform, importance based, block structured etc.) for time series appearing during software life cycle (mainly the software testing phase, and debugging), economics, and environment (air pollution generated by cement plants) in order to help the activity of staff working on risk management for software projects, risk management in finance, and those working on environment risk management.

G.Sh. Tsitsiashvili
DISCRETE TIME MODELS OF FORWARD CONTRACTS INSURANCE
In this paper financial management model of forward contracts insurance suggested in some works is considered by means of risk theory and heavy tailed technique. This model is based on a compensation principle. It attracted large interest and called active discussion among economists. So its mathematical analysis is initiated as economists so mathematicians.

G.Sh. Tsitsiashvili
ANALYSIS OF PORTS RELIABILITIES
This paper is devoted to algorithms of a calculation of ports reliabilities. A port is a no oriented graph with fixed initial and final nodes. As accuracy so asymptotic formulas are considered. Suggested algorithms have minimal numbers of arithmetical operations.

Igor Ushakov, Sumantra Chakravarty
OBJECT ORIENTED COMMONALITIES IN UNIVERSAL GENERATING FUNCTION FOR RELIABILITY AND IN C++
The main idea of Universal Generating Function is exposed in reliability applications.
Some commonalities in this approach and the C++ language are discussed.

Igor Ushakov
METHOD OF OPTIMAL SPARE ALLOCATION FOR MOBILE REPAIR STATION
Method of finding optimal spare stock for Mobile Repair Station is suggested. Numerical calculations are performed with use of real field data. It showed significant improvement: probability of first fix for suggested variant is 0.967 in comparison with 0.534 for existing variant.

Andrey Kostogryzov
MATHEMATICAL MODELS AND SOFTWARE TOOLS FOR QUALITY AND RISK MANAGEMENT ACCORDING STANDARD REQUIREMENTS
The offered mathematical models and supporting them software tools complexes (M&STC) are purposed for a systems analysts from customers, designers, developers, users, experts of testing laboratories and certification bodies, as well as a staff of quality maintenance for any complex system etc. M&STC are focused on providing system standard requirements on the base of modeling random processes that exist for the life cycle of any complex system. Models implement original author's mathematical methodology based on probability theory, theory for regenerating processes and methods for system analysis. M&STC may be also used in training and education for specializations "System engineering", "Software engineering", "System safety and security", "Information systems".

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Solojentsev E.D., Karasev V.V.
SCIENTIFIC SCHOOL «MODELING AND ANALYSIS OF SAFETY AND RISK IN COMPLEX SYSTEMS» - ACTUAL APPROACH TO ACTUAL

Alexej Chovanec
PREDICTION OF NO – FAILURE SYSTEM OPERATION

Alexej Chovanec
PREDICTION OF THE SYSTEM AVAILABILITY USING SIMULATION MODELING
The article deals with the possibility of system availability prediction using the simulation modelling. The system availability determined with system faultlessness and system maintainability is expressed by various parameters of mean time between the failures and the mean time of single elements repair. The system simulations are carried out with more parameters MTBF and MTTR, the results of the simulation course gives a real idea about the system behaviour in time and about changes of the values of asymptotic system availability factor.

Anton Ambrozy, Alexej Chovanec
COST OPTIMIZATION FOR REALISATION OF MAINTENANCE COST
This paper deals with optimal control interval determination using minimization the financial costs. It clears conceptual, mathematical and simulation model of the problem solution. It enumerates and evaluates results of the simulation.

Jozef Bucha, Alexej Chovanec
SIMULATION OF FTA IN SIMULINK
This paper deals with possibility of simulation of reliability block diagrams, failure trees analysis as a time dependent analysis using Matlab/Simulnk

Himanshu Dutt Sharma, Bangale Shreyas Madhukarao
SIMULATED ELECTRICAL NETWORK APPROACH (SENA) TO HARD OPTIMIZATION PROBLEMS
A novel method is proposed for hard optimization type of problem wherein an exact optimal solution is increasingly difficult in terms of run time and memory requirements. Especially for the cases when search graph has higher number of nodes and more number of paths, which increase as factorial of node number. This is based on Simulated Electrical Network Approach (SENA) proposed here, in which the graph is modeled as an electrical network and current distribution is found which is used as a directive for search decisions. The proposed algorithm results in an approximate method that achieves average accuracy of 99.89% to reach close to the most optimal path that is found by ranking all possible paths. Conversely, it can eliminate on average 99.89% paths in polynomial time from consideration if one requires finding the most optimal one.

Milan Holicke
RISK ASSESSMENT AND OPTIMIZATION OF ROAD TUNNELS
Probabilistic methods of risk optimization are applied to specify the most effective arrangements of road tunnels. The total consequences of alternative arrangements are assessed using Bayesian networks supplemented by decision and utility nodes. It appears that the optimization may provide valuable information for a rational decision concerning number of escape routes. Discount rate seems to affect the total consequences and the optimum arrangements of the tunnels more significantly than number of escape routes.

Melnikov V.A.
MODEL OF A RELIABILITY FOR STRUCTURAL - COMPLICATED SYSTEMS, INCLUDING MULTYSTATE ELEMENTS
The problem of development of Boolean models of a reliability for systems, including elements with many states is considered on the basis of multivalued logic, algebra of trains, algebra of groups of incompatible events and classical logistic-probabilistic method (LPM). The inexpediency of development of Boolean models of a reliability on the basis of multivalued logic is displayed. The numerical examples demonstrating serviceability of LPM and their new possibilities are

demonstrated. The perspective of development of methods of an evaluation of effectiveness of operation at different levels of operation rate by formulation of a set of different tasks, solved by the same LPM is underlined

Solojentsev E.D.

SCENARIO MANAGEMENT OF RISKS OF ACCIDENTS AND CATASTROPHES IN BUSINESS AND ENGINEERING

The stages of development of Management and Risk are described. The scenario management of risks of accidents and catastrophes in complex systems on the stages of designing, debugging and exploitation test and exploitation itself are considered. In the scenario management of accidents and catastrophes risks the personnel and the General designer are taken into account. The uniform approach to the modelling of risks in technical, economic and organisational systems is presented on the basis of substantial description of a SCENARIO of an accident or a catastrophe, and then the construction of models of the risk for the purpose of analysis and management. As the intellectual core for the risk quantitative evaluation and analysis and the scenario management of accidents and catastrophes risk, LP-methods and risk LP-models with groups of incompatible events are used.

Romney B. Duffey, John W. Saull

MANAGING AND MEASURING RISK IN TECHNOLOGICAL SYSTEMS

Safety Management is intended to create order out of disorder, to reduce the “information entropy”, for the purpose of improved safety. Our purpose here and now is to try to introduce some predictability and insight into the risk or occurrence of apparently random events, where a general risk prediction we adopt a fundamental must be testable against the world’s existing data. The risk management issues are clear, given the classic features of major human involvement and contribution to accidents, errors and outcomes occurring with modern technological systems. Prior incidents and prior knowledge and experience must be fully incorporated or learned from. If we do not know where we are on the learning curve, we also do not know the probability of such an event, and we have no objective measure of the “safety culture”. Emphasis on defining and finding so-called “lack of safety culture” has resulted in an extensive and detailed study of the safety management and process safety of many global corporations. We utilize the concepts adopted in thermodynamics and Information Theory to establish the information entropy as a finite, physically based and useful measure of risk in technological systems. The results that we demonstrate show that the risk is dynamic, and can be utilized for management and predictive risk analysis purposes.

Jiri Stodola

TRAFFIC ACCIDENTAS INFORMATION SYSTEM AND RISK CRASH EVALUATION

This article analyses the traffic accident rate on roads and highways and possibilities of risk evaluation related to traffic accident occurrence based on factors that were the causes of accidents. A new term – risk of traffic accident occurrence is a product of probability of accident occurrence and its impacts. The results are presented by way of example that uses selected statistical data of the Czech Republic traffic accident rate between 1993 - 2001. The article provides a brief methodological procedure of evaluation of the traffic accident rate using the risk of traffic accident occurrence.

I. Kozine, N.J. Duijm, H. Hagen

THE SEVESO II DIRECTIVE AND DANISH ACTIVITIES SUPPORTING ITS APPLICATION IN SOME EASTERN EUROPEAN COUNTRIES

K. Lauridsen, I. Kozine, A. Amendola, M. Fiori

EU ADVANCES IN IDENTIFYING SOURCES OF UNCERTAINTY IN RISK ANALYSES

This paper presents the scope and some main results of a European project on the ASSessment of Uncertainties in Risk ANalysis of Chemical Establishments (ASSURANCE). The project aims at identifying the uncertainties associated with risk analysis of major industrial hazards and assessing the way these uncertainties can affect the final outcome of risk studies and of the relevant decisions based on that outcome. In order to achieve this goal, a number of benchmark exercises/case studies have been performed by the partners and the results were analysed in a modular and structured way. A reference plant served as the basis for a realistic description of these case studies. For this particular project an ammonia storage plant was selected, consisting of cryogenic and pressurised storage tanks, together with import loading/unloading facilities and the relevant piping. This installation was analysed independently by each partner, using common input data and boundary conditions, but different methods, tools and assumptions. The results were then compared and discrepancies identified, discussed and explained.

