Good memory of Academician
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DUBBED THE REPLACEMENT SYSTEM WITH CONTROL

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1. Model B.V.Gnedenko. The system consists of two elements, one of which is in working condition, and another - in the unloaded reserve. After the failure of the main item, it goes to recovery, back-up substitutes for the failed primary element. If refuses to work element, and the backup does not have time to recover, it is a system failure. If a back-up time to recover during the time of the main element, it becomes redundant and the process then repeats. A mathematical model of system uptime is represented by two equations [1]:

$$\begin{aligned} P_2(t) &= P(t) + \int_0^t W(t-z)a(z)dz, \\ W(t) &= P(t) + \int_0^t G(z)W(t-z)a(z)dz. \end{aligned} \tag{1}$$

In (3) $P_2(t), P(t) -$ probability of failure of the system and a basic element, $a(t) -$ probability density of time to failure of the element, $W(t) -$ conditional probability of failure of the system, provided that the initial time a back-up involved in the work, and the principal immediately began to recover. Control of the state elements continuous and perfect.
Using Laplace transform, we write (1) as:

$$\begin{aligned} \overset{*}{P}_2(s) &= \overset{*}{P}(s) + \overset{*}{W}(s)\overset{*}{a}(s), \\ \overset{*}{W}(s) &= \overset{*}{P}(s) + \overset{*}{W}(s)\overset{*}{b}(s), \end{aligned} \tag{2}$$

Where $\overset{*}{b}(s) = \int_0^\infty e^{-sz}G(z)a(z)dz$. The system of equations (2) has the form:

$$\overset{*}{P}_2(s) = \frac{(1-\overset{*}{a}(s))(1-\overset{*}{b}(s)+\overset{*}{a}(s))}{s(1-\overset{*}{b}(s))}. \tag{3}$$

From (3) we find, for example, mean time to failure of the system:

$$T_2 = \frac{T(1-\overset{*}{b}(0)+\overset{*}{a}(0))}{(1-\overset{*}{b}(0))}. \tag{4}$$

In particular, the exponential distribution of time to failure $P(t) = e^{-\lambda t}$ and recovery time $G(t) = 1 - e^{-\mu t}$ we get:

$$T_2 = \frac{2\lambda + \mu}{\lambda^2}. \tag{5}$$

2. Accounting for non-ideality of control system elements. In [2] proposed to take into account no ideality of the control system to be restored by introducing the probability at this value is multiplied by "resource recovery", namely:

$$G_q(t) = 1 - e^{-q\int_0^t \mu(z)dz}, \quad R_q(t) = 1 - G_q(t),$$

where $R_q(t) = e^{-q\int_0^t \mu(z)dz}$ - probability of unrecovered elements of time, provided that his failure wasdetected with a probability q.

Value of the integral $\vartheta(t) = q\int_0^t \mu(z)dz$ by analogy with the "resource security" professor N.M. Sedyakin $r(t) = \int_0^t \lambda(z)dz$, where $\lambda(t)$ – failure intensity of an element is called a "resource recovery". Taking into account the probability of introducing it in the future will be denoted as the probability $G_q(t), R_q(t),$ and the value of the resource $-\vartheta_q(t)$.

In the above expressions (1) - (5) with a probability q must be directly related $G(t)$, $\overset{*}{b}(s)$, so finally we can write:

$$\overset{*}{P}_{2q}(s) = \frac{(1-\overset{*}{a}(s))(1-\overset{*}{b}_q(s)+\overset{*}{a}(s))}{s(1-\overset{*}{b}_q(s))}, \tag{6}$$

$$T_{2q} = \frac{T(1-\overset{*}{b}_q(0)+\overset{*}{a}(0))}{(1-\overset{*}{b}_q(0))}. \tag{7}$$

In the special case for exponential distributions, will have the formula:

$$T_{2q} = \frac{2\lambda + \mu \cdot q}{\lambda^2}. \tag{8}$$

Thus, the average uptime of the system duplicated in this case is directly proportional to the reliability of monitoring the state of the failed element.

Example 1. Let $\lambda = 0{,}01\ ч^{-1}$, $\mu = 0{,}1\ ч^{-1}$. Then $T_{2q} = 200\ ч$ at $q = 0$, $T_{2q} = 700\ ч$ at $q = 0{,}5$, $T_{2q} = 1200\ ч$ at $q = 1$.

