A GENERAL ANALYTICAL SOLUTION FOR THE OCCURRENCE PROBABILITY OF A SEQUENCE OF ORDERED EVENTS FOLLOWING A POISON STOCHASTIC PROCESS

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ABSTRACT

The author presents a general analytical solution determining "the Occurrence probability of a sequence of events each following Poison Stochastic Process". Generally, this probability is described under the form of an integral equation of order "n". Where "n" is number of the elementary events in the examined sequence.

As far as the author can tell, the solution is original. It will be of a great interest to a wide range of system reliability problems such as: sequential calculations, dominos effects, dynamics fault trees, Markov systems, priority AND gates, events trees, stochastic optimisation, acceleration techniques for Monte-Carlo simulation, …

Key words: Ordered events, sequential events, Poisson stochastic Process, Markov, probability

1 INTRODUCTIONIS

The author is interested in determining "the occurrence probability of a well-defined sequence of ordered events obeying a Poisson stochastic process", $p_n(t)$.

One meets often ordered events in system reliability analyses. Analysts may use "Dynamic Fault Tree" with "Priority Gates", "Markov Graphs" or "Monte-Carlo Simulation" tools in order to deal with the dynamic aspect of this problem.

A fault tree can be described by means of some cut sets. One may calculate the occurrence probability of each cut set. However, the calculated probability does not tell about the occurrence order of the involved events in the cut set. A cut-set, containing n-independent events, may be expressed in $n!$ different ordered sequences. In many reliability and risk assessments, only some given ordered sequences may be of specific concerns. Consequently, it is of great interest to determine the occurrence probabilities of these sequences and their occurrence rates.

In the paper, one describe the problem in the form of a given integral in §3 and a differential equation in §4. In [1], Fussell use the same integral equation as we use in §3, but the events are given in the opposite order. He uses Laplace transformation to find out an exact solution. Although the solution is exact, Fussell switched on to use the asymptotic form of the solution. In [2], Yunge uses the same differential equation given previously in §4 in order to model the sequential occurrence of events in a given priority AND gate (PAG). He uses Laplace transformation and find out the exact solution of the occurrence probability $p_n(t)$. However, in both cases, the authors used complicated forms such that it did not allow putting in evidence the recurrence aspect of the solution. They were more concerned by inserting their models (PAG, ...) in a Dynamic Failure Tree than by other aspects of the solution.

However, if they had not excluded the use of an equivalent Markov Graph, they would have noticed this interesting recurrence aspect of the model, and maybe, they would have used a simpler expression of the solution.

Many other authors followed almost the same way on modelling and produced very interesting applications, [3]-[7], without perceiving that the exact solution could be put in a simpler form.

This would have extended the solution to other categories of problems rather than just the modelling of the PAG's and their inclusion in Dynamic Failure Trees.

Some other researchers could solve the same problem in using numerical techniques such as Petri Nets of Dynamic Bayesian Net (DBN). The use of a numerical technique prevents all possibility to underline the analytical solution and its interest. This is the case of Montani, [8] in an application relative to an active heat rejection system (AHRS) that he used to validate the methodology. The case would have been solved immediately if one had applied the analytical solution given herein. The problem contained only 8 sequences with a maximum length of 4 successive failures.

One may equally mention the case of the applications in [9]-[11], where the general analytical solution would have helped in treating the problems in exact way.

Regarding rare sequential events, many researchers sought answers in developing methods based on Monte-Carlo simulation, [12]-[13]. Some other papers are given in [14]-[21] which have developed interesting methods as well as they developed solutions very close to the one proposed her. Two papers should particularly be underlined are [19] and [20].

The work presented here is limited to Poisson stochastic processes. However, it is of high interest because it will obviously improve the numerical procedures used to treat practical industrial cases.

The analytical solution of this problem presents a particular interest by itself, because of its originality and simplicity. Besides, it suggests some interesting directions of investigation so that it may help in developing analytical solutions for some other specific time distributions different from Poison ones.

