APPLICATION OF RELIABILITY GROWTH MODELS TO SENSOR SYSTEMS

Swajeeth Pilot Panchangam, V. N. A. Naikan

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Reliability Engineering Centre, Indian Institute of Technology,
Kharagpur, West Bengal, India-721302
e-mail: swajeeth@gmail.com, naikan@hijli.iitkgp.ernet.in

ABSTRACT

This paper presents the three reliability growth models namely Duane, AMSAA, and ERG II models briefly. The paper compares the three models for both time and failure terminated tests. Comparisons are carried out by simulating and conducting the statistical hypothesis t-test on twenty sets of failure data. An inference made from statistical hypothesis t-test is that AMSAA model is better choice for time terminated reliability growth test. Duane model is better choice for failure terminated reliability growth test. This is based on comparison with ERG II model which is expected to give best results. Case study on reliability growth tests of strain gauge of pressure sensor used in propulsion systems of satellites is also presented.

NOTATIONS

TT	Time terminated test
FT	Failure terminated test
t_o	$t_{observed}$
IMTBF	Instantaneous mean time between failures
ERG	Exponential reliability growth

KEYWORDS

Duane model; AMSAA model; ERG II model; time terminated test; failure terminated tes; reliability growth.

1 INTRODUCTION

Reliability of sensors used for monitoring various parameters of critical systems is very important for timely assessment of their health and to take appropriate measures for fault diagnosis at incipient stages in order to prevent any catastrophic failures. Reliability growth models help a sensor to undergo series of improvement stages till a final design is frozen for meeting the target reliability and MTBF requirements. This type of modelling enables sensors for successful operation in critical applications such as propulsion systems of a satellite, nuclear power plants, aircraft systems etc. Such systems require continuous reliable monitoring system to avoid unexpected failures which might result in huge economic losses apart from ill effects on environment, health & safety of human beings and other species. Instead of spending huge amounts on replacement/repair of industrial systems due to unreliable sensors it may be better to have high reliable sensors by conducting reliability growth tests. An attempt is made in this paper to compare the three reliability growth models namely Duane, AMSAA, and ERG II models. All these tests are performed for both time and failure terminated test on the strain gauge of a pressure sensor used in propulsion system of a satellite. This is explained in detail with case studies.

The rest of the paper is organized as follows:

In Section-2 the problem statement and reliability growth model approach for sensors are presented. In Section-3, drawback of Duane and AMSAA reliability growth models are presented. A method is proposed in section-4 for comparing reliability growth models for both time terminated as well as failure terminated test data. In section-5, case studies related to time and failure terminated reliability growth tests on a strain gauge of a pressure sensor which is used in propulsion system of a satellite are discussed. Results of this paper are presented in section-6. Conclusions are presented in section-7, followed by selected references.

2 RELIABILITY GROWTH TEST FOR SENSORS

A. Problem statement

Operational health of critical systems like aircrafts, propulsion systems of satellite, nuclear power plants etc, can be assessed by monitoring their critical parameters using sensors. Application of highly reliable sensors is essential for successful operation of the system till the end of mission time. For this purpose, sensor design and development must undergo a series of improvement stages till a final design is frozen for meeting the target reliability and MTBF requirements. A series of life tests need to be conducted after every design improvement. The results of this test will reveal whether the design has reached the target requirements. This procedure is known as reliability growth tests. Several types of reliability growth models such as Duane, AMSAA, and more recently ERG II models have been developed over the years for this purpose [1], [2], [3], [4].

This paper is focused on this important problem. Research objective is to fit the reliability growth model for sensor failure data using ERG II model [4] for both time and failure terminated tests.

Reliability growth model approach

The objective of reliability growth testing of sensors is to improve reliability over time through changes in product design, in manufacturing processes and procedures [5]. This is accomplished through test-fix-test-fix cycle. Reliability growth test is performed on prototypes. During test if any failure occurs due to critical failure mode [6], the failure mode is eliminated through redesign and the test cycle is repeated. Finally, the test continues till the required target MTBF is achieved. In this paper we discussed and compared Duane, AMSAA, and ERG II reliability growth models with case studies.

Redesign of a sensor's component should be in such a way that obeys the following characteristics of a good design.

