
THE REVERSE LOGISTICS FORECASTING MODEL WITH WHOLE PRODUCT RECOVERY

M. Plewa, A. Jodejko-Pietruczuk

•
Wrocław University of Technology, Wrocław, Poland

e-mail: marcin.plewa@pwr.wroc.pl

ABSTRACT

Classical logistics systems supports only processes carried out in material flow from manufacturer to final user. Recently it has been a striking growth of interest in optimizing logistics processes that supports recapturing value from used goods. The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal is called reverse logistics. The main goal of this paper is to create the forecasting model to predict returns quantity for reverse logistics system that supports processes of recapturing value from used products that has been decommissioned (because of failure) before a certain time called an allowable working time before failure.

1 INTRODUCTION

Until recently logistics systems supported only processes carried out in classical material flow from manufacturer to final user. Recently it has been a remarkable growth of interest in optimizing logistics processes that supports recapturing value from used goods. The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal is called reverse logistics. In reverse logistics systems demand can be partially satisfied with new items manufacture or procurement and returned products recovery. Reuse of products can bring direct advantages because company uses recycled materials instead of expensive raw materials. The first part of the article contains literature survey, which is a summary of work that has been done around the theme of research.

The main goal of this paper is to create the model of logistics system that supports processes of recapturing value from used products that has been decommissioned (because of failure) before a certain time called an allowable working time before failure. Article contains forecasting model which explains the process of moving goods from maintenance system to recovery system. In presented model there is an assumption that there is a relationship between demand for new products and the number of returns. There is also an assumption that every product that fail before acceptable time to failure can be recovered and used again. After recovery process products are as good as new ones. Demand for new products can be fulfilled by production or by recovery of used (failed) products. This article deals with single-item products.

This paper objectives are achieved by creating the mathematical model that describe analyzed processes. Some objectives can't be achieved without creating a simulation model, which make possible to describe analyzed processes with various probability distributions. The next step is a sensitivity analysis of created model and verification process. The article ends with conclusions and directions for further research.

2 LITERATURE SURVEY

In literature the field of reverse logistics is usually subdivided into three areas: inventory control, production, recovery and distribution planning. The model presented in this paper is a inventory control model and hence the literature survey refers to this area.

Basic inventory control model in reverse logistics is built on following assumptions (Fleischmann et al. 1997):

- the manufacturer receives and stores the products returned from the final user;
- requirement for the new products is addressed to the final products storage;
- demand for the new products can be fulfilled by the production of the new ones or by the recovery of returned products;
- recovered products are as good as new ones;
- the goal of the model is to minimize the total costs.

Inventory control models in reverse logistics can be divided into deterministic and stochastic models.

2.1 Deterministic models

Deterministic models presented in the literature are mainly the modifications of the classical EOQ model.

First models in this category was created by Schrady (Fleischmann et al. 1997, Schrady 1967, Teunter 2001). In his models demand and return rates are constant. Return rate is described as a part of demand rate. Returns are put into recoverable inventory. Recovered products are as good as new ones and they are stored with new products in serviceable inventory. There is no disposal option, all returns are recovered and used to fulfill the demand. There are fixed lead-times for new products procurement and returns recovery. The goal is to set up optimal lot sizes for procurement and recovery. Schrady considers fixed setup costs for procurement and recovery and holding costs for recoverable and serviceable inventory.

Schrady's work was continued by Steven Nahmias & Henry Rivera (Nahmias & Rivera 1979). In contrast to Schrady they assume limited capacity of the recovery process and zero lead times. Schrady's work was also continued by Teunter. In his model there are multiple lots for procurement and recovery. In Schrady's model there was only one procurement lot. Teunter assume zero lead times but he allows disposal. Teunter considers linear costs for recovery, production, disposal and inventory holding. He also considers fixed setup costs for production and recovery. (Teunter 2001, 2002, 2003, 2004).

Model presented in (Teunter 2004) was extended in (Konstantaras & Papachristos 2006, 2008). In (Konstantaras & Papachristos 2008) authors assume that the parameters determining the number of production and recovery runs are integers. In (Konstantaras & Papachristos 2006) authors add the possibility of occurrence of backorders.

Applicability of the EOQ model in reverse logistics was also studied by Marilyn C. Mabini, Liliane M. Pintelon & Gelders Ludo F. These authors extend model formulated by Schrady. They consider both single and multi-item systems. In single item model in contrast with model proposed by Schrady, Mabini et al. consider backorders. In multi-item model they assume limited capacity of recovery process which must be allocated among multiple products. New products are ordered when recovery capacity is exhausted. They consider the possibility of selling excess of recoverable inventory (Mabini et al. 1992). Another EOQ model in reverse logistics was created by Knut Richter. Richter describes two workshops. In the first manufacturing and recovery processes are carried out and serviceable products are stored. In the second workshop products are used and recoverable items are stored. Recoverable items are periodically transferred to the first workshop. (Richter 1994, 1996a, b, 1997, 1999).

Economic lot size problem was also analyzed by Shie-Gheun Koh, Hark Hwang, Kwon-Ik Sohn and Chang-Seong Ko. They present model similar to the models described by Schrady (1967), Mabini (1992) & Richter (1996). In contrast with previous authors, they take into account the duration of the recovery process. They assume that at least one of the variables describing the number of external orders and the number of starts of the recovery process is equal to one. (Koh et al. 2002).

Production and recovery rates are also taken into account by Knut Richter & Imre Dobos. These authors extend the model formulated previously by Richter by taking into account the quality factor in recovery management. (Dobos 2002, Dobos & Richter 2000, 2003, 2004, 2006, Richter & Dobos 1999).

Model presented by Richter in (Richter 1996a, b) was extended by Ahmed M.A. El Saadany & Mohammad'a Y. Jaber. Ahmed & Jaber believe that in the first period, only the production should be carried out. They also consider fixed switching costs. (El Saadany & Jaber 2008, Jabber & Rosen 2008). In another work (Jaber & El Saadany 2009) the same authors assume that the recovered products differ from new ones. They consider the different demand rates for new products and products of recovery process.

The possibility of applying EOQ models in reverse logistics was also studied by Yong Hui Oh & Hark Hwang. These authors analyze the system in which after recovery returns are treated as a raw material in the manufacturing process (Hwang & Oh 2006).

Particularly noteworthy articles are (Inderfurth et al. 2005, 2006). In these papers authors analyze the impact of recoverable inventory holding time on recovery process.

