DEVELOPMENT AND RESEARCH ON MATHEMATICAL MODELS, MODES OF WIND POWER PLANTS' ELECTROMECHANICAL CONVERTERS

R. I. Mustafayev, L. H. Hasanova

•

Azerbaijan Scientific–Research and Designed–Prospecting
Institute of Energetics, Baku, Azerbaijan, AZ1012, Ave. H.Zardabi-94

e-mail: mustafayevri@mail.ru, gasanovalg@mail.ru

ABSTRACT

A mathematical models of the following electromechanical converters, used in wind power, are presented: the asynchronous generator with a fed through a frequency converter stator, the generator, which is double fed asynchronous machine with rotor winding, fed through a frequency converter, and a synchronous generator with permanent magnets, with stator, fed through the frequency converter. For all converters the optimal control laws of amplitude and frequency of voltage on generator's terminals are obtained for all operating modes of wind power plants.

1. INTRODUCTION

In recent years, the wind-power engineering is developing rapidly. Today the wind power plants of up to 5 MW unit capacity are operating simultaneously with electric power network. By 2010 the total wind power plants capacity in the world constituted 160 GW.

The low-speed synchronous generators, squirrel-cage asynchronous machines, double fed asynchronous machine are used as the electromechanical converters in the wind power plants (WPP).

To raise the WPPs' operating efficiency in a specific range of wind speed variation, it needs also to control accordingly a rotational frequency of wind motor (WM) with jointed generator [1,2,3]. With such control a wind power utilization factor reaches its maximum values for corresponding wind speeds, and so the electric power production increases.

The expressions of wind motor's power and torque are accordingly written as [1]:

$$P_{\mathrm{WM}} = \frac{1}{2}\rho\pi R^2 \cdot \mathrm{V}^3 \cdot C_p \tag{1}$$

$$M_{\mathrm{WM}} = \frac{1}{2}\rho\pi R^3 \cdot \mathrm{V}^2 \cdot \mu \tag{2}$$

where ρ – air mass density, R – radius of wind wheel, V – wind speed; C_p – wind power utilization factor, μ– relative wind motor's driving torque.

A connection between wind power utilization factor C_p and relative driving torque μ is carried out through the relation:

$$C_p = \mu \cdot Z \tag{3}$$

where $Z = \frac{\omega_{\mathrm{WM}} R}{\mathrm{V}}$ – number of modules or specific speed, ω_{WM} – wind motor's angular frequency of revolution.

If to determine μ from the expression (3) and substitute in the expression (2) with taking into account the dependence of specific speed from ω_{WM} and V, we will obtain:

$$M_{\mathrm{WM}} = \frac{P_{\mathrm{WM}}}{\omega_{\mathrm{WM}}} \tag{4}$$

When wind motor rotational frequency control in proportion to wind speed the number of modules Z must theoretically remain constant, i.e.

$$Z = Z_{\mathrm{opt}} = \frac{\omega_{\mathrm{WM}} \cdot R}{\mathrm{V}} = const \tag{5}$$

On the aerodynamic characteristic of WM accordingly Z_{opt}, C_p and μ should remain constant, but that can not be exactly achieved in a real wind power plants. For example, in a WPP of Gamesa G–52 type of 850 kW capacity, where double fed asynchronous machine is used as electromechanical converter, a WM rotational frequency varies from 30,8 rpm (0,513 rev/s) to 14,6 rpm (0,24 rev/s), i.e. in the range of wind speed change from 10,45 m/s to 4,96 m/s (the depth change is 10,45/4,96=2,1) wind motor's rotational frequency is controlled from 0,513 1/s to 0,24 1/s (control depth is 2,1). In a mentioned range the wind power utilization factor Cp varies for considered wind motor from $C_{p\,min} = 0{,}390$ to $C_{p\,max} = 0{,}452$. However, if to operate an average factor value in the range of WM rotational frequency control

$$\mathrm{C}_{p\,mid} = \frac{\sum_{1}^{n} C_{pi}}{n} \tag{6}$$

where C_{pi} – possible fixed values of the wind power utilization factor C_p in the control range, n – number of fixed values, then the error for power calculation does not exceed ±7–8%.

Let's demonstrate Table 1 this on the example of the same WPP. The values of wind speeds, the corresponding them values of C_p, power P and the calculated values of wind motor's rotational frequencies ω_{WM} in the range of rotational frequency control are presented in the Table 1 [4].

Table 1. Dependence of P, C_p and ω_{WM} on the wind speed V.

V, m/s	10,45	9,97	9,44	9,00	8,53	7,97	7,48	7,00	6,51	6,03	5,5	4,96
P, kW	587,9	535,9	479,2	421,2	362,0	277,4	232,0	186,5	149,3	117,4	86,59	61,6
C_p, r.u.	0,398	0,418	0,441	0,446	0,452	0,423	0,429	0,42	0,418	0,415	0,402	0,39
ω_{WM}, 1/s	0,513	0,489	0,463	0,442	0,419	0,391	0,367	0,343	0,319	0,296	0,27	0,243

The calculated average value of wind power utilization factor is:

$$C_{pmid} = 0{,}419$$

If to take now in the whole control range $C_p = C_{p\,mid} = const$, then for determining the powers corresponding to maximum values $C_{p\,max}$ (in presented example it is P_1=362,05 kW at $C_{p\,max} = 0{,}452$) and minimum values $C_{p\,min}$ (P_2=61,6 kW at $C_{p\,min}$=0,39) the error will constitute:

$$\Delta P_{max} = \frac{P_1 - P_1'}{P_1} = +6{,}5\% \quad \text{and} \quad \Delta P_{min} = \frac{P_2 - P_2'}{P_2} = -7{,}8\% \quad ,$$

where $P_1' = 338$ кW and P_2' = 66,46 кW are the power values corresponding to wind speeds at $C_{p\,max}$ and $C_{p\,min}$, but calculated for $C_p = C_{pmid} = 0{,}419$ by formula (1).

It is seen from the calculations, for the vast majority of WPPs an error, when the proposed approach, for power determination does not exceed a value of 7–8%. For example, for «Vestas V–90» WPP of 2 MW capacity $C_{p\,cp} = 0{,}441$ and an error is in the range of from 8% to –2%.

Thus, in the range of rotational frequency control the expression of WM power can be written in the form:

$$P_{\mathrm{WM}} = \frac{1}{2}\rho\pi R^2 \cdot C_{pmid} \cdot \mathrm{V}^3 = K_p \cdot \mathrm{V}^3 \qquad (7)$$

where $K_p = \frac{1}{2}\rho\pi R^2 \cdot C_{p\,mid}$ – power proportionality constant, depending on the WM design parameters and the air mass density, but not depending on wind speed.

