ANTITERRORISM RESOURCES ALLOCATION UNDER FUZZY SUBJECTIVE ESTIMATES

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1. PRELIMINARY

The problem of optimal resources allocation for antiterrorism preventive measures is naturally based on subjective estimates made by experts in this field. Relying on expert estimates is inevitable in this case: there is no other possibility to get input data for the system survivability analysis. There is no such phenomenon like "collecting real data", moreover, there is no "homogenous samples" for consistent statistical analysis of observations, since any case is unique and non-reproducible. Nevertheless, quantitative analysis of necessary level of protection has to be performed.

What are the subjects of such expertise? It seems to us that they are:

- possibility of terrorist attacks on some object or group of objects,
- possible time of such attack,
- expected consequences of the attack and possible losses,
- possible measure of protection and related expenses

Since expert estimates of such complex things are fuzzy due to lack of common understanding the same actions and counter-actions within a group of experts, the question arises: is it possible at all to make any reasonable prognosis and, moreover, say about "optimal allocation of protection resources"?

First of all, we should underline that concept of "optimal solution" relates only to mathematical models. In practice unreliable (and even inconsistent) data and inevitable inaccuracy of the model (i.e. difference between a model and reality) allow us to say only about "rational solutions".

Nevertheless, in practice the problem exists and in any particular case has to be solved with or without using mathematical models. Our objective is to analyze stability of solutions of the optimal resources allocation under fuzziness of experts' estimates.

2. ANALYSIS OF SOLUTION STABILITY: VARIATION OF THE EXPENSE ESTIMATES

First, let us analyze how variation of expenses estimates influence on the solution of the problem on the level of a single object which has to be protested against terrorist attacking. For transparency of explanation, we avoid to consider the influence of defense on the Federal and State levels.

Let us consider some conditional object (Object-1) which can be a subject of a terrorist attack. It is assumed that there may be three different types of enemy's actions (Act.-1, Act.-2 and Act.-3). Defending side can choose several specific protection measures against each type of action {M(i, j), where "i" corresponds to the action type, and "j" corresponds to the type of undertaken protective measure.

Assume that we have three variants of estimates of protection measures costs: lower, middle and upper as it presented in the table below. Here the lower estimates are about 20% lower of the corresponding middle estimates, and the upper ones are also about 20% higher.

There are three types of expert estimates: "optimistic", "moderate" and "pessimistic". First ones assume that success in each situation can be reached by low expenses for protective measures; the last group requires larger expenses for protection in the same situation; and the middle group gives date in between.

Table 1.
Case of optimistic estimates

OBJECT-1		γ_1	C
Act-1	M(1, 1)	0.25	0.8
	M(1, 2)	0.2	2
	M(1, 3)	0.1	2.5
	M(1, 4)	0.01	3.8
Act-2	M(2, 1)	0.2	1.6
	M(2, 2)	0.16	2.8
	M(2, 3)	0.07	3.2
	M(2, 4)	0.02	5.6
Act-3	M(3, 1)	0.11	0.4
	M(3, 2)	0.1	2
	M(3, 3)	0.05	2.4
	M(3, 4)	0.04	3.6
	M(3, 5)	0.01	5.6

Table 2
Case of moderate estimates

OBJECT-1		γ_2	C
Act-1	M(1, 1)	0.25	1
	M(1, 2)	0.2	2.5
	M(1, 3)	0.1	3
	M(1, 4)	0.01	4
Act-2	M(2, 1)	0.2	2
	M(2, 2)	0.16	3
	M(2, 3)	0.07	4
	M(2, 4)	0.02	7
Act-3	M(3, 1)	0.11	0.5
	M(3, 2)	0.1	2.5
	M(3, 3)	0.05	3
	M(3, 4)	0.04	5
	M(3, 5)	0.01	7

Table 3
Case of pessimistic estimates

OBJECT-1		γ_3	C
Act-1	M(1, 1)	0.25	1.2
	M(1, 2)	0.2	3
	M(1, 3)	0.1	6
	M(1, 4)	0.01	7.8
Act-2	M(2, 1)	0.2	2.4
	M(2, 2)	0.16	3.2
	M(2, 3)	0.07	4.8
	M(2, 4)	0.02	8.4
Act-3	M(3, 1)	0.11	2
	M(3, 2)	0.1	3
	M(3, 3)	0.05	3.6
	M(3, 4)	0.04	6
	M(3, 5)	0.01	8.4

How the data of the tables are interpreted?

