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## THEORY&APPLICATIONS

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# RELIABILITY:
## THEORY & APPLICATIONS

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A mathematical models of the following electromechanical converters, used in wind power, are presented: the asynchronous generator with a fed through a frequency converter stator, the generator, which is double fed asynchronous machine with rotor winding, fed through a frequency converter, and a synchronous generator with permanent magnets, with stator, fed through the frequency converter. For all converters the optimal control laws of amplitude and frequency of voltage on generator's terminals are obtained for all operating modes of wind power plants.



In this paper queuing networks with infinite number of servers and exponentially distributed service times are considered. Limit distributions for numbers of customers in aggregated nodes are calculated. Using these calculations statistical estimates of limit distributions parameters are constructed. Mean times of customer sojourn time in opened network and its analog for closed network are estimated also. Obtained results are spread onto estimates of populations and life cycle parameters.



The method and program complex realization for restoration scheme search of power consumption in networks on the basis of algorithm processing of the network count and artificial neural networks (ANN) are offered and given reason. Both algorithms work in a competing mode, mutually smoothing lacks of each separate method. Work complex principles, in particular, concerning application of the generalized error vector for ANN self-training and solution stability are theoretically proved. The program complex realized as a part for 'the adviser of the distributive network dispatcher', was tested on a 201 buses, 7 sources and 227 lines distribution network and one has shown good results.



A problem of a calculation of a graph disconnection probability is considered in a lot of papers. But when a number of arcs increases this problem becomes much more complicated. So it is necessary to construct convenient asymptotic formulas for connectivity or disconnection probability of graph with high reliable arcs. In this paper such problem is solved for planar graphs or graphs arranged on two dimensional manifolds. Such graphs appear in honeycombed structures which are widely used in different applications.



The paper presents the results of the review of methods and models of an estimation and optimization of electric power systems (EPS) reliability. There are classification and characteristic of existing probabilistic methods used for an estimation of EPS reliability in the paper. The methodology of model for an estimation of systems reliability of EPS is described. Theoretical bases of system reliability optimization of EPS are stated. The description of the software YANTAR for estimation of EPS systems reliability is given.


Dear Colleagues!

January 1st of 2012 is the 100th Anniversary of Boris Gnedenko's birthday.

We would like to dedicate two issues of our e-journal "Reliability: Theory and Applications" to this event. The best papers for publishing on these issues will be chosen.

Please send your papers as soon as possible to give us time to evaluate them and make a selection. Remind you that all rights belong to authors, so it can be later presented at any conference or publish in other journals with no restrictions.

To join the Gnedenko Forum send your photo and brief resume

to Professor Igor Ushakov
igusha22@gmail.com,

or to Dr. Alexander Bochkov
a.bochkov@gmail.com

Professor Igor Ushakov
Editor-in-Chief
Reliability: Theory & Applications

1 February 2011

Subject: **Letter to the Editor**

Dear Professor Ushakov,

It came to our attention that we had missed an important reference in our paper titled *G1-Renewal Process as Repairable System Model*, which was published in RTA, Vol.1, No. 3, Sep 2010. The first sentence in the Abstract should have read "This paper considers a point process model with a monotonically decreasing or increasing ROCOF and the underlying distributions from the location-scale family, known as the geometric process (Lam, 1988)". Also, the last sentence in the Introduction should have read: "The considered below geometric process (Lam, 1988, 2009) overcomes this particular drawback".

References:

[1] Lam, Y. (1988), "Geometric Process and Replacement Problem", Acta Math. Appl. Synica, 4, p. 366 – 377.
[2] Lam, Y. (2009), A geometric process d-shock maintenance model, IEEE Transactions on Reliability, V. 58, No. 2, p. 389-396.

Respectfully,

Mark Kamisnkiy and Vasiliy Krivtsov

# ANALYTICAL ESTIMATION OF RELIABILITY OF NETWORKS CONSISTING OF IDENTICAL ELEMENTS.

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## ABSTRACT

In this work the analytical expressions for reliability factor of steady-state network consisting of identical elements are accomplished. For obtaining of the analytical expressions of considered factors, the Markov process characterized by a certain combination of failed elements and by a network state is considered. It is shown that for definition of values of the obtained expressions it is sufficient to define the number of combinations of set capacity elements at which failure the network passes in non-operable state. For small dimension networks this factor may be defined precisely by analysis of all possible combinations of failed elements. For large-scale networks it is necessary to use Monte Carlo method or combined method.

## 1 INTRODUCTION

At network reliability assessment, reliability factors which define dynamics of behavior of the network are of great interest. Following are considered as reliability factors: mean time spent in operable and non-operable conditions within the steady-state mode. The simplest criterion for network operability is its connectivity. The network is considered to be connected if between any pair of nodes exists at least one path.

In the course of deducing inference, we will use connectivity of a network as criterion of its operability. In section 5 the possibility of application of the obtained expressions at use of other criteria of a network operability is justified.

If connectivity is used as a network operability criterion, than mean time spent in the connected state corresponds to mean time to failure (**MTTF**), and network mean time spent in the disconnected state corresponds to mean time to repair (**MTTR**). Therefore **MTTF** and **MTTR** may be considered instead of mentioned reliability factors.

## 2 NETWORK RELIABILITY MODEL AND OBTAINING OF ANALYTICAL EXPRESSIONS

Let us assume that network nodes being absolutely reliable and edges being identical on reliability fail independently from each other and their time to failure and time to repair are exponentially distributed with parameters $\lambda$ and $\mu$.

Let us imply moving off of an edge from the network till its repair to be edge failure. Distinctive feature of networks which complicates definition of reliability factors is that at certain number of failed edges network may be connected as well as disconnected. For example, network presented on Figure 1 may not be precisely defined as connected then two or three edges are removed.

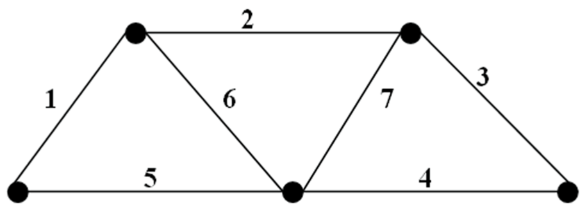


**Figure 1:** Example of the network

The cuts of capacity $i$ is employed as a key parameter which allows us to define considered reliability factors of the network. Let us denote this parameter by $Y_i$.

Let's define $Y_i$ for the network presented on Figure 1. Since considered network is biconnected, therefore one edge moving off cannot break its connectivity. Hence $Y_1=0$. For definition of $Y_2$ and $Y_3$ we consider all possible states of the network when 2 and 3 edges are moved off respectively, Figure 2.

Analyzing the data presented on Figure 2, it is possible to define that 2 cuts of capacity 2, and 14 cuts of capacity 3 are within the considered network, hence $Y_2 = 2$, $Y_3 = 14$. Any combination of $i$ edges at $i > 3$ is a cut, hence $Y_i = \binom{n}{i}$ at $i \geq 4$.

At small $n$, values of $Y_i$ may be defined by exhaustion of all possible states of the network. At great values of $n$, place Monte Carlo method or combined method should be used.

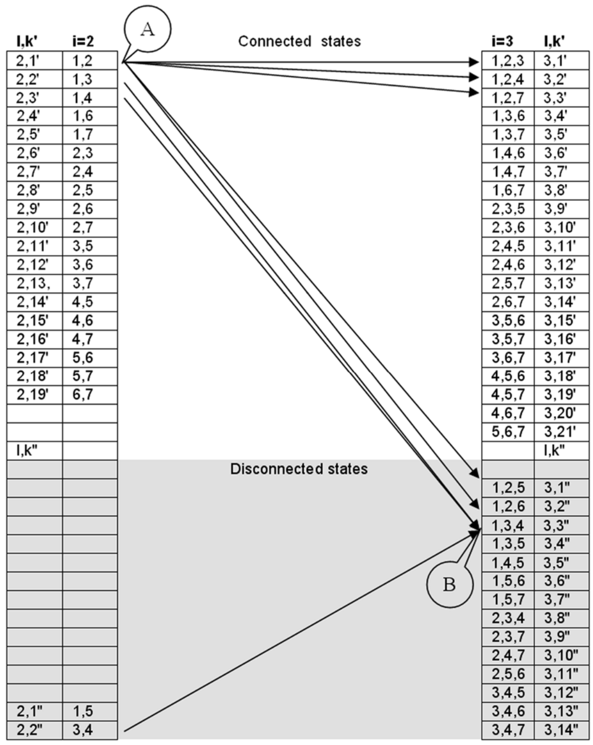


**Figure 2:** Network state transition at failure of i edge (i=3)

Knowing $Y_i$ it is possible to define probability of $i$ edges to become disconnected in case of failure. Let us denote this factor by - $Z_i$.

Value of $Z_i$ is equal to ratio of $Y_i$ to the total number of possible combinations of $i$ of $n$ elements where $n$ is the number of edges in the network, expression (1).

$$Z_i = \frac{Y_i}{\binom{n}{i}} \tag{1}$$

Consider a Markov process which describes network state transition at failure and repair of its edges. Each state is characterized by a certain combination of failed edges and by a network state.

Denoted by $E_+$ is the set of operable states of the network, $E_-$ is the set of non-operable states of the network.

Analytical expressions for definition of an average stay time of Markov and semi-Markov process in a subset of states have been obtained by Ushakov (1969a, 1969b).

For the considered case B.V.Gnedenko (1983), the average stay time of Markov process on a set of states $E_+$ before the first entering to a state of set $E_-$ can be defined from expression:

$$T_{E_+} = \frac{\sum_{i \in E_+} P_i}{\sum_{i \in E_+} (P_i \sum_{j \in E_-} \lambda_{ij})} = \frac{\sum_{i \in E_+} P_i}{\sum_{j \in E_-} (P_j \sum_{i \in E_+} \lambda_{ji})} \tag{2}$$

where $\lambda_{ij}$ - rate of transition from state $i \in E_+$ to state $j \in E_-$,

$\lambda_{ji}$ - rate of transition from state $j \in E_-$ to state $i \in E_+$.

Similarly mean time for the process to be within the set of states $E_-$ is defined

$$T_{E_-} = \frac{\sum_{j \in E_-} P_j}{\sum_{j \in E_-} (P_j \sum_{i \in E_+} \lambda_{ji})} = \frac{\sum_{j \in E_-} P_j}{\sum_{i \in E_+} (P_i \sum_{j \in E_-} \lambda_{ij})} \tag{3}$$

Let us denote connected states by $(i,k^{'})$ pair and disconnected network states by $(i,k^{''})$ pair at $i$ failed edges. Then the left parts of expressions (2) and (3) may be written as follows:

$$T_{E_+} = \frac{\sum_{i=0}^{n} \sum_{k^{'} \in E_+} P_{i,k^{'}}}{\sum_{i=0}^{n} \sum_{k^{'} \in E_+} P_{i,k^{'}} \lambda^{*}_{i,k^{'}}} \tag{4}$$

$$T_{E_-} = \frac{\sum_{i=0}^{n} \sum_{k^{''} \in E_-} P_{i,k^{''}}}{\sum_{i=0}^{n} \sum_{k^{''} \in E_-} P_{i,k^{''}} \mu^{*}_{i,k^{''}}} \tag{5}$$

where $P_{i,k'}$, $P_{i,k''}$ - probability of states; and $\lambda^{*}_{i,k'}$ - rate of transition from state $(i,k^{'})$ to states belonging to the set $E_-$, $\mu^{*}_{i,k^{''}}$ - rate of transition from state $(i,k^{''})$ to states which belong to the set $E_+$.

Summary edge failure rate at $i$ state equals to $(n-i)\lambda$, and summary rate to repair - $i\mu$ therefore:

$$\lambda^{*}_{(i,k^{'})} = (n-i)\lambda Z^{*}_{(i,k^{'})} \tag{6}$$

$$\mu^{*}_{(i,k'')} = i\mu Z^{**}_{(i,k'')} \tag{7}$$

where $Z^{*}_{(i,k')}$ - probability of network transition to disconnected state from $(i,k')$ in case of one edge failure;

$Z^{**}_{(i,k'')}$ - probability of network transition to connected state from $(i,k'')$ state in case of one edge repair.

For definition of $T_{E_+}$ and $T_{E_-}$ from expressions (4) and (5) it is necessary to know values $P_{(i,k')}$ and $P_{(i,k'')}$. Since all edges are identical on reliability therefore all states with $i$ failed edges are equiprobable.

The probability of $i$ edges in the network to be failed is equal to

$$P_i = \frac{(\lambda/\mu)^i \binom{n}{i}}{(1+\lambda/\mu)^n} \tag{8}$$

The total number of such states is $\binom{n}{i}$, therefore

$$P_{(i,k')} = P_{(i,k'')} = \widetilde{P}_i = \frac{P_i}{\binom{n}{i}} = \frac{(\lambda/\mu)^i}{(1+\lambda/\mu)^n} \tag{9}$$

Further, using $Z_i$ defining from expression (1), it is possible to define the number of disconnected states at $i$ failed edges.

$$Y_i = Z_i \binom{n}{i} \tag{10}$$

Similarly, the number of connected states

$$\binom{n}{i} - Y_i = (1-Z_i)\binom{n}{i} \tag{11}$$

If we substitute (6), (7), (8), (10) (11) into (4) and (5), and placing variables independent of $i$ and $k$ outside summation symbols, we obtain

$$T_{E_+} = \frac{\sum_{i=0}^{n}(1-Z_i)\binom{n}{i}\widetilde{P}_i}{\lambda\sum_{i=0}^{n}\widetilde{P}_i(n-i)\sum_{k'\in E_+} Z^{*}_{(i,k')}} \tag{12}$$

$$T_{E_-} = \frac{\sum_{i=0}^{n}(Z_i)\binom{n}{i}\widetilde{P}_i}{\mu\sum_{i=0}^{n}\widetilde{P}_i i\sum_{k''\in E_-} Z^{**}_{(i,k'')}} \tag{13}$$

Practical use of the obtained expressions is made difficult by necessity to consider all possible $2^n$ states of the systems for definition of $Z^{*}_{(i,k')}; Z^{**}_{(i,k'')}$.

In Tkachev (1999, 2010) it is shown, that defining of $Z_i$ value is sufficient to define the values of expressions (12) and (13).

If at $i$ failed edges the network is in one of connected states, then at one of *(n-i)* operational edges failure the network may transit to one *(n-i)* states with $i+1$ number of the failed edges. Let $S_{(i,k')}$ of them be disconnected, then from definition $Z^*_{(i,k')}$ follows that

$$Z^*_{(i,k')} = \frac{S_{(i,k')}}{n-i} \tag{14}$$

For example, $Z^*_{(2,1')} = 2/5$ ***(point A on Figure 2).***

Now, sum expression may be presented as $Z^*_{(i,k')}$.

$$\sum_{k' \in E_+} Z^*_{(i,k')} = \sum_{k' \in E_+} \frac{S_{(i,k')}}{n-i} = \frac{C_i'}{n-i} \tag{15}$$

where $C_i'$ - the total number of transitions from the set of connected states $(i,k')$ to the set of disconnected states $(i+1,k'')$.

For definition of $C_i'$ value the following reasoning may be employed.

If the network is in one of $Y_i$ disconnected states, then at failure of one of *(n-i)* operational edges the network may transit to another disconnected state with the number of failed edges $i+1$. The total number of transitions from disconnected states with the number of failed edges $i$ to disconnected states with the number of failed edges $i+1$ equals to $Y_i(n-i)$. Then value of $Y_{i+1}$ may be expressed through $Y_i$ and $C_i'$ as follows:

$$Y_{i+1} = \frac{Y_i(n-i) + C_i'}{i+1} \tag{16}$$

The denominator equals to $i+1$ because the network can transit in each state with the number of failed edges $i+1$ from $i+1$ states with the number of failed edges $i$ ***(point B on Figure 2)***.

On the other hand, in compliance with definition

$$Z_{i+1} = \frac{Y_{i+1}}{\binom{n}{i+1}} \tag{17}$$

If we substitute $Y_{i+1}$ from (16) to (17) we obtain

$$Z_{i+1} = \frac{Y_i(n-i) + C_i'}{(i+1)\binom{n}{i+1}} \tag{18}$$

In addition,

$$\binom{n}{i+1} = \binom{n}{i}\frac{n-i}{i+1} \tag{19}$$

If we substitute expressions (10) and (19) to (18), then after simplification we obtain

$$Z_{i+1} = \frac{Z_i\binom{n}{i}(n-i) + C_i'}{\binom{n}{i}(n-i)} \tag{20}$$

whence it follows that

$$C_i^{'} = (Z_{i+1} - Z_i)\binom{n}{i}(n-i) \tag{21}$$

Therefore

$$\sum_{k^{'} \in E_+} Z^{*}_{(i,k^{'})} = \frac{C_i^{'}}{n-i} = (Z_{i+1} - Z_i)\binom{n}{i} \tag{22}$$

Substituting (22) to (12), we obtain

$$T_{E_+} = \frac{\sum_{i=0}^{n}(1-Z_i)\binom{n}{i}\widetilde{P}_i}{\lambda\sum_{i=0}^{n}\widetilde{P}_i(n-i)(Z_{i+1}-Z_i)\binom{n}{i}} \tag{23}$$

After notation we obtain

$$R_i = (1-Z_i)\binom{n}{i}\widetilde{P}_i \tag{24}$$

$$Z_i^{*} = \frac{Z_{i+1}-Z_i}{1-Z_i} \tag{25}$$

Thus, expression (23) may be brought to the form

$$T_{E_+} = \frac{\sum_{i=0}^{n} R_i}{\lambda\sum_{i=0}^{n} R_i (n-i) Z_i^{*}} \tag{26}$$

The numerator of this expression is probability for the network to be in connected state at an arbitrary point of time. If we substitute $R_i$ from (24) and $\widetilde{P}_i$ from (9) we obtain

$$R = \frac{1}{(1+\lambda/\mu)^n}\sum_{i=0}^{n}(1-Z_i)\binom{n}{i}(\lambda/\mu)^i \tag{27}$$

By similar reasoning it is possible to obtain expression for system's mean time spent in the disconnected state from (13).

If at $i$ failed edges the network is in one of disconnected states, then at repair of one of $i$ edges the network may transit to one of $i$ states with the number of failed edges *i-1*. Let $S_{(i,k^{''})}$ of them be connected, then from $Z^{**}_{(i,k'')}$ definition follows that

$$Z^{**}_{(i,k^{''})} = \frac{S_{(i,k^{''})}}{i} \tag{28}$$

Now sum expression may be given: $Z^{**}_{(i,k'')}$.

$$\sum_{k^{''} \in E_-} Z^{**}_{(i,k^{''})} = \sum_{k^{''} \in E_-} \frac{S_{(i,k^{''})}}{i} = \frac{C_i^{''}}{i} \tag{29}$$

where $C_i^{''}$ - the total number of transitions from disconnected set $(i,k^{''})$ to connected set $(i-1,k^{'})$.

For definition of $C_i^{''}$ the following reasoning may be used.

If the network is in one of $Y_i$ disconnected states, then at repair of one of $i$ failed edges the network may transit to connected or disconnected state with the number of failed edges *i-1*. The total number of transitions from disconnected states with the number of failed edges $i$, is equal to

$(Y_i)*i$ . $C_i^{''}$ of them are transitions into connected states. Then value of $Y_{i-1}$ be expressed through $Y_i$ and $C_i^{''}$ as follows:

$$Y_{i-1} = \frac{(Y_i)i - C_i^{''}}{n-(i-1)} \tag{30}$$

The denominator equals to $n-(i-1)$ because the network can transit in each state with the number of failed edges *i-1* from $n-(i-1)$ states with the number of failed edges *i*.

From expression (30) we obtain

$$C_i^{''} = (Y_i)i - Y_{i-1}(n-(i-1)) \tag{31}$$

Substituting $Y_i$ from (10) in (31), we obtain

$$C_i^{''} = \binom{n}{i} Z_i - \binom{n}{i-1} Z_{i-1}(n-(i-1)) \tag{32}$$

In addition,

$$\binom{n}{i-1} = \binom{n}{i} \frac{i}{(n-(i-1))} \tag{33}$$

$$C_i^{''} = \binom{n}{i} i(Z_i - Z_{i-1}) \tag{34}$$

If we substitute $C_i^{''}$ value from (34) into (29) and expression (29) into (13), we get

$$T_{E\_} = \frac{\sum_{i=0}^{n} (Z_i) \binom{n}{i} \widetilde{P}_i}{\mu \sum_{i=0}^{n} \widetilde{P}_i i \binom{n}{i} (Z_i - Z_{i-1})} \tag{35}$$

After notation

$$Q_i = Z_i \binom{n}{i} \widetilde{P}_i \tag{36}$$

we obtain

$$T_{E\_} = \frac{\sum_{i=0}^{n} Q_i}{\mu \sum_{i=0}^{n} i Q_i (1 - Z_i^{**})} \tag{37}$$

where

$$Z_i^{**} = \frac{Z_{i-1}}{Z_i} \tag{38}$$

Results of calculation of $Y_i, Z_i, Z_i^{*}, Z_i^{**}$ values for the network on Figure 1 are presented in Table1.

Table 1. $Y_i, Z_i, Z_i^*, Z_i^{**}$ values for the network on Figure 1

| $i$ | $\binom{n}{i}$ | $Y_i$ | $Z_i$ | $Z_i^*$ | $Z_i^{**}$ |
|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 7 | 0 | 0 | 0,095238 | 0 |
| 2 | 21 | 2 | 0,095238 | 0,336842 | 0 |
| 3 | 35 | 14 | 0,4 | 1 | 0,238095 |
| 4 | 35 | 35 | 1 | 0 | 0,4 |
| 5 | 21 | 21 | 1 | 0 | 1 |
| 6 | 7 | 7 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 0 | 1 |

If we substitute $Y_i, Z_i, Z_i^*, Z_i^{**}$ values from Table 1 into expressions (26), (27), (37), where $\lambda$ =0,01 (1/hour) and $\mu$ = 1(1/hour), we obtain following values of considering reliability factors. $T_{E_+} = 23{,}769$ hour.; $R = 0{,}980645$ ; $T_{E_-} = 0{,}469$ hour.

## 3 COMPARISON WITH KNOWN RESULTS

For checking of the obtained expressions we define values of considered system reliability factors (Figure 3) for which analytical estimations are known.

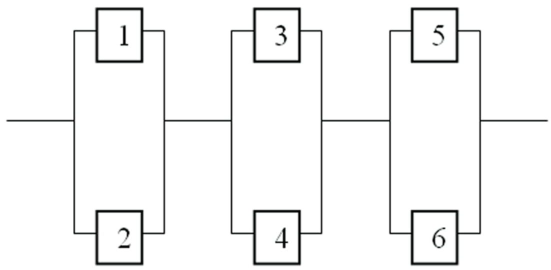


**Figure 3:** Parallel - series system

Following expressions may be found in Gnedenko&Belyayev&Solovyov (1965). For s identical elements connected in parallel with $\lambda$ and $\mu$ parameters.

$$t_{E_+} = \frac{1}{s*\mu}\left[\left(1+\frac{\mu}{\lambda}\right)^s - 1\right]$$

$$t_{E_-} = \frac{1}{s*\mu}$$

$$r = \frac{t_{E_+}}{t_{E_+} + t_{E_-}}$$

For k identical repairing subsystems connected in series.

$$T_{E_+} = \frac{1}{k*(1/t_{E_+})}$$

$$R = r^k$$

$$T_{E_-} = T_{E_+}\frac{1-R}{R}$$

Substituting in this expressions $s = 2, k = 3$, $\lambda$ =0,1(1/hour), $\mu$ =1 (1/hour) we obtain

$T_{E_+} = 20{,}000$ hour.; $R = 0{,}975411$; $T_{E_-} = 0{,}504$ hour.

For definition of these reliability factors using expressions (26), (27), (37), values of $Y_i$ and $Z_i$ should be defined.

It is obvious that $Y_1 = 0$ and $Z_1 = 0{,}0$. In case of 2 elements failure there are three combinations resulting in disconnection (nonoperativity) of the system (1,2), (3,4), (5,6). The total number of probable combinations is 12, hence $Y_2 = 3$ and $Z_2 = 3/12$. For definition of $Y_3$ and $Z_3$ let us consider all probable combinations of 3 elements (Table 2). As followed from the results provided in Table 2, $Y_3 = 12$ and $Z_3 = 12/20$. At failure of 4 or more elements, the system will be disconnected, therefore $Z_4 = Z_5 = Z_6 = 1$.

Table 2. State of the system on Figure 3 at failure of 3 elements.

| | | | | | |
|---|---|---|---|---|---|
| 1 | 123 | - | 11 | 234 | - |
| 2 | 124 | - | 12 | 235 | + |
| 3 | 125 | - | 13 | 236 | + |
| 4 | 126 | - | 14 | 245 | + |
| 5 | 134 | - | 15 | 246 | + |
| 6 | 135 | + | 16 | 256 | - |
| 7 | 136 | + | 17 | 345 | - |
| 8 | 145 | + | 18 | 346 | - |
| 9 | 146 | + | 19 | 356 | - |
| 10 | 156 | - | 20 | 456 | - |

Table 3. Calculation results of system performance (for Figure 3)

| $i$ | $\binom{n}{i}$ | $P_i$ | $Y_i$ | $Z_i$ | $Z_i^*$ | $Z_i^{**}$ | $Q_i$ | $R_i$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0,5644739301 | 0 | 0 | 0 | 0 | 0,0000000000 | 0,5644739301 |
| 1 | 6 | 0,3386843580 | 0 | 0 | 0,2 | 0 | 0,0000000000 | 0,3386843580 |
| 2 | 15 | 0,0846710895 | 3 | 0,2 | 0,5 | 0 | 0,0169342179 | 0,0677368716 |
| 3 | 20 | 0,0112894786 | 12 | 0,6 | 1 | 0,33333 | 0,0067736872 | 0,0045157914 |
| 4 | 15 | 0,0008467109 | 15 | 1 | 0 | 0,6 | 0,0008467109 | 0,0000000000 |
| 5 | 6 | 0,0000338684 | 6 | 1 | 0 | 1 | 0,0000338684 | 0,0000000000 |
| 6 | 1 | 0,0000005645 | 1 | 1 | 0 | 1 | 0,0000005645 | 0,0000000000 |

The rest results are presented in Table 3. Substituting the data from Table 3 into expressions (26), (27), (37), we obtain:

$T_{E_+} = 20{,}000$ hour.; $R = 0{,}975411$; $T_{E_-} = 0{,}504$ hour.

## 4 EXAMPLES OF NETWORK RELIABILITY ANALYSIS

We will consider biconnected networks which node degrees satisfy to the inequality $2 \le k \le 3$.

Let us denote the number of nodes by *m* and the number of edges by *n*. It is necessary to pay attention that for each pair of *(m,n)* there is a topology providing the maximum level of reliability. For definition of such topologies the algorithms offered in Artamonov (1999) have been used.

**Example 1. Ring with one diagonal m=20, n=21**

It is obvious that $Y_1 = 0$ and $Z_1 = 0$. Considering combinations of 2 edges one may see that they form a cut even when they belong to the same chain. In addition, any combination of 2 edges which belong to the same chain is a cut itself. Calculation results of values $Y_i$ and $Z_i$ for the network (Figure4) are presented in Table 4a. Table 4b includes $T_{E_+}$, $T_{E_-}$ and $R$ values calculated for various λ. Value $\mu=1$. Here and elsewhere the unit of measurement for $\lambda$ and $\mu$ is 1/hour, for $T_{E_+}$ and $T_{E_-}$ - hour.

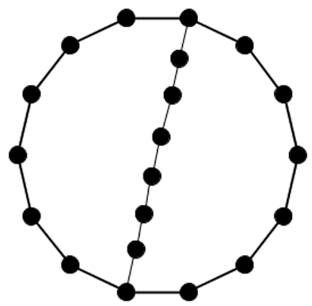

**Figure 4.** Network with parameters m=20, n=21

Table 4a. $Y_i$ and $Z_i$ values for the network presented on Figure 4.

| i | 1 | 2 | ≥3 |
|---|---|---|---|
| Yi | 0 | 63 | $\binom{n}{i}$ |
| Zi | 0 | 0,3000 | 1 |

Table 4b. $T_{E_+}$, $T_{E_-}$, $R$ valuesfor the network presented on Figure 4.

| $\lambda$ | $T_{E_+}$ | $T_{E_-}$ | $R$ |
|---|---|---|---|
| 0,1 | 1,13 | 0,6983 | 0,617547 |
| 0,01 | 79,56 | 0,4997 | 0,993759 |
| 0,001 | 7928,59 | 0,4995 | 0,999937 |

**Example 2. Network with parameters m=20, n=24**

As presented in Artamonov (1999), to reach the maximum reliability it is necessary to find the minimal diameter network with *2(n-m)* nodes of degree 3, and distribute nodes of degree 2 on the edges of this network equally. The network presented on Figure 5 may be found as a result.

$Y_i$ values of this network may be defined by exhaustion of all possible states. Results of calculations are presented Table 5a and Table 5b.

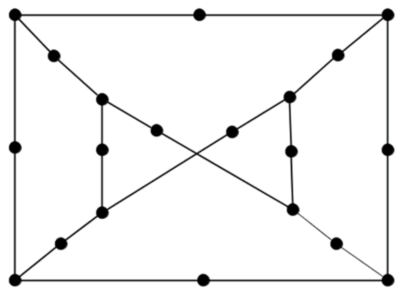

**Figure 5.** Network with parameters m=20, n=24.

Table 5a. Values $Y_i$ and $Z_i$ for the network on Figure 5.

| i | 1 | 2 | 3 | 4 | 5 | ≥ 6 |
|---|---|---|---|---|---|---|
| Yi | 0 | 12 | 328 | 4082 | 29960 | $\binom{n}{i}$ |
| Zi | 0 | 0,043478 | 0,162055 | 0,384252 | 0,704875 | 1 |

Table 5b. Values $T_{E_+}$, $T_{E_-}$, $R$ for the network on Figure 5.

| $\lambda$ | $T_{E_+}$ | $T_{E_-}$ | $R$ |
|---|---|---|---|
| 0,1 | 2,13 | 0,3334 | 0,864576 |
| 0,01 | 390,86 | 0,4845 | 0,998762 |
| 0,001 | 41415,64 | 0,4986 | 0,999988 |

## 5 USE OF OTHER OPERABILITY CRITERIA AND MODELS OF NETWORK RELIABILITY

The connectivity of all nodes of the network has been used as operability criterion so far. However the following question would be appropriate: whether loss of communication with one of nodes is a failure? If we consider the loss of connectivity with *k* nodes to be admissible, then the following network operability criterion may be used: the network is considered operational, if number of connected nodes $\geq (m-k)$.

The obtained results may be also used in this case, but respective alterations in algorithm of definition of values $Y_i$ should be made. In Table 6 values $T_{E_+}$ and *R* at various values of *k* are presented for the network on Figure 5.

Table 6. Values $T_{E_+}$ and $R$ at various values of *k*,
λ=0,01, μ=1 for the network on Figure 5.

| $k$ | $T_{E_+}$ | $R$ |
|---|---|---|
| 1 | 5780,49 | 0,999943591 |
| 2 | 9843,50 | 0,999967012 |
| 3 | 31455,53 | 0,999990157 |

It is possible to weaken restriction on equireliability of edges. It is possible to present each edge in the form of serial connection of certain quantity of equireliable elements. Thus fictitious nodes are entered into network structure. After such transformations values of $Z_i$ are defined. If one of elements making an edge fails, then this edge is removed from the network. Network connectivity is checked without considering of fictitious nodes.

It should be mentioned that obtained results may be used in case of failure of nodes when edges are absolutely reliable. Node failure may be modeled by removal of all edges emanating from this node.

This method of the analysis of networks reliability can be used also when those states are only considered as operable at which value of certain network parameters, for example, the network capacity will satisfy to preset values.

For high dimension networks the statistical estimation of values $Z_i$ can be defined using a Monte-Carlo method. The random combination of i fault elements is generated, and then operability of a network is checked. The ration of an amount of trials at which the network will appear non-operable to the total number of trials will be statistical estimation $Z_i$.

However it is necessary to consider that for reaching of high accuracy of statistical estimation $Z_{i,}$ the amount of trials should be great enough ($10^5$-$10^6$).

## 6 CONCLUSION

Analytical expressions defining reliability factors of networks consisting of identical elements are obtained. These elements fail and repair independently from each other and have exponentially distributed time to failure ad time to repair. Moreover both nodes and edges of the network may be regarded as absolutely reliable. Fidelity of the obtained result is vindicated by calculation of reliability factors of redundant system for which analytical estimations are known.

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# A GENERAL ANALYTICAL SOLUTION FOR THE OCCURRENCE PROBABILITY OF A SEQUENCE OF ORDERED EVENTS FOLLOWING POISSON STOCHASTIC PROCESSES

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## ABSTRACT

The author presents a general analytical solution determining "the Occurrence probability of a sequence of events, each following a Poisson Stochastic Process". Generally, this probability is described under the form of an integral equation of order "$n$". Where "$n$" is number of the elementary independent events in the examined sequence.
As far as the author can tell, the solution is original. It is of a great interest to a wide range of system reliability problems such as: sequential calculations, dominos effects, dynamics fault trees, Markov systems, priority AND gates, stochastic optimization, acceleration techniques for Monte-Carlo simulation, …

Key words: Ordered events, sequential events, Poisson stochastic Process, Markov, probability

## 1 Introduction

The author is interested in determining "the occurrence probability $p_n(t)$ of a well-defined sequence of ordered events obeying Poisson stochastic processes".

One meets often ordered events in system reliability analyses. Analysts may use "Dynamic Fault Tree" with "Priority Gates", "Markov Graphs" or "Monte-Carlo Simulation" tools in order to deal with the dynamic aspect of this problem.

A fault tree can be described by means of some cut sets. One may calculate the occurrence probability of each cut set. However, the calculated probability does not tell about the occurrence order of the involved events in the cut set. A cut-set, containing $n$-independent events, may be expressed in $n!$ different ordered sequences. In many reliability and risk assessments, only some given ordered sequences may be of specific concerns. Consequently, it is of great interest to determine the occurrence probabilities of these sequences and their occurrence rates.

In the paper, one describes the problem in the form of a given integral in section 3 and a differential equation in section 4. Fussell (1976) uses the same integral equation as we do, but the events are arranged in the opposite order. He uses Laplace transformation to find out an exact solution. Although the solution is exact, Fussell switched on to use the asymptotic form of the solution. Yuge (2008) uses the same differential equation given previously in section 4 in order to model the sequential occurrence of events in a given priority AND gate (PAG). He uses Laplace transformation and find out the exact solution of the occurrence probability $p_n(t)$. However, in both cases, the authors used complicated forms such that it did not allow putting in evidence the recurrence aspect of the solution. They were more concerned by inserting their models (PAG, ...) in a Dynamic Failure Tree than by other aspects of the solution.

