FUZZY FAILURE RISK ANALYSIS IN DRINKING WATER TECHNICAL SYSTEM

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ABSTRACT

Drinking water technical system is an essential element of urban infrastructure. The operation of this system is inseparably connected with a risk of failure. The main problem in the risk of failure analysis of water pipe network is the uncertainty of the information collected on the description of failure. In order to consider the uncertainty of information, the theory of fuzzy sets was used. The fuzzification of frequency, severity and the consequences of the incident scenario is basic input for fuzzy risk analysis. The presented model is part of a complex model of risk management of failures in drinking water technical system manly in water pipe network and can be used in practice in system operator's decision-making process. An adaptation of the fuzzy set theory to analyse risk of failure of water mains is not a standard approach for water works. An effect of the analysis of different sources of risk can be used for the design of a more reliable safety system assurance.

1 INTRODUCTION

A drinking water technical system (DWS) belongs to the so called critical infrastructure of cities. That is why it should be a priority task for waterworks and even for the local authorities to ensure the suitable level of its safety. Its aim is to supply consumers with a required amount of water, with a specific pressure and a specific quality, according to binding standards. Disturbances in its operation may be the result of random events, the forces of nature, material defects, mechanical damage or deliberate actions of third parties, including terrorist activities. At the same time this system plays a priority role in the functioning of urban areas, even a short lack of suitable drinking water always causes anxiety or panic among people. In many countries in the world, including the EU countries, the regulations obligate waterworks to prepare accurate water supply management procedures, including an analysis of risk of failure. Failure in water network is one of the most common failure of DWS(Kleiner 2004, Sadig et al 2007, Tchorzewska-Cieślak 2007).

Modelling the risk of failure in water network consists of three main tasks:

- assessment (estimation) of the frequency/ probability of emergency scenarios (undesirable events),
- assessment (estimation) of various consequences of emergency scenarios (undesirable events),
- estimation of water mains protection level and the various types of protection minimizing the possible consequences of emergency scenarios (undesirable events).

The case that occurs most frequently in the risk analysis is a statistical uncertainty caused by the random nature of the studied phenomenon, the influence of external factors, as well as the time factor that determines a change of analysed undesirable event (failure) (Tchorzewksa-Cieślak 2009) .In many cases, data on failures of water pipe network are obtained from experts (water supply system operators, engineers or researchers).

These data are often imprecise and incomplete. The following data, among others, are necessary to perform risk analysis in the DWS (Ezell et al. 2000, Hubbard 2009, Tchorzewska-Cieśłak 2010) :

- data identifying the analysed object (e.g. water treatment station, distribution pipeline), the name and type of the object and its basic technical data,
- data about failures (undesirable events), repairs and other breaks in the DWS's operation (information about the date, time and duration of failure, and a description of the failure),
- data relating to the reasons behind the occurrence of undesirable events,
- data relating to the consequences of these events.

The main aim of this study is to present a risk analysis model using fuzzy set theory and the application of this theory in the risk management process in water network.

2. The risk of failure of water pipe network

Risk assessment includes the so called risk analysis, which is the process aimed at the determination of the consequences of failures (undesirable events) in the DWS, their extend, sources of their occurrence and the assessment of the risk levels. Haimes (Haimes 1998, Heimes 2009) suggests that risk assessment concerns its reasons, as well as its likelihood and consequences.

Drinking water infrastructure system uncertainty or risk is defined as the likelihood or probability that the drinking water service fails to provide water on-demand to its customers (Kleiner et.al 2006, Sadig et. al. 2009).

For purposes of this paper, operational reliability of the DWS is defined as the ability to supply a constant flow of water for various groups of consumers, with a specific quality and a specific pressure, according to consumer demands, in specific operational conditions, at any or at a specific time.

Failure is defined as the event in which the system fails to function with respect to its desired objectives.

Safety of the DWS means the ability of the system to safely execute its functions in a given environment. The measure of DWS safety is risk.

