RELIABILITY, RISK AND AVAILABILITY ANLYSIS OF A CONTAINER GANTRY CRANE

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ABSTRACT

The joint model of the system operation process and the system multi-state reliability is applied to the reliability, risk and availability evaluation of the container gantry crane. The container gantry crane is described and the mean values of the container gantry crane operation process unconditional sojourn times in particular operation states are found and applied to determining this process transient probabilities in these states. The container gantry crane different reliability structures in various its operation states are fixed and their conditional reliability functions on the basis of data coming from experts are approximately determined. Finally, after applying earlier estimated transient probabilities and system conditional reliability functions in particular operation states the unconditional reliability function, the mean values and standard deviations of the container gantry crane lifetimes in particular reliability states, risk function and the moment when the risk exceeds a critical value are found. Next the renewal and availability characteristics for the considered gantry crane are determined.

1 INTRODUCTION

Most real technical systems are very complex and it is difficult to analyze their reliability, availability and safety. Large numbers of components and subsystems and their operating complexity cause that the identification, evaluation, prediction and optimization of their reliability, availability and safety are complicated. The complexity of the systems' operation processes and their influence on changing in time the systems' structures and their components' reliability characteristics are very often met in real practice.

Taking into account the importance of the safety and operating process effectiveness of such systems it seems reasonable to expand the two-state approach to multistate approach (Aven 1985, Kolowrocki 2004, Kołowrocki, Soszyńska-Budny 2011) in their reliability analysis. The assumption that the systems are composed of multistate components with reliability states degrading in time gives the possibility for more precise analysis of their reliability and operation processes' effectiveness. This assumption allows us to distinguish a system reliability critical state (Kolowrocki 2004, Kołowrocki, Soszyńska 2010, Kołowrocki, Soszyńska-Budny 2011, Soszyńska 2010) to exceed which is either dangerous for the environment or does not assure the necessary level of its operation process effectiveness. Then, an important system reliability characteristic is the time to the moment of exceeding the system reliability critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multistate reliability function that is basic characteristics of the multi-state system.

The complexity of the systems' operation processes and these processes influence on changing in time the systems' structures and their components' reliability characteristics (Kołowrocki, Soszyńska 2010, Kołowrocki, Soszyńska-Budny 2010, Kołowrocki, Soszyńska-Budny 2011, Soszynska 2010) is often very difficult to fix and to analyse. A convenient tool for solving this problem is semi-Markov (Grabski 2002, Limnios 2001) modelling of the systems operation processes which is proposed in the paper. Using the joint model of the system multi-state reliability and the system semi-Markov operation process (Kołowrocki, Soszyńska 2010, Kołowrocki, Soszyńska-Budny 2011, Soszyńska 2010) it is possible to point out the variability of system components reliability characteristics by introducing the components' conditional multi-state reliability functions determined by the system operation states. Consequently it is possible to find the system conditional reliability function dependent on the operation states and next it is possible to find its unconditional reliability function and renewal and availability characteristics.

This way the obtained results concerned with multi-state systems in its varying in time operation states can be applied to the reliability, risk and availability evaluation of the container gantry crane.

2 CONTAINER GANTRY CRANE SYSTEM ANALYSIS

We analyse the reliability of the container gantry crane that is operating at the container terminal placed at the seashore (Kołowrocki, Soszyńska-Budny 2010, Kołowrocki, Soszyńska-Budny 2011). The considered container terminal is engaged in trans-shipment of containers. The loading of containers is carried out by using the gantry cranes called Ship-To-Shore (STS).

We consider the STS container gantry crane that is composed of 5 basic subsystems S_1, S_2, S_3, S_4 and S_5 having an essential influence on its reliability. Those subsystems are as follows:

S_1 - the power supply subsystem,

S_2 - the control and monitoring subsystem,

S_3 - the arm getting up and getting down subsystem,

S_4 - the transferring subsystem,

S_5 - the loading and unloading subsystem.

The gantry crane power supply subsystem S_1 consists of:

- a high voltage cable delivering the energy from the substation to the gantry crane $E_1^{(1)}$,
- a drum allowing the cable unreeling during the crane transferring $E_2^{(1)}$,
- an inner crane power supply cable $E_3^{(1)}$,
- a device transmitting the energy from the high voltage cable to the inner crane cable $E_4^{(1)}$,
- main and supporting voltage transformers $E_5^{(1)}$,
- a low voltage power supply cable $E_6^{(1)}$,
- relaying and protective electrical components $E_7^{(1)}$.

The gantry crane control and monitoring subsystem S_2 consists of:

- a crane software controller precisely analyzing the situation and takes suitable actions in order to assure correct work of the crane $E_1^{(2)}$,
- a measuring and diagnostic device sending signals about the crane state to the software controller $E_2^{(2)}$,
- a transmitter of signals from the controller to elements executing the set of commands $E_3^{(2)}$,
- devices carrying out the controller's orders (a permission to work, a blockade of work, etc.) $E_4^{(2)}$,
- control panels (an engine room, an operator's cabin, a crane arm cabin) $E_5^{(2)}$,
- control and steering cables' connections $E_6^{(2)}$.

The gantry crane arm getting up and getting down subsystem S_3 consists of:

- a propulsion unit (an engine, a rope drum, a transmission gear, a clutch, breaks, a rope) $E_1^{(3)}$,
- a set of rollers and multi-wheels $E_2^{(3)}$,
- a crane arm (joints, hooks fastening the arm) $E_3^{(3)}$.