Accordingly, the expression for WM torque takes the form:

$$M_{\mathrm{WM}} = K_p \frac{\mathrm{V}^3}{\omega_{\mathrm{WM}}} \qquad (8)$$

When control the specific speed Z remains constant within the whole control range $Z = Z_{opt} = const$ (for given WPP Z_{opt}=8,0), so we can write

$$\mathrm{V} = \frac{\omega_{\mathrm{W}M} \cdot R}{Z_{opt}} = \frac{R}{Z_{opt}} \cdot \omega_{\mathrm{W}M} \qquad (9)$$

Substituting the expression (9) into (8) we obtain:

$$M_{\mathrm{W}M} = K_p \frac{R^3}{Z_{opt}^3} \cdot \omega_{\mathrm{W}M}^2 = K_{м} \cdot \omega_{\mathrm{W}M}^2 \qquad (10)$$

where $K_{м} = \frac{K_P \cdot R^3}{Z_{opt}^3}$ – torque coefficient of proportionality.

Thus, the wind motor torque, when rotational frequency control in proportion to wind speed, can be considered with defined above error as depending on a square of rotational frequency.

In modern WPPs for smooth rotational frequency control of wind motor is usually used the frequency converters, operating jointly with electromechanical converter. If as electromechanical converters the super-low-speed synchronous generators with «Ringgenerator» permanent magnets (plants of «Enercon», «Vensys» firms), or squirrel-cage asynchronous generators (plants of «Siemens Wind Power» firm) are used, the frequency converters, made with fully controlled semiconductor elements (IGBT – transistors, or a fully controlled GTO – thyristors) with PWM – control are installed in the stator circuits of these machines. If as the electromechanical converters the double fed asynchronous machines are used, these frequency converters are installed in the rotor circuits of these machines (WPPs of «Vestas», «Gamesa» «GE Wind Energy» firms, etc.).

2. STUDY ON MATHEMATICAL MODELS OF OPERATION MODES OF WIND POWER PLANTS CONTAINING ASYNCHRONOUS GENERATORS

Presentation of the mathematical model (model of state) and research on this model the different operating modes of WPPs, containing asynchronous generators, frequency–controlled by both through a stator and through a rotor, will allow to take into account the specific character of this system operation.

The equations of the asynchronous machine in the cell – matrix form are presented in a view [5]:

$$\begin{bmatrix} p\psi_s \\ p\psi_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{\mathrm{s}1} & \boldsymbol{A}_{\mathrm{s}2} \\ \boldsymbol{B}_{\mathrm{r}1} & \boldsymbol{B}_{\mathrm{r}2} \end{bmatrix} \begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix} + \begin{bmatrix} \boldsymbol{U}_{\mathrm{s}} \\ \boldsymbol{U}_{r} \end{bmatrix} \qquad (11)$$

where the expressions for the submatrices are:

$$p\psi_s = \begin{bmatrix} p\psi_{s\alpha} \\ p\psi_{s\beta} \end{bmatrix};\ p\psi_r = \begin{bmatrix} p\psi_{r\alpha} \\ p\psi_{r\beta} \end{bmatrix};\ \psi_s = \begin{bmatrix} \psi_{s\alpha} \\ \psi_{s\beta} \end{bmatrix};\psi_r = \begin{bmatrix} \psi_{r\alpha} \\ \psi_{r\beta} \end{bmatrix};\ \boldsymbol{U}_{\mathrm{s}} = \begin{bmatrix} U_{s\alpha} \\ U_{s\beta} \end{bmatrix};\ \boldsymbol{U}_r = \begin{bmatrix} U_{r\alpha} \\ U_{r\beta} \end{bmatrix}.$$

As for the submatrices $\boldsymbol{A}_{s1}$, A_{s2}, $\boldsymbol{B}_{r1}$, $\boldsymbol{B}_{r2}$, they depend on the form of asynchronous machine equations. When writing the equation in fixed in space axes $\alpha, \beta, 0$, they are equal:

$$\left.\begin{aligned} \boldsymbol{A}_{s1} &= \begin{bmatrix} -r_s k_s & 0 \\ 0 & -r_s k_s \end{bmatrix}; & \boldsymbol{A}_{s2} &= \begin{bmatrix} -r_s k_m & 0 \\ 0 & -r_s k_m \end{bmatrix}; \\ \boldsymbol{B}_{r1} &= \begin{bmatrix} -r_r k_m & 0 \\ 0 & -r_r k_m \end{bmatrix}; & \boldsymbol{B}_{r2} &= \begin{bmatrix} -r_r k_r & -\omega_r \\ \omega_r & -r_r k_r \end{bmatrix} \end{aligned}\right\} \tag{12}$$

If the equations are written in the rotating with a rotor speed axes, (for convenience we denote these axes also by letters α, β, although in the literature they are marked as d, q), then the submatrices $\boldsymbol{A}_{s1}$, $\boldsymbol{B}_{r2}$ will look:

$$\boldsymbol{A}'_{s1} = \begin{bmatrix} -r_s k_s & -\omega_r \\ \omega_r & -r_s k_s \end{bmatrix}; \qquad \boldsymbol{B}'_{r2} = \begin{bmatrix} -r_r k_r & 0 \\ 0 & -r_r k_r \end{bmatrix} \tag{13}$$

In above-cited equations a system of choice of relative units, base values, denominations with taking into account that $p = d / d\tau, \tau = 314 \cdot t$ (time in radian) are generally accepted ones, the factors k_s, k_r, k_m are determined from the inverse matrix of machine parameters, i.e.:

$$\begin{bmatrix} k_s & 0 & k_m & 0 \\ 0 & k_s & 0 & k_m \\ k_m & 0 & k_r & 0 \\ 0 & k_m & 0 & k_r \end{bmatrix} = \begin{bmatrix} x_s & 0 & x_m & 0 \\ 0 & x_s & 0 & x_m \\ x_m & 0 & x_r & 0 \\ 0 & x_m & 0 & x_r \end{bmatrix}^{-1} \tag{14}$$

Following In expressions (12) – (14): r_s, r_r, x_s, x_r, x_m – are accordingly active resistance and inductance of stators (s) and rotors (r) circuits, as well a mutual induction impedance (we take the saturated value of this parameter) ω_r – angular frequency of revolution of asynchronous generator's rotor.

Thus, the expressions (10) – (14) with adding the equations of motion and torques constitute a mathematical model of WPP's asynchronous machine.

$$\left.\begin{aligned} T_j p\omega_r &= m_{\mathrm{EM}} - m_{\mathrm{WM}} \\ m_{\mathrm{EM}} &= k_m \left(\psi_{s\alpha} \cdot \psi_{r\beta} - \psi_{s\beta} \cdot \psi_{r\alpha}\right) \end{aligned}\right\} \tag{15}$$

In expression (15) $\mathrm{T_j}$ – inertia constant of the system (wind motor and generator) in [radian], m_{WM} – wind motor torque, determined by the expression (10), it is, of course, reduced to generator's shaft and represented in the chosen system of relative units. In this case, naturally ω_{WM} is also reduced to generator's shaft with taking into account the transition factor of gear box and also a number of generator's poles pairs.