2 PROBLEM DEFINITION

Let T describe a top event which results from the occurrence of some n basic events e_i ($i = 1,2,...,n$) in a well determined sequential order. Basic events e_i are following Poisson stochastic processes and each is fully characterised by a constant occurrence rate λ_i and by its occurring order 'i'. The e_1 is the 1st occurred and e_n is the last event.

One would like to determine the occurrence probability of the top event T and its occurrence rate.

3 PROBLEM MODELLING

A will defined top event T will occur if and only if some discrete and independent events e_i happen according to a well specified order $[e_1, e_2, e_3, ..., e_n]$. The corresponding occurring instants are defined by $[t_1, t_2, t_3, ..., t_n]$, where $[t_1 < t_2 < t_3 < ... < t_n]$. Each of these instances $[t_1, t_2, t_3, ..., t_n]$ has its distribution probability function (pdf). The first event is e_1 and the last one is e_n.

The probability $p_n(t)$ that Event T happens within the interval [0,t] is given by:

(1) $$p_n(t) = \int_0^t \rho_1(\xi_1) d\xi_1 * \int_{\xi_1}^t \rho_2(\xi_2) d\xi_2 * \int_{\xi_2}^t \rho_3(\xi_3) d\xi_3 * \ldots * \int_{\xi_{n-2}}^t \rho_{n-1}(\xi_{n-1}) d\xi_{n-1} * \int_{\xi_{n-1}}^t \rho_n(\xi_n) d\xi_n$$

Where: $0 \le \xi_1 \le \xi_2 \le \xi_3 \le ... \le \xi_n \le t$ and,
ρ_i is the Poisson density function characterising the event e_i [$\rho_i = \lambda_i * e^{-\lambda_i t}$].

Many authors could previously developed analytical solutions fo Equation (1) when it was a matter of limited number of ordered events obeying a Poison's Stochastic Process, e.g. [1][2][19][20].

Here, the paper develops a simpler form of the exact solution of Equation (1).

4 ANALYTICAL SOLUTION

It is obvious that Equation (1) can equally be expressed on the following differential form:

$$(2) \quad \frac{d}{dt} p_{n+1}(t) \quad = \quad \rho_{n+1}(t)\ p_n(t)$$

Let $p_n(t)$ be the occurrence probability of a sequence T, a set of chronologically ordered events $[e_1, e_2, e_3, ..., e_n]$. The probability $p_n(t)$ is the solution of the Equation (1) and (2).

Let $p_n(t)$ be expressed by following expression:

$$(3) \quad p_n(t) \quad = \sum_{j=1}^{n} C_j^n * (1 - e^{-(\sum_{l=n-j+1}^{n} \lambda_l)t}), \ C_1^1 = 1.0.$$

Where, each event e_i is defined by a constant occurrence rate λ_i, $\{i \in [1,2,...,n]\}$.

The solution of the problem resumes in determining the coefficients C_i^n.

In appendix (1), we demonstrate the solution proposed in Equation (3) and show that the coefficients C_j^i are fully determined thanks to some recurrence pattern, as following:

$$(4) \quad C_1^{i+1} \quad = \sum_{j=1}^{i} C_j^i, \qquad C_{j+1}^{i+1} = -\frac{\lambda_{i+1}}{\sum_{l=i-j+1}^{i+1} \lambda_l} C_j^i, \qquad j = 1,2,...,i, \quad \text{and} \quad i \in [1,2,...,n]$$

Some examples for calculating the parameters C_j^i are given in appendix (2).

