SAFETY OPTIMIZATION OF A FERRY TECHNICAL SYSTEM IN VARIABLE OPERATION CONDITIONS

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ABSTRACT

The general model of the safety of complex technical systems in variable operation conditions linking a semi-Markov modeling of the system operation process with a multi-state approach to system safety analysis and linear programming are applied in maritime transport to safety and risk optimization of a ferry technical system.

1 INTRODUCTION

Most real technical systems are very complex and it is difficult to analyze and optimize their safety. Large numbers of components and subsystems and their operating complexity cause that the evaluation and optimization of their safety is complicated. The complexity of the systems' operation processes and their influence on changing in time the systems' structures and their components' safety characteristics is often very difficult to fix and to analyze. A convenient tool for solving this problem is a semi-markov (Grabski 2002) modeling of the system operation processes linked with a multi-state approach to the system safety analysis (Kolowrocki, Soszynska 2008, Kolowrocki, Soszynska 2009) and a linear programming for the system safety optimization (Kolowrocki, Soszynska 2010). This approach to system safety investigation is based on the multi-state system reliability analysis considered for instance in (Aven 1985, Kolowrocki 2004) and on semi-markov operation processes modeling discussed for instance in (Soszynska 2006, Soszynska 2007). An application of the proposed approach to safety analysis and optimization of maritime ferry technical system is presented in this paper.

2 THE FERRY TECHNICAL SYSTEM SAFETY AND RISK

The considered maritime ferry is a passenger Ro-Ro ship operating in Baltic Sea between Gdynia and Karlskrona ports on regular everyday line. We assume that the ferry is composed of a number of main subsystems having an essential influence on its safety. These subsystems are illustrated in Figure 1.

On the scheme of the ferry presented in Figure 1, there are distinguished its following subsystems:
S_1 - a navigational subsystem,
S_2 - a propulsion and controlling subsystem,
S_3 - a loading and unloading subsystem,
S_4 - a hull subsystem,
S_5 - an anchoring and mooring subsystem,
S_6 - a protection and rescue subsystem,
S_7 - a social subsystem.

In our further analysis of the ferry safety we omit the protection and rescue subsystem S_6 and the social subsystem S_7 and we consider its strictly technical subsystems S_1, S_2, S_3, S_4 and S_5 only, further called the ferry technical system (Kolowrocki, Soszynska 2009).

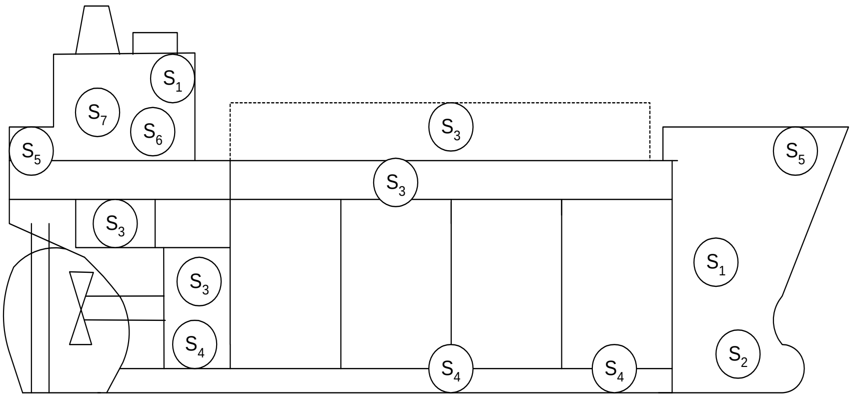

Figure 1. Subsystems having an essential influence on the ferry technical system safety

We assume that the ferry technical system safety structure and the subsystems and components safety depend on its changing in time operation states (Kolowrocki, Soszynska 2010).

Taking into account the experts' opinion on the operation process of the considered ferry, we distinguish the following as its eighteen operation states:

- an operation state z_1 – loading at Gdynia Port,
- an operation state z_2 – unmooring operations at Gdynia Port,
- an operation state z_3 – leaving Gdynia Port and navigation to "GD" buoy,
- an operation state z_4 – navigation at restricted waters from "GD" buoy to the end of Traffic Separation Scheme,
- an operation state z_5 – navigation at open waters from the end of Traffic Separation Scheme to "Angoring" buoy,
- an operation state z_6 – navigation at restricted waters from "Angoring" buoy to "Verko" Berth at Karlskrona,
- an operation state z_7 – mooring operations at Karlskrona Port,
- an operation state z_8 – unloading at Karlskrona Port,
- an operation state z_9 – loading at Karlskrona Port,
- an operation state z_{10} – unmooring operations at Karlskrona Port,
- an operation state z_{11} – ferry turning at Karlskrona Port,
- an operation state z_{12} – leaving Karlskrona Port and navigation at restricted waters to "Angoring" buoy,
- an operation state z_{13} – navigation at open waters from "Angoring" buoy to the entering Traffic Separation Scheme,
- an operation state z_{14} – navigation at restricted waters from the entering Traffic Separation Scheme to "GD" buoy,
- an operation state z_{15} – navigation from "GD" buoy to turning area,
- an operation state z_{16} – ferry turning at Gdynia Port,
- an operation state z_{17} – mooring operations at Gdynia Port,
- an operation state z_{18} – unloading at Gdynia Port.

