
OPTIMAL DISPATCHING OF GENERATORS WITH LOAD DEPENDENT FAILURE RATES

G. Velotto

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e-mail: - giovanni.velotto@libero.it

ABSTRACT

An optimally coordinated energy dispatching among generating units may contribute to attain higher performances of electric networks. Within this wide research field this paper focuses on a theoretical study aimed at proposing an optimal dispatching policy maximizing reliability and Mean Time To Fault of a park of programmable generating units whose failure rate models take into account the dependency from the instantaneous loading condition.

1 INTRODUCTION

The operation environment of power systems is becoming increasingly dynamical due to the continually evolving functions required, Madani & King (2008), Bose (2010).

It is a fact that operational and environmental conditions have a significant effect on accelerating or decelerating the rate of degradation processes which occur prior to failure. Most conventional failure models are developed on the premise that the prevailing environmental and operating conditions either do not change in time, or have no effect on deterioration and failure processes. These hypotheses may give misleading results whose accuracy does not fulfill the requirements imposed by an envisaged technical and economical dynamic environment in which future power systems are called to operate.

In the future scenarios more advanced reliability analysis tools are required.

The intent of this paper is to focus on an optimal dispatching policy of a heterogeneous park of programmable generating units whose failure rate model is supposed dependent on the loading condition.

2 FAILURE RATE AND COVARIATES

A lot of literature has been devoted to the study of failure models for applications in reliability engineering. A comprehensive survey of the settled failure models currently adopted in this field may be found in Singpurwalla (1995).

Among these failure models, in engineering systems analysis the approach based on the definition of a failure rate function $\lambda(t)$ plays a key role in reliability and survival analysis; it has generally turned out to be a useful device for expressing a connection between the physics of failure and the probability of survival. Condition-Dependent Failure Rate (CDFR) models are currently an interesting research issue in reliability engineering also applied to power systems, Pan et al. (2009), Sun et al. (2010), Wang & Xie (2008).

In conventional reliability evaluation of power systems failure rate, derived from the collected statistical data, has usually been assumed to be constant; it has been realised from the real-time

operation that it is not constant and varies with several parameters: in details $\lambda(t)$ may depend upon explanatory variables (covariates) such as loading conditions, installation environment, manufacturer, etc. When such is the case, the explanatory variables become a part of the observable history, and so their effect needs to be incorporated in failure rate expression $\lambda(t, \mathbf{Z}(t), \boldsymbol{\theta}(t))$, where $\mathbf{Z}(t)$ is a vector of covariates and $\boldsymbol{\theta}(t)$ are other model's parameters, generally time dependent, Aalen (1989), Cox (1972), Peña & Hollander (2004), Singpurwalla (2006). The conceptual advantage of this model lies in the representation of a non observable entity, the failure rate, through observable and sometimes controllable entities.

Generating units' failure rates may depend on loading/stress history (e.g. active and reactive power, voltage, peak values, number of occurrence of a peak, highest derivative etc.) and environmental variables history (e.g. temperature, humidity, pollutants' concentration, etc.). Generally some of the loading variables are controllable; some environmental parameters may be controlled only in particular installation conditions.

In this paper the generating units' failure rate is assumed dependent on the active instantaneous loading status, $L(t)$, so $\lambda(t, L(t), \boldsymbol{\theta}(t))$.

Extensive studies in survival analysis, Cox & Oakes (1984), applied for instance to biostatistics, have proposed the following model:

$$\lambda(t, L(t)) = \lambda_o(t) \cdot \exp[\beta(t) \cdot L(t)] \quad (1)$$

Comprehensive studies on these dependences, under static loading conditions, based on data collected for some mechanical elements, e.g. contact bearing, spring, gear, etc., Carderockdiv. (1994), proposed the following model:

$$\lambda(t, L(t)) = \lambda_c(t) \cdot \left[\frac{L(t)}{L_c} \right]^y \quad (2)$$

Where λ_c and L_c are the reference failure rate and the corresponding load, respectively; y denotes the load dependent exponent (up to 10).

2.1 Optimal Control

The covariate induced failure rate process $\lambda(t, \mathbf{Z}(t), \boldsymbol{\theta}(t))$ may be useful to identify optimal control strategies of a subset of $\mathbf{Z}(t)$, \mathbf{Z}_c , of controllable variables capable of optimizing expected reliability performances. The general criterion is to select the control/decision variables, subject to constraints, in order to maximize the expectation of an objective goal functional, over a time horizon.