Let us consider a possibility of action-1 against the object. With no protection at all, the object's vulnerability equals 1 (or 100%). If one would have spent we spent ΔE = 0.8 conditional cost units (c.c.u.) and undertook measure M(1, 1) the object's vulnerability decreases to 0.25. If one does not satisfy such a level of protection, next protective measure (M1, 2) is applied; that leads to decreasing the object vulnerability from 0.25 to 0,3 and costs 2 c.c.u.

Now let us consider all three possible terrorists' actions. In advance nobody knows what kind of action will be undertaken against the defending object. In this situation the most reasonable strategy is providing equal defense levels against all considered types of terrorist attacks, suggested in [1]. It means that if one needs to ensure a level of protection equals □ than one has to consider only such measures against each action that delivers vulnerability level not less than □□□ For instance, in the considered case, if the required level of vulnerability had to be not higher than 0.1, one has to use simultaneously the following measures of protection against possible terrorists' attacks: M(1, 3), M(2, 3) and M(3, 2).

Method of equal protection against the various types of hostile attacks appears to be quite natural. If dealing with natural or other unintended impacts, one can speak about the subjective probabilities of impacts of some particular typef course, in a case of intentional attack from a reasonably thinking enemy, such approach is not appropriate. The fact is, that as soon as the enemy knows about your assumptions about his possible. actions, he takes advantage of this knowledge and choose the hostile action that you expect least of all.

In the example considered above if one chose measures M(1,2) with γ_1=0.2, $M(2, 3)$ with γ_2=0.07 и and M(3, 4) with γ_3=0.04, guaranteed level of the object protection is

$$\gamma_{\text{Object}} = \max(\gamma_1, \gamma_2, \gamma_3) = \max(0.2; 0.07; 0.04) = 0.2.$$

For choosing required (or needed) level of object protection, one can compile a function reflecting the dependence of vulnerability of protection cost.

Table 4. Case of optimistic estimates

Object 1			
Step Number	**Undertaken measures**	**Resulting** γ_{Object}	**Total Expenses,** C_{Object}
1	M(1, 1), M(2, 1), M(3, 1)	max {0.25, 0.2, 0.11}=0.25	0.8+1.6+0.4=2.8
2	M(1, 2), M(2, 1), M(3, 1)	max {0.2, 0.2, 0.11}=0.2	2+1.6+0.4=4
3	M(1, 3), M(2, 2), M(3, 1)	max {0.1, 0.16, 0.11}=0.16	2.5+2.8+0.4=5.7
4	M(1, 3), M(2, 3), M(3, 1)	max {0.1, 0.07, 0.11}=0.11	2.5+3.2+0.4=6.1
5	M(1, 3), M(2, 3), M(3, 2)	max {0.1, 0.07, 0.1}=0.1	2.5+3.2+2=7.7
6	M(1, 4), M(2, 3), M(3, 3)	max {0.01, 0.07, 0.05}=0.07	3.8+3.2+2.4=9.4
7	M(1, 4), M(2, 4), M(3, 3)	max {0.01, 0.02, 0.05}=0.05	3.8+5.6+2.4=11.8
8	M(1, 4), M(2, 4), M(3, 4)	max {0.01, 0.02, 0.04}=0.04	3.8+5.6+3.6=13
9	M(1, 4), M(2, 4), M(3, 5)	max {0.01, 0.02, 0.01}=0.02	3.8+5.6+5.6=15

This function is depicted in Figure 13.8.