4.1 Occurrence Density and Occurrence rate

By definition, the corresponding occurrence density function $\Theta_i(t)$ can directly be deduced via the first derivative of the occurrence probability function as following:

$$(5) \quad \Theta_n(t) \quad = \quad \frac{dp_n(t)}{dt}$$

The occurrence density function will then be defined by:

(6) $$\Theta_n(t) = \sum_{j=1}^{n} C_j^n * (\sum_{l=n-j+1}^{n} \lambda_l) e^{-(\sum_{l=n-j+1}^{n} \lambda_l)t}$$

We may also define an equivalent occurrence rate Λ_i of the whole sequence T, such as:

(7) $$\Lambda_i(t) = \frac{1}{p_i(t)} * \frac{dp_i(t)}{dt} = \frac{\sum_{j=1}^{i} (\sum_{l=i-j+1}^{i} \lambda_l) * C_j^i e^{-(\sum_{l=i-j+1}^{i} \lambda_l)t}}{\sum_{j=1}^{i} C_j^i * (1 - e^{-(\sum_{l=i-j+1}^{i} \lambda_l)t})}$$

As we may expect, although the ordered events are individually governed by a Poisson Stochastic Process, the sequence T is not. The occurrence rate of the sequence T is time dependent, Eq.(7).

4.2 Mean Occurrence Time

One may also determine the mean occurrence time τ_n corresponding to a given sequence (S_n) of n-events $\{e_1, e_2, e_3, ..., e_n\}$, such as:

(8) $$\tau_n = \int_{t=0}^{\infty} t * dp_n(t)$$

The solution of Eq.(8) is elementary and described by:

(9) $$\tau_n = \sum_{j=1}^{n} \frac{C_j^n}{(\sum_{l=n-j+1}^{n} \lambda_l)}$$

4.3 Asymptotic Behaviour

Having demonstrated that the occurrence probability $p_n(t)$ of a given sequence of n-well defined ordered events can be described by Equation (3), it is straightforward to demonstrate that the occurrence probability $p_n(t)$ has an asymptotic value equal to:

(10) $$p_n(t \to \infty) \to \sum_{j=1}^{n} C_j^n$$

The occurrence probability density function Θ_n of the sequence S_n has an asymptotic value equal to:

(11) $$\Theta_n(t \to \infty) \to 0.$$

Similarly, the equivalent occurrence rate Λ_n of the sequence S_n has an asymptotic value equal to:

(12) $\Lambda_n(t \to \infty) \quad \to \quad 0.$

5 NUMERICAL APPLICATION

Two illustrative numerical applications are given in the following in order to help in sizing the interest of having a generic analytical solution determining the occurrence probability of a given sequence (S_n) of n-events $\{e_1, e_2, e_3, ..., e_n\}$ in the given order.

5.1 Occurrence Order

In this application, we are focusing on the dependence of the occurrence probability on the occurrence order of the basic events.

A very simple example is illustrated in figure (1) where the time evolution of the occurrence probability of a sequence of four basic events $[e_1, e_2, e_3, e_4]$ whose occurrence rates are constant and having the following values: 10^{-4} /h, $5*10^{-3}$ /h, $2.5*10^{-2}$ /h, $1.25*10^{-1}$ /h. In Figure (2), we are comparing two configurations represented by a red curve and a blue one.

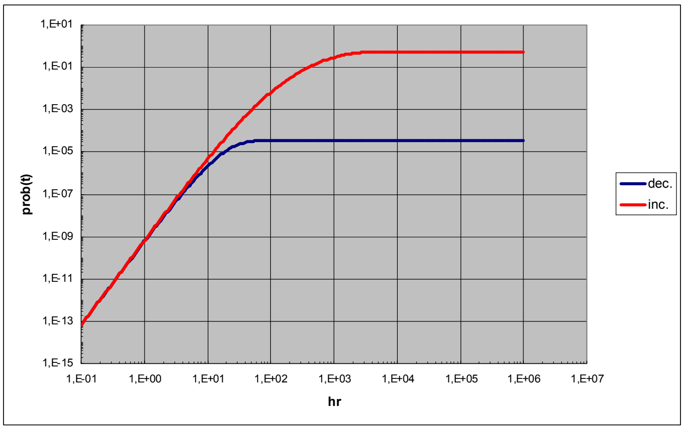

Figure (1) : the occurrence probability of the same set of events in two different orders (dec: decreasing order, inc.: increasing order)

The red curve represents the case where the sequence of events follows the increasing order of the occurrence rates (less frequent occurs first). While, the blue curve describes the case where the sequence of events following the decreasing order of the occurrence rates (more frequent occurs first).