Equally noteworthy is article (Mitra 2009), in which author presents the analysis of the two echelon inventory model.

In the analyzed literature, a separate category are models based on dynamic programming. In most papers, presented models are the extensions of the classical Wagner Whitin model. Applicability of dynamic programming in reverse logistics is discussed in (Inderfurth et al. 2006, Kiesmüller 2003, Konstantaras & Papachristos 2007, Minner & Kleber 2001, Richter & Sombrutzki 2000, Richter & Weber 2001, Van Eijl & van Hoesel 1997). In models described above authors assume one recovery option. Multiple recovery options are considered in (Corbacioğlu & van der Laan 2007, Kleber et al. 2002).

2.2 Stochastic models

In the class of stochastic models there are two model groups:

- models, in which demand for the new product is a consequence of the return;
- models, in which the demand for new products does not depend on the number of returns, but the number of returns may depend on the previous demand.

The first group are models of repair systems. These are closed systems in which the number of elements remains constant. The main goal of such a system is to keep sufficient number of spare parts to provide the required level of availability of the technical system.

There are many literature reviews on repair systems. The most significant papers are (Diaz & Fu 2005, Guide & Srivastava 1997, Kennedy et al. 2002, Mabini & Gelders 1991, Pierskalla & Voelker 2006, Valdez-Flores & Feldman 1989, Wang 2002). The well-known inventory control model in repair systems is METRIC which was firstly presented in (Sherbrooke 1968). Modifications of METRIC model can be found in many subsequent works (e.g. Cheung et al. 1993, Muckstadt 1973, Pyke 1990, Sherbrooke 1971, Sleptchenko et al. 2002, Wang et al. 2000, Zijm & Avsar 2003).

The second group of stochastic inventory control models can be divided into continuous and periodical review models.

2.2.1 Continuous review models

First model in this category was created by Heyman. He built the model, in which demand for the new products and returns quantity are independent Poisson random variables. In Heyman model there is only recoverable inventory, new products are not stored because Heyman assumes zero lead times for external orders and recovery process. Recovered products are as good as new ones. Heyman doesn't consider setup costs for recovery and procurement. Heyman defines disposal level. Newly returned products are disposed of if recoverable inventory level exceed disposal level. Heyman is looking for an optimal solution using the theory of mass service (Heyman 1997).

The work of Heyman was continued by John A. Muckstadt & Michael H. Isaac. These authors consider single and two echelon models. In single echelon model, as in the work of Heyman, the demand for final products and product returns are independent Poisson random variables. In contrast to Heyman, Muckstadt & Isaac consider lead times for recovery and procurement. Procurement lead time is fixed and recovery lead time is a random variable. Muckstadt and Isaac allow backorders. Recovered products are as good as new ones.

In two echelon model, on the upper echelon, there is a central depot where recovery is carried out and new products are delivered. Lower echelon contains the group of sellers who collect returns from customers and send them to the central depot (Muckstadt & Isaac 1981).

Multi echelon reverse logistics model was also created by Aybek Korugan & Surendra M. Gupta. In contrast with previous model Korugan and Gupta consider disposal option but they do not allow backorders (Korugan & Gupta 1998). The work of Korugan & Gupta has been pursued by Mitra S. in (Mitra 2009) by considering fixed setup costs.

The model similar to the single echelon model presented in (Muckstadt & Isaac 1981) has been proposed by Ervin van der Laan, Rommert Dekker, Marc Salomon & Ad Ridder (Van der Laan et al. 1996b). Van der Laan, Dekker & Salomon present also the comparison of alternative inventory control strategies. They compare their models with those presented by Muckstadt et al. & Heyman (van der Laan et al. 1996a). The comparison of alternative control strategies is also presented in (Bayindir et al. 2004). Van der Laan & Salomon also compared PUSH and PULL strategies in reverse logistics. They show that PULL strategy is more profitable only if holding costs in recoverable inventory are significantly lower than in serviceable inventory (Van der Laan & Salomon 1997).

The theory of inventory management in reverse logistics was also developed by Moritz Fleischmann, Reolof Kuik & Rommert Dekker. They built model similar to the one presented by John A. Muckstadt & Michael H. Isaac. They investigate a case in which returned products can be directly reused. They allow backorders and consider fixed setup costs for procurement and linear holding and backorders costs (Fleischmann et al. 2002). Fleischmann & Kuik consider direct reuse of returns also in (Fleischmann & Kuik 2003, Fleischmann et al. 2003).

Particularly noteworthy is an article (Ouyang & Zhu 2006) in which authors present the model that describes the final stage of product life, when the number of returned products can significantly exceed the demand for these products.

2.2.1 Periodical review models

First model in this category was created by V. P. Simpson (1978). He assumes that demand for the new products and returns quantity are independent random variables. Simpson considers separate inventories for new products and returns. He assumes zero lead times for a procurement and recovery. Simpson allows disposal. He considers linear costs for external orders, recovery, inventory holding and unfulfilled demand and does not take into account disposal cost (Simpson 1978).

Model based on similar assumptions was developed by Karl Inderfurth. Unlike his predecessor, he takes into account recovery process and procurement lead time expressed in the number of periods. Unfulfilled demand takes the form of pending orders. Inderfurth investigates two cases: in first one he doesn't allow for returns storage, in the second he eliminates that

constraint. Inderfurth notes that the difference between recovery and procurement lead times is a factor that has significant impact on the model's level of complications (Inderfurth 1997, Inderfurth & van der Laan 2001).

Model presented in (Inderfurth 1997, Sobczyk 2001) is investigated by Kiesmüller & Scherer in (Kiesmüller & Scherer 2003). They use computer simulation to find optimal solution of the model.

G. P. Kiesmüller & S. Miner are the following authors dealing with inventory control in reverse logistics. In their works (Kiesmüller 2003, Kiesmüller & Minner 2003) the authors assume that returns don't depend on demand. Kiesmüller & Minner don't consider disposal. They assume that all the returns are recoverable. They model a production system in which serviceable inventory is replenished with production and recovery. Authors take into consideration production and recovery lead times. They assume that review takes place at the beginning of each period.

Production system is also modelled by Mahadevan, Pyke & Fleischmann. They investigated the system in which returns and demand are described by Poisson distribution. The system consists of recoverable and serviceable inventory. The authors take into account inventory holding and backorders costs. In modelled system a review is performed always after certain number of periods (Mahadevan 2003, Pyke 2001).