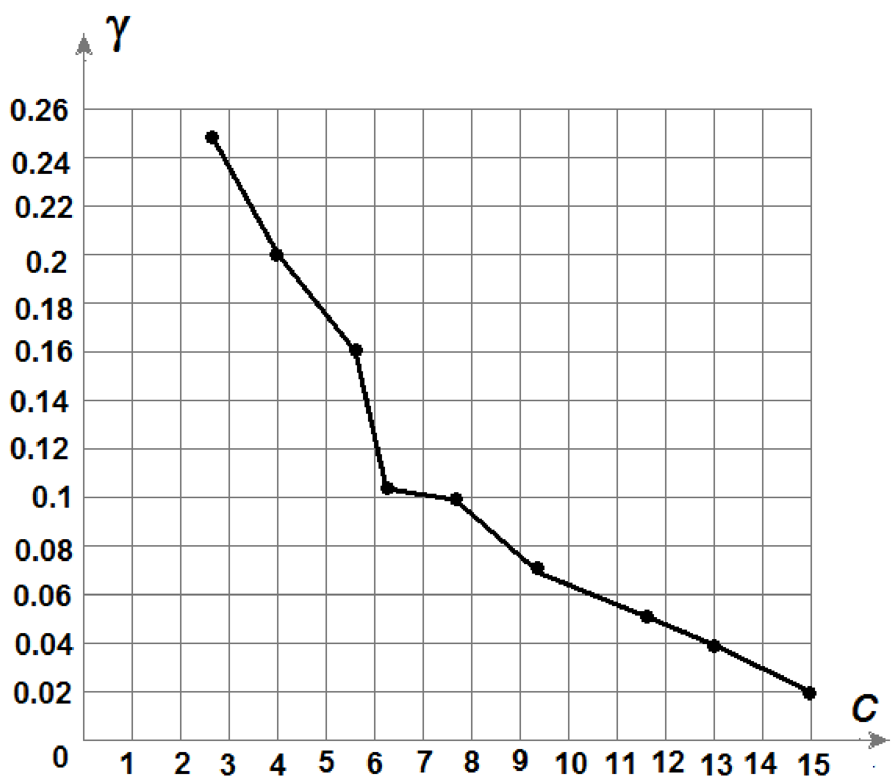

Figure 1. Dependence of the object survivability on cost of protection measures (for "optimistic" estimates).

Without detailed explanations we present numerical results for the cases of "moderate" and "pessimistic" estimates.

Table 5. Case of moderate estimates

Object 1			
Step Number	**Undertaken measures**	**Resulting** γ_{Object}	**Total Expenses,** C_{Object}
1	M(1, 1), M(2, 1), M(3, 1)	max {0.25, 0.2, 0.11}=0.25	1+2+0.5=3.5
2	M(1, 2), M(2, 1), M(3, 1)	max {0.2, 0.2, 0.11}=0.2	2.5+2+0.5=5
3	M(1, 3), M(2, 2), M(3, 1)	max {0.1, 0.16, 0.11}=0.16	3+3+0.5=6.5
4	M(1, 3), M(2, 3), M(3, 1)	max {0.1, 0.07, 0.11}=0.11	3+4+0.5=7.5
5	M(1, 3), M(2, 3), M(3, 2)	max {0.1, 0.07, 0.1}=0.1	3+4+2.5=9.5
6	M(1, 4), M(2, 3), M(3, 3)	max {0.01, 0.07, 0.05}=0.07	4+4+3=11
7	M(1, 4), M(2, 4), M(3, 3)	max {0.01, 0.02, 0.05}=0.05	4+7+3=14
8	M(1, 4), M(2, 4), M(3, 4)	max {0.01, 0.02, 0.04}=0.04	4+7+5=16
9	M(1, 4), M(2, 4), M(3, 5)	max {0.01, 0.02, 0.01}=0.02	4+7+7=18

Data of Table 5 are depicted in Figure 2.

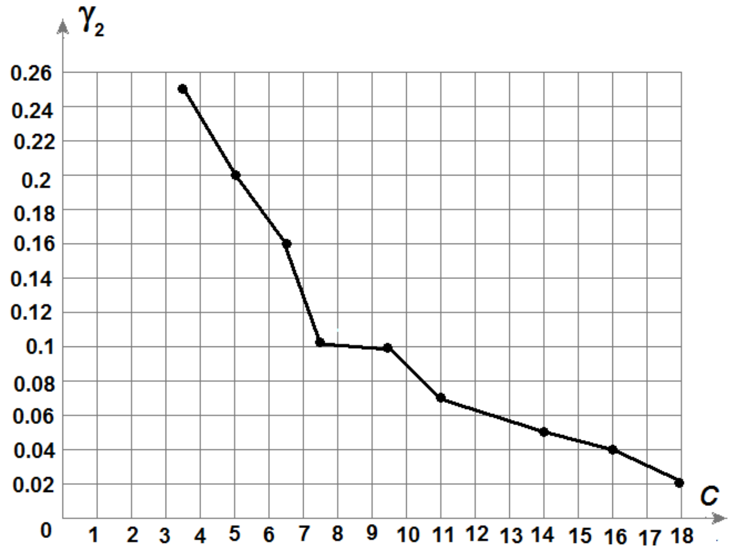

Figure 2. Dependence of the object survivability on cost of protection measures (for “moderate” estimates).