It is obvious that the occurring order of events impacts on the time behaviour of the occurring probability of any sequence of events.

The asymptotic behaviour of the occurrence probability can also be underlined.

5.2 Treatment of a Markov Graph

In this example, a given system is described by Markov graph. The system has 8 possible states. The transitions between different states are fully determined by their transition rates.

In this illustrative example, Figure (2), a unique transitions rate value of $10^{-1}\ h^{-1}$ is considered for all transition rates as following:

$$\lambda_{12} = \lambda_{13} = \lambda_{14} == \lambda_{25} = \lambda_{52} = \lambda_{36} = \lambda_{47} = \lambda_{56} = \lambda_{68} = 10^{-1}\ h^{-1}$$

We are interested in the sequences S_n leading to the absorbing states e_7 or e_8, which are the following:

$$S_3 = e_1 \rightarrow e_4 \rightarrow e_7,$$
$$S_4 = e_1 \rightarrow e_3 \rightarrow e_6 \rightarrow e_8,$$
$$S_5 = e_1 \rightarrow e_2 \rightarrow e_5 \rightarrow e_6 \rightarrow e_8,$$
$$S_{5+2n} = e_1 \rightarrow (e_2 \leftrightarrow e_5)^n \rightarrow e_6 \rightarrow e_8, \qquad n = 0,1,2,\ldots$$

Where;

$$(e_2 \leftrightarrow e_5)^0 = e_2 \rightarrow e_5$$
$$(e_2 \leftrightarrow e_5)^1 = e_2 \rightarrow (e_5 \rightarrow e_2) \rightarrow e_5$$
$$(e_2 \leftrightarrow e_5)^2 = e_2 \rightarrow (e_5 \rightarrow e_2) \rightarrow (e_5 \rightarrow e_2) \rightarrow e_5$$

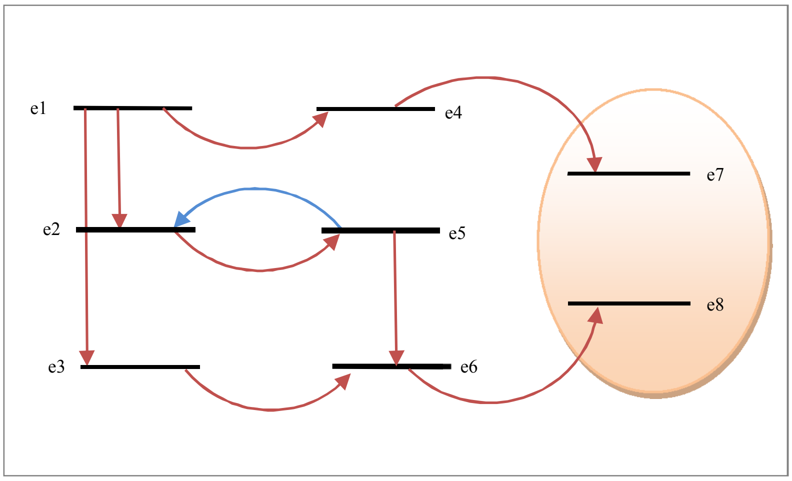

Figure (2) : Schematic presentation of a Markov Graph

One would, then, like to determine for each sequences (S_3, S_4, S_5, S_7) its occurrence probability time-profile, $p_n(t)$. The occurrences probabilities of the sequences S_3, S_4, S_5, S_7 are illustrated in Figure (3). Higher order sequences would have much lower contribution than that of S_7 as illustrated in Figure (3).