Additionally, as in (Kolowrocki, Soszynska 2009, 2010), we assume that subsystems S_υ, $\upsilon = 1,2,...,5,$ of the ferry technical system are composed of five-state, components, and their safety states are 0,1,2,3 and 4. Consequently the components conditional multi-state safety function is the vector (Kolowrocki, Soszynska 2009)

$$[s_{ij}^{(\upsilon)}(t,\cdot)]^{(b)} = [1,\ [s_{ij}^{(\upsilon)}(t,1)]^{(b)}, [s_{ij}^{(\upsilon)}(t,2)]^{(b)}, [s_{ij}^{(\upsilon)}(t,3)]^{(b)}, [s_{ij}^{(\upsilon)}(t,4)]^{(b)}],\ b = 1,2,...,18,$$

with the exponential co-ordinates

$$[s_{ij}^{(\upsilon)}(t,1)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(1)]^{(b)} t],\ [s_{ij}^{(\upsilon)}(t,2)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(2)]^{(b)} t],$$

$$[s_{ij}^{(\upsilon)}(t,3)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(3)]^{(b)} t],\ [s_{ij}^{(\upsilon)}(t,4)]^{(b)} = \exp[-[\lambda_{ij}^{(\vartheta)}(4)]^{(b)} t],\ b = 1,2,...,18,$$

Further, assuming that the ferry technical system is in the safety state subset $\{u, u+1,...,4\}$ $u = 0,1,2,3,4,$ if all its subsystems are in this subset o safety states, we conclude that the ferry is five-state series system (Kolowrocki, Soszynska 2009) of subsystems S_1, S_2, S_3, S_4, S_5 and S_6.

The ferry operation process is very regular in the sense that the operation state changes are from the particular state z_b, $b = 1,2,...,17,$ to the neighboring state z_{b+1}, $b = 1,2,...,17,$ and from z_{18} to z_1 only. Therefore, the probabilities of transitions between the operation states are given by (Kolowrocki, Soszynska 2009)

$$[p_{bl}] = \begin{bmatrix} 0\,1\,0 \ldots 0\,0 \\ 0\,0\,1 \ldots 0\,0 \\ \ldots \\ 0\,0\,0 \ldots 0\,1 \\ 1\,0\,0 \ldots 0\,0 \end{bmatrix}.$$

On the basis of statistical data coming from experts the matrix of the density functions of the ferry technical system operation process conditional sojourn times θ_{bl} $b,l = 1,2,...,18,$ defined in (Kolowrocki, Soszynska 2009), can be evaluated.

Next, the mean values $M_{bl} = E[\theta_{bl}],$ $b,l = 1,2,...,18,$ $b \neq l,$ of the system operation process conditional sojourn times θ_{bl} in particular operation states can be determined and they are:

$$M_{12} = 54.67,\ M_{23} = 2.57,\ M_{34} = 37.33,\ M_{45} = 52.27,\ M_{56} = 526.43,\ M_{67} = 37.16,$$

$$M_{78} = 7.02,\ M_{89} = 23.26,\ M_{910} = 53.69,\ M_{1011} = 2.86,\ M_{1112} = 4.38,\ M_{1213} = 24.12,$$

$$M_{1314} = 508.60,\ M_{1415} = 50.14,\ M_{1516} = 34.43,\ M_{1617} = 4.59,\ M_{1718} = 7.92,\ M_{181} = 18.74.$$

Hence, according to (2) (Kolowrocki, Soszynska 2010), the mean values of the unconditional sojourn times in the operation states are:

$$M_1 = 54.67,\ M_2 = 2.57,\ M_3 = 37.33,\ M_4 = 52.27,\ M_5 = 526.43,\ M_6 = 37.16,\ M_7 = 7.02,$$

$$M_8 = 23.26,\ M_9 = 53.69,\ M_{10} = 2.86,\ M_{11} = 4.38,\ M_{12} = 24.12,\ M_{13} = 508.60,\ M_{14} = 50.14,$$

$$M_{15} = 34.43,\ M_{16} = 4.59,\ M_{17} = 7.92,\ M_{18} = 18.74.$$

Since from the system of equations given in (Kolowrocki, Soszynska 2009, 2010) taking here the form

$$\begin{cases} [\pi_b]_{1x18} = [\pi_b]_{1x18}[p_{bl}]_{18x18} \\ \sum_{b=1}^{v} \pi_b = 1, \end{cases}$$

