A HEURISTIC METHOD FOR RELIABILITY REDUNDANCY OPTIMIZATION OF FLOW NETWORKS

Kumar Pardeep * and Chaturvedi D.K. **

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Department of Instrumentation,
Kurukshetra University, Kurukshetra, – 136 119, India
e-mail: pardeep_kuk@rediffmail.com*, dhiraj_chaturvedi@yahoo.co.in**
*Corresponding author

Pahuja G.L.

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Department of Electrical Engineering,
National Institute of Technology, Kurukshetra – 136 119, India
e-mail: pahuja_gl@yahoo.co.in

ABSTRACT

In flow networks, from the quality and service management point of view, measurement of the transmission ability of a network to meet the customers demand is very important. To meet the ultra high reliable requirements of such networks, a heuristic method for reliability redundancy optimization of flow networks using composite performance measure (CPM) integrating reliability and capacity has been proposed. The method is based upon the selection of main flow paths and backup paths and then optimizing main paths on priority basis. Thus, the reduced computation work makes the proposed algorithm suitable for designing of large, reliable telecommunications networks.

Keywords: flow networks; capacity related reliability; constrained redundancy optimization; heuristic algorithm.

1 INTRODUCTION

Constrained reliability redundancy optimization of networks has generally been studied with reliability as connectivity measure. The practical systems such as computer networks, telecommunication networks, transportation systems, electrical power transmission networks, internet etc. can only transport limited amount of flow therefore these are termed as flow limited networks. To meet the ultra high reliability requirements of such networks, a heuristic method for reliability redundancy optimization of flow networks has been proposed.

Literature is enriched with reliability redundancy optimization of networks with connectivity only as a measure of performance (Sharma & Venkateswaran 1971, Aggarwal 1976, Golden & Magnanti 1977, Gopal et al. 1978, Lee 1980, Bodin et al. 1982, Xue 1985, Dinghua 1987, Fredman & Tarjan 1987, Kim & Yum 1993, Shen 1995, Martins & Santos 1997, Schrijver 1998*,* Ahuja 1998, Kuo & Prasad 2000, Kuo et al. 2001, Park et al. 2004, Pascoal et al. 2005, Kumar et al. 2009, 2010a, b, 2011). However, reliability redundancy optimization of flow networks has rarely been studied. In present days context existing methods do not fulfil the requirements of management of quality of service. The reliability redundancy optimization techniques discussed in Gopal et al. (1978), Dinghua (1987), Kim & Yum (1993), Shen (1995), Park et al. (2004), Kumar et al. (2009, 2010a, b, 2011) are not suited to flow networks. This paper presents a technique for reliability redundancy optimization of flow networks using combined performance measure named capacity related reliability (CRR).

A path is a sequence of arcs and nodes connecting a source to a sink. All the arcs and nodes of network have its own attributes like delay, reliability and capacity etc.. From the quality and service management point of view, measurement of the transmission ability of a network to meet the customers demand is very important (Lin 2006). When a given amount of flow is required to be transmitted through a flow network, it is desirable to optimize the network reliability to carry the desired flow. In such cases, the system reliability is the measure of quality of the system capability to transmit desired flow. The capacity of each arc (the maximum flow passing the arc per unit time) has two levels, 0 and/or a positive integer. The system reliability is the probability that the maximum flow through the network between the source and the sink is not less than the demand (Golden & Magnanti 1977, Lee 1980, Bodin et al. 1982, Fredman & Tarjan 1987, Ahuja 1998, Lin 2003, 2004, Pahuja 2004, Lin 2006, 2007a, b). For determining the reliability it is generally assumed that network is capable of transmitting any required amount of flow between source (s) and terminal (t) nodes of the network. This presumption is neither valid nor justifiable for real life systems as links and nodes can carry only limited amount of flow. In 1980 Lee did the pioneer work of integrated both capacity and reliability & named it combined performance measure as capacity related reliability (CRR) and also termed such networks as flow networks. Max-Flow-Min-Cut theorem has been used to determine the capacity of the network (Sharma & Venkateswaran 1971, Xue 1985, Shen 1995, Martins & Santos 1997, Schrijver 1998, Lin 2003, 2004, Park et al. 2004, Lin 2006, 2007a, b).

2 COMPOSITE PERFORMANCE MEASURE

Reliability under flow constraint is a more realistic performance measure for flow networks. The concept of weighted reliability introduced by Aggarwal (1976) requires that all the successful states qualifying connectivity measure of the network be enumerated. The probability of each success state is evaluated and is multiplied by the normalized weight (Aggarwal 1985). Rushdi (1988) evaluated the same performance index as evaluated by Aggarwal 1985 using decomposition approach. Methods given by Aggarwal (1985), Rushdi (1988), and Shakti (1995) generate both cancelling (failed) and non-cancelling (success) terms.