Henry K Moskatov

ADAPTATION, LEARNING AND INHERENT SAFETY OF 2ND GENERATION AIRSHIPS

Inherent safety of the new generation airships, based on some fundamental laws of Space, is discussed in some detail. An algorithm is proposed to analyze risks, resulting from hazards not compensated by “inherent safety”. Then a thoroughly verified statistical model of learning is used to evaluate results of airship flight testing-the probability of mission success and its confidence limit. The results can be used as a part of evidence for airship airworthiness certification.

Yu. Paramonov, A. Kuznetsov

PLANNING OF INSPECTION PROGRAM OF FATIGUE-PRONE AIRFRAME

To keep the fatigue ageing failure probability of an aircraft fleet on or below the certain level an inspection program is appointed to discover fatigue cracks before they decrease the residual strength of the airframe lower the level allowed by regulations. In this article the Minimax approach with the use one- and two-parametric Monte Carlo modelling for calculating failure probability in the interval between inspections is offered.

Finkelstein M.S.

STOCHASTIC APPROACH TO SAFETY AT SEA ASSESSMENT

A general approach for analysing spatial survival in the plane is suggested. Two types of harmful random events are considered: points with fixed coordinates and moving points. A small normally or tangentially oriented interval is moving along a fixed route in the plane, crossing points of initial Poisson random processes. Each crossing leads to termination of the process with a given probability. The probability of passing the route without termination is derived. A safety at sea application is discussed.

Novosyolov A.

MEASURING RISK

Problem of representation of human preferences among uncertain outcomes by functionals (risk measures) is being considered in the paper. Some known risk measures are presented: expected utility, distorted probability and value-at-risk. Properties of the measures are stated and interrelations between them are established. A number of methods for obtaining new risk measures from known ones are also proposed: calculating mixtures and extremal values over given families of risk measures.

Solojentsev E.D., Rybakov A.V.

RESEARCHES IN IDENTIFICATION OF LOGICAL AND PROBABILISTIC RISK MODELS WITH GROUPS OF INCOMPATIBLE EVENTS

In this paper the results of the researches in identification of the logical and probabilistic (LP) risk models with groups of incompatible events are presented. The dependence of the criterion function on several parameters has been investigated. The parameters include: the total number of optimisations, the amplitude of parameters increments, the initial value of the criterion function (CF), the choice of identical or different amplitudes of increments for different parameters, objects risks distribution. An effective technology of defining the global extreme in the identification of LP-risk model for the calculation time, appreciable to practice has been suggested.

Renzo Righini, Enrique Montiel

A NEW METHOD FOR THE APPLICATION OF RAMS TECHNIQUES TO QUALITY ASSURANCE (QA)

The application of RAMS techniques in all the phases of the lifecycle of each type of installation will surely guarantee its adequate exploitation in terms of production continuity and quality of the obtained products in the respect of prefixed constraints on the security of the working staff, safety and environment impact. In this frame, a particular importance must be attributed to the use of those techniques as support to quality assurance applied in the planning and building phases of the installation and of the products obtained by it. The present paper will include a short description of a method for the application of those techniques in this phase of the lifecycle and of the results that may be obtained by its application in shoes manufacturing, in particular those types where the technical requirements are higher, as it is the cases of certified products like “safety” footwear.

Igor Safonov

ASPECT-ORIENTED SOFTWARE RELIABILITY ENGINEERING

Aspect-Oriented Approach to Software Development allows us effectively to effectively extract, evaluate and solve the main problem of contemporary tendency in Information Technology (particularly, in an Application Software) – a unification is alternated by a personalization. Increasing customer concerns about Performance, Quality, Reliability and Security (PQRS concept) can be satisfied only by symbiosis synergy of adequate models, techniques and tools on all stages of the Software lifecycle. We propose original methodology, formal models and simple methods of Software Reliability Engineering based on our many years experience of concern separation and aspect orientation in Software Development for Specialized Computers, Business Application and Government Institutions.

Brian Bailey, Igor Safonov

TRUST ENGINEERING AND RISK MANAGEMENT FOR SAFETY OF METROPOLIS AND MEGALOPOLIS CITIZENS

The article describes the problems and solutions in the field of safety enhancement in emergency situations of the complex urban agglomerations and analyses of the most actual problem for all metropolises and megalopolises – terrorism, proposing the rational models and techniques of counterterrorism strategy, based on knowledge and experience.

Lev V. Utkin, Thomas Augustin

RISK ANALYSIS ON THE BASIS OF PARTIAL INFORMATION ABOUT QUANTILES

Risk analysis under partial information about probability distributions of states of nature is studied. An efficient method is proposed for a case when initial information is elicited from experts in the form of interval quantiles of an unknown probability distribution. This method reduces a difficult to handle non-linear optimisation problem for computing the optimal action to a simple linear one. A numerical example illustrates the proposed approach.

Lev V. Utkin, Sergey P. Shaburov
RISK ANALYSIS ON THE BASIS OF JUDGMENTS SUPPLIED BY UNKNOWN EXPERTS
The development of a system requires fulfilling the available standards of reliability and safety. Due to possible complexity of the system, its parameters often are determined by experts whose judgements are usually imprecise and unreliable due to the limited precision of human assessments. Therefore, an approach for computing probabilities of expert judgments and for analysing the risk of decision about satisfying the parameters to standards of reliability and safety is proposed in the paper. A numerical example considering a microprocessor system of central train control illustrates the proposed approach.

David VALIS
CONTRIBUTION TO CONSEQUENCES ANALYSIS USING FUZZY PROBABILITY
This article deals both with dependability and risk analysis from a complex point of view. Both these fields seem to be similar in many aspects, but unfortunately no congruence in sources of basic characteristics has been reached, yet. Statistical files are often very vague in terms of monitoring dependability measures or risk factors. There is a great need to use another point of view to describe these factors. One of those measures and fragments of risk or dependability are consequences both in terms of an event occurrence and failure occurrence. By using a new approach, better interconnection between these both fields and deeper applicability would be provided. A theory of fuzzy probability could be one of these new methods that could facilitate modelling of quantitative factors.

David VALIS
CONTRIBUTION TO STOCHASTIC METHODS OF COMPLEX SYSTEMS RISK ANALYSIS
The paper deals with risk assessment of complex systems. As we investigate situations regarding military applications the fragments of risk management are very important for us. Risk and dependability characteristics of military battle equipment have the same importance for us as those measures which have to serve to perform battle missions itself. There is no time on the battle field to solve unpredicted and unexpected situations caused by high risk level or unreliability which might lead to loss of both equipment and crew. Due to high level of risk we face on the battlefield many systems have to be robust enough or have to be redundant to succeed.

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E.M. Farhadzadeh, A.Z. Muradaliyev, Y.Z. Farzaliyev
CRITERION OF THE SUPERVISION ACCURACY OF INDEXES RELIABILITY OF POWER-GENERATING UNITS A STATE DISTRICT POWER STATION
The atomized system is developed, allowing to determine and compare indexes of individual reliability of complex plants in view of a random in character of an initial conditions.

Mikhail Yastrebenetsky
HOW PROFESSOR B.V. GNEDENKO GOT CAUGHT ON A HOOK IN KHARKOV

G.Sh. Tsitsiashvili, M.A. Osipova
DIRECT CALCULATIONS OF A REACHING MOMENT DISTRIBUTION FOR AN AUTOREGRESSIVE RANDOM SEQUENCE BY RECCURENT INTEGRAL EQUALITIES

V. Raizer
RELIABILITY ASSESSMENT DUE TO WEAR
Evaluation of structural reliability under processes of deterioration presents very important problem in design. The structure's wear shows a reduction of bearing capacity in time that for one's turn leads to increasing the probability of failure. The reasons for long duration and irreversible change of structural features can be corrosion in steel structures, decomposition in wood structures, ageing in polymer structures, and processes of abrasion or erosion also. The problem of defects accumulation should be mentioned too, when reduction of the bearing capacity connects with load's value and its duration. The models and peculiarities of corrosion wear and its influence on bearing capacity are discussed in this paper.