Example 2. Let the law of distribution of time to failure is normal. Density distribution is $a(t) = \frac{1}{\sqrt{2\pi}3} e^{-\frac{(t-25)^2}{2\cdot 3^2}}$. Distribution law for the recovery time Weibull. The distribution function of recovery time is $G(t) = 1 - e^{-0{,}2\cdot t^2}$. Then $\overset{*}{b}_q(0) = \int_0^\infty (1 - e^{-q\cdot 0{,}2\cdot z^2}) \cdot \frac{1}{\sqrt{2\pi}3} e^{-\frac{(z-25)^2}{2\cdot 3^2}} dz$. So, $T = 25\ ч., \Theta = 6{,}267\ ч$. Figure 1 shows plots $\overset{*}{b}_q(q), T_{2q}(q),$ plotted for different values of probability q. They imply that an increase in the reliability of control system elements to refuse the value of conditional probability $\overset{*}{b}_q(q)$ increases. With the increase of this probability as the probability q, value of the mean time to failure of the duplicated $T_{2q}(q)$ increases, and increase is nonlinear. This demonstrates the importance of the value of reliability control in the duplicated system.

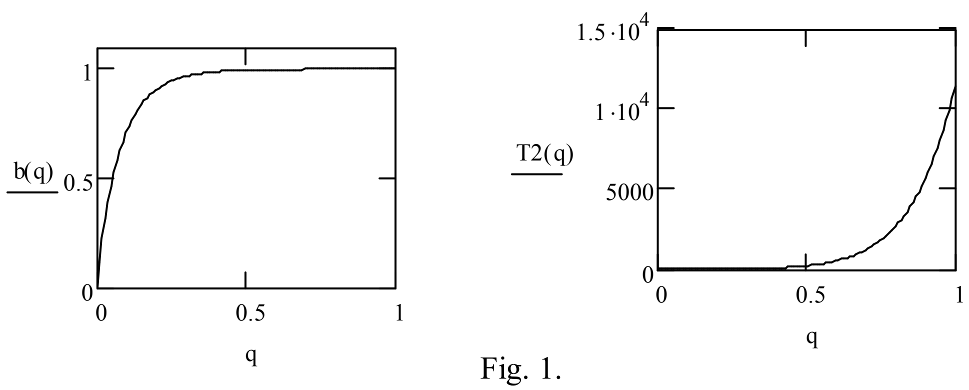

Fig. 1.

3. Willingness duplicated system with control. Obtain equations for the study of readiness duplicated system with arbitrary distributions directly, as was done to determine the probability (1), rather difficult.

Simply use the expression (6) and find a picture of him Laplace distribution density of time to failure duplicated system, using the formula:

$$\overset{*}{P}_{2q}(s) = \frac{1 - \overset{*}{a}_{2q}(s)}{s}, \tag{9}$$

When $\overset{*}{a}_{2q}(s) -$ image of the desired density. Performing the necessary transformations, we obtain:

$$\overset{*}{a}_{2q}(s) = \frac{\overset{*}{a}(s)(\overset{*}{a}(s) - \overset{*}{b}_q(s))}{1 - \overset{*}{b}_q(s)}. \tag{10}$$

Next, use the formula for the image of the function of readiness in the form:

$$\overset{*}{K}_{\Gamma q}(s) = \frac{1 - \overset{*}{a}_{2q}(s)}{s(1 - \overset{*}{a}_{2q}(s)\overset{*}{g}_{2q}(s))}, \tag{11}$$

in which $\overset{*}{g}_{2q}(s) -$ the image probability density recovery duplicated system after its failure.

Pay attention to the fact that this density can take different forms depending on the discipline system recovery, namely the recovery of both elements can be performed by one or two brigades. Consider this further in the analysis of readiness.