Reliability based control applied to power systems may be effective for identifying the most economical policies that can be used to fulfil the expected mission and minimize life cycle costs. Different control concepts and different reliability target formulations can be evaluated; in particular reliability criteria based on crossing rates and on approximations of the extrema of random performance measures could be adopted.

The past decades have seen great development of reliability based optimal control methodologies in several engineering fields (e.g. civil, mechanical, marine, offshore); the research issue is still open because of the various difficulties in stochastic dynamics and reliability theory.

The prospected evolution of power systems paves the way for an efficient integration of these methodologies; moreover the rapid development of the monitoring technologies, particularly sensing techniques, Proceedings of CMD (2010), Proceedings of PHM (2010), non-invasive communication systems (e.g. wireless sensor networks), Baker et al. (2009), Lu et al. (2005), and their expected pervasive diffusion in future power systems for collecting field data may constitute a valid technological support for the actuation of novel reliability-based control techniques.

Implementing this architecture on power systems requires the development of algorithms to perform each of its separate functions. That is where some of the real technical challenges are found.

3 PROBLEM FORMULATION

Assume a set of n programmable generators, all of them characterized by a failure rate model as (1) or (2). The effect of the electrical network is neglected at this initial stage, so a one-node model is considered, Billington & Allan (1984). The objective is to identify a dispatching policy, which optimizes reliability performances. In particular as first objective is identified the minimization of the risk correlated with any intervention on the system; that is the maximization of the reliability of the series of all generators evaluated at the mission time T . This objective may be of interest in case of power systems installed in remote areas, or installed on-board, for which the costs of any intervention are high. Secondly will be evaluated an optimal loading policy maximizing Mean Time To Fault (MTTF) as generally required for improving availability performances and optimal maintenance policies.

In particular let (T_1, T_2, \dots, T_n) be the random variables (r.v.) describing the operating life of each generator; given the failure rate of each generator $\lambda_i(t, L_i(t), \theta_i(t))$. All the covariates are assumed external as defined in Kalbfleisch (2002). If the hazard potential X_i of the generators are supposed independent, and given that $H_i(t)$ are known the r.v. T_i are independent, Singpurwalla (2006), so the probability that all n generators are operating during the mission time T is:

$$R(T) = P(T_1 > T, T_2 > T, \dots, T_n > T | L_i(t), \theta_i(t), i = 1, \dots, n) = \prod_{i=1}^n P(T_i > T | L_i(t), \theta_i(t)) \quad (3)$$

4 DETERMINISTIC CASE

If the total load required to the generators $L(t)$ and the failure rate model's parameters are known deterministic the formulation of the optimization problem is:

$$\begin{aligned} & \max_{L_1(t), \dots, L_n(t)} [R(T)] & (4) \\ & \text{s.t.} \\ & \sum_{i=1}^n L_i(\tau) = L(\tau), \quad \forall \tau \in [0, T] \\ & L_i(\tau) \in [0, L_{iMAX}], \forall i \in [1, 2, \dots, n], \forall \tau \in [0, T] \end{aligned}$$

Considered the monotonicity of the argument function, problem (4) is equivalent to:

$$\max_{L_1(t), \dots, L_n(t)} \left[- \int_0^T \sum_{i=1}^n \lambda_i(\tau, L_i(\tau), \theta_i(\tau)) d\tau \right] \quad (5)$$

So, solving problem (5), the optimal dispatching $L^*(\tau)$ is:

Model (1)

Let $\varphi(\tau)$:

$$\varphi(\tau) = \exp \left[\frac{L(\tau) + \sum_{i=1}^n \frac{\ln(\lambda_{0i}(\tau) \cdot \beta_i(\tau))}{\beta_i(\tau)}}{\sum_{i=1}^n \frac{1}{\beta_i(\tau)}} \right] \quad (6)$$

$$\text{if } \varphi(\tau) \geq \lambda_{0i}(\tau) \cdot \beta_i(\tau) \cdot \exp[\beta_i(\tau) \cdot L_{iMAX}]$$

$$L_i^*(\tau) = L_{iMAX}$$

$$\text{if } \varphi(\tau) \leq \lambda_{0i}(\tau) \cdot \beta_i(\tau)$$

$$L_i^*(\tau) = 0$$

otherwise

$$L_i^*(\tau) = \frac{L(\tau) + \sum_{j=1, j \neq i}^n \ln \left(\frac{\lambda_{0j}(\tau) \cdot \beta_j(\tau)}{\lambda_{0i}(\tau) \cdot \beta_i(\tau)} \right)^{\frac{1}{\beta_j(\tau)}}}{1 + \sum_{j=1, j \neq i}^n \frac{1}{\beta_j(\tau)}} \quad (7)$$

