ANALYTICAL ESTIMATION OF RELIABILITY OF NETWORKS CONSISTING OF IDENTICAL ELEMENTS.

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ABSTRACT

In this work the analytical expressions for reliability factor of steady-state network consisting of identical elements are accomplished. For obtaining of the analytical expressions of considered factors, the Markov process characterized by a certain combination of failed elements and by a network state is considered. It is shown that for definition of values of the obtained expressions it is sufficient to define the number of combinations of set capacity elements at which failure the network passes in non-operable state. For small dimension networks this factor may be defined precisely by analysis of all possible combinations of failed elements. For large-scale networks it is necessary to use Monte Carlo method or combined method.

1 INTRODUCTION

At network reliability assessment, reliability factors which define dynamics of behavior of the network are of great interest. Following are considered as reliability factors: mean time spent in operable and non-operable conditions within the steady-state mode. The simplest criterion for network operability is its connectivity. The network is considered to be connected if between any pair of nodes exists at least one path.

In the course of deducing inference, we will use connectivity of a network as criterion of its operability. In section 5 the possibility of application of the obtained expressions at use of other criteria of a network operability is justified.

If connectivity is used as a network operability criterion, than mean time spent in the connected state corresponds to mean time to failure (**MTTF**), and network mean time spent in the disconnected state corresponds to mean time to repair (**MTTR**). Therefore **MTTF** and **MTTR** may be considered instead of mentioned reliability factors.

2 NETWORK RELIABILITY MODEL AND OBTAINING OF ANALYTICAL EXPRESSIONS

Let us assume that network nodes being absolutely reliable and edges being identical on reliability fail independently from each other and their time to failure and time to repair are exponentially distributed with parameters λ and μ.

Let us imply moving off of an edge from the network till its repair to be edge failure. Distinctive feature of networks which complicates definition of reliability factors is that at certain number of failed edges network may be connected as well as disconnected. For example, network presented on Figure 1 may not be precisely defined as connected then two or three edges are removed.

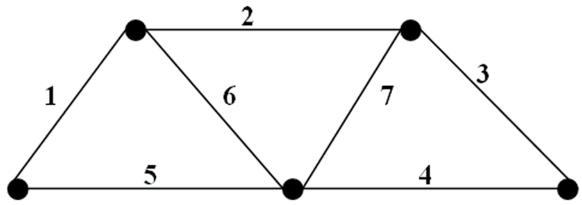

Figure 1: Example of the network

The cuts of capacity *i* is employed as a key parameter which allows us to define considered reliability factors of the network. Let us denote this parameter by Y_i.

Let's define Y_i for the network presented on Figure 1. Since considered network is biconnected, therefore one edge moving off cannot break its connectivity. Hence $Y_1=0$. For definition of Y_2 and Y_3 we consider all possible states of the network when 2 and 3 edges are moved off respectively, Figure 2.

Analyzing the data presented on Figure 2, it is possible to define that 2 cuts of capacity 2, and 14 cuts of capacity 3 are within the considered network, hence $Y_2 = 2$, $Y_3 = 14$. Any combination of *i* edges at *i* > *3* is a cut, hence $Y_i = \binom{n}{i}$ at $i \geq 4$.

At small *n*, values of Y_i may be defined by exhaustion of all possible states of the network. At great values of *n*, place Monte Carlo method or combined method should be used.