If to study a frequency control mode of squirrel–cage asynchronous generator's stator, it is easier to use the equations of the asynchronous generator, written in the fixed in space axes. In this case it is necessary to operate with submatrices $\boldsymbol{A}_{\mathrm{s}1}$, $\boldsymbol{A}_{\mathrm{s}2}$, $\boldsymbol{B}_{r1}$, $\boldsymbol{B}_{r2}$. For squirrel–cage rotor $\boldsymbol{U}_r = \begin{bmatrix} U_{r\alpha} \\ U_{r\beta} \end{bmatrix} = 0$ and for frequency control $\boldsymbol{U}_s = \begin{bmatrix} U_{s\alpha} \\ U_{s\beta} \end{bmatrix} = \begin{bmatrix} k_{us}\cos(k_{fs}\tau) \\ -k_{us}\sin(k_{fs}\tau) \end{bmatrix}$, where k_{us} – the relative value of voltage amplitude as a fraction of the maximum one, bearing in mind that $k_{us\,\boldsymbol{max}} = 1$, k_{fs} – is a relative frequency as a fraction of the nominal one, bearing in mind, that $k_{fs\,н} = 1$.

If to study the frequency control mode of the rotor (double fed asynchronous machine), it is advisable to write the machine's equation in the rotating with rotor speed axes. Then it is necessary to operate submatrices $\boldsymbol{A}'_{s1}, \boldsymbol{B}_{r1}, \boldsymbol{A}_{s2}, \boldsymbol{B}'_{r2}$, and the submatrices of stator and rotor windings voltage will determine as:

$$\boldsymbol{U}_s = \begin{bmatrix} U_{s\alpha} \\ U_{s\beta} \end{bmatrix} = \begin{bmatrix} -U_s \cdot \sin\theta \\ U_s \cdot \cos\theta \end{bmatrix}; \ \boldsymbol{U}_r = \begin{bmatrix} U_{r\alpha} \\ U_{r\beta} \end{bmatrix} = \begin{bmatrix} -k_{ur} \cdot \sin(k_{fr} \cdot \tau) \\ k_{ur} \cdot \cos(k_{fr} \cdot \tau) \end{bmatrix} \quad (16),$$

where θ – is an angle between a vector of synchronously rotating stator voltage U_s and the rotor axis, determined by the relation $p\theta = 1 - \omega_r$; k_{ur} – is a relative amplitude value of rotor winding voltage; k_{fr} – is a relative frequency of rotor winding current.

On these mathematical models the imitation of quasi-stationary mode of «Siemens Wind Power» wind power plants, equipped with squirrel-cage asynchronous generators and controlled by frequency converters, feeding the stator windings of these generators, has been carried out.

The study results in a range of wind motor's rotational frequency control, varied with wind speed change, are presented in Table 2.

It should be noted, that in most part of modern WPPs (of «Gamesa», «Vestas», «Nordex», «GE Wind Energy» type) as electromechanical converters the double fed asynchronous machine (DFAM) is used, owing to its brilliant adjusting characteristics, which is confirmed by the results, calculated on a presented mathematical model. If there is a necessity to take into account at greater length a saturation, a current displacement in the slot, etc. in an asynchronous machine, it needs to refer to [6].

When studying a frequency control of the double fed asynchronous machine's rotor, a current frequency in the rotor winding must be equal to $f_r = f_s \cdot s$, and when the sign of slip during the transition from traction mode to generator one changes, the signs of the amplitude and frequency of the supplying rotor voltage must change, which is taken into account in the parameters of Table 3.

The values of DFAM operating parameters, when control with constant reactive power q output to power network, are shown in Table 3. In this case, the voltages amplitudes k_{ur} varies somewhat with respect to k_{fr}.

With synchronous value of rotational frequency $\omega_r = 1$ $k_{ur} = k_{fr} = 0$, when $k_{ur} = k_{fr}$ changes from 0 to $k_{ur} = k_{fr} = -0{,}22$, the machine operates in generator mode, and rotational frequency ω_r reaches its maximum value $\omega_r = 1{,}22$ (i.e. if the synchronous rotational speed was equal to $\omega_{rs} = 1500$ rpm, it became $\omega_{r\,max} = 1830$ rpm).

The signs before m_{EM} and P_{EM} depend on the value and sign of m_{WM} and show, that the asynchronous machine all time is working in generator mode, although the sign of slip can vary from –0,22 to 0,22. The signs before the reactive power value show, that at $+q$ machine consumes a reactive power from the network, and at $-q$ returns it to network.

Table 2. The study results in a range of wind motor's rotational frequency control, varied with wind speed change

№	V, m/s	m_{EM}, r.u.	ω_r, r.u.	$k_{us}=k_{fs}$, r.u.	P_{EM}, r.u.	q, r.u.	$\beta=\omega_r-k_{fs}$, r.u.
1	7	–0,295	0,679	0,67	–0,201	0,325	0,009
2	8,75	–0,468	0,855	0,84	–0,400	0,356	0,015
3	9,8	–0,588	0,959	0,94	–0,564	0,323	0,019
4	10,5	–0,667	1,021	1	–0,681	0,255	0,021
5	7,7	–0,361	0,751	0,74	–0,272	0,344	0,011
6	9,1	–0,502	0,886	0,87	–0,445	0,352	0,016
7	5,25	–0,163	0,505	0,50	–0,083	0,273	0,005
8	9,1	–0,502	0,886	0,87	–0,445	0,352	0,016
9	8,89	–0,476	0,862	0,847	–0,410	0,355	0,015
10	5,25	–0,163	0,505	0,50	–0,083	0,273	0,005
11	4,9	–0,144	0,475	0,47	–0,068	0,265	0,004
12	4,9	–0,144	0,475	0,47	–0,068	0,265	0,004
13	4,9	–0,144	0,475	0,47	–0,068	0,265	0,004
14	7	–0,295	0,679	0,67	–0,201	0,325	0,009
15	8,75	–0,468	0,855	0,84	–0,400	0,356	0,015
16	9,8	–0,588	0,959	0,94	–0,564	0,323	0,019
17	7	–0,295	0,679	0,67	–0,201	0,325	0,009
18	5,25	–0,163	0,505	0,50	–0,083	0,273	0,005
19	7	–0,295	0,679	0,67	–0,201	0,325	0,009
20	5,25	–0,163	0,505	0,50	–0,083	0,273	0,005
21	9,1	–0,502	0,886	0,87	–0,445	0,352	0,016
22	9,1	–0,502	0,886	0,87	–0,445	0,352	0,016
23	8,75	–0,468	0,855	0,84	–0,400	0,356	0,015
24	6,3	–0,236	0,607	0,60	–0,143	0,303	0,007

Table 3. Studying a frequency control of the double fed asynchronous machine's rotor the transition from traction mode to generator one changes

№	V, m/s	m_{EM}, r.u.	ω_r, r.u.	k_{ur}, r.u.	k_{fr}, r.u.	P_{EM}, r.u.	q, r.u.
1	6,10	–0,304	0,78	0,2515	0,22	–0,237	–0,387
2	6,25	–0,319	0,80	0,2301	0,20	–0,256	–0,387
3	6,42	–0,336	0,82	0,2087	0,18	–0,276	–0,387
4	6,64	–0,360	0,85	0,1766	0,15	–0,307	–0,387
5	6,89	–0,387	0,88	0,1446	0,12	–0,341	–0,387
6	9,49	–0,744	1,22	–0,2259	–0,22	–0,908	–0,387
7	9,39	–0,720	1,20	–0,2042	–0,20	–0,864	–0,387
8	9,23	–0,696	1,18	–0,1824	–0,18	–0,822	–0,387
9	9,00	–0,661	1,15	–0,1500	–0,15	–0,760	–0,387
10	8,61	–0,605	1,10	–0,0965	–0,10	–0,666	–0,387

The dynamic characteristics of WPP's asynchronous generator, when starting by underfrequency relay, before running at rotational frequency control mode, is shown in Figure 1. In this case, the setters define the rates of rise of amplitude and frequency of supplied to stator voltage.