The following section presents a heuristic algorithm for reliability redundancy optimization of flow networks using composite performance measure (CPM) and the method has been utilized to determine the capacity related reliability performance index.

2.1 Notation

$a_l(X)$ Sensitivity factor of l^{th} minimal path set

$b_i(x_i)$ Subsystem selection factor for i^{th} subsystem with x_i components

C_j Total amount of resource j available

$g_i^j(x_i)$ / $c_{ji}(x_i)$ Amount of resources-j consumed in subsystem-i with x_i components / Cost of subsystem i for j^{th} constraint.

$h(.)$ Function yielding system reliability; dependent on number of subsystems (n) and configuration of subsystems

k Number of constraints, $j = 1, 2,\ldots, k$

$L(x)$ $(L_{x_1}, L_{x_2},\ldots, L_{x_n})$, Lower limit of each subsystem i,

m Number of main minimal path sets, $l = 1, 2,., m$

n Number of subsystems, $i = 1, 2,\ldots, n$

P_l l^{th} minimal path set of the system

Ps $(l^1, l^2, \ldots, l^{min})$: priority vector s.t. l^1 and l^{min} are the number of minimal path sets arranged in decreasing order of path selection parameter $a_l(X)$.

$Q_i(x_i)$ Unreliability of subsystem i with x_i components.

r_i Reliability of a component at subsystem i.

$R_i(x_i)$ Reliability of subsystem i with x_i components.

R_r Residual resources [total resource available (C_j) - resources consumed ($\sum g_i^j x_i$)]

$R_s(X)$ System reliability

$S(x)$ Set of variables that have been used as key-elements in a given decomposed expressions

$U(x)$ $(U_{x_1}, U_{x_2}, ..., U_{x_n})$, Upper limit of each of subsystem i,

x^* Optimal solution

x_i Number of components in subsystem i; i = 1,2,….n

X A vector $(x_1, \ldots \ldots x_n)$

ΔR_i Increment in i^{th} stage reliability when a unit is added in parallel to the i^{th} stage

2.2 Assumptions

Following are the assumptions for the rest of the sections:

1. The system and all its subsystems are coherent.
2. Subsystem structures (other than coherence) are not restricted.
3. The networks are modelled with the help of graphs, the paths (ordered pair of arcs and the members of the ordered pair are reliability and capacity respectively) where in are assigned as the weight of each link.
4. Each link can have only two stages up and down.
5. The network nodes are perfect. If the nodes are not perfect, the method needs to be modified to deal with nodes failures.
6. All component states are mutually and statistically independent.
7. All constraints are separable and additive among components.
8. Each constraint is an increasing function of x_i for each subsystem.
9. Redundant components cannot cross subsystem boundaries.

2.3 Composite Performance Measure (CPM)

The weighted reliability measure i.e. composite performance measure (CPM), integrating both capacity and reliability may be stated as by **[27, 28]**:

$$\text{CPM} = \sum_{i \in S(x)} \omega t_i R_i \tag{1}$$

Where ωt_i is the normalized weight and is defined as:

$$\omega t_i = Cap_i / Cap_{max}$$

i.e. the ratio of capacity in the i^{th} state to the maximum capacity (Cap_{max}) of the system and R_i probability of the system being in state S_i and is computed as:

$$R_i = P_r\{S_i\} = \prod_{j / S_{ij}=1} p_j \times \prod_{k / S_{ik}=0} q_k \tag{2}$$

2.4 Capacity Functions of Networks

The capacity function of different arcs connected in parallel is **[25]**:

$$C(X)_{Par} = \sum_{i \in x} Cap_i \tag{3}$$

and the capacity function of different arcs connected in series is:

$$C(X)_{Ser} = \min\{Cap_i\} \tag{4}$$

The rules for connecting series and parallel arcs to integrate capacity and reliability to give composite performance measure are expressed as:

$$CR(X)_{Ser} = \{\min_{i \in x} Cap_i\} \prod_{i=1}^{n} r_i \tag{5}$$

$$CR(X)_{Par} = \sum_{i=1}^{n} Cap_i \cdot \bigcup_{i=1}^{n} r_i \tag{6}$$

CPM for series and parallel networks can be defined as:

$$\text{CPM}_{Par} = CR(X)_{Par} / Cap_{max} \tag{7}$$

and $$\text{CPM}_{Ser} = CR(X)_{Ser} / Cap_{max} \tag{8}$$

3 PROBLEM FORMULATION AND HEURISTIC METHOD

3.1 Problem Formulation

The general constrained redundancy optimization problem in complex systems can be reduced to the following integer programming problem (Kuo et al. 2001):