The gantry crane transferring subsystem S_4 consists of:

- a driving unit (an engine, a clutch, breaks, a transmission gear, gantry crane wheels) $E_1^{(4)}$.

The gantry crane loading and unloading subsystem S_5 consists of the winch unit $E_1^{(5)}$ composed of:
- a propulsion unit (an engine, a clutch, breaks, a transmission gear, ropes),
- a winch head (which a container grab is connected to),
- a container's grab,
- a container's grab stabilizing unit

and the cart unit $E_2^{(5)}$ composed of:
- a propulsion unit (an engine, a clutch, breaks, a transmission gear, cart wheels, ropes),
- rails which cart is moving on during the operation,
- a crane cart.

The subsystems S_1, S_2, S_3, S_4, S_5 are forming a general series gantry crane reliability structure presented in Figure 1.

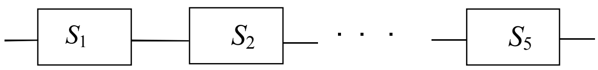

Figure 1. General scheme of gantry crane reliability structure

3 CONTAINER GANTRY CRANE OPERATION PROCESS CHARACTERISTICS PREDICTION

The container gantry crane reliability structure and the subsystems and components reliability depend on its changing in time operation states.

Taking into account expert opinions on the varying in time operation process of the considered container gantry crane we fix the number of the system operation process states $v = 6$ and we distinguish the following as its six operation states:

- an operation state z_1 – the crane standby with the power supply on and the control system off,
- an operation state z_2 – the crane prepared either to starting or finishing the work with the crane arm angle position of 90^o,
- an operation state z_3 – the crane prepared either to starting or finishing the work with the crane arm angle position of 0^o,
- an operation state z_4 – the crane transferring either to or from the loading and unloading area with the crane arm angle position of 90^o,
- an operation state z_5 – the crane transferring either to or from the loading and unloading area with the crane arm angle position of 0^o,
- an operation state z_6 – the containers' loading and unloading with the crane arm angle position of 0^o.

Moreover, we fix that there are possible the transitions between all system operation states.

To identify all parameters of the container gantry crane operation process the statistical data about this process was needed. All statistical data are collected in (Kołowrocki, Soszyńska-Budny 2010). On the basis of statistical data from (Kołowrocki, Soszyńska-Budny 2010) the following matrix

$$[p_{bl}] = \begin{bmatrix} 0 & 0.648 & 0.336 & 0.008 & 0 & 0.008 \\ 0.525 & 0 & 0.373 & 0.093 & 0 & 0.009 \\ 0.105 & 0.111 & 0 & 0 & 0.118 & 0.666 \\ 0.417 & 0.583 & 0 & 0 & 0 & 0 \\ 0.005 & 0 & 0.220 & 0 & 0 & 0.775 \\ 0.012 & 0 & 0.628 & 0 & 0.360 & 0 \end{bmatrix},$$

of the probabilities p_{bl}, $b,l = 1,2,...,6,$ of the container gantry crane operation process transitions from the operation state z_b to the operation state z_l has been fixed.
The mean values $M_{bl} = E[\theta_{bl}],\ b,l = 1,2,...,6,\ b \neq l,$ of the container gantry crane operation process conditional sojourn times at the particular operation states according to (2.12) (Kołowrocki, Soszyńska-Budny 2011) are as follows:

$$M_{12} = 456.98,\ M_{13} = 36.86,\ M_{14} = 50,\ M_{16} = 3,\ M_{21} = 7.89,\ M_{23} = 9.12,\ M_{24} = 1.55,\ M_{26} = 16,$$

$$M_{31} = 5.50,\ M_{32} = 4.34,\ M_{35} = 6.82,\ M_{36} = 7.86,\ M_{41} = 2,\ M_{42} = 2.14,\ M_{51} = 10,\ M_{53} = 2.90,$$

$$M_{56} = 24.68,\ M_{61} = 22.60.\ M_{63} = 23.12,\ M_{65} = 20.51. \tag{1}$$

After considering the results (1) and applying the formula (2.21) from (Kołowrocki, Soszyńska-Budny 2011), the unconditional mean sojourn times of the container gantry crane operation process at the particular operation states are given by:

$$M_1 = E[\theta_1] = 0.648 \cdot 456.98 + 0.336 \cdot 36.86 + 0.008 \cdot 50 + 0.008 \cdot 3 \cong 308.93,$$

$$M_2 = E[\theta_2] = 0.525 \cdot 7.89 + 0.373 \cdot 9.12 + 0.093 \cdot 1.55 + 0.009 \cdot 16 \cong 7.83,$$

$$M_3 = E[\theta_3] = 0.105 \cdot 5.50 + 0.111 \cdot 4.34 + 0.118 \cdot 6.82 + 0.666 \cdot 7.86 \cong 7.09,$$

$$M_4 = E[\theta_4] = 0.417 \cdot 2 + 0.583 \cdot 2.14 \cong 2.08,$$

$$M_5 = E[\theta_5] = 0.005 \cdot 10 + 0.220 \cdot 2.90 + 0.775 \cdot 24.68 \cong 19.82,$$

$$M_6 = E[\theta_6] = 0.012 \cdot 22.60 + 0.628 \cdot 23.12 + 0.360 \cdot 20.51 \cong 22.17.$$

