

# REGIONS OF ACCEPTABILITY APPROXIMATION IN RELIABILITY DESIGN<sup>1</sup>

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## ABSTRACT

An approach to ensure the reliability of engineering systems at design stage is considered in this paper. This approach is associated with construction of an acceptable region inside the system parameter space. A model that describes an acceptable region constructed on the basis of multidimensional grid is offered. The methods for reducing amount of data with respect of resource limitations and particulars of data decomposition for its parallel processing are described.

## 1 INTRODUCTION

The task of feasible parameter region exploration often arises at engineering systems design. This task is associated with a set of specific procedures such as parameters tolerancing and choosing their nominal values, estimation of system sensitivity and parametric reliability. As a rule, a feasible parameter region (region of acceptability) is a domain comprised of parameter vectors which yield proper system performance. Obtaining the region characteristics or its approximation significantly facilitates solutions of design tasks associated with reliability control. Essential difficulty of the region approximation consists in high dimension of parameter space, incomplete prior information and only pointwise exploration of parameter space with system performances calculation.

There are different methods for constructing acceptable region. The method of approximation with a hyper-parallelepiped is in general use (Abramov et al. 2007, Jess et al. 2003). The methods of approximation with ellipsoids and polytopes are more advanced, but more complicated (Conti et al. 1994, Stehr 2005). One more approach consists in approximation with discrete set of elementary figures. Usually, hyper-parallelepipeds are used as elementary figures (Abramov & Nazarov 2011). In this paper, this method of approximation is applied to determine a region of acceptability.

The process of a multidimensional region construction with a discrete set of elementary parallelepipeds (boxes) is associated with two problems. The first problem consists in large amount of data to be processed and stored. The second problem arises from the first one and consists in high computational requirements. The second problem can be solved with application of parallel computing technologies (Abramov et al. 2007).

The application of parallel computing requires decomposition of a task. In this work, it is shown that the decomposition of the process of acceptable region construction can be carried out by splitting data to be processed in parallel. The model of the region approximation allows splitting the data into various parts of various volumes dynamically.

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<sup>1</sup> This work is supported by FEB RAS: Grants 12-I-OEMMITY -01 (Basic Research Program of PMMCP Branch RAS № 14) and 12-III-B-03-023.

The problem of storing large amounts of data is solved by optimization of the data structure, described by the model. The optimization consists in eliminating redundant data using algorithms of lossless data compression. Generally computer memory limitations do not allow keeping all the data describing the region of acceptability. In addition, to increase access speed when using compressed data it is required data splitting with their index ranges assigning. Thus, within the framework of acceptable region construction with respect of data compression and parallel processing, the task of file storage organizing arises.

This work is devoted to solving the problem of storage and processing region of acceptability data with account of computer memory limitations, application of data compression algorithms and parallel processing.

## 2 A REGION OF ACCEPTABILITY

### 2.1 A region of acceptability definition

From a consumer point of view, a system has its performance characteristics (gain, temperature, voltage, etc.). The performances are given as m-vector (1):

$$\mathbf{y} = (y_1, y_2, \dots, y_m). \quad (1)$$

From a design point of view, any system consists of elements/components that perform their functions. These elements are considered to be atomic. Thus, system parameters are considered as the n-vector:

$$\mathbf{x} = (x_1, x_2, \dots, x_n). \quad (2)$$

System performances (1) depend on parameters (2) of system elements (system parameters). System topology is defined by the model (3) which relates system parameters (2) to the system performances (1):

$$\mathbf{y} = \mathbf{y}(\mathbf{x}). \quad (3)$$

System components are influenced by different factors like ambient temperature, supply voltage, radiation, etc. These factors are usually taken into account in the model (3) as operational parameters and cannot be controlled by the designer. Operational parameters and aging factor cause deviations of system parameters which, consequently, cause system performances deviations. Usually, system performances (1) are constrained by performance specifications (4):

$$\mathbf{y}_{\min} \leq \mathbf{y}(\mathbf{x}) \leq \mathbf{y}_{\max} \quad (4)$$

The deviations of system parameters may cause violation of performance specifications (4) that means system failure. The task of parametric synthesis (Abramov et al. 2007) as one of design stages consists in nominal parameters choosing to meet the performance specifications (4) with the account of system parameter deviations during operating cycle. The solution of this task is often associated with determination of a region of acceptability as defined in (5):

$$D_x = \{\mathbf{x} \in \mathbf{R}^n \mid \mathbf{y}_{\min} \leq \mathbf{y}(\mathbf{x}) \leq \mathbf{y}_{\max}\}. \quad (5)$$

The region of acceptability and schematic illustration of system parameter deviation from nominal values  $\mathbf{x}^0$  at the moment  $t_0$  to gradual parametric failure at the moment  $t_2$  are presented in Figure 1.