Model (2)

If $y_i \neq 1$, let $\varphi(\tau)$ defined by:

$$\sum_{i=1}^n \left[\frac{L_{c_i}^{y_i}}{\lambda_{c_i} \cdot y_i} \varphi(\tau) \right]^{y_i-1} = L(\tau) \quad (8)$$

$$\varphi(\tau) \geq 0$$

$$\text{if } \varphi(\tau) \geq \lambda_{c_i} \cdot y_i \cdot \frac{L_{iMAX}^{y_i-1}}{L_{c_i}^{y_i}}$$

$$L_i^*(\tau) = L_{iMAX}$$

otherwise

$$L_i^*(\tau) = \left[\frac{L_{c_i}^{y_i}}{\lambda_{c_i} \cdot y_i} \varphi(\tau) \right]^{\frac{1}{y_i-1}} \quad (9)$$

It can also be proved that $L^*(\tau)$ maximize the MTTF.

4.1 Maximum MTTF for Periodic Deterministic Load

If the load $L(t)$ is supposed deterministic and periodic, with period $h = \Delta T$, λ_i not explicitly depending on the time, $\lambda_i(L_i(t), \theta_i(t))$, and the $\theta_i(t)$ parameters periodic too with the same period of the load (This last hypothesis could be physically justified by periodic maintenance policies), applying Bellman’s optimality principle the maximum MTTF* is given by:

$$\begin{aligned} \text{MTTF}^* &= \max_{L(t)} \left\{ \int_0^{\Delta T} R(t) dt + R(\Delta T) \cdot \max_{L(t)} \{ \text{MTTF} \} \right\} = \\ &= \max_{L(t)} \left\{ \int_0^{\Delta T} R(t) dt + R(\Delta T) \cdot \text{MTTF}^* \right\} \end{aligned} \quad (10)$$

It’s worth noting that this formulation is possible because ageing is neglected.

Equation (10) is the recurrent formulation of the maximization problem.

Banach fixed point theorem guarantees the existence and uniqueness of a fixed point MTTF*:

$$\text{MTTF}^* = \frac{\int_0^{\Delta T} R^*(t) dt}{1 - R^*(\Delta T)} \quad (11)$$

where $R^*(t)$ is the reliability evaluated for the optimal loading policy $L^*(\tau)$.

5 STOCHASTIC CASE

Given a filtered probability space $(\Omega, \mathfrak{F}, P)$ with a filtration $\{\mathfrak{F}_t\}$, the load $L(\tau)$ and the failure rate parameters $[\lambda_{0i}(t), \beta_i(t), \dots]$ are supposed stochastic processes adapted to the filtration, Pham (2009).

If the power system under study is “grid-connected” all the generators may be generally deterministically controlled. If the system is isolated and without storage the controllable variables may be “n-1” among “n”, supposing that the remaining generator, the “slack”, let it be the j-th, is capable supporting the load, that is:

$$L_{jMAX} \geq \max_{t \in [0, +\infty]} [L(t)] - \sum_{i \neq j}^n L_{iMAX} \quad (12)$$

As it is physically reasonable $\max_{t \in [0, +\infty]} [L(t)]$ is supposed to be deterministic; it is assumed also that relation (12) is verified $\forall j \in [1, n]$.

Naturally this hypothesis, in a real case, must be evaluated in the light of load-following regulating performances of the j -th generator, that in this paper are supposed adequate to support the dynamics of the control policy.

Chosen a generator j -th as slack, the objective is to identify a loading policy which maximizes the expected Reliability for the mission time T .

Let:

$$\chi(\tau, L(\tau), \mathbf{L}_j(\tau), \boldsymbol{\Theta}_j(\tau)) = \sum_{i \neq j}^n \lambda_i(t, L_i(t), \boldsymbol{\theta}_i(t)) + \lambda_j(t, L(\tau) - \sum_{i \neq j}^n L_i(t), \boldsymbol{\theta}_i(t)) \quad (13)$$

with

$$\begin{aligned} \mathbf{L}_j(\tau) &= [L_1(t), \dots, L_{j-1}(t), L_{j+1}(t), \dots, L_n(t)] \\ \boldsymbol{\Theta}_j(t) &= [\boldsymbol{\theta}_1(t), \dots, \boldsymbol{\theta}_{j-1}(t), \boldsymbol{\theta}_{j+1}(t), \dots, \boldsymbol{\theta}_n(t)]. \end{aligned}$$