Peter Kelle & Edward A. Silver focus on a periodic review inventory control system in their work as well. They develop an optimal policy for new products purchase using the example of reusable packaging. They assume that all the returns undergo recovery process. The authors don't consider the disposal. Unfulfilled demand takes the form of pending orders and is satisfied during the further periods. The authors don't take into account the cost related with backorders. They substitute it with a customer service level. The authors reduce the stochastic model to a deterministic model which is analytically solved (Kelle & Silver 1989).

D. J. Buchanan & P. L. Abad describe reusable packaging inventory management system as well. The authors create a model similar to that of Kelle & Silver. Buchanan & Abad analyze single- and multi-period model. As for multi-period model, the authors assume that the number of returns at each period is a fraction of all the products found on the market at the beginning of this period. At each period some part of products found on the market isn't suitable for reuse. The authors take into consideration the cost connected with products that were not sold at the end of analyzed planning horizon. The authors assume that the time of product presence on the market is described by the exponential distribution (Buchanan & Abad 1998).

The models presented above are based on the assumptions of a classical periodic review model. Subsequent authors R. H. Teunter & D. Vlachos developed a model similar to continuous review model that had been earlier introduced by Ervin Van der Laan & Marc Salomon. Teunter & Vlachos analyze the model using a computer simulation (Teunter & Vlachos 2002).

Worth mentioning is model developed by Inderfurth. In (Inderfurth 2004) he describes the model in which recovered products differ from new ones and are stored separately. Model presented by Inderfurth is a single period inventory model. Inderfurth assumes that the demand for new products and the demand for recovered returns are independent random variables. Interesting is the assumption, which allows the use of excess inventory of new products to meet unfulfilled demand for recovered products.

Single period inventory model is also presented in (Bhattacharya et al. 2006, Mostard 2005, Toktay 2000, Vlachos & Dekker 2003).

2.3 Summary

Presented review allows to summarize the current state of knowledge. Most authors assume that demand and returns are independent Poisson random variables. Few authors examine the relationship between the demand, and the number of returns but there are no inventory models in reverse logistics that use reliability theory to assess the number of returns. There are few models that use the reliability to assess the reusability of components. In real reverse logistics systems products that fail in a short time after the purchase by the final user are withdrawn from the market.

In most cases these products are returned to the manufacturer. Recovery of such products or components is not labour-intensive, because undamaged components have not been exposed to the aging process and are suitable for direct reuse.

There is a lack of models that consider different recovery options and different return reasons. There are no models which support decision making process about recovery options.

The goal of this paper is to create an inventory control model in which products are recoverable if they are withdrawn because of failure before a specified time from the start of the operation process. The model takes into account the relationship between the size of demand for the new products, and the number of returned objects.

3 MODEL DESCRIPTION

In modelled system demand for the new products at the particular periods D_T is an independent random variable. The demand occurs at the beginning of each period. We assume that failed products are returned to the manufacturer and can be recovered if their operating time was shorter than τ_{dop} . We call it allowable working time before failure. Demand can be fulfilled by production or recovery. Recovered products are as good as new ones. Failed products are returned at the beginning of each period. The number of products returned at the beginning of each period is the following:

$$Z_T = PN_{[t_{T-1}, t_T]} \quad (1)$$

where $PN_{[t_{T-1}, t_T]}$ = the number of products withdrawn because of failure at period $[t_{T-1}, t_T]$; t_{T-1} = the beginning of period $T-1$; t_T = the beginning of period T .

Returned products are stored in recoverable inventory and new products are stored with recovered ones in serviceable inventory. In presented model a review is performed every T_{prz} periods. We assume that the first review occur at the beginning of the first period. The set of review periods can be presented in the following way:

$$V_{T_{prz}} = \{T_{prz,1}, T_{prz,2}, T_{prz,3}, \dots, T_{prz,n}, \dots\} \quad (2)$$

where $T_{prz,n} = 1 + (n-1) \cdot T_{prz}$; $T_{prz,n} \leq H$ for $n=1,2,3,\dots$; n = review number; H = planning horizon.

Inventory position can change as a result of new items production and returns recovery or disposal. Recovery process starts if the serviceable inventory position is equal or lower than s and there is enough products in recoverable inventory to fulfill fixed recovery batch. Otherwise the inventory is replenished owing to production. We assume fixed and equal to one, lead times for production and recovery. We allow the disposal of excess inventory of recoverable items if in the review period the level of recoverable inventory is higher than s_u . We don't allow backorders. Unfulfilled demand is lost.

Framework for this situation is depicted in Figure 1.

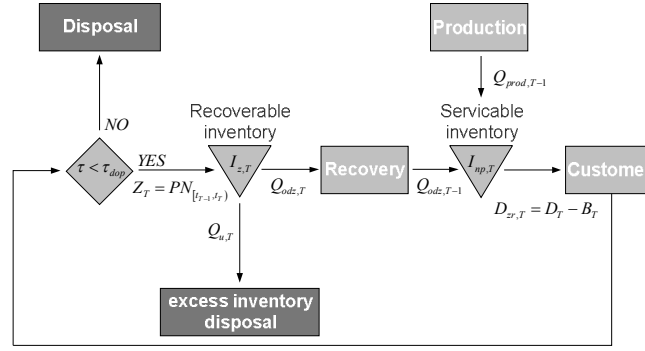


Figure 1. Framework inventory management

The serviceable inventory position in each period T may be presented as follows:

$$I_{np,T} = \max \{ I_{np,T-1} + Q_{odz,T-1} + Q_{prod,T-1} - D_T, 0 \} \quad (3)$$

where $I_{np,T-1}$ = on hand serviceable inventory at the end of period $T-1$; $Q_{odz,T-1}$ = recovery batch size launched in period $T-1$; $Q_{prod,T-1}$ = production batch size launched in period $T-1$; D_T = demand for the new products in period T .

Unfulfilled demand in period T can be presented in the following way:

$$B_T = \left| \min \{ I_{np,T-1} + Q_{odz,T-1} + Q_{prod,T-1} - D_T, 0 \} \right| \quad (4)$$

We don't allow the disposal of products in serviceable inventory. The recoverable inventory level at the end of period T is the following:

$$I_{z,T} = I_{z,T-1} + PN_{[t_{T-1}, t_T]} - Q_{odz,T} - Q_{u,T} \quad (5)$$

where $I_{z,T-1}$ = on hand recoverable inventory at the end of period $T-1$; $Q_{odz,T}$ = recovery batch size launched in period T ; $Q_{u,T}$ = the number of products disposed of in period T .