However, if they had not excluded the use of an equivalent Markov Graph, they would have noticed this interesting recurrence aspect of the model, and maybe, they would have used a simpler expression of the solution.

Many other authors followed almost the same way on modeling and produced very interesting applications, see Walker (2007), Rai (1995), Merle (2010, IEEE Trans.), Merle (2009) and Merle (2010, ESRel), without perceiving that the exact solution could be put in a simpler form.

This would have extended the solution to other categories of problems rather than just the modeling of the PAG's and their inclusion in Dynamic Failure Trees.

Some other researchers could solve the same problem in using numerical techniques such as Petri Nets of Dynamic Bayesian Net (DBN). The use of a numerical technique prevents all possibility to underline the analytical solution and its interest. This is the case of Montani (2008) in an application relative to an active heat rejection system (AHRS) that he used to validate the methodology. The case would have been solved immediately if one had applied the analytical solution given herein. The problem contained only 8 sequences with a maximum length of 4 successive independent failures.

One may equally mention the case of the applications treated in Dobson (2004), Dobson (2005) and Anghel (2007), where the general analytical solution would have helped in treating easily the problems and in an exact way.

Regarding rare sequential events, many researchers sought answers in developing methods based on Monte-Carlo simulation, Adamyan (2002) and Johasen (2006). Some other authors have developed interesting methods as well as they have developed solutions very close to the one proposed here, see Sato (1985), Walker (2009), Yuge (2008), Liu (2007), Manian (1998), Long (2000), Kohda (2003) and the NUREG-094. Two papers should particularly be underlined. These are the papers of Long (2000) and Kohda (2003).

The work presented here is limited to Poisson stochastic processes. However, it is of high interest because it will obviously improve the numerical procedures used to treat practical industrial cases.

The analytical solution of this problem presents a particular interest by itself, because of its originality and simplicity. Besides, it suggests some interesting directions of investigation so that it may help in developing analytical solutions for some other specific time distributions different from Poisson ones.

## 2 Problem definition

Let $T$ describe a top event which results from the occurrence of some $n$ basic events $e_i$ ( $i = 1,2,...,n$ ) in a well determined sequential order. Basic events $e_i$ are following Poisson stochastic processes and each is fully characterized by a constant occurrence rate $\lambda_i$ and by its occurring order '$i$'. The $e_1$ is the 1st occurred and $e_n$ is the last event.

One would like to determine the occurrence probability of the top event $T$ and its occurrence rate.

## 3 Problem modelling

A will defined top event $T$ will occur if and only if some independent events $e_i$ happen according to a well specified order $[e_1, e_2, e_3, ..., e_n]$. The corresponding occurring instants are defined by $[t_1, t_2, t_3, ..., t_n]$, where $[t_1 < t_2 < t_3 < ... < t_n]$. Each of these instances $[t_1, t_2, t_3, ..., t_n]$ has its distribution probability function (pdf). The first event is $e_1$ and the last one is $e_n$.

The probability $p_n(t)$ that Event $T$ happens within the interval [0,t] is given by:

$$p_n(t) = \int_0^t \rho_1(\xi_1)d\xi_1 * \int_{\xi_1}^t \rho_2(\xi_2)d\xi_2 * \int_{\xi_2}^t \rho_3(\xi_3)d\xi_3 * \ldots * \int_{\xi_{n-2}}^t \rho_{n-1}(\xi_{n-1})d\xi_{n-1} * \int_{\xi_{n-1}}^t \rho_n(\xi_n)d\xi_n \quad .........(1)$$

Where: $0 \le \xi_1 \le \xi_2 \le \xi_3 \le ... \le \xi_n \le t$ and, $\rho_i$ is the Poisson density function characterising the event $e_i$ [$\rho_i = \lambda_i * e^{-\lambda_i t}$].

Many authors could previously developed analytical solutions for Equation 1 when it was a matter of limited number of ordered events obeying a Poisson's Stochastic Process, e.g. Fussell (1976), Yuge (2008), Long (2000) and Kohda (2003).
Here, the paper develops a simpler form of the exact solution of Equation 1.

## 4 Analytical Solution

It is obvious that Equation 1 can equally be expressed on the following differential form:

$$\frac{d}{dt} p_{n+1}(t) = \rho_{n+1}(t)\ p_n(t) \quad ............(2)$$

Let $p_n(t)$ be the occurrence probability of a sequence $T$, a set of chronologically ordered events $[e_1, e_2, e_3, ..., e_n]$. The probability $p_n(t)$ is the solution of the Equations 1 and 2.

Let $p_n(t)$ be expressed by following expression:

$$p_n(t) = \sum_{j=1}^{n} C_j^n * (1 - e^{-(\sum_{l=n-j+1}^{n} \lambda_l)t}), \quad ............(3)$$

Where, each event $e_i$ is defined by a constant occurrence rate $\lambda_i$, $\{i \in [1,2,...,n]\}$.

The solution of the problem resumes in determining the coefficients $C_i^n$.

In appendix (1), we demonstrate the solution proposed in Equation 3 and show that the coefficients $C_j^i$ are fully determined thanks to some recurrence pattern, as following:

$$C_1^{i+1} = \sum_{j=1}^{i} C_j^i, \qquad C_{j+1}^{i+1} = -\frac{\lambda_{i+1}}{\sum_{l=i-j+1}^{i+1} \lambda_l} C_j^i, \qquad j = 1,2,...,i, \text{ and } i \in [1,2,...,n] \quad ........(4)$$

Some examples for calculating the parameters $C_j^i$ are given in appendix (2).

### *4.1 Occurrence Density and Occurrence rate*

By definition, the corresponding occurrence density function $\Theta_i(t)$ can directly be deduced via the first derivative of the occurrence probability function as following:

$$\Theta_n(t) = \frac{dp_n(t)}{dt} \qquad ............(5)$$

The occurrence density function $\Theta_n(t)$ will then be determined by:

$$\Theta_n(t) = \sum_{j=1}^{n} C_j^n * (\sum_{l=n-j+1}^{n} \lambda_l) e^{-(\sum_{l=n-j+1}^{n} \lambda_l) t} \qquad ............(6)$$

We may also define an equivalent occurrence rate $\Lambda_i(t)$ of the whole sequence $T$, such as:

$$\Lambda_i(t) = \frac{1}{p_i(t)} * \frac{dp_i(t)}{dt} = \frac{\sum_{j=1}^{i} (\sum_{l=i-j+1}^{i} \lambda_l) * C_j^i e^{-(\sum_{l=i-j+1}^{i} \lambda_l) t}}{\sum_{j=1}^{i} C_j^i * (1 - e^{-(\sum_{l=i-j+1}^{i} \lambda_l) t})} \qquad ............(7)$$

As we may expect, although the ordered events are individually governed by a Poisson Stochastic Process, the sequence $T$ is not. The occurrence rate of the sequence $T$ is time dependent, Equation 7.

### *4.2 Mean Occurrence Time*

One may also determine the mean occurrence time $\tau_n$ corresponding to a given sequence ($S_n$) of n-events $\{e_1, e_2, e_3, ..., e_n\}$, such as:

$$\tau_n = \int_{t=0}^{\infty} t * dp_n(t) \qquad ............(8)$$

The solution of Equation 8 is elementary and can be determined by:

$$\tau_n = \sum_{j=1}^{n} \frac{C_j^n}{(\sum_{l=n-j+1}^{n} \lambda_l)} \qquad ............(9)$$

### *4.3 Asymptotic Behaviour*

Having demonstrated that the occurrence probability $p_n(t)$ of a given sequence of n-well defined ordered events can be described by Equation 3, it is straightforward to demonstrate that the occurrence probability $p_n(t)$ has an asymptotic value equal to:

$$p_n(t \to \infty) \quad \to \sum_{j=1}^{n} C_j^n \qquad \text{...........(10)}$$

The occurrence probability density function $\Theta_n$ of the sequence $S_n$ has an asymptotic value equal to:

$$\Theta_n(t \to \infty) \quad \to \quad 0. \qquad \text{...........(11)}$$

Similarly, the equivalent occurrence rate $\Lambda_n$ of the sequence $S_n$ has an asymptotic value equal to:

$$\Lambda_n(t \to \infty) \quad \to \quad 0. \qquad \text{...........(12)}$$

## 5 Numerical Application

Two illustrative numerical applications are given in the following in order to help in sizing the interest of having a generic analytical solution determining the occurrence probability of a given sequence ($S_n$) of n-events $\{e_1, e_2, e_3, ..., e_n\}$ in the given order.

### *5.1 Occurrence Order*

In this application, we are focusing on the dependence of the occurrence probability on the occurrence order of the basic events.

A very simple example is illustrated in figure 1 where the time evolution of the occurrence probability of a sequence of four basic events $[e_1, e_2, e_3, e_4]$ whose occurrence rates are constant and having the following values: $10^{-4}$ /h, $5*10^{-3}$ /h, $2.5*10^{-2}$ /h, $1.25*10^{-1}$ /h. In Figure 1, we are comparing two configurations represented by a red curve and a blue one.

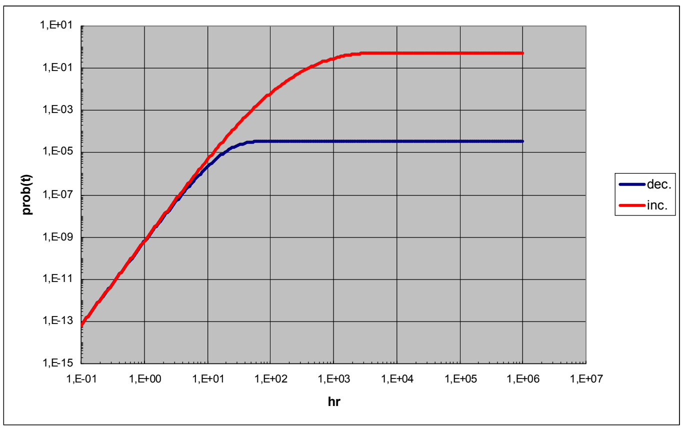


**Figure 1.** The occurrence probability of the same set of events in two different orders (dec: decreasing order, inc.: increasing order)

The red curve represents the case where the sequence of events follows the increasing order of the occurrence rates (less frequent occurs first). While, the blue curve describes the case where the sequence of events following the decreasing order of the occurrence rates (more frequent occurs first).

It is obvious that the occurring order of events impacts on the time behaviour of the occurring probability of any sequence of events.

The asymptotic behaviour of the occurrence probability can also be underlined.

### *5.2 Treatment of a Markov Graph*

In this example, a given system is described by Markov graph. The system has 8 possible states. The transitions between different states are fully determined by their transition rates.

In this illustrative example, Figure 2, a unique transitions rate value of $10^{-1}\ h^{-1}$ is considered for all transition rates as following:

$$\lambda_{12} = \lambda_{13} = \lambda_{14} == \lambda_{25} = \lambda_{52} = \lambda_{36} = \lambda_{47} = \lambda_{56} = \lambda_{68} = 10^{-1}\ h^{-1}$$

We are interested in the sequences $S_n$ leading to the absorbing states $e_7$ or $e_8$, which are the following:

$$S_3 = e_1 \rightarrow e_4 \rightarrow e_7,$$
$$S_4 = e_1 \rightarrow e_3 \rightarrow e_6 \rightarrow e_8,$$
$$S_5 = e_1 \rightarrow e_2 \rightarrow e_5 \rightarrow e_6 \rightarrow e_8,$$
$$S_{5+2n} = e_1 \rightarrow (e_2 \leftrightarrow e_5)^n \rightarrow e_6 \rightarrow e_8, \qquad n = 0,1,2,...$$

Where;

$$(e_2 \leftrightarrow e_5)^0 = e_2 \rightarrow e_5$$
$$(e_2 \leftrightarrow e_5)^1 = e_2 \rightarrow (e_5 \rightarrow e_2) \rightarrow e_5$$
$$(e_2 \leftrightarrow e_5)^2 = e_2 \rightarrow (e_5 \rightarrow e_2) \rightarrow (e_5 \rightarrow e_2) \rightarrow e_5$$

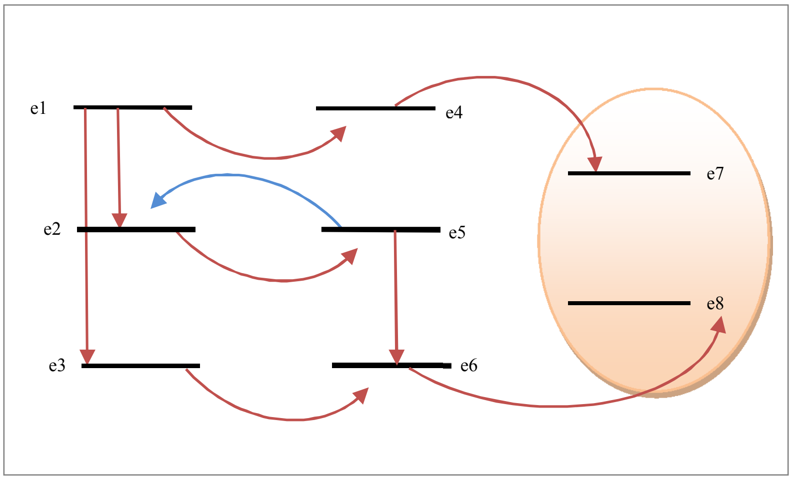


**Figure 2.** Schematic presentation of a Markov Graph

One would, then, like to determine for each sequences $(S_3, S_4, S_5, S_7)$ its occurrence probability time-profile, $p_n(t)$. The occurrences probabilities of the sequences $S_3, S_4, S_5, S_7$ are illustrated in Figure 3. Higher order sequences would have much lower contribution than that of $S_7$ as illustrated in Figure 3.

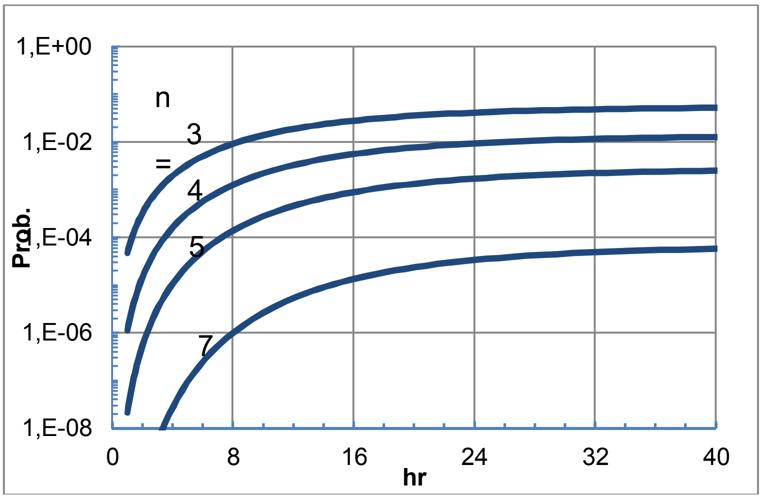


**Figure 3.** Occurrence probability as a time-function of each sequence rank ($S_3, S_4, S_5, S_7$)

The asymptotic values of the occurrence probabilities of the sequences ($S_3, S_4, S_5, S_7,...$) are illustrated for the application in Figure 4.

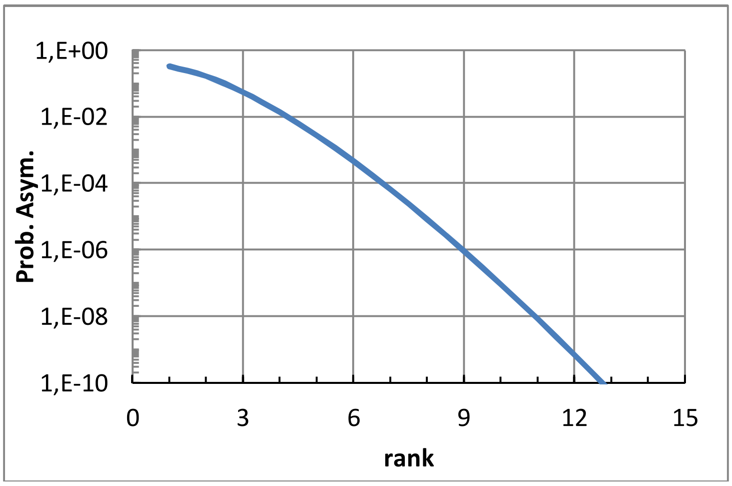


**Figure 4.** The asymptotic probability as a function of the sequence rank

Finally, one would also determine the mean occurrence time of the sequences ($S_3, S_4, S_5, S_7,...$) as a function of the sequence order, Figure 5.

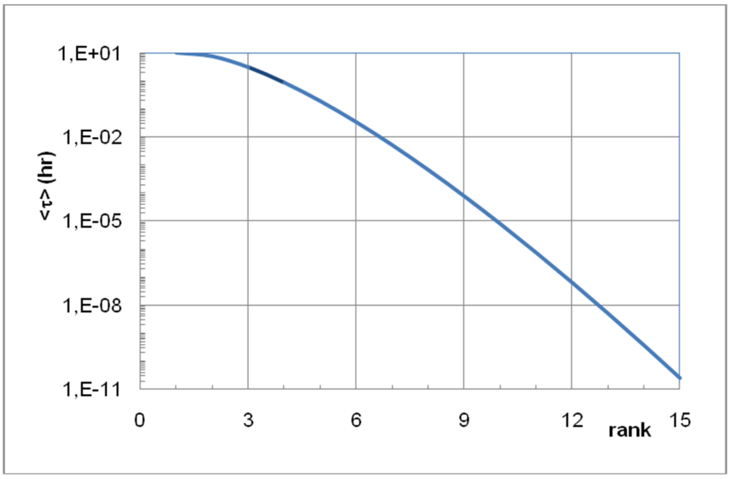


**Figure 5.** Meantime of occurrence as a function of the sequence length

## 6 Conclusion

The paper proposes a general solution in order to determine the occurrence probability of a given sequence of n-events following Stochastic Poisson's Processes. The solution is analytical and original.

Some interesting asymptotic characteristics of this analytical solution have been assessed.

Two simple numerical applications are illustrated in order to underline the interest of possessing an analytical generic solution to this problem.

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## Appendix (1)

Let $p_n(t)$, the occurrence probability of the sequence T, be the solution of the Equation 0 and 0 and be expressed as:

$$p_i(t) \; = \sum_{j=1}^{i} C_j^i * (1 - e^{-(\sum_{l=i-j+1}^{i} \lambda_l)t}), \; C_1^1 = 1.0 \text{ and } i \in [1,2,...,n]$$

$$p_{i+1}(t) = \sum_{j=1}^{i+1} C_j^{i+1} * (1 - e^{-(\sum_{l=i-j+2}^{i+1} \lambda_l)t}), \; C_1^1 = 1.0 \text{ and } i \in [1,2,...,n]$$

And,

$$\frac{d}{dt} p_{n+1}(t) = \rho_{n+1}(t) \; p_n(t)$$

Where Sequence T contains $i$ ordered events, each is defined by an occurrence rate $\lambda_j$.

$$\frac{d}{dt}p_{i+1}(t)=\frac{d}{dt}\sum_{j=1}^{i+1}C_j^{i+1}*(1-e^{-(\sum_{l=i-j+2}^{i+1}\lambda_l)t})=\sum_{j=1}^{i+1}C_j^{i+1}(\sum_{l=i-j+2}^{i+1}\lambda_l)*e^{-(\sum_{l=i-j+2}^{i+1}\lambda_l)t}$$

$$=C_1^{i+1}*\lambda_{i+1}e^{-\lambda_{i+1}t}+\sum_{j=2}^{i+1}C_j^{i+1}(\sum_{l=i-j+2}^{i+1}\lambda_l)*e^{-(\sum_{l=i-j+2}^{i+1}\lambda_l)t}$$

$$=C_1^{i+1}*\lambda_{i+1}e^{-\lambda_{i+1}t}+\sum_{k=1}^{i}C_{k+1}^{i+1}(\sum_{l=i-k+1}^{i+1}\lambda_l)*e^{-(\sum_{l=i-k+1}^{i+1}\lambda_l)t}$$

While;

$$\rho_{n+1}(t)\ p_n(t)=\lambda_{i+1}*e^{-\lambda_{i+1}t}\sum_{j=1}^{i}C_j^i*\left(1-e^{-(\sum_{l=i-j+1}^{i}\lambda_l)t}\right)$$

$$=\sum_{j=1}^{i}C_j^i*\lambda_{i+1}*\left(e^{-\lambda_{i+1}t}-e^{-(\sum_{l=i-j+1}^{i+1}\lambda_l)t}\right)$$

$$=\left(e^{-\lambda_{i+1}t}*\sum_{j=1}^{i}C_j^i*\lambda_{i+1}\right)-\left(\sum_{j=1}^{i}C_j^i*\lambda_{i+1}*e^{-(\sum_{l=i-j+1}^{i+1}\lambda_l)t}\right)$$

So,

$$C_1^{i+1}*\lambda_{i+1}e^{-\lambda_{i+1}t}+\sum_{k=1}^{i}C_{k+1}^{i+1}(\sum_{l=i-k+1}^{i+1}\lambda_l)*e^{-(\sum_{l=i-k+1}^{i+1}\lambda_l)t}=\left(e^{-\lambda_{i+1}t}*\sum_{j=1}^{i}C_j^i*\lambda_{i+1}\right)-\left(\sum_{j=1}^{i}C_j^i*\lambda_{i+1}*e^{-(\sum_{l=i-j+1}^{i+1}\lambda_l)t}\right)$$

1st Condition

$$\lambda_{i+1}*\left(C_1^{i+1}-\sum_{j=1}^{i}C_j^i\right)*e^{-\lambda_{i+1}t}=0$$

Then;

$$C_1^{i+1}=\sum_{j=1}^{i}C_j^i\ ,\quad i\in[1,2,...,n]$$

2nd Condition

$$\sum_{j=1}^{i}\left(C_{j+1}^{i+1}(\sum_{l=i-j+1}^{i+1}\lambda_l)+C_j^i*\lambda_{i+1}\right)*e^{-(\sum_{l=i-j+1}^{i+1}\lambda_l)t}=0$$

Then,

$$C_{j+1}^{i+1} = -\frac{\lambda_{i+1}}{\sum_{l=i-j+1}^{i+1} \lambda_l} * C_j^i, \ \forall \ j \in [1,i] \quad \text{and} \quad i \in [1,2,...,n]$$

## Appendix (2):

Consider the case of a sequence containing 4 basic events in some well-determined order, so we have :

$$C_1^{i+1} = \sum_{j=1}^{i} C_j^i, \text{ and } \quad C_1^1 = 1$$

$$C_{j+1}^{i+1} = -\lambda_{i+1} \cdot \frac{C_j^i}{(\sum_{l=i-j+1}^{i+1} \lambda_l)} \quad j = 1,2,...,i$$

We may, then, find out the coefficients $C_j^i$ as following:

$N = 1$

$C_1^1 = 1$

$N = 2$

$C_1^2 = 1,$

$$C_2^2 = -\frac{\lambda_2}{\lambda_2 + \lambda_1}$$

$N = 3$

$$C_1^3 = (1 - \frac{\lambda_2}{\lambda_2 + \lambda_1}),$$

$$C_2^3 = -\frac{\lambda_3}{\lambda_3 + \lambda_2},$$

$$C_3^3 = +\frac{\lambda_3}{\lambda_3 + \lambda_2 + \lambda_1} * \frac{\lambda_2}{\lambda_2 + \lambda_1}$$

$N = 4$

$$C_1^4 = (1 - \frac{\lambda_2}{\lambda_2 + \lambda_1}) - (\frac{\lambda_3}{\lambda_3 + \lambda_2}) + (\frac{\lambda_3}{\lambda_3 + \lambda_2 + \lambda_1} * \frac{\lambda_2}{\lambda_2 + \lambda_1})$$

$$C_2^4 = -\frac{\lambda_4}{\lambda_4 + \lambda_3} * (1 - \frac{\lambda_2}{\lambda_2 + \lambda_1})$$

$$C_3^4 = \frac{\lambda_4}{\lambda_4 + \lambda_3 + \lambda_2} * \frac{\lambda_3}{\lambda_3 + \lambda_2}$$

$$C_4^4 = -\frac{\lambda_4}{\lambda_4 + \lambda_3 + \lambda_2 + \lambda_1} * \frac{\lambda_3}{\lambda_3 + \lambda_2 + \lambda_1} * \frac{\lambda_2}{\lambda_2 + \lambda_1}$$

So, the occurrence probability of this given sequence will, then, be determined by:

$$
\begin{aligned}
p_4(t) &= \sum_{j=1}^{4} C_j^4 * (1 - e^{-(\sum_{l=4-j+1}^{4} \lambda_l)t}) \\
&= C_1^4 * (1 - e^{-\lambda_4 t}) \\
&+ C_2^4 * (1 - e^{-(\lambda_4 + \lambda_3)t}) \\
&+ C_3^4 * (1 - e^{-(\lambda_4 + \lambda_3 + \lambda_2)t}) \\
&+ C_4^4 * (1 - e^{-(\lambda_4 + \lambda_3 + \lambda_2 + \lambda_1)t})
\end{aligned}
$$

# OPTIMAL DISPATCHING OF GENERATORS WITH LOAD DEPENDENT FAILURE RATES

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## ABSTRACT

An optimally coordinated energy dispatching among generating units may contribute to attain higher performances of electric networks. Within this wide research field this paper focuses on a theoretical study aimed at proposing an optimal dispatching policy maximizing reliability and Mean Time To Fault of a park of programmable generating units whose failure rate models take into account the dependency from the instantaneous loading condition.

## 1 INTRODUCTION

The operation environment of power systems is becoming increasingly dynamical due to the continually evolving functions required, Madani & King (2008), Bose (2010).

It is a fact that operational and environmental conditions have a significant effect on accelerating or decelerating the rate of degradation processes which occur prior to failure. Most conventional failure models are developed on the premise that the prevailing environmental and operating conditions either do not change in time, or have no effect on deterioration and failure processes. These hypotheses may give misleading results whose accuracy does not fulfill the requirements imposed by an envisaged technical and economical dynamic environment in which future power systems are called to operate.

In the future scenarios more advanced reliability analysis tools are required.

The intent of this paper is to focus on an optimal dispatching policy of a heterogeneous park of programmable generating units whose failure rate model is supposed dependent on the loading condition.

## 2 FAILURE RATE AND COVARIATES

A lot of literature has been devoted to the study of failure models for applications in reliability engineering. A comprehensive survey of the settled failure models currently adopted in this field may be found in Singpurwalla (1995).

Among these failure models, in engineering systems analysis the approach based on the definition of a failure rate function $\lambda(t)$ plays a key role in reliability and survival analysis; it has generally turned out to be a useful device for expressing a connection between the physics of failure and the probability of survival. Condition-Dependent Failure Rate (CDFR) models are currently an interesting research issue in reliability engineering also applied to power systems, Pan et al. (2009), Sun et al. (2010), Wang & Xie (2008).

In conventional reliability evaluation of power systems failure rate, derived from the collected statistical data, has usually been assumed to be constant; it has been realised from the real-time operation that it is not constant and varies with several parameters: in details λ(t) may depend upon explanatory variables (covariates) such as loading conditions, installation environment, manufacturer, etc. When such is the case, the explanatory variables become a part of the observable history, and so their effect needs to be incorporated in failure rate expression $\lambda(t, \boldsymbol{Z}(t), \boldsymbol{\theta}(t)$, where $\mathbf{Z}$(t) is a vector of covariates and $\boldsymbol{\theta}$(t) are other model's parameters, generally time dependent, Aalen (1989), Cox (1972), Peña & Hollander (2004), Singpurwalla (2006). The conceptual advantage of this model lies in the representation of a non observable entity, the failure rate, through observable and sometimes controllable entities.

Generating units' failure rates may depend on loading/stress history (e.g. active and reactive power, voltage, peak values, number of occurrence of a peak, highest derivative etc.) and environmental variables history (e.g. temperature, humidity, pollutants' concentration, etc.). Generally some of the loading variables are controllable; some environmental parameters may be controlled only in particular installation conditions.

In this paper the generating units' failure rate is assumed dependent on the active instantaneous loading status, $L(t)$, so $\lambda(t, L(t), \boldsymbol{\theta}(t)$.

Extensive studies in survival analysis, Cox & Oakes (1984), applied for instance to biostatistics, have proposed the following model:

$$\lambda(t, L(t)) = \lambda_O(t) \cdot \exp\left[\beta(t) \cdot L(t)\right] \tag{1}$$

Comprehensive studies on these dependences, under static loading conditions, based on data collected for some mechanical elements, e.g. contact bearing, spring, gear, etc., Carderockdiv. (1994), proposed the following model:

$$\lambda(t, L(t)) = \lambda_C(t) \cdot \left[\frac{L(t)}{L_c}\right]^y \tag{2}$$

Where $\lambda_c$ and $L_c$ are the reference failure rate and the corresponding load, respectively; $y$ denotes the load dependent exponent (up to 10).

## 2.1 Optimal Control

The covariate induced failure rate process $\lambda(t, \boldsymbol{Z}(t), \boldsymbol{\theta}(t)$ may be useful to identify optimal control strategies of a subset of $\boldsymbol{Z}(t)$, $\boldsymbol{Z}_C$, of controllable variables capable of optimizing expected reliability performances. The general criterion is to select the control/decision variables, subject to constraints, in order to maximize the expectation of an objective goal functional, over a time horizon.

Reliability based control applied to power systems may be effective for identifying the most economical policies that can be used to fulfil the expected mission and minimize life cycle costs. Different control concepts and different reliability target formulations can be evaluated; in particular reliability criteria based on crossing rates and on approximations of the extrema of random performance measures could be adopted.

The past decades have seen great development of reliability based optimal control methodologies in several engineering fields (e.g. civil, mechanical, marine, offshore); the research issue is still open because of the various difficulties in stochastic dynamics and reliability theory.

The prospected evolution of power systems paves the way for an efficient integration of these methodologies; moreover the rapid development of the monitoring technologies, particularly sensing techniques, Proceedings of CMD (2010), Proceedings of PHM (2010), non-invasive communication systems (e.g. wireless sensor networks), Baker et al. (2009), Lu et al. (2005), and their expected pervasive diffusion in future power systems for collecting field data may constitute a valid technological support for the actuation of novel reliability-based control techniques.

Implementing this architecture on power systems requires the development of algorithms to perform each of its separate functions. That is where some of the real technical challenges are found.

## 3 PROBLEM FORMULATION

Assume a set of n programmable generators, all of them characterized by a failure rate model as (1) or (2). The effect of the electrical network is neglected at this initial stage, so a one-node model is considered, Billington & Allan (1984). The objective is to identify a dispatching policy, which optimizes reliability performances. In particular as first objective is identified the minimization of the risk correlated with any intervention on the system; that is the maximization of the reliability of the series of all generators evaluated at the mission time T. This objective may be of interest in case of power systems installed in remote areas, or installed on-board, for which the costs of any intervention are high. Secondly will be evaluated an optimal loading policy maximizing Mean Time To Fault (MTTF) as generally required for improving availability performances and optimal maintenance policies.

In particular let $(T_1, T_2, \dots . T_n)$ be the random variables (r.v.) describing the operating life of each generator; given the failure rate of each generator $\lambda_i(t, L_i(t), \boldsymbol{\theta_i}(t)$. All the covariates are assumed external as defined in Kalbfleisch (2002). If the hazard potential $X_i$ of the generators are supposed independent, and given that $H_i(t)$ are known the r.v. $T_i$ are independent, Singpurwalla (2006), so the probability that all n generators are operating during the mission time T is:

$$R(T) = P(T_1 > T, T_2 > T, \dots . T_n > T | L_i(t), \boldsymbol{\theta_i}(t), \mathrm{i} = 1, \dots, \mathrm{n}) = \prod_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{P}(T_i > T | \, L_i(t), \boldsymbol{\theta_i}(\mathrm{t})) \tag{3}$$

## 4 DETERMINISTIC CASE

If the total load required to the generators L(t) and the failure rate model's parameters are known deterministic the formulation of the optimization problem is:

$$\max_{L_1(t),\dots,L_n(t)} [R(T)] \tag{4}$$
$$\text{s.t.}$$
$$\sum_{i=1}^{n} L_i(\tau) = L(\tau), \qquad \forall \tau \in [0, T]$$
$$L_i(\tau) \in [0, L_{iMAX}], \forall i \in [1,2,..,n], \forall \tau \in [0, T]$$

Considered the monotonicity of the argument function, problem (4) is equivalent to:

$$\max_{L_1(t),\dots,L_n(t)} \left[ -\int_0^T \sum_{i=1}^{n} \lambda_i(\tau, L_i(\tau), \boldsymbol{\theta_i}(t)) d\tau \right] \tag{5}$$

So, solving problem (5), the optimal dispatching $\mathbf{L}^*(\tau)$ is:

<u>*Model (1)*</u>

Let $\varphi(\tau)$:

$$\varphi(\tau) = \exp\left[ \frac{\mathrm{L}(\tau) + \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\ln(\lambda_{0\mathrm{i}}(\tau) \cdot \beta_{\mathrm{i}}(\tau))}{\beta_{\mathrm{i}}(\tau)}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{1}{\beta_{\mathrm{i}}(\tau)}} \right] \tag{6}$$

$$\text{if } \varphi(\tau) \geq \lambda_{0\mathrm{i}}(\tau) \cdot \beta_{\mathrm{i}}(\tau) \cdot \exp[\beta_{\mathrm{i}}(\tau) \cdot L_{iMAX}]$$
$$L_i^*(\tau) = L_{iMAX}$$

$$\text{if } \varphi(\tau) \leq \lambda_{0\mathrm{i}}(\tau) \cdot \beta_{\mathrm{i}}(\tau)$$

$$L_i^*(\tau) = 0$$

otherwhise

$$L_i^*(\tau) = \frac{L(\tau) + \sum_{\substack{j=1 \\ j\neq i}}^{n} ln\left(\frac{\lambda_{0j}(\tau)\cdot\beta_j(\tau)}{\lambda_{0i}(\tau)\cdot\beta_i(\tau)}\right)^{\frac{1}{\beta_j(\tau)}}}{1+\sum_{\substack{j=1 \\ j\neq i}}^{n}\frac{1}{\beta_j(\tau)}} \quad (7)$$

*Model (2)*

If $y_i \neq 1$ , let $\varphi(\tau)$ defined by:

$$\sum_{i=1}^{n}\left[\frac{L_{c_i}^{y_i}}{\lambda_{c_i}\cdot y_i}\varphi(\tau)\right]^{\frac{1}{y_i-1}} = L(\tau) \quad (8)$$

$$\varphi(\tau) \geq 0$$

$$\text{if } \varphi(\tau) \geq \lambda_{c_i}\cdot y_i \cdot \frac{L_{iMAX}^{y_i-1}}{L_{c_i}^{y_i}}$$

$$L_i^*(\tau) = L_{iMAX}$$

otherwhise

$$L_i^*(\tau) = \left[\frac{L_{c_i}^{y_i}}{\lambda_{c_i}\cdot y_i}\varphi(\tau)\right]^{\frac{1}{y_i-1}} \quad (9)$$

It can also be proved that $\boldsymbol{L}^*(\tau)$ maximize the MTTF.