G.Sh. Tsitsiashvili, M.A. Osipova
ACCURACY SOLUTION OF A.A. NOVIKOV PROBLEM

O.V.Abramov, Y.V.Katueva and D.A.Nazarov
DISTRIBUTED COMPUTING ENVIRONMENT FOR RELIABILITY-ORIENTED DESIGN
A theoretical approach and applied techniques for designing analogous electronic devices and systems with due account of random variations in system parameters and reliability specifications are considered. The paper discusses the problem of choosing nominal values of parameters of electronic devices and systems for which the system survival probability or the performance assurance probability for the predetermined time period is maximized. Several algorithms for region of acceptability location, modelling and discrete optimization using parallel and distributed processing are discussed. For seeking a numerical solution of the parametric design problem a distributed computer-aided reliability-oriented design system is proposed.

N.N. Radaev, A.V. Bochkov
DETERMINING A PRIORI DISTRIBUTION OF ERROR-FREE RUNNING TIME FOR HIGH-RELIABILITY COMPONENTS BY DELPHI METHOD
We have considered the approach to determining a priori distribution of error-free running time for high-reliability components by the method of paired comparisons useful for the increase of their reliability indicators. We have introduced the distinct variables, whose grades of membership are interpreted as subjective probabilities of finding the error-free running time and its characteristics at various time intervals. The method of recording the expert evaluation accuracy has been suggested.

T. Aven
RISK ANALYSIS AND MANAGEMENT. BASIC CONCEPTS AND PRINCIPLES
This paper reviews and discusses some key concept and principles of risk analysis and risk management, based on a set of statements, formulated as myths about risk. Examples of such myths are: risk is equal to the expected value, risk equals uncertainty, risk can be expressed by probabilities, risk is equal to an event, risk acceptance criteria contribute to obtaining a high safety standard, and ALARP can be verified by cost-benefit analyses. It is concluded that risk needs to address both the consequences and the uncertainties about the consequences, and that it is necessary to see beyond expected values and probabilities.

R. Guo, D. Guo

CREDIBILISTIC FUZZY REGRESSION

In reliability, quality control and risk analysis, fuzzy methodologies are more and more involved and inevitably introduced difficulties in seeking fuzzy functional relationship between factors. In this paper, we propose a scalar variable formation of fuzzy regression model based on the credibility measure theoretical foundation. It is expecting our scalar variable treatments on fuzzy regression models will greatly simplify the efforts to seeking fuzzy functional relationship between fuzzy factors. An M-estimator for the regression coefficients is obtained and accordingly the properties and the variance-covariance for the coefficient M-estimators are also investigated in terms of weighted least-squares arguments. Finally, we explore the asymptotic membership function for the coefficient M-estimators.

Krzysztof Kolowrocki

RELIABILITY AND RISK ANALYSIS OF MULTI-STATE SYSTEMS WITH DEGRADING COPONENTS

Applications of multi-state approach to the reliability evaluation of systems composed of independent components are considered. The main emphasis is on multi-state systems with degrading components because of the importance of such an approach in safety analysis, assessment and prediction, and analysing the effectiveness of operation processes of real technical systems. The results concerned with multi-state series systems are applied to the reliability evaluation and risk function determination of a homogeneous bus transportation system. Results on homogeneous multi-state "m out of n" systems are applied to durability evaluation of a steel rope. A non-homogeneous series-parallel pipeline system composed of several lines of multi-state pipe segments is estimated as well. Moreover, the reliability evaluation of the model homogeneous multi-state parallel-series electrical energy distribution system is performed.

S. Guze

TRELIABILITY ANALYSIS OF TWO-STATE CONSECUTIVE "M OUT OF L: F"-SERIES SYSTEMS

A non-stationary approach to reliability analysis of two-state series and consecutive "m out of k: F" systems is presented. Further, the consecutive "m out of k: F"-series system is defined and the recurrent formulae for its reliability function evaluation are proposed.

Paramonov Yuri, Andersons Janis, Kleinhofs Martinsh

MINMAXDM DISTRIBUTION FAMILY FOR TENSILE STRENGTH OF COMPOSITE

Generalization of extended family of weakest-link distributions with application to the composite specimen strength analysis is presented. Composite (specifically, monolayer) specimen for tensile strength is modeled as series system but every "link" of this system is modeled as parallel system. Results of successful attempts of using some specific distribution from this family for fitting of experimental dataset of strength of some carbon fiber reinforced specimens are presented.

Viorel Gh. Voda

SOME COMMENTS ON STATISTICAL RISKS

In this work we make a detailed analysis of the concept of risk, the stress being focused then on various kinds of statistical risks: producer and consumer risks, technical risk, Taguchi`s risk (making a connection with Cpm capability index) and a risk arising in SPC practice.

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Roger Flage & Terje Aven
EXPRESSING AND COMMUNICATING UNCERTAINTY IN RELATION TO QUANTITATIVE RISK ANALYSIS
A quantitative risk analysis (QRA) should provide a broad, informative and balanced picture of risk, in order to support decisions. To achieve this, a proper treatment of uncertainty is a prerequisite. Most approaches to treatment of uncertainty in QRA seem to be based on the thinking that uncertainty relates to the calculated probabilities and expected values. This causes difficulties when it comes to communicating what the analysis results mean, and could easily lead to weakened conclusions if large uncertainties are involved. An alternative approach is to hold uncertainty, not probability, as a main component of risk, and regard probabilities purely as epistemic-based expressions of uncertainty. In the paper the latter view is taken, and we describe what should be the main components of a risk description when following this approach. We also indicate how this approach relates to decision-making. An important issue addressed is how to communicate the shortcomings and limitations of probabilities and expected values. Sensitivity analysis plays a key role in this regard. Examples are included to illustrate ideas and findings.

A. Blokus-Roszkowska, K. Kołowrocki
MODELLING ENVIRONMENT AND INFRASTRUCTURE OF SHIPYARD TRANSPORTATION SYSTEMS AND PROCESSES
In the paper an analytical model of port transportation systems environment and infrastructure influence on their operation processes is constructed and presented in an example of shipyard rope transportation systems in Naval Shipyard in Gdynia. A general semi-markov model of a system operation process is proposed and the methods of its parameters statistical identification are presented. Further, the shipyard rope transportation system and the ship rope elevator operation processes are analyzed and their operation states are defined. A preliminary collection of statistical data necessary to the ship transportation systems' operation processes identification is included.

P. Baraldi, E. Zio, M. Compare
IMPORTANCE MEASURES IN PRESENCE OF UNCERTAINTIES
This paper presents a work on the study of importance measures in presence of uncertainties originating from the lack of knowledge and information on the system (epistemic uncertainties). A criterion is proposed for ranking the risk contributors in presence of uncertainties described by probability density functions.

L. Gucma, M. Schoeneich
MONTE CARLO METHOD OF SHIP'S UNDERKEEL CLEARANCE EVALUATION FOR SAFETY OF FERRY APPROACHING TO YSTAD PORT DETERMINATION
The paper is concerned with the analysis of simulation research results of newly design Piast ferry entering to modernized Ystad Port. The ship simulation model is described. After execution of real time simulations the Monte Carlo method of underkeel clearance evaluation is applied to asses the probability of ferry touching the bottom. The results could be used in risk assessment of ships entering to the ports.

L. Gucma
METHODS OF SHIP-BRIDGE COLLISION SAFETY EVALUATION
The paper presents methods and models used nowadays for risk assessment of ship-bridge collisions.

R. Guo, D. Guo

DEAR THEORY IN SYSTEM DYNAMIC ANALYSIS

In this paper, we introduce our newly created DEAR (an abbreviation of Differential Equation Associated Regression) theory, which merges differential equation theory, regression theory and random fuzzy variable theory into a new rigorous small sample based inferential theoretical foundation. We first explain the underlying idea of DEAR modelling, its classification, and then the M-estimation of DEAR model. Furthermore, we explore the applicability of DEAR theory in the analysis in system dynamics, for example, repairable system analysis, quality dynamics analysis, stock market analysis, and ecosystem analysis, etc.

S. Guze

RELIABILITY ANALYSIS OF TWO-STATE SERIES-CONSECUTIVE "M OUT OF K: F" SYSTEMS

A non-stationary approach to reliability analysis of two-state series and consecutive "m out of k: F" systems is presented. Further, the series-consecutive "m out of k: F" system is defined and the recurrent formulae for its reliability function evaluation are proposed. Moreover, the application of the proposed formulae to reliability evaluation of the radar system composed of two-state components is illustrated.

B. Kwiatuszewska-Sarnecka, K. Kołowrocki, J. Soszyńska

MODELLING OF OPERATIONAL PROCESSES OF BULK CARGO TRANSPORTATION SYSTEM

A general analytical model of industrial systems infrastructure influence on their operation processes is constructed. Next, as its particular case a detailed model of port infrastructure influence on port transportation systems operation processes is obtained to apply and test it to selected transportation systems used in Gdynia Port.