Creating a motor's driving torque, wind facilitates the unit acceleration, which is carried out by program frequency and voltage ramps, after lock in synchronous speed the machine passes into generator mode with $m_{\text{EM}} = -0{,}144$ and $\omega_r = 0{,}475$ (which corresponds to $\text{V} = 4{,}9$ m/s) (Figure 1*a*). Ibidem, the programmed changes of amplitude and frequency of voltage, supplied to generator's stator are shown (Figure 1*b*).

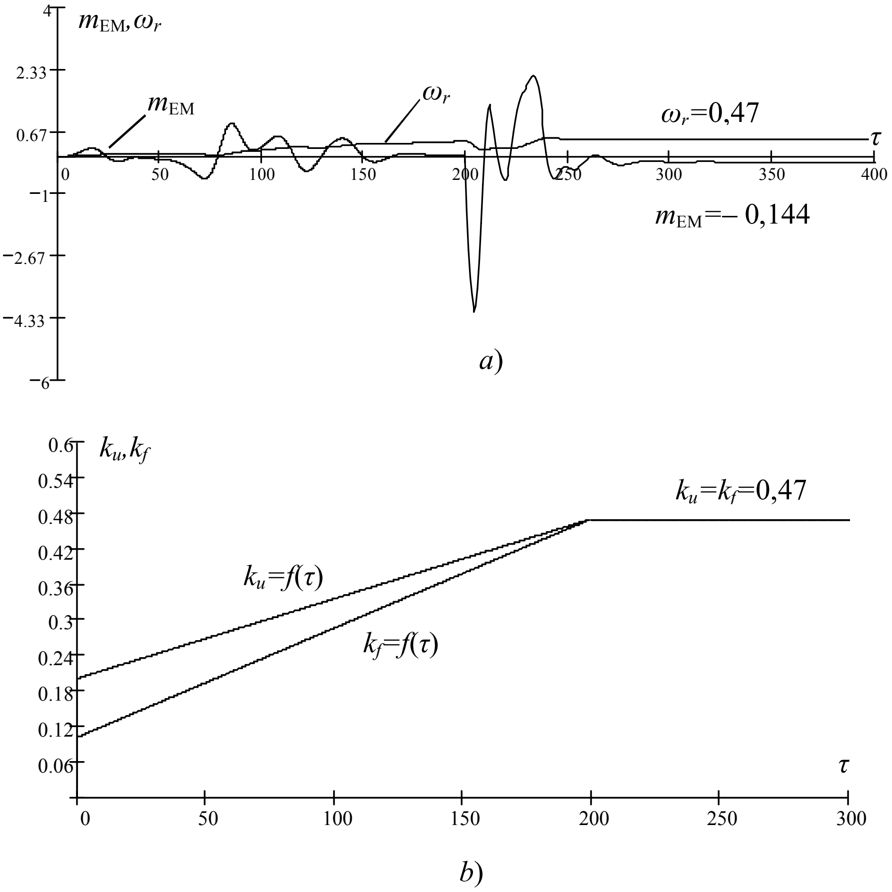


Figure 1. The curves of the transient process in the frequency start (*a*) and linear variation of the amplitude $k_{us} = k_{s0} + a \cdot \tau = 0{,}2 + 0{,}00135 \cdot \tau$ and frequency $k_{fs} = k_{f0} + b \cdot \tau = 0{,}1 + 0{,}00185 \cdot \tau$ of voltage (*b*).

3. DEVELOP A DIGITAL MODEL OF WPP'S SYNCHRONOUS GENERATOR BOTH WITH ELECTROMAGNETIC EXCITATION AND PERMANENT MAGNETS

Besides asynchronous generators the synchronous generators are used in WPPs, more often with permanent magnets (plants of Enercon, Vensys, GE Energy type) and rarely with electromagnetic excitation (Frisia type, etc.). For all this, they may be super-low-speed ones (so–called Ringgenerator) for gearless WPPs of Enercon and Vensys type and "normal" ones for geared WPPs of GE Energy and Frisia type [4]. And they both are equipped with frequency converters with IGBT-transistors, supplying the generators' stator circuit.

The purpose of this article is to develop a digital model of WPP's synchronous generator both with electromagnetic excitation and permanent magnets (most part) for investigation of transient and steady-state operating modes under frequency control.

The torque characteristics of wind motors, which are recommended to determine for 4 ranges of wind speeds change, are presented in [7]: from the beginning of WPP's operation (i.e. from a minimum wind speed at which a power output into the power network begins) to the beginning of

WPP's rotational frequency control; a range of WPP's rotational frequency control; a range from the end of rotational frequency control to design wind speed (i.e. the wind speed at which the rated power is supplied to network); and finally, the range from design wind speed to the maximum possible wind speed, at which the output of WPP's rated power to the network still continue.

Thus, the expressions for wind motor torque, reduced to generator's shaft, are determined by the above characteristics, and frequency-controlled range is a second one of wind speeds variations.

To study the static and dynamic characteristics of WPPs, equipped with synchronous generators with electromagnetic excitation, the well-known equations of Park can be used for synchronous machines. If it is necessary, the saturation accounting of main machine flow can be carried out according to the method, presented in [8]. In contrast to conventional form, these equations are written in so-called "mixed" form – that is, the currents are exactly marked out, which must be monitored, for all this they are accordingly determined by other currents and flux linkages. Besides that, the applied to excitation winding voltage, as well as in [8], is presented as a fraction of no-load voltage, and wind motor torque in the range of WPP's rotational frequency control in proportion to wind speed is determined by the expression:

$$m_{\text{WM}} = k_{м} \cdot \omega_r^2 \tag{17}$$

where $k_{м}$ – coefficient of proportionality of wind motor's torque.

To model a synchronous generator with permanent magnets, found a preferential use in WPPs, the Park equations needs to be transformed as follows. Assuming $p\psi_{df}$ is equal to zero $p\psi_{df} = 0$, from the equation of this system excitation we have:

$$i_{df} = \frac{U_{df}^*}{x_{ad}} \tag{18}$$

Substituting the value of this current to remaining equations, and removing the flux linkage equation ψ_{df} for its uselessness, we obtain the equations of synchronous machine with permanent magnets. It should be noted, that this proposed form of Park equations record for synchronous machine with electromagnetic excitation allows passing easily to the equations of machine with permanent magnets. But U_{df}^* in them should naturally be interpreted not as the excitation voltage, but as a value of magnetic energy of magnet per unit volume, or when small values of residual induction as a coercive force of the magnet. Thus, for example, it is necessary for $U_{df}^* = 1$ to take such value of magnetic energy of the magnet, which is capable to provide for no-load generator operation the value of the e.m.f. at the stator terminals equal to $\ell_{xx}=1$.