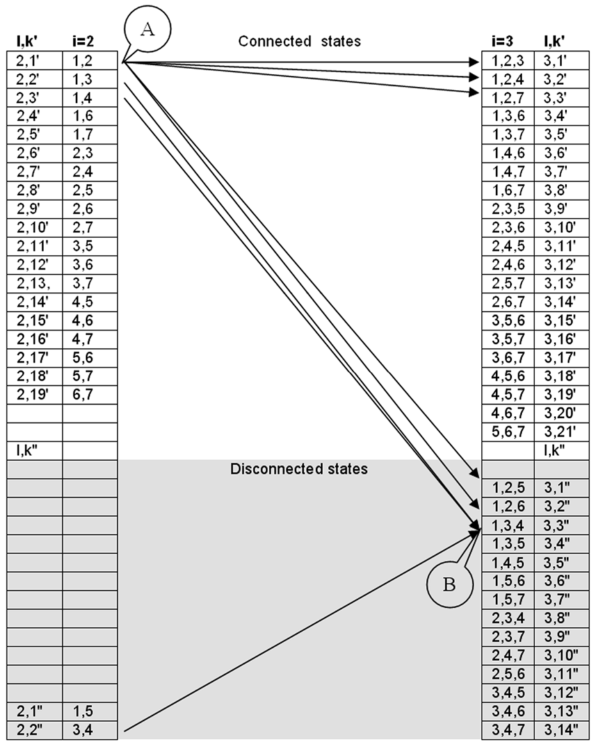

Figure 2: Network state transition at failure of i edge (i=3)

Knowing Y_i it is possible to define probability of *i* edges to become disconnected in case of failure. Let us denote this factor by - Z_i.

Value of Z_i is equal to ratio of Y_i to the total number of possible combinations of *i* of *n* elements where *n* is the number of edges in the network, expression (1).

$$Z_i = \frac{Y_i}{\binom{n}{i}} \tag{1}$$

Consider a Markov process which describes network state transition at failure and repair of its edges. Each state is characterized by a certain combination of failed edges and by a network state.

Denoted by E_+ is the set of operable states of the network, E_- is the set of non-operable states of the network.

Analytical expressions for definition of an average stay time of Markov and semi-Markov process in a subset of states have been obtained by Ushakov (1969a, 1969b).

For the considered case B.V.Gnedenko (1983), the average stay time of Markov process on a set of states E_+ before the first entering to a state of set E_- can be defined from expression:

$$T_{E_+} = \frac{\sum_{i\in E_+} P_i}{\sum_{i\in E_+}(P_i \sum_{j\in E_-}\lambda_{ij})} = \frac{\sum_{i\in E_+} P_i}{\sum_{j\in E_-}(P_j \sum_{i\in E_+}\lambda_{ji})} \tag{2}$$

where λ_{ij} - rate of transition from state $i \in E_+$ to state $j \in E_-$,

λ_{ji} - rate of transition from state $j \in E_-$ to state $i \in E_+$.

Similarly mean time for the process to be within the set of states E_- is defined

$$T_{E_-} = \frac{\sum_{j\in E_-} P_j}{\sum_{j\in E_-}(P_j \sum_{i\in E_+}\lambda_{ji})} = \frac{\sum_{j\in E_-} P_j}{\sum_{i\in E_+}(P_i \sum_{j\in E_-}\lambda_{ij})} \tag{3}$$

Let us denote connected states by $(i,k^{'})$ pair and disconnected network states by $(i,k^{''})$ pair at i failed edges. Then the left parts of expressions (2) and (3) may be written as follows:

$$T_{E_+} = \frac{\sum_{i=0}^{n}\sum_{k^{'}\in E_+} P_{i,k^{'}}}{\sum_{i=0}^{n}\sum_{k^{'}\in E_+} P_{i,k^{'}}\lambda^*_{i,k^{'}}} \tag{4}$$

$$T_{E_-} = \frac{\sum_{i=0}^{n}\sum_{k^{''}\in E_-} P_{i,k^{''}}}{\sum_{i=0}^{n}\sum_{k^{''}\in E_-} P_{i,k^{''}}\mu^*_{i,k^{''}}} \tag{5}$$

where $P_{i,k'}$, $P_{i,k''}$ - probability of states; and $\lambda^*_{i,k'}$ - rate of transition from state $(i,k^{'})$ to states belonging to the set E_-, $\mu^*_{i,k^{''}}$ - rate of transition from state $(i,k^{''})$ to states which belong to the set E_+.