we get

$\pi_b \cong 0.056$ for $b = 1,2,...,18.$

Thus, according to the results contained in (Kolowrocki, Soszynska 2009, 2010), the long term proportion of transients p_b at the operational states z_b, can be approximated by

$$p_1 = 0.038,\ p_2 = 0.002,\ p_3 = 0.026,\ p_4 = 0.036,\ p_5 = 0.363,\ p_6 = 0.026,\ p_7 = 0.005,$$

$$p_8 = 0.016,\ p_9 = 0.037,\ p_{10} = 0.002,\ p_{11} = 0.003,\ p_{12} = 0.016,\ p_{13} = 0.351,\ p_{14} = 0.034,$$

$$p_{15} = 0.024,\ p_{16} = 0.003,\ p_{17} = 0.005,\ p_{18} = 0.013. \tag{1}$$

Under the assumption that the changes of the ferry operation states have an influence on the subsystems S_υ, $\upsilon = 1,2,...,5,$ components safety and on the ferry technical system safety structures as well, on the basis of expert opinions and statistical data given in (Soszynska et al. 2009), the ferry technical system safety structures and their components safety functions and the ferry technical system conditional safety functions at different operation states can be determined (Kolowrocki, Soszynska 2010). Namely, in the case when the system operation time is large enough, the unconditional fife-state safety function of the ferry technical system is given by the vector

$$\boldsymbol{s}(t,\ \cdot) = [1,\ \boldsymbol{s}(t,1),\ \boldsymbol{s}(t,2),\ \boldsymbol{s}(t,3),\ \boldsymbol{s}(t,4)],\ t \ge 0, \tag{2}$$

where, after considering the values of p_b, $b = 1,2,...,18,$ given by (1), its co-ordinates are

$$\boldsymbol{s}(t,u) = 0.038 \cdot [\boldsymbol{s}(t,u)]^{(1)} + 0.002 \cdot [\boldsymbol{s}(t,u)]^{(2)} + 0.026 \cdot [\boldsymbol{s}(t,u)]^{(3)} + 0.036 \cdot [\boldsymbol{s}(t,u)]^{(4)}$$

$$+ 0.363 \cdot [\boldsymbol{s}(t,u)]^{(5)} + 0.026 \cdot [\boldsymbol{s}(t,u)]^{(6)} + 0.005 \cdot [\boldsymbol{s}(t,u)]^{(7)} + 0.016 \cdot [\boldsymbol{s}(t,u)]^{(8)}$$

$$+ 0.037 \cdot [\boldsymbol{s}(t,u)]^{(9)} + 0.002 \cdot [\boldsymbol{s}(t,u)]^{(10)} + 0.003 \cdot [\boldsymbol{s}(t,u)]^{(11)} + 0.016 \cdot [\boldsymbol{s}(t,u)]^{(12)}$$

$$+ 0.351 \cdot [\boldsymbol{s}(t,u)]^{(13)} + 0.034 \cdot [\boldsymbol{s}(t,u)]^{(14)} + 0.024 \cdot [\boldsymbol{s}(t,u)]^{(15)} + 0.003 \cdot [\boldsymbol{s}(t,u)]^{(16)}$$

$$+0.005\cdot[\bar{s}(t,u)]^{(17)}+0.013\cdot[s(t,u)]^{(18)},$$

(3)

for $t \geq 0$, $u = 1,2,3,4$, where $[s(t,u)]^{(b)}$, $b = 1,2,...,18$, are the system conditional safety functions at particular operation states z_b, $b = 1,2,...,18$, determined in (Kolowrocki, Soszynska 2010).

The safety function of the ferry technical system is presented in Figure 2

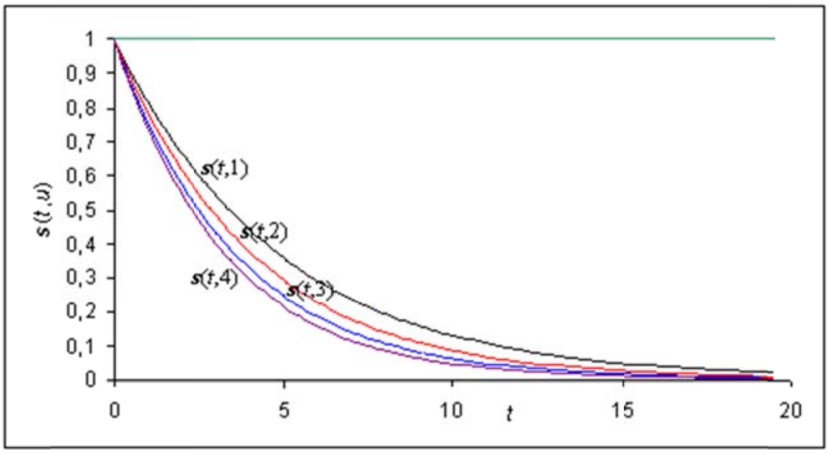

Figure 2. Graph of the safety function $[s(t,\cdot)]$ coordinates

From (3), the mean values of the ferry technical system unconditional lifetimes in the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4} respectively are:

$\mu(1) \cong 4.70$ years,

$$\mu(2) \cong 0.038\cdot 6.45 + 0.002\cdot 2.43 + 0.026\cdot 3.9 + 0.036\cdot 3.80 + 0.363\cdot 3.80 + 0.026\cdot 3.24$$
$$+ 0.005\cdot 2.43 + 0.016\cdot 7.69 + 0.037\cdot 7.69 + 0.002\cdot 2.43 + 0.003\cdot 3.37 + 0.016\cdot 3.80$$
$$+ 0.351\cdot 3.80 + 0.034\cdot 3.80 + 0.024\cdot 3.90 + 0.003\cdot 3.37 + 0.005\cdot 2.43 + 0.013\cdot 6.45$$
$$\cong 4.11 \text{ years}, \qquad (4)$$

$\mu(3) \cong 3.66$ years, $\mu(4) \cong 3.29$ years.

From the above (Kolowrocki, Soszynska 2009, 2010), the mean values of the system lifetimes in the particular safety states are:

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 0.59,\ \bar{\mu}(2) = \mu(2) - \mu(3) = 0.77 \text{ years},$$

$$\bar{\mu}(3) = \mu(3) - \mu(4) = 0.45,\ \bar{\mu}(4) = \mu(4) = 2.29 \text{ years}. \qquad (5)$$

If we assume that the critical safety state is $r = 2$, then the system risk function (Kolowrocki, Soszynska 2009, 2010), is given by

$$\boldsymbol{r}(\mathrm{t}) = 1 - s(t,2) \qquad (6)$$

where $s(t,2)$ is given by (3) for $u = 2$.

The moment when the system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.21 \text{ year.} \tag{7}$$

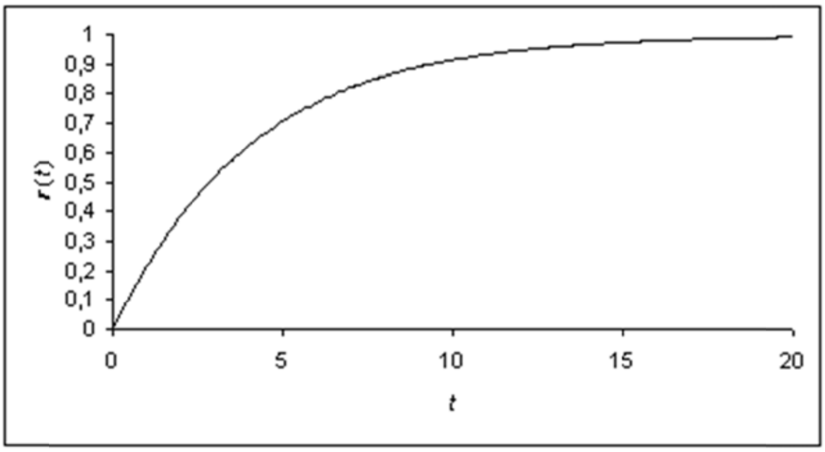

Figure 3. Graph of the ferry technical system risk function $r(t)$

3 OPTIMIZATION OF THE FERRY TECHNICAL SYSTEM OPERATION PROCESS

Considering the equation (3), it is natural to assume that the system operation process has a significant influence on the system safety. This influence is also clearly expressed in the equation (4), for the mean values of the system unconditional lifetimes in the safety state subsets.

The objective function defined in (Kolowrocki, Soszynska 2010), in this case as the system critical state is $r = 2$, takes the form

$$\mu(2) = p_1 \cdot 6.45 + p_2 \cdot 2.43 + p_3 \cdot 3.90 + p_4 \cdot 3.80 + p_5 \cdot 3.80 + p_6 \cdot 3.24$$

$$+ p_7 \cdot 2.43 + p_8 \cdot 7.69 + p_9 \cdot 7.69 + p_{10} \cdot 2.43 + p_{11} \cdot 3.37 + p_{12} \cdot 3.80$$

$$+ p_{13} \cdot 3.80 + p_{14} \cdot 3.80 + p_{15} \cdot 3.90 + p_{16} \cdot 3.37 + p_{17} \cdot 2.43 + p_{18} \cdot 6.45. \tag{8}$$

