ASYMPTOTIC OPTIMUM QUANTIZATION OF THE CASUAL SIGNAL WITH BLANKS BETWEEN QUANTA

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ABSTRACT

In article generalization of a problem of optimum quantization of a casual signal with blanks between quanta is presented. Unlike known works the law of distribution of a casual signal with quanta is received. Instead of the integer decision of a problem the approached asymptotic decision is offered at a great number of quanta and the estimation of its accuracy is given. Besides, the decision of the given problem is received at fuzzy values of parameters of a blank and a population mean of an initial random variable with the normal law of distribution. Bibl. 8 nam., 5 fig.

Keywords: a random variable, asymptotic the optimum decision, quantum, probability density, uniform and normal distributions, fuzzy parameter, accessory function.

Introduction. The problem of quantization of a casual signal, apparently, has been put for the first time and asymptotic is solved in article [1]. Article has been connected with a problem of value of the information in the theory of the information [2], it differed high mathematical level. Application of results of this article to the decision of applied problems of a technical profile inconveniently enough. However, the idea of the asymptotic approach to the decision can form a basis for the decision of various practical problems.

In article [3] as criterion function of quantization of a casual signal the population mean, the most simple and rough characteristic of a random variable is accepted. The problem consisted in search of size of such quantum at which the population mean quantized random variable would reach a minimum. It has been shown that the problem of search of a minimum is a problem of integer optimization. For the numerical decision of a problem original enough algorithm is offered. Use of this algorithm, in our opinion, is expedient for the decision of unique problems of the raised accuracy, especially at small number of quanta. The basic lack – a combination of rough criterion function and enough bulky algorithm of the decision of a problem. The algorithm of the decision represents the basic value of article. It can be used as the sample of programming of the decision of challenges by students of technical colleges. For the decision of engineering problems it restrictedly is suitable because of high labor input of search of result, especially at rough criterion function.

The idea of article [3] was used successfully at the decision of various problems of protection of the information [4,5], and also in problems of reliability of switching structures of systems.

Interest to a quantization problem, in our opinion, is connected now with a problematics of the theory of fuzzy sets. Really, many parametry technical, program and social systems can't be having appearedleny unequivocally, accurately, "is not washed away". The author of given article in [6] the decision of a problem of optimization by a method uncertain mnozhitelja was offered to Lagranzha at fuzzy restriction.

The decision of a problem of quantization at fuzzy parameters practically it is necessary. However, to promote in this area it is not simple enough. In is given the article some generalization criterial the approach to quantization of a casual signal, and also model of the account of an illegibility parametra a blank between quanta is offered.

The author realizes that article is connected only with a question kvantovanija a casual signal, but not quantizations in the information theory.

Density of probability of size quantized a signal. Despite the stated critical remark concerning roughness of criterion function, we will use value of its formal representation and we will make attempt of development of idea of authors [3]. So, the size population mean quantezed a casual signal is presented by authors as:

$$M(x) = (x+c)\int_0^\infty [trunc(\frac{z}{x}) + 1] dF(z), \tag{1}$$

Where size of quantum of a signal, blank size between quanta, the greatest whole part of number, and function of distribution of a random variable no quantized a signal. Further we will be a floor-corduroy road that distribution function has continuous density and a final population mean. We will construct formally following expression for probability density, using application delta-function [7]:

$$\varphi(w) = \int_0^\infty \Delta(w - (x+c)[trunc(\frac{z}{x}) + r]) f(z) dz, \tag{2}$$

where $\Delta -$ delta-function, $f(z) = F^{'}(z)$ and $r -$ let some constant number, satisfying to a condition $0 \le r \le 1$. Further, we will be released from integer relations $\frac{z}{x}$, that is we believe that division is presented with some on-sinfulness from integer value. At $r = 0$ size of the relation of the tsebark, some lack, at $r = 1 -$ whole with some surplus. It is asked, at enough great number of quanta as itself will conduct conducted-rank r? Obviously, we have the right to believe with the big degree of confidence that the random variable $\hat{r}$ between the next quanta will be distributed in regular intervals with probability density $\frac{1}{x}$ on an interval x. Certainly, at small number of quanta and necessity of search of the integer decision of a problem of quantization it is an assumption unacceptably. Thus, it is a question of asymptotic representation of density of probability (2). For search of density (2) it is necessary to execute double integration at a random $\hat{r}$, but we won't do it, and we will be limited to size population mean expectations for it $v_{1,r} = \frac{1}{2}$. Then (2) it will be presented in a kind:

$$\varphi(w) = \int_0^\infty \Delta(w - (x+c)[\frac{z}{x} + \frac{1}{2}]) f(z) dz. \tag{3}$$

As to transform the size in integral is monotonous, after performance of simple transformations (2) is received:

$$\varphi(w) = \frac{x}{x+c} f(\frac{x}{x+c} w - rx), \quad x + c \le w \langle \infty. \tag{4}$$

It is easy to be convinced, that $\int_{r(x+c)}^\infty \varphi(z) dz = 1$, having made variable replacement $\frac{x}{x+c} w - rx = u$.