D. Montoro-Cazorla

SHOCK MODELS UNDER POLICY N

We present the life distribution of a device subject to shocks governed by phase-type distributions. The probability of failures after shocks follows discrete phase-type distribution. Lifetimes between shocks are affected by the number of cumulated shocks and they follow continuous phase-type distributions. The device can support a maximum of N shocks. We calculate the distribution of the lifetime of the device and illustrate the calculations by means of a numerical application. Computational aspects are introduced. This model extends other previously considered in the literature.

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Joanna Soszynska

ASYMPTOTIC APPROACH TO RELIABILITY EVALUATION OF LARGE "M OUT OF L"-SERIES SYSTEM IN VARIABLE OPERATION CONDITIONS

The semi-markov model of the system operation process is proposed and its selected parameters are defined. There are found reliability and risk characteristics of the multi-state "m out of l"-series system. Next, the joint model of the semi-markov system operation process and the considered multi-state system reliability and risk is constructed. The asymptotic approach to reliability and risk evaluation of this system in its operation process is proposed as well.

C. Tanguy

ASYMPTOTIC DEPENDENCE OF AVERAGE FAILURE RATE AND MTTF FOR A RECURSIVE, MESHED NETWORK ARCHITECTURE

The paper is concerned with the exact and asymptotic calculations of the availability, average failure rate and MTTF (Mean Time To Failure) for a recursive, meshed architecture proposed by Beichelt and Spross. It shows that the asymptotic size dependences of average failure rate and MTTF are different, but not inverse of each other, as is unfortunately assumed too frequently. Besides, the asymptotic limit is reached for rather small networks.

D Valis

CONTRIBUTION TO FAILURE DESCRIPTION

In our lives we meet many events which have very diverse causes, mechanisms of development and consequences. We frequently work with the events´ description besides other assessments in safety/risk assessment. In pure technical applications these events are related with the failure occurrence of equipment, a device, a system or an item. The theory speaks about failure itself, its mechanisms, circumstances of occurrence, etc. but at the same time we need appropriate terminology to describe these conditions. Our basic approaches into observing, dealing and handling failure may fall into two groups. We either talk about a probabilistic approach or about a deterministic (logic) approach. As we need to get some information about a failure we need to find it or transfer it from different sources. This contribution can be a complex problem for the term “failure” and its related characteristics. In the paper there are mentioned functions of an object and their description, classification of failures, main characteristics of failure, possible causes of failure, mechanisms of failure and consequences of failure and also other contributions related with failure very closely.

D. Valis

CONTRIBUTION TO AVAILABILITY ASSESSMENT OF COMPLEX SYSTEMS

As we use complex systems with one shot items in many technical applications we need to know basic characteristics of such system. Performance, safety and other are as much important as dependability measures. In real applications we have to take into account a related distribution of an observed variable. In terms of complex systems with one shot items it is a discrete random variable related to one shot item. The whole system and its failures (unexpected and inadvertent events) may have two typical types of distributions and their characteristics. We either consider a continuous variable (such as time, mileage, etc.) or a counting variable (such as number of cycles, sequences, etc.) regarding to a failure occurrence. As the one shot items is supposed to back up the main system function the total reliability of the system should be higher than. The main issue regarding the system using one shot items in their construction is to determine the probability of the task (mission) success. The paper presents both theoretical approach and practical example of the solution.

Mateusz Zajac, Tymoteusz Budny

ON DETERMINATION OF SOME CHARACTERISTICS OF SEMI-MARKOV PROCESS FOR DIFFERENT DISTRIBUTIONS OF TRANSIENT PROBABILITIES

There is a model of transport system presented in the paper. The possible semi - Markov process definitions are included. The system is defined by semi – Markov processes, while functions distributions are assumed. There are attempts to assess factors for other than exponential functions distributions. The paper consist discussion on Weibull and Gamma distribution in semi – Markov calculations. It appears that some forms of distribution functions makes computations extremely difficult.

Xuejing Zhao, Mitra Fouladirad, Christophe Bérenguer, Laurent Bordes

MAINTENANCE POLICY FOR DETERIORATING SYSTEM WITH EXPLANATORY VARIABLES

This paper discusses the problem of the optimization of maintenance threshold and inspection period for a continuously deteriorating system with the influence of covariates. The deterioration is modeled by an increasing stochastic process. The process of covariates is assumed to be a temporally homogeneous finite-state Markov chain. A model similar to the proportional hazards model is used to represent the influence of the covariates. Parametric estimators of the unknown parameters are obtained by using Least Square Method. The optimal maintenance threshold and the optimal inspection interval are derived to minimize the expected average cost. Comparisons of the expected average costs under different conditions of covariates and different maintenance policies are given by numerical results of Monte Carlo simulation.

J. Okulewicz, T. Salamonowicz

PREVENTIVE MAINTENANCE WITH IMPERFECT REPAIRS OF A SYSTEM WITH REDUNDANT OBJECTS

An object ability to realise tasks may be restored by repairing only failed components. This is called imperfect repair as the object is not as good as new after such a repair. Preventive replacement is an example of imperfect repair as well. The advantage of such maintenance is that it enables controlling a reliability level of a system. Sets of objects' components which should be replaced are derived on a basis of statistical diagnosing with use of data about components failures. The acceptable level of a failure risk while executing transportation tasks has been taken as a criterion of choosing elements to be replaced. An algorithm of selecting components for preventive replacement has been developed. It was shown that a level of a system reliability can be controlled by changing an order of a quantile function in coordination and a number of redundant objects. A computer simulation model of the system was used to illustrate derived dependencies.

Jakub Nedbalek

RBF NETWORKS FOR FUNCTION APPROXIMATION IN DYNAMIC MODELLING

The paper demonstrates the comparison of Monte Carlo simulation algorithm with neural network enhancement in the reliability case study. With regard to process dynamics, we attempt to evaluate the tank system unreliability related to the initiative input parameters setting. The neural network is used in equation coefficients calculation, which is executed in each transient state. Due to the neural networks, for some of the initial component settings we can achieve the results of computation faster than in classical way of coefficients calculating and substituting into the equation.

M.F. Milazzo, G. Maschio, G. Uguccioni

FREQUENCY ASSESSMENT OF LOSS OF CONTAINMENT INCLUDING THE EFFECTS OF MEASURES OF RISK PREVENTION

This paper presents a method for the quantification of the effects of measures of risk prevention of the frequency for rupture of pipework. Some methodologies, given in the literature for this purpose, assume that each plant under analysis is characterized by the same combinations of causes of failure and prevention mechanisms but this assumption is not always true. The approach suggested here is based on the methodology proposed in 1999 by Papazoglou for the quantification of the effects of organizational and managerial factors. Taking advantage of this methodology the objective of the assessment of the influence of measures of risk prevention in pipework has been achieved through the definition of the links between the causes of failure and the measures adopted by the company in order to prevent and/or to mitigate them.

Krzysztof Kolowrocki, Joanna Soszynska

MODELLING ENVIRONMENT AND INFRASTRUCTURE INFLUENCE ON RELIABILITYAND OPERATION PROCESSES OF PORT OIL TRANSPORTATION SYSTEM

In the paper a probabilistic model of industrial systems environment and infrastructure influence on their operation processes is proposed. Semi-markov processes are used to construct a general model of complex industrial systems' operation processes. Main characteristics of this model are determined as well. In particular case, for a port oil transportation system, its operation states are defined, the relationships between them are fixed and particular model of its operation process is constructed and its main characteristics are determined. Further, the joint model of the system operation process and the system reliability is defined sand applied to the reliability evaluation of the port oil transportation system.

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V. Raizer

NATURAL DISASTERS AND STRUCTURAL SURVIVABILITY

The term "disaster" is known to denote any environmental changes putting human lives under treat or materially deteriorating living conditions. A considerable part of disasters comprises natural calamities. These disasters can originate inside Earth (earthquakes, volcanic processes), near or on its surface (disturbance of slope stability, karsts, considerable changes in soil conditions and ground's settlements). The causes of disasters can as well be associated with a water, either at a liquid (flood, tsunami) or at a frozen state (complex or glacier avalanches), and, finally with atmospheric conditions. In many cases successions of interdependent disasters are possible, including these occurring in different media (earthquake-tsunami, earthquake-landslide, and lands-flood etc.).

Gasanenko V. A., Chelobitchenko O. O.

DYNAMIC MODEL OF AIR APPARATUS PARK

The article is devoted to construction and research of dynamic stochastic model of park of aircrafts. A stochastic is enclosed in all of natural characteristic exploitations of this set of apparatuses: times of flight and landing, possibility of receipt of damage on flight, including the past recovery air apparatus; times of repair. The estimations of total possible flights are got for the any fixed interval of time.

Tsitsiashvili G.Sh., Losev A.S.