Maximise

$$R_s(X) = h(R_1(x_1), \ldots, R_n(x_n)), \tag{9}$$

subject to $$\sum_{i=1}^{n} g_i^j(x_i) \le C_j, \quad j = 1, 2, \ldots, k \tag{10}$$

and $1 \le x_i \le U_{xi}$, $\quad i = 1, 2, \ldots, n.$

3.2 Proposed Heuristic Method

In real life systems all the arcs are not simultaneously connected to carry flow from source to sink. Hence a flow path set is the arcs and nodes that actually carry traffic. In these practical systems all the path sets are not utilized for transfer of information (Hayashi & Abe 2008). The flow is transmitted through the main path(s) and in case of failure of this path(s), a backup path completes the task of main path. The backup path(s) come in operation only when the main paths fail thus, enhancing the reliability of the network, it is presumed that the main path(s) and backup path(s) for the network are known. The proposed algorithm first optimizes the main path(s) and then back up path(s) using redundancy optimization technique. This leads to more efficient use of resources which are generally limited. Unlike existing heuristics, a stopping criterion has been applied to switch from reliability redundancy optimization of main path(s) to reliability redundancy optimization of backup paths. The algorithm considers sensitivity factor as the criteria for selecting the main path from the list of given main paths and then a subsystem for applying redundancy within the chosen flow path.

3.3 Steps of the Proposed Method

Step1: Firstly select the main path sets *m* and back up flow paths of the flow Network.

Step2: Let $x_i = 1$ for all *i; i* = 1, 2,…, *n*.

Step3: Calculate $a_l(X)$, *l* = 1, 2,…, *m* for each minimal path set and find *l** such that $a_{l*}(X) = max\ [a_l(X)]$ and

$$a_l(X) = \frac{\prod_{i\in P_l} R_i(x_i)}{\sum_{i\in P_l}\sum_{j=1}^{k}(g_i^j(x_i)/kC_j)},$$

$$l = 1,2,3........m.$$

Step4: For the chosen minimal path set – *l** find i^* such that $b_{i^*}(x_{i^*}) = max\ [b_i(x_i)]$ and

$$b_i(x_i) = \frac{\Delta R_i}{\sum_{j=1}^{k}(g_i^j(x_i)/kC_j)}, \quad \text{for each } i \in P_l$$

where $\Delta R_i = \left(1 - Q^2{}_i(x_i)\right) - R_i(x_i)$

Step5: Check, by adding one redundant subsystem to unsaturated subsystem i^* :

i) if no constraints are violated, add one redundant subsystem to unsaturated subsystem i^* by replacing x_{i^*} with $x_{i^*}+1$, and go to step 3.

ii) if at least one constraint is exactly satisfied and others are not violated, then add one redundant subsystem to unsaturated subsystem i^* by replacing x_{i^*} with $x_{i^*}+1$. The $x^* = X$ is the optimal solution. Go to step 6.

iii) if at least one constraint is violated, then remove minimal path set *l** from further consideration and consider the next path having maximum $a_{l*}(X)$ value and go to step 4.

iv) if all minimal path sets are now excluded from further consideration, then $x^* = X$ is the optimal solution; else go to step 3

Step6: In case any resources are still available optimize the backup paths as discussed in Step2.

Step4: Evaluate the composite performance measure (CPM) for each subsystem of the network.

Step5: Evaluate the system reliability using the CPM of the each subsystem.

4 COMPUTATION AND RESULTS

To illustrate the performance of the proposed algorithm a network having six arcs {x_1, x_2, x_3, x_4, x_5, x_6} and five minimal path sets {y_1, y_2, y_3, y_4, y_5} as shown in the Figure 1 is considered and solved for capacity related redundancy reliability optimization using CPM (7 and 8). System reliability is determined using Bayes method. The network shown in Figure 1 is a bench mark problem, considered by Hayashi & Abe (2008).

Using Baye's method, the Reliability of the above system can be expressed as:

$$R_s(X) = R_3\,[1- Q_6\,\{1-(1-Q_1Q_2)(1-Q_4Q_5)\}] + Q_3[1-(1-R_2R_5)(1-R_1R_4)]*Q6 \quad (11)$$