Let's find a size population mean $\hat{w}$, using (4):

$$v_{1,\varphi}(x) = \frac{x+c}{x} v_{1,Z} + r(x+c), \tag{5}$$

where $v_{1,Z} = \int_0^\infty f(z) dz$.

We investigate special cases for $r = 0, \frac{1}{2}, 1,$. At $r = 0$ it is had $v_{1,\varphi}(x) = \frac{x+c}{x} v_{1,Z}$, $v'_{1,\varphi}(x) = -\frac{c v_{1,Z}}{x^2}$. Function of $v_{1,\varphi}(x)$ a minimum has no. At $r = \frac{1}{2}$ $v_{1,\varphi}(x) = \frac{x+c}{x} v_{1,Z} + \frac{1}{2}(x+c)$, $v'_{1,\varphi}(x) = -\frac{c v_{1,Z}}{x^2} + \frac{1}{2}$.

There is a minimum at $x_0 = \sqrt{2c v_{1,Z}}$, equal $v_{1,\varphi}(x_0) = (1 + \frac{c}{\sqrt{2c v_{1,Z}}}) v_{1,Z} + \frac{1}{2}(\sqrt{2c v_{1,Z}} + c)$. At $r = 1$ $v_{1,\varphi}(x) = \frac{x+c}{x} v_{1,Z} + x + c$, $v'_{1,\varphi}(x) = -\frac{c}{x^2} v_{1,Z} + 1$. There is a minimum a $x_0 = \sqrt{c v_{1,Z}}$, equal $v_{1,\varphi}(x_0) = (1 + \frac{c}{\sqrt{c v_{1,Z}}}) v_{1,Z} + \sqrt{c v_{1,Z}} + c$.

Making calculations under the resulted formulas for normal distribution with parameters $v_{1,Z} = 100$ *ед.*, $\sigma_Z = 20$ *ед.*, and $c = 5$ *ед.* , we receive:

- $r = \frac{1}{2}$, $x_0 \approx 31{,}6$ *ед.*, $v_{1,\varphi}(x_0) \approx 134{,}1$ *ед.*, $\approx 3{,}7$ *квантов*;
- $r = 1$, $x_0 \approx 22{,}4$ *ед.*, $v_{1,\varphi}(x_0) \approx 149{,}7$ *ед.*, $\approx 5{,}5$ *квантов*;
- $r = 2$, $x_0 \approx 15{,}8$ *ед.*, $v_{1,\varphi}(x_0) \approx 173{,}2$ *ед.*, $\approx 8{,}3$ *квантов* .

The calculation executed under the formula (1), shows that for resulted values parameters of the normal law is received $x_0 \approx 31{,}6$ *ед.*, $M(x_0) \approx 134{,}3$ *ед.*, $\approx 3{,}7$ *квантов* . It testifies about satisfactorym coincidence of calculation to the calculation executed under the formula (4) at $r = \frac{1}{2}$.

Let's find expression for the second initial moment quantized a size case:

$$v_{2,\varphi}(x) = \int_{r(x+c)}^{\infty} z^2 \varphi(z) dz = \frac{(x+c)^2}{x^2} (v_{2,Z} + x v_{1,Z} + r^2 x^2), \tag{6}$$

where $v_{2,Z} = \int_0^{\infty} z^2 f(z) dz$. We will result expression for an average quadratic deviation at $r = \frac{1}{2}$ – $\sigma_\varphi = \sqrt{\frac{(x+c)^2}{x} (\frac{\sigma_Z{}^2}{x} + \frac{v_{1,Z}}{2})}$, we will calculate its value and value variation factor in a minimum point $x_0 = 31{,}6$ *ед.*, it is received $\sigma_\varphi = 51{,}5$ *ед.*, $\eta_\varphi = 0{,}38$.