Recovery batch size launched in period T is the following:

$$Q_{odz,T} = Q_{odz} \cdot X_T \quad (6)$$

where

$$X_T = \begin{cases} 1 \text{ if } \begin{cases} I_{np,T} \leq s \\ \text{and} \\ I_{z,T-1} + PN_{[t_{T-1}, t_T]} \geq Q_{odz} \\ \text{and} \\ T \in V_{T_{prz}} \end{cases} \\ 0 \text{ if } \begin{cases} I_{np,T} > s \\ \text{or} \\ (I_{np,T} \leq s \text{ and } I_{z,T-1} + PN_{[t_{T-1}, t_T]} < Q_{odz}) \\ \text{or} \\ T \notin V_{T_{prz}} \end{cases} \end{cases} \quad (7)$$

Production batch size launched in period T is the following:

$$Q_{prod,T} = Q_{prod} \cdot Y_T \tag{8}$$

where

$$Y_T = \begin{cases} 1 \text{ if } \left\{ \begin{array}{l} I_{np,T} \leq s \text{ and } I_{z,T-1} + PN_{[t_{T-1},t_T]} < Q_{odz} \\ \text{and} \\ T \in V_{T_{prz}} \end{array} \right. \\ 0 \text{ if } \left\{ \begin{array}{l} (I_{np,T} \leq s \text{ and } I_{z,T-1} + PN_{[t_{T-1},t_T]} \geq Q_{odz}) \\ \text{or} \\ I_{np,T} > s \text{ or } T \notin V_{T_{prz}} \end{array} \right. \end{cases} \tag{9}$$

The number of products disposed of in period T is the following:

$$Q_{u,T} = (I_{z,T-1} + PN_{[t_{T-1},t_T]} - Q_{odz,T} - s_u) \cdot U_T \tag{10}$$

where

$$U_T = \begin{cases} 1 \text{ if } \left\{ \begin{array}{l} (I_{z,T-1} + PN_{[t_{T-1},t_T]} - Q_{odz,T}) > s_u \\ \text{and} \\ T \in V_{T_{prz}} \end{array} \right. \\ 0 \text{ if } \left\{ \begin{array}{l} (I_{z,T-1} + PN_{[t_{T-1},t_T]} - Q_{odz,T}) \leq s_u \\ \text{or} \\ T \notin V_{T_{prz}} \end{array} \right. \end{cases} \tag{11}$$

Behavior of the system for $T_{prz}=1$ is depicted in Figure 2.

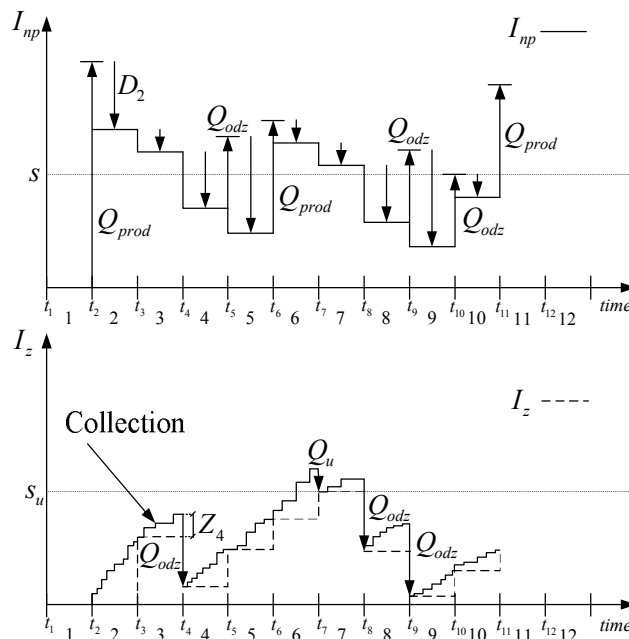


Figure 2. System behavior

3.1 The method of forecasting the number of objects returned to the system

Presented model of reverse logistics system required to develop methods for forecasting the number of objects returned to the system, within a specified period of time, at a fixed time limit τ_{dop} . Proposed forecasting model is an extension of model presented by Murayama in the (Murayama & Shu 2001, Murayama et al. 2004, 2005, 2006). The value of a τ_{dop} depends on aging process and total costs. Therefore, whether the object at the time t is located in a recovery system depends on:

- random variable D_{zr,t_e} specifying the number of objects placed on the operating system at the moment t_e ,
- random variable determining the probability of a technical object failure until τ_{dop} .

The probability of the object failure in the time interval from x_1 to x_2 can be written as follows (Migdalski 1982):

$$F(x_1, x_2) = P(x_1 \leq \tau < x_2) \tag{12}$$

where τ = random variable that describes the time to object failure.

For $t_b - t_a < \tau_{dop}$ the number of objects $PN_{[t_a, t_b]}$, returned to the recovery system in the time period $[t_a, t_b]$ is the sum of three cases:

$$PN_{[t_a, t_b]} = PN_{1,[t_a, t_b]} + PN_{2,[t_a, t_b]} + PN_{3,[t_a, t_b]} \tag{13}$$

The first two are associated with objects, for which $t_e \leq t_a$. The third one describes the situation in which objects are returned in interval (t_a, t_b) .

In the first case (Fig. 3): $x_1 = t_a - t_e$ and $x_2 = \tau_{dop}$. The variable t_e can be in the range from $\max(t_a - \lceil \tau_{dop} \rceil, 0)$ to $t_b - \lceil \tau_{dop} \rceil$ and the number of returned objects is the following:

$$PN_{1,[t_a, t_b]} = \sum_{t_e = \max(t_a - \lceil \tau_{dop} \rceil, 0)}^{t_b - \lceil \tau_{dop} \rceil} (D_{zr,t_e} \cdot F(t_a - t_e, \tau_{dop})) \tag{14}$$

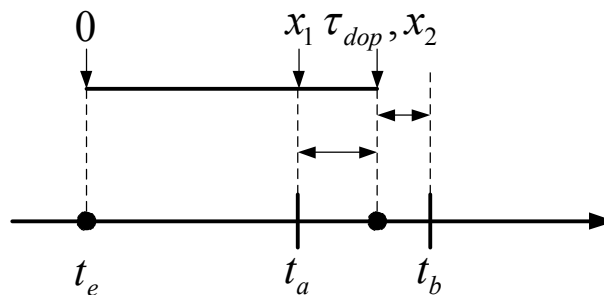


Figure 3. Predicted quantity of returned products between t_a and t_b described by Equation 14