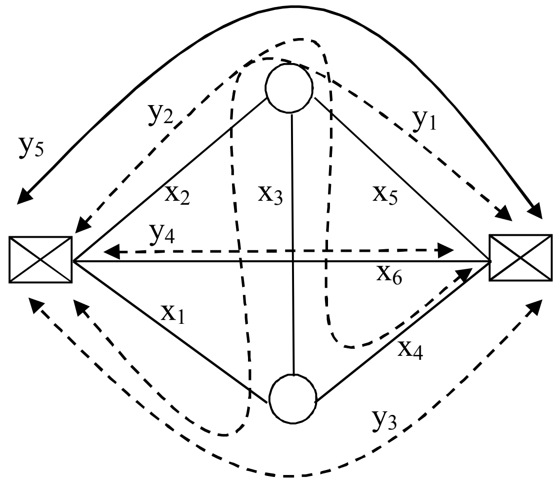

Figure 1 Illustration Network

The simple minimal path sets of the Network are

P1 = {1, 3, 5}, **P2** = {2, 3, 4}, **P3** = {1, 4}, **P4** = {6}, **P5** = {2, 5}

The problem is solved for data given in Table 1. Using this initial data the general problem of constrained reliability redundancy allocation has been solved using the steps discussed in Section 3.3 above. The problem is solved by considering the flow path sets P1, P2, P3 and P4 as main path sets and P5 as backup path. The proposed algorithm gives the optimal solution (3, 1, 1, 2, 1, 4) with system reliability R_s = 0.999998458, the optimized subsystem reliability probability R_i and unreliability probabilities Q_i are shown in Table 2.

Table 1 Data for Fig. 1

i	**1**	**2**	**3**	**4**	**5**	**6**
r_i	0.70	0.75	0.8	0.85	0.7	0.9
g_i^1 / c_{1i}	2	3	2	3	1	3
C_1	30					

Table 2 Optimized subsystem reliability/unreliability for Fig. 1

i	x_1	x_2	x_3	x_4	x_5	x_6
X*	3	1	1	2	1	4
R_i	0.973	0.75	0.8	0.9775	0.7	0.9999
Qi	0.027	0.25	0.2	0.0225	0.3	0.0001

The capacity of each subsystem of the flow path is taken as 100 and the capacity of flow paths of the network is determined using proposed approach as:

$$\left.\begin{array}{l} P1 = \min\{3*100, 100, 100\} = 100 \\ P2 = \min\{100, 100, 100\} = 100 \\ P3 = \min\{3*100, 2*100\} = 200 \\ P4 = \min\{4*100\} = 400 \end{array}\right\} \tag{12}$$

Next the composite performance measure CPM expression (13) is derived using (7 and 8) and the value for CPM for an assumed flow of 200 is suppose to pass through the flow path and it comes out to be 1.0000 .

$$CPM_{P1} = \frac{\min\ Cap_i}{Cap_{\max}}[R_1 R_3 R_5] \tag{13}$$

$$= (100/200) \text{ x } 0.973 \text{ x } 0.8 \text{ x } 0.7 = 0.27244$$

$$CPM_{P2} = \frac{\min\ Cap_i}{Cap_{\max}}[R_2 R_3 R_4] \tag{14}$$

$$= (100/200) \text{ x } 0.75 \text{ x } 0.8 \text{ x } 0.9775 = 0.29325$$

$$CPM_{P3} = \frac{\min\ Cap_i}{Cap_{\max}}[R_1 R_4] \tag{15}$$

$$= (200/200) \text{ x } 0.973 \text{ x } 0.9775 = 0.9511075$$

$$CPM_{P4} = \frac{\min\ Cap_i}{Cap_{\max}}[R_6] \tag{16}$$

$$= (400/200) \text{ x } 0.9999$$

$$= (2) \text{ x } 0.9999 \quad \text{as } 0 \le (\min Cap_i / Cap_{max}) \le 1$$

$$\text{so} \quad = 1 \text{x } 0.9999 = 0.9999$$

Composite performance measure integrating the reliability with capacity is calculated as:

$$CPM_{Network} = 1 - (1-CPM_{P1})(1-CPM_{P2})(1-CPM_{P3})(1-CPM_{P4})$$

$$= 1 - (1-0.27244) \text{ x } (1-0.29325) \text{ x } (1-0.9511) \text{ x } (1-0.9999)$$

$$= 1.0000$$

The above result shows that proposed method is capable of optimizing the flow network to transport the desired capacity through the network with highest reliability. However, the selection of main paths and backup paths will affect the quality of composite performance measure. Hence the proper choice of these paths may be done using cardinality criteria (Kumar et al. 2010b) or any other hierarchical measures of importance.

5 CONCLUSIONS

This paper has presented a new model for designing reliable flow networks capable of transmitting required flow. The proposed algorithm utilizes the concept of main and backup flow paths. The choice of backup and flow paths is application specific and paths with minimum cardinality may be selected as main path and disjoint paths can be the backup paths. The numerical example demonstrates that the proposed algorithm is fast for designing large, reliable telecommunications networks because the task of optimization is reduced, as only few paths are selected as main paths.

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