Since the lower $\breve{p}_b$ and upper $\hat{p}_b$ bounds of the unknown transient probabilities p_b, $b = 1,2,...,18$, coming from experts are respectively:

$$\breve{p}_1 = 0.0006,\ \breve{p}_2 = 0.001,\ \breve{p}_3 = 0.018,\ \breve{p}_4 = 0.027,\ \breve{p}_5 = 0.286,\ \breve{p}_6 = 0.018,$$

$$\breve{p}_7 = 0.002,\ \breve{p}_8 = 0.001,\ \breve{p}_9 = 0.001,\ \breve{p}_{10} = 0.001,\ \breve{p}_{11} = 0.002,\ \breve{p}_{12} = 0.013,$$

$$\breve{p}_{13} = 0.286,\ \breve{p}_{14} = 0.025,\ \breve{p}_{15} = 0.018,\ \breve{p}_{16} = 0.002,\ \breve{p}_{17} = 0.002,\ \breve{p}_{18} = 0.001,$$

$$\hat{p}_1 = 0.056,\ \hat{p}_2 = 0.002,\ \hat{p}_3 = 0.027,\ \hat{p}_4 = 0.056,\ \hat{p}_5 = 0.780,\ \hat{p}_6 = 0.024,$$

$$\hat{p}_7 = 0.018,\ \hat{p}_8 = 0.018,\ \hat{p}_9 = 0.056,\ \hat{p}_{10} = 0.003,\ \hat{p}_{11} = 0.004,\ \hat{p}_{12} = 0.024,$$

$$\hat{p}_{13} = 0.780,\ \hat{p}_{14} = 0.043,\ \hat{p}_{15} = 0.024,\ \hat{p}_{16} = 0.004,\ \hat{p}_{17} = 0.007,\ \hat{p}_{18} = 0.018,$$

then we assume, for the objective function defined by (8), the following bounds constraints

$$0.0006 \le p_1 \le 0.056,\ 0.001 \le p_2 \le 0.002,\ 0.018 \le p_3 \le 0.027,$$

$$0.027 \le p_4 \le 0.056,\ 0.286 \le p_5 \le 0.780,\ 0.018 \le p_6 \le 0.024,$$

$$0.002 \le p_7 \le 0.018,\ 0.001 \le p_8 \le 0.018,\ 0.001 \le p_9 \le 0.056,$$

$$0.001 \le p_{10} \le 0.003,\ 0.002 \le p_{11} \le 0.004,\ 0.013 \le p_{12} \le 0.024,$$

$$0.286 \le p_{13} \le 0.780,\ 0.025 \le p_{14} \le 0.043,\ 0.018 \le p_{15} \le 0.024,$$

$$0.002 \le p_{16} \le 0.004,\ 0.002 \le p_{17} \le 0.007,\ 0.001 \le p_{18} \le 0.018,$$

(9)

$$\sum_{b=1}^{18} p_b = 1,$$

Now, in order to find the optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,...,18$, that maximize the objective function (8), w arrange the system conditional lifetimes mean values $\mu_b(2)$, $b = 1,2,...,18$, in non-increasing order

$$\mu_8(2) \ge \mu_9(2) \ge \mu_1(2) \ge \mu_{18}(2) \ge \mu_3(2) \ge \mu_{15}(2) \ge \mu_4(2) \ge \mu_5(2) \ge \mu_{12}(2)$$

$$\ge \mu_{13}(2) \ge \mu_{14}(2) \ge \mu_{11}(2) \ge \mu_{16}(2) \ge \mu_6(2) \ge \mu_2(2) \ge \mu_7(2) \ge \mu_{10}(2) \ge \mu_{17}(2),$$

and we substitute

$$x_1 = p_8 = 0.016,\ x_2 = p_9 = 0.037,\ x_3 = p_1 = 0.038,\ x_4 = p_{18} = 0.013,$$

$$x_5 = p_3 = 0.026,\ x_6 = p_{15} = 0.024,\ x_7 = p_4 = 0.036,\ x_8 = p_5 = 0.363,$$

$$x_9 = p_{12} = 0.016,\ x_{10} = p_{13} = 0.351,\ x_{11} = p_{14} = 0.034,\ x_{12} = p_{11} = 0.003,$$

$$x_{13} = p_{16} = 0.003,\ x_{14} = p_6 = 0.026,\ x_{15} = p_2 = 0.002,\ x_{16} = p_7 = 0.005,$$

$$x_{17} = p_{10} = 0.002,\ x_{18} = p_{17} = 0.005. \qquad (10)$$

Afterwards, we maximize with respect to x_i, $i = 1,2,...,18$, the linear form (8) that after considering the substitution (10) takes the form

$$\mu(2) = x_1 \cdot 7.69 + x_2 \cdot 7.69 + x_3 \cdot 6.45 + x_4 \cdot 6.45 + x_5 \cdot 3.90 + x_6 \cdot 3.90$$

$$+ x_7 \cdot 3.80 + x_8 \cdot 3.80 + x_9 \cdot 3.80 + x_{10} \cdot 3.80 + x_{11} \cdot 3.80 + x_{12} \cdot 3.37 + x_{13} \cdot 3.37$$