In the second case (Fig. 4): $x_1 = t_a - t_e$ and $x_2 = t_b - t_e$. The variable t_e can be in the range from $t_b - \lceil \tau_{dop} \rceil + 1$ to t_a and the number of returned objects is the following:

$$PN_{2,[t_a, t_b]} = \sum_{t_e = t_b - \lceil \tau_{dop} \rceil + 1}^{t_a} (D_{zr,t_e} \cdot F(t_a - t_e, t_b - t_e)) \tag{15}$$

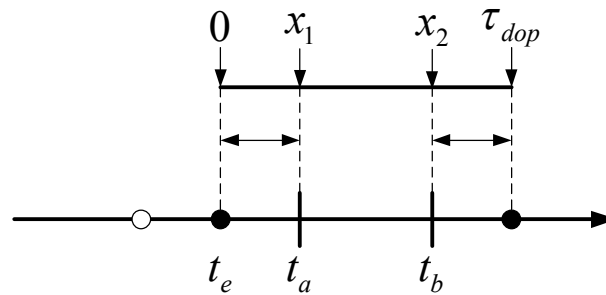


Figure 4. Predicted quantity of returned products between t_a and t_b described by Equation 15

In the third case (Fig. 5): $x_1=0$ and $x_2=t_b-t_e$. The variable t_e can be in the range from t_a+1 to t_b and the number of returned objects is the following:

$$PN_{3,[t_a,t_b]} = \sum_{t_e=t_a+1}^{t_b} (D_{zr,t_e} \cdot F(0, t_b - t_e)) \tag{16}$$

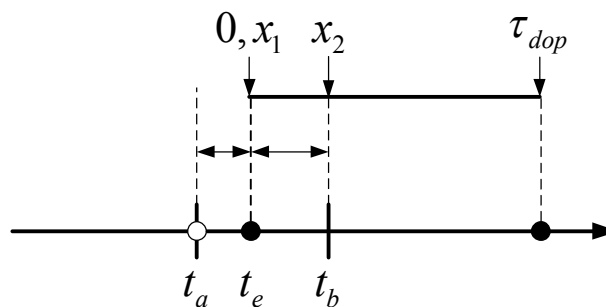


Figure 5. Predicted quantity of returned products between t_a and t_b described by Equation 16

The joint cost function in any period T takes the following form:

$$TC_T = \sum_{i=1}^T \left(\begin{array}{l} k_{skz} \cdot I_{z,i} + k_{sknp} \cdot I_{np,i} + \\ + k_{odz} \cdot Q_{odz,i} + k_{prod} \cdot Q_{prod,i} + \\ + k_b \cdot B_i + k_u \cdot Q_{u,i} \end{array} \right) \tag{17}$$

where: k_{skz} = recoverable inventory holding cost; k_{sknp} = serviceable inventory holding cost; k_{odz} = recovery cost; k_{prod} = production cost; k_b = lack-of-inventory cost; k_u = excess inventory disposal cost.

In presented model we don't consider production and recovery fixed costs and don't allow backorders.

4 SIMULATION AND SENSITIVITY ANALYSIS

In presented model we don't consider production and recovery fixed costs and don't allow backorders In simulation we assume that demand D_T is a Poisson random variable with parameter λ_D and the time to failure is described with Weibull distribution with scale parameter α_r and shape parameter β_r . In simulation we use Monte Carlo methods with random number generators

implemented in Matlab software. Table 1 shows the range of variation of parameters of the simulation model.

Table 1. Range of variation of parameters of the simulation model

Parameter	Range of variation			
	input	min	charge	Max
λ_D	1000	1000	-	1000
α_r	40	1.0	4.0	117
β_r	1.5	0.5	0.5	15
τ_{dop}	25	1.0	3.0	88
Q_{prod}	3000	250	250	7500
Q_{odz}	2000	100	100	3000
s	0.0	0.0	100	2900
s_u	2000	0.0	250	7250
T_{prz}	1.0	1.0	1.0	30
k_{sknp}	2.0	0.0	1.0	29
k_{skz}	1.0	0.0	1.0	29
k_{prod}	20	0.0	1.0	29
k_{odz}	10	0.0	1.0	29
k_u	0.5	0.0	1.0	29
k_b	5.0	0.0	1.0	29

Figures 6-9 show exemplary results of sensitivity analysis. Chosen example presents the impact of changes in the size of production batch on the output model parameters. Increasing batch leads to raise of the average level of serviceable inventory. Which reduces the amount of unfulfilled demand and thus increases the level of customer service. The increase of fulfilled demand leads to an increase in the number of returns to the system. Increasing number of returns initially increase the number of starts of the recovery process. However, a substantial extension of the production cycle results in reduction of recovery and thus raise the recoverable inventory level. This leads to an increase in disposal of products.

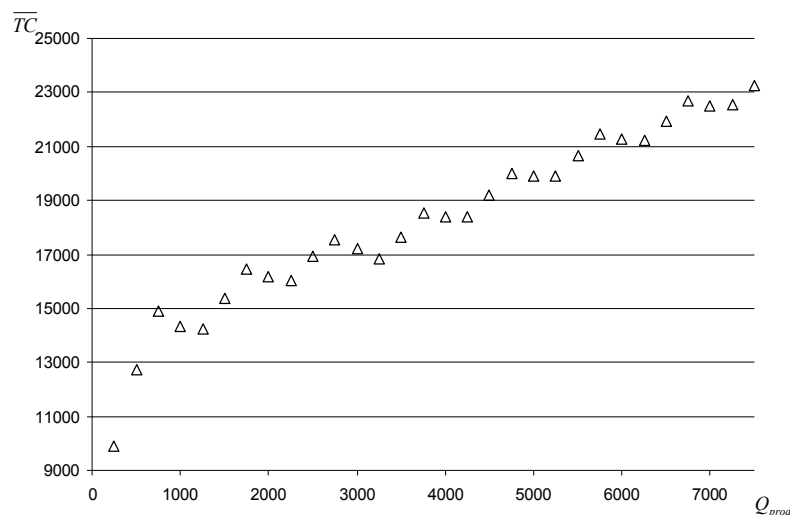


Figure 6. Variation of the average size of the total cost of the system, as a function of changes in production batch

