APPROXIMATION OF DOMAINS OF SERVICEABILITY AND ATTAINABILITY OF CONTROL SYSTEM ON THE BASIS OF THE INDUCTIVE APPROACH

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ABSTRACT

One of fundamental problems of the dynamic systems control theory is the problem of a finding or estimation of attainability sets. Their practical construction, especially in nonlinear systems of the high dimensionality, usually turns out hugely the complex problem demanding considerable computing resources. In this connection possibility of use of adaptive partition by means of a uniform grid as one of effective methods of approximation of such sets is analyzed. The choice of received net descriptions for specification of their unknown borders in the conditions of uncertainty on the basis of the inductive approach is substantiated, using available aprioristic and current information.

1 INTRODUCTION

Problems of a choice of optimum control and reduction of space of its search arise at the decision of problems of designing, modeling of real processes or the facts, the analysis of the data in various areas of a science and technique. Choice of control by dynamic objects, in particular, optimum parametrical synthesis is connected from the computing viewpoint with very big costs. It demands effective numerical methods for the decision in the conditions of the high dimensionality and uncertainty. Use of domains of serviceability and attainability which in actual practice usually have no analytical description allows decreasing computing labour input. The various numerical methods reducing space of search and time of calculations are applied to their approximation depending on the available aprioristic information (See Chernousko 1988, Abramov 1992, Katueva & Nazarov 2005).

Nowadays a number of methods for approximation of attainability domains of the various kinds of controllable dynamical systems are known. One of them offered by Chernousko (1988) is based on use of ellipsoidal approximations and is applicable basically for nonlinear dynamic systems. This method does not demand excessively big computational cost, however it does not allow receiving result with beforehand pre-set accuracy of calculations. Approaches of Gusejnov et al. (1998) allow approximating barely boundaries of attainability domains and only in case of their convexity. The method offered by Yevseyev & Usachov's (2001) is similar to them. It uses the task of a set of evenly distributed nodal points belonging to approximable attainability domain in phase space. On these points the approximating set formed by association of spheres with the centers in nodal points and certain identical radius on which accuracy of the received approach is calculated is under construction. Accuracy of the received approach is defined by radius of these spheres.

The serviceability domains used in parametrical synthesis can be constructed, on the basis of mathematical model of investigated system, system of geometrical restrictions for the output values and internal parameters. In some instances the system of linear inequalities which generally speaking are nonlinear is applied for their description, and presence of the aprioristic information on convexity allows applying methods of convex optimization. In many situations there is effective approximation methods by some geometrical figures (inscribed or described hyperparallelepiped with the facets parallel to co-ordinate planes (Abramov et al. 2004, convex polyhedrons and ellipsoids (Digo G. & Digo N. 2006, 2008)). Besides, at performance of certain requirements as respects objective function, conditions of serviceability and restrictions on internal parameters, methods of non-uniform coverings, such as coverings by the n-dimensional parallelepipeds (cubes) inscribed in spheres (Zhigljavsky & Zhilinskas 1991), and the coverings n-dimensional parallelepipeds received by adaptive diagonal partition (Jones et al. 1993, Sergeev & Kvasov 2006, 2008) can be used.

In the presence of geometrical restrictions the approach based on adaptive partition of a feasible parallelepiped on a vector of internal parameters or on a state vector of investigated system is of interest for approximation of serviceability domains and attainability domains. In the conditions of uncertainty it allows to trace unknown borders of domains with bigger accuracy.

In the paper on the basis of the inductive approach (Ivakhnenko & Madala 1994) ways of carrying out of adaptive partition and a choice of points in derivable cells for an estimation of their quality are investigated.

2 THE BASIC CONCEPTS, DEFINITIONS AND EXPLANATORIES

Set of the problems connected with definition of requirements to parameters of object, nominal values of parameters and their permissible variations concerns problems of parametrical synthesis. Optimum parametrical synthesis (parametrical optimization) allows finding values of parameters of elements which are the best from the point of view of satisfaction to technical project requirements for invariable structure of projected object. Taking into account random parametrical perturbations it consists in choice nominal values $x_{ном} = (x_{1\,ном},..., x_{n\,ном})$ of internal parameters of investigated system or device, providing a maximum of probability of its non-failure operation during the set time interval:

$$x_{ном} = \arg\max P\{\mathbf{X}(x_{ном}, t) \in D_{\mathbf{x}}, \forall t \in [0, T]\}, \tag{1}$$

where $\mathbf{X}(x_{ном}, t)$ - random process of change of parameters; $D_{\mathbf{x}}$ **-** serviceability domain; T - the set of system working or the device.

