SENSITIVITY ANALYSIS OF OPTIMAL REDUNDANCY SOLUTIONS

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Solving practical optimal redundancy problems, one can ponder: what is the sense of optimizing if input data are taken “from the ceiling”? Indeed, statistical data are so unreliable (especially in reliability problems) that such doubts have a very good ground.

Not found any sources after searching the answer for this question, the author decided to make some investigation of optimal solutions sensitivityunder influence of data scattering.

A simple series system of six units has been considered (see Figure 1). For reliability increasing, one uses a loaded redundancy, i.e. if a main unit k has xk redundant units, its reliability is found from

$$P_k(x_k) = 1-(1-p_k)^{x_k+1}$$

where pk is a probability of failure free operation (PFFO) of a single unit *k*. And the total cost of x_k redundant units is equal to $c_k \cdot x_k$, where c_k is the cost of a single unit of type *k*.

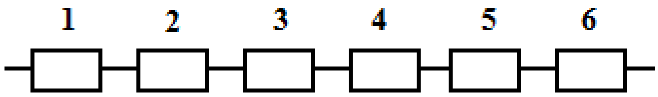


Figure 1. Series system underwent analysis

Units’ parameters are presented in Table 1.

Table 1. Input data

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
p_k	0.8	0.8	0.8	0.9	0.9	0.9
c_k	5	5	5	1	1	1

Assumed that units are mutually independent, i.e. system’s reliability is defined as

$$P_{System}(x_k, 1\le k\le 6) = \prod_{1\le k\le 6} P_k(x_k)$$

And the total system’s redundant units cost is:

$$C_{System}(x_k, 1\le k\le 6) = \sum_{1\le k\le 6} c_k x_k$$

Below presented solutions of both problems of optimal redundancy: direct:

$$\min_{1\le x_k<\infty}\left\{C(x_k, 1\le k\le 6) \mid P(x_k, 1\le k\le 6) \ge P^*\right\}$$

and inverse:

$$\max_{1 \le x_k < \infty} \left\{ P(x_k, 1 \le k \le 6) \mid C(x_k, 1 \le k \le 6) \le C^* \right\}$$

For finding the optimal solutions, the Steepest Descent Method was applied. For this "base" system the solutions for several sets of parameters are presented for Direct Problem in Table 2 and for Inverse Problem in Table 3. (Numbers are given with high accuracy only for demonstration purposes; in practice, one has to use only significant positions after a row of nines.)

Table 2. Solution for Direct problem

P*	x_1	x_2	x_3	x_4	x_5	x_6	Achieved P	System C
0.95	3	3	3	3	2	2	0.9559520	52
0.99	4	4	3	3	3	3	0.991187	69
0.995	5	4	4	4	3	3	0.995229	75
0.999	6	5	5	4	4	4	0.999218	93

Table 3. Solution for Inverse problem

C^*	x_1	x_2	x_3	x_4	x_5	x_6	Achieved C	System P
50	3	3	2	2	2	2	46	0.931676
75	4	4	3	3	3	3	75	0.995229
100	5	4	4	4	3	3	99.5	0.999602

The questions of interest are: how optimal solution will change if input data are changed? Two types of experiments have been performed: in the first series of experiments, different unit's costs with fixed probabilities were considered (see Figure 2) and in another one different unit's probabilities with fixed costs were considered (see Figure 3).

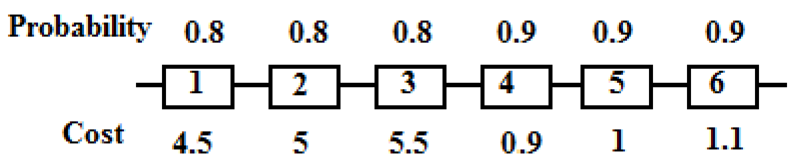


Figure 2. Input data for the first series of experiments

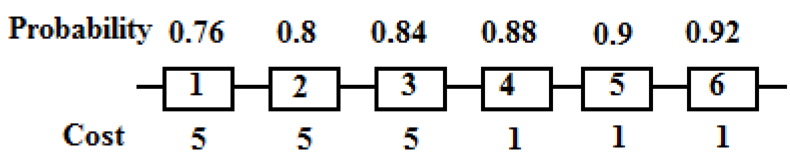


Figure 3. Input data for the second series of experiments

The results of calculations are as follows:

Table 4. Values of Probabilities of Failure-free operations

	0.999	0.995	0.99	0.95
Initial	0.999218	0.99566	0.9922	0.955952
Various *C*	0.998996	0.99566	0.9922	0.955952
Various *P*	0.999218	0.99566	0.9922	0.955952

In conclusion , there was performed a Monte Carlo simulation where parameter of the PFFO and cost were changed simultaneously. In this case, parameters of probabilities of each unit were calculated (in Excel) as:

p_k=0.8p_k+0.4p_k*RAND()
and
c_k= 0.8c_k+0.4*RAND(),

i.e. considered a random variation of the values within ±20% limits..

The final results for this case are presented in Tables 5 – 8.

Table 5. Results of Monte Carlo simulations for P*=0.999

No.	**P* = 0.999**							
	P	**C**	x_1	x_2	x_3	x_4	x_5	x_6
1	0.999352	100	6	6	6	4	4	4
2	0.999218	102	6	6	6	5	4	4
3	0.999313	102	6	6	6	4	4	4
4	0.999212	97	5	6	6	4	4	4
5	0.999182	102	6	6	6	4	4	4
6	0.999171	97	6	6	5	4	4	4
7	0.999171	103	6	6	6	4	5	4
8	0.999596	100	6	6	6	4	4	4
9	0.999526	100	6	6	6	4	4	4
10	0.999399	100	6	6	6	4	4	4

Table 6. Results of Monte Carlo simulations for P*=0.995

No.	**P* = 0.995**							
	P	**C**	x_1	x_2	x_3	x_4	x_5	x_6
1	0.995478	84	5	5	5	3	3	3
2	0.996755	85	5	4	4	4	3	3
3	0.995026	85	5	4	5	4	3	3
4	0.996777	79	4	5	5	3	3	3
5	0.996777	84	5	5	5	3	3	3
6	0.995525	79	5	5	4	3	3	3
7	0.996732	85	5	5	5	3	4	3
8	0.996732	85	5	5	5	3	4	3
9	0.995645	84	5	5	5	3	3	3
10	0.99567	84	5	5	5	3	3	3

Table 7. Results of Monte Carlo simulations for P*=0.99

No.	P* = 0.99							
	P	**C**	x_1	x_2	x_3	x_4	x_5	x_6
1	0.990147	69	4	4	4	3	3	3
2	0.990965	70	4	4	4	4	3	3
3	0.990229	70	4	4	4	4	3	3
4	0.99185	69	4	4	4	3	3	3
5	0.990389	71	4	4	4	4	4	3
6	0.99107	69	4	4	4	3	3	3
7	0.992185	74	5	4	4	3	3	3
8	0.990422	71	4	4	4	3	4	3
9	0.990893	71	5	4	4	3	3	3
10	0.990466	69	4	4	4	3	3	3

Table 7. Results of Monte Carlo simulations for P*=0.95

No.	P* = 0.95							
	P	**C**	x_1	x_2	x_3	x_4	x_5	x_6
1	0.950045	52	3	3	3	3	2	2
2	0.955842	52	3	3	3	3	2	2
3	0.951936	52	3	3	3	3	2	2
4	0.951711	54	3	3	3	2	2	2
5	0.957883	50	3	3	3	3	3	2
6	0.951908	51	3	3	3	2	2	2
7	0.962227	51	3	3	3	2	2	2
8	0.962227	51	3	3	3	3	3	2
9	0.95261	50	3	3	3	3	2	3
10	0.950393	52	3	3	3	3	2	2

Analysis of data presented in Tables 5-8 shows relatively significant difference in numerical results (see Figure 4).