Given the problem:

$$\begin{aligned} \max_{L_j[0, T]} \{E[R_j(T)]\} \\ \text{s.t.} \\ L_i(\tau) \in [0, L_{iMAX}], \forall i \in [1, \dots, j-1, j+1, \dots, n], \forall \tau \in [0, T], \end{aligned} \quad (14)$$

considered that the processes describing loads and failure rate parameters have been supposed adapted to the filtration $\{\mathfrak{F}_t\}$, according to the principle of optimality (14) may be written as:

$$0 = \min_{L_j[0, T]} \left\{ E \left[\chi(\tau, L(\tau), \mathbf{L}_j(\tau), \boldsymbol{\Theta}_j(\tau)) \right] - E \left[\chi(\tau, L(\tau), \mathbf{L}_j(\tau), \boldsymbol{\Theta}_j(\tau)) \right] \cdot J(t) + j(t) \right\} \quad (15)$$

with

$$\begin{aligned} J(t) &= \min_{\substack{L_j(\tau) \\ t \leq \tau \leq T}} \left\{ E \left[\int_t^T \chi(\tau, L(\tau), \mathbf{L}_j(\tau), \boldsymbol{\Theta}_j(\tau)) \exp \left[- \int_t^\tau \chi(s, L(s), \mathbf{L}_j(s), \boldsymbol{\Theta}_j(s)) ds \right] d\tau \right] \right\} \\ J(T) &= 0 \\ J(0) &= 1 - \max_{L_j[0, T]} \{E[R_j(T)]\}. \end{aligned}$$

The equation (15) implies that the optimal loading policy $L_j^*(\tau)$ satisfies:

$$\begin{aligned} \min_{L_j(\tau)} \left\{ E \left[\chi(\tau, L(\tau), \mathbf{L}_j(\tau), \boldsymbol{\Theta}_j(\tau)) \right] \right\} \\ \text{s.t.} \\ L_i(\tau) \in [0, L_{iMAX}], \forall i \in [1, \dots, j-1, j+1, \dots, n], \forall \tau \in [0, T]. \end{aligned} \quad (16)$$

So taking into account the Taylor expansion of (16) around the expected value of $[L(\tau), \boldsymbol{\Theta}_j(\tau)]$, and adopting a multi-index notation, Marti (2008),:

$$\min_{L_j(\tau)} \left\{ \chi(\tau, \overline{L(\tau)}, \mathbf{L}_j(\tau), \overline{\boldsymbol{\Theta}_j(\tau)}) + \frac{1}{2} \sum_{|\alpha|=2} D^\alpha \left(\chi(\tau, \overline{L(\tau)}, \mathbf{L}_j(\tau), \overline{\boldsymbol{\Theta}_j(\tau)}) \right) \cdot E \left[([L(\tau), \boldsymbol{\Theta}_j(\tau)] - [L(\tau), \boldsymbol{\Theta}_j(\tau)])^\alpha \right] \right\} \quad (17)$$

where $E\left([L(\tau), \Theta_j(\tau)] - [L(\tau), \Theta_j(\tau)]^\alpha\right)$ are the covariances in multi-index notation; applying Lagrange multipliers method, the particularized equations for failure rate model (1) are:

$$\left\{ \begin{array}{l} \psi_j(\tau) + \exp[\beta_i(\tau) \cdot L_i(\tau)] \cdot [L_i^2(\tau) \cdot \vartheta_i(\tau) + L_i(\tau) \cdot \rho_i(\tau) + \kappa_i(\tau)] + \mu_i - \gamma_i = 0 \\ \psi_j(\tau) = \exp[\beta_j(\tau) \cdot z_j(\tau)] \cdot [z_j^2(\tau) \cdot \nu_j(\tau) + z_j(\tau) \cdot \xi_j(\tau) + \zeta_j(\tau)] \\ z_j(\tau) = \bar{L}(\tau) - \sum_{\substack{k=1 \\ k \neq j}}^n L_k(\tau) \\ \nu_j(\tau) = -\frac{\beta_j(\tau) \cdot \sigma_{\beta_i \beta_j}(\tau) \cdot \bar{\lambda}_{oj}(\tau)}{2} \mathbf{0} \\ \xi_j(\tau) = -\sigma_{\beta_j \beta_j}(\tau) \cdot \bar{\lambda}_{oj}(\tau) - 4\sigma_{\beta_i \beta_j}(\tau) \cdot \bar{\beta}_j(\tau) \\ \zeta_j(\tau) = -\bar{\lambda}_{oj}(\tau) \cdot \bar{\beta}_j(\tau) \cdot \left[\frac{1}{2} \sigma_{LL}(\tau) \bar{\beta}_j(\tau)^2 + 2\sigma_{L\beta_j}(\tau) \cdot (1 + \bar{\beta}_j(\tau)^2) + 1 \right] + \\ -2\bar{\beta}_j(\tau)^2 \cdot \sigma_{L\lambda_{oj}}(\tau) - 4\sigma_{\lambda_{oj} \beta_j}(\tau) \\ \vartheta_i(\tau) = \frac{\bar{\beta}_i(\tau) \cdot \bar{\lambda}_{oj}(\tau) \cdot \sigma_{\beta_i \beta_j}(\tau)}{2} \\ \rho_i(\tau) = \bar{\lambda}_{oi}(\tau) \sigma_{\beta_i \beta_i}(\tau) + 2\bar{\beta}_i(\tau) \sigma_{\lambda_{oi} \beta_i}(\tau) \\ \kappa_i(\tau) = \bar{\lambda}_{oi}(\tau) \bar{\beta}_i(\tau) + 2\sigma_{\lambda_{oi} \beta_i}(\tau) \\ \mu_i \cdot (L_i(\tau) - L_{iMAX}) = 0 \\ \gamma_i \cdot L_i(\tau) = 0 \\ \mu_i \geq 0 \quad \gamma_i \geq 0 \end{array} \right. \quad (18)$$

$$\forall i \in [1, \dots, j-1, j+1, \dots, n]$$

$\sigma_{SD}(\tau)$ is the covariance between the r. v. S and D.

For model (2), for sake of simplicity of the analytical expression, are reported only the particularized equations with known and constant failure rate parameters and $y_j > 2, \forall j$:

$$\min_{L_j(\tau)} \left\{ \sum_{\substack{i=1 \\ i \neq j}}^n \left[\lambda_{c_i} \left(\frac{L_i(\tau)}{L_{c_i}} \right)^{y_i} \right] + \frac{\lambda_{c_i}}{L_{c_i} \lambda_{c_i}} \left[\left(\bar{L}(\tau) - \sum_{\substack{i=1 \\ i \neq j}}^n L_i(\tau) \right)^{y_j} + \right. \right. \\ \left. \left. + \frac{1}{2} y_j (y_j - 1) \left(\bar{L}(\tau) - \sum_{\substack{i=1 \\ i \neq j}}^n L_i(\tau) \right)^{y_j-2} \sigma^2_L(\tau) \right] \right\} \quad (19)$$

s.t.

$$L_j(\tau) \in [0, L_{jMAX}]$$

λ_{c_i} and L_{c_i} are the parameters for the i-th generator

$\sigma^2_L(\tau)$ is variance of $L(\tau)$.

The analysis performed allows, also, identifying the i-th optimal slack generator which optimizes expected reliability, determined by:

$$i: E[R_i(T)] = \max_{j=1, \dots, n} [E[R_j(T)]] \quad (20)$$

Applying an analogous procedure used for the solution of problem (14) it is possible to show that:

$$L_j^*(\tau) \text{ maximize } E[R_j(T)] \forall T \Leftrightarrow L_j^*(\tau) \text{ maximize } E[MTTF_j] \quad (21)$$

5.1 Maximum expected MTTF for Stochastic-Load with periodicity

Following the procedure applied in the deterministic case, and similarly, supposing that λ_i is not explicitly time dependent, and the processes describing load and failure rate parameters are supposed periodic with fixed period $h = \Delta T$, the maximum expected MTTF* is given by the following recurrent equation:

$$MTTF_j^* = \max_{L_j[0, \Delta T]} \left\{ E \left[\int_0^{\Delta T} R(\tau, L_j[0, \Delta T], L(\tau)) d\tau + R(\Delta T, L_j[0, \Delta T], L[0, \Delta T]) \cdot MTTF_j^* \right] \right\}. \quad (22)$$

Banach fixed point theorem guarantees the existence and uniqueness of a fixed point $MTTF_j^*$ given by:

$$MTTF_j^* = \frac{E \left[\int_0^{\Delta T} R(\tau, L_j^*[0, \Delta T], L(\tau)) d\tau \right]}{1 - E[R(\Delta T, L_j^*[0, \Delta T], L[0, \Delta T])]} \quad (23)$$

Even in this case the analysis performed allows identifying the i -th optimal slack generator which optimizes the expected MTTF. It coincides with that one given by (20).