### 4.1 Maximum MTTF for Periodic Deterministic Load

If the load $\mathrm{L(t)}$ is supposed deterministic and periodic, with period $h = \Delta T$, $\lambda_i$ not explicitly depending on the time, $\lambda_i(L_i(t), \boldsymbol{\theta_i}(t))$, and the $\boldsymbol{\theta_i}(t)$ parameters periodic too with the same period of the load (This last hypothesis could be physically justified by periodic maintenance policies), applying Bellman's optimality principle the maximum MTTF* is given by:

$$\mathrm{MTTF}^* = \max_{\substack{\mathbf{L}(t) \\ t\in[0,\Delta T]}}\left\{\int_0^{\Delta T}\mathrm{R(t)dt} + \mathrm{R}(\Delta T)\cdot \max_{\substack{\mathbf{L}(t) \\ t\in[\Delta T,+\infty]}}\{\mathrm{MTTF}\}\right\} =$$

$$= \max_{\substack{\mathbf{L}(t) \\ t\in[0,\Delta T]}}\left\{\int_0^{\Delta T}\mathrm{R(t)dt} + \mathrm{R}(\Delta T)\cdot \mathrm{MTTF}^*\right\} \quad (10)$$

It's worth noting that this formulation is possible because ageing is neglected.

Equation (10) is the recurrent formulation of the maximization problem.

Banach fixed point theorem guarantees the existence and uniqueness of a fixed point MTTF*:

$$MTTF^* = \frac{\int_0^{\Delta T} R^*(t)dt}{1-R^*(\Delta T)} \quad (11)$$

where $R^*(t)$ is the reliability evaluated for the optimal loading policy $\mathbf{L}^*(\tau)$.

## 5 STOCHASTIC CASE

Given a filtered probability space $(\Omega, \Im, P)$ with a filtration $\{\Im_t\}$, the load $L(\tau)$ and the failure rate parameters $[\lambda_{0i}(t), \beta_i(t), \dots]$ are supposed stochastic processes adapted to the filtration, Pham (2009).

If the power system under study is "grid-connected" all the generators may be generally deterministically controlled. If the system is isolated and without storage the controllable variables may be "n-1" among "n", supposing that the remaining generator, the "slack", let it be the j-th, is capable supporting the load, that is:

$$L_{jMAX} \geq \max_{t \in [0,+\infty]}[L(t)] - \sum_{\substack{i=1 \\ i \neq j}}^{n} L_{iMAX} \tag{12}$$

As it is physically reasonable $\max_{t \in [0,+\infty]}[L(t)]$ is supposed to be deterministic; it is assumed also that relation (12) is verified $\forall j \in [1, n]$.

Naturally this hypothesis, in a real case, must be evaluated in the light of load-following regulating performances of the j-th generator, that in this paper are supposed adequate to support the dynamics of the control policy.

Chosen a generator j-th as slack, the objective is to identify a loading policy which maximizes the expected Reliability for the mission time T.

Let:

$$\chi\left(\tau, L(\tau), \boldsymbol{L_j}(\tau), \boldsymbol{\Theta}_{\mathbf{j}}(\tau)\right) = \sum_{\substack{i=1 \\ i \neq j}}^{n} \lambda_i(t, L_i(t), \boldsymbol{\theta_i}(t) + \lambda_j(t, L(\tau) - \sum_{\substack{i=1 \\ i \neq j}}^{n} L_i(t), \boldsymbol{\theta_i}(t)) \tag{13}$$

with

$$\boldsymbol{L_j}(\tau) = [L_1(t), \dots, L_{j-1}(t), L_{j+1}(t), \dots, L_n(t)]$$
$$\boldsymbol{\Theta}_{\mathbf{j}}(t) = \left[\boldsymbol{\theta_1}(t), \dots, \boldsymbol{\theta_{j-1}}(t), \boldsymbol{\theta_{j+1}}(t), \dots., \boldsymbol{\theta_n}(t)\right].$$

Given the problem:

$$\max_{\boldsymbol{L_j}[0,T]}\left\{E\left[R_j(T)\right]\right\} \tag{14}$$
s.t.
$$L_i(\tau) \in [0, L_{iMAX}], \forall i \in [1,..j-1, j+1,..n], \forall \tau \in [0, T],$$

considered that the processes describing loads and failure rate parameters have been supposed adapted to the filtration $\{\mathfrak{I}_t\}$ , according to the principle of optimality (14) may be written as:

$$0 = \min_{\boldsymbol{L_j}[0,T]}\left\{E\left[\chi\left(\tau, L(\tau), \boldsymbol{L_j}(\tau), \boldsymbol{\Theta}_{\mathbf{j}}(\tau)\right)\right] - E\left[\chi\left(\tau, L(\tau), \boldsymbol{L_j}(\tau), \boldsymbol{\Theta}_{\mathbf{j}}(\tau)\right)\right] \cdot J(t) + \dot{J}(t)\right\} \tag{15}$$

with

$$J(t) = \min_{\substack{\boldsymbol{L_j}(\tau) \\ t \leq \tau \leq T}}\left\{E\left[\int_t^T \chi\left(\tau, L(\tau), \boldsymbol{L_j}(\tau), \boldsymbol{\Theta}_{\mathbf{j}}(\tau)\right) exp\left[-\int_t^\tau \chi\left(s, L(s), \boldsymbol{L_j}(s), \boldsymbol{\Theta}_{\mathbf{j}}(s)\right) ds\right] d\tau\right]\right\}$$
$$J(T) = 0$$
$$J(0) = 1 - \max_{\boldsymbol{L_j}[0,T]}\left\{E\left[R_j(T)\right]\right\}.$$

The equation (15) implies that the optimal loading policy $\boldsymbol{L_j^*}(\tau)$ satisfies:

$$\min_{\boldsymbol{L_j}(\tau)}\left\{E\left[\chi\left(\tau, L(\tau), \boldsymbol{L_j}(\tau), \boldsymbol{\Theta}_{\mathbf{j}}(\tau)\right)\right]\right\} \tag{16}$$
s.t.
$$L_i(\tau) \in [0, L_{iMAX}], \forall i \in [1,..j-1, j+1,..n], \forall \tau \in [0, T].$$

So taking into account the Taylor expansion of (16) around the expected value of $\left[L(\tau), \boldsymbol{\Theta}_{\mathbf{j}}(\tau)\right]$, and adopting a multi-index notation, Marti (2008),:

$$\min_{\boldsymbol{L_j}(\tau)}\left\{\begin{matrix} \chi\left(\tau, \overline{L(\tau)}, \boldsymbol{L_j}(\tau), \overline{\boldsymbol{\Theta}_{\mathbf{j}}(\tau)}\right) + \\ \frac{1}{2}\sum_{|\alpha|=2} D^\alpha\left(\chi\left(\tau, \overline{L(\tau)}, \boldsymbol{L_j}(\tau), \overline{\boldsymbol{\Theta}_{\mathbf{j}}(\tau)}\right)\right) \cdot E\left[\left(\left[L(\tau), \boldsymbol{\Theta}_{\mathbf{j}}(\tau)\right] - \left[L(\tau), \boldsymbol{\Theta}_{\mathbf{j}}(\tau)\right]\right)^\alpha\right]\end{matrix}\right\} \tag{17}$$

where $E\left[\left(\left[L(\tau),\mathbf{\Theta_j}(\tau)\right]-\left[L(\tau),\mathbf{\Theta_j}(\tau)\right]\right)^{\alpha}\right]$ are the covariances in multi-index notation; applying Lagrange multipliers method, the particularized equations for failure rate model (1) are:

$$\begin{cases}
\psi_j(\tau)+exp[\beta_i(\tau)\cdot L_i(\tau)]\cdot\left[{L_i}^2(\tau)\cdot\vartheta_i(\tau)+L_i(\tau)\cdot\rho_i(\tau)+\kappa_i(\tau)\right]+\mu_i-\gamma_i=0\\
\psi_j(\tau)=exp\left[\beta_j(\tau)\cdot z_j(\tau)\right]\cdot\left[{z_j}^2(\tau)\cdot\nu_j(\tau)+z_j(\tau)\cdot\xi_j(\tau)+\zeta_j(\tau)\right]\\
z_j(\tau)=\overline{L(\tau)}-\sum_{\substack{k=1\\k\neq j}}^{n}L_k(\tau)\\
\nu_j(\tau)=-\frac{\beta_j(\tau)\cdot\sigma_{\beta_i\beta_j}(\tau)\cdot\overline{\lambda_{oJ}}(\tau)}{2}0\\
\xi_j(\tau)=-\sigma_{\beta_j\beta_j}(\tau)\cdot\overline{\lambda_{oJ}}(\tau)-4\sigma_{\beta_i\beta_j}(\tau)\cdot\overline{\beta_J}(\tau)\\
\zeta_j(\tau)=-\overline{\lambda_{oJ}}(\tau)\cdot\overline{\beta_J}(\tau)\cdot\left[\frac{1}{2}\sigma_{LL}(\tau)\overline{\beta_J}(\tau)^2+2\sigma_{L\beta_j}(\tau)\cdot\left(1+\overline{\beta_J}(\tau)^2\right)+1\right]+\\
-2\overline{\beta_J}(\tau)^2\cdot\sigma_{L\lambda_{oj}}(\tau)-4\sigma_{\lambda_{oj}\beta_j}(\tau)\\
\vartheta_i(\tau)=\frac{\overline{\beta_\iota}(\tau)\cdot\overline{\lambda_{oJ}}(\tau)\cdot\sigma_{\beta_i\beta_j}(\tau)}{2}\\
\rho_i(\tau)=\overline{\lambda_{o\iota}}(\tau)\sigma_{\beta_i\beta_i}(\tau)+2\overline{\beta_\iota}(\tau)\sigma_{\lambda_{oi}\beta_i}(\tau)\\
\kappa_i(\tau)=\overline{\lambda_{o\iota}}(\tau)\overline{\beta_\iota}(\tau)+2\sigma_{\lambda_{oi}\beta_i}(\tau)\\
\mu_i\cdot(L_i(\tau)-L_{iMAX})=0\\
\gamma_i\cdot L_i(\tau)=0\\
\mu_i\geq 0\;\gamma_i\geq 0
\end{cases}\tag{18}$$

$$\forall i\in[1,..j-1,j+1,..n]$$
$$\sigma_{SD}(\tau)\ is\ the\ covariance\ between\ the\ r.v.S\ and\ D.$$

For model (2), for sake of simplicity of the analytical expression, are reported only the particularized equations with known and constant failure rate parameters and $y_j>2,\forall j$:

$$\min_{L_j(\tau)}\left\{\sum_{\substack{i=1\\i\neq j}}^{n}\left[\lambda_{c_i}\left(\frac{L_i(\tau)}{L_{c_i}}\right)^{y_i}\right]+\frac{\lambda_{c_i}}{{L_{c_i}}^{\lambda_{c_i}}}\left[\begin{array}{c}\left(\bar{L}(\tau)-\sum_{\substack{i=1\\i\neq j}}^{n}L_i(\tau)\right)^{y_j}+\\+\frac{1}{2}y_j(y_j-1)\left(\bar{L}(\tau)-\sum_{\substack{i=1\\i\neq j}}^{n}L_i(\tau)\right)^{y_j-2}\sigma^2{}_L(\tau)\end{array}\right]\right\}\tag{19}$$

s.t.

$$L_j(\tau)\in\left[0,L_{jMAX}\right]$$

$\lambda_{c_i}$and $L_{c_i}$ are the parameters for the i-th generator

$\sigma^2{}_L(\tau)$ is variance of $L(\tau)$.

The analysis performed allows, also, identifying the i-th optimal slack generator which optimizes expected reliability, determined by:

$$i:E[R_i(T)]=\max_{j=1,..,n}\left[E\left[R_j(T)\right]\right]\tag{20}$$

Applying an analogous procedure used for the solution of problem (14) it is possible to show that:

$$\boldsymbol{L_j}^*(\tau) \text{ maximize } E\left[R_j(T)\right] \forall T \iff \boldsymbol{L_j}^*(\tau) \text{ maximize } E\left[MTTF_j\right] \tag{21}$$

### 5.1 Maximum expected MTTF for Stochastic-Load with periodicity

Following the procedure applied in the deterministic case, and similarly, supposing that $\lambda_i$ is not explicitly time dependent, and the processes describing load and failure rate parameters are supposed periodic with fixed period $h = \Delta T$, the maximum expected MTTF* is given by the following recurrent equation:

$$MTTF_j^* = \max_{\boldsymbol{L_j}[0,\Delta T]} \left\{ E \begin{bmatrix} \int_0^{\Delta T} R(\tau, \boldsymbol{L_j}[0,\Delta T], L(\tau) d\tau + \\ +R(\Delta T, \boldsymbol{L_j}[0,\Delta T], L[0,\Delta T]) \cdot MTTF_j^* \end{bmatrix} \right\}. \tag{22}$$

Banach fixed point theorem guarantees the existence and uniqueness of a fixed point $MTTF_j^*$ given by:

$$MTTF_j^* = \frac{E\left[\int_0^{\Delta T} R(\tau, \boldsymbol{L_j}^*[0,\Delta T], L(\tau) d\tau\right]}{1 - E\left[R(\Delta T, \boldsymbol{L_j}^*[0,\Delta T], L[0,\Delta T])\right]} \tag{23}$$

Even in this case the analysis performed allows identifying the i-th optimal slack generator which optimizes the expected MTTF. It coincides with that one given by (20).

## 6 CONCLUSION

Reliability-based control applied to power systems may be effective for identifying the most economical policies that can be used to fulfill the expected mission and minimize life cycle costs. In this field adequate methodological and technological development may have very high potential value for future applications in electrical power systems. Recent development of the monitoring technologies, particularly sensing techniques, non-invasive communication systems and implementation framework, and their expected pervasive diffusion in future power systems for collecting field data may constitute a valid technological support for the actuation of novel reliability-based control techniques.

The theoretical discussion of this paper focuses on a study of a reliability-based optimal dispatching policy of a heterogeneous park of programmable generating units whose failure model takes into account the dependency from the instantaneous loading condition. In deterministic analysis rigorous analytical expressions of the dispatching policy has been obtained for a generic set of heterogeneous components within simplification assumptions. Some general results have also been obtained in presence of stochastic loads and failure rate's parameters.

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# APPROXIMATION OF DOMAINS OF SERVICEABILITY AND ATTAINABILITY OF CONTROL SYSTEM ON THE BASIS OF THE INDUCTIVE APPROACH

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## ABSTRACT

One of fundamental problems of the dynamic systems control theory is the problem of a finding or estimation of attainability sets. Their practical construction, especially in nonlinear systems of the high dimensionality, usually turns out hugely the complex problem demanding considerable computing resources. In this connection possibility of use of adaptive partition by means of a uniform grid as one of effective methods of approximation of such sets is analyzed. The choice of received net descriptions for specification of their unknown borders in the conditions of uncertainty on the basis of the inductive approach is substantiated, using available aprioristic and current information.

## 1 INTRODUCTION

Problems of a choice of optimum control and reduction of space of its search arise at the decision of problems of designing, modeling of real processes or the facts, the analysis of the data in various areas of a science and technique. Choice of control by dynamic objects, in particular, optimum parametrical synthesis is connected from the computing viewpoint with very big costs. It demands effective numerical methods for the decision in the conditions of the high dimensionality and uncertainty. Use of domains of serviceability and attainability which in actual practice usually have no analytical description allows decreasing computing labour input. The various numerical methods reducing space of search and time of calculations are applied to their approximation depending on the available aprioristic information (See Chernousko 1988, Abramov 1992, Katueva & Nazarov 2005).

Nowadays a number of methods for approximation of attainability domains of the various kinds of controllable dynamical systems are known. One of them offered by Chernousko (1988) is based on use of ellipsoidal approximations and is applicable basically for nonlinear dynamic systems. This method does not demand excessively big computational cost, however it does not allow receiving result with beforehand pre-set accuracy of calculations. Approaches of Gusejnov et al. (1998) allow approximating barely boundaries of attainability domains and only in case of their convexity. The method offered by Yevseyev & Usachov's (2001) is similar to them. It uses the task of a set of evenly distributed nodal points belonging to approximable attainability domain in phase space. On these points the approximating set formed by association of spheres with the centers in nodal points and certain identical radius on which accuracy of the received approach is calculated is under construction. Accuracy of the received approach is defined by radius of these spheres.

The serviceability domains used in parametrical synthesis can be constructed, on the basis of mathematical model of investigated system, system of geometrical restrictions for the output values and internal parameters. In some instances the system of linear inequalities which generally speaking are nonlinear is applied for their description, and presence of the aprioristic information on convexity allows applying methods of convex optimization. In many situations there is effective approximation methods by some geometrical figures (inscribed or described hyperparallelepiped with the facets parallel to co-ordinate planes (Abramov et al. 2004, convex polyhedrons and ellipsoids (Digo G. & Digo N. 2006, 2008)). Besides, at performance of certain requirements as respects objective function, conditions of serviceability and restrictions on internal parameters, methods of non-uniform coverings, such as coverings by the n-dimensional parallelepipeds (cubes) inscribed in spheres (Zhigljavsky & Zhilinskas 1991), and the coverings $n$-dimensional parallelepipeds received by adaptive diagonal partition (Jones et al. 1993, Sergeev & Kvasov 2006, 2008) can be used.

In the presence of geometrical restrictions the approach based on adaptive partition of a feasible parallelepiped on a vector of internal parameters or on a state vector of investigated system is of interest for approximation of serviceability domains and attainability domains. In the conditions of uncertainty it allows to trace unknown borders of domains with bigger accuracy.

In the paper on the basis of the inductive approach (Ivakhnenko & Madala 1994) ways of carrying out of adaptive partition and a choice of points in derivable cells for an estimation of their quality are investigated.

## 2 THE BASIC CONCEPTS, DEFINITIONS AND EXPLANATORIES

Set of the problems connected with definition of requirements to parameters of object, nominal values of parameters and their permissible variations concerns problems of parametrical synthesis. Optimum parametrical synthesis (parametrical optimization) allows finding values of parameters of elements which are the best from the point of view of satisfaction to technical project requirements for invariable structure of projected object. Taking into account random parametrical perturbations it consists in choice nominal values $x_{ном} = (x_{1\,ном},..., x_{n\,ном})$ of internal parameters of investigated system or device, providing a maximum of probability of its non-failure operation during the set time interval:

$$x_{ном} = \arg\max P\{\mathbf{X}(x_{ном}, t) \in D_{\mathbf{x}}, \forall t \in [0,T]\}, \tag{1}$$

where $\mathbf{X}(x_{ном}, t)$ - random process of change of parameters; $D_{\mathbf{x}}$ **-** serviceability domain; $T$ - the set of system working or the device.

Let us give the serviceability conditions () on output parameters $\mathbf{y} = (y_1,..., y_m)$

$$a_j \le y_j(\mathbf{x}) \le b_j, \quad j = 1,..., m, \tag{2}$$

and

$$y_j = F_j(x_1,..., x_n), \tag{3}$$

$F_j(\cdot)$ is known operator which depends on topology of investigated device. Dependence (3) is usually set not in obvious, and in the algorithmic form, in particular, through numerical decisions of systems of the equations (differential or algebraic), describing functioning of investigated system (Abramov 1992, Abramov at el. 2004, Katueva & Nazarov 2005).

Let, besides, direct restrictions

$$x_{i\min} \le x_i \le x_{i\max}, \quad x_i \ge 0, i = \overline{1,n}, \tag{4}$$

on the internal (controlled) parameters generating feasible domain in orthogonal system of co-ordinates and having the form of an $n$-dimensional feasible parallelepiped $B_d$:

$$B_d = \{\mathbf{x} \in R^n \mid x_{i\min} \le x_i \le x_{i\max}, i = 1,..., n\} \tag{5}$$

are known. These restrictions give expression of a physical or technological realizability conditions.

The set of points $\mathbf{x} \in B_d$ in which all given conditions of serviceability (2) are satisfied, is called serviceability domain $D_{\mathbf{x}}$ in space of controlled parameters $\mathbf{x} = (x_1,..., x_n)$, i.e.

$$D_{\mathbf{x}} = \{\mathbf{x} \in D \mid a_j \le y_j(\mathbf{x}) \le b_j, \quad j = 1,...,m\}. \tag{6}$$

This domain, as a rule, is unknown and there is no information on its form and orientation in space of internal parameters.

In specified conditions the problem of construction of serviceability domain in a general view has no analytical decision, and the various numerical methods, reducing space of search and reducing labour input of calculations by them multisequencing, are applied to its approximation (Abramov et al. 2004).

Let us suppose that changing of a state of some controllable dynamic system which is subject to action of uncertain factors is described by the equation

$$\dot{\mathbf{z}}(t) = f(t, \mathbf{z}, \mathbf{u}), \quad \mathbf{z} \in R^n, \ \mathbf{u} \in R^m, \tag{7}$$

on span of time $[t_0, T]$, $t_0$ - the initial moment of time. In formula (7) $\mathbf{z} = (z_1,...,z_n)$ - *n*-dimensional vector of a system state with terminal restrictions

$$z_{i\min} \le z_i \le z_{i\max}, \quad z_i \ge 0, i = \overline{1,n}, \tag{8}$$

generating feasible domain in orthogonal system of co-ordinates the feasible domain *D*:

$$D = \{\mathbf{z} \in R^n \mid z_{i\min} \le z_i \le z_{i\max}, \quad z_i \ge 0, \quad i = \overline{1,n}\}, \tag{9}$$

$\mathbf{u} = (u_1,...,u_m)$ - *m*-dimensional vector of control (or uncontrollable disturbances).

Let, besides, geometrical restrictions on a kind vector are given as

$$\mathbf{u} \in U(t, \mathbf{z}). \tag{10}$$

As the attainability domain $D_{\mathbf{z}}$ of system (7) for the given restrictions (8), (10) and $t \ge t_0$ is called (See Kostousova 1998, Bobyleva 2002) set of points $\mathbf{z} \in D$, for each of which exist an initial condition $\mathbf{z}_0 \in D$ and the control $\mathbf{u}(\cdot)$ satisfying (10) which generate decision $\mathbf{z}(\cdot)$ for system (7) such that $\mathbf{z}(t) = \mathbf{z}$, i.e.

$$D_{\mathbf{z}} = \{\mathbf{z} \in D \mid \mathbf{u} \in U(t, \mathbf{z}), t \in [t_0, T]\}. \tag{11}$$

Approximation of attainability domains of nonlinear dynamic systems at restrictions (10) is connected with the same computing difficulties, as well as at numerical construction of serviceability domains.

From the analysis of definitions and concepts which are described above follows that there is an interrelation between problems of a finding of sets of admissible variations of parameters and sets of attainability of controllable systems. Restrictions (4) and (8) generate accordingly *n*-dimensional the feasible parallelepiped $B_d$ from (5) and *n*-dimensional the attainability parallelepiped *D* from (9) in orthogonal system of co-ordinates. Each of them contains the required domain (the serviceability domain $D_{\mathbf{x}} \subset B_d$ and the attainability domain $D_{\mathbf{z}} \subset D$**),** therefore for approximation of these domains can be used one and the same algorithm. In this connection further we will operate only with a parallelepiped $B_d$ and domain $D_{\mathbf{x}}$.

## 3 THE FORMULATION OF A PROBLEM AND ITS ANALYSIS

Let's assume that *n*-dimensional parallelepiped $B_d$ is given by expression (5) and contains domain $D_{\mathbf{x}}$ from the formula (6), but this parallelepiped is not described for this domain $D_{\mathbf{x}}$. The problem is stated to describe $B_d$ on the basis of algorithm of adaptive partition (Jones et al. 1993, Sergeev & Kvass 2006) for application at approximation of domain $D_{\mathbf{x}}$ for the purpose of reduction of space of search and time of calculations.

It is obvious that the parallelepiped $B_{\text{o}}$ having sides, parallel to corresponding sides $B_d$ and touching on borders of domain $D_{\mathbf{x}}$**,** will be described around $D_{\mathbf{x}}$. Its construction as the first step at an approximation stage, allows few to reduce search space. One of possible algorithms of construction $B_o$ and the matrix description $D_{\mathbf{x}}$ are resulted in Katueva & Nazarov (2005), Digo G. & Digo N. (2008). According to it such matrix description of domain $D_{\mathbf{x}}$ turns out imposing of an n-dimensional uniform grid on the described parallelepiped $B_o$ by means of quantization of domain of values of each $i$-th parameter. As a result described parallelepiped $B_o$ is represented in the form of set of non-crossing parallelepipeds (cells)

$$B_o = \bigcup_{k_1=1}^{l_1} \bigcup_{k_2=1}^{l_2} \dots \bigcup_{k_n=1}^{l_n} B_{k_1,k_2,\dots k_n}\,, \tag{12}$$

where $l_i$ - quantity of partition on a co-ordinate axis of $i$-th parameter. The information on the description $B_o$ (the information on each cell of representation (12)) for the subsequent use is written down in an array $A[K]$ in the form of co-ordinates of its borders and quantity of partitions $l_i$ on each $i$-th co-ordinate axis. It allows calculating co-ordinates of each cell $B_{k_1,k_2,\dots k_n}$ from (12).

Other approach to use of the uniform grid, connected with partition of domain $B_d$ into $n$-dimensional parallelepipeds on the basis of adaptive partition (See Jones et al. 1993) is of interest. According to it $B_d$ divides on $n$-dimensional parallelepipeds, and in each of them some point is selected. Two moments thus are important: it is necessary to define that a way to produce splitting and on what points to estimate quality received $n$-dimensional parallelepipeds.

For application of the description of the parallelepiped $B_d$ received by means of this approach, at area approximation $D_{\mathbf{x}}$ compact and accessible storage of the information on its demanded characteristics should be provided. Borders co-ordinates and partitions quantity on each co-ordinate axis which provide calculation of co-ordinates of borders of each cell, co-ordinates of points in cells as quality indicators of these cells (the cell is considered good if the point chosen in it belongs $D_{\mathbf{x}}$, and bad otherwise) concern such characteristics.

## 4 ADAPTIVE PARTITION OF FEASIBLE PARALLELEPIPED

To approximate attainability domain $D_{\mathbf{x}}$ adaptive partition (Jones et al. 1993, Sergeev & Kvasov 2006, 2008) of feasible domain $B_d$ on $n$-dimensional parallelepipeds (cells) is made by means of a uniform mesh. In each cell the central point gets out. For acceleration of process of calculations new parallelepipeds are formed by division already available on 3 parts on various measurements. It allows spending calculations only in the central points of the left and right thirds, considering earlier received results. For simplification of computing process it is supposed that everyone a component $x_i$ of vector $\mathbf{x}$ in a feasible parallelepiped $B_d$ from (4) has the bottom border 0 and the top border 1, i.e. $B_d$ always is a $n$-dimensional init cube (it without generality loss always can achieve by means of linear transformation of technological restrictions (3)). Then partition begins with a unique $n$-dimensional parallelepiped, the init $n$-dimensional cube, which each side has the length equal 1.

During approximation of domain $D_{\mathbf{x}}$ such characteristics of a feasible parallelepiped $B_d$ as coordinates of its borders and quantity $l_i$ of partition on each $i$-th co-ordinate axis, which provide calculation of borders coordinates of each cell, coordinates of the centers of cells as indicators of these cells quality (the cell is considered good if its centre belongs domain $D_{\mathbf{x}}$, and bad otherwise) are required. With their help mesh representation $B_d$ is formed. The information about this representation is stored in a file $A[K]$ of the length $K=\prod_{i=1}^{n} l_i$. Its elements accept values 0 (a cell bad) or 1 (a cell good). Such way of storage is compact and easily accessible.

Since new parallelepipeds are formed by dividing existing ones into thirds on various dimensions, the only possible lengths for their sides are $3^{-k}$, ( $k=0,1,2,...$). Each received parallelepiped always is divided on the largest side. No its side of length $3^{-(k+1)}$ can be divided until all of those of length $3^{-k}$ have been divided. After carrying out *r* divisions in a parallelepiped $B_d$ it will be received the $j=\mathrm{mod}\,(r,n)$ sides of length $3^{-(k+1)}$ and $(n-j)$ the sides of length $3^{-k}$ where $k=(r-j)/n$, and, hence, the distance from the center of a parallelepiped to the vertexes is given by the formula $d=[j3^{-2(k+1)}+(n-j)3^{-2k}]^{0.5}/2$. The described process of adaptive partition is ended, when for the given accura $\varepsilon>0$ inequality $3^{-(k+1)}<\varepsilon$ is executed.

As a result of adaptive partition the feasible parallelepiped $B_d$ becomes covered by the n-dimensional uniform mesh that each cell has the own characteristic in terms of «good-bad» concerning a corresponding central point. The received mesh representation has matrix analogue in the form of the *n*-dimensional array $C[l_1,l_2,...,l_n]$ which dimension coincides with dimension of a vector of internal parameters or a state vector **x** of investigated system.

## 5 POSSIBILITIES OF REDUCTION OF SPACE OF SEARCH

The information on a parallelepiped $B_d$ which is contained in a file $C[l_1,l_2,...,l_n]$, allows reducing search space at the domain description $D_{\mathbf{x}}$. One of ways of such reduction is transition from the feasible parallelepiped $B_d$ to a parallelepiped $B_0$ described around domain $D_{\mathbf{x}}$ on the basis of the analysis of the information from a file $A[K]$ about quality of cells, received in the course of partition. For this purpose value of co-ordinate $x_n$ is established equal 0 ( $x_n=0$ ).Check is carried out, whether there are among cells of this layer good cells (the corresponding element of a file $A[K]$ is equal 1). If yes, transition to a layer of cells in which equality is carried out $x_n=1$, is made. Otherwise this layer is excluded from the further consideration, and transition to the analysis of a following layer is carried out. Process goes on until there will be at least one good cell in a checked layer.

The similar analysis is made and for the others (*n*-1) variables. As a result is, within a preset value $\varepsilon$, there is the *n*-dimensional parallelepiped $B_0$ described around domain $D_{\mathbf{x}}$:

$$B_0=\{\mathbf{x}\in R^n \mid 0\le c_i\le x_i\le d_i\le 1, i=1,...,n\}. \tag{13}$$

As well as and $B_d$, the parallelepiped $B_0$ has its matrix representation $\widehat{C}[\widehat{l}_1,\widehat{l}_2,...,\widehat{l}_n]$ and a corresponding array of quality cells $\widehat{A}[\widehat{K}]$ of length $\widehat{K}=\prod_{i=1}^{n}\widehat{l}_i$. This information provides more exact description $D_{\mathbf{x}}$ which, in turn, can be approximated geometrical figures (See Chernousko 1988, Abramov 1992, Digo G. and N. 2008).

## 6 CONCLUSION

The method of adaptive partition of a feasible parallelepiped is applied to serviceability domains or attainability domain on components of a vector of internal parameters or on a state vector of investigated system. He allows describing these domains a set of cells with the sizes of cells of pre-set accuracy, received on the basis of idea of a uniform net covering.

The chosen using strategy of the central points for the characteristic of the received cells leads to mesh representation of approximable domains in terms of «good-bad» concerning corresponding central points and provides preservation of coordinates of already received centers.

The found mesh representation of a feasible parallelepiped is compactly, and its storage in the form of an *n*-dimensional array is accessible to the subsequent use.

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# TECHNIQUE OF FORMATION HOMOGENEOUS SAMPLE SAME OBJECTS

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**Importance of a problem.** Reliability of electro power systems (EPS) in many respects depends on reliability of the capital equipment and devices (objects EPS). Modern objects EPS concern to a category of complex objects (CO). CO - multivariate, in other words, characterized greater number of attributes and their versions. As first approximation, these attributes presented by two groups. The first group of attributes characterizes a design CO and the second presented in characteristics sheet, and the second group -conditions of operation CO.

If necessary estimations of a technical condition CO (for example, at an estimation of expediency of a conclusion CO on major overhaul), data on a technical condition of it CO, as a rule, it appears insufficient. Insufficient there are not only data on emergency and scheduled switching-off, but also data on size of deterioration and speed of its change. For this reason normative materials attraction of retrospective data about reliability same CO [1] recommended. At this classification influence, the second group of attributes that increases risk of the erroneous decision is not considered.

Classification CO represents not only important, but also a difficult problem. Its difficulty caused by necessity of research of homogeneous (unpresentable) sample CO from a final data set (FDS).

According to GOST 15895-77 «Statistical methods of product quality control» representative sample is sample (test), which sufficiently reflects properties of the given set (in our case FDS) as a whole. But what such "sufficiently"?

Let's enter some definitions of terms:

- under unpresentable we shall understand sample on a number of versions of attributes (VA), statistical function of distribution (s.f.d.) which not casually differs from s.f.d. FDS [2].
- under FDS we shall understand the retrospective information presented in the form of the empirical table (ET).
- under ET we shall understand the table which lines contain the information on conditions (results of test) CO, and columns - versions of each of examined attributes [3].

In the illustrative purposes as CO examine power transformers and autotransformers (further: transformers (TR)). On table 1 are resulted passport data TR, and in table 2 - conditions of operation TR in AISTR [3]. These data examined as attributes TR.

Tables 1 and 2 testify that alongside with quantitative estimations VA, there are also quality standards. For example, a factory manufacturer TR, system of cooling, a consequence of refusal, etc. Thus, attributes CO have various scales of measurement

Table 1

| NAMEPLATE DATA THE TRANSFORMER | |
|---|---|
| Name of the enterprise * **NNN** | |
| Name of substation * **MMM** | |
| Dispatching designation **AT1** | |
| Type the transformer * **ATDCTN-200000/220** | |
| Kind the transformer * **autotransformer** | |
| Factory-manufacturer * **ZTZ** | Factory number |
| Year of manufacturing **1984** | |
| Rated voltage, кv **330** | Rated power, Mva **133** |
| Number of phases **1** | Phases * **A** |
| Number of windings **3** | |
| Type of a RE * **RNOA-220/1250** | Drive of a RE * **PDP-4U1** |
| Type a winding of LP * | |
| System of cooling * **DC** | |
| Way protection of butter * **film** | |
| Execution butter filled inputs * | |

Table 2

| PASSPORT CONDITIONS OF OPERATION THE TRANSFORMER | |
|---|---|
| Purpose of the transformer * **Connections SC** | |
| Name of the enterprise * **NNN** | |
| Name of substation * **MMM** | |
| Dispatching designation **AT1** | |
| Type the transformer * **ATDCTN-200000/220** | |
| Year of manufacturing **1984** | |
| Year of commissioning **1985** | Year of dismantle |
| Condition a neutral * **neutral is earthed** | |
| Condition of a RE * **Working** | |
| Loading of a winding of LP * **synchronous equalizer** | |
| Maximal loading, r.u. **0.77** | Number of c.s.c. **3** |
| Consequence of refusal * **Decrease in structural reliability** | |
| Results of test: | |
| CADG - **Condition of risk** | Inputs - **Admissible** |
| PCAB - **Admissible** | RE - **Condition of risk** |
| Active part - **Admissible** | System of cooling - **Admissible** |
| Area of storm protection **1** | Date of last MO **14.02.2003** |

Formally, at classification of retrospective data on set VA, the information on technical condition TR sharply decreases, and in a greater part of cases does not allow have not only quantitative, but also quality standard of parameters of reliability TR. At the same time it is clear, what not all attributes TR are equally significant (from the point of view of reliability of work), and a functional divergence s.f.d. FDS and samples should be not less guaranteed value of its statistical component.