Summary edge failure rate at i state equals to $(n-i)\lambda$, and summary rate to repair - $i\mu$ therefore:

$$\lambda^*_{(i,k^{'})} = (n-i)\lambda Z^*_{(i,k^{'})} \tag{6}$$

$$\mu^{*}_{(i,k'')} = i\mu Z^{**}_{(i,k'')} \tag{7}$$

where $Z^{*}_{(i,k')}$ - probability of network transition to disconnected state from (i,k') in case of one edge failure;

$Z^{**}_{(i,k'')}$ - probability of network transition to connected state from (i,k'') state in case of one edge repair.

For definition of T_{E_+} and T_{E_-} from expressions (4) and (5) it is necessary to know values $P_{(i,k')}$ and $P_{(i,k'')}$. Since all edges are identical on reliability therefore all states with i failed edges are equiprobable.

The probability of *i* edges in the network to be failed is equal to

$$P_i = \frac{(\lambda/\mu)^i \binom{n}{i}}{(1+\lambda/\mu)^n} \tag{8}$$

The total number of such states is $\binom{n}{i}$, therefore

$$P_{(i,k')} = P_{(i,k'')} = \widetilde{P}_i = \frac{P_i}{\binom{n}{i}} = \frac{(\lambda/\mu)^i}{(1+\lambda/\mu)^n} \tag{9}$$

Further, using Z_i defining from expression (1), it is possible to define the number of disconnected states at *i* failed edges.

$$Y_i = Z_i \binom{n}{i} \tag{10}$$

Similarly, the number of connected states

$$\binom{n}{i} - Y_i = (1 - Z_i)\binom{n}{i} \tag{11}$$

If we substitute (6), (7), (8), (10) (11) into (4) and (5), and placing variables independent of *i* and *k* outside summation symbols, we obtain

$$T_{E_+} = \frac{\sum_{i=0}^{n} (1-Z_i)\binom{n}{i}\widetilde{P}_i}{\lambda \sum_{i=0}^{n} \widetilde{P}_i (n-i) \sum_{k' \in E_+} Z^{*}_{(i,k')}} \tag{12}$$

$$T_{E_-} = \frac{\sum_{i=0}^{n} (Z_i)\binom{n}{i}\widetilde{P}_i}{\mu \sum_{i=0}^{n} \widetilde{P}_i i \sum_{k'' \in E_-} Z^{**}_{(i,k'')}} \tag{13}$$

Practical use of the obtained expressions is made difficult by necessity to consider all possible 2^n states of the systems for definition of $Z^{*}_{(i,k')}; Z^{**}_{(i,k'')}$.

In Tkachev (1999, 2010) it is shown, that defining of Z_i value is sufficient to define the values of expressions (12) and (13).

If at i failed edges the network is in one of connected states, then at one of *(n-i)* operational edges failure the network may transit to one *(n-i)* states with $i+1$ number of the failed edges. Let $S_{(i,k^{'})}$ of them be disconnected, then from definition $Z^{*}_{(i,k^{'})}$ follows that

$$Z^{*}_{(i,k')} = \frac{S_{(i,k^{'})}}{n-i} \tag{14}$$

For example, $Z^{*}_{(2,1^{'})} = 2/5$ ***(point A on Figure 2).***

Now, sum expression may be presented as $Z^{*}_{(i,k^{'})}$.

$$\sum_{k^{'} \in E_{+}} Z^{*}_{(i,k^{'})} = \sum_{k^{'} \in E_{+}} \frac{S_{(i,k^{'})}}{n-i} = \frac{C_i^{'}}{n-i} \tag{15}$$

where $C_i^{'}$ - the total number of transitions from the set of connected states $(i,k^{'})$ to the set of disconnected states $(i+1,k^{''})$.

For definition of $C_i^{'}$ value the following reasoning may be employed.