Since, according to (2.23) (Kołowrocki, Soszyńska-Budny 2011), from the system of equations

$$\begin{cases} [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6] = [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6][p_{bl}]_{6x6} \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 = 1, \end{cases}$$

we get

$$\pi_1 = 0.0951,\ \pi_2 = 0.1020,\ \pi_3 = 0.3100,\ \pi_4 = 0.0102,\ \pi_5 = 0.1547,\ \pi_6 = 0.3280.$$

Then, the limit values of the transient probabilities $p_b(t)$ of the gantry crane operation process at the operation states z_b, according to (2.22) in (Kołowrocki, Soszyńska-Budny 2011), are given by

$$p_1 = 0.6874,\ p_2 = 0.0187,\ p_3 = 0.0515,\ p_4 = 0.0005,\ p_5 = 0.0717,\ p_6 = 0.1702. \quad (2)$$

4 CONTAINER GANTRY CRANE IN VARIABLE OPERATION CONDITION RELIABILITY AND RISK EVALUATION

After discussion with experts, taking into account the effectiveness of the operation of the container gantry crane, we fix that the system and its components have four reliability states 0, 1, 2, 3, i.e. $z = 3$. And consequently, at all operation states z_b, $b = 1,2,...,6$, we distinguish the following reliability states of the system and its components:

- a reliability state 3 – the gantry operation is fully effective,
- a reliability state 2 – the gantry operation is less effective because of ageing,
- a reliability state 1 – the gantry operation is less effective because of ageing and more dangerous,
- a reliability state 0 – the gantry is destroyed.

We assume that there are possible the transitions between the components reliability states only from better to worse ones and we fix that the system and components critical reliability state is $r = 2$.
Consequently, we assume that the gantry crane subsystems S_υ, $\upsilon = 1,2,...,5$ are composed of four-state components, i.e. z =3, with the multi-state reliability functions

$$[R_i^{(\upsilon)}(t,\cdot)]^{(b)} = [1,\ [R_i^{(\upsilon)}(t,1)]^{(b)}, [R_i^{(\upsilon)}(t,2)]^{(b)}, [R_i^{(\upsilon)}(t,3)]^{(b)}\ ],\ b = 1,2,...,6,$$

with exponential co-ordinates $[R_i^{(\upsilon)}(t,1)]^{(b)}$, $[R_i^{(\upsilon)}(t,2)]^{(b)}$, $[R_i^{(\upsilon)}(t,3)]^{(b)}$ different in various operation states z_b, $b = 1,2,...,6$.
In (Kołowrocki, Soszyńska-Budny 2010), on the basis of expert opinions, the reliability functions of the gantry crane components in different operation states are approximately determined. We will use them in our further system reliability analysis and evaluation.

At the system operation state z_1, the container gantry crane is composed of the subsystem S_1 which is a series system composed of $n = 7$ components $E_i^{(1)}$, $i = 1,2,..,7$ (subsystems) with the structure showed in Figure 2.

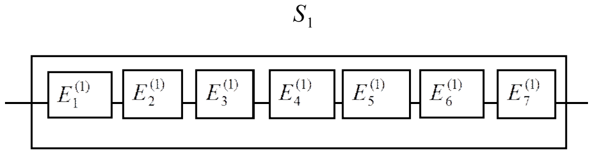

Figure 2. The scheme of the container gantry crane at operation state z_1

Thus, at the system operation state z_1, the container gantry crane is identical with subsystem S_1, that is a four-state series system with its structure shape parameter $n = 7$ and according to (1.22)-(1.23) (Kołowrocki, Soszyńska-Budny 2011), its four-state reliability function is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(1)} = [1, [\boldsymbol{R}(t,1)]^{(1)}, [\boldsymbol{R}(t,2)]^{(1)}, [\boldsymbol{R}(t,3)]^{(1)}], \quad t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(1)} = \exp[-0.020t]\exp[-0.040t]\exp[-0.040t]\exp[-0.020t]\exp[-0.020t]$$

$$\exp[-0.018t]\exp[-0.033t] = \exp[-0.191t], \tag{3}$$

$$[\boldsymbol{R}(t,2)]^{(1)} = \exp[-0.033t]\exp[-0.050t]\exp[-0.050t]\exp[-0.033t]\exp[-0.030t]$$

$$\exp[-0.028t]\exp[-0.040t] = \exp[-0.264t], \tag{4}$$

$$[\boldsymbol{R}(t,3)]^{(1)} = \exp[-0.050t]\exp[-0.066t]\exp[-0.066t]\exp[-0.050t]\exp[-0.040t]$$

$$\exp[-0.040t]\exp[-0.050t] = \exp[-0.362t]. \tag{5}$$

The expected values of the container gantry crane conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_1, calculated from the results given by (3)-(5), according to (3.8) (Kołowrocki, Soszyńska-Budny 2011), respectively are:

$$\mu_1(1) \cong 5.24, \ \mu_1(2) \cong 3.79, \ \mu_1(3) \cong 2.76 \text{ years.}$$
(6)

At the system operation states z_2 and z_3, the container gantry crane is composed of the subsystems S_1, S_2 and S_3 forming a series structure shown in Figure 3. The subsystem S_1 is a series system composed of $n = 7$ components $E_i^{(1)}$, $i = 1,2,..,7$, the subsystem S_2 is a series system composed of $n = 6$ components $E_i^{(2)}$, $i = 1,2,..,6$, and the subsystem S_3 is a series system composed of $n = 3$ components $E_i^{(3)}$, $i = 1,2,3$.