Let us give the serviceability conditions () on output parameters $\mathbf{y} = (y_1,..., y_m)$

$$a_j \le y_j(\mathbf{x}) \le b_j, \quad j = 1,..., m, \tag{2}$$

and

$$y_j = F_j(x_1,..., x_n), \tag{3}$$

$F_j(\cdot)$ is known operator which depends on topology of investigated device. Dependence (3) is usually set not in obvious, and in the algorithmic form, in particular, through numerical decisions of systems of the equations (differential or algebraic), describing functioning of investigated system (Abramov 1992, Abramov at el. 2004, Katueva & Nazarov 2005).

Let, besides, direct restrictions

$$x_{i\min} \le x_i \le x_{i\max}, \quad x_i \ge 0, i = \overline{1, n}, \tag{4}$$

on the internal (controlled) parameters generating feasible domain in orthogonal system of co-ordinates and having the form of an n-dimensional feasible parallelepiped B_d:

$$B_d = \{\mathbf{x} \in R^n \mid x_{i\min} \le x_i \le x_{i\max}, i = 1,..., n\} \tag{5}$$

are known. These restrictions give expression of a physical or technological realizability conditions.

The set of points $\mathbf{x} \in B_d$ in which all given conditions of serviceability (2) are satisfied, is called serviceability domain $D_{\mathbf{x}}$ in space of controlled parameters $\mathbf{x} = (x_1,..., x_n)$, i.e.

$$D_{\mathbf{x}} = \{\mathbf{x} \in D \mid a_j \le y_j(\mathbf{x}) \le b_j, \quad j = 1,...,m\}. \tag{6}$$

This domain, as a rule, is unknown and there is no information on its form and orientation in space of internal parameters.

In specified conditions the problem of construction of serviceability domain in a general view has no analytical decision, and the various numerical methods, reducing space of search and reducing labour input of calculations by them multisequencing, are applied to its approximation (Abramov et al. 2004).

Let us suppose that changing of a state of some controllable dynamic system which is subject to action of uncertain factors is described by the equation

$$\dot{\mathbf{z}}(t) = f(t, \mathbf{z}, \mathbf{u}), \quad \mathbf{z} \in R^n, \ \mathbf{u} \in R^m, \tag{7}$$

on span of time $[t_0, T]$, t_0 - the initial moment of time. In formula (7) $\mathbf{z} = (z_1,...,z_n)$ - n-dimensional vector of a system state with terminal restrictions

$$z_{i\min} \le z_i \le z_{i\max}, \quad z_i \ge 0, i = \overline{1,n}, \tag{8}$$

generating feasible domain in orthogonal system of co-ordinates the feasible domain D:

$$D = \{\mathbf{z} \in R^n \mid z_{i\min} \le z_i \le z_{i\max}, \quad z_i \ge 0, \quad i = \overline{1,n}\}, \tag{9}$$

$\mathbf{u} = (u_1,...,u_m)$ - m-dimensional vector of control (or uncontrollable disturbances).

Let, besides, geometrical restrictions on a kind vector are given as

$$\mathbf{u} \in U(t, \mathbf{z}). \tag{10}$$

As the attainability domain $D_{\mathbf{z}}$ of system (7) for the given restrictions (8), (10) and $t \ge t_0$ is called (See Kostousova 1998, Bobyleva 2002) set of points $\mathbf{z} \in D$, for each of which exist an initial condition $\mathbf{z}_0 \in D$ and the control $\mathbf{u}(\cdot)$ satisfying (10) which generate decision $\mathbf{z}(\cdot)$ for system (7) such that $\mathbf{z}(t) = \mathbf{z}$, i.e.

$$D_{\mathbf{z}} = \{\mathbf{z} \in D \mid \mathbf{u} \in U(t, \mathbf{z}), t \in [t_0, T]\}. \tag{11}$$

Approximation of attainability domains of nonlinear dynamic systems at restrictions (10) is connected with the same computing difficulties, as well as at numerical construction of serviceability domains.