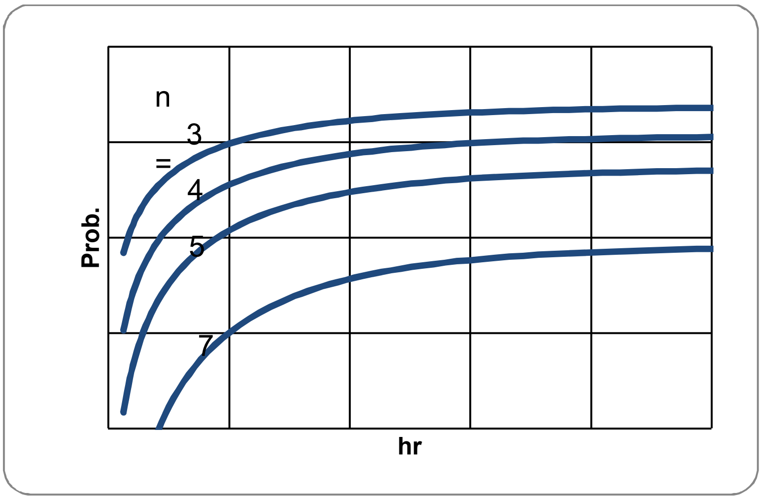

Figure (3) Occurrence probability as a time-function of each sequence rank
(S_3, S_4, S_5, S_7)

The asymptotic values of the occurrence probabilities of the sequences ($S_3, S_4, S_5, S_7, ...$) are illustrated for the application in Figure (4).

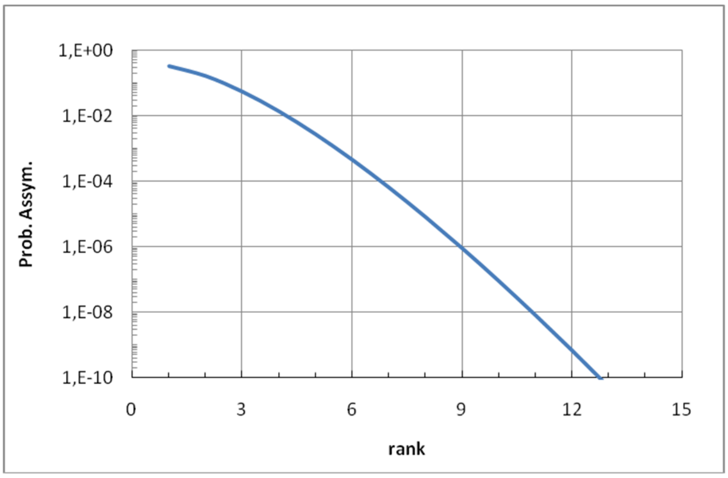

Figure (4) : the asymptotic probability as a function of the sequence rank

Finally, one would also determine the mean occurrence time of the sequences (S_3, S_4, S_5, S_7 ,...) as a function of the sequence order.

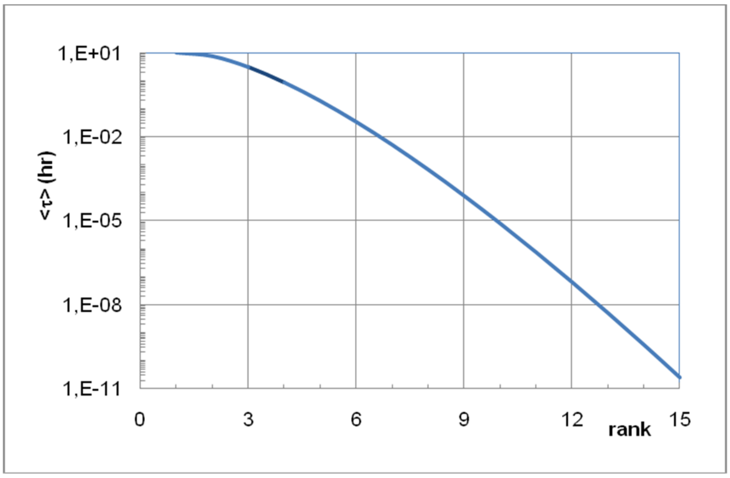

Figure (5) : Meantime of occurrence as a function of the sequence length

6 CONCLUSIONS

The paper proposes a general solution in order to determine the occurrence probability of a given sequence of n-events following Stochastic Poison's Processes. The solution is analytical and original.