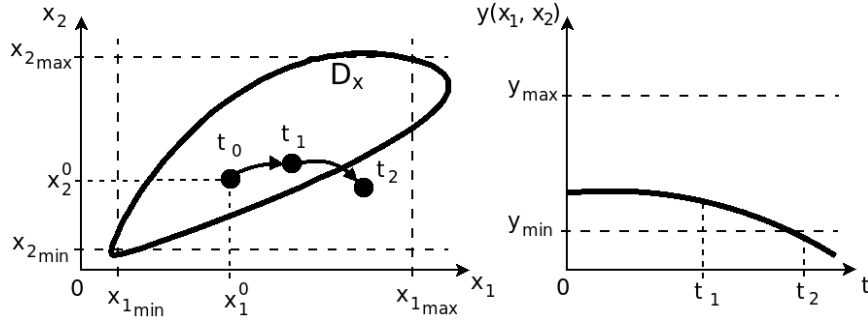


Figure 1. The region of acceptability and gradual parametric failure.

### 2.2 Grid Approximation of a Region of Acceptability

As it was said before, the approximation of a n-dimensional region with a discrete set of elementary hyper-parallelipeds (boxes) is used in this work. The basis of such representation of a region is a n-dimensional regular grid inside a bounding box (circumscribed box (Abramov et al. 2007) or parameter tolerances box) defined by the constraints (6):

$$\mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{x}_{\max} . \tag{6}$$

Grid nodes (7) define corners of the elementary boxes:

$$x_{ij} = x_{i0} + j \cdot h_i , \tag{7}$$

where  $i=1,2,\dots,n$  is an index of a parameter,  $j=0,1,\dots,Q_i$  is the node index for  $i$ -th parameter (the first node  $x_{i0} = x_{i \min}$ ),  $h_i = (x_{i \max} - x_{i \min}) / Q_i$  is the grid spacing for  $i$ -th parameter,  $Q_i$  – is the amount of “quanta” – the atomic intervals into which the range  $[x_{i \min}, x_{i \max}]$  is divided. For each  $x_i$  inside a “quantum”  $\partial y / \partial x_i = 0$  is supposed. Every “quantum” is indexed with  $k_i = 1, 2, \dots, Q_i$ , thus the set of indices  $(k_1, k_2, \dots, k_n)$  identifies an elementary box. It is supposed that every point  $\mathbf{x}$  inside an elementary box yields the same performances as its central point  $\mathbf{x}_c(k_1, k_2, \dots, k_n)$  with the coordinates defined in (8):

$$x_{i_c}^{k_i} = x_{i0} + k_i \cdot \frac{h_i}{2}, \quad \forall i = 1, 2, \dots, n . \tag{8}$$

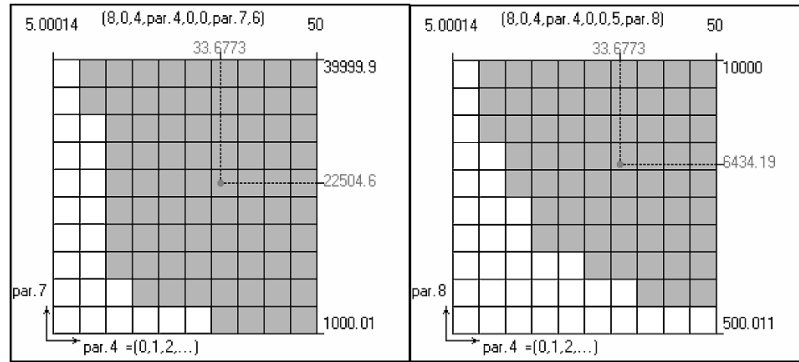
Every point  $\mathbf{x}_c(k_1, k_2, \dots, k_n)$  acts as a sampling point for elementary box identified by the indices  $(k_1, k_2, \dots, k_n)$ . System performances (1) are calculated for every elementary box’s sampling point using the model (3). Then these performances are compared with performance specifications (4). Thus, for every elementary box, the binary function (9) is calculated:

$$F_{D_x}(k_1, k_2, \dots, k_n) = \begin{cases} 1, & \mathbf{y}_{\min} \leq \mathbf{y}(\mathbf{x}_c(k_1, \dots, k_n)) \leq \mathbf{y}_{\max} \\ 0, & \text{otherwise} \end{cases} . \tag{9}$$