$$+ x_{14} \cdot 3.24 + x_{15} \cdot 2.43 + x_{16} \cdot 2.43 + x_{17} \cdot 2.43 + x_{18} \cdot 2.43, \tag{11}$$

with the following bound constraints

$$0.001 \le x_1 \le 0.018,\ 0.001 \le x_2 \le 0.056,\ 0.0006 \le x_3 \le 0.056,$$

$$0.001 \le x_4 \le 0.018,\ 0.018 \le x_5 \le 0.027,\ 0.018 \le x_6 \le 0.024,$$

$$0.027 \le x_7 \le 0.056,\ 0.286 \le x_8 \le 0.780,\ 0.013 \le x_9 \le 0.024,$$

$$0.286 \le x_{10} \le 0.780,\ 0.025 \le x_{11} \le 0.043,\ 0.002 \le x_{12} \le 0.004,$$

$$0.002 \le x_{13} \le 0.004,\ 0.018 \le x_{14} \le 0.024,\ 0.001 \le x_{15} \le 0.002,$$

$$0.002 \le x_{16} \le 0.018,\ 0.001 \le x_{17} \le 0.003,\ 0.002 \le x_{18} \le 0.007,$$

$$\sum_{i=1}^{18} x_i = 1.$$

Further, according to the procedure given in (Kolowrocki, Soszynska 2010), we calculate

$$\breve{x} = \sum_{i=1}^{18} \breve{x}_i = 0.7046,\ \ \breve{y} = 1 - \breve{x} = 1 - 0.7046 = 0.2954 \tag{12}$$

and we find

$$\breve{x}^0 = 0,\ \hat{x}^0 = 0,\ \hat{x}^0 - \breve{x}^0 = 0,$$

$$\breve{x}^1 = 0.001\ \ \hat{x}^1 = 0.018,\ \hat{x}^1 - \breve{x}^1 = 0.017$$

$$\breve{x}^2 = 0.002,\ \hat{x}^2 = 0.074,\ \hat{x}^2 - \breve{x}^2 = 0.072,$$

$$\breve{x}^3 = 0.0026,\ \hat{x}^3 = 0.13,\ \hat{x}^3 - \breve{x}^3 = 0.1274,$$

$$\breve{x}^4 = 0.0036,\ \hat{x}^4 = 0.148,\ \hat{x}^4 - \breve{x}^4 = 0.1444,$$

$$\breve{x}^5 = 0.0216,\ \hat{x}^5 = 0.175,\ \hat{x}^5 - \breve{x}^5 = 0.1534,$$

$$\breve{x}^6 = 0.0396,\ \hat{x}^6 = 0.199,\ \hat{x}^6 - \breve{x}^6 = 0.1594,$$

$$\breve{x}^7 = 0.0666,\ \hat{x}^7 = 0.255,\ \hat{x}^7 - \breve{x}^7 = 0.1884,$$

$$\breve{x}^{8} = 0.3526, \ \ \hat{x}^{8} = 1.035, \ \ \hat{x}^{8} - \breve{x}^{8} = 0.6824. \tag{13}$$

From the above, as according to (13), after considering the inequality

$$\hat{x}^{I} - \breve{x}^{I} < 0.295 \ , \tag{14}$$

it follows that the largest value $I \in \{0,1,...,18\}$ such that this inequality holds is $I = 7$.

Therefore, we fix the optimal solution that maximize linear function (11) according to the rule given in (Kolowrocki, Soszynska 2010). Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.018\,, \ \ \dot{x}_2 = \hat{x}_2 = 0.056\,, \ \dot{x}_3 = \hat{x}_3 = 0.056\,, \ \ \dot{x}_4 = \hat{x}_4 = 0.018\,,$$

$$\dot{x}_5 = \hat{x}_5 = 0.027\,, \ \ \dot{x}_6 = \hat{x}_6 = 0.024\,, \ \ \dot{x}_7 = \hat{x}_7 = 0.056\,,$$

$$\dot{x}_8 = y - \hat{x}^{7} + \breve{x}^{7} + \breve{x}_8 = 0.2954 - 0.255 + 0.0666 + 0.286 = 0.393,$$

$$\dot{x}_9 = \breve{x}_9 = 0.013, \ \ \dot{x}_{10} = \breve{x}_{10} = 0.286, \ \ \dot{x}_{11} = \breve{x}_{11} = 0.025, \ \ \dot{x}_{12} = \breve{x}_{12} = 0.002,$$

$$\dot{x}_{13} = \breve{x}_{13} = 0.002, \ \dot{x}_{14} = \breve{x}_{14} = 0.018, \ \ \dot{x}_{15} = \breve{x}_{15} = 0.001, \ \ \dot{x}_{16} = \breve{x}_{16} = 0.002,$$

$$\dot{x}_{17} = \breve{x}_{17} = 0.001, \ \ \dot{x}_{18} = \breve{x}_{18} = 0.002.$$