If the network is in one of Y_i disconnected states, then at failure of one of *(n-i)* operational edges the network may transit to another disconnected state with the number of failed edges $i+1$. The total number of transitions from disconnected states with the number of failed edges i to disconnected states with the number of failed edges $i+1$ equals to $Y_i(n-i)$. Then value of Y_{i+1} may be expressed through Y_i and $C_i^{'}$ as follows:

$$Y_{i+1} = \frac{Y_i(n-i) + C_i^{'}}{i+1} \tag{16}$$

The denominator equals to $i+1$ because the network can transit in each state with the number of failed edges $i+1$ from $i+1$ states with the number of failed edges i ***(point B on Figure 2)***.

On the other hand, in compliance with definition

$$Z_{i+1} = \frac{Y_{i+1}}{\binom{n}{i+1}} \tag{17}$$

If we substitute Y_{i+1} from (16) to (17) we obtain

$$Z_{i+1} = \frac{Y_i(n-i) + C_i^{'}}{(i+1)\binom{n}{i+1}} \tag{18}$$

In addition,

$$\binom{n}{i+1} = \binom{n}{i}\frac{n-i}{i+1} \tag{19}$$

If we substitute expressions (10) and (19) to (18), then after simplification we obtain

$$Z_{i+1} = \frac{Z_i\binom{n}{i}(n-i) + C_i^{'}}{\binom{n}{i}(n-i)} \tag{20}$$

whence it follows that

$$C_i^{'} = (Z_{i+1} - Z_i)\binom{n}{i}(n-i) \quad (21)$$

Therefore

$$\sum_{k^{'} \in E_+} Z^*_{(i,k^{'})} = \frac{C_i^{'}}{n-i} = (Z_{i+1} - Z_i)\binom{n}{i} \quad (22)$$

Substituting (22) to (12), we obtain

$$T_{E_+} = \frac{\sum_{i=0}^{n}(1-Z_i)\binom{n}{i}\widetilde{P}_i}{\lambda\sum_{i=0}^{n}\widetilde{P}_i(n-i)(Z_{i+1}-Z_i)\binom{n}{i}} \quad (23)$$

After notation we obtain

$$R_i = (1-Z_i)\binom{n}{i}\widetilde{P}_i \quad (24)$$

$$Z_i^* = \frac{Z_{i+1}-Z_i}{1-Z_i} \quad (25)$$

Thus, expression (23) may be brought to the form

$$T_{E_+} = \frac{\sum_{i=0}^{n} R_i}{\lambda\sum_{i=0}^{n} R_i(n-i)Z_i^*} \quad (26)$$

The numerator of this expression is probability for the network to be in connected state at an arbitrary point of time. If we substitute R_i from (24) and $\widetilde{P}_i$ from (9) we obtain

$$R = \frac{1}{(1+\lambda/\mu)^n}\sum_{i=0}^{n}(1-Z_i)\binom{n}{i}(\lambda/\mu)^i \quad (27)$$

By similar reasoning it is possible to obtain expression for system's mean time spent in the disconnected state from (13).

If at i failed edges the network is in one of disconnected states, then at repair of one of i edges the network may transit to one of i states with the number of failed edges *i-1*. Let $S_{(i,k^{''})}$ of them be connected, then from $Z^{**}_{(i,k'')}$ definition follows that

$$Z^{**}_{(i,k^{''})} = \frac{S_{(i,k^{''})}}{i} \quad (28)$$

Now sum expression may be given: $Z^{**}_{(i,k'')}$.

$$\sum_{k^{''} \in E_-} Z^{**}_{(i,k^{''})} = \sum_{k^{''} \in E_-} \frac{S_{(i,k^{''})}}{i} = \frac{C_i^{''}}{i} \quad (29)$$

where $C_i^{''}$ - the total number of transitions from disconnected set $(i,k^{''})$ to connected set $(i-1,k^{'})$.

For definition of $C_i^{''}$ the following reasoning may be used.

If the network is in one of Y_i disconnected states, then at repair of one of i failed edges the network may transit to connected or disconnected state with the number of failed edges *i-1*. The total number of transitions from disconnected states with the number of failed edges i, is equal to

$(Y_i)*i$. $C_i^{''}$ of them are transitions into connected states. Then value of Y_{i-1} be expressed through Y_i and $C_i^{''}$ as follows:

$$Y_{i-1} = \frac{(Y_i)i - C_i^{''}}{n-(i-1)} \tag{30}$$

The denominator equals to $n-(i-1)$ because the network can transit in each state with the number of failed edges *i-1* from $n-(i-1)$ states with the number of failed edges *i*.