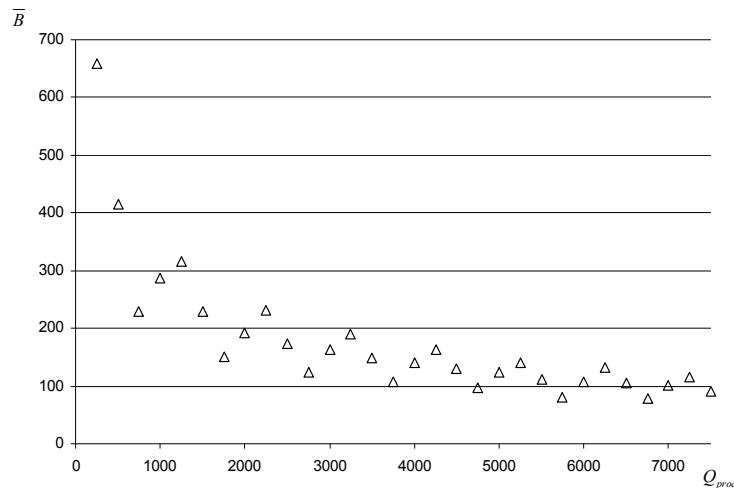


Figure 7. Variation of the average size of unfulfilled demand, as a function of changes in production batch

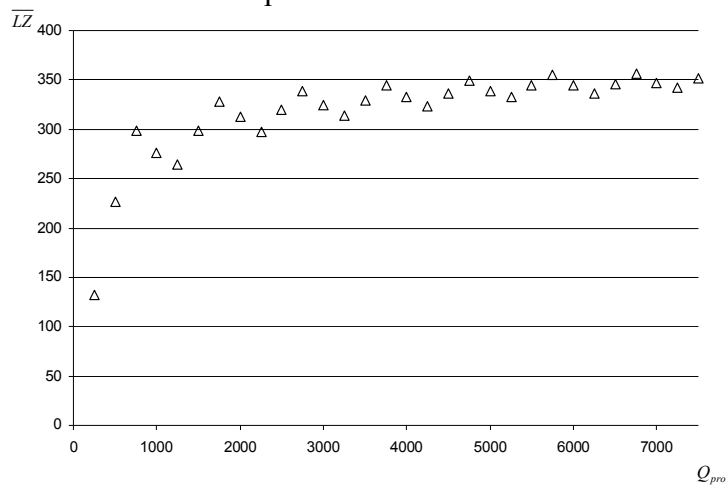


Figure 8. Variation of the average number of returns, as a function of changes in production batch

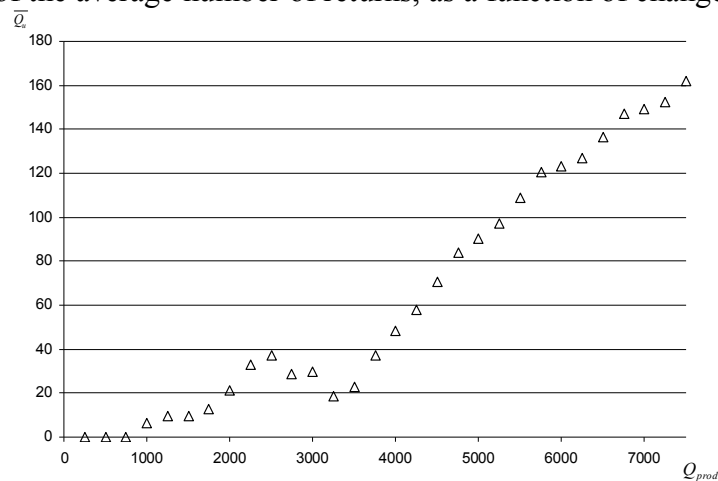


Figure 9. Variation of the average number of disposed products, as a function of changes in production batch

5 SIMULATION AND SENSITIVITY ANALYSIS

The analysis shows that the selection of appropriate parameter values has a significant impact on the returns management. Determining the appropriate value of the recovery and production batch helps to achieve a high level of reuse of recoverable products, increased service levels and low operating costs of the system.

Sensitivity analysis has shown that with the assumed values of input parameters, the level of the total cost of the system depends primarily on the level of the unit cost of storage of recoverable items. A much smaller impact on economic performance of the analyzed model was observed for the unit cost of storage of serviceable products and the unit cost of production. The smallest impact on the overall costs of the system has the change of the unit cost of disposal. Slightly more important impact proved to have a unit cost of the process of recovery and the cost of penalties associated with the occurrence of the lack of final products.

There are many possible extensions of the presented model by taking into account:

- parameters associated with the process of collection and transportation of returns. Particularly important may be the consideration of different return sources;
- ability to deliver returns in batches;
- longer than a period, time of recovery and production;
- random times of recovery;
- complex reliability structure of the technical objects.

5 REFERENCES

- Bayindir, Z.P., Teuner, R.H. & Dekker, R.A. 2004. Comparison of Inventory Control Policies for a Joint Manufacturing/Remanufacturing Environment with Remanufacturing Yield Los., Proceedings of the 13th International Symposium on Inventories, Hungary: Budapest.
- Bhattacharya, S., Guide Jr. V.D.R. & Van Wassenhove L.N. 2006. Closed-Loop Supply Chains: An Introduction to the Feature Issu., *Production and Operations Management* 15 (3): 421-431.
- Buchanan, D.J. & Abad, P.L. 1998. Optimal policy for periodic review returnable inventory system. *IIE Transactions* 30: 1049-1055.
- Cheung, K.L. & Hausman, W.H. 1993. A Multiechelon Inventory Model with Multiple Failures. *Naval Research Logistics*: 40: 593-602.
- Corbacioğlu, U. & van der Laan, E.A. 2007. Setting the holding cost rates in two-product system with remanufacturing. *International Journal of Production Economics* 109: 185-194.
- Diaz, A. & Fu, M.C. 2005. Multi-echelon models for repairable items: A review, *Decision & Information Technologies Research Works Collection*. Digital Repository at the University of Maryland.
- Dobos, I. & Richter, K. 2004. An extended production/recycling model with stationary demand and return rate., *International Journal of Production Economics* 90: 311-323.
- Dobos, I. & Richter, K. 2006. A production/recycling model with quality consideration. *International Journal of Production Economics* 104: 571-579.
- Dobos, I. & Richter, K. 2003. A production/recycling model with stationary demand and return rates. *Central European Journal of Operations Research* 11: 35-46.
- Dobos, I. & Richter, K. 2000. The integer EOQ repair and waste disposal model – Further analysis. *Central European Journal of Operations Research* 8: 173-194.
- Dobos, I. 2002. The generalization of Schrady's model: a model with repair, Working Paper No. 7, Department of Business Economics, Budapest University of Economics and Public Administration.
- El Saadany, A. & Jaber, M.Y. 2008. The EOQ repair and waste disposal model with switching costs. *Computers & Industrial Engineering* 55: 219-233.
- Fleischmann, M., Bloemhof-Ruwaard, J.M., Dekker, R., van der Laan, E., van Nunen, J.A.E.E. & van Wassenhove, L.N. 1997. Quantitative models for reverse logistics: A review. *European Journal of Operational Research* 103: 1-17.
- Fleischmann, M., Dekker, R. & Kuik, R. 2002. Controlling inventories with stochastic item returns: A basic model. *European Journal of Operational Research* 138: 63-75.
- Fleischmann, M. & Kuik, R. 2003. On optimal inventory control with independent stochastic item returns. *European Journal of Operational Research* 151: 25-37.
- Fleischmann, M., van Nunen, J. & Gräve, B. 2003. Integrating Closed-Loop Supply Chains and Spare Parts Management at IBM, *Interfaces* 33(6): s. 44-56.
- Guide, Jr. V.D.R. & Srivastava, R. 1997. Repairable inventory theory: Models and applications. *European Journal of Operational Research* 102: 1-20.
- Heyman, D.P. 1997. Optimal disposal policies for a single-item inventory system with return., *Naval Research Logistics Quarterly* 24: 385-405.
- Hwang, H. & Oh, Y.H., Deterministic inventory model for recycling system. *Journal of Intelligent Manufacturing* 17: 423-428.