Expression(11) after substituting in it (10) reduces to:

$$\overset{*}{K}_{\Gamma q}(s) = \frac{1 - \overset{*}{b}_q(s) - \overset{*}{a}(s)(\overset{*}{a}(s) - \overset{*}{b}_q(s))}{s(1 - \overset{*}{b}_q(s) - \overset{*}{a}(s)(\overset{*}{a}(s) - \overset{*}{b}_q(s))\overset{*}{g}_{2q}(s))}. \tag{12}$$

Recall that $K_\Gamma = K_\Gamma(\infty) = \lim\limits_{s \to 0} s\overset{*}{K}_\Gamma(s)$. Performing the limit, we find the coefficient of readiness:

$$K_{\Gamma q} = \frac{T(2 - \overset{*}{b}_q(0))}{T(2 - \overset{*}{b}_q(0)) + \Theta_{2q}(1 - \overset{*}{b}_q(0))}, \tag{13}$$

$\Theta_{2q} = \int\limits_0^\infty g_{2q}(z)dz -$ average recovery time of one or two brigades.

Example 3. Assume that the distributions of time to failure and recovery of elements exponential. Failure rate and recovery elements are equal λ, μ. If System Restore is one team, then by (13) we obtain:

$$K_{\Gamma q} = \frac{q\mu(2\lambda + q\mu)}{q^2\mu^2 + 2\lambda q\mu + 2\lambda^2}. \tag{14}$$

If system restore is performed by two teams, then the coefficient of readiness will be:

$$K_{\Gamma q} = \frac{2q\mu(2\lambda + q\mu)}{2q^2\mu^2 + 4\lambda q\mu + 3\lambda^2}. \tag{15}$$

Correctness (14) and (15) can be checked by applying a system of differential equations.

Example 4. Determine the availability of the system if the probability density of time to failure of the element $a(t) = \frac{1}{\sqrt{2\pi}3} e^{-\frac{(t-25)^2}{2\cdot 3^2}}$, and the distribution function of his recovery time $G_q(t) = 1 - e^{-q\cdot 0,2\cdot t^2}$.

Then $\overset{*}{b}_q(0) = \int_0^\infty (1 - e^{-q\cdot 0,2\cdot z^2}) \cdot \frac{1}{\sqrt{2\pi}3} e^{-\frac{(z-25)^2}{2\cdot 3^2}} dz$.

Maintenance of two failed elements is one brigade. The average recovery time of both elements is equal to: $\Theta_{2q}(q) = \int_0^\infty g_{2q}(z)dz$, $g_{2q}(t) = \int_0^t g_q(z)g_q(t-z)dz$. $g_q(t) = 0,4qze^{-0,2z^2}$.

$$\Theta_{2q}(q) = \frac{10}{q} \cdot \Gamma(\frac{3}{2}).$$

$$\overset{*}{b}_q(q) = \int_0^\infty (1 - e^{-q\cdot 0,2\cdot z^2}) \cdot \frac{1}{\sqrt{2\pi}3} e^{-\frac{(z-25)^2}{2\cdot 3^2}} dz.$$

Substituting these values into the formula

$$K_{\Gamma q}(q) = \frac{T(2 - \overset{*}{b}_q(0))}{T(2 - \overset{*}{b}_q(0)) + \Theta_{2q}(1 - \overset{*}{b}_q(0))},$$

obtain the dependence of the system from the parameter control. It is shown in Picture 2. It can be seen that even for relatively small values of control availability becomes sufficiently close to unity.

When servicing two teams of recovery, this effect will increase.

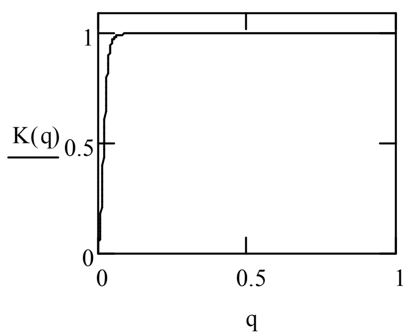

Fig. 2.

Conclusion. An expression for the function of readiness duplicated system in the Laplace transform and its significance is the steady state for arbitrary distributions of time to failure and recovery of constituent elements.

In these expressions have introduced the parameter reliability monitoring the state of the elements after their refusal. The value of this option allows you to take into account the duration of the recovery elements after their refusal.

Because of this, a generalization of the result obtained B.V.Gnedenko, ready for the duplicated system with control of the state elements.

Literature

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2. Smagin V.A. By a probabilistic model of control - ABT. -2010. - № 6.-S.25-33.