- Reliability
- Long useful life
- Low maintenance and noise level (if any)
- High accuracy
- Low cost
- Attractive appearance
- Trouble free
- simplicity

Reliability growth test eliminates the sensor components which obey the following characteristics:

- Poor accuracy
- High noise level and non adjustable
- Poor reliability
- Non-repairable and flimsy
- Wear out
- Rattles and rusts
- Cracks and corrodes
- High maintenance costs
- Short useful life

Design/redesign process of sensor components should include the following steps:

Step 1: Perform analysis requirements.
Step 2: Define the scope, objectives, and pertinent restraints with respect to the requirements; identify any significant problem, which has to be solved.
Step 3: Develop alternative design
Step 4: Perform feasibility analysis of alternative design
Step 5: Optimize the promising design
Step 6: Select the design for use
Step 7: Implement the design

3 DRAW BACK OF DUANE AND AMSAA RELIABILITY GROWTH MODELS

A. Sensor and Sensor System

Duane [1] and AMSAA [2] are frequently used reliability growth models. These models when applied to sensors, the failure intensity versus time of operation graph is continuous. So, the exact failure intensity of a sensor after fixing an occurred failure through redesign in not known. Exact test-fix-test-fix cycles are not known in these models. ERG II model [4], in which failure intensity versus time of operation plot for sensor is discontinuous (step wise). Exact failure intensity after fixing failure can be known in this model. Therefore, ERG II model is an accurate reliability growth model for sensors. This is further explained in detail in section 4 & 5 with case studies.

ERG II model is briefly discussed in this section. Sen et.al., proposed a constant-step failure-rate model called ERG II model [4], amounting to the assumption that inter failure times $X_i = T_i - T_{i-1}$, (i= 1, 2…) denoting the successive failure times are independent exponential random variables with hazard λi , (i= 1, 2….), the structure of λ_i is assumed to be of the form as shown below [4].

$$\lambda_i = \frac{\mu}{i^{\delta} - (i-1)^{\delta}}, \mu > 0, \delta \geq 1 \qquad (1)$$

Where, ‘μ’ is scale parameter and ‘δ’ is shape parameter of ERG II model.

MTBF (estimated) in time terminated test is:

$$\theta(t^*) = \frac{n^{\delta} - (n-1)^{\delta}}{\mu} \qquad (2)$$

MTBF (estimated) in failure terminated test is:

$$\theta(t_n) = \frac{n^{\delta} - (n-1)^{\delta}}{\mu} \qquad (3)$$

For more details about this model readers may refer [4].

4 SELECTION OF RELIABILITY GROWTH MODEL

Error in estimation using any model is defined as the deviation of the estimated parameter from the desired parameter. Any natural random errors usually follow normal distribution [7], [8] if the sample size for estimation is large (at least 30). For smaller sample sizes t-test is used as an approximation of normal distribution for hypothesis testing. In this section, a method based on statistical sampling technique is used to find the better model among Duane and AMSAA models for time and failure terminated reliability growth tests by comparing with accurate model i.e. ERG II model. A t-test is applied for statistically concluding which is the better model for two test cases namely time terminated and failure terminated tests.

4.1. t-test

The t-test is a statistical method for testing the degree of difference between two means in small sample. It uses t-distribution theory to deduce the probability when difference happens, then judges whether the difference between two means is significant.

For further details on t-test refer [9].

4.2. Time terminated test

In time terminated test, we conduct reliability growth test on a sensor up to certain mission time and then we terminate the test. After termination we estimate the instantaneous MTBF from the test data obtained. In time terminated test, given n successive failure times $t_1 < t_2 < \cdots < t_n$ that occur prior to the accumulated test time or observed system time, T.

Twenty sets of failure data (assumed for hypotheses) are simulated for conducting the hypothesis test for comparison of reliability growth models. The failure data includes sets of 3, 4, 5, 6, 7, 8, 9, and 10 numbers of failures. The range of failure data is between 0 and 30 minutes i.e. the mission time for time terminated test. The failure data is simulated for evaluating IMTBF in time terminated Duane, AMSAA, and ERG II models. The results obtained after simulating twenty samples are tabulated as shown in table 1. A t-test is performed between ERG II and Duane models and the results are compared with a similar t-test between ERG II and AMSAA models for selecting a more appropriate model.

The $T_{critical}$ value with degree of freedom of 19, with 95% confidence interval is obtained as **2.093**.

From t-test the t_0 value obtained for Duane model in comparing with ERG II model is **-2.100**.