From expression (30) we obtain

$$C_i^{''} = (Y_i)i - Y_{i-1}(n-(i-1)) \tag{31}$$

Substituting Y_i from (10) in (31), we obtain

$$C_i^{''} = \binom{n}{i} Z_i - \binom{n}{i-1} Z_{i-1}(n-(i-1)) \tag{32}$$

In addition,

$$\binom{n}{i-1} = \binom{n}{i} \frac{i}{(n-(i-1))} \tag{33}$$

$$C_i^{''} = \binom{n}{i} i(Z_i - Z_{i-1}) \tag{34}$$

If we substitute $C_i^{''}$ value from (34) into (29) and expression (29) into (13), we get

$$T_{E\_} = \frac{\sum_{i=0}^{n}(Z_i)\binom{n}{i}\widetilde{P}_i}{\mu\sum_{i=0}^{n}\widetilde{P}_i i\binom{n}{i}(Z_i - Z_{i-1})} \tag{35}$$

After notation

$$Q_i = Z_i \binom{n}{i} \widetilde{P}_i \tag{36}$$

we obtain

$$T_{E\_} = \frac{\sum_{i=0}^{n} Q_i}{\mu \sum_{i=0}^{n} i Q_i (1 - Z_i^{**})} \tag{37}$$

where

$$Z_i^{**} = \frac{Z_{i-1}}{Z_i} \tag{38}$$

Results of calculation of $Y_i, Z_i, Z_i^*, Z_i^{**}$ values for the network on Figure 1 are presented in Table1.

Table 1. $Y_i, Z_i, Z_i^*, Z_i^{**}$ values for the network on Figure 1

i	$\binom{n}{i}$	Y_i	Z_i	Z_i^*	Z_i^{**}
0	1	0	0	0	0
1	7	0	0	0,095238	0
2	21	2	0,095238	0,336842	0
3	35	14	0,4	1	0,238095
4	35	35	1	0	0,4
5	21	21	1	0	1
6	7	7	1	0	1
7	1	1	1	0	1

If we substitute $Y_i, Z_i, Z_i^*, Z_i^{**}$ values from Table 1 into expressions (26), (27), (37), where λ =0,01 (1/hour) and μ= 1(1/hour), we obtain following values of considering reliability factors. $T_{E_+} = 23{,}769$ hour.; $R = 0{,}980645$; $T_{E_-} = 0{,}469$ hour.

3 COMPARISON WITH KNOWN RESULTS

For checking of the obtained expressions we define values of considered system reliability factors (Figure 3) for which analytical estimations are known.

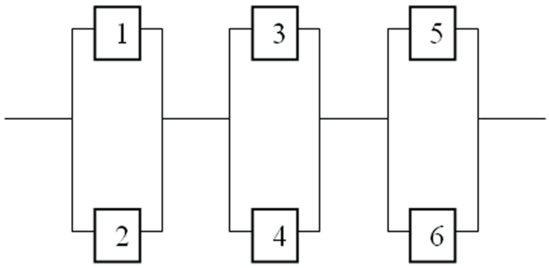

Figure 3: Parallel - series system

Following expressions may be found in Gnedenko&Belyayev&Solovyov (1965). For s identical elements connected in parallel with λ and μ parameters.

$$t_{E_+} = \frac{1}{s*\mu}\left[\left(1+\frac{\mu}{\lambda}\right)^s - 1\right]$$

$$t_{E_-} = \frac{1}{s*\mu}$$

$$r = \frac{t_{E_+}}{t_{E_+} + t_{E_-}}$$

For k identical repairing subsystems connected in series.

$$T_{E_+} = \frac{1}{k*(1/t_{E_+})}$$

$$R = r^k$$

$$T_{E_-} = T_{E_+}\frac{1-R}{R}$$

Substituting in this expressions $s = 2, k = 3$, λ =0,1(1/hour), μ =1 (1/hour) we obtain

$T_{E_+} = 20{,}000$ hour.; $R = 0{,}975411$; $T_{E_-} = 0{,}504$ hour.