6 CONCLUSION

Reliability-based control applied to power systems may be effective for identifying the most economical policies that can be used to fulfill the expected mission and minimize life cycle costs. In this field adequate methodological and technological development may have very high potential value for future applications in electrical power systems. Recent development of the monitoring technologies, particularly sensing techniques, non-invasive communication systems and implementation framework, and their expected pervasive diffusion in future power systems for collecting field data may constitute a valid technological support for the actuation of novel reliability-based control techniques.

The theoretical discussion of this paper focuses on a study of a reliability-based optimal dispatching policy of a heterogeneous park of programmable generating units whose failure model takes into account the dependency from the instantaneous loading condition. In deterministic analysis rigorous analytical expressions of the dispatching policy has been obtained for a generic set of heterogeneous components within simplification assumptions. Some general results have also been obtained in presence of stochastic loads and failure rate's parameters.

REFERENCES

- Aalen O.O. 1989. A linear regression model for analysis of life times. *Statistics in Medicine*, vol. 8: 907–25.
- Baker P.C., Catterson V.M., McArthur S.D.J. 2009. Integrating an Agent-Based Wireless Sensor Network within an Existing Multi-Agent Condition Monitoring System. *Proc. 15th International Conference on Intelligent System Applications to Power Systems, ISAP'09*, Curitiba, Brazil, November 8 - 12.
- Billington R., Allan R.N. 1984. *Reliability evaluation of power systems*. New York: Plenum Press.
- Bose A. 2010. New Smart Grid Applications for Power System Operations. *Proceedings of The 2010 IEEE PES General Meeting*. Minneapolis, Minnesota, USA, July 25-29.
- Carderockdiv. 1994. *Handbook of reliability prediction procedures for mechanical equipment*. Naval Surface Warfare Center Report, Bethesda, Maryland.

- Cox D.R. 1972. Regression models and life tables. *Journal of the Royal Statistical Society*, vol. B, no. 34:187–220.
- Cox D.R., Oakes D. 1984. *Analysis of Survival Data*. Chapman & Hall.
- Kalbfleisch J. D., Prentice R.L. 2002. *The Statistical Analysis of Failure Time Data*, New York: Wiley.
- Lu B., Wu L., Habetler T.G., Harley R.G., Gutierrez J.A. 2005. On the application of wireless sensor networks in condition monitoring and energy usage evaluation for electric machines. *Proc. 31st Annual Conference of, IEEE Industrial Electronics Society, IECON 2005*, Raleigh, North Carolina, USA, November 6-10.
- Madani V., King R.L. 2008. Strategies to meet grid challenges for safety and reliability. *International Journal of Reliability and Safety*, vol. 2, no. 1-2/2008: 146-165.
- Marti K. 2008. *Stochastic Optimization Methods*, New York: Springer-Verlag.
- Pan J., Wang J. Z., Lubkeman D. 2009. Condition based failure rate modeling for electric network components. *Proc. Power Systems Conference and Exposition, PSCE '09. IEEE/PES*, USA, 15-18 March.
- Peña E., Hollander M. 2004. Models for recurrent events in reliability and survival analysis. In Soyer R., Mazzuchi T. and Singpurwalla N. (eds.) *Mathematical Reliability: An Expository Perspective*. Boston, MA: Kluwer, ch. 6: 105-123.
- Pham H. 2009. *Continuous-time Stochastic Control and Optimization with Financial Applications*. Berlin: Springer Verlag.
- Proceedings of The 2010 International Conference on Condition Monitoring and Diagnosis, CMD '10*, Toyosu, Tokyo, Japan, 6–11 Sept.
- Proceedings of The 2010 International Conference on Prognostics and System Health Management, PHM '10*, Macao, China, 12-14 Jan.
- Singpurwalla N. D. 1995. Survival in Dynamic Environments. *Statistical Science*, vol.10, no. 1: 86-103.
- Singpurwalla N. D. 2006. *Reliability and risk: a Bayesian perspective*. Wiley.
- Sun Y., Wang P., Cheng L., Liu H. 2010. Operational reliability assessment of power systems considering condition-dependent failure rate. *Generation, Transmission & Distribution, IET* , vol.4, no.1: 60-72.
- Wang Z., Xie L. 2008. Dynamic Reliability Model of Components under Random Load. *IEEE Transactions on Reliability*, vol.57, no.3: 474-479.