For determined distributions, that is at $f(z) = \Delta(z - v_{1,Z})$, $v_{1,Z} = 100$ *ед.*, we will receive the same values for a population mean as it depends only from $v_{1,Z}$ and doesn't depend from σ_Z.

Likelihood estimation of size quantized a random variable to mozh the approximately to define, using the formula of density of probability:

$$\phi(x) = \frac{1}{\sqrt{2\pi}\sigma_\varphi} e^{-\frac{(x - v_{1,\varphi}(x_0))^2}{2\sigma_\varphi^2}} . \tag{7}$$

Thus its third initial moment will be equal:

$$\nu_{3,\varphi} = \int_{r(x+c)}^{\infty} z^3 \varphi(z) dz = \frac{(x+c)^3}{x^3}(\nu_{3,Z} + 3rx\nu_{2,Z} + 3r^2x^2\nu_{1,Z} + r^3x^3) \ . \quad (8)$$

The relative error of value of the third initial moment, defined under the formula (8), in comparison with the third moment found on method [3] is equal $(2{,}674 - 2{,}629)\cdot 10^6 \cdot 100 / 2{,}674 \cdot 10^6 \approx 1{,}7\%$.

Any initial moment i an order for probability density can be defined under the formula :

$$\nu_{i,\varphi} = \frac{(x+c)^i}{x^i} \sum_{j=0}^{i} C_i^j \nu_{i-j,Z} (rx)^j \ . \quad (10)$$

Believing size of optimum quantum a random variable, it is possible to find its approached value mean squared deviations σ_κ and using the normal law of distribution to receive a necessary likelihood estimation of size of quantum. Exact definition σ_κ is inconvenient enough, as quanta as random variables, are dependent. Believing their independent, the approached estimation from above for σ_κ can be found from following reasons. We will find transformation of Laplas of density of probability (4)

$$L(s) = e^{-r(x+c)} \Psi(\frac{x+c}{x} s), \quad (11)$$

where $\Psi -$ transformation of Laplas $f(z)$. Further, believing known x_0 and average of quanta n_0, at the given optimum decision we will write down:

$$[g(s)]^{n_0} = L(s), \quad (12)$$

where $g(s) -$ the image of Laplasa of density of probability of size of quantum. Let's find it $g(s) = \sqrt[n_0]{L(s)}$. The first and second initial moments will be defined as $\nu_{1,k} = -g'(0), \nu_{,k} = g''(0),$ and $\sigma_k = \sqrt{\nu_{2,k} - \nu_{1,k}^2}$. Without resulting bulky calculations for our example with $r = \frac{1}{2}$, $x_0 = 31{,}62$ *ед.*, $n_0 = 4$ we will receive $\sigma_k \approx 13{,}58; \eta = \frac{\sigma_k}{\nu_{1,k}} = 0{,}43$.

For real, dependent quanta, these values will be slightly less.

Algorithm of fuzzy quantization of a casual signal. In many cases quantizations of a casual signal it is necessary to take into consideration a fuzzy of the separate parameters influencing received decisions of a problem. Such parameters can be a little. In our example in such parameters can to be parameters of the law of distribution quantized a random variable, blank size between quanta.

The decision of a problem of fuzzy quantization at several fuzzy parameters inconveniently enough because of necessity of construction of multidimensional function of an accessory. Therefore we will be limited to influence consideration only one fuzzy parameter. As such parameter we will accept blank size between quanta c. We will be set by an interval of fuzzy and function of an accessory of parameter in the conditions of a considered example with normal function of distribution of a random variable. Let they are represented by the function $\mu(c) = 0{,}111 \cdot (8-c) \cdot (c-2), \quad c \in [2,8]$ which schedule is represented in drawing 1.

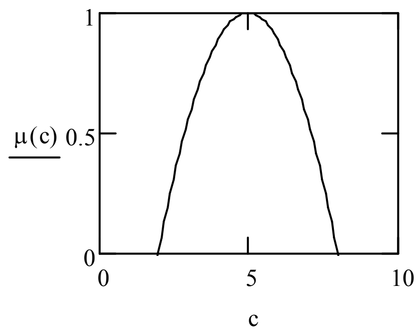


Fig. 1

To construct schedules of functions of an accessory to a population mean (5) and $\nu_{1,\varphi}(x_0) = \frac{x_0 + c}{x_0}\nu_{1,Z} + r(x_0 + c)$, $x_0 = \sqrt{2c\nu_{1,Z}}$ and at $r = \frac{1}{2}$. Then it is necessary to be set by degree « illegibility» these functions and to define their admissible borders illegibility».