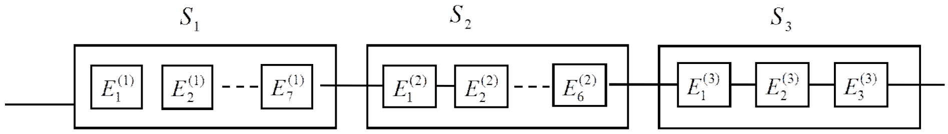

Figure 3. The scheme of the container gantry crane at operation states z_2 and z_3

Thus, at the system operation state z_2, the container gantry crane is composed of the subsystems S_1, S_2 and S_3 forming a series structure.
At this operation state, the subsystem S_1 is a four-state series system with its structure shape parameter $n = 7$ and according to (1.22)-(1.23) (Kołowrocki, Soszyńska-Budny 2011),
its four-state reliability function is given by the vector

$$[\boldsymbol{R}^{(1)}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}^{(1)}(t,1)]^{(2)}, [\boldsymbol{R}^{(1)}(t,2)]^{(2)}, [\boldsymbol{R}^{(1)}(t,3)]^{(2)}], \ t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}^{(1)}(t,1)]^{(2)} = \exp[-0.020t]\,\exp[-0.040t]\,\exp[-0.040t]\,\exp[-0.020t]$$

$$\exp[-0.022t]\,\exp[-0.018t]\,\exp[-0.033t] = \exp[-0.193t], \tag{7}$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(2)} = \exp[-0.033t]\,\exp[-0.050t]\,\exp[-0.050t]\,\exp[-0.033t]$$

$$\exp[-0.027t]\,\exp[-0.028t]\,\exp[-0.040t] = \exp[-0.261t], \tag{8}$$

$$[\boldsymbol{R}^{(1)}(t,3)]^{(2)} = \exp[-0.050t]\,\exp[-0.066t]\,\exp[-0.066t]\,\exp[-0.050t]$$

$$\exp[-0.048t]\,\exp[-0.040t]\,\exp[-0.050t] = \exp[-0.370t]. \tag{9}$$

The subsystem S_2 at the operation state z_2, is a four-state series system with its structure shape parameter $n = 6$ and according to (1.22)-(1.23) (Kołowrocki, Soszyńska-Budny 2011), its four-state reliability function is given by the vector

$$[\boldsymbol{R}^{(2)}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}^{(2)}(t,1)]^{(2)}, [\boldsymbol{R}^{(2)}(t,2)]^{(2)}, [\boldsymbol{R}^{(2)}(t,3)]^{(2)}],\ t \ge 0,$$

with the coordinates

$$[\boldsymbol{R}^{(2)}(t,1)]^{(2)} = \exp[-0.053t]\exp[-0.048t]\exp[-0.048t]\,\exp[-0.048t]$$

$$\exp[-0.020t]\exp[-0.018t] = \exp[-0.235t], \tag{10}$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(2)} = \exp[-0.059t]\,\exp[-0.053t]\,\exp[-0.053t]\,\exp[-0.053t]$$

$$\exp[-0.025t]\,\exp[-0.029t] = \exp[-0.272t], \tag{11}$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(2)} = \exp[-0.066t]\,\exp[-0.059t]\,\exp[-0.059t]\,\exp[-0.059t]$$

$$\exp[-0.033t]\exp[-0.040t] = \exp[-0.316t]. \tag{12}$$

The subsystem S_3 at the operation state z_2, is a four-state series system with its structure shape parameter $n = 3$ and according to (1.22)-(1.23) (Kołowrocki, Soszyńska-Budny 2011), its four-state reliability function is given by the vector

$$[\boldsymbol{R}^{(3)}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}^{(3)}(t,1)]^{(2)}, [\boldsymbol{R}^{(3)}(t,2)]^{(2)}, [\boldsymbol{R}^{(3)}(t,3)]^{(2)}],\ t \ge 0,$$

with the coordinates

$$[\boldsymbol{R}^{(3)}(t,1)]^{(2)} = \exp[-0.025t]\,\exp[-0.033t]\,\exp[-0.033t] = \exp[-0.091t], \tag{13}$$

$$[\boldsymbol{R}^{(3)}(t,2)]^{(2)} = \exp[-0.040t]\,\exp[-0.040t]\,\exp[-0.040t] = \exp[-0.120t], \tag{14}$$

$$[\boldsymbol{R}^{(3)}(t,3)]^{(2)} = \exp[-0.066t]\exp[-0.066t]\exp[-0.066t] = \exp[-0.198t]. \quad (15)$$

Considering that the container gantry crane at the operation state z_2 is a four-state series system composed of subsystems S_1, S_2 and S_3, after applying (1.22)–(1.23) (Kołowrocki, Soszyńska-Budny 2011), its conditional four-state reliability function is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}(t,1)]^{(2)}, [\boldsymbol{R}(t,2)]^{(2)}, [\boldsymbol{R}(t,3)]^{(2)}], \ t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(2)} = \exp[-0.193t]\exp[-0.235t]\exp[-0.091t] = \exp[-0.519t], \quad (16)$$

$$[\boldsymbol{R}(t,2)]^{(2)} = \exp[-0.261t]\exp[-0.272t]\exp[-0.120t] = \exp[-0.653t], \quad (17)$$

$$[\boldsymbol{R}(t,3)]^{(2)} = \exp[-0.370t]\exp[-0.316t]\exp[-0.198t] = \exp[-0.884t]. \quad (18)$$