**Recommend method.** Assume that:

- nameplate data and data on conditions of operation of all TR power supply systems, and on dispatching magazines and certificates of failures data on scheduled and emergency switching-off TR are received. On these data two are made ET, first of which (ET1) the second includes data on design features and conditions of operation TR, and (ET2) - data on switching-off TR and minimally necessary data for an input in ET1;

- is known deduced on major overhaul TR for which it is necessary to find homogeneous sample of the same objects.

Let's designate number examined TR through $M_{TP}$, number of attributes- $n_{np}$, number VA $i$-th TR - $r_i$ with $i = 1, n_{np}$.

Following calculations are spent:

1. For each of $M_{TP}$ TR the interval of time during which the characteristic of emergency and scheduled switching-off TR known is determined. Designate this interval through $\Delta\tau_i$ with $i$= 1, $M_{TP}$.
2. In each of $\Delta\tau_i$ an interval, it is determined total duration of scheduled switching-off (switching-off on CR, operating repair, in a reserve, etc.). Designate this size as $\Delta\tau^{\Pi}_{i,\Sigma}$;
3. Determine size $\Delta\tau_\Sigma = \sum_{i=1}^{M_{TP}}\Delta\tau_i - \sum_{i=1}^{M_{TP}}\Delta\tau^{\Pi}_{i,\Sigma}$ - total duration of work and idle time in emergency repair.
4. Determined total number of emergency switching-off TR. Considering, that emergency switching-off happen automatic, manually and under the emergency application, and consequences of switching-off TR under the emergency application, as a rule, it is essential less consequences of first two kinds of switching-off, expedient to spend separate classification of switching-off. Designate number of examined emergency switching-off (for example, under the emergency application) through $n^{ав}_\Sigma$;
5. Calculate an estimation of specific number of emergency switching-off TR: $\lambda^*_{TP} = n^{ав}_\Sigma / \Delta\tau_\Sigma$.

In a basis of estimation, $\lambda^*_{TP}$ there is assumption of constant probability occurrence emergency switching-off during an individual interval of time (year). Operating experience shows, that non-failure operation TR depends both on their design, and from conditions of operation. Therefore, the average value of size $\lambda^*_{TP}$ has no practical sense. Concrete TR can have in park TR analogues, in the form of same TR. However, this sample is non-uniform, since TR can differ with conditions of operation (for example, service life). Therefore, recommendation [1] about expediency of the account of data on a technical condition of the same objects demands the specification. If to agree with the above-stated, it is possible to make following next steps:

6. For everyone VA set TR the specific number of emergency switching-off $\lambda^*_{TP,j}$ with $j = 1, n_{np}$ is calculated. The purpose of these calculations is the finding

$$\lambda^*_{TP,max} = max\left(\lambda^*_{TP,1};\ \lambda^*_{TP,2};\ \lambda^*_{TP,3};\quad ;\ \lambda^*_{TP,j};\qquad \lambda^*_{TP,n_{np}}\right);$$

7. Estimations $\lambda^*_{TP}$ are compared and $\lambda^*_{TP,max}$. Comparison spent in view of casual character of these estimations, and in view of not only errors of the first, but also the second sort. The methodology of this comparison is based on the theory of statistical modeling and the theory of check statistical hypotheses and stated by us in [5]. If it appears, that the hypothesis about a casual divergence $\lambda^*_{TP}$ and $\lambda^*_{TP,max}$ is more preferable (less sum errors of the first and second sort), than an alternative hypothesis, having in view of, that a casual divergence $\lambda^*_{TP}$ and $\lambda^*_{TP,max}$ simultaneously means a casual divergence $\lambda^*_{TP}$ and $\lambda^*_{TP,j}$ with $j = 1, n_{np}$ within the limits of had statistical data about emergency switching-off TR any of VA insignificant and all TR is represented with a file of homogeneous same objects, specific which number of refusals equally $\lambda^*_\Sigma$. Though at classification of retrospective data only to one attribute it is improbable, the statement at a casual divergence $\lambda^*_{TP}$ and $\lambda^*_{TP,max}$ remain constant for any number VA. Moreover, process of search of homogeneous

sample same TR stops, and management is (if necessary) transferred to a press of their list.

8. If the hypothesis about not casual distinction $\lambda^*_{TP}$ and $\lambda^*_{TP,max}$, sample TR for which the specific quantity of emergency switching-off equally $\lambda^*_{TP,max}$ is accepted for FDS and on these statistical data is more preferable:

9. Estimations $\lambda^*_{TP,j}$ are calculated with $j = 1,(n_{np} - 1)$;

10. Determined $\lambda^*_{TP,max} = max\{\lambda^*_{TP,j}\}_{(n_{np}-1)}$;

11. Estimations $\lambda^*_{TP}$ and $\lambda^*_{TP,max}$ (see item 7) are compared;

12. See item 8. Recurrence of cycles of classification of data and checks of hypotheses is carried out up to an opportunity of the proved acceptance of a hypothesis about a casual divergence of estimations $\lambda^*_{TP}$ and $\lambda^*_{TP,max}$. The integrated block diagram of algorithm of formation of homogeneous sample same TR is resulted on fig.1.

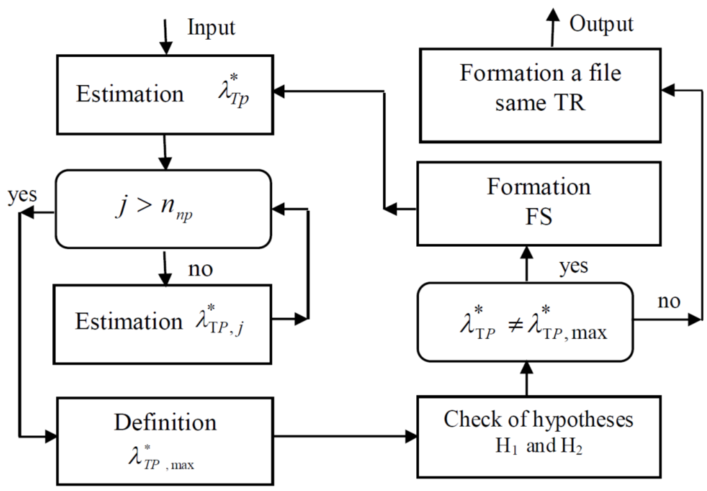


Fig.1. Integrated block diagram of algorithm of formation of homogeneous sample of the same objects

## CONCLUSION

1. Control of a technical condition concrete TR retrospective data about reliability same TR not always can far appear useful. It concerns, first, to TR with the service life exceeding normative value, working unlike controllable TR in conditions with the intensive storm activity, through currents of short circuit exposed to frequent dynamic influence, etc.
2. Method and algorithm of formation of homogeneous sample same TR is developed.
3. Practical realization of algorithm and the program of formation of homogeneous sample same TR allows to specify list TR, the information on which technical condition promotes recognition of a technical condition controllable TR and by that to raise objectivity of recommendations on increase of reliability TR.

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# DEVELOPMENT AND RESEARCH ON MATHEMATICAL MODELS, MODES OF WIND POWER PLANTS' ELECTROMECHANICAL CONVERTERS

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## ABSTRACT

A mathematical models of the following electromechanical converters, used in wind power, are presented: the asynchronous generator with a fed through a frequency converter stator, the generator, which is double fed asynchronous machine with rotor winding, fed through a frequency converter, and a synchronous generator with permanent magnets, with stator, fed through the frequency converter. For all converters the optimal control laws of amplitude and frequency of voltage on generator's terminals are obtained for all operating modes of wind power plants.

## 1. INTRODUCTION

In recent years, the wind-power engineering is developing rapidly. Today the wind power plants of up to 5 MW unit capacity are operating simultaneously with electric power network. By 2010 the total wind power plants capacity in the world constituted 160 GW.

The low-speed synchronous generators, squirrel-cage asynchronous machines, double fed asynchronous machine are used as the electromechanical converters in the wind power plants (WPP).

To raise the WPPs' operating efficiency in a specific range of wind speed variation, it needs also to control accordingly a rotational frequency of wind motor (WM) with jointed generator [1,2,3]. With such control a wind power utilization factor reaches its maximum values for corresponding wind speeds, and so the electric power production increases.

The expressions of wind motor's power and torque are accordingly written as [1]:

$$P_{\text{WM}} = \frac{1}{2}\rho\pi R^2 \cdot \text{V}^3 \cdot C_p \tag{1}$$

$$M_{\text{WM}} = \frac{1}{2}\rho\pi R^3 \cdot \text{V}^2 \cdot \mu \tag{2}$$

where ρ – air mass density, R – radius of wind wheel, V – wind speed; $C_p$ – wind power utilization factor, $\mu$– relative wind motor's driving torque.

A connection between wind power utilization factor $C_p$ and relative driving torque μ is carried out through the relation:

$$C_p = \mu \cdot Z \tag{3}$$

where $Z = \frac{\omega_{\text{WM}} R}{\text{V}}$ – number of modules or specific speed, $\omega_{\text{WM}}$ – wind motor's angular frequency of revolution.

If to determine $\mu$ from the expression (3) and substitute in the expression (2) with taking into account the dependence of specific speed from $\omega_{\text{WM}}$ and V, we will obtain:

$$M_{\mathrm{WM}} = \frac{P_{\mathrm{WM}}}{\omega_{\mathrm{WM}}} \quad (4)$$

When wind motor rotational frequency control in proportion to wind speed the number of modules Z must theoretically remain constant, i.e.

$$Z = Z_{opt} = \frac{\omega_{\mathrm{WM}} \cdot R}{\mathrm{V}} = const \quad (5)$$

On the aerodynamic characteristic of WM accordingly $Z_{opt}$, $C_p$ and $\mu$ should remain constant, but that can not be exactly achieved in a real wind power plants. For example, in a WPP of Gamesa G–52 type of 850 kW capacity, where double fed asynchronous machine is used as electromechanical converter, a WM rotational frequency varies from 30,8 rpm (0,513 rev/s) to 14,6 rpm ( 0,24 rev/s), i.e. in the range of wind speed change from 10,45 m/s to 4,96 m/s (the depth change is 10,45/4,96=2,1) wind motor's rotational frequency is controlled from 0,513 1/s to 0,24 1/s (control depth is 2,1). In a mentioned range the wind power utilization factor Cp varies for considered wind motor from $C_{p\,min} = 0{,}390$ to $C_{p\,max} = 0{,}452$. However, if to operate an average factor value in the range of WM rotational frequency control

$$\mathrm{C}_{p\,mid} = \frac{\sum_{1}^{n} C_{pi}}{n} \quad (6)$$

where $C_{pi}$ – possible fixed values of the wind power utilization factor $C_p$ in the control range, n – number of fixed values, then the error for power calculation does not exceed ±7–8%.

Let's demonstrate Table 1 this on the example of the same WPP. The values of wind speeds, the corresponding them values of $C_p$, power $P$ and the calculated values of wind motor's rotational frequencies $\omega_{\mathrm{WM}}$ in the range of rotational frequency control are presented in the Table 1 [4].

Table 1. Dependence of $P$, $C_p$ and $\omega_{\mathrm{WM}}$ on the wind speed V.

| V, m/s | 10,45 | 9,97 | 9,44 | 9,00 | 8,53 | 7,97 | 7,48 | 7,00 | 6,51 | 6,03 | 5,5 | 4,96 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $P$, kW | 587,9 | 535,9 | 479,2 | 421,2 | 362,0 | 277,4 | 232,0 | 186,5 | 149,3 | 117,4 | 86,59 | 61,6 |
| $C_p$, r.u. | 0,398 | 0,418 | 0,441 | 0,446 | 0,452 | 0,423 | 0,429 | 0,42 | 0,418 | 0,415 | 0,402 | 0,39 |
| $\omega_{\mathrm{WM}}$, 1/s | 0,513 | 0,489 | 0,463 | 0,442 | 0,419 | 0,391 | 0,367 | 0,343 | 0,319 | 0,296 | 0,27 | 0,243 |

The calculated average value of wind power utilization factor is:

$$C_{pmid} = 0{,}419$$

If to take now in the whole control range $C_p = C_{p\,mid} = const$, then for determining the powers corresponding to maximum values $C_{p\,max}$ (in presented example it is $P_1$=362,05 kW at $C_{p\,max} = 0{,}452$) and minimum values $C_{p\,min}$ ($P_2$=61,6 kW at $C_{p\,min}$=0,39) the error will constitute:

$$\Delta P_{max} = \frac{P_1 - P_1'}{P_1} = +6{,}5\% \quad \text{and} \quad \Delta P_{min} = \frac{P_2 - P_2'}{P_2} = -7{,}8\% \quad ,$$

where $P_1' = 338$ кW and $P_2'$ = 66,46 кW are the power values corresponding to wind speeds at $C_{p\,max}$ and $C_{p\,min}$, but calculated for $C_p = C_{pmid} = 0{,}419$ by formula (1).

It is seen from the calculations, for the vast majority of WPPs an error, when the proposed approach, for power determination does not exceed a value of 7–8%. For example, for «Vestas V–90» WPP of 2 MW capacity $C_{p\,cp} = 0{,}441$ and an error is in the range of from 8% to –2%.

Thus, in the range of rotational frequency control the expression of WM power can be written in the form:

$$P_{\text{WM}} = \frac{1}{2}\rho\pi R^2 \cdot C_{pmid} \cdot \text{V}^3 = K_p \cdot \text{V}^3 \quad (7)$$

where $K_p = \frac{1}{2}\rho\pi R^2 \cdot C_{p\,mid}$ – power proportionality constant, depending on the WM design parameters and the air mass density, but not depending on wind speed.

Accordingly, the expression for WM torque takes the form:

$$M_{\text{WM}} = K_p \frac{\text{V}^3}{\omega_{\text{WM}}} \quad (8)$$

When control the specific speed Z remains constant within the whole control range $Z = Z_{opt} = const$ (for given WPP $Z_{opt}$=8,0), so we can write

$$\text{V} = \frac{\omega_{\text{W}M} \cdot R}{Z_{opt}} = \frac{R}{Z_{opt}} \cdot \omega_{\text{W}M} \quad (9)$$

Substituting the expression (9) into (8) we obtain:

$$M_{\text{W}M} = K_p \frac{R^3}{Z_{opt}^3} \cdot \omega_{\text{W}M}^2 = K_м \cdot \omega_{\text{W}M}^2 \quad (10)$$

where $K_м = \frac{K_P \cdot R^3}{Z_{opt}^3}$ – torque coefficient of proportionality.

Thus, the wind motor torque, when rotational frequency control in proportion to wind speed, can be considered with defined above error as depending on a square of rotational frequency.

In modern WPPs for smooth rotational frequency control of wind motor is usually used the frequency converters, operating jointly with electromechanical converter. If as electromechanical converters the super-low-speed synchronous generators with «Ringgenerator» permanent magnets (plants of «Enercon», «Vensys» firms), or squirrel-cage asynchronous generators (plants of «Siemens Wind Power» firm) are used, the frequency converters, made with fully controlled semiconductor elements (IGBT – transistors, or a fully controlled GTO – thyristors) with PWM – control are installed in the stator circuits of these machines. If as the electromechanical converters the double fed asynchronous machines are used, these frequency converters are installed in the rotor circuits of these machines (WPPs of «Vestas», «Gamesa» «GE Wind Energy» firms, etc.).

## 2. STUDY ON MATHEMATICAL MODELS OF OPERATION MODES OF WIND POWER PLANTS CONTAINING ASYNCHRONOUS GENERATORS

Presentation of the mathematical model (model of state) and research on this model the different operating modes of WPPs, containing asynchronous generators, frequency–controlled by both through a stator and through a rotor, will allow to take into account the specific character of this system operation.

The equations of the asynchronous machine in the cell – matrix form are presented in a view [5]:

$$\begin{bmatrix} p\psi_s \\ p\psi_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{\text{s1}} & \boldsymbol{A}_{\text{s2}} \\ \boldsymbol{B}_{\text{r1}} & \boldsymbol{B}_{\text{r2}} \end{bmatrix} \begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix} + \begin{bmatrix} \boldsymbol{U}_{\text{s}} \\ \boldsymbol{U}_r \end{bmatrix} \quad (11)$$

where the expressions for the submatrices are:

$$p\psi_s = \begin{bmatrix} p\psi_{s\alpha} \\ p\psi_{s\beta} \end{bmatrix};\ p\psi_r = \begin{bmatrix} p\psi_{r\alpha} \\ p\psi_{r\beta} \end{bmatrix};\ \psi_s = \begin{bmatrix} \psi_{s\alpha} \\ \psi_{s\beta} \end{bmatrix};\psi_r = \begin{bmatrix} \psi_{r\alpha} \\ \psi_{r\beta} \end{bmatrix};\ \boldsymbol{U}_s = \begin{bmatrix} U_{s\alpha} \\ U_{s\beta} \end{bmatrix};\ \boldsymbol{U}_r = \begin{bmatrix} U_{r\alpha} \\ U_{r\beta} \end{bmatrix}.$$

As for the submatrices $\boldsymbol{A}_{s1}$, $A_{s2}$, $\boldsymbol{B}_{r1}$, $\boldsymbol{B}_{r2}$, they depend on the form of asynchronous machine equations. When writing the equation in fixed in space axes $\alpha, \beta, 0$ , they are equal:

$$\left.\begin{aligned} \boldsymbol{A}_{s1} &= \begin{bmatrix} -r_s k_s & 0 \\ 0 & -r_s k_s \end{bmatrix}; & \boldsymbol{A}_{s2} &= \begin{bmatrix} -r_s k_m & 0 \\ 0 & -r_s k_m \end{bmatrix}; \\ \boldsymbol{B}_{r1} &= \begin{bmatrix} -r_r k_m & 0 \\ 0 & -r_r k_m \end{bmatrix}; & \boldsymbol{B}_{r2} &= \begin{bmatrix} -r_r k_r & -\omega_r \\ \omega_r & -r_r k_r \end{bmatrix} \end{aligned}\right\} \quad (12)$$

If the equations are written in the rotating with a rotor speed axes, (for convenience we denote these axes also by letters $\alpha, \beta$, although in the literature they are marked as d, q), then the submatrices $\boldsymbol{A}_{s1}$, $\boldsymbol{B}_{r2}$ will look:

$$\boldsymbol{A}'_{s1} = \begin{bmatrix} -r_s k_s & -\omega_r \\ \omega_r & -r_s k_s \end{bmatrix}; \qquad \boldsymbol{B}'_{r2} = \begin{bmatrix} -r_r k_r & 0 \\ 0 & -r_r k_r \end{bmatrix} \quad (13)$$

In above-cited equations a system of choice of relative units, base values, denominations with taking into account that $p = d / d\tau, \tau = 314 \cdot t$ (time in radian) are generally accepted ones, the factors $k_s, k_r, k_m$ are determined from the inverse matrix of machine parameters, i.e.:

$$\begin{bmatrix} k_s & 0 & k_m & 0 \\ 0 & k_s & 0 & k_m \\ k_m & 0 & k_r & 0 \\ 0 & k_m & 0 & k_r \end{bmatrix} = \begin{bmatrix} x_s & 0 & x_m & 0 \\ 0 & x_s & 0 & x_m \\ x_m & 0 & x_r & 0 \\ 0 & x_m & 0 & x_r \end{bmatrix}^{-1} \quad (14)$$

Following In expressions (12) – (14): $r_s, r_r, x_s, x_r, x_m$ – are accordingly active resistance and inductance of stators (s) and rotors (r) circuits, as well a mutual induction impedance (we take the saturated value of this parameter) $\omega_r$ – angular frequency of revolution of asynchronous generator's rotor.

Thus, the expressions (10) – (14) with adding the equations of motion and torques constitute a mathematical model of WPP's asynchronous machine.

$$\left.\begin{aligned} T_j p\omega_r &= m_{\text{EM}} - m_{\text{WM}} \\ m_{\text{EM}} &= k_m \left(\psi_{s\alpha} \cdot \psi_{r\beta} - \psi_{s\beta} \cdot \psi_{r\alpha}\right) \end{aligned}\right\} \quad (15)$$

In expression (15) $T_j$ – inertia constant of the system (wind motor and generator) in [radian], $m_{\text{WM}}$ – wind motor torque, determined by the expression (10), it is, of course, reduced to generator's shaft and represented in the chosen system of relative units. In this case, naturally $\omega_{\text{WM}}$ is also reduced to generator's shaft with taking into account the transition factor of gear box and also a number of generator's poles pairs.

If to study a frequency control mode of squirrel–cage asynchronous generator's stator, it is easier to use the equations of the asynchronous generator, written in the fixed in space axes. In this case it is necessary to operate with submatrices $\boldsymbol{A}_{s1}$, $\boldsymbol{A}_{s2}$, $\boldsymbol{B}_{r1}$, $\boldsymbol{B}_{r2}$. For squirrel–cage rotor $\boldsymbol{U}_r = \begin{bmatrix} U_{r\alpha} \\ U_{r\beta} \end{bmatrix} = 0$ and for frequency control $\boldsymbol{U}_s = \begin{bmatrix} U_{s\alpha} \\ U_{s\beta} \end{bmatrix} = \begin{bmatrix} k_{us}\cos(k_{fs}\tau) \\ -k_{us}\sin(k_{fs}\tau) \end{bmatrix}$, where $k_{us}$ – the relative value of voltage amplitude as a fraction of the maximum one, bearing in mind that $k_{us\,\boldsymbol{max}} = 1$, $k_{fs}$ – is a relative frequency as a fraction of the nominal one, bearing in mind, that $k_{fs\,н} = 1$.

If to study the frequency control mode of the rotor (double fed asynchronous machine), it is advisable to write the machine's equation in the rotating with rotor speed axes. Then it is necessary to operate submatrices $\boldsymbol{A}'_{s1}, \boldsymbol{B}_{r1}, \boldsymbol{A}_{s2}, \boldsymbol{B}'_{r2}$, and the submatrices of stator and rotor windings voltage will determine as:

$$\boldsymbol{U}_s = \begin{bmatrix} U_{s\alpha} \\ U_{s\beta} \end{bmatrix} = \begin{bmatrix} -U_s \cdot \sin\theta \\ U_s \cdot \cos\theta \end{bmatrix}; \; \boldsymbol{U}_r = \begin{bmatrix} U_{r\alpha} \\ U_{r\beta} \end{bmatrix} = \begin{bmatrix} -k_{ur} \cdot \sin(k_{fr} \cdot \tau) \\ k_{ur} \cdot \cos(k_{fr} \cdot \tau) \end{bmatrix} \quad (16),$$

where $\theta$ – is an angle between a vector of synchronously rotating stator voltage $U_s$ and the rotor axis, determined by the relation $p\theta = 1 - \omega_r$; $k_{ur}$ – is a relative amplitude value of rotor winding voltage; $k_{fr}$ – is a relative frequency of rotor winding current.

On these mathematical models the imitation of quasi-stationary mode of «Siemens Wind Power» wind power plants, equipped with squirrel-cage asynchronous generators and controlled by frequency converters, feeding the stator windings of these generators, has been carried out.

The study results in a range of wind motor's rotational frequency control, varied with wind speed change, are presented in Table 2.

It should be noted, that in most part of modern WPPs (of «Gamesa», «Vestas», «Nordex», «GE Wind Energy» type) as electromechanical converters the double fed asynchronous machine (DFAM) is used, owing to its brilliant adjusting characteristics, which is confirmed by the results, calculated on a presented mathematical model. If there is a necessity to take into account at greater length a saturation, a current displacement in the slot, etc. in an asynchronous machine, it needs to refer to [6].

When studying a frequency control of the double fed asynchronous machine's rotor, a current frequency in the rotor winding must be equal to $f_r = f_s \cdot s$, and when the sign of slip during the transition from traction mode to generator one changes, the signs of the amplitude and frequency of the supplying rotor voltage must change, which is taken into account in the parameters of Table 3.

The values of DFAM operating parameters, when control with constant reactive power q output to power network, are shown in Table 3. In this case, the voltages amplitudes $k_{ur}$ varies somewhat with respect to $k_{fr}$.

With synchronous value of rotational frequency $\omega_r = 1$ $k_{ur} = k_{fr} = 0$, when $k_{ur} = k_{fr}$ changes from 0 to $k_{ur} = k_{fr} = -0{,}22$, the machine operates in generator mode, and rotational frequency $\omega_r$ reaches its maximum value $\omega_r = 1{,}22$ (i.e. if the synchronous rotational speed was equal to $\omega_{rs} = 1500$ rpm, it became $\omega_{r\,max} = 1830$ rpm).

The signs before $m_{\mathrm{EM}}$ and $P_{\mathrm{EM}}$ depend on the value and sign of $m_{\mathrm{WM}}$ and show, that the asynchronous machine all time is working in generator mode, although the sign of slip can vary from –0,22 to 0,22. The signs before the reactive power value show, that at +*q* machine consumes a reactive power from the network, and at –*q* returns it to network.

Table 2. The study results in a range of wind motor's rotational frequency control, varied with wind speed change

| № | V, m/s | $m_{EM}$, r.u. | $\omega_r$, r.u. | $k_{us}=k_{fs}$, r.u. | $P_{EM}$, r.u. | $q$, r.u. | $\beta=\omega_r-k_{fs}$, r.u. |
|---|---|---|---|---|---|---|---|
| 1 | 7 | –0,295 | 0,679 | 0,67 | –0,201 | 0,325 | 0,009 |
| 2 | 8,75 | –0,468 | 0,855 | 0,84 | –0,400 | 0,356 | 0,015 |
| 3 | 9,8 | –0,588 | 0,959 | 0,94 | –0,564 | 0,323 | 0,019 |
| 4 | 10,5 | –0,667 | 1,021 | 1 | –0,681 | 0,255 | 0,021 |
| 5 | 7,7 | –0,361 | 0,751 | 0,74 | –0,272 | 0,344 | 0,011 |
| 6 | 9,1 | –0,502 | 0,886 | 0,87 | –0,445 | 0,352 | 0,016 |
| 7 | 5,25 | –0,163 | 0,505 | 0,50 | –0,083 | 0,273 | 0,005 |
| 8 | 9,1 | –0,502 | 0,886 | 0,87 | –0,445 | 0,352 | 0,016 |
| 9 | 8,89 | –0,476 | 0,862 | 0,847 | –0,410 | 0,355 | 0,015 |
| 10 | 5,25 | –0,163 | 0,505 | 0,50 | –0,083 | 0,273 | 0,005 |
| 11 | 4,9 | –0,144 | 0,475 | 0,47 | –0,068 | 0,265 | 0,004 |
| 12 | 4,9 | –0,144 | 0,475 | 0,47 | –0,068 | 0,265 | 0,004 |
| 13 | 4,9 | –0,144 | 0,475 | 0,47 | –0,068 | 0,265 | 0,004 |
| 14 | 7 | –0,295 | 0,679 | 0,67 | –0,201 | 0,325 | 0,009 |
| 15 | 8,75 | –0,468 | 0,855 | 0,84 | –0,400 | 0,356 | 0,015 |
| 16 | 9,8 | –0,588 | 0,959 | 0,94 | –0,564 | 0,323 | 0,019 |
| 17 | 7 | –0,295 | 0,679 | 0,67 | –0,201 | 0,325 | 0,009 |
| 18 | 5,25 | –0,163 | 0,505 | 0,50 | –0,083 | 0,273 | 0,005 |
| 19 | 7 | –0,295 | 0,679 | 0,67 | –0,201 | 0,325 | 0,009 |
| 20 | 5,25 | –0,163 | 0,505 | 0,50 | –0,083 | 0,273 | 0,005 |
| 21 | 9,1 | –0,502 | 0,886 | 0,87 | –0,445 | 0,352 | 0,016 |
| 22 | 9,1 | –0,502 | 0,886 | 0,87 | –0,445 | 0,352 | 0,016 |
| 23 | 8,75 | –0,468 | 0,855 | 0,84 | –0,400 | 0,356 | 0,015 |
| 24 | 6,3 | –0,236 | 0,607 | 0,60 | –0,143 | 0,303 | 0,007 |

Table 3. Studying a frequency control of the double fed asynchronous machine's rotor the transition from traction mode to generator one changes

| № | V, m/s | $m_{EM}$, r.u. | $\omega_r$, r.u. | $k_{ur}$, r.u. | $k_{fr}$, r.u. | $P_{EM}$, r.u. | $q$, r.u. |
|---|---|---|---|---|---|---|---|
| 1 | 6,10 | –0,304 | 0,78 | 0,2515 | 0,22 | –0,237 | –0,387 |
| 2 | 6,25 | –0,319 | 0,80 | 0,2301 | 0,20 | –0,256 | –0,387 |
| 3 | 6,42 | –0,336 | 0,82 | 0,2087 | 0,18 | –0,276 | –0,387 |
| 4 | 6,64 | –0,360 | 0,85 | 0,1766 | 0,15 | –0,307 | –0,387 |
| 5 | 6,89 | –0,387 | 0,88 | 0,1446 | 0,12 | –0,341 | –0,387 |
| 6 | 9,49 | –0,744 | 1,22 | –0,2259 | –0,22 | –0,908 | –0,387 |
| 7 | 9,39 | –0,720 | 1,20 | –0,2042 | –0,20 | –0,864 | –0,387 |
| 8 | 9,23 | –0,696 | 1,18 | –0,1824 | –0,18 | –0,822 | –0,387 |
| 9 | 9,00 | –0,661 | 1,15 | –0,1500 | –0,15 | –0,760 | –0,387 |
| 10 | 8,61 | –0,605 | 1,10 | –0,0965 | –0,10 | –0,666 | –0,387 |

The dynamic characteristics of WPP's asynchronous generator, when starting by underfrequency relay, before running at rotational frequency control mode, is shown in Figure 1. In this case, the setters define the rates of rise of amplitude and frequency of supplied to stator voltage.

Creating a motor's driving torque, wind facilitates the unit acceleration, which is carried out by program frequency and voltage ramps, after lock in synchronous speed the machine passes into generator mode with $m_{\text{EM}} = -0{,}144$ and $\omega_r = 0{,}475$ (which corresponds to $\text{V} = 4{,}9$ m/s) (Figure 1*a*). Ibidem, the programmed changes of amplitude and frequency of voltage, supplied to generator's stator are shown (Figure 1*b*).

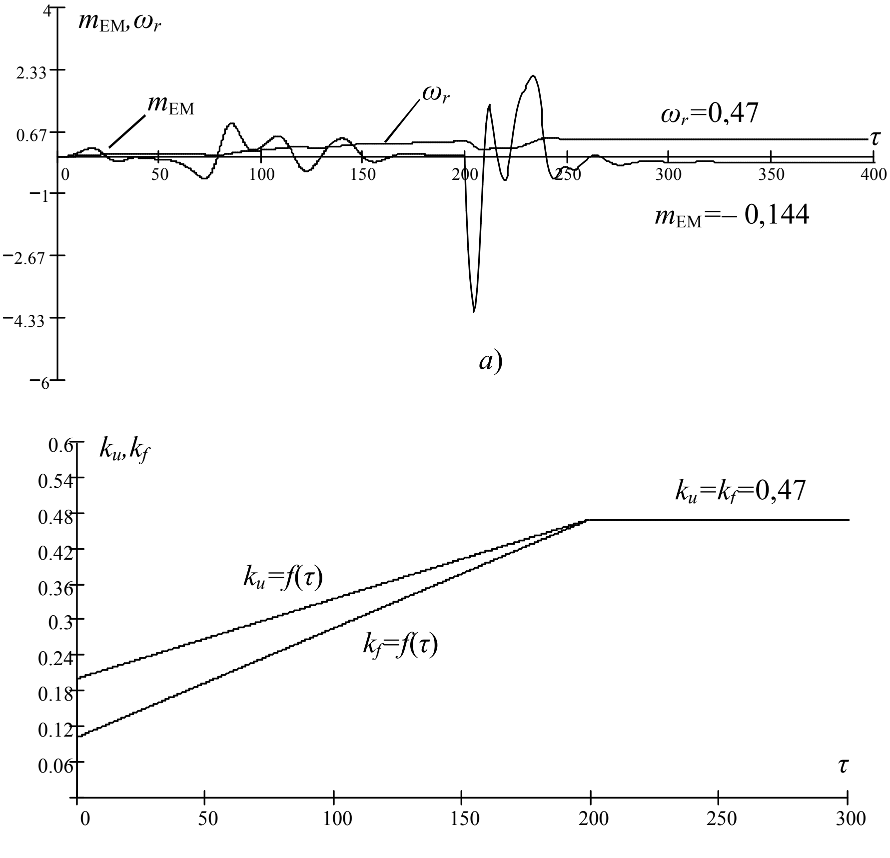



**Figure 1.** The curves of the transient process in the frequency start (*a*) and linear variation of the amplitude $k_{us} = k_{s0} + a\cdot\tau = 0{,}2 + 0{,}00135\cdot\tau$ and frequency $k_{fs} = k_{f0} + b\cdot\tau = 0{,}1 + 0{,}00185\cdot\tau$ of voltage (*b*).

## 3. DEVELOP A DIGITAL MODEL OF WPP'S SYNCHRONOUS GENERATOR BOTH WITH ELECTROMAGNETIC EXCITATION AND PERMANENT MAGNETS

Besides asynchronous generators the synchronous generators are used in WPPs, more often with permanent magnets (plants of Enercon, Vensys, GE Energy type) and rarely with electromagnetic excitation (Frisia type, etc.). For all this, they may be super-low-speed ones (so–called Ringgenerator) for gearless WPPs of Enercon and Vensys type and "normal" ones for geared WPPs of GE Energy and Frisia type [4]. And they both are equipped with frequency converters with IGBT-transistors, supplying the generators' stator circuit.

