NUMERICAL EXPERIMENT FOR RUIN PROBABILITY IN RISK MODEL WITH DEPENDENT FINANCIAL AND INSURANCE RISKS

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ABSTRACT

For discrete time risk model with dependent financial and insurance risks numerical experiment with recurrent procedure of ruin probability calculation is made. It shows that suggested recurrent procedure is much faster than application of usual Monte-Carlo method.

INTRODUCTION

In (Tsitsiashvili, 2010) for discrete time risk model with dependent financial and insurance risks recurrent algorithm of ruin probability calculation is constructed on a base of hyperexponential approximation of insurance losses distributions. Special methods of symbol forms transformation and economical procedure of enumeration of vectors with integer components and fixed sum are developed

In this paper constructed recurrent algorithm is tested in numerical experiment. Comparison analysis showed that suggested algorithm is significantly faster then application of usual Monte-Carlo method.

1. PRELIMINARIES

In (Tsitsiashvili 2010) recurrent discrete time risk model with initial capital $x,\ x \geq 0,$ nonnegative losses Z_n, $n = 1,2,...,$ and inflation factor $Y_n^{-1} > 1$ from $n-1$ to n year:

$$S_0 = x,\ \ S_n = Y_n^{-1} S_{n-1} + 1 - Z_n,\ \ n = 1,2...,$$

is considered. Here $X_n = Z_n - 1$ is insurance risk and Y_n is financial risk. Suppose that the sequence $\{Y_n, n \geq 0\}$ is stationary reversible Markov chain with state set $\{r_q^{-1}, q \in Q\}$, $Q = \{1,...,m\}$, with transition matrix $\|\pi_{q'q}\|_{q,q' \in Q}$ and with initial distribution

$$P\left(Y_n = r_q^{-1}\right) = p_q > 0,\ \ \sum_{q \in Q} p_q = 1\,.$$

Introduce dependence between financial and insurance risks by a conditional distribution of random variable Z_n

$$\bar{F}_q(t) = P\left(Z_n > t \big/ Y_n = r_q^{-1}\right),\ \ q \in Q,\ \ t \geq 0.$$

In this model finite horizon ruin probability

$$\psi_n(x) = P\left(\inf\{n = 1,2,\dots : S_n \le 0 | S_0 = x\} \le n\right).$$

Following recurrent algorithm of its calculation is constructed. Denote (δ_{ij} is Kronecker symbol)

$$1_q = \left(\delta_{1q},\dots,\delta_{mq}\right), \quad K = (k_1,\dots,k_m),\ k_i \in \{0,1,\dots\},\ i = 1,\dots,m, \quad r^K = \prod_{q\in Q} r_q^{k_q},\ K_\Sigma = \sum_{q\in Q} k_q.$$

Theorem. *If*

$$r^K \lambda_i \ne \lambda_j,\ i \ge 1,\ j \le l,\ K_\Sigma \ge 1,$$

and

$$\bar{F}_q(t) = \sum_{i=1}^{l} a_{qi} \exp(-\lambda_i t), -\infty < a_{qi} < \infty, \sum_{i=1}^{l} a_{qi} = 1,\ q \in Q,\ t \ge 0,$$

when for $n \ge 1$

$$\psi_n(t) = \begin{cases} \sum_{q\in Q} \sum_{1\le K_\Sigma \le n} \sum_{i=1}^{l} B_{n,i,q}^K,\ t > 0, \\ 1 + \sum_{q\in Q} \sum_{1\le K_\Sigma \le n} \sum_{i=1}^{l} B_{n,i,q}^K \left(\exp\left(-r^K \lambda_i t\right) - 1\right),\ t \le 0, \end{cases} \tag{1}$$

and for $q, q' \in Q,\ i = 1,\dots,l$

$$B_{1,i,q}^{1_q} = p_q a_{qi} \exp(-\lambda_i),\ B_{1,i,q}^{1_{q'}} = 0,\ q \ne q',$$

$$B_{n+1,i,q}^{1_q} = \sum_{q'\in Q} \left[\pi_{q'q} a_{qi} \exp(-\lambda_i) \left(\lambda_i \sum_{1\le K_\Sigma \le n} \sum_{j=1}^{l} \frac{B_{n,j,q'}^K}{r^K \lambda_j - \lambda_i} + p_{q'}\right)\right],\ q = q',\ B_{n+1,i,q}^{1_{q'}} = 0,\ q \ne q',$$

$$B_{n+1,j,q}^K = -\sum_{q'\in Q} \left[\pi_{q'q} I\left(k_q > 0\right) \exp\left(-r^{K-1_q} \lambda_i\right) \sum_{j=1}^{l} \frac{B_{n,j,q'}^{K-1_q} a_{qj} \lambda_j}{r^{K-1_q} \lambda_i - \lambda_j}\right],\ 1 < K_\Sigma \le n+1.$$

Remark 1. In (Tsitsiashvili 2010) there is some inaccuracy in recurrent formulas for coefficients $B_{n+1,i,q}^{1_q}, B_{n+1,j,q}^K$. Here odd multiplier $p_{q'}$ is cancelled from these formulas.