For construction of calculations in the environment of Mathcad it is necessary to use indexes representations of functions [8]:

$$i = 2,3:8;\ \delta = 1;\ c_i = \delta \cdot i;\ \mu_i = 0.111\cdot(8 - c_i)\cdot(c_i - 2);\ \nu_{1,\varphi,i} = (x + c_i)\cdot(\frac{\nu_{1,Z}}{x} + \frac{1}{2});\ x_0 = \sqrt{2\cdot c_i\cdot\nu_{1,Z}}.$$

Calculating, we will receive two vectors – a vector - argument and function vector. Substituting in a vector-argument numerical values $\nu_{1,Z} = 100$ *ед.*, $r = \frac{1}{2}$, writing down values both functions in shape it is transposed vectors-lines, we will construct the functions of an accessory of size population mean quantezed the random variable, represented on figure 2.

$$\nu 1\phi := (0\ \ 122.7\ \ 126.5\ \ 130.3\ \ 134.2\ \ 138\ \ 141.8\ \ 145.7)^T$$

$$\mu := (0\ \ 0\ \ 0.56\ \ 0.89\ \ 1.00\ \ 0.89\ \ 0.56\ \ 0)^T$$

We will be set, for example, by trust level to the size $\nu_{1,\varphi}$, equal 0,8 and we will find values of the bottom and top borders for this level:

$$A(t) = l\operatorname{int}erp(\nu_{1,\varphi}, \mu, t);\ l\operatorname{int}erp(\nu_{1,\varphi}, \mu, 129.26) = 0.8;\ l\operatorname{int}erp(\nu_{1,\varphi}, \mu, 139.04) = 0.8.$$

Thus, at level trust a size $\alpha = 0,8$ population mean quantezed a signal will be in limits $129,26\ un. \le \nu_{1,\varphi} \le 139,04\ un.$.

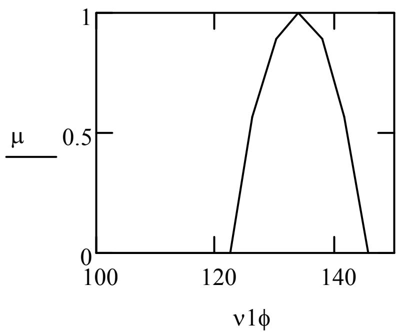


Fig. 2.

$$\nu1\phi \rightarrow \begin{bmatrix} 0 \\ 0 \\ (x+2)\cdot\left(\frac{\nu_{1,z}}{x}+r\right) \\ (x+3)\cdot\left(\frac{\nu_{1,z}}{x}+r\right) \\ (x+4)\cdot\left(\frac{\nu_{1,z}}{x}+r\right) \\ (x+5)\cdot\left(\frac{\nu_{1,z}}{x}+r\right) \\ (x+6)\cdot\left(\frac{\nu_{1,z}}{x}+r\right) \\ (x+7)\cdot\left(\frac{\nu_{1,z}}{x}+r\right) \\ (x+8)\cdot\left(\frac{\nu_{1,z}}{x}+r\right) \end{bmatrix} \qquad \mu \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ .555 \\ .888 \\ .999 \\ .888 \\ .555 \\ 0 \end{pmatrix}$$

Arriving similarly, we find the function an accessory for optimum size of quantum at fuzzy size of a blank between quanta which is represented in figure 3. Function and argument vectors are equal:

$$x_0 = (0\ 20\ 24.49\ 28.28\ 31.62\ 34.64\ 37.42\ 40)^T,$$
$$\mu = (0\quad 0\quad 0.56\quad 0.89\quad 1.00\quad 0.89\quad 0.56\quad 0)^T.$$

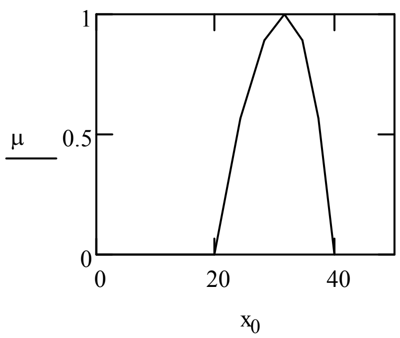


Fig. 3.

Interval accessories of optimum size of quantum at $\alpha = 0{,}8$ trust level should satisfy to inequality $27{,}25\ un. \le x_0 \le 35{,}40\ un.$.