AN ASYMPTOTIC ANALYSIS OF A RELIABILITY OF INTERNET TYPE NETWORKS

In this paper a problem of a construction of accuracy and asymptotic formulas for a reliability of internet type networks is solved. Analogously to [1] such network is defined as a tree where each node is connected directly with a circle scheme on a lower level with n>0 nodes. A construction of accuracy and asymptotic formulas for probabilities of an existence of working ways between each pair of nodes of the internet type network is based on a recursive definition of these networks and on asymptotic formulas for a reliability of a random port. This asymptotic formula represents the port reliability as a sum of probabilities of a work for all ways between initial and final nodes of this port. An estimate of a relative error and a complexity of these asymptotic calculations for a radial-circle scheme are shown.

Salem Bahri, Fethi Ghribi, Habib Ben Bacha

A STUDY OF ASYMPTOTIC AVAILABILITY MODELING FOR A FAILURE AND A REPAIR RATES FOLLOWING A WEIBULL DISTRIBUTION

The overall objective of the maintenance process is to increase the profitability of the operation and optimize the availability. However, the availability of a system is described according to lifetime and downtime. It is often assumed that these durations follow the exponential distribution. The work presented in this paper deals with the problem of availability modeling when the failure and repair rates are variable. The lifetime and downtime were both governed by models of Weibull (the exponential model is a particular case). The differential equation of the availability was formulated and solved to determine the availability function. An analytical model of the asymptotic availability was established as a theorem and proved. As results deduced from this study, a new approach of modeling of the asymptotic availability was presented. The developed model allowed an easy evaluation of the asymptotic availability. The existence of three states of availability for a system has been confirmed by this evaluation. Finally, these states can be estimated by comparing the shape parameters of the Weibull model for the failure and repair rates.

Igor Ushakov, Sumantra Chakravarty

OBJECT ORIENTED COMMONALITIES IN UNIVERSAL GENERATING FUNCTION FOR RELIABILITY AND IN C++

The main idea of Universal Generating Function is exposed in reliability applications. Some commonalities in this approach and the C++ language are discussed.

Alexandru ISAIC-MANIU, Viorel Gh. VODĂ

SOME INFERENCES ON THE RATIO AVERAGE LIFETIME/TESTING TIME IN ACCEPTANCE SAMPLING PLANS FOR RELIABILITY INSPECTION

In this paper we construct effective single sampling plans for reliability inspection, when the distribution of failure times of underlying objects obey a Weibull law. To this purpose we use the index average lifetime (E (T)/testing time (T) for two values of E(T) - acceptable and non acceptable ones - and known shape parameter (K) of the Weibull cdf. We derive also a relationship between this index and reliability function R(t) of the assumed statistical law. A numerical illustrations is provided in the case of Rayleigh cdf - that is for a Weibull shape
k = 2.

Tsitsiashvili G.Sh., Losev A.S.

AN ACCURACY OF ASYMPTOTIC FORMULAS IN CALCULATIONS OF A RANDOM NETWORK RELIABILITY

In this paper a problem of asymptotic and numerical estimates of relative errors for different asymptotic formulas in the reliability theory are considered. These asymptotic formulas for random networks are similar to calculations of Feynman integrals. A special interest has analytic and numerical comparison of asymptotic formulas for the most spread Weibull and Gompertz distributions in life time models. In the last case it is shown that an accuracy of asymptotic formulas is much higher.

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Armen. S. Stepanyants, Valentina S. Victorova

FAILURE FREQUENCY CALCULATION TECHNIQUE IN LOGICAL-PROBABILISTIC MODELS

The technique for calculation failure frequency measure of reliability in class of logical-probabilistic-models is proposed. The technique is applicable for models of redundant repairable systems which are not limited by serial-parallel structures. In conjunction with system decomposition the techniques makes it possible to analyze high dimensional systems very efficiently.

Tsitsiashvili G. Sh.

PHASE TRANSITION IN RENEWAL SYSTEMS WITH COMMON RESERVE

Mathematical models of renewal systems with a common reserve have been introduced and analyzed detailed in the monograph [1]. In [2] a phenomenon of a phase transition in the aggregated renewal system with the unload reserve is analyzed as analytically so numerically. But the mathematical method applied in this paper is too specific to analyze the phase transition phenomenon in general renewal systems with the common reserve. This phenomenon is connected with a reform of municipal engineering systems.

In this paper a method based on a definition of a state in which a birth and death process describing this system has a maximal limit probability is suggested. This method allows to construct convenient upper bounds of the limit probability for other states and to analyze phase transition phenomenon. The obtained bounds depend on transition intensities of the birth and death processes which describe aggregations of renewal systems with unload, under load and load reserves. The suggested method allows an analyzing of a renewal system with a competition between the repair places also.

Mr. Marc Antoni MEng

THE AGEING OF SIGNALLING EQUIPMENT AND THE IMPACT ON MAINTENANCE STRATEGIES

Research projects of SNCF (French railway) aim at reducing the costs of infrastructure possessions and improving the operational equipment availability and safety. This permanent search for a better regularity led the SNCF to analyse the maintenance approach of signalling equipment in detail. Until now, it was commonly acknowledged that signalling equipment, which consists of many electronic devices, is not subject to ageing. In this study, a Weibull lifetime model, able to describe an ageing phenomenon, is used and it can be shown that the deterioration is statistically significant. The validity of the model is tested. We also analyse the influence of environmental covariates. We simulate different scenarios in order to investigate the impact of several maintenance strategies as well as on future maintenance costs, on the amount of components to replace based on the mean age of the network. It can be shown that in most cases a systematic replacement strategy offers the best solution.

E.B. Abrahamsen & T. Aven, W. Røed

COMMUNICATION OF COST-EFFECTIVENESS OF SAFETY MEASURES BY USE OF A NEW VISUALIZING TOOL

A cost-effectiveness analysis (CEA) is often used as basis for comparisons between competing safety measures. In a CEA indices such as the expected cost per expected number of lives saved are calculated. These indices are presented to the decision-makers, and seen in relation to reference values, they form the basis for assessment of the effectiveness of the safety measures.

The appropriateness of using cost-effectiveness indices based on expected values have been thoroughly discussed in the literature. It is argued that uncertainty is not properly taken into account by the CEA, and extended frameworks for CEA are required. This paper represents a contribution to this end, by presenting a diagram that visualizes uncertainty in addition to the expected values as in the traditional CEA. The diagram is meant to be a presentation tool for semi-quantitative cost-effectiveness analyses used as a part of a screening process to identify safety measures to be assessed in a more detailed analysis. In the paper we discuss the use of the diagram in communication between analysts and other stakeholders, in particular the decision-makers. An example is presented to illustrate the applicability of the tool.

G. Albeanu, A. Averian, I. Duda

WEB SOFTWARE RELIABILITY ENGINEERING

There is an increasing request for web software systems, some of them to be used very intensive. The customers ask not only for fast design and implementation, but also for a high quality product. Considering reliability as an important quality attribute, this paper describes the current state of the art in designing, implementing, and testing web software. An important attention is given to software vulnerabilities and how to deliver secure software.

H.-P. Berg

CORROSION MECHANISMS AND THEIR CONSEQUENCES FOR NUCLEAR POWER PLANTS WITH LIGHT WATER REACTORS

It is well known that operational conditions in light water reactors strongly influence the corrosion processes. This paper gives an overview which types of corrosion are identified in operating practice based on the evaluation of events which are reported to the authorities in line with the German reporting criteria. It has been found that the main contributor is the stress corrosion cracking. Several examples of different corrosion mechanisms and their consequences are provided for PWR although a high standard of quality of structures, systems and components has been achieved. Recommendations have been given to check the plant specifications concerning the use of auxiliary materials or fluids during maintenance as well as to examine visually the outer surfaces of austenitic piping with regard to residua of adhesive or adhesive tapes within the framework of in-service inspections. However, events in the last two years show that such problems cannot be totally avoided.

H.-P. Berg, R. Gersinska,J. Sievers

IMPROVED APPROACH FOR ESTIMATING LEAK AND BREAK FREQUENCIES OF PIPING SYSTEMS IN PROBABILISTIC SAFETY ASSESSMENT

The estimation of leak and break frequencies in piping systems is part of the probabilistic safety assessment of technical plants. In this paper, the statistical method based on the evaluation of the German operational experience for piping systems with different diameters is described because an earlier estimation has been updated and extended introducing new methodical aspects and data. Major point is the inclusion of structure reliability models based on fracture mechanics calculation procedures. As an example of application the statistical estimation method for leak and break frequencies of piping systems with a nominal diameter of 50 mm (the volume control system of a German pressurized water reactor) was updated. Moreover, the evaluation of the operational experience was extended to 341 years with respect to cracks, leaks and breaks in the volume control system of German pressurized water reactors (PWR). Using the actual data base, new calculations of leak and break frequencies have been performed and the results have been compared with the previous values.