The function (9) determines the membership of a sampling point in the region  $D_x$ . Let us denote the set of elementary boxes  $B_g$ , then the function (9) defines a partitioning (10) of this set:

$$B_g = B_g^0 \cup B_g^1, \quad B_g^0 \cap B_g^1 = \emptyset . \tag{10}$$

The subset  $B_g^1$  is the approximation of the region of acceptability  $D_x$ , constructed with a discrete set of elementary boxes, defined with a regular grid. The example of 2-dimensional sections of 8-dimensional region of acceptability for an amplifier parameters choosing is illustrated in Figure 2.



**Figure 2.** 2D sections of 8D region of acceptability for nominal sizing.

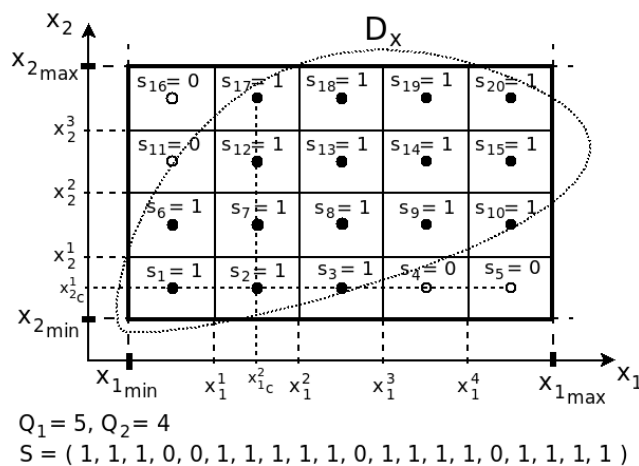
The region of acceptability approximation with a grid is defined with the model (11):

$$G_R = (n, B, Q, S), \tag{11}$$

where  $n$  is the amount of designable system parameters (2),  $B = \{(x_i \min, x_i \max), \forall i=1,2,\dots,n\}$  is a bounding box, defined by the constraints (6) in system parameter space,  $S = (s_1, s_2, \dots, s_n)$  is a set of membership indicators that store results of membership function (9). Every indicator  $s_p \in \{0,1\}$  displays the membership of the corresponding elementary box in subset  $B_g^1$  or  $B_g^0$ ,  $R=Q_1 \cdot Q_2 \cdot \dots \cdot Q_n$  is the amount of elementary boxes and, consequently, the amount of membership indicators. The one-to-one correspondence between indices  $(k_1, k_2, \dots, k_n)$  and the index  $p$  of an indicator in the set  $S$  is defined in (12). It is evident, that zero-based indices are preferable.

$$p = k_1 + Q_1 \cdot (k_2 - 1) + Q_1 \cdot Q_2 \cdot (k_3 - 1) \times \dots \times Q_1 \cdot Q_2 \cdot \dots \cdot Q_{n-1} \cdot (k_n - 1) \tag{12}$$

The process of the region of acceptability construction on the basis of the model (11) was described in the work (Abramov & Nazarov 2011). Briefly, this process consists in complete enumeration of the values of index  $p$  with calculation of corresponding indices  $(k_1, k_2, \dots, k_n)$ , calculation its sampling point (8), calculation of membership function (9) and assigning its result to the indicator  $s_p$ . The illustration of the result of this process and indicators assignment is presented in Figure3.



**Figure 3.** A region of acceptability approximation with a regular grid.

The usage of 1-dimensional structure for storing indicators is explained both by the convenience of the data storage and transmission and by its flexibility for task decomposition for parallel processing. The flexibility consists in the opportunity to splitting of the indicators array into arbitrary amount of portions of various volumes (e.g. for load balancing).

### 3 DECOMPOSITION OF THE DATA

The problem of the region  $D_x$  approximation with the model (11) consists in large amount of data. In addition, the indicators array data are redundant. This redundancy is induced by the way of its consecutive storage and its inner binary representation. The redundancy associated with the array data storage consists in occurrence of long sequences of indicators of the same value on the back of the way of consecutive storage of 1-dimensional sections of a multidimensional region (Fig. 3). The redundancy related to inner array data representation consists in the follows. Every indicator requires only one binary bit, but the basic addressable memory element is a byte, that consists of several binary bits (usually, eight bits). Thus when using usual byte array to store one byte per an indicator, a kind of “rarefaction” in the data occurs, that is highly undesirable when storing large amount of data. The solution to this problem is utilizing of every binary bit of a continuous byte array that can be achieved with the use of binary arithmetic (Abramov & Nazarov 2011).