Finally, after making the substitution inverse to (10), we get the optimal transient probabilities

$$\dot{p}_8 = \dot{x}_1 = 0.018\,, \ \dot{p}_9 = \dot{x}_2 = 0.056\,, \ \dot{p}_1 = \dot{x}_3 = 0.056\,, \ \dot{p}_{18} = \dot{x}_4 = 0.018,$$

$$\dot{p}_3 = \dot{x}_5 = 0.027\,, \ \dot{p}_{15} = \dot{x}_6 = 0.024, \ \ \dot{p}_4 = \dot{x}_7 = 0.056, \ \ \dot{p}_5 = \dot{x}_8 = 0.393,$$

$$\dot{p}_{12} = \dot{x}_9 = 0.013, \ \ \dot{p}_{13} = \dot{x}_{10} = 0.286, \ \ \dot{p}_{14} = \dot{x}_{11} = 0.025, \ \ \dot{p}_{11} = \dot{x}_{12} = 0.002,$$

$$\dot{p}_{16} = \dot{x}_{13} = 0.002, \ \ \dot{p}_6 = \dot{x}_{14} = 0.018, \ \ \dot{p}_{12} = \dot{x}_{15} = 0.001, \ \ \dot{p}_7 = \dot{x}_{16} = 0.002,$$

$$\dot{p}_{10} = \dot{x}_{17} = 0.001, \ \ \dot{p}_{17} = \dot{x}_{18} = 0.002, \tag{15}$$

that maximize the system mean lifetime in the safety state subset $\{2,3,4\}$ expressed by the linear form (8) giving its optimal value

$$\begin{aligned}\dot{\mu}(2) \cong\ & 0.056 \cdot 6.45 + 0.001 \cdot 2.43 + 0.027 \cdot 3.90 + 0.056 \cdot 3.80 + 0.393 \cdot 3.80 \\ & + 0.018 \cdot 3.24 + 0.002 \cdot 2.43 + 0.018 \cdot 7.69 + 0.056 \cdot 7.69 + 0.001 \cdot 2.43 \\ & + 0.002 \cdot 3.37 + 0.013 \cdot 3.80 + 0.286 \cdot 3.80 + 0.025 \cdot 3.80 + 0.024 \cdot 3.90 \\ & + 0.002 \cdot 3.37 + 0.002 \cdot 2.43 + 0.018 \cdot 6.45 = 4.27. \end{aligned} \tag{16}$$

4 THE FERRY TECHNICAL SYSTEM OPTIMAL SAFETY CHARACTERISTICS

Further, using the optimal transient probabilities (15), we obtain the optimal solution for the mean value of the system unconditional lifetime in the safety state subset $\{1,2,3,\}$, $\{3,4\}$ and $\{4\}$

$$\dot{\mu}(1) \cong 4.92, \ \dot{\mu}(3 \cong) 3.79, \ \dot{\mu}(4) \cong 3.42, \tag{17}$$

and the optimal solutions for the mean values of the system unconditional lifetimes in the particular safety states 1,2,3 and 4 are as follows

$$\dot{\bar{\mu}}(1) = 0.65, \ \dot{\bar{\mu}}(2) = 0.48, \ \dot{\bar{\mu}}(3) = 0.37, \quad \dot{\bar{\mu}}(4) = 3.42. \tag{18}$$

Moreover, according to (2), the corresponding optimal unconditional multistate safety function of the system is of the form of the vector

$$\dot{\boldsymbol{s}}(t,\cdot) = [1, \ \dot{\boldsymbol{s}}(t,1), \dot{\boldsymbol{s}}(t,2), \dot{\boldsymbol{s}}(t,3), \dot{\boldsymbol{s}}(t,4)], \ t \geq 0, \tag{19}$$

with the coordinates given by

$$\dot{\boldsymbol{s}}(t,1) = 0.056 \cdot [\boldsymbol{s}(t,u)]^{(1)} + 0.001 \cdot [\boldsymbol{s}(t,u)]^{(2)} + 0.027 \cdot [\boldsymbol{s}(t,u)]^{(3)} + 0.056 \cdot [\boldsymbol{s}(t,u)]^{(4)}$$

$$+ 0.393 \cdot [\boldsymbol{s}(t,u)]^{(5)} + 0.018 \cdot [\boldsymbol{s}(t,u)]^{(6)} + 0.002 \cdot [\boldsymbol{s}(t,u)]^{(7)} + 0.018 \cdot [\boldsymbol{s}(t,u)]^{(8)}$$

$$+ 0.056 \cdot [\boldsymbol{s}(t,u)]^{(9)} + 0.001 \cdot [\boldsymbol{s}(t,u)]^{(10)} + 0.002 \cdot [\boldsymbol{s}(t,u)]^{(11)} + 0.013 \cdot [\boldsymbol{s}(t,u)]^{(12)}$$