A. Blokus-Roszkowska, K. Kołowrocki

RELIABILITY AND AVAILABILITY OF A GROUND SHIP-ROPE TRANSPORTER IN VARIABLE OPERATION CONDITIONS

In the paper the environment and infrastructure influence of the ground ship-rope transporter operating in Naval Shipyard in Gdynia on its operation processes is considered. The results are presented on the basis of a general model of technical systems operation processes related to their environment and infrastructure. The transporter operation process is described and its statistical identification is given. Next, the reliability, risk and availability evaluation of the transporter in variable operation conditions is presented. In addition, the reliability and availability basic characteristics of the system assuming its components' failure dependence are determined. Finally, the obtained results for the ground ship-rope transporter under the assumption that its components are dependent and independent are compared.

R. Briš

UNAVAILABILITY CALCULATIONS WITHIN THE LIMITS OF COMPUTER ACCURACY

The paper presents a new analytical algorithm which is able to carry out direct and exact reliability quantification of highly reliable systems with maintenance (both preventive and corrective). A directed acyclic graph is used as a system representation. The algorithm is based on a special new procedure which permits only summarization between two or more non-negative numbers that can be very different. If the summarization of very small positive numbers transformed into the machine code is performed effectively no error is committed at the operation. Reliability quantification is demonstrated on a real system from practice.

F. Cadini, E. Zio, L.R. Golea, C.A. Petrescu

ANALYSIS AND OPTIMIZATION OF POWER TRANSMISSION GRIDS BY GENETIC ALGORITHMS

Two applications of multi-objective genetic algorithms (MOGAs) are reported with regards to the analysis and optimization of electrical transmission networks. In a first case study, an analysis of the topological structure of a network system is carried out to identify the most important groups of elements of different sizes in the network. In the second case study, an optimization method is devised to improve the reliability of power transmission by adding lines to an existing electrical network.

R. Guo, D. Guo, C. Thiart

POISSON PROCESSES WITH FUZZY RATE

Poisson processes, particularly the time-dependent extension, play important roles in reliability and risk analysis. It should be fully aware that the Poisson modeling in the current reliability engineering and risk analysis literature is merely an ideology under which the random uncertainty governs the phenomena. In other words, current Poisson Models generate meaningful results if randomness assumptions hold. However, the real world phenomena are often facing the co-existence reality and thus the probabilistic Poisson modeling practices may be very doubtful. In this paper, we define the random fuzzy Poisson process, explore the related average chance distributions, and propose a scheme for the parameter estimation and a simulation scheme as well. It is expecting that a foundational work can be established for Poisson random fuzzy reliability and risk analysis.

S. Guze, L. Smolarek

MODELLING THE SHIP SAFETY ON WATERWAY ACCORDING TO NAVIGATIONAL SIGNS RELIABILITY

An approach to safety analysis connected with consecutive “m out of n” systems is presented. Further, the consecutive “m out of n: G” system is defined and the recurrent formulae for its reliability function evaluation are proposed. Next the IALA buoys and leading lights system are introduced. Moreover, the safety states model for ship navigation are defined. Further, analysis of safety during manoeuvre in restricted area with curved draws is illustrated.

K. Kolowrocki, J. Soszynska

RELIABILITY, RISK AND AVAILABILITY BASED OPTIMIZATION OF COMPLEX TECHNICAL SYSTEMS OPERATION PROCESSES. PART 1. THEORETICAL BACKGROUNDS

A convenient new tool for solving the problem of reliability and availability evaluation and optimization of complex technical systems is presented. Linking a semi-markov modeling of the system operation processes with a multi-state approach to system reliability and availability analysis is proposed to construct the joint general model of reliability and availability of complex technical systems in variable operation conditions. This joint model and a linear programming is proposed to complex technical systems reliability and availability evaluation and optimization respectively.

K. Kolowrocki, J. Soszynska

RELIABILITY, RISK AND AVAILABILITY BASED OPTIMIZATION OF COMPLEX TECHNICAL SYSTEMS OPERATION PROCESSES. PART 2. APPLICATION IN PORT TRANSPORTATION

The joint general model of reliability and availability of complex technical systems in variable operation conditions linking a semi-markov modeling of the system operation processes with a multi-state approach to system reliability and availability analysis and linear programming considered in the paper Part 1 are applied in maritime industry to reliability, risk and availability optimization of a port piping oil transportation system.

Krzysztof Kolowrocki, Joanna Soszynska

SAFETY AND RISK EVALUATION OF STENA BALTICA FERRY IN VARIABLE OPERATION CONDITIONS

Basic safety structures of multi-state systems of components with degrading safety states related to their variable operation conditions are defined. For these systems the conditional and unconditional multi-state safety functions are determined. A semi-markov process for the considered systems operation modelling is applied. Further, the paper offers an approach to the solution of a practically important problem of linking the multi-state systems safety models and the systems operation processes models.

Theoretical definitions and results are illustrated by the example of their application in the safety and risk evaluation of the Stena Baltica ferry operating at the Baltic Sea. The ferry transportation system has been considered in varying in time operation conditions. The system safety structure and its components safety functions were changing in variable operation conditions.

Mohamed Salahuddin Habibullah, Ernest Lumanpauw, Kołowrocki Krzysztof, Joanna Soszyńska, Ng Kien Ming

A COMPUTATIONAL TOOL FOR A GENERAL MODEL OF OPERATION PROCESSES IN INDUSTRIAL SYSTEMS

The complexities of real industrial systems operation processes require computational methods that can analyze the large data and evaluate the behaviours of these systems. The use of methods such as Bayesian Network, Formal Safety Assessment and Statistical-Model based method were discussed as possibilities. Of which, a computational tool, based on the Semi-Markov model, was developed. This tool was then applied to analyze the behaviour of the operation processes of the oil transportation system in Dębogórze, Poland. The analyses showed that the computational solutions generated compared favourably well with some well-established analytical formulae, enabling possible extensions of the tool to include reliability and optimization evaluations to be explored.

D. Valis, Z. Vintr, M. Koucky

SELECTED APPROACHES FOR RELIABILITY COMPARISON OF HIGHLY RELIABLE ITEMS

The application of electronic elements introduces a number of advantages as well as disadvantages. The paper deals with advanced method of dependability - reliability analysis procedure of a highly reliable item. The data on manufacturing and operating of a few hundred thousands pieces of the highly reliable devices are available and from the statistical point of view they are very important collection/set. However, concerning some pieces of the items the manufacturing procedure of them was not made, controlled and checked accurately. The procedure described in the paper is based on the thorough data analysis aiming at the operation and manufacturing of these electronic elements. As the data sets collected are statistically non-coherent the objective of the paper is to make a statistical assessment and evaluation of the results. Failure rates calculations and their relation comparability regarding the both sets are presented in the paper.

Christian Tanguy

MEAN TIME TO FAILURE FOR PERIODIC FAILURE RATE

The paper is concerned with the determination of the Mean Time To Failure (MTTF) in configurations where the failure rate is periodical. After solving two configurations exactly, we show that when the period of the failure rate oscillations is small with respect to the average failure rate, the MTTF is essentially given by the inverse of the average failure rate, give or take corrections that can be expressed analytically. This could be helpful in the description of systems the environment of which is subject to changes.

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Bożena Babiarz

RELIABILITY ASSESSMENT OF HEAT SUPPLY SYSTEMS IN THEIR OPERATIONAL PROCESS

This paper presents an analysis of the operational process of heat supply system, taking into consideration its reliability. The specific character of the operation of heat-supply systems has been considered in this work. In the process of exploitation of heat-supply systems five operational states have been distinguished, using as a criterion the level of indoor temperature decrease in residential rooms. The method of modelling the reliability of heat-supply systems is worked out. The methodology of determining the overall index of heat-supply system reliability has been presented. The measure of heat-supply system reliability has been taken to be as the scale/quantity of inadequate supply of heat power at a given state. Calculations have been carried out regarding the changeability of exterior conditions for one of the groups of customers – residential users. On the basis of the operational data for the heat supply system with two heat sources, shortfalls of heat power and the probability of their occurrence have been calculated as an application of this methodology.

R. V. Kakubava

MULTI-LINE MARKOV CLOSED QUEUING SYSTEM FOR TWO MAINTENANCE OPERATIONS

In the given paper multi-component standby system with renewable elements is considered. For it multi-line closed Markov queuing model for two maintenance operations – replacements and renewals, is constructed and investigated. In this model the numbers of main elements as well as standby ones, also the numbers of replacement units as wall as renewal ones are arbitrary. An economic criterion for dependability planning (structural control) of considered system is introduced, the optimization problem is stated and partially investigated.