The purpose of this article is to develop a digital model of WPP's synchronous generator both with electromagnetic excitation and permanent magnets (most part) for investigation of transient and steady-state operating modes under frequency control.

The torque characteristics of wind motors, which are recommended to determine for 4 ranges of wind speeds change, are presented in [7]: from the beginning of WPP's operation (i.e. from a minimum wind speed at which a power output into the power network begins) to the beginning of

WPP's rotational frequency control; a range of WPP's rotational frequency control; a range from the end of rotational frequency control to design wind speed (i.e. the wind speed at which the rated power is supplied to network ); and finally, the range from design wind speed to the maximum possible wind speed, at which the output of WPP's rated power to the network still continue.

Thus, the expressions for wind motor torque, reduced to generator's shaft, are determined by the above characteristics, and frequency-controlled range is a second one of wind speeds variations.

To study the static and dynamic characteristics of WPPs, equipped with synchronous generators with electromagnetic excitation, the well-known equations of Park can be used for synchronous machines. If it is necessary, the saturation accounting of main machine flow can be carried out according to the method, presented in [8]. In contrast to conventional form, these equations are written in so-called "mixed" form – that is, the currents are exactly marked out, which must be monitored, for all this they are accordingly determined by other currents and flux linkages. Besides that, the applied to excitation winding voltage, as well as in [8], is presented as a fraction of no-load voltage, and wind motor torque in the range of WPP's rotational frequency control in proportion to wind speed is determined by the expression:

$$m_{\mathrm{WM}} = k_{м} \cdot \omega_r^2 \tag{17}$$

where $k_{м}$ – coefficient of proportionality of wind motor's torque.

To model a synchronous generator with permanent magnets, found a preferential use in WPPs, the Park equations needs to be transformed as follows. Assuming $p\psi_{df}$ is equal to zero $p\psi_{df} = 0$, from the equation of this system excitation we have:

$$i_{df} = \frac{U_{df}^*}{x_{ad}} \tag{18}$$

Substituting the value of this current to remaining equations, and removing the flux linkage equation $\psi_{df}$ for its uselessness, we obtain the equations of synchronous machine with permanent magnets. It should be noted, that this proposed form of Park equations record for synchronous machine with electromagnetic excitation allows passing easily to the equations of machine with permanent magnets. But $U_{df}^*$ in them should naturally be interpreted not as the excitation voltage, but as a value of magnetic energy of magnet per unit volume, or when small values of residual induction as a coercive force of the magnet. Thus, for example, it is necessary for $U_{df}^* = 1$ to take such value of magnetic energy of the magnet, which is capable to provide for no-load generator operation the value of the e.m.f. at the stator terminals equal to $\ell_{xx}=1$.

Comparing the WPP's synchronous generators with electromagnetic excitation and with permanent magnets excitation on their adjusting properties, it becomes obvious, that the generators with permanent magnets are inferior to the ones with electromagnetic excitation. For example, to stabilize the voltage of synchronous generators with permanent magnets when load changes at constant rotational frequency, the parametric and direct methods of influence the permanent magnets on magnetic flux are used [9]. But they are only suitable for small-capacity machines and micromachines.

Everything changes cardinally with a frequency converter in the WPP's synchronous generator stator circuit. In this case the output voltage can not only be stabilized, but adjusted to ensure the selected law of optimal control, i.e. current frequency control in the generator stator circuit provides a change of rotational frequency of WPP's shaft in proportion to wind speed, which allows maximizing the utilization factor of wind power in the whole control range. And amplitude control will allow providing the specified operating mode of either the generator itself or electric power network at the point of its connection.

For frequency controlled generator with electromagnetic excitation one more control channel is added – its excitation current.

To study the transient and steady-state modes of WPP synchronous generator's operation, when control both amplitude and frequency of stator voltage, it needs to adapt so the Park equations, which is known to be written in rotating with rotor speed d, q axes, that they should reflect the changes of amplitude and frequency of generator's stator voltage.

For this purpose the basic equations of WPP's synchronous machine is offered to leave recorded in d, q axes, and only the components of stator voltage $U_{ds}$ and $U_{qs}$, which without frequency control are written in the form $U_{ds} = -U_s \cdot \sin\theta$ and $U_{qs} = U_s \cdot \cos\theta$**,** to present with frequency control in such form, that they should reflect the changes of amplitude and frequency of voltage at the terminals of WPP's synchronous generator.

For all this it needs to refer to shown in Figure 2 diagram. Here $\alpha_0$, $\beta_0$ – are fixed in space axes of coordinates, $\alpha_s$, $\beta_s$ – synchronously rotating axes with angular frequency $\omega_s$, which is corresponded to frequency of current at the output of frequency converter, d, q – coordinate axes, rotating with a rotor speed $\omega_r$. Angle $\alpha = \omega_r \cdot \tau$ – is the angle between the axes $\alpha_s$, $\beta_s$ and $\alpha_0$, $\beta_0$, the angle between d, q axes and fixed axes $\alpha_0$, $\beta_0$ is $\alpha = \omega_r \cdot \tau$, and finally $\theta$ – is the angle between d, q axes and $\alpha_s$, $\beta_s$, which is the interior angle of synchronous machine. From the diagram in Figure 2 we have:

$$\theta = \alpha + \alpha_s = \omega_r \cdot \tau + \omega_s \cdot \tau \tag{19}$$

where $\tau$ – synchronous time in radian, equal to $\tau = \omega_{баз} \cdot t = 314 \cdot \mathrm{t}$, t – time in seconds.

If to dispose the vector of stator voltage in initial (source) mode at $45^0$ angle to the axes $\alpha_s$, $\beta_s$, its projections on these axes in the source mode will be the same and equal to $U_{s\alpha_0} = U_{s\beta_0} = 0{,}707$.

In accordance with the diagram in Figure 2, the projections of vectors $U_{s\alpha}$ and $U_{s\beta}$ on d, q axes in their turn will be written as:

$$\left.\begin{aligned} U_{d\alpha} &= U_{s\alpha} \cdot \cos\theta = U_{s\alpha} \cdot \cos(\alpha + \omega_s \cdot \tau) \\ U_{q\alpha} &= U_{s\alpha} \cdot \sin\theta = U_{s\alpha} \cdot \sin(\alpha + \omega_s \cdot \tau) \\ U_{d\beta} &= U_{s\beta} \cdot \sin\theta = U_{s\beta} \cdot \sin(\alpha + \omega_s \cdot \tau) \\ U_{q\beta} &= U_{s\beta} \cdot \cos\theta = U_{s\beta} \cdot \cos(\alpha + \omega_s \cdot \tau) \end{aligned}\right\} \tag{20}$$

General projections of these components on d, q axes pursuant to Figure 2 are determined as

$$\left.\begin{aligned} U_{ds} &= U_{d\alpha} - U_{d\beta} \\ U_{qs} &= U_{q\alpha} + U_{q\beta} \end{aligned}\right\} \tag{21}$$

Substituting the voltage components from the expression (20) to expression (21) and bearing in mind, that $U_{s\alpha} = U_{s\beta} = U_{s\alpha 0} \cdot k_u = U_{s\beta 0} \cdot k_u = 0{,}707 \cdot k_u$ and $\omega_s = k_f \cdot \omega_{s0}$, where $k_u = \frac{U_s}{U_{s0}}$ and $k_f = \frac{\omega_s}{\omega_{s0}}$ in one's turn $U_{s0} = \omega_{s0} = 1$ [in relative unit], we finally get::

$$\begin{aligned} U_{ds} &= 0{,}707 \cdot k_u \left[\cos(\alpha + k_f \cdot \tau) - \sin(\alpha + k_f \cdot \tau)\right] \\ U_{qs} &= 0{,}707 \cdot k_u \left[\sin(\alpha + k_f \cdot \tau) + \cos(\alpha + k_f \cdot \tau)\right] \end{aligned} \tag{22}$$

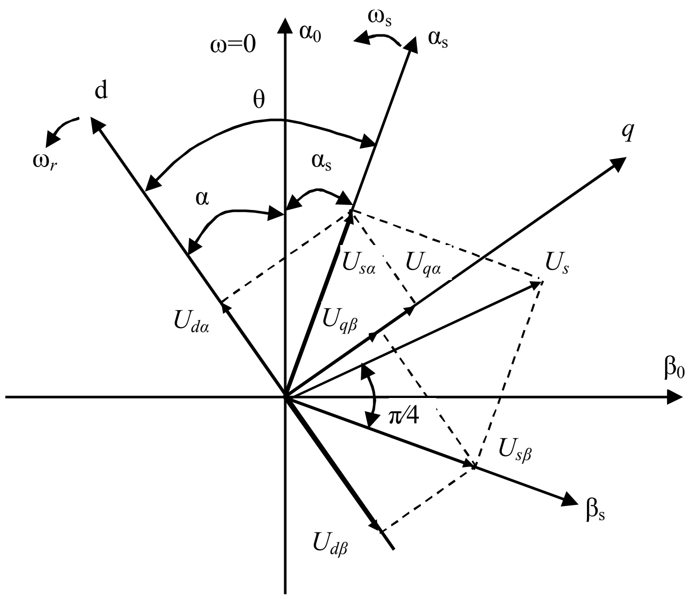



**Figure 2.** Diagram of the location rotating axes of synchronous machines.

And finally, if it is necessary, these equations by the trivial conversions can be written in a more convenient for using form:

$$\left.\begin{aligned} U_{ds} &= AC - BD \\ U_{qs} &= AD + BC \end{aligned}\right\} \qquad (23)$$

where $A = 0{,}707 \cdot k_u \cdot \cos(k_f \cdot \tau)$ $\quad C = \cos\alpha - \sin\alpha$

$B = 0{,}707 \cdot k_u \cdot \sin(k_f \cdot \tau)$ $\quad D = \cos\alpha + \sin\alpha$

During the study of parallel operation of several WPPs, consisting the wind power farm (WPF), the expressions for the stator currents $i_{ds}$ and $i_{qs}$, written in d, q axes, can be represented in the fixed coordinate system. For all this it's enough to determine the currents components in accordance with the diagram by the expressions:

$$\left.\begin{aligned} i_{\alpha 0} &= i_{ds} \cdot \cos\alpha + i_{qs} \cdot \sin\alpha \\ i_{\beta 0} &= -i_{ds} \cdot \sin\alpha + i_{qs} \cdot \cos\alpha \end{aligned}\right\}$$

(24)

It should be borne in mind, that these currents in steady-state mode have a frequency at the output of frequency converter, i.e. they flow between the windings of stator and frequency converter output, if it is necessary to determine the currents before the converter, then $\alpha = \omega_r \cdot \tau$ needs to be replaced by $\omega_{s0} \cdot \tau$, i.e. as $\omega_{s0} = 1$, that simply replace by $\tau$:

$$\begin{aligned} i_{\alpha 0} &= i_{ds} \cdot cos\tau + i_{qs} \cdot sin\tau \\ i_{\beta 0} &= -i_{ds} \cdot sin\tau + i_{qs} \cdot cos\tau \end{aligned} \qquad (25)$$

The above conversions allow, without changing the structure of Park equations, which are written in the rotating with rotor speed axes, in voltage components $U_{ds}$ and $U_{qs}$ to take into account the change of both amplitude and voltage frequency of WPP's synchronous generator stator circuit, obtained after output from the frequency converter.

Thus, the system of Park equations together with the equations (22) or (23) constitute the digital mathematical model of WPP's synchronous generator with electromagnetic excitation under frequency control.

A final version of digital model of WPP's synchronous generator with permanent magnets under frequency control will appear in the following form:

$$\left.\begin{aligned}
&p\Psi_{ds} = 0{,}707 \cdot k_u \left[\cos(k_f \cdot \tau)\cdot(\cos\alpha - \sin\alpha) - \sin(k_f \cdot \tau)\cdot(\cos\alpha + \sin\alpha)\right] - \omega_r \cdot \psi_{qs} - r_s \cdot i_{ds}\\
&p\Psi_{qs} = 0{,}707 \cdot k_u \left[\cos(k_f \cdot \tau)\cdot(\cos\alpha + \sin\alpha) + \sin(k_f \cdot \tau)\cdot(\cos\alpha - \sin\alpha)\right] + \omega_r \cdot \psi_{ds} - r_s \cdot i_{qs}\\
&p\psi_{dr} = -\frac{r_{dr}}{x_{dr}}\cdot\psi_{dr} + \frac{r_{dr}\cdot x_{ad}}{x_{dr}}\cdot i_{ds} + \frac{r_{dr}}{x_{dr}}\cdot U^*_{df}\\
&p\psi_{qr} = -\frac{r_{qr}}{x_{qr}}\cdot\psi_{qr} + \frac{r_{qr}\cdot x_{aq}}{x_{qr}}\cdot i_{qs}\\
&p\omega_r = \frac{1}{T_j}\cdot m_{\text{вд}} - \frac{1}{T_j}\cdot m_{EM}\\
&p\alpha = \omega_r\\
&m_{EM} = \psi_{ds}\cdot i_{qs} - \psi_{qs}\cdot i_{ds}\\
&i_{ds} = \frac{x_{dr}}{\Delta d}\cdot\psi_{ds} - \frac{x_{dr} - x_{ad}}{\Delta d}\cdot U^*_{df} - \frac{x_{ad}}{\Delta d}\cdot\psi_{dr}\\
&i_{qs} = \frac{x_{qr}}{\Delta q}\cdot\psi_{qs} - \frac{x_{aq}}{\Delta q}\cdot\psi_{qr}
\end{aligned}\right\}\quad (26)$$

Values $\Delta d = x_{ds}\cdot x_{dr} - x_{ad}^2$ and $\Delta q = x_{qs}\cdot x_{qr} - x_{aq}^2$.

Besides these basic equations, the equations of active and reactive power of WPP's synchronous generator can be added for complete analysis to these equations:

$$\left.\begin{aligned}
p &= U_{ds}\cdot i_{ds} + U_{qs}\cdot i_{qs}\\
q &= U_{qs}\cdot i_{ds} - U_{ds}\cdot i_{qs}
\end{aligned}\right\}\quad (27)$$

The study of model synchronous generator with imitation of operating mode of Vensys 77 type WPP have been carried out by equations (26). The specified WPP is equipped with low-speed synchronous generator with permanent magnets, controlled by a frequency converter with IGBT – transistors of 1500 kW capacity, design wind speed is 13 m/s, the initial wind speed is 3 m/s, and maximum allowable operating wind speed is 22 m/s, a range of WPP's rotational frequency control is 9–17,3 rpm, i.e., almost 1:2 [4]. The studies have been carried out at $U^*_{df} = 1{,}2 = const$.

To solve (26) system the Matcad program has been used.

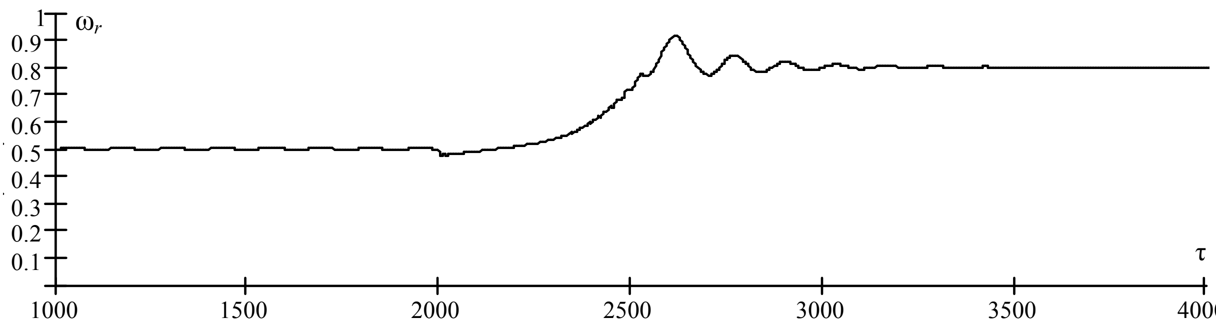
The parameters of steady–state mode for control $k_u = k_f$ are given in table 4; when control with constant reactive power output *q*=–0,18=*const* (negative sign corresponds to generator mode) and when control with constant power factor $\cos\varphi \approx 1$. Analysis of the results shows, that torque is determined by rotational frequency, and consequently by network frequency, and for all cases when frequency changes $k_f$ from 0,5 to 1, it varies from –0,15 to –0,6. This torque is corresponded to active power P, which also varies from –0,074 to –0,6, and the active current $i_{qs}$, which range of variation is from –0,136 to 0,543, these values are identical for all modes. Control with a constancy of reactive power output *q*=*const* and a constancy of $\cos\varphi \approx 1$ in a synchronous machine with permanent magnets can be achieved by separate control of voltage amplitude at the output of frequency converter in accordance with the values $k_u$, presented in clauses (a) and (b) of table 4. It should be noted, that within the frequency change range from 0,6 to 0,9 the control $k_u = k_f$ practically coincides with the control with a constancy of *q*=*const*.

Table 4. The parameters of steady–state mode for control $k_u = k_f$, when control with constant reactive power output *q=const* and when control with constant power factor $\cos\varphi \approx 1$.

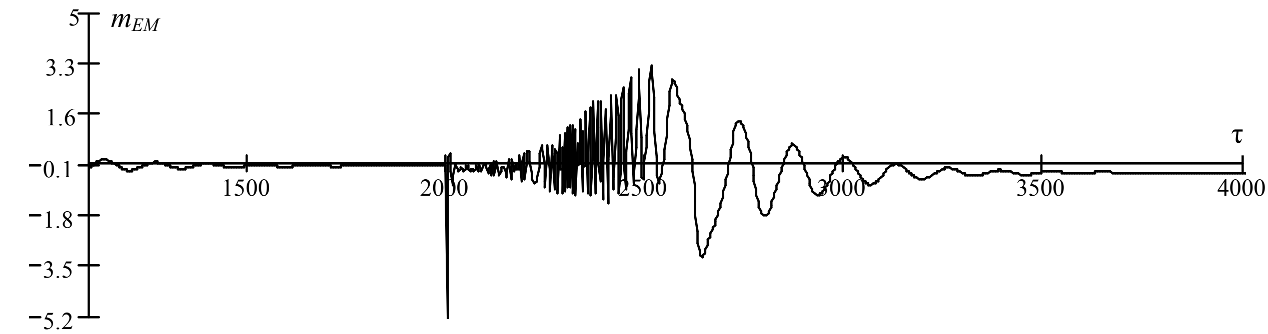
| a) | Control with $k_u = k_f$ | | | | | |
|---|---|---|---|---|---|---|
| $k_u=k_f$ | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1,0 |
| $m$ | –0,150 | –0,215 | –0,296 | –0,389 | –0,486 | –0,598 |
| $P$ | –0,074 | –0,128 | –0,205 | –0,304 | –0,435 | –0,596 |
| $q$ | –0,137 | –0,158 | –0,175 | –0,180 | –0,177 | –0,155 |
| $i_{ds}$ | –0,200 | –0,200 | –0,210 | –0,220 | –0,234 | –0,257 |
| $i_{qs}$ | –0,136 | –0,197 | –0,265 | –0,346 | –0,441 | –0,543 |
| b) | Control with the constant reactive power output $q = const$ | | | | | |
| $k_f$ | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1,0 |
| $k_u$ | 0,484 | 0,592 | 0,698 | 0,799 | 0,899 | 0,991 |
| $q$ | –0,179 | –0,180 | –0,180 | –0,181 | –0,180 | –0,179 |
| $i_{ds}$ | –0,270 | –0,232 | –0,216 | –0,217 | –0,237 | –0,257 |
| c) | Control with the constant power factor $\cos\varphi \approx 1$ | | | | | |
| $k_f$ | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1,0 |
| $k_u$ | 0,545 | 0,650 | 0,758 | 0,860 | 0,960 | 1,055 |
| $q$ | –0,0017 | –0,0017 | –0,00173 | –0,0017 | –0,0018 | –0,0017 |
| $i_{ds}$ | –0,01 | –0,0187 | –0,033 | –0,055 | –0,089 | –0,137 |

Among the dynamic characteristics the mode of sharp increase of driving torque on generator's shaft (wind gust) is of interest, with working-off the mode of output to a new frequency of frequency converter.

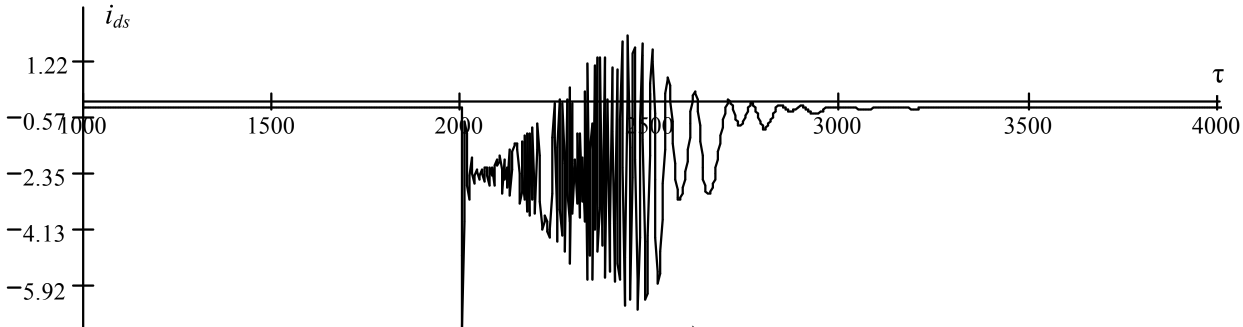
The fluktogramms of WPP generator's regime parameters change $\omega_r$, $m_{EM}$, $i_{ds}$, $i_{qs}$, $q$, $P_{EM}$ are accordingly presented in Figure 3 (*a, b, c, d, e, f*) when operating in steady-state mode, corresponding to the rotation frequency $\omega_r$=0,5 ($k_u=k_f$=0,5) with wind gust (at 2000 radian), increasing the driving torque approximately 2,5 times (from $m_{EM} = -0{,}15$ to $m_{EM} = -0{,}385$), and this new torque value should correspond to WPP's shaft rotational frequency, which is equal to $\omega_r$=0,8 ($k_u=k_f$=0,8). When assignment the new values of $k_u$ and $k_f$, a control path for simplicity of analysis is assumed to be an aperiodic link with the time constant $T_j$=100 rad (~ 0,3 s). A system, as it is seen from fluktogramm, is stable one, although the amplitude of oscillations of some regime parameters is rather significant one ($q_{max}$=6, $i_{ds\ max}$=6,5), but a change of their average value during the transient is not greater than 3-fold value.



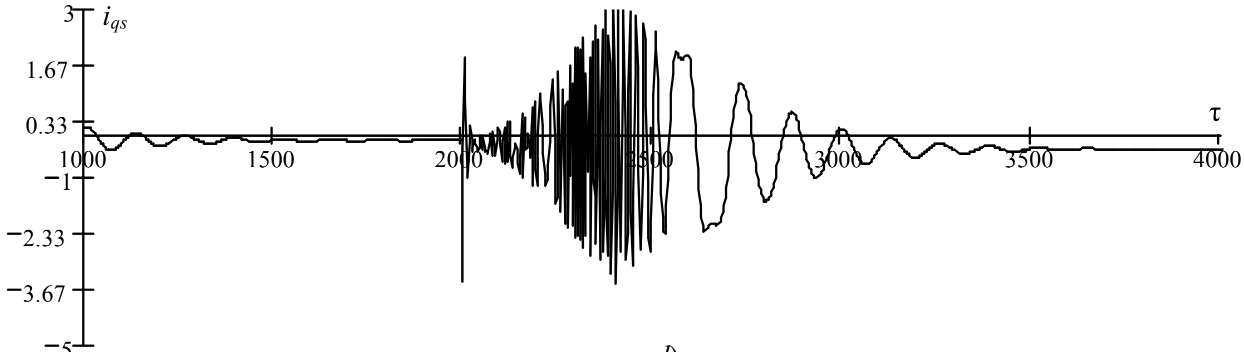

*a*)


$m_{EM}$
5
3.3
1.6
−0.1
−1.8
−3.5
−5.2
τ
1500
2000
2500
3000
3500
4000

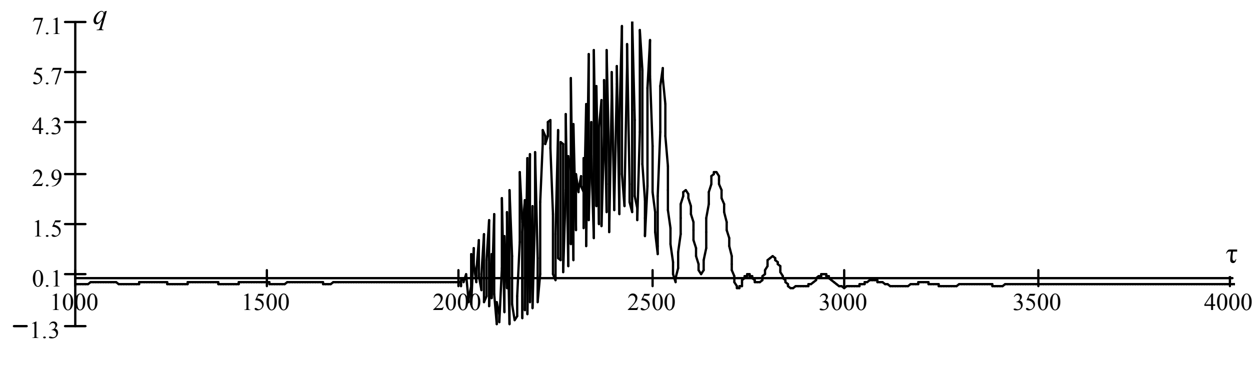
*b*)


$i_{ds}$
1.22
−0.57
−2.35
−4.13
−5.92
−7.7
τ
1000
1500
2000
2500
3000
3500
4000

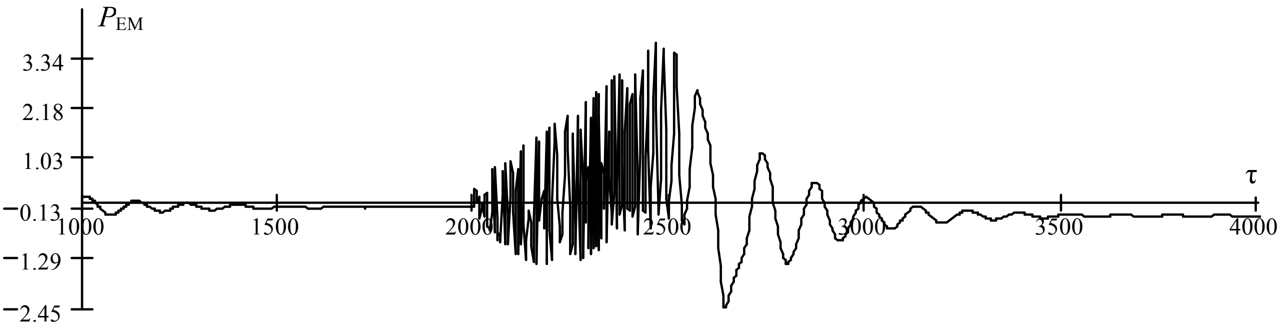
*c*)


$i_{qs}$
3
1.67
0.33
−1
−2.33
−3.67
−5
τ
1000
1500
2000
2500
3000
3500
4000

*d*)


$q$
7.1
5.7
4.3
2.9
1.5
0.1
−1.3
τ
1000
1500
2000
2500
3000
3500
4000

*e*)




*f)*

**Figure 3.** The fluktogramms of WPP generator's regime parameters change $\omega_r$ (*a*), $m_{EM}$ (*b*), $i_{ds}$ (*c*), $i_{qs}$ (*d*), $q$(*e*), $P_{EM}$ (*f*) are accordingly when operating in steady-state mode.

## 4. CONCLUSION

1. It is determined, that with an acceptable in engineering calculations error (no more than 8%) in the area of speed control directly proportional to speed of wind - WPP's energy resource, the wind motor torque can be considered as proportional to the square of its rotational frequency.
2. The matrix form of equations of WPP's asynchronous machine condition under frequency control both of stator and rotor, allowing obtaining the uniform study results, have been presented.
3. The operating modes when the frequency control of WPP's asynchronous generator stator have been studied for voltage control both with constant generator's relative slip and with constant consumed reactive power. It was revealed, that the first mode was consistent with the known law of frequency control by academician M.P.Kostenko, and the second mode allowed to compensate the consumed by WPP reactive power with the help of uncontrolled static capacitors, installed in the place of WPP's connection to electric power network.
4. It was demonstrated, the reactive power output to network can be controlled in quasi-stationary modes with frequency control of rotor of double fed asynchronous machines.
5. The dynamic regimes of WPP with asynchronous generator when frequency dispersal of plant have been studied; it was revealed, the starting torques and currents were minimized in this case.
6. Frequency-controlled mathematical model of WPP's synchronous generator both with the permanent magnets and an electromagnetic excitation have been developed. It was shown, in WPP's synchronous generator with permanent magnets the operating modes with constant reactive power output $q = const$ and constant $\cos\varphi \approx 1 = const$ could be provided for the separate control of amplitude and frequency of generator stator voltage.
7. The effectiveness of start by underfrequency relay of WPP's synchronous generator with permanent magnets, providing the smooth acceleration, the small values of operational parameters during start-up and "soft" locking in synchronism has been demonstrated.

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# PROBABILITY-STATISTICAL ANALYSIS OF QUEUING NETWORKS WITH INFINITE NUMBER OF SERVERS

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## ABSTRACT

In this paper queuing networks with infinite number of servers and exponentially distributed service times are considered. Limit distributions for numbers of customers in aggregated nodes are calculated. Using these calculations statistical estimates of limit distributions parameters are constructed. Mean times of customer sojourn time in opened network and its analog for closed network are estimated also. Obtained results are spread onto estimates of populations and life cycle parameters.

**Keywords:** ruin probability, transportation problem, asymptotic formula, enumeration problem

## INTRODUCTION

In (Tsitsiashvili & Osipova, 2009) a problem of estimates of product limit distributions parameters is formulated and is solved for opened and closed queuing networks with single server nodes. In this paper opened and closed queuing networks with infinite number of servers in each node are considered. A problem of an aggregation of nodes in these networks is analyzed and limit product distributions are represented via some analogs of nodes load coefficients. Such approach allows to obtain formulas for efficiency indexes of networks and to construct algorithms of their estimates by observations of numbers of customers in different nodes. We speak about a customer mean sojourn time in opened network and about mean time between customer successive appearances in some node for closed network. Obtained results allow analyzing a dynamics of a population with individuals which have many states (Hun & Gulevich, 2008) and life cycle of technical systems with many states (Yung-Wen Liu & Kailash C. Kapur, 2008).

## 1. OPENED EXPONENTIAL NETWORKS

Consider opened queuing network with the states set $S=\{0,1,...,m\}$ and irresolvable matrix of transition probabilities $\Theta=\|\theta_{ij}\|_{i,j=0}^{m}$ and Poisson input flow with intensity $\lambda>0$. In each node $i\in I$, $I=\{1,...,m\}$, there is infinite number of servers with service time which has exponential distribution with parameter $\mu_i>0$. The node 0 is dummy, it is a source of arriving customers and is a runoff of customers which depart from network. For fixed $\lambda>0$ the system of linear algebraic equations (Basharin & Tolmachev, 1983)

$$(\lambda,\lambda_1,...,\lambda_m)=(\lambda,\lambda_1,...,\lambda_m)\Theta \qquad (1)$$

has single solution $\lambda_i>0$, $i\in I$. Markov process $(n_1(t),...,n_m(t))$ which describes numbers of customers in nodes of opened network is ergodic (Foss, 1991) and its limit distribution has the following form (Serfozo, 1999, Example 1.29) (see (Basharin & Tolmachev, 1983) also)

$$p(n_1,...,n_m)=\prod_{i=1}^{m}\exp(-\rho_i)\frac{\rho_i^{n_i}}{n_i!},\ \rho_i=\frac{\lambda_i}{\mu_i},\ i\in I, \tag{2}$$

**Theorem 1.** Almost surely we have

$$\lim_{T\to\infty} n_i^T=\rho_i,\ n_i^T=\frac{\int_0^T n_i(t)dt}{T},\ i\in I. \tag{3}$$

Theorem 1 statement is obtained from the law of large numbers for discrete Markov processes (Serfozo, 1999, Theorem 1.2).

Construct a model of aggregated network uniting nodes from not intersected subsets $I_1,...,I_r$ of set $I$ and conserve initial order of customers service and routing. Denote $(N_1(t),...,N_r(t))$ random process (non Markov) describing numbers of customers in united nodes, $N_k(t)=\sum_{i\in I_k} n_i(t)$, $k=1,...r$.

**Theorem 2.** The process $(N_1(t),...,N_r(t))$ has limit distribution

$$P(N_1,...,N_r)=\prod_{k=1}^{r}\exp(-R_k)\frac{R_k^{N_k}}{N_k!},\ R_k=\sum_{i\in I_k}\rho_i \tag{4}$$

and almost surely

$$\lim_{T\to\infty} N_j^T=R_j,\ N_j^T=\frac{\int_0^T N_j(t)dt}{T},\ j=1,...,r. \tag{5}$$

**Proof.** Formula (4) is a corollary of formula (2) and well known fact that (Feller, 1984, Chapter XI) a sum of independent random variables with Poisson distributions and parameters $a,b$ has Poisson distribution with parameter $a+b$. Formula (5) is a corollary of formulas (3), (4).

Suppose that $f$ is mean sojourn time of customer in opened network which equals with mean time of customer service in network nodes. The quantity $f$ may be interpreted as a mean life time of individual represented by a customer of input flow. Then we have (Borovkov, 1972, § 31, Theorem 6)

$$f=\frac{R}{\lambda},\ R=\sum_{i\in I}\rho_i. \tag{6}$$

So if we estimate the input intensity $\lambda$ then theorem 2 allows to estimate the parameter $R$ and consequently the parameter $f$.

## 2. DYNAMICS OF POPULATION OF INDIVIDUALS WITH MANY STATEDS

In (Hun & Gulevich, 2008) a model of a dynamics of a population with $n$ individuals is considered. A state of $k$-th individual (a stage of illness) is described by Markov chain $x_k(t)$, $t=0,1,...$, with the states set $I=\{1,...,m\}$ and with the transition probabilities

$$\theta_{i,i},\ \theta_{i,1}>0,\ i=1,...,m,\ \theta_{i,i+1}>0,\ i=1,...,m-1.$$

A transition of an individual into state 1 means that it dies and immediately new individual burns with same number. If the process $x_k(t)$ is interpreted as a displacement of particle then it is in $i$-th state random time $\tau_i$ with geometric distribution and with parameter $1-\theta_{i,i}$. For ergodic process $x_k(t)$ it is possible easily to calculate stationary distribution $\pi(i)$, $i\in I$.