Similarly, from t-test, the t_0 value obtained for AMSAA model in comparing with ERG II model is **+1.769**.

Fig.1 shows the results of t-test between ERG II and Duane, ERG II and AMSAA models in time terminated test for twenty samples.

Table 1. IMTBF in TT reliability growth test for different simulations

Simulation Number	Number of failures	IMTBF (in min)		
		Duane	AMSAA	ERG II
1	3	9.4507	11.012	12.245
2	3	10.1418	10.357	10.335
3	3	11.9593	9.3678	8.6458
4	4	8.2706	11.997	12.425
5	4	17.5087	13.057	12.417
6	4	12.0615	9.1239	11.563
7	4	9.9432	12.799	11.548
8	5	8.9326	7.3740	6.8882
9	5	15.4799	9.6276	11.479
10	5	12.5313	8.7551	9.1577
11	6	11.5370	8.0476	9.4361
12	6	9.2731	7.3194	7.1354
13	6	6.7138	5.5993	5.8971
14	7	7.4591	5.7930	6.2416
15	7	8.3275	5.6956	6.5554
16	8	6.8532	5.6475	4.9845
17	8	6.9742	5.3884	5.7987
18	9	6.2077	4.6915	5.3578
19	9	7.8065	4.9501	5.4523
20	10	6.3343	4.8001	5.5487

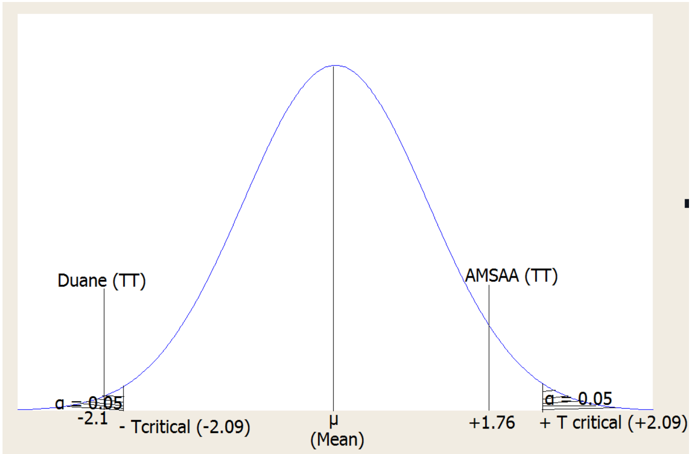

Fig.1. Critical values and t_o values of T-test between ERG II and Duane, ERG II and AMSAA models in TT test.

From fig.1, the inferences are as following:

a) t-test has shown that the Duane model is falling beyond the critical value, and
b) AMSAA model is within the critical limits.

Therefore, statistically AMSAA model is better model than Duane model for time terminated reliability growth test.

This is further explained using a case study of reliability growth test on a strain gauge of a pressure sensor which is used in propulsion system of a satellite in section 5. The strain gauge is selected for this test due to its high failure rate compared to other components of a pressure sensor [10]. This strain gauge has to operate successfully in propulsion system of a satellite for 30 minutes, by sustaining huge amounts of pressure.

4.3. Failure terminated test

In failure terminated test, we conduct reliability growth test on a sensor up to certain number of failure occurs irrespective of time and then we terminate the test. After termination we estimate the IMTBF from the test data obtained. In failure terminated test, given n successive failure times $t_1 < t_2 < \cdots < t_n$ following accumulated test time or observed system time $T = t_N$.

Similar analysis as in section 4.2 has been carried out on failure terminated simulation data which is presented in table 2.

From t-test, the t_0 value obtained for Duane model in comparing with ERG II model is **+2.03**.

Similarly, from t-test, the t_0 value obtained for AMSAA models in comparing with ERG II model is **-4.12**.

Fig.2 shows the results of t-test between ERG II and Duane, ERG II and AMSAA models in time terminated test for twenty samples.