R. Lisi, M. F. Milazzo, G. Maschio

RISK ASSESSMENT OF EXPLOSIVE ATMOSPHERES IN WORKPLACES

The application of the Directive 99/92/EC deals with the safety and health protection of workers potentially exposed to explosive atmospheres and requires the assessment of explosion risks. These can arise by the release of inflammable substances typical of industries classified as major hazards, but they often may be generated in other industries where inflammable materials are handled. Risk assessment of explosive atmospheres is required in both cases, for this purpose, in this article a quantitative approach has been proposed. The paper describes the main aspects of the methodology, based on a probabilistic risk assessment, and finally its application to a case-study.

R. Dubčáková, P. Praks

STATISTICAL MODELLING OF INDOOR RADON CONCENTRATION USING METEOROLOGICAL PARAMETERS

The radon volume activity in buildings is generally time variable. Its variability is caused by many natural and man-made factors. An example of these factors includes meteorological parameters, soil properties, characteristics of the building construction, properties of water used in the building and also the behavior of inhabitants. These factors can influence each other and also they are related with the exposition of inhabitants.

This article reports a continual indoor radon monitoring and a statistical evaluation of a dataset obtained by the 18 days-long measuring in a house located in the Czech Republic.

The contributions of carefully selected meteorological parameters and human influences were also observed. Results of the observation were divided into two parts (inhabited, uninhabited) and analyzed in relation with the indoor radon concentration. The multiplied linear regression was applied to model obtained datasets.

Results of time series analyses of the continual indoor radon concentration and the meteorological monitoring are presented and discussed as well.

G.Tsitsiashvili, A. Losev

CALCULATION OF CONNECTIVITY PROBABILITY IN RECURSIVELY DEFINED RANDOM NETWORKS

In this paper a problem of a construction of new and practically interesting classes of recursively defined networks, including internet type networks, with sufficiently fast algorithms of calculation of connectivity probability is considered. For this aim recursive and asymptotic formulas of connectivity probability calculation are constructed. Asymptotic formulas are based on assumptions that all network arcs are low reliable or there are high reliable and low reliable arcs in considered network. For example in radial-circle scheme radial arcs may be high reliable and circle arcs – low reliable.

Yakov Genis
RELIABILITY ASSESSMENT OF SYSTEMS WITH PERIODIC MAINTENANCE UNDER RARE FAILURES OF ITS ELEMENTS

There is investigated a model of a system with the highly reliable elements, where the periods of functioning are changed by the periods of maintenance. The system must be operational only in the periods of functioning although the restoration in these periods is not provided. The system is completely restored in the nearest period of maintenance. Since the elements of system are highly reliable, the reserve of system is rarely exhausted during each period of functioning. Therefore it is possible to use the results, obtained for the systems with fast restoration, for the reliability assessment of system, which is not restorable in the periods of the functioning. The estimations of indices of failure-free performance and maintainability of such systems are obtained.

M. Zajac, D. Valis
FUNDAMENTAL RISK ASSESSMENT IN EXAMPLE OF TRANSSHIPMENT SYSTEM

The paper represents discussion about risk assessment for transhipment system in reduces data condition. As a particular example transhipment system is presented. Article can be treated as first estimation. Future work and objectives are characterized in the end.

G.Sh. Tsitsiashvili
ASYMPTOTIC ANALYSIS OF LATTICE RELIABILITY

Asymptotic formulas for connection probabilities in a rectangular lattice with identical and independent arcs are obtained.. For a small number of columns these probabilities may be calculated by the transfer matrices method. But if the number of columns increases then a calculation complexity increases significantly. A suggested asymptotic method allows to make calculations using a sufficiently simple geometric approach in a general case.

Al. Isaic-Maniu, I. M. Dragan
THE RISK OF OPERATIONAL INCIDENTS IN BANKING INSTITUTIONS

Banking-financial institutions are organizations which might be included in the category of complex systems. Consequently, they can be applied after adaptation and particularization, in the general description and assessment methods of the technical or organizational systems. The banking-financial system faces constrains regarding the functioning continuity. Interruptions in continuity as well as operational incidents represent risks which can lead to the interruption of financial flows generation and obviously of profit. Banking incidents include from false banknote, cloned cards, informatics attacks, false identity cards to ATM attacks. The functioning of banking institutions in an incident-free environment generates concern from both risk assessment and forecasting points of view.

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Z. Bluvband & S. Porotsky
PARAMETER ESTIMATIONS FOR AVAILABILITY GROWTH

The reliability growth process applied to a complex system undergoing development and field test involves surfacing failure modes, analyzing the modes, and, in addition to repair, in some cases implementing corrective actions to the surfaced modes. In such a manner, the system configuration is matured with respect to reliability. The conventional procedure of reliability growth implies evaluation of two principal parameters of the NHPP process only for failure rate. Since standard NHPP does not take into account parameters of repairs, it is necessary to develop expanded procedure as the basis for the Availability Growth. It implies evaluation of both: a) the parameters of failure rate and, b) the parameters of repair rate. Authors suggest a model and numerical method to search these parameters.

Farhadzadeh E.M., Muradaliyev A.Z., Farzaliyev Y.Z.
CONTROL THE IMPORTANCE OBSERVABLE LAWS OF CHANGE RELIABILITY OVER OPERATION

The problem of the control of the importance of observable laws of change of parameters of reliability (PR) at small statistical data of operating experience or experiment in conditions when the argument has a serial or nominal scale of measurement, concerns to number of the most difficult and insufficiently developed. In particular, at operation of electro installations the important role-played with data on reliability of units of the same equipment, on the reasons of occurrence and character of their damage, law of change PR of the equipment for various classes of a pressure and so forth

A. I. Pereguda, D. A. Timashov
A FUZZY RELIABILITY MODEL FOR “SAFETY SYSTEM-PROTECTED OBJECT” COMPLEX

The paper presents a new fuzzy reliability model for automated “safety system-protected object” complex. It is supposed that parameters of reliability model and reliability indices are fuzzy variables. Scheduled periodic inspections of safety system are also taken into account. Asymptotic estimates of mean time to accident membership function are proposed.

A. I. Pereguda, D. A. Timashov
AN ADVANCED RELIABILITY MODEL FOR AUTOMATED “SAFETY SYSTEM-PROTECTED OBJECT” COMPLEX WITH TIME REDUNDANCY

The paper presents a new reliability model for an automated “safety system-protected object” complex with time redundancy. It is supposed that the time redundancy is caused by a protected object inertia. Scheduled periodic inspections of the safety system are also taken into account. Two-sided estimates of the mean time to accident are proposed.

Armen S. Stepanyants, Valentina S. Victorova

RELIABILITY AND CAPABILITY MODELING OF TECHNOLOGICAL SYSTEMS WITH BUFFER STORAGE

The paper is devoted to reliability and capability investigation of technological systems, inclusive of development of dynamic reliability model for two-phase product line with buffer storages and multiphase line decomposition

G.Tsitsiashvili

COMPARISON ANALYSIS OF RELIABILITY OF NETWORKS WITH IDENTICAL EDGES

Efficient and fast algorithms of parameters calculation in Burtin-Pittel asymptotic formula for networks with identical and high reliable edges are constructed. These algorithms are applied to a procedure of a comparison of networks obtained from a radial-circle network by a cancelling of some edges or their collapsing into nodes or by a separate reservation of these edges.

G.Tsitsiashvili

ASYMPTOTIC FORMULAS IN DISCRETE TIME RISK MODEL WITH DEPENDENCE OF FINANCIAL AND INSURANCE RISKS

Asymptotic formulas for a ruin probability in discrete time risk model with a dependence of financial and insurance risks are obtained. These formulas are constructed in a suggestion which is adequate to economical crisis: the larger is a financial risk the larger is an insurance risk.

Heinz-Peter Berg

QUANTITATIVE SAFETY GOALS AND CRITERIA AS A BASIS FOR DECISION MAKING

Internationally, probabilistic safety analyses represent the state of the art in the licensing process for new industrial facilities, but increasingly also for evaluating the safety level of older industrial plants, e. g. as part of periodic safety reviews of nuclear power plants. Quantitative safety goals have not yet reached the same level of acceptance. However, this depends on the type of industry. Most of the countries consider those criteria as safety targets rather than as sharply defined boundary values. The Netherlands and the United Kingdom are exceptions, they require demonstration of compliance with legally binding safety goals in the licensing procedure.

Heinz-Peter Berg

RISK MANAGEMENT: PROCEDURES, METHODS AND EXPERIENCES

Risk management is an activity which integrates recognition of risk, risk assessment, developing strategies to manage it, and mitigation of risk using managerial resources. Some traditional risk managements are focused on risks stemming from physical or legal causes (e.g. natural disasters or fires, accidents, death). Financial risk management, on the other hand, focuses on risks that can be managed using traded financial instruments. Objective of risk management is to reduce different risks related to a pre-selected domain to an acceptable. It may refer to numerous types of threats caused by environment, technology, humans, organizations and politics. The paper describes the different steps in the risk management process which methods are used in the different steps, and provides some examples for risk and safety management.