Risk (r) is a function of the parameters: the probability or frequency (f_i) that representative emergency scenario occurs (S), the magnitude of losses (C_j) caused by RES and the degree of sensitivity (E_k) to RES, according to equation (1) (Rak 2009,Rak et al 2006).

$$r = \sum_{S=1}^{N} (P_i \quad C_j \quad E_k)$$
(1)

where:

S- a series of the successive undesirable events (failures),

 P_i - a point value depending on the frequency of RES or a single failure,

i- a number of the scale for the frequency,

 C_i --a point value of losses caused by RES or a single failure,

j- a number of the scale for the losses,

 E_k -a point value for the parameter of exposure (sensitivity of water mains) associated with RES or a single failure,

k- a number of the scale for the sensitivity,

N-number of *RES*.

To analyse risk defined in this way the matrix methods can be used (Markowski et.al 2008, Rak et al 2006). According to equation (1) the qualitative risk matrix was developed, assuming a descriptive point scale for the particular risk parameters (Tchorzewska-Cieślak 2010). Depending on the frequency of a given failure the point weights for the parameter f are presented in Table 1.

P_i	Probability of failure
D	very low probability
P ₁	once in 10 years and less often
D	low probability,
P_2	once in (10÷5) years
P ₃	medium probability,
	once in (5÷1)years
P ₄	probability,
	once in (1÷0.5) year
P ₅	once a month and more often,

Table 1. Criteria for a descriptive point scale for the parameter P_{i} (*i*=1,2,3,4,5)

The criteria and the point weights for the assumed descriptive point scale for the parameter of losses C_j and sensitivity E_k are presented in Tables 2 and 3.

Table 2. Criteria for	a descriptive p	oint scale for th	e parameter C	(j=1,2,3)
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C_j	Description					
$C_1=1$	small losses :					
	• perceptible organoleptic changes in water,					
	• isolated consumer complaints,					
	• financial losses up to 5 10 ³ EUR					
C ₂ =2	medium losses:					
	• considerable organoleptic problems (odour, changed colour and turbidity),					
	• consumer health problems, numerous complaints,					
	• information in local public media, financial losses up to 10 ⁵ EUR					
C ₃ =3	large losses:					
	• endangered people require hospitalisation,					
	• professional rescue teams involved, serious toxic effects in test organisms,					
	• information in nationwide media, financial losses over 10 ⁵ EUR					

Table 3.	Criteria	for a des	criptive	point	scale fo	or the	parameter	Ek. (k={	1.2.3})
14010 5.	Criteria	101 a aco		pome	Seare re	1 1110	parameter	$-\kappa$	(··,-,-,	,

E _k	Description							
E ₁ =1	small sensitivity to failure (high resistance):							
	• the network in the ring system,							
	• the ability to cut off the damaged section of the network by means of gates (for repair),							
	• the ability to avoid interruptions in water supply to customers,							
	• full monitoring of water mains, continuous measurements of pressure and flow rate at strategic							
	points of the network covering the entire area of water supply, utilising SCADA and GIS software,							
	the possibility to remote control network hydraulic parameters							
$E_2=2$	medium sensitivity to failure:							
	• the network in the radial or mixed system,							
	• the possibility to cut off the damaged section of the network by means of gates, but the network							
	capacity limits water supply to customers,							
	water mains standard monitoring, measurements of pressure and flow rate							
E ₃ =3	large sensitivity to failure (low resistance):							
	• the network in the radial system,							
	• the inability to cut off the damaged section of the network by means of gates (for repair) without							
	interrupting water supply to customers,							
	limited water mains monitoring							