- Inderfurth. K., Janiak. A., Kovalyov. M.Y. & Werner. F. 2006. Batching Work and Rework Processes with Limited Deterioration of Reworkables. *Computers and Operations Research* 33 (6): 1595-1605.
- Inderfurth. K., Lindner. G. & Rachaniotis. N.P. 2005. Lot sizing in a production system with rework and product deterioration., *International Journal of Production Research* 43 (7): 1355-1374.
- Inderfurth. K. 2004. Optimal policies in hybrid manufacturing/remanufacturing system with product substitution., *International Journal of Production Economics* 90: 325-343.
- Inderfurth. K., Simple optimal replenishment and disposal policies for a product recovery system with leadtimes. *OR Spektrum* 19: 111-122.
- Inderfurth. K. & van der Laan. E. 2001. Leadtime effects and policy improvement for stochastic inventory control with remanufacturing., *International Journal of Production Economics* 71: 381-390.
- Jaber. M.Y. & El Saadany. A.M.A. 2009. The production, remanufacture and waste disposal model with lost sales. *International Journal of Production Economics* 120: 115-124.
- Jaber. M.Y. & Rosen. M.A. 2008. The economic order quantity repair and waste disposal model with entropy cost. *European Journal of Operational Research* 188: 109-120.
- Kelle. P. & Silver. E.A. 1989. Purchasing Policy of New Containers Considering the Random Returns of Previously Issued Container. *IIE Transactions* 21 (4): 349-354.
- Kennedy. W.J., Patterson. J.W. & Fredendall. L.D. 2002. An overview of recent literature on spare parts inventories. *International Journal of Productions Economics* 76: 201-215.
- Kiesmüller. G.P. 2003. A new approach for controlling a hybrid stochastic manufacturing/remanufacturing system with inventories and different leadtimes. *European Journal of Operational Research* 147: 62-71.
- Kiesmüller. G.P. & Minner. S. 2003. Simple expression for finding recovery system inventory control parameter values. *Journal of Operational Research Society* 54: 83-88.
- Kiesmüller. G.P. 2003. Optimal control of a one product recovery system with leadtimes. *International Journal of Production Economics* 81-82: 333-340.
- Kiesmüller. G.P. & Scherer C.W. 2003. Computational issues in a stochastic finite horizon one product recovery inventory model. *European Journal of Operational research* 146: 553-579.
- Kleber. R., Minner. S. & Kiesmüller. G. 2002. A continuous time inventory model for a product recovery system with options. *International Journal of Production Economics* 79: 121-141.
- Koh. S.G., Hwang. H., Sohn. K. I. & Ko C.S. 2002. An optimal ordering and recovery policy for reusable items. *Computers & Industrial Engineering* 43: 59-73.
- Konstantaras. I. & Papachristos. S. 2008. A note on: Developing an exact for an inventory system with product recovery. *International Journal of production economics* 111: 707-712.
- Konstantaras. I. & Papachristos. S. 2006. Lot-sizing for a single-product recovery system with backordering. *International Journal of Production Research* 44: 2031-2045.
- Konstantaras. I. & Papachristos. S. 2007. Optimal policy and holding cost stability regions in a periodic review inventory system with manufacturing and remanufacturing options. *European Journal of Operational Research* 178: 433-448.
- Korugan. A. & Gupta. S. M. 1998. A Multi-Echelon Inventory System with Returns. *Computers & Industrial Engineering* 53 (1-2): 145-148.
- Mabini. M.C. & Gelders. L.F. 1991. Repairable item inventory systems: A literature review. *Belgian Journal Operations Research, Statistics and Computer Science* 30 (4): 58-69.
- Mabini. M.C., Pintelon. L.M. & Gelders. L.F. 1992. EOQ type formulations for controlling repairable inventories. *International Journal of Production Economics* 28: 21-33.
- Mahadevan. B., Pyke. D.F. & Fleischmann. M. 2003. Periodic review, push inventory policies for remanufacturing. *European Journal of Operational Research* 151: 536-551.
- Migdalski. J. 1982. *Poradnik niezawodności. Podstawy matematyczne*. Wyd. WEMA. Warszawa.
- Minner. S. & Kleber. R. 2001. Optimal control of production and remanufacturing in a simple recovery model with linear cost functions. *OR Spectrum* 23: 3-24.
- Mitra. S. 2009. Analysis of a two-echelon inventory system with returns. *Omega* 37: 106-115.
- Mostard. J., de Koster. R. & Teuner. R. 2005. The distribution-free newsboy problem with resalable returns. *International Journal of Production Economics* 97: 329-342.
- Muckstadt. J.A. 1973. A model for a multi-item, multi-echelon, multi-indenture inventory system. *Management Science* 20 (4): 472-481.
- Muckstadt. J.A. & Isaac. M.H. 1981. An analysis of single item inventory systems with returns. *Naval Research Logistics Quarterly* 28: 237-254.
- Murayama. T. & Shu. L.H. 2001. Treatment of Reliability for Reuse and Remanufacture. *Proceedings of the 2nd International Symposium on Environmentally Conscious Design and Inverse Manufacturing (EcoDesign'01)*: 287-292. Japan: Tokyo.
- Murayama. T., Yamamoto. S. & Oba. F. 2004. Mathematical Model of Reusability. *Proceedings of the 12th IEEE International Symposium on Electronics and the Environment (ISEE)*: 183-188. USA: Scottsdale AZ.