The expected values of the container gantry crane conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_2, calculated from the results given by (16)-(18), according to (3.8) (Kołowrocki, Soszyńska-Budny 2011), respectively are:

$$\mu_2(1) \cong 1.93, \ \mu_2(2) \cong 1.53, \mu_2(3) \cong 1.13 \text{ year}. \quad (19)$$

After proceeding in the analogous way in the system reliability analysis and evaluation at the remaining operation states z_3, z_4, z_5 and z_6, we may determine the system conditional reliability function that are presented below.

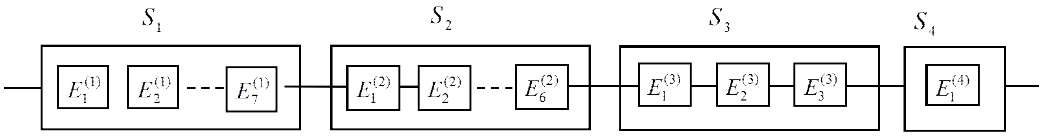

Figure 4. The scheme of the container gantry crane at operation states z_4 and z_5

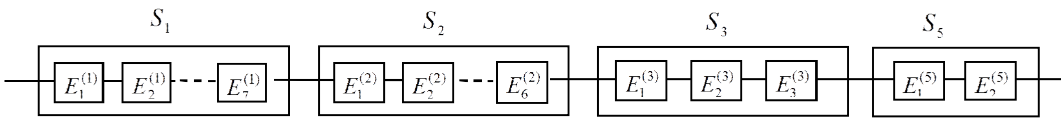

Figure 5. The scheme of the container gantry crane at operation state z_6

At the operation state z_3, the container gantry crane conditional reliability function of the system is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}(t,1)]^{(3)}, [\boldsymbol{R}(t,2)]^{(3)}, [\boldsymbol{R}(t,3)]^{(3)}], \ t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(3)} = \exp[-0.196t]\exp[-0.235t]\exp[-0.091t] = \exp[-0.522t], \tag{20}$$

$$[\boldsymbol{R}(t,2)]^{(3)} = \exp[-0.264t]\exp[-0.272t]\exp[-0.120t] = \exp[-0.656t], \tag{21}$$

$$[\boldsymbol{R}(t,3)]^{(3)} = \exp[-0.375t]\exp[-0.316t]\exp[-0.198t] = \exp[-0.889t]. \tag{22}$$

The expected values of the container gantry crane conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_3, calculated from the results given by (20)-(22), according to (3.8) (Kołowrocki, Soszyńska-Budny 2011), respectively are:

$$\mu_3(1) \cong 1.91,\ \mu_3(2) \cong 1.52,\ \mu_3(3) \cong 1.12 \text{ year}. \tag{23}$$

At the system operation states z_4 and z_5, the container gantry crane is composed of the subsystems S_1, S_2, S_3 and S_4 forming a series structure shown in Figure 4. The subsystem S_1 is a series system composed of $n = 7$ components $E_i^{(1)}$, $i = 1,2,..,7$, the subsystem S_2 is a series system composed of $n = 6$ components $E_i^{(2)}$, $i = 1,2,..,6$, the subsystem S_3 is a series system composed of $n = 3$ components $E_i^{(3)}$, $i = 1,2,3$, and the subsystem S_4 consists of a component $E_1^{(4)}$.
Thus, at the operation state z_4, the container gantry crane conditional reliability function of the system is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(4)} = [1, [\boldsymbol{R}(t,1)]^{(4)}, [\boldsymbol{R}(t,2)]^{(4)}, [\boldsymbol{R}(t,3)]^{(4)}],\ t \ge 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(4)} = \exp[-0.216t]\exp[-0.241t]\exp[-0.061t]\exp[-0.029t] = \exp[-0.547t], \tag{24}$$

$$[\boldsymbol{R}(t,2)]^{(4)} = \exp[-0.289t]\exp[-0.278t]\exp[-0.091t]\exp[-0.04t] = \exp[-0.698t], \tag{25}$$

$$[\boldsymbol{R}(t,3)]^{(4)} = \exp[-0.428t]\exp[-0.328t]\exp[-0.133t]\exp[-0.066t] = \exp[-0.955t]. \tag{26}$$

The expected values of the container gantry crane conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_4, calculated from the results given by (24)-(26), according to (3.8) (Kołowrocki, Soszyńska-Budny 2011), respectively are:

$$\mu_4(1) \cong 1.83,\ \mu_4(2) \cong 1.43,\ \mu_4(3) \cong 1.05 \text{ year}. \tag{27}$$