Comparing the WPP's synchronous generators with electromagnetic excitation and with permanent magnets excitation on their adjusting properties, it becomes obvious, that the generators with permanent magnets are inferior to the ones with electromagnetic excitation. For example, to stabilize the voltage of synchronous generators with permanent magnets when load changes at constant rotational frequency, the parametric and direct methods of influence the permanent magnets on magnetic flux are used [9]. But they are only suitable for small-capacity machines and micromachines.

Everything changes cardinally with a frequency converter in the WPP's synchronous generator stator circuit. In this case the output voltage can not only be stabilized, but adjusted to ensure the selected law of optimal control, i.e. current frequency control in the generator stator circuit provides a change of rotational frequency of WPP's shaft in proportion to wind speed, which allows maximizing the utilization factor of wind power in the whole control range. And amplitude control will allow providing the specified operating mode of either the generator itself or electric power network at the point of its connection.

For frequency controlled generator with electromagnetic excitation one more control channel is added – its excitation current.

To study the transient and steady-state modes of WPP synchronous generator's operation, when control both amplitude and frequency of stator voltage, it needs to adapt so the Park equations, which is known to be written in rotating with rotor speed d, q axes, that they should reflect the changes of amplitude and frequency of generator's stator voltage.

For this purpose the basic equations of WPP's synchronous machine is offered to leave recorded in d, q axes, and only the components of stator voltage U_{ds} and U_{qs}, which without frequency control are written in the form $U_{ds} = -U_s \cdot \sin\theta$ and $U_{qs} = U_s \cdot \cos\theta$, to present with frequency control in such form, that they should reflect the changes of amplitude and frequency of voltage at the terminals of WPP's synchronous generator.

For all this it needs to refer to shown in Figure 2 diagram. Here α_0, β_0 – are fixed in space axes of coordinates, α_s, β_s – synchronously rotating axes with angular frequency ω_s, which is corresponded to frequency of current at the output of frequency converter, d, q – coordinate axes, rotating with a rotor speed ω_r. Angle $\alpha = \omega_r \cdot \tau$ – is the angle between the axes α_s, β_s and α_0, β_0, the angle between d, q axes and fixed axes α_0, β_0 is $\alpha = \omega_r \cdot \tau$, and finally θ – is the angle between d, q axes and α_s, β_s, which is the interior angle of synchronous machine. From the diagram in Figure 2 we have:

$$\theta = \alpha + \alpha_s = \omega_r \cdot \tau + \omega_s \cdot \tau \tag{19}$$

where τ – synchronous time in radian, equal to $\tau = \omega_{баз} \cdot t = 314 \cdot t$, t – time in seconds.

If to dispose the vector of stator voltage in initial (source) mode at 45^0 angle to the axes α_s, β_s, its projections on these axes in the source mode will be the same and equal to $U_{s\alpha_0} = U_{s\beta_0} = 0{,}707$.

In accordance with the diagram in Figure 2, the projections of vectors $U_{s\alpha}$ and $U_{s\beta}$ on d, q axes in their turn will be written as:

$$\left.\begin{aligned} U_{d\alpha} &= U_{s\alpha} \cdot \cos\theta = U_{s\alpha} \cdot \cos(\alpha + \omega_s \cdot \tau) \\ U_{q\alpha} &= U_{s\alpha} \cdot \sin\theta = U_{s\alpha} \cdot \sin(\alpha + \omega_s \cdot \tau) \\ U_{d\beta} &= U_{s\beta} \cdot \sin\theta = U_{s\beta} \cdot \sin(\alpha + \omega_s \cdot \tau) \\ U_{q\beta} &= U_{s\beta} \cdot \cos\theta = U_{s\beta} \cdot \cos(\alpha + \omega_s \cdot \tau) \end{aligned}\right\} \tag{20}$$

General projections of these components on d, q axes pursuant to Figure 2 are determined as

$$\left.\begin{aligned} U_{ds} &= U_{d\alpha} - U_{d\beta} \\ U_{qs} &= U_{q\alpha} + U_{q\beta} \end{aligned}\right\} \tag{21}$$

Substituting the voltage components from the expression (20) to expression (21) and bearing in mind, that $U_{s\alpha} = U_{s\beta} = U_{s\alpha 0} \cdot k_u = U_{s\beta 0} \cdot k_u = 0{,}707 \cdot k_u$ and $\omega_s = k_f \cdot \omega_{s0}$, where $k_u = \dfrac{U_s}{U_{s0}}$ and $k_f = \dfrac{\omega_s}{\omega_{s0}}$ in one's turn $U_{s0} = \omega_{s0} = 1$ [in relative unit], we finally get::

$$\begin{aligned} U_{ds} &= 0{,}707 \cdot k_u \left[\cos(\alpha + k_f \cdot \tau) - \sin(\alpha + k_f \cdot \tau)\right] \\ U_{qs} &= 0{,}707 \cdot k_u \left[\sin(\alpha + k_f \cdot \tau) + \cos(\alpha + k_f \cdot \tau)\right] \end{aligned} \tag{22}$$

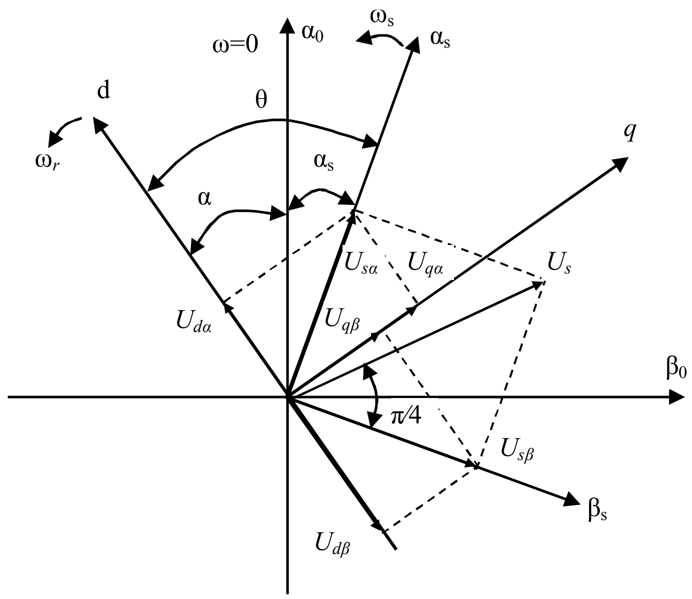


Figure 2. Diagram of the location rotating axes of synchronous machines.