A. Blokus-Roszkowska, K. Kołowrocki
RELIABILITY AND AVAILABILITY OF A SHIPYARD SHIP-ROPE ELEVATOR IN VARIABLE OPERATION CONDITIONS

In the paper the environment and infrastructure influence of the ship-rope elevator operating in Naval Shipyard in Gdynia on its operation processes is considered. The results are presented on the basis of a general model of technical systems operation processes related to their environment and infrastructure. The elevator operation process is described and its statistical identification is given. Next, the elevator is considered in varying in time operation conditions with different its components' reliability functions in different operation states. Finally, the reliability, risk and availability evaluation of the elevator in variable operation conditions is presented.

F. Cadini, D. Avram, E. Zio
A MONTE CARLO-BASED TECHNIQUE FOR ESTIMATING THE OPERATION MODES OF HYBRID DYNAMIC SYSTEMS

Many real systems are characterized by a hybrid dynamics of transitions among discrete modes of operation, each one giving rise to a specific continuous dynamics of evolution. The estimation of the state of these hybrid dynamic systems is difficult because it requires keeping track of the transitions among the multiple modes of system dynamics corresponding to the different modes of operation. A Monte Carlo-based estimation method is here illustrated through an application to a case study of literature.

F. Grabski, A. Załęska-Fornal
BOOTSTRAP METHODS FOR THE CENSORED DATA IN EMPIRICAL BAYES ESTIMATION OF THE RELIABILITY PARAMETERS

Bootstrap and resampling methods are the computer methods used in applied statistics. They are types of the Monte Carlo method based on the observed data. Bradley Efron described the bootstrap method in 1979 and he has written a lot about it and its generalizations since then. Here we apply these methods in an empirical Bayes estimation using bootstrap copies of the censored data to obtain an empirical prior distribution.

R. Guo, D. Guo, T. Dunne
RANDOM FUZZY CONTINUOUS-TIME MARKOV JUMP PROCESSES

Continuous-time Markov chains are an important subclass in stochastic processes, which have facilitated many applications in business decisions, investment risk analysis, insurance policy making and reliability modeling. One should be fully aware that the existing continuous-time Markov chains theory is merely a framework under which the random uncertainty governs the phenomena. However, the real world phenomena often reveal a reality in which randomness and vagueness co-exist, and thus probabilistic continuous-time Markov chains modeling practices may be not wholly adequate. In this paper, we define random fuzzy continuous-time Markov chains, explore the related average chance distributions, and propose both a scheme for parameter estimation and a simulation scheme. It is expected that a foundational base can be established for reliability modeling and risk analysis, particularly, repairable system modeling.

S. Guze, B, Kwiatuszewska-Sarnecka, J. Soszyńska

THE COMPUTER PROGRAM TO VERIFY THE HYPOTHESIZES AND TO PREDICT OF THE PARAMETERS FOR OPERATIONAL PROCESS

The theoretical background and technical information for the program are presented. Further, the components of the program are described and user manual is given.

O. Hryniewicz

CONFIDENCE BOUNDS FOR THE RELIABILITY OF A SYSTEM FROM SUBSYSTEM DATA

The paper is concerned with the construction of lower bounds for the reliability of a system when statistical data come from independent tests of its elements. The overview of results known from literature and obtained under the assumption that elements in a system are independent is given. It has been demonstrated using a Monte Carlo experiment that in case when these elements are dependent and when their dependence is described by Clayton and Gumbel copulas these confidence bounds are not satisfactory. New simple bounds have been proposed which in some practical cases perform better than the classical ones.

M. Jurdziński, S. Guze, P. Kamiński

TIME DIFFERENCES IN PERATION STATES OF STENA BALTICA FERRY DURING THE OPEN WATER AREAS PASSAGE

The paper deals with analysis of ships operation stages in open water areas effected by environmental constraints influencing on ship sea keeping parameters in application to ferry “Stena Baltica” operated in the Baltic Sea between Gdynia and Karlskrona harbors.

K. Kołowrocki, B. Kwiatuszewska-Sarnecka, J. Soszyńska

PRELIMINARY RELIABILITY, RISK AND AVAILABILITY ANALYSIS AND EVALUATION OF BULK CARGO TRANSPORTATION SYSTEM IN VARIABLE OPERATION CONDITIONS

In the paper, definitions and theoretical results on system operations process, multi-state system reliability, risk and availability modelling are illustrated by the example of their application to a bulk cargo transportation system operating in Gdynia Port Bulk Cargo Terminal. The bulk cargo transportation system is considered in varying in time operation conditions. The system reliability structure and its components reliability functions are changing in variable operation conditions. The system reliability structures are fixed with a high accuracy. Whereas, the input reliability characteristics of the bulk cargo transportation system components and the system operation process characteristics are not sufficiently exact because of the lack of statistical data. Anyway, the obtained evaluation may be a very useful example in simple and quick systems reliability characteristics evaluation, especially during the design and improving the transportation systems operating in ports.

K. Kolowrocki, J. Soszynska

METHODS AND ALGORITHMS FOR EVALUATING UNKNOWN PARAMETERS OF OPERATION PROCESSES OF COMPLEX TECHNICAL SYSTEMS (part 1)

The paper objectives are to present the methods and tools useful in the statistical identifying unknown parameters of the operation models of complex technical systems and to apply them in the maritime industry. There are presented statistical methods of determining unknown parameters of the semi-markov model of the complex technical system operation processes. There is also presented the chi-square goodness-of-fit test applied to verifying the distributions of the system operation process conditional sojourn times in the particular operation states. Applications of these tools to identifying and predicting the operation characteristics of a ferry operating at the Baltic Sea waters are presented as well.

K. Kolowrocki, J. Soszynska

METHODS AND ALGORITHMS FOR EVALUATING UNKNOWN PARAMETERS OF COMPONENTS RELIABILITY OF COMPLEX TECHNICAL SYSTEMS (part 2)

The paper objectives are to present the methods and tools useful in the statistical identifying the unknown parameters of the components reliability and safety of complex industrial systems and to apply them in the maritime industry. There are presented statistical methods of estimating the unknown intensities of departure from the reliability state subsets of the exponential distribution of the component lifetimes of the multistate systems operating in various operation states. The goodness-of-fit method applied to testing the hypotheses concerned with the exponential form of the multistate reliability function of the particular components of the complex technical system in variable operations conditions is suggested. An application of these tools to reliability characteristics of a ferry operating at the Baltic Sea waters is presented as well.

B. Kwiatuszewska-Sarnecka

MODELS OF RELIABILITY AND AVAILABILITY IMPROVEMENT OF SERIES AND PARALLEL SYSTEMS RELATED TO THEIR OPERATION PROCESSES

Integrated general models of approximate approaches of complex multi-state series and parallel systems, linking their reliability and availability improvement models and their operation processes models caused changing reliability and safety structures and components reliability characteristics in different operation states, are constructed. These joint models are applied to determining improved reliability and availability characteristics of the considered multi-state series and parallel systems related to their varying in time operation processes. The conditional reliability characteristics of the multi-state systems with hot, cold single reservation of component and the conditional reliability characteristics of the multi-state systems with reduced rate of departure by a factor of system components are defined.

M.S. Habibullah, Fu Xiuju, K. Kolowrocki, J. Soszynska

CORRELATION AND REGRESSION ANALYSIS OF SPRING STATISTICAL DATA OF MARITIME FERRY OPERATION PROCESS

These are presented statistical methods of correlation and regression analysis of the operation processes of complex technical systems. The collected statistical data from the Stena Baltica ferry operation process are analysed and used for determining correlation coefficients and linear and multiple regression equations, expressing the influence of the operation process conditional sojourn times in particular operation states on the ferry operation process total conditional sojourn time.

D. Valis, Z. Vintr, M. Koucky

CONTRIBUTION TO RELIABILITY ANALYSIS OF HIGHLY RELIABLE ITEMS

In recent years the intensive efforts in developing and producing electronic devices have more and more critical inference in many areas of human activity. Engineering is one of the areas which have been also importantly affected. The paper deals with dependability namely reliability analysis procedure of a highly reliable item. The data on manufacturing and operating of a few hundred thousands pieces of electronic item are available and they are statistically a very important collection/set. However, concerning some items the manufacturing procedure was not checked and controlled accurately. The procedure described in the paper is based on the thorough data analysis aiming at the operating and manufacturing of these electronic elements. The results indicate some behaviour differences between correctly and incorrectly made elements. It was proved by the analysis that dependability and safety of these elements was affected to a certain degree. Although there is a quite big set of data the issue regarding the statistical comparability is very important.

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