From the analysis of definitions and concepts which are described above follows that there is an interrelation between problems of a finding of sets of admissible variations of parameters and sets of attainability of controllable systems. Restrictions (4) and (8) generate accordingly n-dimensional the feasible parallelepiped B_d from (5) and n-dimensional the attainability parallelepiped D from (9) in orthogonal system of co-ordinates. Each of them contains the required domain (the serviceability domain $D_{\mathbf{x}} \subset B_d$ and the attainability domain $D_{\mathbf{z}} \subset D$**),** therefore for approximation of these domains can be used one and the same algorithm. In this connection further we will operate only with a parallelepiped B_d and domain $D_{\mathbf{x}}$.

3 THE FORMULATION OF A PROBLEM AND ITS ANALYSIS

Let's assume that n-dimensional parallelepiped B_d is given by expression (5) and contains domain $D_{\mathbf{x}}$ from the formula (6), but this parallelepiped is not described for this domain $D_{\mathbf{x}}$. The problem is stated to describe B_d on the basis of algorithm of adaptive partition (Jones et al. 1993, Sergeev & Kvass 2006) for application at approximation of domain $D_{\mathbf{x}}$ for the purpose of reduction of space of search and time of calculations.

It is obvious that the parallelepiped B_0 having sides, parallel to corresponding sides B_d and touching on borders of domain $D_{\mathbf{x}}$**,** will be described around $D_{\mathbf{x}}$. Its construction as the first step at an approximation stage, allows few to reduce search space. One of possible algorithms of construction B_o and the matrix description $D_\mathbf{x}$ are resulted in Katueva & Nazarov (2005), Digo G. & Digo N. (2008). According to it such matrix description of domain $D_\mathbf{x}$ turns out imposing of an n-dimensional uniform grid on the described parallelepiped B_o by means of quantization of domain of values of each i-th parameter. As a result described parallelepiped B_o is represented in the form of set of non-crossing parallelepipeds (cells)

$$B_o = \bigcup_{k_1=1}^{l_1} \bigcup_{k_2=1}^{l_2} \dots \bigcup_{k_n=1}^{l_n} B_{k_1,k_2,\dots k_n}\ , \tag{12}$$

where l_i - quantity of partition on a co-ordinate axis of i-th parameter. The information on the description B_o (the information on each cell of representation (12)) for the subsequent use is written down in an array $A[K]$ in the form of co-ordinates of its borders and quantity of partitions l_i on each i-th co-ordinate axis. It allows calculating co-ordinates of each cell $B_{k_1,k_2,\dots k_n}$ from (12).

Other approach to use of the uniform grid, connected with partition of domain B_d into n-dimensional parallelepipeds on the basis of adaptive partition (See Jones et al. 1993) is of interest. According to it B_d divides on n-dimensional parallelepipeds, and in each of them some point is selected. Two moments thus are important: it is necessary to define that a way to produce splitting and on what points to estimate quality received n-dimensional parallelepipeds.

For application of the description of the parallelepiped B_d received by means of this approach, at area approximation $D_\mathbf{x}$ compact and accessible storage of the information on its demanded characteristics should be provided. Borders co-ordinates and partitions quantity on each co-ordinate axis which provide calculation of co-ordinates of borders of each cell, co-ordinates of points in cells as quality indicators of these cells (the cell is considered good if the point chosen in it belongs $D_\mathbf{x}$, and bad otherwise) concern such characteristics.

4 ADAPTIVE PARTITION OF FEASIBLE PARALLELEPIPED

To approximate attainability domain $D_\mathbf{x}$ adaptive partition (Jones et al. 1993, Sergeev & Kvasov 2006, 2008) of feasible domain B_d on n-dimensional parallelepipeds (cells) is made by means of a uniform mesh. In each cell the central point gets out. For acceleration of process of calculations new parallelepipeds are formed by division already available on 3 parts on various measurements. It allows spending calculations only in the central points of the left and right thirds, considering earlier received results. For simplification of computing process it is supposed that everyone a component x_i of vector $\mathbf{x}$ in a feasible parallelepiped B_d from (4) has the bottom border 0 and the top border 1, i.e. B_d always is a n-dimensional init cube (it without generality loss always can achieve by means of linear transformation of technological restrictions (3)). Then partition begins with a unique n-dimensional parallelepiped, the init n-dimensional cube, which each side has the length equal 1.