And finally, if it is necessary, these equations by the trivial conversions can be written in a more convenient for using form:

$$\left.\begin{aligned} U_{ds} &= AC - BD \\ U_{qs} &= AD + BC \end{aligned}\right\} \tag{23}$$

where $A = 0{,}707 \cdot k_u \cdot \cos(k_f \cdot \tau)$ $\quad C = \cos\alpha - \sin\alpha$

$B = 0{,}707 \cdot k_u \cdot \sin(k_f \cdot \tau)$ $\quad D = \cos\alpha + \sin\alpha$

During the study of parallel operation of several WPPs, consisting the wind power farm (WPF), the expressions for the stator currents i_{ds} and i_{qs}, written in d, q axes, can be represented in the fixed coordinate system. For all this it's enough to determine the currents components in accordance with the diagram by the expressions:

$$\left.\begin{aligned} i_{\alpha 0} &= i_{ds} \cdot \cos\alpha + i_{qs} \cdot \sin\alpha \\ i_{\beta 0} &= -i_{ds} \cdot \sin\alpha + i_{qs} \cdot \cos\alpha \end{aligned}\right\} \tag{24}$$

It should be borne in mind, that these currents in steady-state mode have a frequency at the output of frequency converter, i.e. they flow between the windings of stator and frequency converter output, if it is necessary to determine the currents before the converter, then $\alpha = \omega_r \cdot \tau$ needs to be replaced by $\omega_{s0} \cdot \tau$, i.e. as $\omega_{s0} = 1$, that simply replace by τ:

$$\begin{aligned} i_{\alpha 0} &= i_{ds} \cdot cos\tau + i_{qs} \cdot sin\tau \\ i_{\beta 0} &= -i_{ds} \cdot sin\tau + i_{qs} \cdot cos\tau \end{aligned} \tag{25}$$

The above conversions allow, without changing the structure of Park equations, which are written in the rotating with rotor speed axes, in voltage components U_{ds} and U_{qs} to take into account the change of both amplitude and voltage frequency of WPP's synchronous generator stator circuit, obtained after output from the frequency converter.

Thus, the system of Park equations together with the equations (22) or (23) constitute the digital mathematical model of WPP's synchronous generator with electromagnetic excitation under frequency control.

A final version of digital model of WPP's synchronous generator with permanent magnets under frequency control will appear in the following form:

$$\left.\begin{aligned}
&p\Psi_{ds} = 0{,}707 \cdot k_u \left[\cos(k_f \cdot \tau) \cdot (\cos\alpha - \sin\alpha) - \sin(k_f \cdot \tau) \cdot (\cos\alpha + \sin\alpha)\right] - \omega_r \cdot \psi_{qs} - r_s \cdot i_{ds} \\
&p\Psi_{qs} = 0{,}707 \cdot k_u \left[\cos(k_f \cdot \tau) \cdot (\cos\alpha + \sin\alpha) + \sin(k_f \cdot \tau) \cdot (\cos\alpha - \sin\alpha)\right] + \omega_r \cdot \psi_{ds} - r_s \cdot i_{qs} \\
&p\psi_{dr} = -\frac{r_{dr}}{x_{dr}} \cdot \psi_{dr} + \frac{r_{dr} \cdot x_{ad}}{x_{dr}} \cdot i_{ds} + \frac{r_{dr}}{x_{dr}} \cdot U_{df}^{*} \\
&p\psi_{qr} = -\frac{r_{qr}}{x_{qr}} \cdot \psi_{qr} + \frac{r_{qr} \cdot x_{aq}}{x_{qr}} \cdot i_{qs} \\
&p\omega_r = \frac{1}{T_j} \cdot m_{вд} - \frac{1}{T_j} \cdot m_{EM} \\
&p\alpha = \omega_r \\
&m_{EM} = \psi_{ds} \cdot i_{qs} - \psi_{qs} \cdot i_{ds} \\
&i_{ds} = \frac{x_{dr}}{\Delta d} \cdot \psi_{ds} - \frac{x_{dr} - x_{ad}}{\Delta d} \cdot U_{df}^{*} - \frac{x_{ad}}{\Delta d} \cdot \psi_{dr} \\
&i_{qs} = \frac{x_{qr}}{\Delta q} \cdot \psi_{qs} - \frac{x_{aq}}{\Delta q} \cdot \psi_{qr}
\end{aligned}\right\} \quad (26)$$

Values $\Delta d = x_{ds} \cdot x_{dr} - x_{ad}^2$ and $\Delta q = x_{qs} \cdot x_{qr} - x_{aq}^2$.

Besides these basic equations, the equations of active and reactive power of WPP's synchronous generator can be added for complete analysis to these equations:

$$\left.\begin{aligned}
p &= U_{ds} \cdot i_{ds} + U_{qs} \cdot i_{qs} \\
q &= U_{qs} \cdot i_{ds} - U_{ds} \cdot i_{qs}
\end{aligned}\right\} \quad (27)$$

The study of model synchronous generator with imitation of operating mode of Vensys 77 type WPP have been carried out by equations (26). The specified WPP is equipped with low-speed synchronous generator with permanent magnets, controlled by a frequency converter with IGBT – transistors of 1500 kW capacity, design wind speed is 13 m/s, the initial wind speed is 3 m/s, and maximum allowable operating wind speed is 22 m/s, a range of WPP's rotational frequency control is 9–17,3 rpm, i.e., almost 1:2 [4]. The studies have been carried out at $U_{df}^{*} = 1{,}2 = const$.

To solve (26) system the Matcad program has been used.

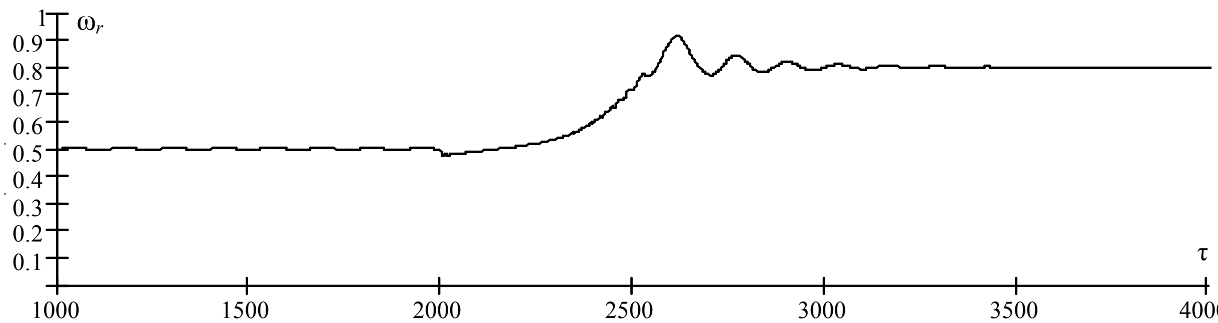
The parameters of steady–state mode for control $k_u = k_f$ are given in table 4; when control with constant reactive power output q=–0,18=*const* (negative sign corresponds to generator mode) and when control with constant power factor $\cos\varphi \approx 1$. Analysis of the results shows, that torque is determined by rotational frequency, and consequently by network frequency, and for all cases when frequency changes k_f from 0,5 to 1, it varies from –0,15 to –0,6. This torque is corresponded to active power P, which also varies from –0,074 to –0,6, and the active current i_{qs}, which range of variation is from –0,136 to 0,543, these values are identical for all modes. Control with a constancy of reactive power output q=*const* and a constancy of $\cos\varphi \approx 1$ in a synchronous machine with permanent magnets can be achieved by separate control of voltage amplitude at the output of frequency converter in accordance with the values k_u, presented in clauses (a) and (b) of table 4. It should be noted, that within the frequency change range from 0,6 to 0,9 the control $k_u = k_f$ practically coincides with the control with a constancy of q=*const*.