Table 13.6. Case of pessimistic estimates

Object 1			
Step Number	**Undertaken measures**	**Resulting** □$_{Object}$	**Total Expenses,** C_{Object}
1	M(1, 1), M(2, 1), M(3, 1)	max {0.25, 0.2, 0.11}=0.25	1.2+2.4+2=5.6
2	M(1, 2), M(2, 1), M(3, 1)	max {0.2, 0.2, 0.11}=0.2	3+2.4+2=7.4
3	M(1, 3), M(2, 2), M(3, 1)	max {0.1, 0.16, 0.11}=0.16	3+3.2+2=8.2
4	M(1, 3), M(2, 3), M(3, 1)	max {0.1, 0.07, 0.11}=0.11	3+4.8+2=9.8
5	M(1, 3), M(2, 3), M(3, 2)	max {0.1, 0.07, 0.1}=0.1	3+4.8+3=10.8
6	M(1, 4), M(2, 3), M(3, 3)	max {0.01, 0.07, 0.05}=0.07	4+4.8+3.6=12.4
7	M(1, 4), M(2, 4), M(3, 3)	max {0.01, 0.02, 0.05}=0.05	4+8.4+3.6=16
8	M(1, 4), M(2, 4), M(3, 4)	max {0.01, 0.02, 0.04}=0.04	4+8.4+6=18.4
9	M(1, 4), M(2, 4), M(3, 5)	max {0.01, 0.02, 0.01}=0.02	4+8.4+8.4=20.8

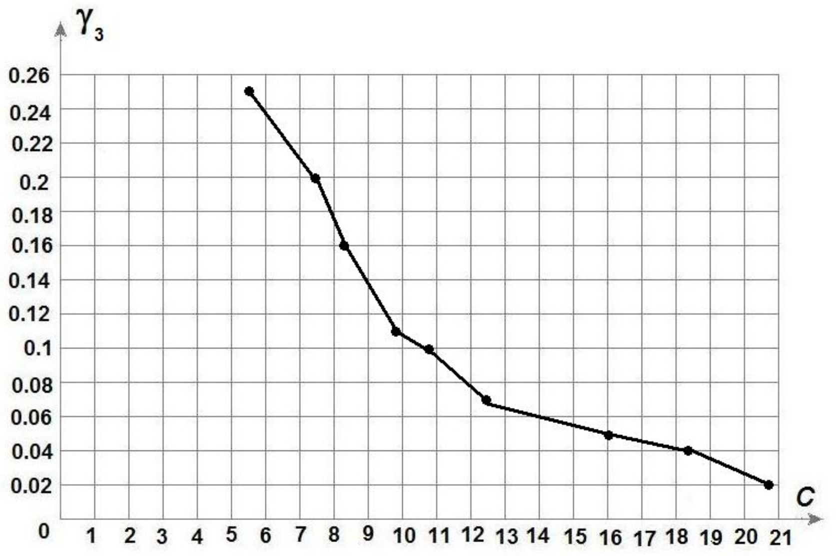

Figure 3. Dependence of the object survivability on cost of protection measures (for "pessimistic" estimates).

Such analysis gives a possibility to find what measures should be undertaken for each required level of protection (or admissible level of vulnerability) and given limited resources.

The final "trajectory" of the dependency "Expenses vs. Vulnerability" presented below.