Some interesting asymptotic characteristics of this analytical solution have been assessed.

Two simple numerical applications are illustrated in order to underline the interest of possessing an analytical generic solution to this problem.

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Appendix (1)

Let $p_n(t)$, the occurrence probability of the sequence T, be the solution of the Equation (1) and (2) and be expressed as:

$$p_i(t) = \sum_{j=1}^{i} C_j^i * (1 - e^{-(\sum_{l=i-j+1}^{i} \lambda_l)t}),\ C_1^1 = 1.0 \text{ and } i \in [1,2,...,n]$$

$$p_{i+1}(t) = \sum_{j=1}^{i+1} C_j^{i+1} * (1 - e^{-(\sum_{l=i-j+2}^{i+1} \lambda_l)t}),\ C_1^1 = 1.0 \text{ and } i \in [1,2,...,n]$$

And,

$$\frac{d}{dt} p_{n+1}(t) = \rho_{n+1}(t)\ p_n(t)$$

Where Sequence T contains i ordered events, each is defined by an occurrence rate λ_j.

$$\frac{d}{dt} p_{i+1}(t) = \frac{d}{dt} \sum_{j=1}^{i+1} C_j^{i+1} * (1 - e^{-(\sum_{l=i-j+2}^{i+1} \lambda_l)t}) = \sum_{j=1}^{i+1} C_j^{i+1} (\sum_{l=i-j+2}^{i+1} \lambda_l) * e^{-(\sum_{l=i-j+2}^{i+1} \lambda_l)t}$$

$$= C_1^{i+1} * \lambda_{i+1} e^{-\lambda_{i+1} t} + \sum_{j=2}^{i+1} C_j^{i+1} (\sum_{l=i-j+2}^{i+1} \lambda_l) * e^{-(\sum_{l=i-j+2}^{i+1} \lambda_l)t}$$

$$= C_1^{i+1} * \lambda_{i+1} e^{-\lambda_{i+1} t} + \sum_{k=1}^{i} C_{k+1}^{i+1} (\sum_{l=i-k+1}^{i+1} \lambda_l) * e^{-(\sum_{l=i-k+1}^{i+1} \lambda_l)t}$$

While;

$$\rho_{n+1}(t)\ p_n(t) = \lambda_{i+1} * e^{-\lambda_{i+1} t} \sum_{j=1}^{i} C_j^i * \left(1 - e^{-(\sum_{l=i-j+1}^{i} \lambda_l)t}\right)$$

$$= \sum_{j=1}^{i} C_j^i * \lambda_{i+1} * \left(e^{-\lambda_{i+1} t} - e^{-(\sum_{l=i-j+1}^{i+1} \lambda_l)t}\right)$$

$$= \left(e^{-\lambda_{i+1} t} * \sum_{j=1}^{i} C_j^i * \lambda_{i+1}\right) - \left(\sum_{j=1}^{i} C_j^i * \lambda_{i+1} * e^{-(\sum_{l=i-j+1}^{i+1} \lambda_l)t}\right)$$

So,

$$C_1^{i+1} * \lambda_{i+1} e^{-\lambda_{i+1} t} + \sum_{k=1}^{i} C_{k+1}^{i+1} (\sum_{l=i-k+1}^{i+1} \lambda_l) * e^{-(\sum_{l=i-k+1}^{i+1} \lambda_l)t} =$$

$$\left(e^{-\lambda_{i+1} t} * \sum_{j=1}^{i} C_j^i * \lambda_{i+1}\right) - \left(\sum_{j=1}^{i} C_j^i * \lambda_{i+1} * e^{-(\sum_{l=i-j+1}^{i+1} \lambda_l)t}\right)$$