Table 4. The parameters of steady–state mode for control $k_u = k_f$, when control with constant reactive power output *q=const* and when control with constant power factor $\cos\varphi \approx 1$.

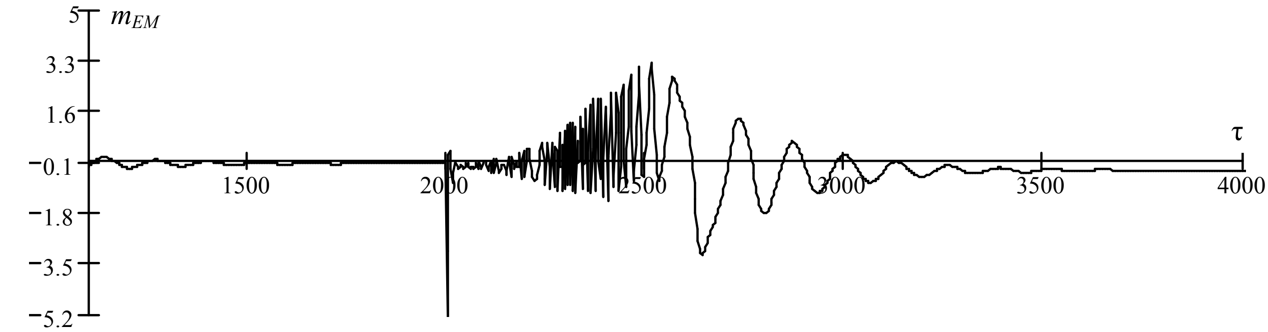
a)	Control with $k_u = k_f$					
$k_u=k_f$	0,5	0,6	0,7	0,8	0,9	1,0
m	–0,150	–0,215	–0,296	–0,389	–0,486	–0,598
P	–0,074	–0,128	–0,205	–0,304	–0,435	–0,596
q	–0,137	–0,158	–0,175	–0,180	–0,177	–0,155
i_{ds}	–0,200	–0,200	–0,210	–0,220	–0,234	–0,257
i_{qs}	–0,136	–0,197	–0,265	–0,346	–0,441	–0,543
b)	Control with the constant reactive power output $q = const$					
k_f	0,5	0,6	0,7	0,8	0,9	1,0
k_u	0,484	0,592	0,698	0,799	0,899	0,991
q	–0,179	–0,180	–0,180	–0,181	–0,180	–0,179
i_{ds}	–0,270	–0,232	–0,216	–0,217	–0,237	–0,257
c)	Control with the constant power factor $\cos\varphi \approx 1$					
k_f	0,5	0,6	0,7	0,8	0,9	1,0
k_u	0,545	0,650	0,758	0,860	0,960	1,055
q	–0,0017	–0,0017	–0,00173	–0,0017	–0,0018	–0,0017
i_{ds}	–0,01	–0,0187	–0,033	–0,055	–0,089	–0,137

Among the dynamic characteristics the mode of sharp increase of driving torque on generator's shaft (wind gust) is of interest, with working-off the mode of output to a new frequency of frequency converter.

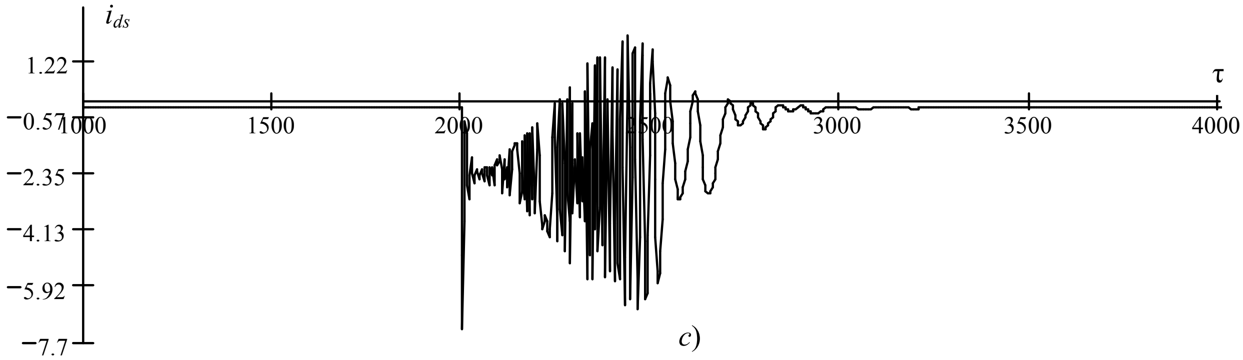
The fluktogramms of WPP generator's regime parameters change ω_r, m_{EM}, i_{ds}, i_{qs}, q, P_{EM} are accordingly presented in Figure 3 (*a, b, c, d, e, f*) when operating in steady-state mode, corresponding to the rotation frequency ω_r=0,5 (k_u=k_f=0,5) with wind gust (at 2000 radian), increasing the driving torque approximately 2,5 times (from $m_{EM} = -0{,}15$ to $m_{EM} = -0{,}385$), and this new torque value should correspond to WPP's shaft rotational frequency, which is equal to ω_r=0,8 (k_u=k_f=0,8). When assignment the new values of k_u and k_f, a control path for simplicity of analysis is assumed to be an aperiodic link with the time constant T_j=100 rad (~ 0,3 s). A system, as it is seen from fluktogramm, is stable one, although the amplitude of oscillations of some regime parameters is rather significant one (q_{max}=6, $i_{ds\ max}$=6,5), but a change of their average value during the transient is not greater than 3-fold value.



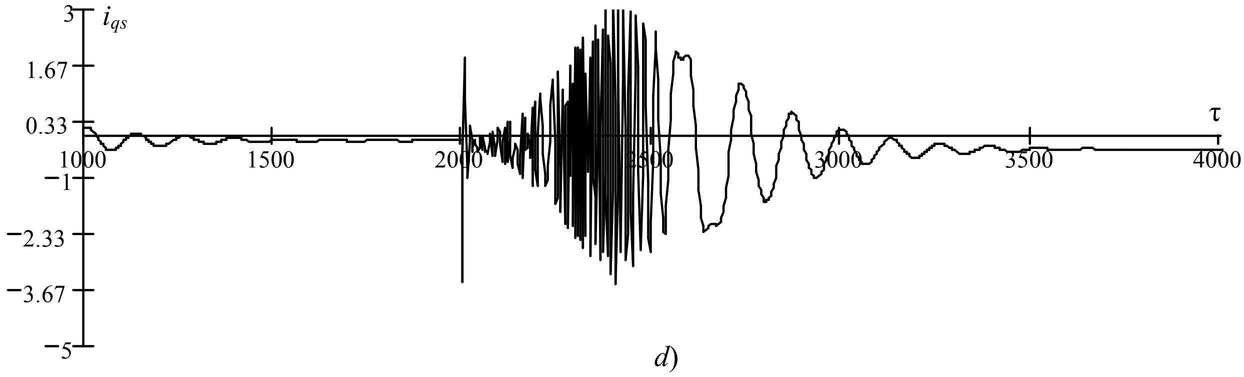
a)