The use of fuzzy set theory in the analysis of risk of water mains failure

The notion of fuzzy sets was introduced in 1965 by L.A Zadeh of the University of Berkeley. Unlike in the classical set theory, the limit of the fuzzy set is not precisely determined, but there is a gradual transition from non-membership of elements in a set, through their partial membership, to membership (Dubois et.al 1980, Kluska 2009, lee 1999). This gradual transition is described by the so called membership function μ_{A} , where A is a set of fuzzy numbers. Risk analysis is based largely on expert opinions and uses such linguistic terms as small losses, high risk and can be described by means of fuzzy sets (Braglia et.al 2003a, Karwowski 1986,Kleiner 2004,markowski 2008, Sadig 2009, Shang-Lien 2002, Tchórzewska-Cieślak 2010). For risk analysis of water mains failure the membership function class type *t* (a triangular function) according to equation (2), the membership function class type γ according to equation (3) and the membership function class type *L* according to equation (4), were proposed.

$$\mu_A(x,a,b,c) = \begin{cases} 0 & \text{for } x \le a \\ \frac{x-a}{b-a} & \text{for } a \le x \le b \\ \frac{c-x}{c-b} & \text{for } b \le x \le c \\ 0 & \text{for } x \ge c \end{cases}$$
(2)

$$\mu_A(x,a,b) = \begin{cases} 0 & \text{for } x \le a \\ \frac{x-a}{b-a} & \text{for } a \le x \le b \\ 1 & \text{for } x > b \end{cases}$$
(3)

$$\mu_A(x,a,b) = \begin{cases} 1 & \text{for } x \le a \\ \frac{b-x}{b-a} & \text{for } a < x \le b \\ 0 & \text{for } x > b \end{cases}$$
(4)

where:

x- variable, parameter value,

 μ_A - the membership function of variable x in the fuzzy set A ,

a, b, c -the membership function parameters (minimal, median (central) and maximum value of fuzzy number),

For the probability parameter the set of possible linguistic characterization is defined as:

 $\overline{P} = \{P_i\}, i = \{1, 2, 3, 4, 5\}.$

Table 4 shows the linguistic characterization, type and parameters of membership function.

Table 4. The linguistic characterization,	type and parameters of membership function, for	or P
parameter,	$\overline{P} = \{P_i\}, i = \{1, 2, 3, 4, 5\}$	

Fuzzy	lin and the share standard in a		membership function parameters			
set	linguistic characterization	type of membership function	а	b	с	
P ₁	very low probability	type L, acc.to eq.(4)	0.125	0.25	-	
P ₂	low probability	triangular t, acc.to eq.(2)	0.125	0.25	0.375	
P ₃	medium probability	triangular t, acc.to eq.(2)	0.25	0.375	0.5	
P ₄	probability	triangular t, acc.to eq.(2)	0.375	0.5	0.625	
P ₅	high probability	type γ , <i>acc.to eq.(3)</i>	0.5	0.625	-	

Figure 1 shows forms of membership function for the *P* parameter.



Figure1. Form of membership function for the parameter P.

For the losses parameter the set of possible linguistic characterization is defined as:

$$\overline{C} = \{C_j\}, j = \{1, 2, 3\}.$$

Table 5 shows the linguistic characterization, type and parameters of membership function for C parameter.

Table	e 5. The In	nguistic	charact	terizat	10n, 1	type	and	parame	eters o	of mem	bership	function	on,
			for	C para	amet	er, \overline{C}		$C_{1}, C_{2}, C_{2}, C_{2}$	3,}				
												-	

Fuzzy	linguistic characterization	type of membership function	membership function parameters			
set		type of memoership function	а	b	с	
C ₁	small	triangular t, acc. to eq.(2)	0.0	0.0	1.5	
C ₂	medium	triangular t, acc. to eq.(2)	0.5	1.5	2.5	
C ₃	large	triangular t, acc. to eq.(2)	1.5	3.0	3.0	

For the sensitivity parameter the set of possible linguistic characterization is defined as: $\overline{E} = \{E_j\}, j = \{1,2,3\}.$

Table 6 shows the possible linguistic characterization for the sensitivity parameter E, type and parameters of membership function.