In (Hun & Gulevich, 2008) if Markov chains $x_k(t)$, $k=1,...,n,$ are independent then numbers of individuals which are in state $i$-th satisfy the law of large numbers (Serfozo, 1999, Theorem 1.2) and almost surely

$$\lim_{T\to\infty}\frac{n_i^T}{n}=\pi_i,\ i\in I.$$

These formulas allowed to analyze optimal strategies of a prevention of non infectious diseases.

In this paper a model of a population dynamics in continuous time is constructed. In constructed model non realistic assumption of constant number of individuals in a population is cancelled.

We consider population model as opened queuing network with infinite number of servers in each node. Then a vector of numbers of individuals in different stages of diseases is described by Markov process $(n_1(t),...,n_m(t))$. In this case sojourn time of individual in $i$-th state has exponential distribution with the parameter $\mu_i$. A presence of infinite number of servers in each node guarantees independent individuals displacements along network nodes.

Analogously to (Hun & Gulevich, 2008) assume that the route matrix $\Theta$ of opened queuing network contains following non zero elements:

$$\theta_{i,i+1},\ \theta_{i,0}=1-\theta_{i,i+1},\ i=1,...,m-1,\ \theta_{m,0}=\theta_{0,1}=1.$$

Then it is easy to obtain from (1) that

$$\lambda_{i+1}=\lambda_i\theta_{i,i+1},\ i=1,...,m,$$

And consequently from (6) we have that mean individual life time is

$$f=\sum_{i=1}^{m}\prod_{k=0}^{i-1}\frac{\theta(k,k+1)}{\mu_i}.$$

**Remark 1.** Suppose that customer-individual passing through $i$-th node may receive additional service-prevention in parallel node $i'$ with intensity $\mu_{i'}<\mu_i$. That increases mean sojourn time of customer in this node. If customer arrives in node $i$ then with probability $a_i$ it is served on server of this node and with probability $b_i=1-a_i$ it is served in node $i'$. As result equality $\rho_i=\lambda_i/\mu_i$ is replaced by equality

$$\rho_i=\frac{a_i\lambda_i}{\mu_i},\ \rho_{i'}=\frac{b_i\lambda_i}{\mu_{i'}}.$$

Using these equalities it is possible to estimate influence of prevention on main indexes of this network.

## 3. CLOSED EXPONENTIAL NETWORKS

Consider closed queuing network with the states set $S$, the route matrix $\Theta$, $n$ customers circulating along this network and $n$ servers in each node with service intensity $\mu_i$ on a server of $i$-th node, $i\in S$. Suppose that for fixed $\lambda_0>0$ the system of linear algebraic equations (1) has single solution $\lambda_1>0,...,\lambda_m>0$. Then Markov process $(n_0(t),...,n_m(t))$ characterizing numbers of customers in nodes of closed network is ergodic (Ivchenko & Kashtanov & Kovalenko, 1982, Theorem 2.4) and has limit multinomial distribution (Serfozo, 1999, Example 1.29)

$$P(n_0,...,n_m)=n!\prod_{i=0}^{m}\frac{d_i^{m_i}}{n_i!},\ d_i=\frac{\rho_i}{\rho_0+...+\rho_m},\ \rho_0=\frac{\lambda_0}{\mu_0},\ i\in S,\ \sum_{i=0}^{m}n_i=n. \quad (7)$$

**Theorem 3.** Almost surely

$$\lim_{T\to\infty}\frac{n_i^T}{n}=d_i,\ i\in S. \quad (8)$$

**Proof.** From binomial theorem we have that the process

$$(n_0(t),n_1(t),...,n_{m-2}(t),n_{m-1}+n_m(t))$$

has limit distribution

$$P(n_0, n_1, \dots n_{m-2}, n_{m-1} + n_m) = n! \frac{(d_{m-1} + d_m)^{n_{m-1}+n_m}}{(n_{m-1} + n_m)!} \prod_{i=0}^{m-2} \frac{d_i^{m_i}}{n_i!}. \quad (9)$$

Using mathematical induction it is easy to prove that the process $\left( n_0(t), \sum_{k \in I} n_k(t) \right)$ has limit distribution

$$P(n_0, n_1, \dots, n_m) = \frac{n! d_0^{n_0} (d_1 + \dots + d_m)^{n-n_0}}{n_0!(n - n_0)!}.$$

Consequently the process $n_0(t)$ has binomial limit distribution with mean $nd_0$. Analogously it is possible to prove that random process $n_i(t)$ has binomial limit distribution with mean $nd_i$, $i = 1, \dots, m$. Then from the law of large numbers for ergodic Markov processes (Serfozo, 1999, Theorem 1.2) we obtain the formula (8).

Construct a model of aggregated network uniting nodes in not intersected subsets $I_1, \dots, I_r$ of the set $I$, conserving initial characteristics of customers service and routing. Denote $(N_1(t), \dots, N_r(t))$ random process (non Markov) which describes numbers of customers in united nodes, $N_k(t) = \sum_{i \in I_k} n_i(t)$, $k = 1, \dots, r$.

**Theorem 4.** The process $(N_1(t), \dots, N_r(t))$ has multinomial limit distribution

$$P(N_1, \dots N_r) = n! \prod_{k=1}^{r} \frac{D_k^{N_k}}{N_k!}, \ D_k = \sum_{i \in I_k} d_i, \ k = 1, \dots, r, \quad (10)$$

and almost surely

$$\lim_{T \to \infty} \frac{N_k^T}{n} = D_k, \ k = 1, \dots, r. \quad (11)$$

**Proof.** The formula (10) may be obtained from the formula (9) by mathematical induction. The formula (11) is a corollary of the formula (10) and the theorem 3.

Denote $f_i$ mean time between successive appearances of a customer in $i$-th node of closed network. To calculate $f_0$ consider opened network with route matrix $\Theta$ and with input intensity $\lambda_0$. It is obvious that

$$f_0 = \frac{1}{\mu_0} + f = \frac{1}{\mu_0} + \frac{R}{\lambda_0} = \frac{\rho_0 + \dots + \rho_m}{\lambda_0} = \frac{\rho_0}{d_0 \lambda_0} = \frac{1}{\mu_0 d_0}. \quad (12)$$

Consequently if we know statistical estimate of service intensity $\mu_0$ then using the theorem 3 it is possible to estimate $d_0$ and consequently $f_0$. Analogously it is possible to obtain the equality

$$f_i = \frac{1}{\mu_i d_i}, \quad (13)$$

and using it to estimate mean $f_i$ by estimate of service intensity $\mu_i$, $i \in I$.

## CONCLUSION

Theorems 2, 4 allow to aggregate nodes of initial network and consequently to simplify a procedure of statistical estimates of its parameters. It means that we may delete a solution of the system (1) and to simplify a procedure of the network observation. Besides of product distribution parameters suggested approach allows to estimate mean sojourn times of customers in the network. Obtained results may be generalized onto queuing networks with non exponential service time distributions using (Harrison & Lemoine, 1981) and may be applied in technical gerontology and demography to estimate mean time of endowment for persons of fixed age.

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# POWER SUPPLY RESTORATION IN DISTRIBUTIVE NETWORKS

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## ABSTRACT

The method and program complex realization for restoration scheme search of power consumption in networks on the basis of algorithm processing of the network count and artificial neural networks (ANN) are offered and given reason. Both algorithms work in a competing mode, mutually smoothing lacks of each separate method. Work complex principles, in particular, concerning application of the generalized error vector for ANN self-training and solution stability are theoretically proved. The program complex realized as a part for 'the adviser of the distributive network dispatcher', was tested on a 201 buses, 7 sources and 227 lines distribution network and one has shown good results.

Keywords: *distributive network, power restoration, automation, dispatcher adviser.*

Reliability operation performances of distributive network restoration are its time and completeness of a consumer supply in postemergency state and limiting conditions. A number of foreign [1-4, 5-13] and Russian [5-8] works, including author's [19-26] of given paper, is devoted the specified problem. We offer a combined supply restoration method on the basis of the graph research algorithm and the artificial neural network with training and self-training modes.

The distributive network consumer majority can receive the electric power on several various circuits. At emergency state of one or groups of feeding communications in an existing mode it is necessary quickly to find a reserve circuit in a network for a power supply to the currentless consumer. Following requirements should be allowed: the consumer element overload and voltage drop cannot exceed permissible limits; in the loading restriction need less responsible consumers are disconnected the first of all.

Existing distributive network restoration methods can be united conditionally in three groups: 1) on the basis of the graph theory and the combinatory mathematics; 2) with knowledge base creation at training and the subsequent sample of scheme condition image for concrete event; 3) with artificial neural network use (ANN).

The variant finding concerns advantages of the first of them practically for any mode, including well-founded refusal in the consumer supply of those buses, which cannot be connected on physical or mode conditions, for example, at the unique and damaged feeder [2-5, 8]. However at seeming simplicity method demands viewing of variant set, hence, decision reception time increases. For reduction of variant quantity those or other restrictions are entered usually: combinational, mode etc., reducing their number to comprehensible values, but worsening their quality.

At the second approach the set of possible modes is modeled off-line mode and their target images, consumers satisfying to maintenance by a supply (communication switch state), are remembered in the knowledge base. In on-line mode the target image to gets out from base the corresponding to entrance image (conditions of an incorrect mode) is the network scheme. Such approach allows finding the decision rather quickly. Its lack is great volume of the knowledge base which growth depends practically exponentially on number of distributive network communications. So for a distributive network on 1000 communications the volume under the knowledge base is estimated in $10^{21}$ Gb. Taking into account various restrictions this figure decreases, but nevertheless remains considerable. Nevertheless, works in this direction proceed [9-13].

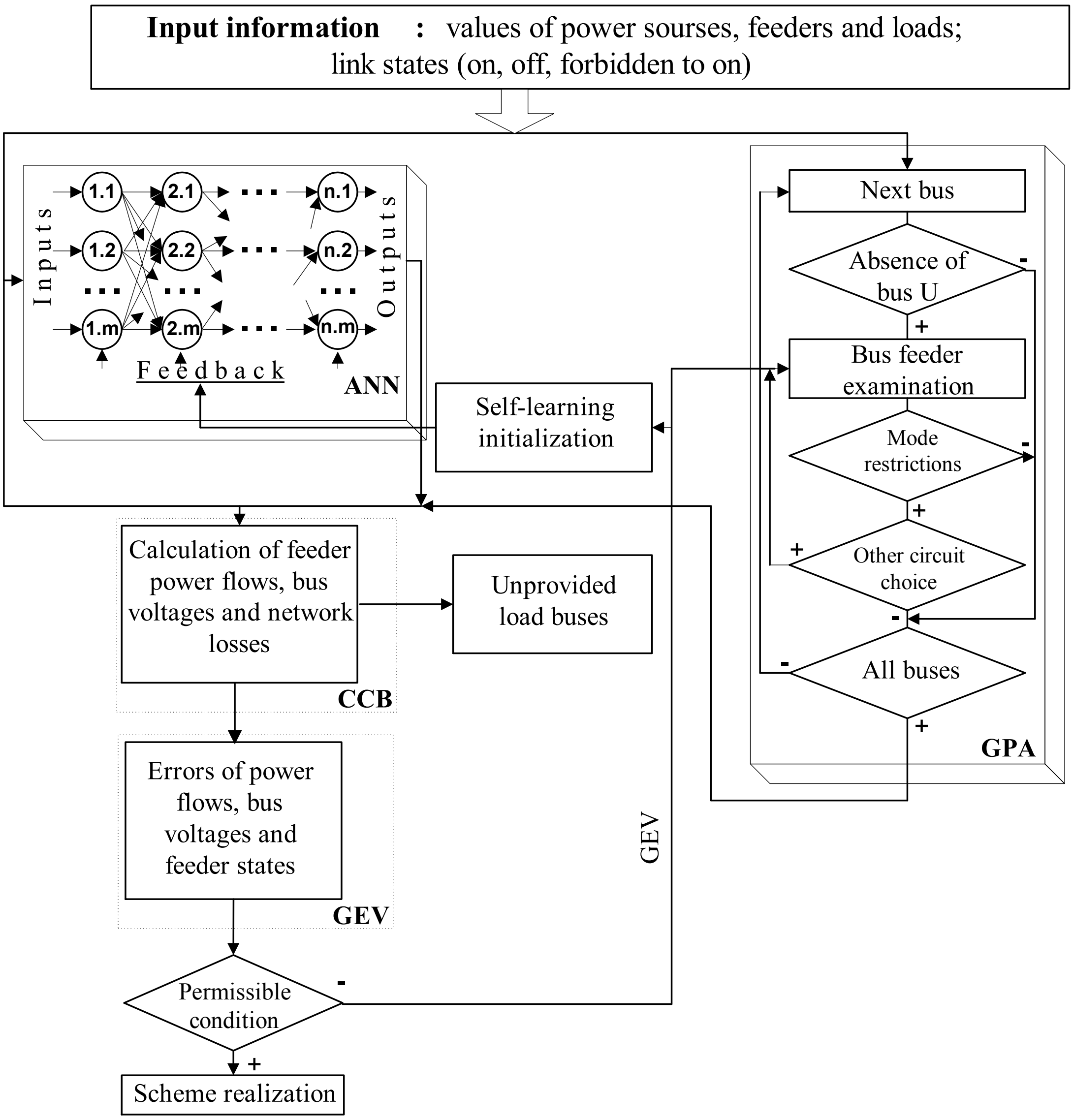
The third way, with use ANN, under the form in many respects models the second approach, but as the knowledge base the functional nonlinear converter (neural network) is used. In it a method after a well-founded structure choice and ANN long training the decision results is obtained quickly enough. However their interpretation represents certain complexity to a concrete mode in most cases and demands additional actions on mode correctness check. From our experience the decision method ANN represents good approach for its subsequent specification by methods of the first approach.

Problem in the specified methods is modernization of a distributive network. If in the first case an input or addition of its new elements increases number of considered variants, then for two last cases network area which modernization of communications and buses influences, it is necessary to repeat all process of training [11].

Certain complexity is made by coordination of computer complex work on power supply circuit search for distributive network consumers with information base of its dispatching point and with mode management automatics. Regarding the information are usually necessary: full scheme model of a distributive network, switch states (switched on, switched off, switched off and forbidden to on), loading levels for moment of prefault conditions (it is considered that whenever possible it is necessary to provide such loadings in postfault mode) and power flows on feeders. Here the network condition estimations, in detail, considered in a number of works [14-17] are not discussed. As to automatics it is considered to be that the complex searches for the network scheme decision after automatic reclose and fallback, and other automatics working off, taking into account the condition which have been with feeders after work of automatics.

We offer a *complex restoration scheme search method for consumer power supply* (fig. 1), working on a decision combination on graph processing algorithm (GPA) and by means of an ANN. Comparative simplicity used GPA, on the one hand, allows to find quickly the scheme for the majority of buses, on another - does not lead to cycling of its search in a difficult network configuration. ANN works faster in the presence of the decision in training sample. Such combination allows to lower requirements to computing complex resources for scheme search in comparison with known methods and to reduce decision time. Scheme restoration co-ordinates with information base of a control office and mode control automatics, particularly, on feeder states and loading levels for a mode prefault moment.

The absent data is restored by settlement methods [19]. The account of "mode weighting" before restoration of normal conditions in a network is carried out by corresponding factors of "load weighting". Conditions-restrictions in decision search are: mode – inadmissibility of an feeder overload and a voltage drop in buses more than specified value; priority – in load restriction need the less responsible consumers [20] first of all should be disconnected. GPA substantially models operation personnel actions in a concrete situation taking into account listed above restrictions. In parallel with it on the basis of training sample the ANN tries to find the new scheme of the network satisfying specified above restrictions.




All offered schemes are checked by the condition calculating block (CCB) which participates in formation of the generalized error vector (GEV). GEV specifies GPA and ANN acceptability / unacceptability of the offered scheme and in the latter case a search direction of the new decision. GPA searches for the new scheme taking into account the specified GEV restrictions. ANN passes in a self-training mode, being retargeted according to GEV. GPA and ANN blocks in a competing mode find the power supply scheme, satisfying to mode admissible conditions. Thus redistribution of consumers between power sources for load alignment is considered.

Advantage GPA is decision completeness without dependence from feeder initial conditions. Buses, which can be provided by energy, are provided with it. On those buses, which are not provided with energy because of mode or configuration conditions, the information on the reasons of such situation is outputted. A method lack is rather big decision search time to computer measures (from shares to tens seconds depending on scheme complexity).

Advantage ANN – almost instant decision on scheme conditions, which have entered into sample of training. In the condition absence in training sample the decision search time in the course of self-training is comparable with similar GPA time. Revealing of the unconnection reasons is more difficult than for loadings in GPA method. However, input of the decision for a new scheme condition in training sample at condition repetition leads to fast decision definition further.

*The condition calculating block* uses the information on necessary consumer loadings, had capacities and voltages in root network buses, feeder parameters, their states (switched on, switched off, switched off and forbidden to on) and restrictions in a network configuration. The mode calculation basis in view of its traditional character is not resulted. Its result is a finding of power flows on feeders and voltages in buses.

During decision, having received the current entrance function close to **r(k)**, the network restores on a target function **u(k)**. For the cases which have not got to training sample, target function not always leads to an admissible mode. CCB carries out check on mode admissibility; in result the error vector GEV is formed (fig. 2). Last initializes ANN self-training [23].

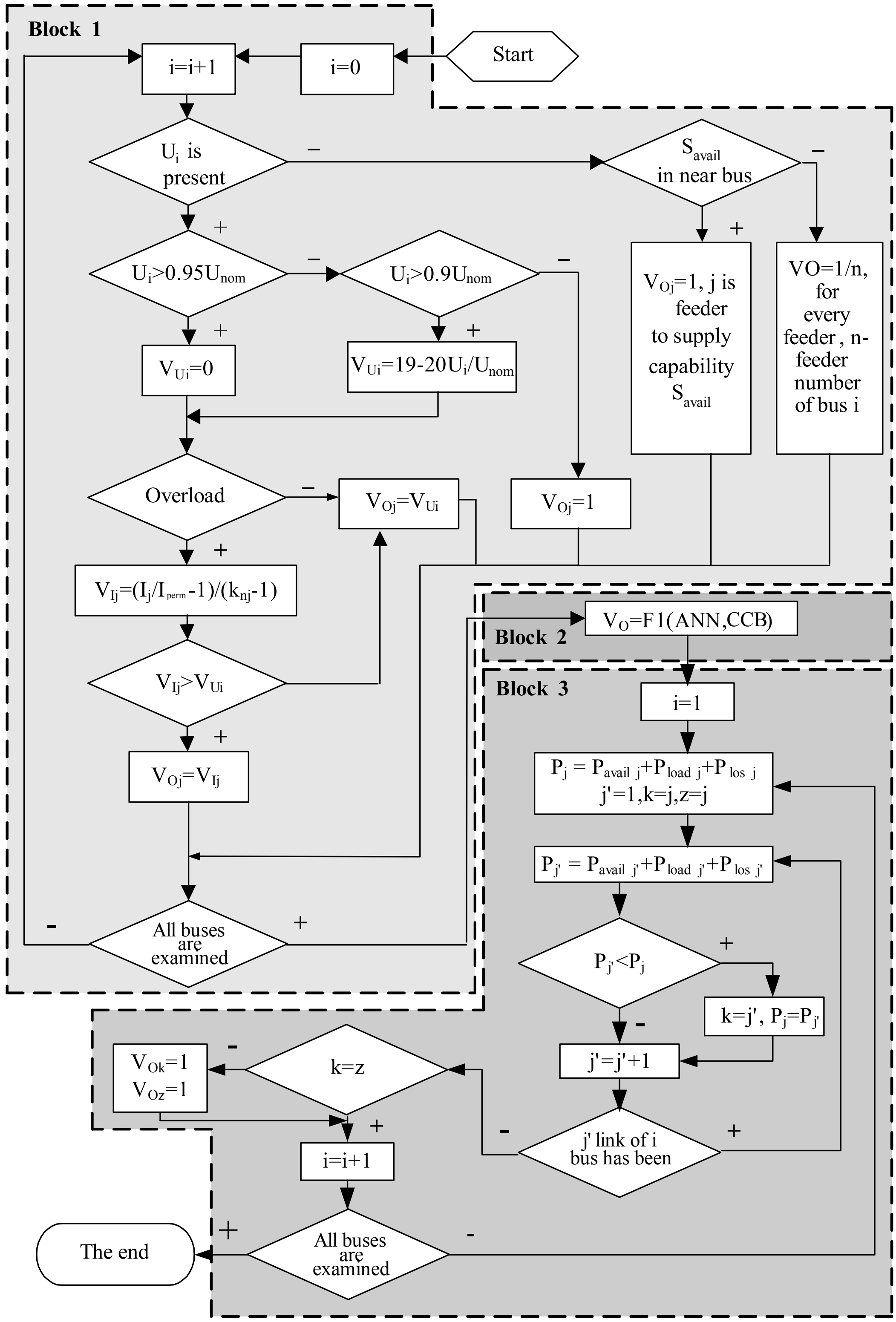



Fig. 2. Flow chart of the generalized error vector forming.

*The structure of GPA work* is reflected by the integrated block-diagram (fig. 1, block GPA). After damage switch-off the information on a breaker state and voltage in buses of a network at the calculating moment in GPA, and also consumer loadings in buses before network disturbance are entered. The disconnected buses are defined, and their number comes to light. If quantity of the disconnected buses is more than half they are carried out by algorithm of initial load connection [21], allowing quickly to restore the basic scheme part with the account of bus supply importance. If number of the disconnected buses small the bus, having the highest supply reliability importance and the greatest demanded load capacity, gets out first from their entire list. From the chosen bus to the nearest bus, having necessary capacity, the chain is found. The chain search algorithm is based on source search in the scheme graph [22]. Difference is that search is conducted at first at width, and then in depth. Such algorithm reorganization allows connecting bus to the nearest one, having necessary capacity that reduces losses on power flows. Regime parameter calculation occurs in CCB and the regime condition checking – in GEV block. If necessary link is not found, the information on the disconnected bus and the reasons are deduced on the monitor.

*The restoration scheme search block on ANN* (fig. 1, ANN block) represents multilayered fully connected neuron network into which the information on current network conditions is entered in first layer. The target layer forms the offered decision of feeder states. The quantity of internal elements and layers is being determinates by researches.

At training mode the ANN adjusts weight element factors by means of back propagation algorithm (BP) [18] so that for entrance function **r(k)** to approximate function of decisions **u(k)** training function $\mathbf{u}^*\mathbf{(k)}$ for $\mathbf{k} \in \mathbf{[1, N]}$, where **N** is a set of training sample examples.

Application of ANN algorithm with self-training for a solved problem is connected with a convergence substantiation of its decision. Such analysis is spent on the basis of the elementary scheme from three links with three switching devices (fig. 3). Examination of all possible conditions of its switching devices and definition of the generalized error vector in each set of states for influence on ANN makes it possible [24]. The specified approach is connected with possibility of equivalent transformation of a network to the elementary scheme. It is shown that after several iterations (from one to three for the resulted example) GEV becomes zero, hence, for the specified scheme the decision convergence is provided. But as in GEV definition each link is considered separately any other possible schemes with various switch state combinations and available or/and consumed power are reduced to one of the cases considered on the elementary scheme.

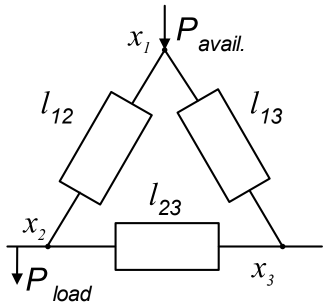



Fig.3. Elementary cell of
of distributive network.

As it is noted above, at restoration of a power supply it is important to consider their importance on supply reliability. Last characteristic is considered through a priority of bus in a network. Priority values of each bus taking into account its importance are initially set by the expert and (tab. 1 [22]) are stored in the data table on buses. In algorithms with serial processing of buses (GPA, CCB) their order is defined by current priority value on decrease.

It is necessary to notice that for correct ANN work the bus priority rationing system is entered. All bus priorities are limited by frameworks from 0.7 to 1. Values less than 0.7 are not used, as their efficiency at algorithm work is low, it is especially at presence of bus with several communication lines. Transformation of real priorities in rationing ones occurs automatically, agrees simple linear dependence, i.e. to maximum priority bus corresponds $\boldsymbol{Q}$ =1, and to minimum priority bus – $\boldsymbol{Q}$ =0.7. It is obvious that priorities of buses should not differ in hundreds and thousand times, otherwise distinction between buses with close priorities for algorithm will be weak and inefficient from the point of view of program time expenses.

In a manual mode of bus priority formation for the higher priority it is recommended to use values $\boldsymbol{Q}$ = 0.9 and above, for buses of the average priority – $\boldsymbol{Q}$ = 0.8 … 0.85, for the lower priority – $\boldsymbol{Q}$ = 0.7... 0.75. It is obvious that among buses of an identical priority, for example, buses which are necessary for connecting first from all exist higher priority ones. For possibility of realization of such feature the interval of values $\boldsymbol{q}$ also is entered. Thus, the above value $\boldsymbol{q}$, the bus will be more necessarily connected. For optimum algorithm work (from the point of view of time expenses) it is not recommended to specify the bus priority value of lower within 0.75-0.8, and for the average 0.85-0.9 as it will lead to boundary effect when on the importance of maintenance with the power the lower priority bus will come nearer to average priority buses, and the average priority bus will come nearer to the higher priority buses, accordingly.

Such difference in bus connection error formation, on the one hand, "will force" algorithm to connect higher priority buses even at capacity available deficiency for maintenance of all network buses, and on the other hand, will not admit switching off of knot with a high priority for connection of buses with a smaller priority.

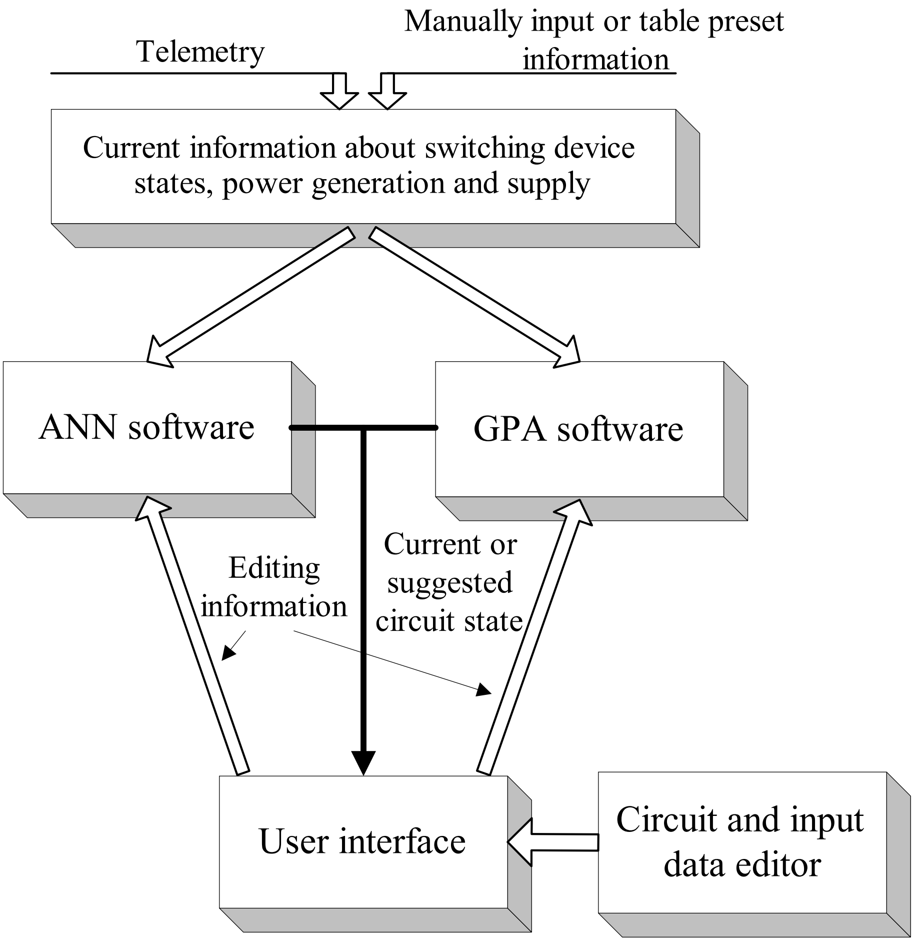



Fig. 4. Restoration system software.

Definition process of link state error weight $\boldsymbol{q}$ depends on a bus priority which feeder can connect. We will admit, feeder $\boldsymbol{j}$ can connect buses $\boldsymbol{i}_1$ (priority $\boldsymbol{Q}_1$) and $\boldsymbol{i}_2$ (priority $\boldsymbol{Q}_2$). Then a priority of this communication $\boldsymbol{q}_j = \boldsymbol{Q}_1$ if feeding bus will be $\boldsymbol{i}_2$, and $\boldsymbol{q}_j = \boldsymbol{Q}_2$ if feeding bus will be $\boldsymbol{i}_1$.

The program part can be divided into conditionally independent five blocks (fig. 4). The basis of search algorithm is made by GPA and ANN blocks. Initially in algorithm structure competitive processing of two various processes by decision search has been put. Processes are carried out mutually independently, but use the same entrance information. Thus, decision finding average time for enough big sample of required decisions should be reduced essentially.

The information is formed in the table of the bus and feeder characteristic (tab. 1). In its integer part (*uzi, lni*) it is described the network topology, bus states (is included/is disconnected) also bus supply priorities. Regarding values from a floating comma (*uzf, lnf*) remain parameters for buses and feeders on the voltage, available and consumed power, their power flows, admissions on an overload etc. Here are allocated in the italics the parameters set by the operator, or received from an operative measuring complex (SCADA). The direct font notes the parameters counted by programs.

*Table 1*

***Parameters, links and states of scheme elements***

| Variable | s | ***Bus specification*** | Bus number, (*n*) | | | |
|---|---|---|---|---|---|---|
| | | | 1 | 2 | … | $n_u$ |
| ***uzi[n,s]*** | 1 | State (attribute + switch off/switch on) | | | | |
| | 2 | Supply feeder number | | | | |
| | 3 | Plug-in bus priority | | | | |
| ***uzf[n,s]*** | 1 | *Gener./load active power (load with +)* | | | | |
| | 2 | *Gener./load reactive power (inductive with +)* | | | | |
| | 3 | Bus active power (total) | | | | |
| | 4 | Bus reactive power (total) | | | | |
| | 5 | Bus available active power | | | | |
| | 6 | Bus available reactive power | | | | |
| | 7 | Bus voltage | | | | |
| Variable | s | ***Feeder specification*** | Bus number, (*n*) | | | |
| | | | 1 | 2 | … | $n_l$ |
| ***lni[n,s]*** | 1 | Feeder starting bus | | | | |
| | 2 | Feeder end bus | | | | |
| | 3 | Feeder state (switched on, switched off or forbidden to switch) | | | | |
| ***lnf[n,s]*** | 1 | *Resistance* | | | | |
| | 2 | *Reactance* | | | | |
| | 3 | *Admissible continuous current* | | | | |
| | 4 | Carrying current value | | | | |
| | 5 | Active power flow | | | | |
| | 6 | Reactive power flow | | | | |
| | 7 | Voltage loss | | | | |
| | 8 | Active power loss | | | | |
| | 9 | Reactive power loss | | | | |
| | 10 | Admissible overload factor on preset time | | | | |

Program realization of the considered approach is based on use MS Visual $C^{++}$. By this time it was possible to realize two independent GPA and ANN modules which are united in one program however can be executed in parallel on different processors of multiprocessor computers or computing clusters. Program code feature is convenience of additional module inclusion (competing processes) to search the decision with other method use. Hence, it is possible not only to modernize constantly already used algorithms, but also to estimate quality, computing and time expenses for new algorithms. One more positive side of the chosen program architecture is that the interface of the user on formation, change and addition of test distributive networks is completely separated from decision search problems, serves only for preparation of the entrance data and can change irrespective of the specified tasks.

The time estimation of its performance during algorithm working out was a separate problem as in known publications to us time for decision search sharply increased at increase in elements of a distributive network [25]. There was a danger that positively proved on this parameter at small quantity of bus and interbus links the algorithm will sharply worsen the results at increase in network elements. Check of this parameter is shown below at algorithm approbation.

To estimate requirements to memory at increase in a distributive network at one *i* an element (bus, feeder, switching element) it is possible proceeding from the formula: $\Delta M=k\cdot T_i$, where $k$=1.07 – defined experimentally factor, and $T_i$ – memory byte quantity, necessary for a data storage about *i* element of the scheme. For bus ($i$=1) $T_1$ = 130 byte; for feeder ($i$=2) $T_2$ = 234 bytes; for a switching element ($i$=3) $T_3$=30 byte. Let a test scheme contains 201 buses, 227 feeders, 7 power supply source, including 5 diesel generators, and 165 switching elements. Then in the test scheme total used memory amount for tested distributive network scheme are easy for counting up under the formula:

$$M = 1.07\cdot[201\cdot(130 + 7) + 227\cdot234 + 165\cdot30] + R \approx 91598 + R,$$

where $R$ is a memory size occupied without dependence from element quantity, present at the scheme. This memory is not a constant, its size fluctuates depending on what dimension is used ANN.

Debugging of a program complex was spent on the test scheme [21]. Despite the simplicity, the scheme gave the chance to check up all basic cases arising in real distributive networks, such as: use of a reserve line at an outage the basic one, loading switching on bus with greater available capacity and/or with the least losses, impossibility of consumer maintenance by the electric power in connection with mode restrictions or feeder malfunction of the given bus. On the other hand, scheme simplicity did program complex work the evident, allowing to check at once program results, verifying them with the received manually data.

Transition to more difficult scheme was a following stage of testing of the program. It has been decided to use the part of power distributive network of Komi power system. Its parameters are given above. As it was already marked, authors were ready to increase in time delays and increase of requirements to memory resources, but as has shown experiment, serious changes have not occurred. It is obvious that with increase in scheme elements, each of which contains some data about the structure and physical properties, the memory size increases inevitably, but this dependence has linear character, and its factor is approximately equal to unit (the computer operative memory expense are considered necessary for program work).

During algorithm work speed measurement on various dimension schemes it was found out that the increase in time expenses, as well as in a case with memory resources, has almost linear character with multiplication to the constant factor close to two. It is caused first of all by inevitable increase in ANN dimension both on entrance and target layers, and on internal structure. Time expenses for initial ANN training in calculations were not considered, as take place in offline mode with exponential growth at increase in dimension and number of examples for processing.

Program interface feature is multiwindow intuitively clear approach, as at scheme creation and its processing, and at reflexion of current processes in a distributive network (fig. 5). He allows displaying on the monitor as a preemergency scheme state with corresponding conditions of switching devices, power flows and loadings, and the offered decision on supply scheme restoration. Nevertheless, there is number of principles, which it is necessary to adhere at scheme formatting and editing. Some conditions are not critical, i.e. obligatory, at the same time can in the subsequent essentially (by 5-10 %) to increase decision finding speed and also to increase scheme display information capability.