m_{EM}
5
3.3
1.6
−0.1
−1.8
−3.5
−5.2
1500
2000
2500
3000
3500
4000
τ

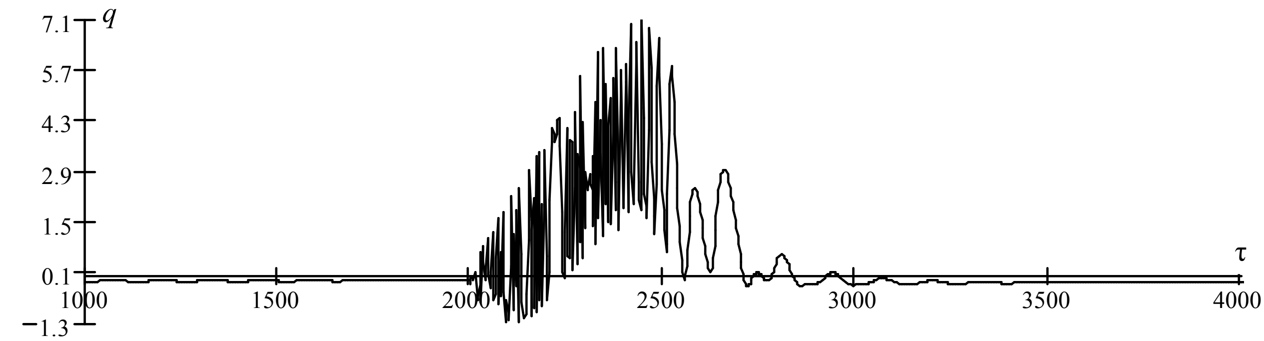
b)


i_{ds}
1.22
−0.57
−2.35
−4.13
−5.92
−7.7
1000
1500
2000
2500
3000
3500
4000
τ

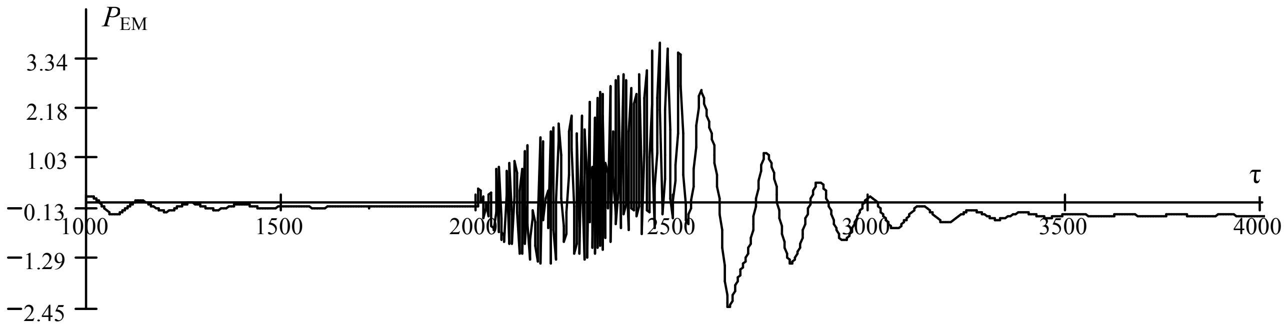
c)


i_{qs}
3
1.67
0.33
−1
−2.33
−3.67
−5
1000
1500
2000
2500
3000
3500
4000
τ

d)


q
7.1
5.7
4.3
2.9
1.5
0.1
−1.3
1000
1500
2000
2500
3000
3500
4000
τ

e)



f)

Figure 3. The fluktogramms of WPP generator's regime parameters change ω_r (*a*), m_{EM} (*b*), i_{ds} (*c*), i_{qs} (*d*), q(*e*), P_{EM} (*f*) are accordingly when operating in steady-state mode.

4. CONCLUSION

1. It is determined, that with an acceptable in engineering calculations error (no more than 8%) in the area of speed control directly proportional to speed of wind - WPP's energy resource, the wind motor torque can be considered as proportional to the square of its rotational frequency.
2. The matrix form of equations of WPP's asynchronous machine condition under frequency control both of stator and rotor, allowing obtaining the uniform study results, have been presented.
3. The operating modes when the frequency control of WPP's asynchronous generator stator have been studied for voltage control both with constant generator's relative slip and with constant consumed reactive power. It was revealed, that the first mode was consistent with the known law of frequency control by academician M.P.Kostenko, and the second mode allowed to compensate the consumed by WPP reactive power with the help of uncontrolled static capacitors, installed in the place of WPP's connection to electric power network.
4. It was demonstrated, the reactive power output to network can be controlled in quasi-stationary modes with frequency control of rotor of double fed asynchronous machines.
5. The dynamic regimes of WPP with asynchronous generator when frequency dispersal of plant have been studied; it was revealed, the starting torques and currents were minimized in this case.
6. Frequency-controlled mathematical model of WPP's synchronous generator both with the permanent magnets and an electromagnetic excitation have been developed. It was shown, in WPP's synchronous generator with permanent magnets the operating modes with constant reactive power output $q = const$ and constant $\cos\varphi \approx 1 = const$ could be provided for the separate control of amplitude and frequency of generator stator voltage.
7. The effectiveness of start by underfrequency relay of WPP's synchronous generator with permanent magnets, providing the smooth acceleration, the small values of operational parameters during start-up and "soft" locking in synchronism has been demonstrated.

REFERENCES

1. Andrianov V., Bistriskiy D., Bashkevich K., Sektorov V. 1960. Wind electric plant. Edited by prof. Andrianov V. StEnerPubl., Moscow.
2. Wind power engineering. Edited by D-de Renzo. Tran. from Eng. AtomEnerPubl., M. 1982.

3. Mustafaev R., Kurdukyov Y., Listenqarten B. Systems analysis of optimal control power output wind power systems. Journal of Electricity, 1984, №7. Moscow.
4. Wind energy 2006. Catalog. 2006 BWE – Service CmbH.
5. Mustafayev R. On the universal structure of a mathematical model of electric machines. SSSR of news IHEs. Journal of Electromechanics 1989. №4.
6. Kopylov I. Mathematical modeling of electrical machines. M. «Higher school», 1987.
7. Mustafaev R., Hasanova L. Torque-power characteristic of Up-to-Date wind power plants. Journal of Electrotechnics, 2009, №7, p. 53–58. Moscow.
8. Application of analog computers in power systems. Edited by Sokolov N. Publ.Ener., M. 1964, 408 p.
9. Special electric machines. Edited by Bertinov A. Ener.Publ. M.1982. p.552.