Table 6. The linguistic characterization, type and parameters of membership function,

Fuzzy	linguistic characterization	nistic characterization turns of membership function		membership function parameters			
set		type of memoership function	а	b	с		
E ₁	small	triangular t, acc. to eq.(2))	0.0	0.0	1.5		
E ₂	medium	triangular t, acc. to eq.(2)	0.5	1.5	2.5		
E ₃	large	triangular t, acc. to eq.(2)	1.5	3.0	3.0		

for	Е	parameter,	$E = \{$	$\{E_1\}$	$, E_{2}, $,E ₃ ,	}

Figure 2 shows forms of membership function for the parameters *C* and *E*.



Figure 2. Form of membership function for the parameters E and C.

For risk the set of possible linguistic characterization is defined as: $\overline{R} = \{R_l\}, l = \{1, 2, 3\}.$

Table 7 shows the linguistic characterization of risk, type and parameters of membership function.

Table 7. The linguistic characterization, type and parameters of membership function,

for risk, $\overline{R} = \{R_1, R_2, R_3\}$

Fuzzy	linguistic abaracterization	linguistic characterization type of membership		membership function parameters			
set	iniguistic characterization	function	а	b	с		
D	tolerable risk (TR)	triangular	0.0	0.0	18		
κ ₁		<i>t</i> , <i>acc eq</i> .(2)	0.0	0.0	10		
D	controlled risk (CR)	triangular	5	22.5	40.5		
K ₂		<i>t</i> , <i>acc eq</i> .(2)	5	22.5	40.5		
р	unacceptable risk(Ur)	triangular	27	45	45		
К3		<i>t, acc eq.</i> (2)	27	43	43		

4. THE DECISION MODEL

Decision-making tools help in the selection of prudent, technically feasible, and scientifically justifiable actions to protect the environment and human health in a cost-effective way. Processing of information obtained from experts is carried out by the following steps: fuzzification, inference with the use of the rules, and, in case of Mamdani-type inference (Mamdani 1977), defuzification, to get the result in a discrete form. For each linguistic variable a set of membership functions that correspond to the values of this variable is defined.

The Mamdani – Zadeh type decision model was proposed:

- The input base of the proposed model consists of three values of risk parameters:
- the probability of failure $x_1(P_i)$: five possible fuzzy sets: $P = \{P_1, P_2, P_3, P_4, P_5\},\$
- losses associated with the occurrence of failure $x_2(C_j)$, three possible fuzzy sets: C={C₁, C₂, C₃}
- and a degree of exposure (resistance) to failure $x_3(E_k)$, three possible fuzzy sets $E = \{E_1, E_2, E_3\}$
- The output of the model, which allows making an operational decision, is the index risk value for water mains failure $y(R_{l_i})$, three possible fuzzy sets: $R = \{R_1, R_2, R_3\}$.

• The fuzzification, which converts a vector of numbers (the crisp input values of risk parameters) into a vector of degrees of membership (a singleton method was used).

• The inference – the determination of a fuzzy conclusion model in form of the resulting membership function. In this block all rules whose premises are satisfied, are activated. At this moment the following steps are used:

- determination of the so called rule base, which provides a knowledge base for qualitative knowledge and consists in determining the relationship between the particular parameters of the model. A rule base determines the relationships between the inputs and outputs of a system using linguistic *antecedent* and *consequent* propositions in a set of IF-THEN rules, where if premise, then- conclusion.
- the rule base of a complex system usually requires a large number of rules to describe the behaviour of a system for all possible values of the input variables. The base of rules contains the logical rules which determine cause and effect relationship between the particular risk parameters in water mains. Based on the risk matrix shown in Table 4, the base of rules was determined. It is a set of rules: $R_M = \{R_{MI}, R_{M2}, \dots, R_{M45}\}$, in a general form:

If probability is P_i and possible consequences are C_i and sensitivity is E_k then risk is R_i