During approximation of domain $D_\mathbf{x}$ such characteristics of a feasible parallelepiped B_d as coordinates of its borders and quantity l_i of partition on each i-th co-ordinate axis, which provide calculation of borders coordinates of each cell, coordinates of the centers of cells as indicators of these cells quality (the cell is considered good if its centre belongs domain $D_\mathbf{x}$, and bad otherwise) are required. With their help mesh representation B_d is formed. The information about this representation is stored in a file $A[K]$ of the length $K=\prod_{i=1}^{n} l_i$. Its elements accept values 0 (a cell bad) or 1 (a cell good). Such way of storage is compact and easily accessible.

Since new parallelepipeds are formed by dividing existing ones into thirds on various dimensions, the only possible lengths for their sides are 3^{-k}, ($k=0,1,2,...$). Each received parallelepiped always is divided on the largest side. No its side of length $3^{-(k+1)}$ can be divided until all of those of length 3^{-k} have been divided. After carrying out *r* divisions in a parallelepiped B_d it will be received the $j=\mathrm{mod}\,(r,n)$ sides of length $3^{-(k+1)}$ and $(n-j)$ the sides of length 3^{-k} where $k=(r-j)/n$, and, hence, the distance from the center of a parallelepiped to the vertexes is given by the formula $d=[j3^{-2(k+1)}+(n-j)3^{-2k}]^{0.5}/2$. The described process of adaptive partition is ended, when for the given accura $\varepsilon>0$ inequality $3^{-(k+1)}<\varepsilon$ is executed.

As a result of adaptive partition the feasible parallelepiped B_d becomes covered by the n-dimensional uniform mesh that each cell has the own characteristic in terms of «good-bad» concerning a corresponding central point. The received mesh representation has matrix analogue in the form of the *n*-dimensional array $C[l_1,l_2,...,l_n]$ which dimension coincides with dimension of a vector of internal parameters or a state vector $\mathbf{x}$ of investigated system.

5 POSSIBILITIES OF REDUCTION OF SPACE OF SEARCH

The information on a parallelepiped B_d which is contained in a file $C[l_1,l_2,...,l_n]$, allows reducing search space at the domain description $D_{\mathbf{x}}$. One of ways of such reduction is transition from the feasible parallelepiped B_d to a parallelepiped B_0 described around domain $D_{\mathbf{x}}$ on the basis of the analysis of the information from a file $A[K]$ about quality of cells, received in the course of partition. For this purpose value of co-ordinate x_n is established equal 0 ($x_n=0$).Check is carried out, whether there are among cells of this layer good cells (the corresponding element of a file $A[K]$ is equal 1). If yes, transition to a layer of cells in which equality is carried out $x_n=1$, is made. Otherwise this layer is excluded from the further consideration, and transition to the analysis of a following layer is carried out. Process goes on until there will be at least one good cell in a checked layer.

The similar analysis is made and for the others (*n*-1) variables. As a result is, within a preset value ε, there is the *n*-dimensional parallelepiped B_0 described around domain $D_{\mathbf{x}}$:

$$B_0=\{\mathbf{x}\in R^n \mid 0\le c_i\le x_i\le d_i\le 1, i=1,...,n\}. \quad (13)$$

As well as and B_d, the parallelepiped B_0 has its matrix representation $\widehat{C}[\widehat{l}_1,\widehat{l}_2,...,\widehat{l}_n]$ and a corresponding array of quality cells $\widehat{A}[\widehat{K}]$ of length $\widehat{K}=\prod_{i=1}^{n}\widehat{l}_i$. This information provides more exact description $D_{\mathbf{x}}$ which, in turn, can be approximated geometrical figures (See Chernousko 1988, Abramov 1992, Digo G. and N. 2008).

6 CONCLUSION

The method of adaptive partition of a feasible parallelepiped is applied to serviceability domains or attainability domain on components of a vector of internal parameters or on a state vector of investigated system. He allows describing these domains a set of cells with the sizes of cells of pre-set accuracy, received on the basis of idea of a uniform net covering.

The chosen using strategy of the central points for the characteristic of the received cells leads to mesh representation of approximable domains in terms of «good-bad» concerning corresponding central points and provides preservation of coordinates of already received centers.

The found mesh representation of a feasible parallelepiped is compactly, and its storage in the form of an n-dimensional array is accessible to the subsequent use.

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