Within the framework of the array data decomposition task, consider reducing of redundancy related to long sequences of the same indicator values. The most evident solution is storing lengths of the sequences of the same values, e.g. the array of characters BBBBWWBBWWWWWW will be reduced to B4W3B2W5. This is well-know algorithm called Run-Length Encoding (RLE) and usually it is used in computer graphics and communication technologies. The advantage of this algorithm is speed of compression and decompression, but only when enumerating sequentially (Salomon 2007).

With respect to storing of indicators array, the algorithm of RLE is described as follows. The array of membership indicators  $S = (s_1, s_2, \dots, s_n)$ ,  $s_p \in \{0,1\} \forall p = 1, 2, \dots, R$  is stored as a set of pairs like (13):

$$S_{RLE} = ((c_1, l_1), (c_2, l_2), \dots, (c_{R^*}, l_{R^*})), \tag{13}$$

where  $c_i \in \{0,1\}$  is an indicator value,  $l_i$  is the amount of its repetitions (the length of the sequence),  $R^*$  is the amount of pairs  $(c_i, l_i)$  in the array  $S_{RLE}$  that can not be evaluated without enumeration over all the elements of initial indicators array.

The algorithm of RLE reduces the data size of indicator array from 10 to 1000 times (Fig.4). The compression ratio depends on the grid spacing (“quanta” amount) and the actual region  $D_x$  configuration.

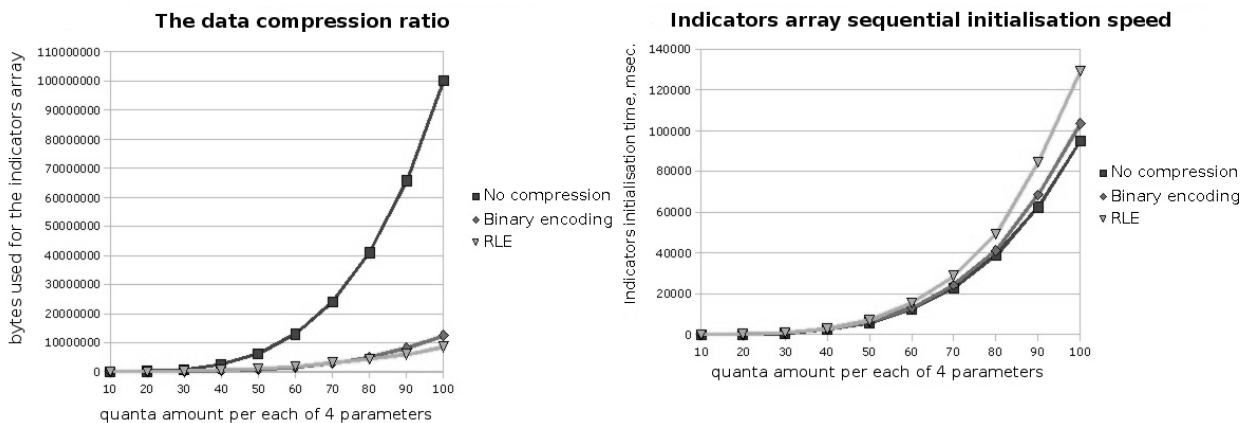


Figure 4. The indicators array compression ratio with RLE algorithm.

The problem of RLE algorithm is low speed of random indicator access. Since the array (13) includes only sequences codes, it is required to obtain the index  $q$  of the pair  $(c_q, l_q)$  which encodes

the indicator  $s_p$  identified by random index  $p$ . The searching for the pair  $(c_q, l_q)$  requires sequential addition of the lengths until  $p$  satisfies (14):

$$L_{q-1} < p \leq L_q, \quad L_q = \sum_{i=1}^q l_i, \quad L_0 = 0. \tag{14}$$

Then  $q$  is the index of the desired pair  $(c_q, l_q)$ , and  $s_p = c_q$ . As can be seen, the access to random indicator takes  $q$  operations of addition. That is the problem of using this algorithm to reduce data redundancy. One of the solutions to this problem is offered in this work. The amount of addition operations when searching  $q$  in (14) can be reduced if the search is started from pair  $(c_r, l_r)$  and previously calculated sum  $L_r < p$ . Thus it is proposed to split the array  $S_{RLE}$  into  $T$  portions with assigning corresponding ranges (15) of indicator indices:

$$(p_{i\min}, p_{i\max}), \quad \forall i = 1, 2, \dots, T, \tag{15}$$

Moreover,  $p_{i\min} = p_{(i-1)\max} + 1, \forall i = 2, 3, \dots, T$  and  $p_{T\max} = R$ . In this case, the search of the desired pair  $(c_q, l_q)$  starts from the search of the portion  $t$ , which encodes the indicators with the indices from the range (15) where index  $p$  falls:

$$p_{t\min} \leq p \leq p_{t\max}. \tag{16}$$

After the portion  $t$  of pairs is found, the search of desired pair  $(c_q, l_q)$  is performed according to (14) as it was described before, but with significant improvement in the form of  $L_0 = p_{t\min} - 1$ . If the algorithm of binary search is used for finding the portion according to (16), it takes  $O(\log_2 T)$  operations, that is much less than linear sum (14).

Another advantage of indicator array splitting is the possibility of its parallel processing. As was said before, high computational requirements are another problem of the region of acceptability approximation. Thus, the solving of this task is almost incogitable without parallel computations. The algorithm (Abramov & Nazarov 2011) of region of acceptability approximation on the basis of the model (11) represents the same instructions performed for every elementary box (SPMD – Single Process, Multiple Data). This fact allows for task decomposition on the basis of indicators array splitting.

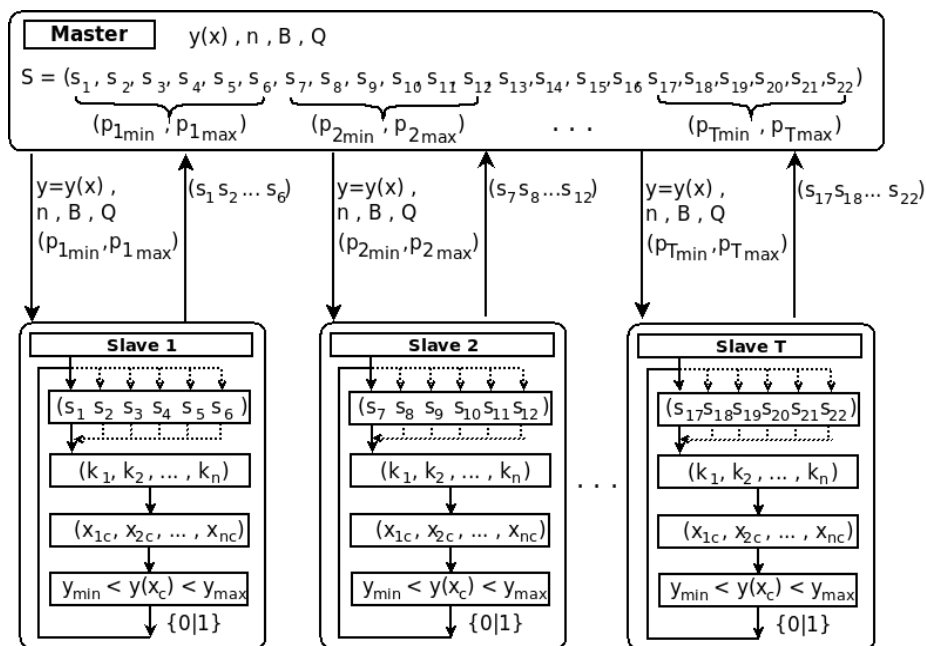


Figure 5. The indicators array decomposition and its parallel processing.

The construction of the region of acceptability approximation on the basis of the model (11) in parallel processes requires passing the model parameters  $n$ ,  $B$ ,  $Q$  and the range (15) to every parallel process. Using these parameters, every process is able to restore univalent indices  $(k_1, k_2, \dots, k_n)$  on the every iteration of indicators enumeration inside the range, and, consequently, to calculate the performances (1) and membership function (9). This process is illustrated in Figure 5.

#### 4 CONCLUSIONS

The problem of large amount of data in the framework of the region of acceptability construction on the basis of approximation with a discrete set of elementary boxes is considered in this work. The ways of reducing of the data redundancy and corresponding problem of data access are considered. The methods of increasing random access speed on compressed data are offered. The efficiency of indicators array partitioning both for access speed and for the task decomposition for using of parallel computations technique is presented in this work.

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