At the operation state z_5, the container gantry crane conditional reliability function of the system is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(5)} = [1, [\boldsymbol{R}(t,1)]^{(5)}, [\boldsymbol{R}(t,2)]^{(5)}, [\boldsymbol{R}(t,3)]^{(5)}],\ t \ge 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(5)} = \exp[-0.216t]\exp[-0.241t]\exp[-0.061t]\exp[-0.025t] = \exp[-0.543t], \quad (28)$$

$$[\boldsymbol{R}(t,2)]^{(5)} = \exp[-0.289t]\exp[-0.278t]\exp[-0.091t]\exp[-0.029t] = \exp[-0.687t], \quad (29)$$

$$[\boldsymbol{R}(t,3)]^{(5)} = \exp[-0.428t]\exp[-0.328t]\exp[-0.133t]\exp[-0.050t] = \exp[-0.939t]. \quad (30)$$

The expected values of the container gantry crane conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_5, calculated from the results given by (28)-(30), according to (3.8) (Kołowrocki, Soszyńska-Budny 2011), respectively are:

$$\mu_5(1) \cong 1.84,\ \mu_5(2) \cong 1.46,\ \mu_5(3) \cong 1.06 \text{ year.}$$
(31)

At the system operation state z_6, the container gantry crane is composed of the subsystems S_1, S_2, S_3 and S_5 forming a series structure shown in Figure 5. The subsystem S_1 is a series system composed of $n = 7$ components $E_i^{(1)}$, $i = 1,2,..,7$, the subsystem S_2 is a series system composed of $n = 6$ components $E_i^{(2)}$, $i = 1,2,..,6$, the subsystem S_3 is a series system composed of $n = 3$ components $E_i^{(3)}$, $i = 1,2,3$ and the subsystem S_5 is a series system composed of $n = 2$ components $E_i^{(5)}$, $i = 1,2$.

Thus, at the operation state z_6, the container gantry crane conditional reliability function of the system is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(6)} = [1, [\boldsymbol{R}(t,1)]^{(6)}, [\boldsymbol{R}(t,2)]^{(6)}, [\boldsymbol{R}(t,3)]^{(6)}],\ t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(6)} = \exp[-0.201t]\exp[-0.250t]\exp[-0.087t]\exp[-0.080t] = \exp[-0.618t], \quad (32)$$

$$[\boldsymbol{R}(t,2)]^{(6)} = \exp[-0.273t]\exp[-0.29t]\exp[-0.12t]\exp[-0.1t] = \exp[-0.783t], \quad (33)$$

$$[\boldsymbol{R}(t,3)]^{(6)} = \exp[-0.396t]\exp[-0.337t]\exp[-0.16t]\exp[-0.132t] = \exp[-1.025t]. \quad (34)$$

The expected values of the container gantry crane conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_6, calculated from the results (32)-(34), according to (3.8) (Kołowrocki, Soszyńska-Budny 2011), respectively are:

$$\mu_6(1) \cong 1.62,\ \mu_6(2) \cong 1.28,\ \mu_6(3) \cong 0.98 \text{ year.}$$
(35)

In the case when the operation time is large enough the unconditional four-state reliability function of the container gantry crane is given by the vector

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), \boldsymbol{R}(t,2), \boldsymbol{R}(t,3)],\ t \geq 0, \quad (36)$$

where according to (3.5)-(3.6) (Kołowrocki, Soszyńska-Budny 2011) and considering (2), the vector coordinates are given respectively by

$$R(t,1) = 0.6874\cdot[R(t,1)]^{(1)} + 0.0187\cdot[R(t,1)]^{(2)} + 0.0515\cdot[R(t,1)]^{(3)} + 0.0005\cdot[R(t,1)]^{(4)}$$

$$+ 0.0717\cdot[R(t,1)]^{(5)} + 0.1702\cdot[R(t,1)]^{(6)} \text{ for } t \geq 0,$$

(37)

$$R(t,2) = 0.6874\cdot[R(t,2)]^{(1)} + 0.0187\cdot[R(t,2)]^{(2)} + 0.0515\cdot[R(t,2)]^{(3)} + 0.0005\cdot[R(t,2)]^{(4)}$$

$$+ 0.0717\cdot[R(t,2)]^{(5)} + 0.1702\cdot[R(t,2)]^{(6)} \text{ for } t \geq 0, \tag{38}$$

$$R(t,3) = 0.6874\cdot[R(t,3)]^{(1)} + 0.0187\cdot[R(t,3)]^{(2)} + 0.0515\cdot[R(t,3)]^{(3)} + 0.0005\cdot[R(t,3)]^{(4)}$$

$$+ 0.0717\cdot[R(t,3)]^{(5)} + 0.1702\cdot[R(t,3)]^{(6)} \text{ for } t \geq 0, \tag{39}$$

and the coordinates $[R(t,u)]^{(b)}$, $b = 1,2,...,6$, $u = 1,2,3$, are given by (3)-(5), (16)-(18), (20)-(22), (24)-(26), (28)-(30), (32)-(34). The graphs of the coordinates of the container gantry crane reliability function are presented in Figure 6.