- Aggregation of rules - in the process of aggregation a degree of fulfilment of each rule is calculated based on the degree of fulfilment of its premises. For this purpose, the fuzzy logic operations: (*and, or*), are used. Based on the presented base of rules, the inference min-max, using the operator S-norms and T-norms, was proposed (Dubois et al. 1980, Mamdani 1977). The aggregating output membership function of a resultant output fuzzy risk category is expressed as:

$$\mu_{R}(R_{1}) = \max\{\min \mu_{F}^{m}(f_{1}), \mu_{C}^{m}(C_{j}), \mu_{E}^{m}(E_{k}), \mu_{R}^{m}(r_{1})\}$$

where m is the number of rules, i the number of fuzzy frequency sets, j the number of loss parameter sets, k the number of sensitivity and l is the number of fuzzy risk sets.

• The defuzzification, whose aim is to obtain a specific value of risk.

This process is the final stage of the model and provides the basis for the water supply system operator's decision-making process. For example, if the risk value corresponds to the category of unacceptable risk an operator undertakes some measures to reduce risk of failure (water mains modernization). The transformation of fuzzy set into not fuzzy value (determined) can be performed by various methods. For the proposed model the centre of gravity method was used:

$$R = \frac{\sum_{l=1}^{5} R_{l} - \mu_{l}(r_{l})}{\sum_{l=1}^{3} \mu_{l}(r_{l})}$$
(5)

Using the operating data of water supply system in a city with population of 200 thousand, the risk analysis, using the fuzzy software, was performed. Data on water main failures for five years of water supply network operation were collected and analysed in terms of frequency of failures and their consequences.

2

Risk assessment was performed for three diameter ranges:

- up to ϕ 150mm,
- $\phi(150-400)$ mm,

The result of the analysis for defined membership functions according to tables 1,2,3 and 45 rules of interference is presented in figure 3(using Matlab fuzzy toolbox). Defined rules are in the form:

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1. If (P is P1) and (C is C1) and (E is E1) then (R is R1) (1)
2. If (P is P1) and (C is C1) and (E is E2) then (R is R1) (1)
3. If (P is P1) and (C is C1) and (E is E3) then (R is R2) (1)
4. If (P is P1) and (C is C2) and (E is E1) then (R is R1) (1)
5. If (P is P1) and (C is C2) and (E is E2) then (R is R2) (1)
6. If (P is P1) and (C is C3) and (E is E3) then (R is R2) (1)
7. If (P is P1) and (C is C3) and (E is E1) then (R is R1) (1)
8. If (P is P1) and (C is C3) and (E is E2) then (R is R2) (1)
9. If (P is P1) and (C is C3) and (E is E3) then (R is R2) (1)
10. If (P is P2) and (C is C1) and (E is E1) then (R is R1) (1)
11. If (P is P2) and (C is C1) and (E is E2) then (R is R2) (1)
12. If (P is P2) and (C is C1) and (E is E3) then (R is R2) (1)
13. If (P is P2) and (C is C2) and (E is E1) then (R is R1) (1)
14. If (P is P2) and (C is C2) and (E is E2) then (R is R2) (1)
15. If (P is P2) and (C is C2) and (E is E3) then (R is R2) (1)
16. If (P is P2) and (C is C3) and (E is E1) then (R is R2) (1)
17. If (P is P2) and (C is C3) and (E is E2) then (R is R3) (1)
18. If (P is P2) and (C is C3) and (E is E3) then (R is R2) (1)
19. If (P is P3) and (C is C1) and (E is E1) then (R is R1) (1)
20. If (P is P3) and (C is C1) and (E is E2) then (R is R2) (1)
21. If (P is P3) and (C is C1) and (E is E3) then (R is R2) (1)
22. If (P is P3) and (C is C2) and (E is E1) then (R is R2) (1)
23. If (P is P3) and (C is C2) and (E is E2) then (R is R2) (1)
24. If (P is P3) and (C is C2) and (E is E3) then (R is R2) (1)
25. If (P is P3) and (C is C3) and (E is E1) then (R is R2) (1)
26. If (P is P3) and (C is C3) and (E is E2) then (R is R2) (1)
27. If (P is P3) and (C is C3) and (E is E3) then (R is R3) (1)
28. If (P is P4) and (C is C1) and (E is E1) then (R is R2) (1)
29. If (P is P4) and (C is C1) and (E is E2) then (R is R2) (1)
30. If (P is P4) and (C is C1) and (E is E3) then (R is R2) (1)
31. If (P is P4) and (C is C2) and (E is E1) then (R is R2) (1)
32. If (P is P4) and (C is C2) and (E is E2) then (R is R2) (1)
33. If (P is P4) and (C is C2) and (E is E3) then (R is R3) (1)
34. If (P is P4) and (C is C3) and (E is E1) then (R is R2) (1)
35. If (P is P4) and (C is C3) and (E is E2) then (R is R2) (1)
36. If (P is P4) and (C is C3) and (E is E3) then (R is R3) (1)
37. If (P is P5) and (C is C1) and (E is E1) then (R is R2) (1)
38. If (P is P5) and (C is C1) and (E is E2) then (R is R2) (1)
39. If (P is P5) and (C is C1) and (E is E3) then (R is R2) (1)
40. If (P is P5) and (C is C2) and (E is E1) then (R is R2) (1)
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41. If (P is P5) and (C is C2) and (E is E2) then (R is R2) (1) 42. If (P is P5) and (C is C2) and (E is E3) then (R is R3) (1) 43. If (P is P5) and (C is C3) and (E is E1) then (R is R2) (1) 44. If (P is P5) and (C is C3) and (E is E2) then (R is R3) (1) 45. If (P is P5) and (C is C3) and (E is E3) then (R is R3) (1)