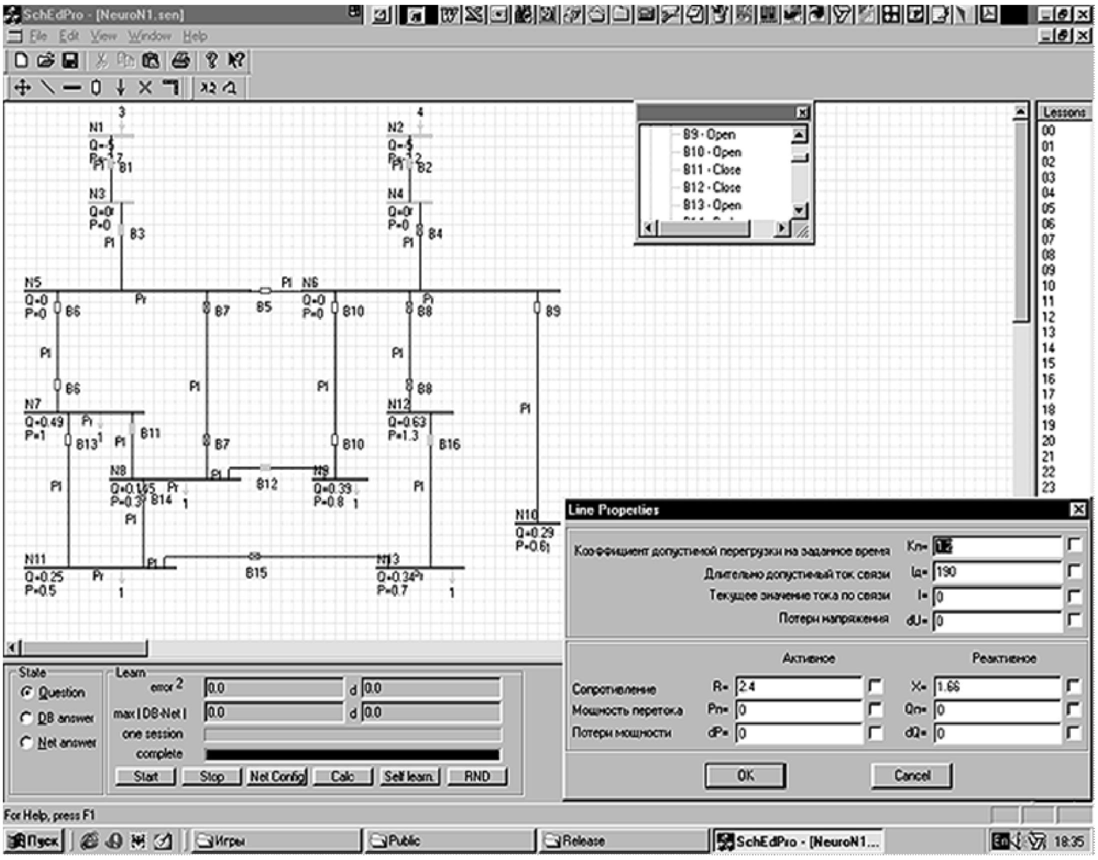



Fig. 5. Window of restoration system for distributive network.

To the indispensable conditions connected with editing of initial schemes and data, concern: 1) presence of two switching elements (breakers) on each feeder between the generating bus and any another; 2) feeder should be from both ends connected to buses; 3) on any line can be noted no more than two switching elements and if on a line two switching elements they work synchronously, i.e. both are simultaneously switched on, switched off or forbidden to switch. The line segment between two tapings in programs is presented as a line without switching devices. In this case its state is defined on voltage presence on its ends which can or be absent on both ends and then the line is considered disconnected, or to be present, and then the line is considered switched on; 4) buses on the monitor screen can settle down only horizontally that is connected with bus programming specificity; 5) switching elements and lines can vertically settle down or horizontally; 6) the bus with generated capacity is set as bus with negative consuming capacity.

To desirable, but not to editing indispensable conditions the following concerns: 1) the line should begin by bus which is usually feeding, and to come to an end in bus which is usually consuming that accelerates restoration scheme finding process; 2) between the generating bus and the first bus with consumption should settle down "the empty" or "virtual" bus, i.e. bus with zero consumption and zero generation that is connected with program technology of a generating bus finding; 3) necessary element scheme parameters, such as should be set: line type, length and wire type, line potential levels. Admissible values of currents, feeder resistance and reactance are defined automatically under the help data in the scheme editor.

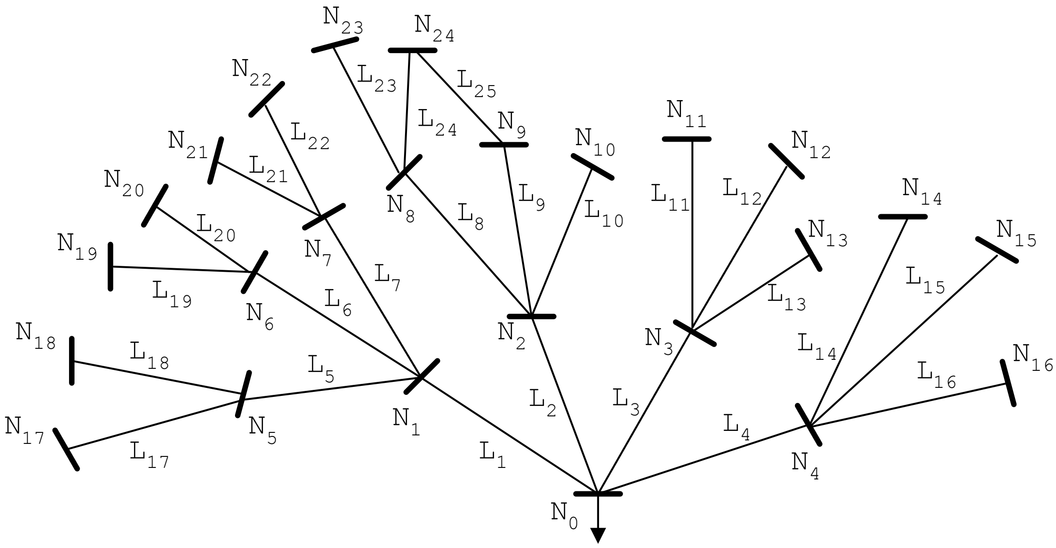



Fig.6. Part of distribution network.

Values of generated and consumed active and reactive power in buses, and also states of switching devices whenever possible arrive on telemetry channels of system. For buses and feeders on which the current data cannot be received in the specified form, there is a possibility to set their tabular form depending on the set conditions, or to correct manually. The full list of parameters necessary for calculation and their placing in information tables are given in [22]; 4) it is desirable to reduce quantity of the information demanded for display about each scheme element, having reduced it to is minimum necessary.

All operating modes were checked on the computer of class Intel Pentium MMX-166, with RAM volume in 64 Mb. The time tests were carried out at start of the program from compiler Visual $C^{++}$ 6.0 in a debugging mode. Even in such not optimum conditions the program has shown good results on decision finding time which made no more than 0.5 s where on a share of display of the information it was necessary 0.3 s. In agreement with statistics of expense time increase rate at increase in scheme element quantity with program start in the worker, instead of a debugging mode, it is possible to make the assumption that time for the problem decision in schemes with element quantity reflected in programs, an order of 10 thousand (in their considered example more than 600) can make no more than 6 s. At increase in processor and subsystem video capacity three times, that corresponds to possibilities of modern computers, it is possible to realize supply restoration scheme search in complicated networks in a real time mode.

## Conclusion

The offered method allows automating distributive network consumer supply restoration by computer means at failures and mode restrictions and to lower error probability of the personnel.

Method feature is joint application of competing decision search processes by algorithms of GPA and ANN, using advantage of each and reducing search time in a concrete situation.

In the absence of the ready decision in ANN training sample its dynamic self-training is made in real time on developed a technique and GEV definition algorithm. The found new decision fills up the specified sample.

Method approbation on test schemes of a distributive network has shown its working capacity.

**Appendix 1**

### Processing algorithm of the scheme graph

In the scheme (fig. 6) it is necessary to find a circuit, including nodes and connections from the disconnected node $N_0$ to node $N_x$, which has necessary capacity. All nodes are determined over available capacity and loading. All connections are described over power capacity. The distribution network is described over a node matrix (**NM**) connecting nodes and lines.

The algorithm carries out search $N_x$ at first into breadth, next into depth, what allows connecting $N_0$ to the nearest node. Decision tracking occurs in tab. 2. Search begins into **NM** with depth of step S=1. Nodes of a first step (fig. 7) are looked thru and the opportunity of connection to node $N_1$ is being estimated. The available capacity in $N_1$ and the power capacity of connection $L_1$ are being checked.

*Table 2*

**Solution search**

| Step | Loading node | Previous connection | Analyzed connection | Required active power | Required reactive power |
|---|---|---|---|---|---|
| 1 | $N_0$ | … | … | $P_0$ | $Q_0$ |
| … | … | … | … | … | … |
| n | $N_x$ | … | … | … | … |

Here it is being checked, whether is boundary (the last in an analyzed circuit, for example, nodes $N_{10}$-$N_{24}$). At non-compliance with the conditions (**Cond**.) the following node $N_2$ is being analyzed and so on up to $N_4$. If on a first step the supply node is not found, S=2, nodes $N_5$-$N_{16}$ are being analyzed and further for S=3 – $N_{17}$-$N_{24}$.

At condition executions in forming of nodes and of connections a necessary circuit are being defined from tab. 2. On example (fig. 7) this circuit is formed to node $N_{19}$.

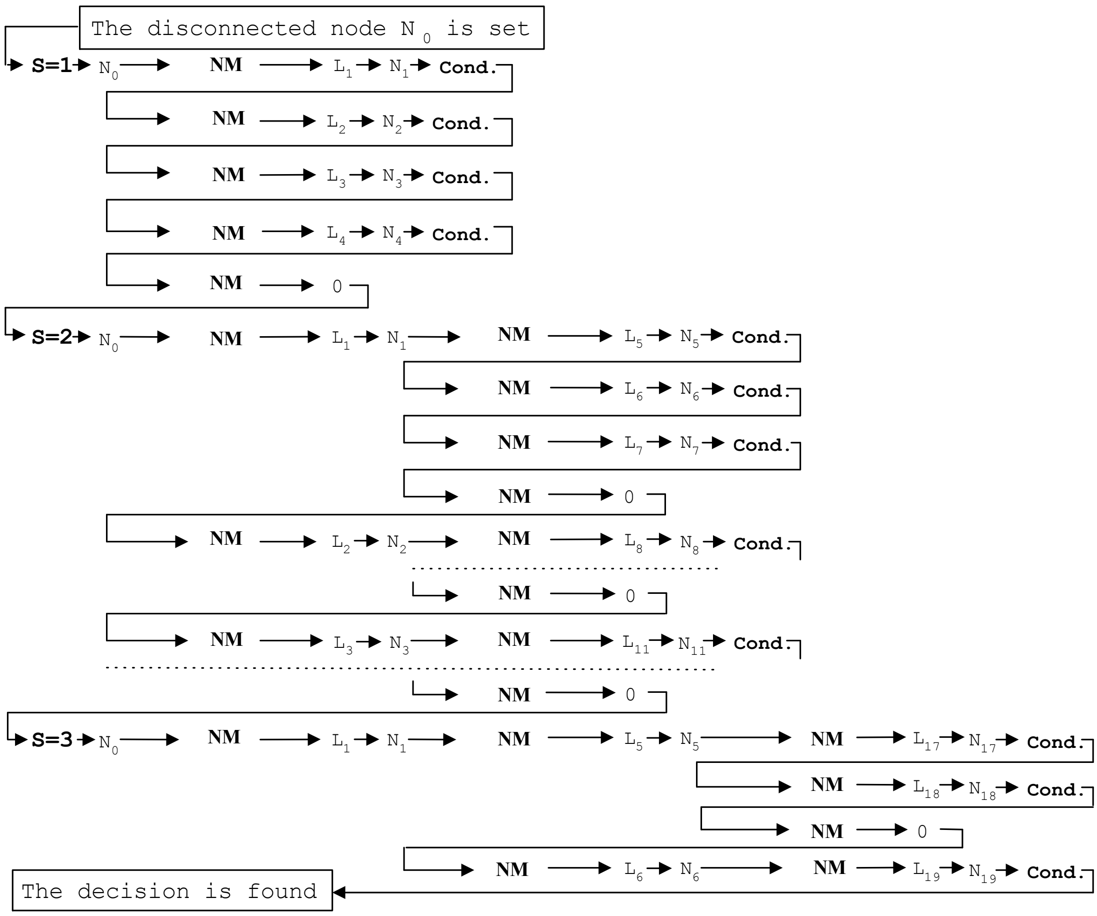



Fig.7 Process of solution search

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# ASYMPTOTIC FORMULA FOR DISCONNECTION PROBABILITY OF GRAPH ON TWO DIMENSIONAL MANIFOLD

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## 1. INTRODUCTION

A problem of a calculation of a graph disconnection probability is considered in a lot of papers, see for example [1]-[4]. In [1] upper and low bounds of the graph disconnection probability (a reliability polynomial) are constructed using maximal systems of disjoint cross sections. For a graph with sufficiently small number of arcs in [2] accelerated algorithms are constructed. These algorithms showed good results in a comparison with Maple 11. In [3] this problem is solved using Monte-Carlo method and some specific combinatory indexes and formulas.

But when a number of arcs increases this problem becomes much more complicated. So it is necessary to construct convenient asymptotic formulas for connectivity or disconnection probability of graph with high reliable arcs. In this paper such problem is solved for planar graphs or graphs arranged on two dimensional manifolds. Such graphs appear in honeycombed structures which are widely used in different applications.

## 2. PRELIMINARIES

Consider unoriented and connected graph $G$ with finite sets of nodes $U$ and of arcs $W$. Denote $\mathcal{L}(u,v)$ the set of all cross sections in $G$ which divide nodes $u,v \in U, u \neq v$,

$$\mathcal{L} = \bigcup_{u \neq v} \mathcal{L}(u,v).$$

Put $d(L)$ a number of arcs in cross section $L$ and define

$$D(u,v) = \min\left(d(L) : L \in \mathcal{L}(u,v)\right), \quad D = \min_{u \in v} D(u,v), \quad \mathcal{L}_* = \{L \in \mathcal{L} : d(L) = D\},$$

$C$ is a number of cross sections from the set $\mathcal{L}_*$

**Theorem 1.** Suppose that graph arcs $w \in W$ fail independently with the probability $h$ then the probability $P$ of the graph $G$ disconnection satisfies the formula

$$P \sim Ch^D, \ h \to 0. \tag{1}$$

So to calculate asymptotic of graph disconnection probability it is necessary to find the constants $C, D$. Suppose that $G$ is two dimensional integer rectangle with the size $M \times N$. If $M, N > 1$ then the set $\mathcal{L}_*$ consists of four angle cross sections with two arcs [5] and so

$$P \sim 4h^2, \ h \to 0.$$

In general case for $M > 0, N > 0$ we obtain the formula

$$P \sim \left(4 + I(M = 1)N + I(N = 1)M\right)h^2, \ h \to 0.$$

But to make asymptotic analysis of disconnection probability in honeycombed structures it is necessary to pass from integer rectangle to more general graphs.

## 3. MAIN RESULTS

This generalization is based on a concept of a graph $G$ arranged on connected and two dimensional smooth manifold without edge $\mathcal{T}$ [6,chapter 1].

Suppose that between two nodes of the graph $G$ there is not more than two arcs and there are not arcs beginning and ending at the same node (loops). Arcs do not intersect and may have only common nodes. Each node and each arc belong to some cycle with more than two arcs and more than two nodes.

Call faces (or cells) areas $S_i, i = 0,...,m$, of the manifold $\mathcal{T}$ limited by its cycles minimal by the set theory inclusions. So faces may have common nodes, common arcs but have not common internal points. Put two faces adjacent if there is their common arc. Each arc belongs to two faces (is adjacent to two faces). Denote by $\delta S_i$ the face $S_i$ boundary.

Suppose that faces $S_i,...,S_m$ are bounded and call them internal. Then the face

$$S_0 = \mathcal{T} \setminus \bigcup_{i=1}^{m} S_i$$

may be called external. The face $S_0$ may be unbounded if for example the manifold $\mathcal{T}$ is a plane. It may be bounded also if for example $\mathcal{T}$ is a sphere or a torus.

(A). Suppose that each two internal faces $S_i, S_j, 1 \le i \le m$ , may have no more than single common arc.

Examples of graphs satisfied Condition (A) are connected aggregations of quadrates from rectangular lattice or connected aggregations of hexagons from hexagonal lattice.

Denote $A_{i,j}$ the set of arcs adjacent to faces $S_i, S_j, 0 \le i \ne j \le m$, and put $n_{i,j}$ a number of arcs in the set $A_{i,j}$. Designate $M_{i,j} = C^2_{n_{i,j}}$, if $n_{i,j} > 1$ and $M_{i,j} = 0$ if $n_{i,j} \le 1$. Define $N = \Sigma_{1 \le i \le m} M_{i,0}$, $M = \Sigma_{0 \le i < j \le m} M_{i,j}$.

**Theorem 2.** Suppose that Condition (A) and the inequality $N > 0$ are true then $C = N, D = 2$.

An example of a graph satisfied Theorem 2 conditions is integer rectangle.

**Theorem 3.** If $M > 0$ then the equalities $C = M, D = 2$ are true.

Remark that Condition (A) is absent in Theorem 3. Denote $U_3$ the set of the graph $G$ nodes which are connected with three arcs and put $K_3$ the number of elements in $U_3$.

**Theorem 4.** If $M = 0$, $K_3 > 0$, then $C = K_3, D = 3$.

Examples of graphs which satisfy Theorem 3 conditions are the dodecahedron [3,Chapter 4, Figure 4.2] and integer tube obtained by a gluing of a pair of opposite sides in an integer rectangle with a size $M \times N$, $M > 1$, $N > 1$.

**Theorem 5.** If $M = 0$, $K_3 = 0$, $K_4 > 0$ then $C = K_4, D = 4$.

An example of a graph satisfies Theorem 5 conditions is a graph arranged on two dimensional torus and obtained by a gluing of two pairs of opposite sides in an integer rectangle with a size $M \times N$, $M > 1$, $N > 1$.

## 4. PROOF OF MAIN RESULTS

**Theorem 1.**

**Proof.** From the Burtin-Pittel formula [3] the probability $P(u,v)$ of the nodes $u,v$ disconnection in $G$ satisfies the relation $P(u,v)\sim C(u,v)h^{D(u,v)}$, $h\to 0$ where $C(u,v)$ is a number of sections $L\in\mathcal{L}(u,v): d(L)=D(u,v)$. Assume that $V_L$ is random event when all arcs of cross section $L$ fail then

$$P=P\left(\bigcup_{L\in\mathcal{L}} V_L\right)=P\left(\left(\bigcup_{L\in\mathcal{L}_*} V_L\right)\cup\left(\bigcup_{L\in\mathcal{L}\setminus\mathcal{L}_*} V_L\right)\right)\sim P\left(\bigcup_{L\in\mathcal{L}_*} V_L\right),\ h\to 0,$$

as

$$P(V_L)=o\left(h^D\right),\ L\in\mathcal{L}\setminus\mathcal{L}_*,\ P\left(\bigcup_{L\in\mathcal{L}_*} V_L\right)\sim Ch^D.$$

**Theorem 2.**

**Proof.** 1. Prove at first that $D>1$. If $D=1$ then there are nodes $u,v\in U$ dividing by single arc $w\in W$. Suppose that $\Gamma$ is a way in the graph $G$ connecting nodes $u,v$ and the arc $w$ belongs to the face $S_i$. Without a restriction of a generality assume that the nodes $u,v\in S_i$. Then it is possible to construct a way $\Gamma'$ in which the arc $w$ is replaced by a way along the face $S_i$ boundary which bypasses the arc $w$ (see fig. 1). Consequently the arc $w$ does not divide the nodes $u,v$ and so minimal number of arcs which create the graph $G$ cross section is larger than one.

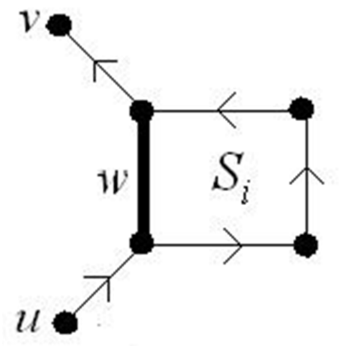


Fig. 1. Item 1 illustration

2. Suppose that $w_1,w_2$ is a pair of graph arcs. Prove that if these arcs do not belong to common internal face then the set $\{w_1,w_2\}\notin\mathcal{L}$. Indeed the arc $w_1$ (the arc $w_2$) may be bypassed by its internal face boundary (see fig. 1). And a way around the arc $w_1$ (around the arc $w_2$) does not contain the arcs $w_1,w_2$. So the set of arcs $\{w_1,w_2\}\notin\mathcal{L}$.

3. Assume that the arcs $w_1,w_2$ belong to internal face $S_i$ but do not belong simultaneously to external face $S_0$. Prove that the set $\{w_1,w_2\}\notin\mathcal{L}$.

a) At first consider the case when there are internal faces $S_j,S_k$, $j\neq i,k\neq i$ so that $w_1\in S_j$, $w_2\in S_k$. As any two internal faces have not more than single common arc then $j\neq k$.

So the arc $w_1$ (the arc $w_2$) may be bypassed along $\delta S_j$ (along $\delta S_k$) without the arcs $w_1, w_2$. Consequently the set $\{w_1, w_2\} \notin \mathcal{L}$.

b) Suppose now that the arc $w_1 = (u_1, v_1) \in S_0$ and the arc $w_2 = (u_2, v_2) \notin S_0$. Then the arc $w_1$ may be bypassed by the way along $\delta S_0$. The arc $w_2$ may be bypassed along $\delta S_i$ from the node $v_2$ to the node $v_1$. Then this way may be prolonged along $\delta S_0$ to the node $u_1$ and then it may return along $\delta S_i$ from the node $u_1$ to the node $u_2$ (see fig. 2). Consequently the set $\{w_1, w_2\} \notin \mathcal{L}$.

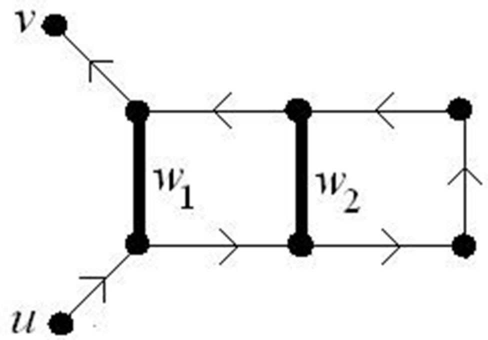


Fig. 2. Item 3, subitem b) illustration.

4. Suppose that $w_1 \in S_i \cap S_0$, $w_2 \in S_i \cap S_0$. Prove that $\{w_1, w_2\} \in \mathcal{L}$. Take a cycle around the face $S_i$ and suppose that $u, v$ are first nodes of this cycle touching $w_1, w_2$ appropriately. Extract from the face $S_i$ the set $\delta' S_i$ of arcs which do not belong to $S_0$ Exclude trivial case when the arcs $w_1, w_2$ have common node. Contrast each arc $w$ from $\delta' S_i$ the set of internal faces $\{S_k, k \in J_w\}$ defined by the symbols set $J_w, i \in J_i$. This set satisfies recurrent conditions: if $k \in J_w, t \neq k$ and $S_k \cap S_t \neq 0$ then $t \in J_w$. Denote $R_w = \bigcup_{k \in J_w} S_k$ and obtain that if $u \in R_{w'}$, $v \in R_{w''}$ then $R_{w'} \cap R_{w''} = \varnothing$ (see fig. 3). In opposite case $w_1 \notin S_i \cap S_0$ (see fig. 4). This contradiction proves that the set $\{w_1, w_2\} \in \mathcal{L}$.

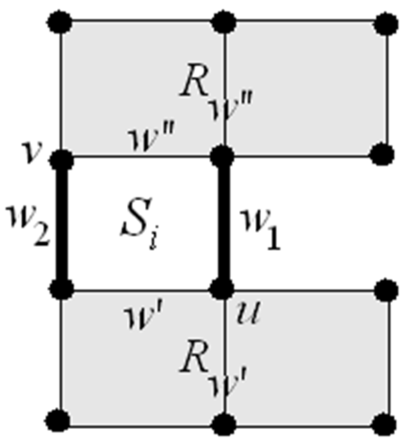


Fig. 3. The case $R_{w'} \cap R_{w''} = \varnothing$.

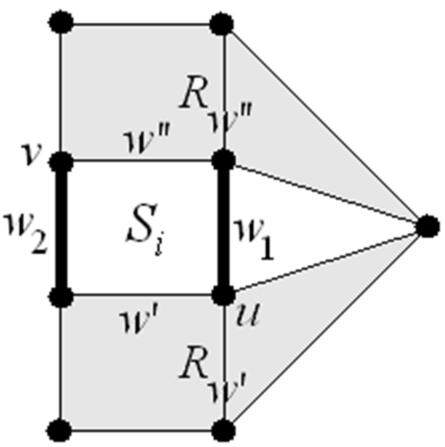


Fig. 4. The case $R_{w'} \cap R_{w''} \neq \varnothing$.

The illustration of the figure 4 may be added by following formal inference. Assume that the arcs $w', w_1$ have common node $u$ and the arcs $w'', w_2$ have common node $v$ and $w' \notin S_i$, $w'' \notin S_i$. Suppose that the node $z \in R_{w'} \bigcap R_{w''}$.

From the recurrent definition of the set $J_{w'}$ it is easy to prove that between the nodes $u, z$ there is acyclic way $\Gamma(u,z)$ consisting of the faces $S_t, t \in J_{w'}$ arcs excluding the face $S_i$ arcs. Analogously between the nodes $z, v$ there is acyclic way $\Gamma(z,v)$ consisting of the faces $S_t, t \in J_{w''}$ arcs without the face $S_i$ arcs. Consider a polygon bounded by the way $\Gamma(u,z) \bigcup \Gamma(z,v)$ and by the way $\Gamma(u,v)$which connects the nodes $u, v$ and passes along $\delta S_i, w_1 \in \Gamma(u,v)$. It is obvious that this polygon consists of internal faces and does not contain the face $S_i$. So the arc $w_1$ is adjacent to some internal face which does not coincide with $S_i$. This statement contradicts with initial suggestion that $w_1 \in S_i$. Consequently $R_{w'} \bigcap R_{w''} \neq \emptyset$.

**Theorem 3.**
**Proof.** This statement may be proved analogously to Theorem 2 statement if to pass from the faces $S_0, S_i$ to the faces $S_j, S_k$, see item 3, subitem a) in Theorem 2 proof.

**Theorem 4.**
**Proof.** As $M = 0$ and each face may have no more than single common arc with the face $S_0$ then $D > 0$. From Theorem 2the graph $G$ which satisfies Theorem 5 conditions has not cross sections with two arcs. Prove that there are graph cross sections with three arcs and all these cross sections $\{w_1, w_2, w_3\}$contain arcs connected with a node from $U_3$ . Indeed if arcs $w_1, w_2, w_3$ are connected with a node from $U_3$ then the set $\{w_1, w_2, w_3\} \in \mathcal{L}$. Prove that if arcs $w_1, w_2, w_3$ are not connected with a node from $U_3$ then the set $\{w_1, w_2, w_3\} \notin \mathcal{L}$. Then Theorem 4 will be proved.

As $M = 0$ so each two faces of the graph $G$ may have no more than single common arc and each node connects with more than two arcs. Assume that arcs $w_1, w_2, w_3$ belong to the faces $S_1, S_2, S_3$ appropriately and some of these faces may coincide.

1. Suppose that there is not a pair of arcs from $w_1, w_2, w_3$ in the same face. Then each arc $w_i$ may be bypassed by a way along $\delta S_i$ (see fig. 1). So any way $\Gamma(u,v)$ from $u$ to $v$ which contains arcs from $w_1, w_2, w_3$ may be replaced by a way $\Gamma'(u,v)$ which does not contain arcs from $w_1, w_2, w_3$. For this aim it is enough to replace arc from this set by a way which bypasses this arc. Consequently the set $\{w_1, w_2, w_3\} \notin \mathcal{L}$.

2. Suppose that all three arcs from $w_1, w_2, w_3$ belong to common face $S$. Then each arc $w_i$ may be bypassed by a way along $\delta S_i$ without arcs from $w_1, w_2, w_3$ (see fig. 1). Consequently the set $\{w_1, w_2, w_3\} \notin \mathcal{L}$ .

3. Suppose that faces $S_1, S_2, S_3, S$ are different and $w_1 \in S_2$, $w_2 \in S_3$, $w_3 \in S$. Then the arc $w_1$ may be bypassed by a way along $\delta S_1$, the arc $w_3$ may be bypassed by a way along $\delta S$ (see fig. 1) and the arc $w_2$ - by a way from $\delta S_3$ to the arc $w_3$, then around the arc $w_3$ along $\delta S$

and then along $\delta S_3$ to the arc $w_2$ second node (see fig. 2). And the way around $w_i$ does not contain arcs from $w_1, w_2, w_3$. Consequently the set $\{w_1, w_2, w_3\} \notin \mathcal{L}$.

4. Suppose that the faces $S_1, S_2, S_3, S$ are different and $w_1 \in S$, $w_2 \in S$, $w_3 \notin S$. Then each arc $w_i$ may be bypassed by a way along $\delta S_i$ without arcs from the set $w_1, w_2, w_3$ (see fig. 1). Consequently the set $\{w_1, w_2, w_3\} \notin \mathcal{L}$.

**Theorem 5.**

**Proof.** From Theorem 4 we have that the graph $G$ satisfying Theorem 5 has not cross sections with three arcs. Prove that in $G$ there are cross sections with four arcs $w_1, w_2, w_3, w_4$ and all these cross sections consist of arcs connected with some node from $U_4$. Indeed if the arcs $w_1, w_2, w_3, w_4$ are connected with some node from $U_4$ t$\{w_1, w_2, w_3\} \notin \mathcal{L}$hen the set $w_1, w_2, w_3, w_4 \in \mathcal{L}$.Prove that all other sets are not cross sections in $G$.

From $M = 0$ we have that each two faces in $G$ may have no more than single common arc. Denote faces $S_1, S_2, S_3, S_4$ which contain arcs $w_1, w_2, w_3, w_4$ appropriately.

1. Suppose that there is not a pair of arcs from the set $\{w_1, w_2, w_3, w_4\}$ which belong to the same face. Then each arc $w_i$ may be bypassed by a way along $\delta S_i$ without arcs from the set $\{w_1, w_2, w_3, w_4\}$. Consequently each way $\Gamma(u, v)$ from $u$ to $v$ which contains some arcs from the set $\{w_1, w_2, w_3, w_4\}$may be replaced by a way $\Gamma'(u, v)$ without arcs from the set $\{w_1, w_2, w_3, w_4\}$ (see fig. 1). Consequently the set $\{w_1, w_2, w_3, w_4\} \notin \mathcal{L}$.

2. Suppose that all arcs from $\{w_1, w_2, w_3, w_4\}$ belong to the same face $S$. Then each arc $w_i$ may be bypassed by a way along $\delta S_i$ adjacent with $w_i$ (see fig. 1) without arcs from the set $\{w_1, w_2, w_3, w_4\}$. Consequently the set $\{w_1, w_2, w_3, w_4\} \notin \mathcal{L}$.

3. Suppose that the faces $S_1, S_2, S_3, S_4, S$ are different and $w_1 \in S_2$, $w_3 \in S_4$, $w_4 \notin S$. Then the arc $w_1$ may be bypassed by a way along $S_1$ the arc $w_2$ - by parts of $\delta S_2$ and along $\delta S_1$ around $w_1$ (see fig. 2), the arc $w_4$ - along $S$ and the arc $w_3$ - by a way along parts of $S_4$ and along $S$ around the arc $w_4$ (see fig. 2). Consequently the set $\{w_1, w_2, w_3, w_4\} \notin \mathcal{L}$.

4. Suppose that faces $S_1, S_2, S_3, S_4, S$ are different and

$$w_1 \in S,\ w_2 \in S, w_3 \in S, w_1 \notin S_4,\ w_2 \notin S_4,\ w_3 \notin S_4.$$

Then each arc from $w_i$, $i = 1,2,3$, may be bypassed by a way along $\delta S_i$ without arcs from $\{w_1, w_2, w_3, w_4\}$. Analogously the arc $w_4$ may be bypassed along $S_4$ (see fig. 1). Consequently the set $\{w_1, w_2, w_3, w_4\} \notin \mathcal{L}$.

5. Suppose that the faces $S_1, S_2, S_3, S_4, S$ are different and

$$w_1 \in S,\ w_2 \in S, w_3 \in S, w_4 \in S_1.$$

Then the arc $w_2$ may be bypassed by a way along $\delta S_2$ the arc $w_3$ - along $\delta S_3$ the arc $w_4$ - along $\delta S_4$ (see fig. 1). The arc $w_1$ may be bypassed along parts of $\delta S_1$ and around the arc $w_4$ along

$\delta S_4$ (see fig. 2). Each way bypassing the arc $w_i$ does not contain arcs from the set $\{w_1, w_2, w_3, w_4\}$. Consequently $\{w_1, w_2, w_3, w_4\} \notin \mathcal{L}$.

## 5. CONCLUSION

So we obtain asymptotic formulas for disconnection probability of a wide variety of graphs with high reliable arcs. This problem is sufficiently complicated especially for the coefficient $C$ because it is necessary to solve some N-P problem. But a consideration of planar graphs or graphs arranged on two dimensional smooth manifold without edge simplifies this calculations significantly. It takes place because though a problem to arrange a graph on a manifold is sufficiently complicated but in different applications this problem is solved by a designer without any calculations.

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# MODELS AND METHODS FOR ESTIMATION AND OPTIMIZATION OF ELECTRIC POWER SYSTEM RELIABILITY

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## ABSTRACT

The paper presents the results of the review of methods and models of an estimation and optimization of electric power systems (EPS) reliability. There are classification and characteristic of existing probabilistic methods used for an estimation of EPS reliability in the paper. The methodology of model for an estimation of systems reliability of EPS is described. Theoretical bases of system reliability optimization of EPS are stated. The description of the software YANTAR for estimation of EPS systems reliability is given.

## 1. INTRODUCTION

Providing reliability of electric power system (EPS) is a complex multifaceted problem to be solved at different levels of territorial, temporal, and technological hierarchy of control. The reliability models are mainly intended to obtain the reliability indices that could be directly or indirectly applied to make decisions on ensuring reliability of the entire power system and its facilities. The problems of EPS reliability can be considered in terms of both estimation and optimization.