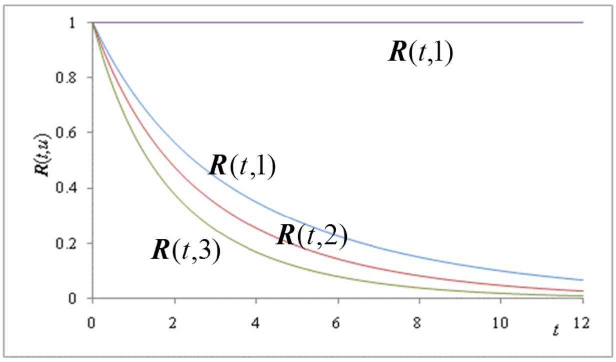

Figure 6. The graph of the container gantry crane reliability function $R(t,\cdot)$ coordinates

The expected values and standard deviations of the container gantry crane unconditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ calculated from the results given by (37)-(39), according to (3.7)-(3.9) (Kołowrocki, Soszyńska-Budny 2011) and considering (2), (6), (19), (23), (27), (31), (35), respectively are:

$$\mu(1) = 0.6874\cdot 5.24 + 0.0187\cdot 1.93 + 0.0515\cdot 1.91 + 0.0005\cdot 1.83 + 0.0717\cdot 1.84$$

$$+ 0.1702\cdot 1.62 \cong 4.14 \text{ years}, \tag{40}$$

$$\sigma(1) \cong 4.71 \text{ years},$$

$$\mu(2) = 0.6874\cdot 3.79 + 0.0187\cdot 1.53 + 0.0515\cdot 1.52 + 0.0005\cdot 1.43 + 0.0717\cdot 1.46$$

$$+ 0.1702 \cdot 1.28 \cong 3.04 \text{ years,} \tag{41}$$

$$\sigma(2) \cong 3.43 \text{ years,} \tag{42}$$

$$\mu(3) = 0.6874 \cdot 2.76 + 0.0187 \cdot 1.13 + 0.0515 \cdot 1.12 + 0.0005 \cdot 1.05 + 0.0717 \cdot 1.06$$

$$+ 0.1702 \cdot 0.98 \cong 2.22 \text{ years,} \tag{43}$$

$$\sigma(3) \cong 2.50 \text{ years,}$$

Further, considering (3.10) from (Kołowrocki, Soszyńska-Budny 2011) and (40), (41), (43), the mean values of the unconditional lifetimes in the particular reliability states 1, 2, 3 respectively are:

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 1.10, \ \bar{\mu}(2) = \mu(2) - \mu(3) = 0.82, \ \bar{\mu}(3) = \mu(3) = 2.22 \text{ years.} \tag{44}$$

Since the critical reliability state is $r = 2$, then the system risk function, according to (3.11) (Kołowrocki, Soszyńska-Budny 2011), is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t, 2), \tag{45}$$

where $\boldsymbol{R}(t,2)$ is given by (38).

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (3.12) (Kołowrocki, Soszyńska-Budny 2011), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.126 \text{ year.}$$

(46)

The graph of the risk function $\boldsymbol{r}(t)$ of the container gantry crane operating at the variable conditions is given in Figure 7.

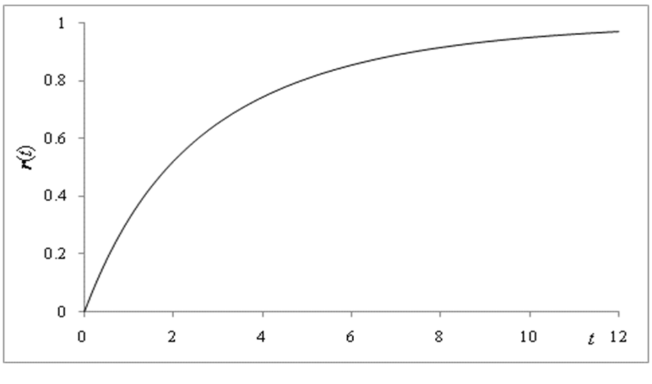

Figure 7. The graph of the container gantry crane risk function $\boldsymbol{r}(t)$

5 CONTAINER GANTRY CRANE AVAILABILITY PREDICTION

Using the results of the container gantry crane reliability prediction given by (41)-(42) and the results of the classical renew theory presented in (Kołowrocki, Soszyńska-Budny 2011), we may predict the renewal and availability characteristics of this system in the case when it is repairable and its time of renovation is either ignored or non-ignored.

First, assuming that the container gantry crane is repaired after the exceeding its reliability critical state r = 2 and that the time of the system renovation is ignored we obtain the following results:

a) the time $S_N(2)$ until the *Nth* exceeding by the system the reliability critical state r = 2, for sufficiently large *N*, has approximately normal distribution $N(3.04N, 3.43\sqrt{N})$, i.e.,

$$F^{(N)}(t,2) = P(S_N(2) < t) \cong F_{N(0,1)}(\frac{t - 3.04N}{3.43\sqrt{N}}), \; t \in (-\infty, \infty);$$

b) the expected value and the variance of the time $S_N(2)$ until the *Nth* exceeding by the system the reliability critical state r = 2 are respectively given by

$$E[S_N(2)] \cong 3.04N, \; D[S_N(2)] \cong 11.76N;$$

c) the number $N(t,2)$ of exceeding by the system the reliability critical state r = 2 up to the moment $t, t \geq 0,$ for sufficiently large t, approximately has the distribution of the form

$$P(N(t,2) = N) \cong F_{N(0,1)}(\frac{3.04(N+1) - t}{1.967\sqrt{t}}) - F_{N(0,1)}(\frac{3.04N - t}{1.967\sqrt{t}}), \; N = 0,1,...;$$

d) the expected value and the variance of the number $N(t,2)$ of exceeding by the system the reliability critical state r = 2 up to the moment $t, t \geq 0,$ for sufficiently large t, approximately are respectively given by