$$+ 0.286 \cdot [\boldsymbol{s}(t,u)]^{(13)} + 0.025 \cdot [\boldsymbol{s}(t,u)]^{(14)} + 0.024 \cdot [\boldsymbol{s}(t,u)]^{(15)} + 0.002 \cdot [\boldsymbol{s}(t,u)]^{(16)}$$

$$+ 0.002 \cdot [\bar{\boldsymbol{s}}(t,u)]^{(17)} + 0.018 \cdot [\boldsymbol{s}(t,u)]^{(18)}, \tag{20}$$

for $t \geq 0$, $u = 1,2,3,4$, where $[\boldsymbol{s}(t,u)]^{(b)}$, $b = 1,2,...,18$, are the system conditional safety functions at particular operation states z_b, $b = 1,2,...,18$, given in (Kolowrocki, Soszynska 2010).

The safety function of the ferry technical system is presented in Figure 4.

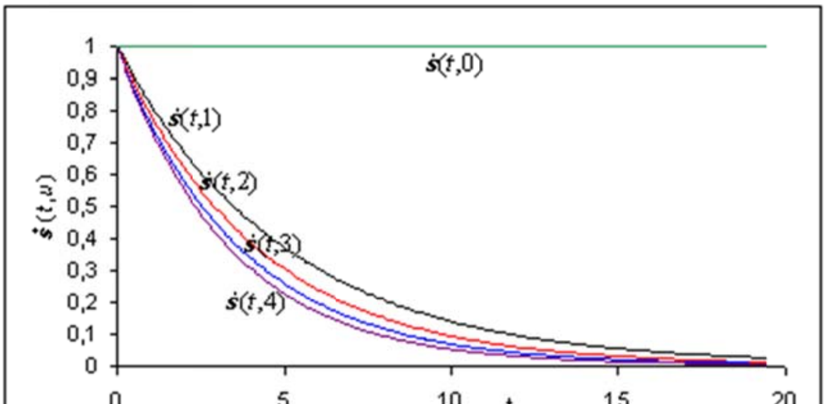

Figure 4. Graph of the optimal safety function $[\dot{s}(t,\cdot)]$ coordinates

If the critical safety state is $r = 2$, then the system risk function (Kolowrocki, Soszynska 2010) is given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{s}}(t,2) \text{ for } t \geq 0, \tag{21}$$

where $\dot{\boldsymbol{s}}(t,2)$ is given by (20) for $u = 2$.

Hence, the moment when the optimal system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 0.22 \text{ year.} \tag{22}$$

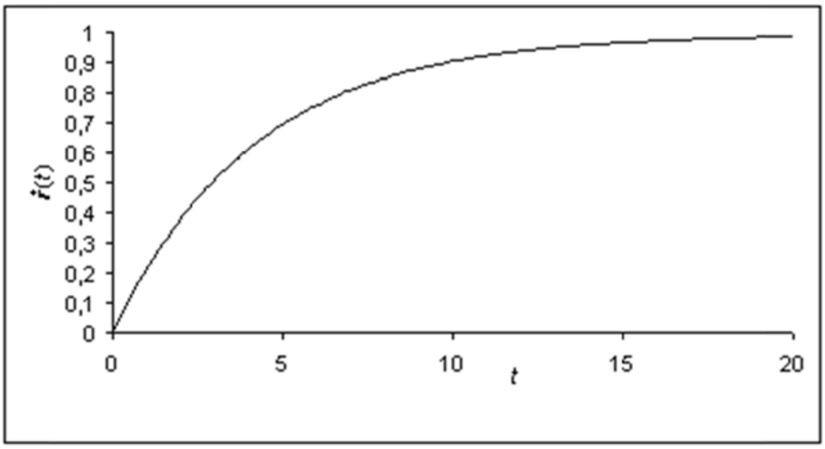

Figure 5. The graph of the ferry technical system optimal risk function $\dot{\boldsymbol{r}}(t)$

The comparison of the ferry technical system safety characteristics after its operation process optimization given by (16)-(22) with the corresponding characteristics before this optimization determined by (2)-(7) justifies the sensibility of this action.

5 CONCLUSION

The joint model of the safety of complex technical systems in variable operation conditions linking a semi-Markov modeling of the system operation processes with a multi-state approach to system safety analysis was applied to the maritime ferry technical system safety characteristics evaluation. Next, the final results obtained from this joint model and a linear programming were used to perform this complex technical system safety optimization. These tools practical application to safety and risk evaluation and optimization of a technical system of a ferry operating in variable operation conditions at the Baltic Sea waters and the results achieved are interesting for safety practitioners from maritime transport industry and from other industrial sectors as well.

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