In Table 8 the results of the analysis are presented.

φ	Р	С	Е	Risk	Risk
[mm]				value	category
up to 150	0.223	1.97	2.0	24.4	CR
150-400	0.173	2.08	2.15	23.8	CR
≥ 400	0.114	2.66	1.07	20.4	TR

Table 8. Risk assessment for the analysed water mains.



Figure 3. The fuzzy risk graph

6. CONCLUSIONS

- Technical systems for drinking water supply require specific procedures for supervision and monitoring. Safety analyses of such systems are based on risk analyses and assessments primarily for the health safety of urban residents.
- Now, however, new trends in the technical management of water supply systems are heavily focused on security issues related to the design, operation and management of these systems. Of particular interest in the future will be the evaluation of risk and reliability issues of the various components, subsystems and the systems as a whole, from the viewpoint of their susceptibility to terrorism.
- Effective risk management requires operators to monitor actively the entire water supply system, to develop emergency plans, to response fast in emergency, and to be able to analyse and assess risk.
- One of the methods to deal with uncertainties in risk assessment is a fuzzy logic where fuzzy sets are a fundamental issue.

- In the study the application of fuzzy logic theory to analyse risk of failure of DWS was proposed. In case of having inaccurate or various (eg from different experts) data on particular risk parameters, there is the possibility to describe them by a linguistic variables.
- In contrast to the traditional risk analysis, all variables of the risk parameters (according to equation (1)) are expressed in fuzzy sets defined by appropriate membership functions
- The probability or frequency of failures and their possible consequences can be defined as fuzzy values, particularly when they are estimated and not precisely determined, which often occurs at the analysis of failures in water supply network.
- The decision model presented in the study, based on assumptions of Mamdani's fuzzy modelling, may be used in practice in water mains as an element of a complex management of risk of failures of water mains.
- A certain limitation of the proposed method is the need to develop a database of the rules, based on the knowledge of experts, whose opinions on the assumed criteria may differ from each other.
- In order to develop the complete and most reliable database of the rules (the knowledge base), as much information as possible about failures of water mains, their possible consequences and causes, should be collected.
- Proposed method provides an alternative to other methods for assessing and managing risk of water network failure (subjective probability theory, mathematical theory of records) and its use is justified if you have a subjective assessment of risk parameters.

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