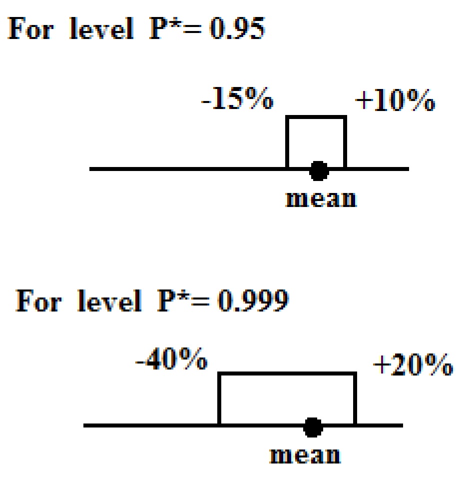


Figure 4. Deviation of maximum and minimum values of probability of failure-free operation in results of Monte Carlo simulation

However, the problem is not in coincidence of final values of PFFO or cost. The problem is: how change of parameters influences on the optimal values of x_1, x_2, … .

However, one can observe that even with a system of six units (redundant groups) a visual analysis of sets ($\boldsymbol{x_1, x_2, \ldots, x_6}$) is extremely difficult and, at the same time, deductions based on some averages or deviations of various x_k are almost useless.

The author was forced to invent some kind of a special presentation of sets of x_k's. Since there is no official name for such kind of graphical presentation, it is called "multiple polygons" (in Russian "мульти-звездограммы»). On such multiple polygon there are numbers of "rays" corresponding to the number of redundant of units (groups). Each ray has several levels corecponded to the number of calculated redundant units for considered case (see Figure 5).

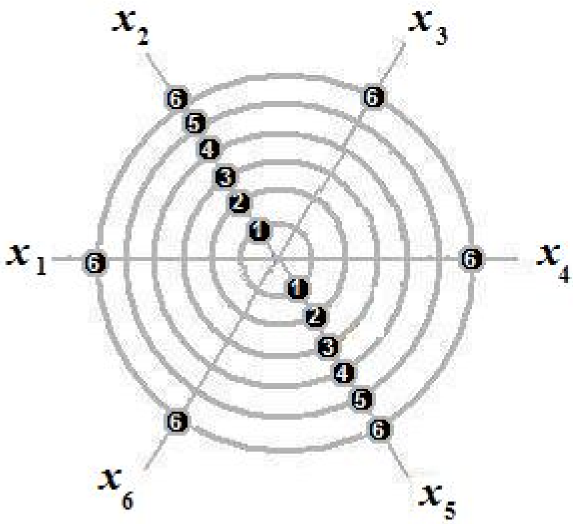


Figure 5. Multiple polygon axes with numbered levels

These multiple polygons give a perfect visualization of "close-to-optimal" solutions and characterize observed deviation of particular solutions. Such multiple polygons for considered example are given in Figure 6. (Here bold lines re used for connection xk obtained as optimal solution for units with parameters given in Table 1.)

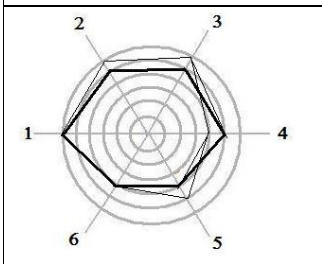
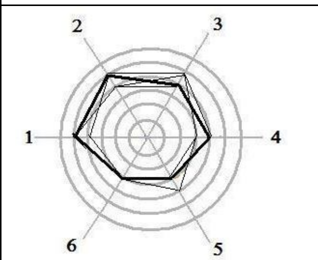
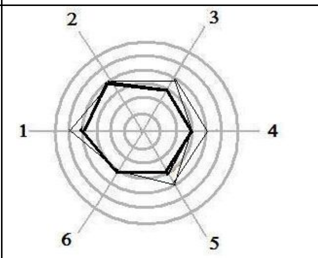
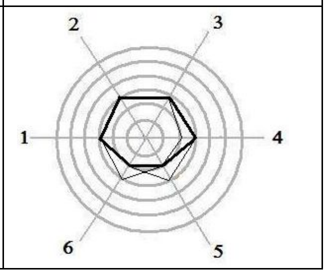
***P** = 0.999**	***P** = 0.999**	***P** = 0.999**	***P** = 0.999**
			

Figure 6. Deviations of optimal solutions for randomly variation of parameters from the optimal solution obtained for parameters given in Table 1

Thus, one can notice that input parameters variation may influence enough significantly enough on probability of failure-free operation and the total system cost from run to run of Monte Carlo simulation though optimal solution remains more or less stable.