$$H(t,2) = 0.3289t, \; D(t,2) = 0.419t.$$

Further, assuming that the container gantry crane is repaired after the exceeding its reliability critical state r = 2 and that the time of the system renovation is not ignored and it has the mean value $\mu_0(2) = 0.0027$ and the standard deviation $\sigma_0(2) = 0.0014$, we obtain the following results:

a) the time $\overline{S}_N(2)$ until the *Nth* exceeding by the system the reliability critical state r = 2, for sufficiently large *N*, has approximately normal distribution $N(3.04N + 0.0027(N-1), \sqrt{11.76N - 0.0000019(N-1)})$, i.e.,

$$\overline{F}^{(N)}(t,2) = P(\overline{S}_N(2) < t) \cong F_{N(0,1)}(\frac{t - 3.0427N + 0.0027}{\sqrt{11.760002N - 0.0000019}}), \; t \in (-\infty, \infty);$$

b) the expected value and the variance of the time $\overline{S}_N(2)$ until the *Nth* exceeding by the system the reliability critical state r = 2, for sufficiently large *N*, are respectively given by

$$E[\overline{S}_N(2)] \cong 3.04N + 0.0027(N-1), \; D[\overline{S}_N(2)] \cong 11.76N + 0.0000019(N-1);$$

c) the number $\overline{N}(t,2)$ of exceeding by the system the reliability critical state r = 2 up to the moment $t, t \geq 0,$ for sufficiently large t, has approximately distribution of the form

$$P(\overline{N}(t,2)=N)\cong F_{N(0,1)}(\frac{3.0427(N+1)-t-0.0027}{1.966\sqrt{t+0.0027}})$$

$$-F_{N(0,1)}(\frac{3.0427N-t-0.0027}{1.966\sqrt{t+0.005}}),\ \ N=0,1,...;$$

d) the expected value and the variance of the number $\overline{N}(t,2)$ of exceeding by the system the reliability critical state *r* = 2 up to the moment $t, t\geq 0,$ for sufficiently large *t*, are respectively given by

$$\overline{H}(t,2)\cong 0.329(t+0.0027)\,,\ \overline{D}(t,2)\cong 0.417(t+0.0027);$$

e) the time $\overline{\overline{S}}_N(2)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution $N(3.0427N, 3.429\sqrt{N})$, i.e.,

$$\overline{\overline{F}}^{(N)}(t,2)=P(\overline{\overline{S}}_N(2)<t)\cong F_{N(0,1)}(\frac{t-3.0427N)}{3.429\sqrt{N}}),\ \ t\in(-\infty,\infty);$$

f) the expected value and the variance of the time $\overline{\overline{S}}_N(2)$ until the *Nth* system's renovation, for sufficiently large *N*, are respectively given by

$$E[\overline{\overline{S}}_N(2)]\cong 3.0427N\,,\ D[\overline{\overline{S}}_N(2)]\cong 11.76002N\,;$$

g) the number $\overline{\overline{N}}(t,2)$ of the system's renovations up to the moment $t, t\geq 0,$ for sufficiently large *t*, has approximately distribution of the form

$$P(\overline{\overline{N}}(t,2)=N)\cong F_{N(0,1)}(\frac{3.0427(N+1)-t}{1.966\sqrt{t}})-F_{N(0,1)}(\frac{3.0427N-t}{1.966\sqrt{t}}),\ N=0,1,...;$$

h) the expected value and the variance of the number $\overline{\overline{N}}(t,2)$ of system's renovations up to the moment $t, t\geq 0,$ for sufficiently large *t*, are respectively given by

$$\overline{\overline{H}}(t,2)\cong 0.3286t,\ \ \overline{\overline{D}}(t,2)\cong 0.417t;$$

i) the steady availability coefficient of the system at the moment $t, t\geq 0,$ for sufficiently large *t*, is given by

$$A(t,2)\cong 0.9989,\ \ t\geq 0;$$

j) the steady availability coefficient of the system in the time interval $<t, t+\tau), \tau>0,$ for sufficiently large *t*, is given by

$$A(t,\tau,2) \cong 0.329\int_{\tau}^{\infty} \boldsymbol{R}(t,2)dt, \quad t \geq 0, \quad \tau > 0,$$

where $\boldsymbol{R}(t,2)$ is given by (7.97).

6 CONCLUSION

In the paper the multi-state approach to the analysis and evaluation of systems' reliability and risk has been practically applied. The container gantry crane has been considered at varying in time operation conditions. The system reliability structure and its components reliability functions were changing at variable operation conditions. The paper proposed an approach to the solution of practically very important problem of linking the systems' reliability and their operation processes. To involve the interactions between the systems' operation processes and their varying in time reliability structures a semi-markov model of the systems' operation processes and the multi-state system reliability functions were applied. This approach gives practically important in everyday usage tool for reliability evaluation of the systems with changing reliability structures and components reliability characteristics during their operation processes what exemplary was illustrated in its application to the gantry crane.

The characteristics of the gantry crane operation process are of high quality because of the very good statistical data necessary for their estimation. Unfortunately, the reliability characteristics of the gantry crane components are evaluated on non sufficiently exact data coming from experts and concerned with the mean values of the components lifetimes only that because of the complete lack of statistical data about their failures are strongly lowered. Also, the system and its components reliability states are defined on a high level of generality and should be described more precisely. All these inaccuracies causes that the evaluation of the gantry crane reliability, risk and availability characteristics should be consider as an illustration of the proposed approach application.

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