Reliability of power systems and their facilities is a complex property. It should be considered as a set of some single properties that are essential for a certain object. This makes the problem of analysis and synthesis of EPS reliability increasingly more complicated. It does not seem possible to create some unified model for solving all reliability problems, i.e.:

- at all stages of control (while forecasting, designing, making decisions on expansion, for long-term, short-term and current control of operation);
- at all territorial levels (from a piece of equipment to a unit, facility and system of various degree of integration: regional, national, etc.);
- for all production stages (primary energy resources, generation, transportation, conversion and distribution of electric energy), division of reliability problems by production stage is topical due to replacement of vertically integrated systems by partly horizontally integrated sectors of the industry as a result of emerging market relations in the electric power industry;
- for all individual properties (security, longevity, failure-free operation, maintainability, stabilability, survivability, controllability and storability).

Levels and stages of reliability problem solving may considerably vary in accuracy and completeness. Moreover, the models applied to solve one and the same problem may differ in completeness and accuracy in terms of initial data representation and solutions to be obtained. Thus, the decomposition of a unified universal model into a great number of models that solve partial problems of reliability is objectively conditioned. However, the need arises to coordinate solutions to the partial reliability problems.

The specific and unique nature of EPS in the majority of cases does not allow the use of mathematical models and algorithms intended for calculation of reliability indices that are suggested in the general theory of reliability. This theory is sufficiently well developed yet for a limited class of systems. These models can not completely reflect the technology particularities of electric power systems, the great number of functions to be performed by them, numerous purposes they are intended for, their multiple probable states and dominating number of partial failures. This is why special mathematical models are developed to estimate reliability of electric power system facilities.

The problem of power system reliability estimation is formulated and solved depending on:

- the research goal;
- the time constraints (on-line and for a long-term perspective);
- the assumed calculated scheme (the degree of equivalenting);
- the validity and forms of initial information representation;
- the nomenclature of reliability indices to be calculated;
- the requirements for accuracy of results to be obtained;
- the applied mathematical tools.

There are a lot of techniques and methods for calculating reliability of EPS and its facilities in the national and international practice [1-4]. Along with a great number of "nuances" that make them distinct they also have certain elements of commonality and similar principal approaches. The diversity of mathematical models and methods makes it useful to analyze their specific features and potential capabilities in order to reveal the areas of their primary application.

## 2. CHARACTERISTIC OF MODELS INTENDED TO ESTIMATE (SYNTHESIZE) RELIABILITY OF FACILITIES (SYSTEMS) IN ELECTRIC POWER INDUSTRY

For methodological purposes all the models that are intended to estimate (synthesize) reliability of the EPS facilities can be classified in terms of their application in accordance with the criteria presented in Table 1.

Solving the control problems of EPS and its facilities in terms of reliability, the reliability indices are calculated either within an individual (independent) problem or in the process of solving the main problem.

Individual models intended for reliability estimation (synthesis) that are applied when solving individual problems on EPS expansion and operation make it possible to limit ourselves to a rough preliminary consideration of reliability (i.e. apply indirect standards and rules for considering reliability factor) but with further mandatory analysis of obtained solutions in terms of reliability. Here the values of reliability indices are specified only for the best variants identified.

There can be another way of applying reliability models. The reliability optimization can be implemented as a software, in which an independent model can be represented by a submodule for reliability estimation of a variant of the considered EPS scheme. This certainly requires that the model should be satisfactorily fast, accurate and sufficiently complete. Such a model can be used within the software either in each iteration or at the final stage of formation of some variant.

Table 1. Classification of reliability models of power systems and their facilities

| Type of model | Extent to which influencing factors are taken into account | Accuracy of considered factors representation | Speed of software applied | Purpose of model |
|---|---|---|---|---|
| Independent reliability model | Virtually completely | Most accurate representation | Low or average | Study of reliability properties of EPSs and their facilities, as well as properties of respective models |
| | Sufficiently completely | Sufficiently accurate | Average | Comparative analysis of reliability of variants of synthesis of facilities to be controlled<br>------------------------<br>Reliability optimization of EPSs and their facilities |
| Reliability model as part of optimization model for solving a specific control problem | Sufficiently completely | Approximate | Sufficiently high | Reliability estimation in the process of optimization calculation |
| | Insufficiently | Approximate, even rough | High | Approximated consideration for reliability in optimization model |

Independent models are essential when **studying** reliability as a property of facilities and systems in the electric power industry. Along with comparative analysis of the variants these models can be used in two more aspects shown in Table 1. The studies conducted give an opportunity to formulate new problems in a more substantiated way.

## 3. CHARACTERISTIC OF THE MAIN METHODS APPLIED TO ESTIMATE RELIABILITY OF ELECTRIC POWER FACILITIES AND SYSTEMS (Table 2)

The methods underlying the models intended for estimation of reliability of power systems and their facilities are known in the general theory of reliability, but their application in this case is somewhat different.

The studies on reliability are characterized by application of a great number of various methods.

When solving any problem, including the problem of EPS reliability estimation, the preference for the method to be applied should be determined by the content of the problem solved. Here, in every case the objective is to obtain a sufficiently fast and convenient computational software that generates satisfactory results.

All the methods can be classified by several principal features. And first of all in terms of information support and the mathematical tool applied for estimation of reliability indices.

The following two **groups** of methods are suggested: the methods for experimental reliability estimation (group A), and the methods for reliability calculation and forecasting (group B) (Table 2, column 1).

**Methods for experimental estimation of reliability** (**group A**) are based on the study of results obtained during special tests (”tests for reliability”) that are carried out at facilities themselves or their physical analogues. Special tests are taken to mean the process of determining or checking reliability indices by experiment. The main objective of such tests is to create an information basis (i.e. to obtain more complete and reliable data on the facility as a physical reality, however, the representativeness of the data is limited). Experimental determination and check of reliability indices can be carried out at all stages of control: designing, manufacturing and operating, but not for all facilities. The database in question can be used for calculation methods intended for analysis and synthesis of reliability or for industrial control of the reliability level to be provided.

A special place in this group is occupied by the methods for studies to be conducted while carrying out the tests of physical-chemical and other causes of failures that require multi-purpose experiments under certain conditions.

Table 2. Classification of methods for determination of reliability indices of power systems and their facilities

<table>
<tr><th>Groups</th><th>Classes</th><th colspan="3">Types</th><th>Primary application</th></tr>
<tr><td rowspan="2">A. Methods of experimental reliability estimation</td><td rowspan="2">I. Tests for reliability of a real facility</td><td colspan="3">1. Long-term tests</td><td rowspan="2">Determination and check of EPS equipment reliability</td></tr>
<tr><td colspan="3">2. Accelerated tests</td></tr>
<tr><td rowspan="10">B. Methods of calculation-based reliability determination</td><td rowspan="3">II. Methods that do not require modeling a facility by component</td><td colspan="3">3. Retrospective methods</td><td>Short-term EPS operation scheduling</td></tr>
<tr><td colspan="3">4. Extrapolation methods</td><td rowspan="2">Prospective and long-term expansion planning of power systems and their facilities</td></tr>
<tr><td colspan="3">5. Expert methods</td></tr>
<tr><td rowspan="7">III. Methods based on modeling a studied facility by component</td><td rowspan="4">6. Deterministic methods</td><td colspan="2">6.1. Methods of physical simulation</td><td rowspan="7">At all territorial levels temporal stages of control of EPS expansion and operation as a main or an auxiliary tool</td></tr>
<tr><td colspan="2">6.2. Criterion n–i</td></tr>
<tr><td colspan="2">6.3. Method of the worst case</td></tr>
<tr><td colspan="2">6.4. Standardization</td></tr>
<tr><td rowspan="3">7. Probabilistic methods</td><td rowspan="2">7.1. Analytical methods based on representation of real stochastic phenomena by</td><td>a) random events</td></tr>
<tr><td>b) random processes</td></tr>
<tr><td>7.2. Statistical methods based on representation of real stochastic phenomena by</td><td>a) random events<br>b) random processes</td></tr>
</table>

**Methods for reliability calculation (group B)** are used to determine numerical characteristics of reliability of an object to be studied under known structure, operating conditions and reliability indices of its components.

The methods are divided into **classes** depending on the methodological principles and mathematical tools applied (Table 2, column 2):

I. *Tests for reliability* are subdivided into long-term and accelerated (Table 2, columns 2 and 3). The main principle of long-term tests is restoration of real conditions of facility operation. Accelerated tests are characterized by short-term facility operation under large loads with a view to receive the required information on reliability in the maximum possible short period of time (compared to the operating conditions).

II. *Methods that do not require a detailed* (by-component) modeling of the facility:

- The retrospective methods are based on generalization of the previous experience;
- The extrapolation methods are based on the analysis and forecasting of the existing trends;
- The expert methods are based on the knowledge, experience and intuition of experts.

These methods are used in forecasting to estimate numerical reliability indices of a facility under conditions of incomplete certainty of both quantitative reliability characteristics of components constituting the facility and conditions of its operation.

III. *Methods based on the by-component modeling* of an object to be studied. These methods are subdivided into the so called deterministic and probabilistic. The probabilistic methods include: **analytical** methods based on 1) functional relationships in the form of mathematical dependences, 2) analytical expressions of probabilistic processes, 3) complete or reduced search of potential states of an object, and "**statistical**" methods that are based on the Monte-Carlo method and pseudostatistical methods of LPι programming type, etc. On the other hand the probabilistic methods can be based on representation of stochastic phenomena by random events or random processes. This class also includes the methods of physical simulation that require mostly by-component representation of a complex facility.

Further we will present a short characteristic of the methods in terms of their types and areas of primary application (Table 2, columns 3 and 4).

**Methods of group A** (Table 2) are not widely used for well-known reasons (unique nature and other specific features of EPS). Special tests for reliability require either very long tests of a small number of components, or short tests of a rather large amount of equipment (facilities). For most of electric power facilities none of the foregoing is acceptable for economic reasons and for the reason of uniqueness of certain types of equipment (facility). According to the rules existing in the electric power industry development tests of facilities are carried out to reveal and eliminate design shortcomings rather than estimate reliability.

However, the method of long-term tests is applied in the electric power industry in the form of analysis of operation experience (statistical data) of real facilities (systems) and their equipment. For these reason EPS should have information systems and reliability services, besides, great and laborious work should be carried out to collect, process, store and use the data on reliability of electric power facilities.

Application of accelerated tests in electric power industry is limited by the level of simple equipment and individual nodes, and components of more complex equipment.

**Methods of group B** are preferred for use in the models for reliability estimation of power systems and their facilities because of their commonality, rigor and lesser dependence on subjective factors. These methods are used at all territorial and temporal levels as a main or an auxiliary tool.

The methods that do not require by-component modeling of an object to be studied are divided into the following types (Table 2, p.II (3,4,5):

**II.3. The retrospective methods** are calculation methods intended for reliability estimation. They are based on the analysis of previous operating experience of a facility and reasonable use of this experience to forecast the conditions of its expansion. In doing so most often the experimental planning method and regression analysis are applied.

These methods can be applied provided the properties and structure of an object are sufficiently stable over time, which is generally characteristic of power systems and their facilities. The primary application area of these methods is analysis of power systems or their individual components operation, which is aimed at prospective and long-term planning of their expansion.

**II.4. The extrapolation methods** are the methods intended for forecasting reliability. They are based on the analysis of change in reliability indices of the facility operation depending on change in its individual parameters or operating conditions. These methods are widely applied to forecast reliability of EPS equipment. They are also sometimes applied to estimate reliability of facilities, subsystems and entire systems. The methods suggest using representative statistical data that are collected over a long-term period of time or concern a large number of facilities. However, power systems as objects to be studied change over time in a very complex way, therefore there are problems with accuracy of the results obtained, particularly when reliability of EPS facilities is estimated for a remote future.

**II.5 Application of the expert methods** is sensible in the cases where the process of determining reliability can not be formalized or the data on the facility expansion are very much uncertain.

The methods based on the by-component modeling of an object to be studied are divided into two large classes: deterministic and probabilistic (Table 2, p.III 6,7).

**III.6. The deterministic methods** are the methods and criteria for analysis and synthesis of reliability of a facility (system) which do not suggest modeling the probabilistic characteristics of failures of the facility components but are rather intended for analysis of the facility capability to resist any disturbance from an a priori specified class, i.e. operation of the facility after such a disturbance should meet the required conditions and parameters. For power systems and their facilities these are admissible levels of voltage, frequency, equipment loading, shortage-free operation. We will dwell on some of the deterministic methods.

**III.6.1. The methods of physical simulation** can be applied for experimental estimation of reliability, particularly at a system level. However, this possibility so far has been used only for studying individual states or operating conditions of a system in deterministic form, for example when studying survivability. And it will hardly be widely used for probabilistic analysis due to complexity of such kind of experiments.

**III.6.2.** Reliability of a facility at failure of any i components (reliability by the **criterion (rule) n-i**) is the property of the facility to perform the main functions, when *i* = 1, 2, 3 … components out of ***n*** fail. Here loading of all components should lie within the admissible limits, and state variables should not go beyond the standard ranges, however, decrease in the efficiency of facility operation is as a rule allowed (increase in fuel consumption, power losses, etc.)

**III.6.3. The "worst case" method** suggests that a facility should perform its functions in the situation where mix and parameters of its components, taking into account operating conditions, have limiting values.

**III.6.4. Standardization** suggests that reliability factor is taken into account by applying the rules and reliability indices specified for structure and parameters of a facility.

**III.7. The probabilistic methods** are the methods and criteria that can be used to estimate how often and for how long a facility (system) will be unavailable due to failures of its components. This estimation should certainly include modeling of probabilistic processes in the system and probabilistic characteristics of failures of components. The probabilistic approaches are considered to provide more versatile, deep and accurate characteristics of facility reliability as compared to the deterministic methods, however they are more complex and labor intensive.

**III.7.1. The analytical methods** of reliability calculation employ as a rule the main principles of probability theory, combinatorial analysis, algebra, logics, queuing theory, etc.

The analytical methods, with available mathematical description of functional relationships between individual factors, allow one to solve any problem related to estimation of reliability in the electric power industry with a required accuracy. The absolute advantage of these methods over the other methods in practical application is diminished by the following factors: the absence of description of functional relationships or its awkwardness in some cases; "the curse of dimensionality" that makes it impossible sometimes to carry out computations even with modern computers within an acceptable time; difficulties, related to computation of some indices.

**III.7.2. The statistical methods** are the methods intended for calculation of reliability on the basis of statistical modeling, where the main processes of the facility operation, including stochastic ones are represented by a probabilistic model tested many times. These methods are in fact the methods of "mathematical test for reliability".

The statistical methods are used in the cases where there are obstacles to application of the analytical methods. In these methods the factors that determine EPS reliability are taken into account relatively easily and the constraints on the form of distribution laws of the considered events are not imposed. The required accuracy of the obtained results is sometimes achieved through a considerable increase in the number of experiments which decreases the merit of these methods. The research dealing with search of new techniques and methods of "playing" various states remains urgent.

The methods (analytical and statistical) that are based on representation of probabilistic phenomena in the form of *random events* are used in the reliability models more often than the methods based on the notion of random process. This is explained by the fact that many phenomena in EPS can be described at the level of random events, and the mathematical tools for random events are better developed and simpler.

Nevertheless, the reliability models employ the analytical and statistical methods that are based on representation of real stochastic phenomena *by random processes*, for example, the analytical methods based on combinatorial approach, the Markov and semi-Markov circuits, and the statistical methods based on statistical modeling. The methods based on the analysis of random processes in combination with statistical modeling on fast computers make it possible to obtain a more complete and sometimes more accurate information on reliability of a studied facility, which is necessary due to constant expansion and complication of power systems.

Classification of the methods could be continued at a lower hierarchical level. However, the practical value of this classification will not be high. At this level we would have to characterize a constantly expanding set of applied methods that differ in formalization of techniques intended for representation of operating conditions, structure of systems, characteristics of components, and simplifications and assumptions made.

For example, the following modifications of the analytical methods are observed:

- *the logic-and-probabilistic methods* (functions of the algebra of logic, table logic method, the "fault tree" method), in which the structure of facility and specific features of its operation are described by means of the algebra of logic and reliability is calculated with the help of probability theory. The table logic method is a partial case for the method of random events, based on making up tables or matrices of relations between operating conditions of a facility, failures of its components and failures of the entire facility;
- *the method of structural schemes (flowcharts)* employs the conditional graphical representation of system components and relations among them (a structural scheme). The reliability indices are calculated with the help of graph theory or by sequential equivalenting.
- *the phase-space method* studies a random operation process of a complex facility on the phase space, i.e. a set of states that differ in the mix of faultless and faulty components;
- *the Markov process method* (*the state-space method)* is a particular case of the phase-space method, in which the process of object operation is described by a system of differential equations for transitions of components from one state to another;
- *the network method (the method of minimum paths and cutsets)*;
- *the topological methods,*

etc.

The statistical methods include:

- *the functionally statistical method,* in which the system operation process is described by the probabilistic model tested on computer many times;
- *the logically statistical method,* in which the system structure and specific features of its operation are described by means of the Boolean algebra and reliability is calculated by statistical modeling;

etc.

The methods for reliability calculation also differ by whether or not they consider the equipment restorability, whether they consider only the time instants of *failure* occurrence or *failure states* of an object. Besides they differ by optimization methods of deficient conditions, etc.

Note that classification of the methods in Table 2 is formal and abstract in a sense. In practice one and the same model can combine different methods, each being applied to solve particular subproblems. Experience shows that such models prove to be most acceptable in terms of required accuracy and speed of calculations.

In recent past the described methods and models were applied only to study reliability in terms of failure-free operation and maintainability (restorability), however their use in the study of other single properties of reliability is becoming increasingly frequent.

When constructing corresponding models it should be remembered that the *single properties of reliability* that are important for EPS have different degrees of significance. Therefore, there should be a certain order in their studies.

This is not so important for operating systems and each property can be estimated (synthesized) separately. In designing, however, the order of providing single properties is essential. It is senseless, for example, to provide high survivability unless the required level of security, longevity, failure-free operation, maintainability and stabilability is provided.

It is known that:

- the single properties are interrelated and interdependent;
- the relative cost of ensuring a required level of certain single properties is different;
- with the single properties ranked in a proper order the cost of providing every subsequent

single property is lower if all previous properties are provided.

Hence, the rational sequence of single reliability properties of EPS should be as follows:

1. Security.
2. Longevity.
3. Failure-free operation.
4. Maintainability.
5. Stabilability.
6. Survivability.
7. Controllability.
8. Provision with resources.
9. Storability.

*The temporal aspect* of reliability is characterized by such notions as adequacy, security, current (switching) reliability, etc. The above described models and methods can also be applied to them, yet considering a specific character of the problem solved.

Reliability estimated on the basis of one or another model is "calculated" reliability that cannot exactly coincide with actual reliability of an object. It will only roughly correspond to it because of assumptions and simplifications made in the model. Therefore, from a set of models one should choose those which could have a status of "*normative models*". Application of such models in relevant calculations would make it possible to compare reliability of objects on a normative ("legal") basis.

## 4. THE MODEL FOR ESTIMATION OF FACILITY RELIABILITY

**Generally, the model for estimation of facility reliability** consists of three modules:

1. The module for probability estimation of the considered calculated object states.
2. The module for optimization of object operation in the calculated states determined in the first module.
3. The module for determination of reliability indices of an object by the data obtained in the first two modules.

All the above methods are applicable in the first module intended for solution of the reliability estimation problem.

In the module for optimization of operation (**module 2**) under liberalization of market in the electric power industry along with technical requirements it is necessary to take into account the contract obligations to be fulfilled by different production stages of the system (production, transmission, distribution) while interacting with each other. Reliability estimation in the system should be oriented to the market and profit and hence the corresponding reliability indices should be determined. This module is central in the reliability estimation model. Greater requirements are placed on it in terms of complete consideration of the main factors, accuracy and high speed.

**The third module** of the model has no special complexities and is an ordinary solver using the calculation results obtained in the first two modules.

For the horizontally integrated system of EPS control the reliability estimation problem can be supplemented by estimation of contribution made by each production stage of EPS to the system reliability. Since the number of economic entities taking part in the market is great solving this problem will make it possible to determine the boundaries of their responsibility for reliability and adjust reliability of individual production stages of EPS to the required system reliability index. Among the production stages of particular interest are the stages of the main EPS structure, namely the stage of electricity generation and transmission (a network stage).

## 5. RELIABILITY OPTIMIZATION OF LARGE EPSs

Consider **reliability optimization of large EPSs** in greater detail. Large EPSs are the systems, whose calculated schemes consist of sets of power nodes connected by transmission lines with limited transfer capabilities. Power nodes are represented by concentrated subsystems with generalized load and generating facilities, and the internal network in them does not impede power transmission from power sources to load buses in all possible conditions. Large EPSs (in particular, UPS of Russia or even interstate interconnections) make it possible to achieve a certain economic effect due to mutual assistance (in comparison with their isolated operation). The mutual assistance is provided owing to creation of intersystem lines with sufficient transfer capability. Therefore, the optimal value of generation capacity in the interconnection and its distribution among EPSs are determined jointly with transfer capabilities of tie lines.

The computational model for optimization of a large EPS expansion variant in terms of reliability is intended to choose the main structure of EPS and includes both the optimization of generation capacities and transfer capabilities of tie lines, and the stage of EPS provision with primary energy resources. In the general case the constraints on all types of resources (material, manpower, financial) should be taken into account.

The model should envisage two criteria to consider reliability. These are 1) technical and economic minimization of discounted costs of backing up capacity, primary energy resources and networks with account taken of compensation for the probable economic loss because of unreliable electricity supply to consumers and 2) provision of a standard reliability level.

Technical and economic optimization of reliability must be based on the following principles.

The optimal reliability level of the multi-nodal EPS corresponds to the minimum total discounted costs:

$$C_{\Sigma} = C^G + C^L + C^E + LOSS \rightarrow min, \quad (4)$$

where

$C^G = \sum_{m=1}^{M} C_m^G \cdot R_m^G,\ C^L = \sum_{n=1}^{N} C_n^L \cdot R_n^L,\ C^E = \sum_{m=1}^{M} C_m^E \cdot R_m^E$ – costs of generation capacity reserve at all $M$ system nodes, costs of reinforcement of transfer capabilities of all $N$ system lines and costs of reserves of primary energy resources, respectively;

$C_m^G,\ C_n^L,\ C_m^E$ – specific costs for generation capacity reserve of the $m$-th node, for reinforcement of transfer capability of the $n$-th line and for reserves of primary energy resources, respectively;

$R_m^G,\ R_n^L,\ R_m^E$ –value of generation capacity reserve at the $m$-th node, additional transfer capability of the $n$-th line, reserves of primary energy resources, respectively;

$LOSS = \sum_{m=1}^{M} loss_{re} \cdot E_m^{un}(R_m^G,\ R_n^L,\ R_m^E) \quad m=\overline{1,M},\ n=\overline{1,N}$ – total costs of the entire system to compensate for the economic loss because of average annual electricity undersupply to consumers at all $M$ nodes of a system;

$loss_{re}$ – specific value of the averaged loss due to interruptions in electricity supply to consumers of the $m$-th node;

$E_m^{un}$ – mean value of electricity undersupply to consumers of the $m$-th node.

Conditions of the minimum of functional (4) in this case are represented by the equality to zero of partial derivatives with respect to reserves for individual nodes $R_m^{G}$, $R_m^{E}$ and tie lines $R_n^{Л}$

$$\frac{\partial C_{\Sigma}}{\partial R_m^G} = \frac{\partial C^G}{\partial R_m^G} + \frac{\partial LOSS}{\partial R_m^G} = C_m^G + \frac{\partial LOSS}{\partial R_m^G} = 0, \quad m = \overline{1,M};$$
$$\frac{\partial C_{\Sigma}}{\partial R_m^E} = \frac{\partial C^E}{\partial R_m^E} + \frac{\partial LOSS}{\partial R_m^E} = C_m^E + \frac{\partial LOSS}{\partial R_m^E} = 0, \quad m = \overline{1,M}; \qquad (5)$$
$$\frac{\partial C_{\Sigma}}{\partial R_n^L} = \frac{\partial C^L}{\partial R_n^L} + \frac{\partial LOSS}{\partial R_n^L} = C_n^L + \frac{\partial LOSS}{\partial R_n^L} = 0, \quad n = \overline{1,N}.$$

The aim of the calculations is to obtain the values $R_m^G$, $R_m^E$ and $R_n^L$ for all nodes and lines, that meet condition (5).

In the optimization calculations based on set (standard) reliability indices the optimality criterion looks as follows.

The optimal reliability level corresponds to the minimum costs

$$C_{\Sigma} = C^G + C^L + C^E \rightarrow min \qquad (6)$$

at $\Pi_m = \Pi_{stand\ m}$, $m = \overline{1,M}$ or $\Pi_m \rightarrow max$ $m = \overline{1,M}$ at constraints on resources when provision of $\Pi_m = \Pi_{stand\ m}$, $m = \overline{1,M}$ is impossible, i.e. when the problem of optimal distribution of limited resources is solved.

Here $\Pi$ – calculated reliability index, $\Pi_{stand\ m}$ – standard value of this index.

As in the first case (estimation of loss due to electricity undersupply) in the second case (direct use of standard reliability index) the calculation of reliability indices is a submodule of the optimization model of EPS.

Along with special optimization programs that contain a module for reliability estimation the independent interactive models can be used for reliability estimation for optimization purposes. In some cases this produces a positive effect (the number of iterations decreases, the accuracy of reliability indices increases).

Nowadays the estimation models applied in optimization programs should satisfy the requirements presented below.

The *estimation model* for calculation of EPS reliability indices is recommended when the temporal or resources constraints are insufficient to completely provide economically sound standards of redundancy and the choice of measures to provide reliability is so limited that only several alternative variants can be formed by expert. In such cases the estimation model is applied to technical analysis of the variants. The analysis results in calculation and comparison of the obtained reliability indices with the standards. As a rule, preference is given to the variant with a higher reliability level and lower costs.

A specialized estimation model should calculate reliability characteristics of large EPS that can be represented by any calculated scheme, in particular a mutiloop one with limited transfer capabilities of tie lines connecting nodes. The model should calculate current and capital maintenances. Also the calculated operating conditions generated in the model should be optimized.

The results obtained with such a model should include:

a) reliability indices;

b) the values of calculated reserves of different types;

c) the values of maximum load of tie lines for mutual redundancy of EPS and the functions of load distribution for these tie lines;

d) the values of required primary energy resources (water for HPP and fuel for TPP);

e) the estimates of deficiency of basic resources (generation capacity and primary energy resources at nodes and transfer capabilities of tie lines) for ensuring reliability.

The information obtained should allow an expert to thoroughly analyze the calculated reliability as a property of the concrete EPS in the considered specific conditions and detect available "bottlenecks" in EPS in terms of reliability.

Since the estimation program makes it possible to consider all possible system states and operating conditions, the obtained characteristics of power flows by tie lines represent sufficiently full information about their loading and do not require specification by analysis of calculated long-term and maximum conditions, as is the case with simplified techniques.

Characteristic of the software YANTAR to be considered as an example of EPS modeling for reliability calculation is presented below [5].

## 6. THE SOFTWARE YANTAR

### 6.1 The software goal

The softwar*e YANTAR* is intended for reliability estimation in terms of failure-free operation and maintainability (restorability) of large EPS of any (radial, looped) multinode calculated scheme with limited transfer capabilities of tie lines among nodes. The problem is solved for the conditions of long-term planning of operation and expansion at the levels of Unified, interconnected and local power systems.

In the interactive mode the software YANTAR optimizes the value and allocation of generation capacity reserves, transfer capabilities of tie lines and reserves of energy resources in terms of reliability factor.

### 6.2 Required initial data

- The calculated EPS scheme (equivalent nodes and tie lines among them);
- Characteristic daily load curves at each node referred to common time, duration of the corresponding calculated intervals;
- Mean square deviations of loads from regular hourly values;
- Monthly maximum load curves by node;
- Composition and parameters of generation units for each node and each calculated interval;
- Composition and parameters of transmission lines for each line in each calculated interval;
- Probabilities of emergency outages of generators and transmission lines;
- Standards for current maintenance of EPS components during a year;
- Standards for capital and medium maintenance of the EPS components during a year;
- Limitations on the use of primary energy resources (fuel for TPP and water for HPP);
- Specific discounted costs of EPS equipment by node and by tie line;
- Compensation costs due to electricity undersupply to consumers by node.

### 6.3 Calculation results

The reliability indices for nodes and the whole system for each considered interval and the whole year are:

- Probability of shortage-free operation;
- The coefficient of electricity provision for consumers;
- Mathematical expectation of electricity undersupply;
- Mathematical expectation of power shortage.

The values of calculated reserves of different types for nodes and the entire system are:

- Reserve intended for capital (medium) maintenance;
- Reserve intended for current maintenance;
- Reserve for updating;
- Operating reserve;
- The total volume of reserves of all types;

Results of the calculation also include;

- The values of additional transfer capabilities of tie lines to provide mutual redundancy of EPS;
- The functions of load distribution among tie lines during a year;
- The values of required primary energy resources.
- The dual (objectively conditioned) estimates of deficiency of main resources (generation capacities and energy resources at nodes and transfer capabilities of tie lines) to ensure reliability of electricity supply to consumers.

### 6.4 *Solution method*

The problem is solved by the simulation modeling of EPS operation for the considered period (year). The series of capacity distribution among generators at nodes and transmission lines in tie lines are calculated for their emergency outages. Binomial expression is applied as generating function. The state of loads and capacities of system facilities are obtained by the Monte Carlo method. The calculated operating conditions are optimized by the interior point method [6], where a principle of minimization and distribution of capacity shortage among nodes is used as a functional. The principle is chosen in accordance with specific features of the problem from the following four variants [7]:

- in proportion to loads at nodes;
- in terms of power losses in transmission lines and damages caused by power undersupply by node;
- in terms of electricity price in regional markets and its production cost;
- in terms of electricity price in the spot and wholesale markets.

The model also calculates capacities and mix of units under current and capital (medium) maintenance in each calculated interval by the known techniques.

The module for optimization of the calculated EPS state in which account is taken of the wholesale electricity market characteristics is described as an example.

The operation optimization problem can be formulated as follows:

*For the known values of available generation capacity, specific costs of electricity generation, required levels of load supply and coefficients of load significance at nodes, specified transfer capabilities of tie lines and coefficients of power losses in them, as well as for the values of electricity tariffs for the internal market of each power node and external wholesale markets*

*determine the value of loading of generating facilities and the values of load covered at nodes according to the given optimality criterion in terms of constraints on the ranges of possible changes in generation capacity and loads at nodes, power flows over tie lines and power balances to be met at nodes in terms of power losses in the networks.*

**Mathematical formulation of the problem and a method suggested for its solution.**

The calculated scheme of EPS is represented as a connected graph with $M$ nodes (vertices) and $N$ ties (arcs) among the nodes. Each node is characterized by the amount of required load $\overline{P}_m^l$ and the value of operable generation capacity $\overline{P}_m^g$, and each tie – by the transfer capabilities in the direct and reverse directions $\overline{P}_n$ and $\underline{P}_n$ respectively, and by the loss factor $K_{loss\,n}$.

Denote the covered load power at the $m$-th node by $P_m^l$, the generation capacity participating in load meeting by $P_m^g$, the flow along the $n$-th tie by $P_n$. The optimization problem functional of any (deficit and deficit-free) system condition is described by the following expression:

$$\max \sum_{m=1}^{M} (c_m P_m^g + c_e \sum_{n \in N_0} P_n + g_m \Delta P_m^g - d_m P_m^g - f \Delta P_m^g) \quad (1)$$

subject to:

$$\sum_{n=1}^{N} (a_{mn} P_n + b_{mn} k_{loss\,n} P_n^2) + P_m^l - P_m^g = 0$$

$$0 \le P_m^g \le \overline{P}_m^g \qquad m = \overline{1, M} \quad (2)$$

$$0 \le P_m^l \le \overline{P}_m^l$$

$$\underline{P}_n \le P_n \le \overline{P}_n \;, \qquad n = \overline{1, N}\,, \quad (3)$$

where $\Delta P_m^l = \overline{P}_m^l - P_m^l$ – power deficit at the $m$-th node;

$\Delta P_m^g = \overline{P}_m^g - P_m^g$ – excess of generation capacity at the $m$-th node;

$a_{mn}$ – elements of the matrix $A$ of ties.

$b_{mn}$ – elements of the matrix $B$, such that

$$b_{mn} = \begin{cases} 1, & if \;\; P_n < 0; \\ 0, & if \;\; a_{mn} \;\; P_n \ge 0. \end{cases}$$

$N_0$ – set of ties, which are used by the $m$-th node to sell (buy) the electricity of the higher-level wholesale market as compared to the local (internal) market.

In functional (1) there are the following technical and economic coefficients:

$c_m$ – price of electricity supplied to consumers at node $m$ (Rub/kWh);

$c_e$ – electricity price in the wholesale interregional market (Rub/kWh);

$d_m$ – costs on production of 1 kWh of electricity at the corresponding node (Rub/kWh);

$g_m$ – cost of fuel used for generation of 1 kWh of electricity (Rub/kWh);

$f_m$ – specific loss, compensation costs or penalties due to electricity undersupply (Rub/kWh).

In functional (1) dimensionality is violated, as far as the costs per 1 kWh are multiplied by the capacity (kW), this being its specific feature. The time for existence of this condition that is common for all functional components is factorized and determined beyond the considered optimization block of the software package.

In general the optimal solution can be obtained for real relations between the functional coefficients $f_m > c_m$, $c_m > d_m$, $d_m > g_m$ in the majority of system nodes.

Based on the calculation of operating conditions that were made with the model we can obtain:

- Price of electricity supplied to consumers;
- Loss due to electricity undersupply;
- Electricity production costs;
    - Economic benefit for the entire system and for nodes;

- Commercial benefit (profit) for the entire system and for nodes (power companies).

The form of functional (1) may vary depending on conditions of the competitive market operation and a set of its players.

The results of the analysis of all calculated conditions are used to calculate reliability indices (p.3).

The software is developed in Windows OS in the Fortran programming language in batch mode. The maximum parameters of the calculated scheme are 100 nodes and 160 tie lines.

## 7. CONCLUSION

Objectively there should be a great number of reliability models. This is associated with a great number of control problems solved in terms of reliability at different territorial, temporal and production levels.

Vertically integrated systems focus on technical and economic aspects of control.

For the liberalized market environment the models should be supplemented by the methods for solving financial problems, the problems of competition among numerous independent market players, etc.

Thus, the need arises to solve the problem of coordination of reliability solutions for different levels and stages.

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