For definition of these reliability factors using expressions (26), (27), (37), values of Y_i and Z_i should be defined.

It is obvious that $Y_1 = 0$ and $Z_1 = 0{,}0$. In case of 2 elements failure there are three combinations resulting in disconnection (nonoperativity) of the system (1,2), (3,4), (5,6). The total number of probable combinations is 12, hence $Y_2 = 3$ and $Z_2 = 3/12$. For definition of Y_3 and Z_3 let us consider all probable combinations of 3 elements (Table 2). As followed from the results provided in Table 2, $Y_3 = 12$ and $Z_3 = 12/20$. At failure of 4 or more elements, the system will be disconnected, therefore $Z_4 = Z_5 = Z_6 = 1$.

Table 2. State of the system on Figure 3 at failure of 3 elements.

1	123	-	11	234	-
2	124	-	12	235	+
3	125	-	13	236	+
4	126	-	14	245	+
5	134	-	15	246	+
6	135	+	16	256	-
7	136	+	17	345	-
8	145	+	18	346	-
9	146	+	19	356	-
10	156	-	20	456	-

Table 3. Calculation results of system performance (for Figure 3)

i	$\binom{n}{i}$	P_i	Y_i	Z_i	Z_i^*	Z_i^{**}	Q_i	R_i
0	1	0,5644739301	0	0	0	0	0,0000000000	0,5644739301
1	6	0,3386843580	0	0	0,2	0	0,0000000000	0,3386843580
2	15	0,0846710895	3	0,2	0,5	0	0,0169342179	0,0677368716
3	20	0,0112894786	12	0,6	1	0,33333	0,0067736872	0,0045157914
4	15	0,0008467109	15	1	0	0,6	0,0008467109	0,0000000000
5	6	0,0000338684	6	1	0	1	0,0000338684	0,0000000000
6	1	0,0000005645	1	1	0	1	0,0000005645	0,0000000000

The rest results are presented in Table 3. Substituting the data from Table 3 into expressions (26), (27), (37), we obtain:

$T_{E_+} = 20{,}000$ hour.; $R = 0{,}975411$; $T_{E_-} = 0{,}504$ hour.

4 EXAMPLES OF NETWORK RELIABILITY ANALYSIS

We will consider biconnected networks which node degrees satisfy to the inequality $2 \le k \le 3$.

Let us denote the number of nodes by *m* and the number of edges by *n*. It is necessary to pay attention that for each pair of *(m,n)* there is a topology providing the maximum level of reliability. For definition of such topologies the algorithms offered in Artamonov (1999) have been used.

Example 1. Ring with one diagonal m=20, n=21

It is obvious that $Y_1 = 0$ and $Z_1 = 0$. Considering combinations of 2 edges one may see that they form a cut even when they belong to the same chain. In addition, any combination of 2 edges which belong to the same chain is a cut itself. Calculation results of values Y_i and Z_i for the network (Figure4) are presented in Table 4a. Table 4b includes T_{E_+}, T_{E_-} and R values calculated for various λ. Value $\mu=1$. Here and elsewhere the unit of measurement for λ and μ is 1/hour, for T_{E_+} and T_{E_-} - hour.

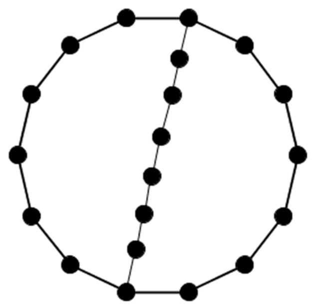

Figure 4. Network with parameters m=20, n=21

Table 4a. Y_i and Z_i values for the network presented on Figure 4.

i	1	2	≥ 3
Yi	0	63	$\binom{n}{i}$
Zi	0	0,3000	1

Table 4b. T_{E_+}, T_{E_-}, R valuesfor the network presented on Figure 4.