In (Tsitsiashvili 2010) recurrent algorithm of matrix $\|\pi_{q'q}\|_{q,q'\in Q}$.generation is replaced by a random choice of elements of symmetric matrix $\|A_{ij}\|_{i,j=1}^{m}$ for fixed probabilities $\{p_1,\dots,p_m\}$ from formulas

$$\max\left[0,\ p_i - \sum_{k=1}^{j-1} A_{ik} - \sum_{s=j+1}^{m} p_s + \sum_{s=1}^{i-1} \sum_{k=j+1}^{m} A_{sk}\right] < A_{ij} < \min\left[p_j - \sum_{k=1}^{i-1} A_{kj},\ p_i - \sum_{k=1}^{j-1} A_{ik}\right],\ i \le j \le m-1,$$

$$A_{im} = p_i - \sum_{k=1}^{m-1} A_{ik},\ 1 \le i \le m,$$

then $\pi_{ij} = \dfrac{A_{ij}}{p_i},\ 1 \le i, j \le m.$

In (Tsitsiashvili 2010) a problem of an enumeration of all vectors of the set

$$\boldsymbol{K} = \{K = (k_1,\dots,k_m) :\ k_i = 0,1,\dots,\ i = 1,\dots,m, 1 \le K_\Sigma \le n\}$$

is solved via recurrent calculation of the sets of vectors

$$\boldsymbol{K}_i^j = \{K = (k_1,\dots,k_j) :\ k_i = 0,1,\dots,\ i = 1,\dots,j,\ K_\Sigma = i\},\ 0 \le i \le n,\ 1 \le j \le m,$$

$$\boldsymbol{K_0^1} = \{0\},\ \boldsymbol{K_i^1} = \{i\},\ 1 \le i \le n,\ \boldsymbol{K_i^{j+1}} = \bigcup_{t=0}^{i} \{(K,t) : K \in \boldsymbol{K_{i-t}^j}\},\ 0 \le i \le n, 1 \le j \le m-1.$$

Then

$$\boldsymbol{K} = \bigcup_{i=1}^{n} \boldsymbol{K_i^m}$$

and calculation complexity of this algorithm is not larger than

$$(n+1)^{m+1}.$$

2. NUMERICAL EXPERIMENT

Suppose that $m = 2,\ Q = \{1, 2\},\ p_1 = 0.25,\ p_2 = 0.75,\ r_1 = 1.03,\ r_2 = 1.08,$

$$\pi_{11} = 5/9,\ \pi_{12} = 4/9, \pi_{21} = 4/27, \pi_{22} = 23/27$$

and consider Pareto distributions of insurance losses

$$\bar{F}_1(t) = (1+5x)^{-1.2},\ \bar{F}_2(t) = (1+0.83x)^{-2.2},\ t > 0.$$

We approximate Pareto distributions by hyperexponential (Anja Feldman & Ward Whitt 1998)

$$\bar{F}_1(t) \approx \sum_{i=1}^{27} a_{1i} \exp(-\lambda_i t),\ \bar{F}_2(t) \approx \sum_{i=1}^{27} a_{2i} \exp(-\lambda_i t),$$

with parameters

i	a_{1i}	a_{2i}	λ_i
1	0.089437	0	23.304
2	0.533823	0	6.516
3	0.307218	0	1.546
4	0.059768	0	0.306
5	0.008462	0	0.057
6	0.001122	0	0.01
7	0.000147	0	0.002
8	0.0000192	0	0.00035
9	2.5×10^{-6}	0	0.000065
10	3.27×10^{-7}	0	0.000012
11	4.27×10^{-9}	0	2.2×10^{-6}
12	5.56×10^{-10}	0	3.9×10^{-7}
13	7.18×10^{-10}	0	6.8×10^{-8}
14	8.37×10^{-11}	0	8.3×10^{-9}
15	0	0.193963	4.491
16	0	0.651199	1.422
17	0	0.147814	0.371
18	0	0.006832	0.076
19	0	0.000188	0.014
20	0	4.61×10^{-6}	0.003
21	0	1.11×10^{-7}	0.0005
22	0	2.65×10^{-9}	0.000088

i	a_{1i}	a_{2i}	λ_i
23	0	6.35×10^{-11}	0.000016
24	0	1.52×10^{-12}	2.9×10^{-6}
25	0	3.36×10^{-14}	5.4×10^{-7}
26	0	8.51×10^{-16}	9.7×10^{-8}
27	0	1.72×10^{-17}	1.58×10^{-8}

Results of numerical experiments for ruin probability $\psi_{10}(x)$ with step h=0.5 by x using Monte-Carlo method with N=10000000 realizations and by the formula (1) are represented on the table

x	Monte-Carlo method	the formula (1)
0.5	0.421138	0.422476
1	0.347786	0.348598
1.5	0.291625	0.292944
2	0.247464	0.249402
2.5	0.212054	0.214347
3	0.183264	0.185541
3.5	0.159391	0.161535
4	0.139627	0.141338
4.5	0.123078	0.124237
5	0.109101	0.109692

Time of calculation by Monte-Carlo method is approximately 2 hours and 15 minutes and time of calculation by the formula (1) is 15 seconds.

Results of numerical experiments for ruin probability $\psi_{50}(x)$ with step h=0.5 by x using Monte-Carlo method with N=10000000 realizations and by the formula (1) are represented on the table

X	Monte-Carlo method	the formula (1)
0.5	0.442933	0.444793
1	0.38219	0.38395
1.5	0.335	0.336951
2	0.296502	0.298993
2.5	0.264418	0.267366
3	0.237423	0.240461
3.5	0.214501	0.217261
4	0.19457	0.197082
4.5	0.177139	0.179427
5	0.162125	0.163917

Time of calculation by Monte-Carlo method is approximately 10 hours and 30 minutes and time of calculation the formula (1) is 26 minutes.

REFERENCES

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