THE AMENDMENT TO AMDAHL ` S THE LAW

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INTRODUCTION

Law Amdahl, sometimes also law Amdahl-Uer, shows restriction of growth of productivity of the computing system with increase of quantity of calculators. It also is applicable to collective of people solving a problem, admitting parallels decisions between its members. John Amdahl has formulated the law in 1967, having found out idle time in essence, but insuperable restriction under the maintenance on growth of productivity at parallels calculations: «In a case when the problem is divided on some parts, total time of its performance for parallel system can not be less time of performance of the longest fragment ». According to this law, acceleration of performance of the program for the account parallels its instructions on set of calculators is limited to time necessary for performance of its consecutive instructions.

If to assume, that it is necessary to solve some computing problem with computing algorithm which α share from total amount of calculations can be received only by consecutive calculations, and the share $1-\alpha$ can parallels be ideal (that is time of calculation will be in inverse proportion to number of the involved calculators p) then acceleration on the computing system from p processors, in comparison with the uniprocessor decision will not exceed size

$$S_p = \frac{1}{\alpha + \frac{1-\alpha}{p}} \ . \tag{1}$$

Law Amdahl shows, that the gain of efficiency of calculations depends on algorithm of a problem and is limited from above for any problem with $\alpha \neq 0$. Not for any problem escalating number of processors in the computing system is meaningful. Moreover, if to take into account time necessary for data transmission between processors of the computing system dependence of time of calculations on number of processors will have a maximum. It imposes restriction on we scaleing the computing system that means, that from the certain moment addition of new processors in system will increase time of the decision of a problem.

As analogue of law Amdahl law Gustavson-Barsis serves. It agrees to it an estimation of maximum achievable acceleration of performance of the parallel program depending on quantity of simultaneously carried out streams of calculations (processors) and shares of consecutive calculations of the program it is defined by the formula:

$$S_p = \alpha + (1-\alpha)p = p + (1-p)\alpha \ . \tag{2}$$

The given estimation of acceleration name acceleration of scaling (scaled speedup) as this characteristic shows as far as parallel calculations can be effectively organized at increase of complexity of decided(solved) problems.

The proof (2) uses the attitude $S_p = \frac{T_1}{T_p}$, in which T_1, T_p – time of the decision of a problem for one and p processors. The size of a share of consecutive calculations $\alpha = \frac{\tau(n)}{\tau(n) + \pi(n)/p}$, where

$\tau(n)$ time of a consecutive part of the program, and $\pi(n)$ time of a part of the program which can be распараллелена is entered. Then $S_p = \frac{T_1}{T_p} = \frac{(\tau(n)+\pi(n)/p)(\alpha+(1-\alpha)p}{\tau(n)+\pi(n)/p}$, the formula (2) whence follows.

It is necessary to notice, that formulas (1) and (2) are not equivalent. The values calculated on them coincide only at $\alpha = 0$ and $\alpha = 1$. In a range of values $0 \langle \alpha \langle 1$ formula (1) in comparison with the formula (2) gives higher result of acceleration.

The purpose of present article is corrective action in law Amdahl represented by the formula (1).

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We shall take into account the remark that « if to take into account time necessary for data transmission between processors of the computing system dependence of time of calculations on number of processors will have a maximum. It imposes restriction on we scaleing the computing system that means, that from the certain moment addition of new processors in system will increase time of the decision of a problem ».

Let's make the following assumption: the share of time of the parallel decision of a problem on p processors will develop of two components. The first component corresponds to valid or real time of the parallel decision of a problem to each of p processors. She is less, than the specified size in law Amdahl $\frac{1-\alpha}{p}$ on size $\Delta(\alpha, p)\frac{1-\alpha}{p}$. This, second component, defines total expected expenses of time for definition of an opportunity paralleling algorithm of a problem and the organization paralleling with transfer of the necessary information to each processor of system for the subsequent performance by all processors of their independent parts of full algorithm of a problem. It is obvious, that with increase of quantity of processors in system the share of an expense of time for performance of parallel work of processors should be increased. It is finally possible to write down the following expression for acceleration of the computing system:

$$S_p = \frac{1}{\alpha + (1-\alpha)(\frac{1}{p} + kp^n)} \quad . \qquad (3)$$

In the formula (3) k, n uncertain constants, which sizes depend on a kind of the program which is necessary for realizing in the computing system. Definition of values of these factors in given article is not considered it is considered. It represents a separate independent problem. The decision of this problem, in our opinion, should to be based on experimental data or analytical researches with subsequent use of necessary model numerical estimating.

The optimum number of processors at the found values of sizes k, n is defined under the formula $p_0 = 1 + trunc(\sqrt[n+1]{\frac{1}{kn}})$, where $trunc(x)$ the greatest whole part of number x.

EXAMPLE OF CALCULATION OF ACCELERATION

Let at research of some program it is established, that $\alpha = 0{,}2;\ k = 0{,}01;\ n = 3$. The result of calculation of size of acceleration is shown in figure 1.

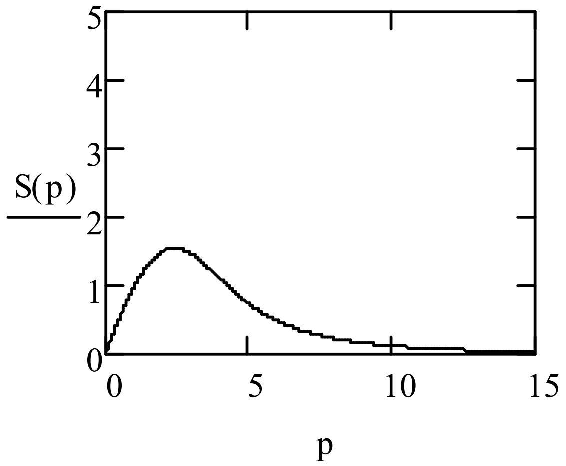

Fig. 1.

From him follows, that the maximal value $S_p = 1{,}465$. It is achieved at the number of processors equal $p_0 = 3$. Thus $\frac{1}{p_0} = 0{,}333;\ kp_0{}^n = 0{,}272$ and shares of expenses of time for actually parallel calculations and their organization will make accordingly $(1-\alpha)/p_0 = 0{,}267;\ (1-\alpha)kp_0^n = 0{,}216$.

CONCLUSION

Law Amdahl is known, allowing to establish the maximal size of acceleration of calculations of the system consisting of given quantity of processors, at the certain parity of a consecutive and parallel part of calculations of the program.

The given law does not allow to take into account system expenses of time for decision making and manufacture of parallel calculations in system.

The amendment to law Amdahl is offered, allowing in a quantitative kind to take into account the specified expenses of time. At presence of the given amendment acceleration of calculations of system reaches the maximal value at some optimum number of processors. The further increase of their quantity does not result in increase of acceleration of calculations.

Use of law Amdahl with the entered amendment allows to define optimum number of processors of the computing system for the organization in it of parallel calculations. Results of article can be used and in other systems with the parallel organization of work of their parts.

REFERENCES

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