λ	T_{E_+}	T_{E_-}	R
0,1	1,13	0,6983	0,617547
0,01	79,56	0,4997	0,993759
0,001	7928,59	0,4995	0,999937

Example 2. Network with parameters m=20, n=24

As presented in Artamonov (1999), to reach the maximum reliability it is necessary to find the minimal diameter network with *2(n-m)* nodes of degree 3, and distribute nodes of degree 2 on the edges of this network equally. The network presented on Figure 5 may be found as a result.

Y_i values of this network may be defined by exhaustion of all possible states. Results of calculations are presented Table 5a and Table 5b.

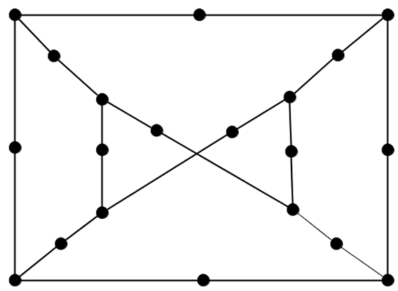

Figure 5. Network with parameters m=20, n=24.

Table 5a. Values Y_i and Z_i for the network on Figure 5.

i	1	2	3	4	5	≥ 6
Yi	0	12	328	4082	29960	$\binom{n}{i}$
Zi	0	0,043478	0,162055	0,384252	0,704875	1

Table 5b. Values T_{E_+}, T_{E_-}, R for the network on Figure 5.

λ	T_{E_+}	T_{E_-}	R
0,1	2,13	0,3334	0,864576
0,01	390,86	0,4845	0,998762
0,001	41415,64	0,4986	0,999988

5 USE OF OTHER OPERABILITY CRITERIA AND MODELS OF NETWORK RELIABILITY

The connectivity of all nodes of the network has been used as operability criterion so far. However the following question would be appropriate: whether loss of communication with one of nodes is a failure? If we consider the loss of connectivity with *k* nodes to be admissible, then the following network operability criterion may be used: the network is considered operational, if number of connected nodes $\geq (m-k)$.

The obtained results may be also used in this case, but respective alterations in algorithm of definition of values Y_i should be made. In Table 6 values T_{E_+} and *R* at various values of *k* are presented for the network on Figure 5.

Table 6. Values T_{E_+} and R at various values of *k*,
λ=0,01, μ=1 for the network on Figure 5.

k	T_{E_+}	R
1	5780,49	0,999943591
2	9843,50	0,999967012
3	31455,53	0,999990157

It is possible to weaken restriction on equireliability of edges. It is possible to present each edge in the form of serial connection of certain quantity of equireliable elements. Thus fictitious nodes are entered into network structure. After such transformations values of Z_i are defined. If one of elements making an edge fails, then this edge is removed from the network. Network connectivity is checked without considering of fictitious nodes.

It should be mentioned that obtained results may be used in case of failure of nodes when edges are absolutely reliable. Node failure may be modeled by removal of all edges emanating from this node.

This method of the analysis of networks reliability can be used also when those states are only considered as operable at which value of certain network parameters, for example, the network capacity will satisfy to preset values.

For high dimension networks the statistical estimation of values Z_i can be defined using a Monte-Carlo method. The random combination of i fault elements is generated, and then operability of a network is checked. The ration of an amount of trials at which the network will appear non-operable to the total number of trials will be statistical estimation Z_i.

However it is necessary to consider that for reaching of high accuracy of statistical estimation Z_i, the amount of trials should be great enough (10^5-10^6).

6 CONCLUSION

Analytical expressions defining reliability factors of networks consisting of identical elements are obtained. These elements fail and repair independently from each other and have exponentially distributed time to failure ad time to repair. Moreover both nodes and edges of the network may be regarded as absolutely reliable. Fidelity of the obtained result is vindicated by calculation of reliability factors of redundant system for which analytical estimations are known.

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