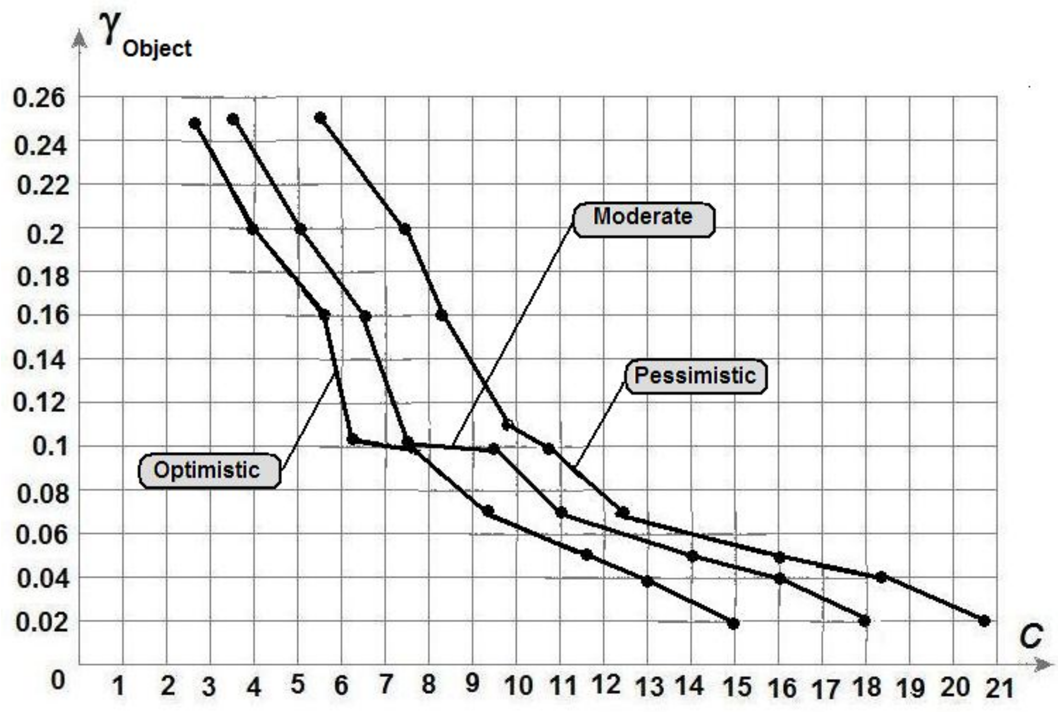

Figure 4. Comparison of solutions for three types of estimates.

Using the tables above, consider two solutions of the Direct Problem with required levels of vulnerability 0.1 and 0.02.

However, decision makers are interested mostly in correctness of undertaken measures, rather than in difference in absolute values of the estimated costs of the object protection. In other words, in other words, he is interested in how, for example, the solution adopted for the optimistic scenario, will be wealthy in case, if in fact the situation is better described pessimistic scenarios.

It is obvious that if the goal is to reach some given level of vulnerability the vector of solution (i.e. set of undertaken measures for protecting the object against terrorist attacks) in the frame of considered conditions the vector of solution will be the same, though will led to different expenses.

Consider solutions for required level of object vulnerability not higher than 0.1 and not higher than 0.02. They are:

Table 7. Comparison of solutions for three types of scenarios.

Type of scenario	**Undertaken protection measures**	
	$\gamma_{\text{Object}} \leq 0.1$	$\gamma_{\text{Object}} \leq 0.02$
Optimistic	M(1, 3), M(2, 3), M(3, 2)	M(1, 4), M(2, 4), M(3, 5)
Moderate	M(1, 3), M(2, 3), M(3, 2)	M(1, 4), M(2, 4), M(3, 5)
Perssimistic	M(1, 3), M(2, 3), M(3, 2)	M(1, 4), M(2, 4), M(3, 5)

One can see that solutions for all three scenarios coincides for both levels of object protection! Of course, such situation occurs not always, however we should underline that vectors of solution for minimax criterion $\gamma_{\text{Object}}=\max(\gamma_1,\gamma_2,\gamma_3)$ is much more stable than vector for probabilistic criterion $1-\gamma_{\text{Object}} = 1-\prod_{1\leq k\leq n}(1-\gamma_k)$.

CONCLUSION

The presented analysis shows that presented model of optimal allocation of counter-terrorism resources, suggested in the previous section, is working stably enough.

Development of improved computer model will allow analyzing more realistic situations, including random instability of input data. However, it seems that such "one-side biased" expert estimates should lead to more serious errors than random variations of the parameters.

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