- Murayama, T., Yoda, M., Eguchi, T. & Oba, F. 2005. Adaptive Production Planning by Information Sharing for Reverse supply Chain. Proceedings of the 4th International Symposium on Environmentally Conscious Design and Inverse Manufacturing (EcoDesign'05): 326-331. Japan: Tokyo.
- Murayama, T., Yoda, M., Eguchi, T. & Oba, F. 2006. Production Planning and Simulation for Reverse Supply Chain. Japan Society Mechanical Engineering International Journal, Series C, 49 (2): 281-286.
- Nahmias, S., & Rivera, H. 1979. A deterministic model for a repairable item inventory system with a finite repair rate. International Journal of Production Research 17: 215-221.
- Ouyang, H. & Zhu, X. 2006. An Inventory Control System for Remanufacture with Disposal. RAM: 1-5.
- Pierskalla, W.P. & Voelker, J.A. 2006. A survey of maintenance models: The control and surveillance of deteriorating system. Naval Research Logistics Quarterly 23 (3): 353-388.
- Pyke, D.F. 2001. Periodic Review Inventory Policies for Remanufacturing. Tuck School of Business Working Paper No. 1-6.
- Pyke, D.F. 1990. Priority Repair and Dispatch Policies for Repairable-Item Logistics Systems. Naval Research Logistics 37: 1-30.
- Richter, K. 1994. An EOQ repair and waste disposal model. Proceedings of the Eight International Working Seminar on Production Economics 3: 83-91. Ingls/Innsbruck.
- Richter, K. & Dobos, I. 1999. Analysis of the EOQ repair and waste disposal model with integer setup numbers. International Journal of Production Economics 59: 463-467.
- Richter, K. 1997. Pure and mixed strategies for the EOQ repair and waste disposal problem. OR Spectrum 19: 123-129.
- Richter, K. & Sombrutzki, M. 2000. Remanufacturing planning for the reverse Wagner/Whitin models. European Journal of Operational Research 121 (2): 304-315.
- Richter, K. 1996a. The EOQ repair and waste disposal model with variable setup numbers. European Journal of Operational Research 95: 313-324.
- Richter, K. 1996b. The extended EOQ repair and waste disposal model. International Journal of Production Economics 45: 443-447.
- Richter, K. & Weber, J. 2001. The reverse Wagner/Whitin model with variable manufacturing and remanufacturing cost. International Journal of Production Economics 71: 447-456.
- Schrady, D.A. 1967. A deterministic inventory model for repairable items. Naval Research Logistics Quarterly 14 (3): 391-398.
- Sherbrooke, C.C. 1971. An Evaluator for the Number of Operationally Ready Aircraft in a Multilevel Supply System. Operations Research 19: 618- 635.
- Sherbrooke, C.C. 1968. METRIC: A Multi-echelon Technique for Recoverable Item Control. Operations Research 16: 122-141.
- Simpson V.P. 1978. Optimum solution structure for a repairable inventory problem. Operations Research 26 (2): 270-281.
- Sleptchenko, A., van der Heijden, M.C. & van Harten, A. 2002. Effects of finite repair capacity in multi-echelon, multi-indenture service part supply system. International Journal of Production Economics 79: 209-230.
- Sobczyk, M. 2001. Statystyka, Wyd. Naukowe PWN, Warszawa.
- Teunter, R.H. 2001. Economic ordering quantities for recoverable item inventory system. Naval Research Logistics 48: 484-495.
- Teunter, R.H. 2002. Economic order quantities for stochastic discounted cost inventory system with remanufacturing., International Journal of Logistics: Research and Applications 5 (2): 161-175.
- Teunter, R.H. & Flapper, S.D.P. 2003. Lot-sizing for a single-stage single-product production system with rework of perishable production defectives. OR Spectrum 25: 85-96.
- Teunter, R.H. 2004. Lot-sizing for inventory systems with product recovery. Computers & Industrial Engineering, 46: 431-441.
- Teunter, R.H. & Vlachos, D. 2002. On the necessity of a disposal option for returned items that can be remanufactured. International Journal of Production Economics 75: 257-266.
- Toktay, L.B., Wein, L.W. & Zenios, S.A. 2000. Inventory Management of Remanufacturable Products. Management Science 46 (11): 1412-1426.
- Valdez-Flores, C. & Feldman, R.M. 1989. A survey of Preventive Maintenance Models for Stochastically Deteriorating Single-Unit System. Naval Research 66: 419-446.
- Van der Laan, E.A., Dekker, R. & Salomon, M. 1996. Product remanufacturing and disposal: A numerical comparison of alternative control strategies. International Journal of Production Economics 45: 489-498.
- Van der Laan, E.A., Dekker, R., Salomon, M. & Ridder, A. 1996. An (S,Q) inventory model with remanufacturing and disposal. International Journal of Production Economics 46-47: 339-350.
- Van der Laan, E.A. & Salomon, M. 1997. Production planning and inventory control with remanufacturing and disposal. International Journal of Operational Research 102: 264-278.

-
- Van Eijl, C.A. & van Hoesel, C.P.M. 1997. On the discrete lot-sizing and scheduling problem with Wagner-Whitin costs. *Operations Research Letters* 20: 7-13.
- Vlachos, D. & Dekker, 2003. R. Return handling options and order quantities for single period products. *European Journal of Operational Research* 151: 38-52.
- Wang, H. 2002. A survey of maintenance policies of deteriorating system. *European Journal of Operational Research* 139: 469-489.
- Wang, Y., Cohen, M.A. & Zheng, Y. 2000. A Two-Echelon Repairable Inventory System with Stocking-Center-Dependent Depot Replenishment Lead Times. *Management Science* 46 (11): 1441-1453.
- Zijm, W.H. & Avsar, Z.M. 2003. Capacitated two-indenture models for repairable item systems. *International Journal of Production Economics* 81-82: 573-588.