For comparison we will calculate function an accessory for a population mean (1) resulted in [3]. The schedule of this function is shown in figure 4, and at the left it is an interval of an accessory of argument $M(x_0)$ for $x_0 = 31{,}62$ *ед.*, at which $M(x_0) = \min$.

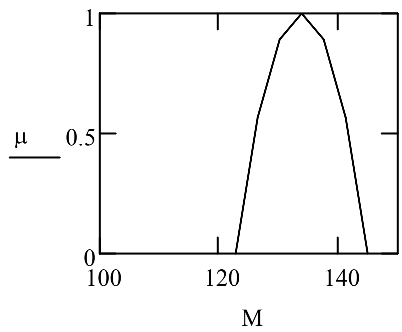
At trust level $\alpha = 0.8$ $\ 129.41\ un. \le M \le 138.73\ un.$

The received results coincide practically with split-hair accuracy that testifies to a correctness of the offered model quantization at the account of an fuzzy of parameter.

$$M := (123.09\ \ 126.75\ \ 130.41\ \ 134.07\ \ 137.73\ \ 141.39\ \ 145.05)^T$$

$$\mu := (0\ \ 0.56\ \ 0.89\ \ 1.00\ \ 0.89\ \ 0.56\ \ 0)^T$$

Fig. 4.



Let's give an example calculations of two-dimensional function of an accessory to fuzzy definition of size of optimum quantum:

$$r_{i,j} = \int_2^8 \Delta(u - c_i) \int_{80}^{120} \sqrt{2uz}\Delta(z - m_j) dzsu, \quad \omega_{i,j} = \min(\mu_i, \nu_j), \tag{13}$$

where $r_{i,j}$ − an element of a two-dimensional matrix of values of arguments, and $\omega_{i,j}$ − an element of a matrix of function of the accessory, corresponding i, j − to value of argument. Both matrixes are shown in drawing 5.

Fig. 5.

$$\omega := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0.56 & .056 & 0.56 & 0 \\ 0 & 0.75 & 0.89 & 0.75 & 0 \\ 0 & 0.75 & 1.00 & 0.75 & 0 \\ 0 & 0.75 & 0.89 & 0.75 & 0 \\ 0 & 0.56 & 0.56 & 0.56 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad r := \begin{pmatrix} 17.9 & 19.0 & 20.0 & 21.0 & 21.9 \\ 21.9 & 23.3 & 24.5 & 25.7 & 26.8 \\ 25.3 & 26.8 & 28.3 & 29.7 & 31.0 \\ 28.3 & 30.0 & 31.6 & 33.2 & 34.6 \\ 31.0 & 32.9 & 34.6 & 36.3 & 37.9 \\ 33.5 & 35.5 & 37.4 & 39.2 & 41.0 \\ 35.8 & 38.0 & 40.0 & 42.0 & 43.8 \end{pmatrix} \tag{14}$$

From matrixes (14) follows that value the accessory function, equal 1.00, corresponds to optimum value of size of quantum $x_0 = 31.6\ ed.$ On values of elements of a matrix r it is possible to define degree fuzzy quantum sizes at certain level of trust to accessory function in a matrix ω. For example, for $\alpha_c = 0.75$ on parameter of a blank c the quantum size should be in limits $30.0\ un. \le x_0 \le 33.2\ un.$, and for $\alpha_\nu = 0.89$ on value of a population mean ν_{1Z} – in limits $28.3\ un. \le \nu_{1Z} \le 34.6\ un.$. For achievement it is more necessary for accuracy to raise accuracy of calculations under formulas (13). Similarly it is possible to define requirements to fuzzy size of a population mean with quanta $\nu_{1\varphi}$.

The conclusion. On the basis of use of expression for a population mean of the random variable presented in the form of sequence of equal quanta on size with blanks between them, expression for density of probability of a random variable with quanta is received.

Asymptotic representation of the given density of probability under condition of replacement of integer number of quanta with the sum of the relation of realization of an initial random variable to size of quantum and a population mean of in regular intervals distributed random variable on an interval of size of quantum is offered. It allows to define optimum size of quantum, to find a likelihood estimation of a random variable with quanta and values of its initial moments at optimum size of quantum.

On an example for an fuzzy blank on size between quanta definition of functions of an accessory of fuzzy values of sizes of optimum quantum and a population mean of a random variable with quanta is shown.

Given article has no direct relation to the information. Further it is expedient to consider the problem on quantization of a casual signal with syntactic, semantic and pragmatical forms of the static information.

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