1st Condition

$$\lambda_{i+1} * \left(C_1^{i+1} - \sum_{j=1}^{i} C_j^i \right) * e^{-\lambda_{i+1} t} \quad =0$$

Then;

$$C_1^{i+1} = \sum_{j=1}^{i} C_j^i \text{ , } i \in [1,2,...,n]$$

2nd Condition

$$\sum_{j=1}^{i} \left(C_{j+1}^{i+1} (\sum_{l=i-j+1}^{i+1} \lambda_l) + C_j^i * \lambda_{i+1} \right) * e^{-(\sum_{l=i-j+1}^{i+1} \lambda_l) t} \quad =0$$

Then,

$$C_{j+1}^{i+1} \quad = - \frac{\lambda_{i+1}}{\sum_{l=i-j+1}^{i+1} \lambda_l} * C_j^i \text{ , } \forall \ j \in [1,i] \text{ and } i \in [1,2,...,n]$$

Appendix (2):

Consider the case of a sequence containing 4 basic events in some well-determined order, so we have :

$$C_1^{i+1} \quad = \sum_{j=1}^{i} C_j^i \text{ ,} \qquad \text{and} \qquad C_1^1 = 1$$

$$C_{j+1}^{i+1} \quad = - \lambda_{i+1} \cdot \frac{C_j^i}{(\sum_{l=i-j+1}^{i+1} \lambda_l)} \qquad j = 1,2,...,i$$

We may, then, find out the coefficients C_j^i as following:

$N = 1$

$C_1^1 = 1$

$N = 2$

C_1^2 **= 1,**

$C_2^2 = - \frac{\lambda_2}{\lambda_2 + \lambda_1}$

$$N=3$$
$$C_1^3=(1-\frac{\lambda_2}{\lambda_2+\lambda_1}),$$
$$C_2^3=-\frac{\lambda_3}{\lambda_3+\lambda_2},$$
$$C_3^3=+\frac{\lambda_3}{\lambda_3+\lambda_2+\lambda_1}*\frac{\lambda_2}{\lambda_2+\lambda_1}$$

$$N=4$$
$$C_1^4=(1-\frac{\lambda_2}{\lambda_2+\lambda_1})-(\frac{\lambda_3}{\lambda_3+\lambda_2})+(\frac{\lambda_3}{\lambda_3+\lambda_2+\lambda_1}*\frac{\lambda_2}{\lambda_2+\lambda_1})$$
$$C_2^4=-\frac{\lambda_4}{\lambda_4+\lambda_3}*(1-\frac{\lambda_2}{\lambda_2+\lambda_1})$$
$$C_3^4=\frac{\lambda_4}{\lambda_4+\lambda_3+\lambda_2}*\frac{\lambda_3}{\lambda_3+\lambda_2}$$
$$C_4^4=-\frac{\lambda_4}{\lambda_4+\lambda_3+\lambda_2+\lambda_1}*\frac{\lambda_3}{\lambda_3+\lambda_2+\lambda_1}*\frac{\lambda_2}{\lambda_2+\lambda_1}$$

So, the occurrence probability of this given sequence will, then, be determined by:

$$\begin{aligned} p_4(t) &= \sum_{j=1}^{4} C_j^4 *(1-e^{-(\sum_{l=4-j+1}^{4}\lambda_l)t}) \\ &= C_1^4 * (1-e^{-\lambda_4 t}) \\ &+ C_2^4 * (1-e^{-(\lambda_4+\lambda_3)t}) \\ &+ C_3^4 * (1-e^{-(\lambda_4+\lambda_3+\lambda_2)t}) \\ &+ C_4^4 * (1-e^{-(\lambda_4+\lambda_3+\lambda_2+\lambda_1)t}) \end{aligned}$$