Table 2. IMTBF in FT reliability growth test for twenty different simulations

Simulation Number	Number of failures	IMTBF (in min)		
		Duane	AMSAA	ERG II
1	3	9.4507	5.2117	9.3179
2	3	10.1418	5.9038	10.283
3	3	11.9593	6.2871	11.472
4	4	9.9432	5.7126	8.4592
5	4	17.5087	8.7312	13.147
6	4	12.0615	6.9772	10.635
7	4	8.2706	5.2157	7.8853
8	5	8.9326	6.9316	9.1426
9	5	15.4799	9.3680	12.900
10	5	12.5313	8.5101	11.167
11	6	11.5370	7.8308	10.1502
12	6	9.2731	7.1148	10.133
13	6	6.7138	4.9040	7.0484
14	7	7.4591	5.7259	7.1179
15	7	8.3275	5.6292	10.250
16	8	6.8532	5.3363	6.5220
17	8	6.9742	5.3884	6.5592
18	9	6.2077	4.6378	5.7306
19	9	7.8065	4.9501	6.5423
20	10	6.3343	4.8001	5.8250

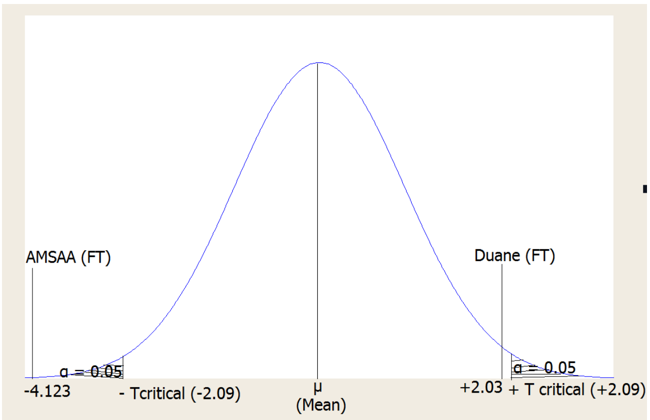

Fig.2. Critical values and t_o values of t-test between ERG II and Duane, ERG II and AMSAA models in FT test.

From fig.2, the inferences are as following:

a) t-test has been shown that the AMSAA model is falling beyond the critical value, and
b) Duane model is within the critical limits.

Therefore, statistically Duane model is better model than AMSAA model for failure terminated reliability growth test.

This is further explained using a case study of reliability growth test on a strain gauge of a pressure sensor which is used in propulsion system of a satellite in section 5.

5 CASE STUDY

Case study- I: Consider a time terminated reliability growth test on a strain gauge of a pressure sensor. Assume that the test is conducted on it for a mission time of 30 minutes. The strain gauge is subjected to as high pressures as in the actual propulsion system. For each failure occurred, failure cause is analysed & eliminated. It is redesigned and again the reliability growth test is conducted. The table 3 shows the assumed times at which the initial deigned strain gauge and redesigned same strain gauge failed until 30 minutes.

Table 3. Failure times of strain gauge for each redesign

S.No	Design	Failure time (in min)
1	D1 (initial design)	1.5
2	D2	4.6
3	D3	10.5
4	D4	18.6

We require evaluating the following parameters using Duane, AMSAA, & ERG II models.

a) IMTBF at the end of mission time, and additional test time required to achieve an IMTBF of 30 minutes.
b) Based on above results estimate & comment on the better fitting model between Duane and AMSAA models for a pressure sensor next to ERG II model.

Solution:

a) Adopting the mathematical forms of Duane, AMSAA, and EG II models from [1], [2], & eq. 2 respectively the IMTBF obtained using above three models are as shown in table 4.

Table 4. IMTBF of a strain gauge in TT reliability growth test

S.No	Reliability growth model	IMTBF (in min)
1	Duane	8.2706
2	AMSAA	11.9976
3	ERG II	12.4256

The failure intensity versus time plot for above three models is as shown in fig.3, fig.4, & fig.5 respectively.

Fig.3. Failure intensity vs time plot using Duane model for TT test

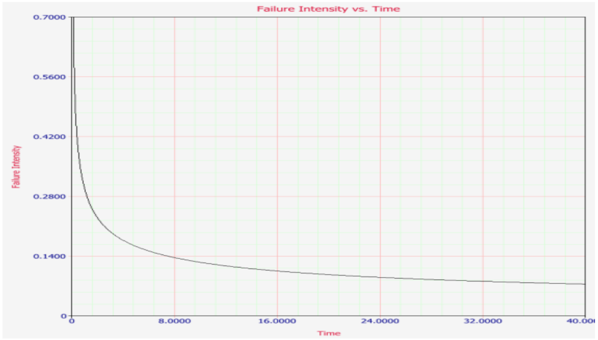

Fig.4. Failure intensity vs time plot using AMSAA model for TT test

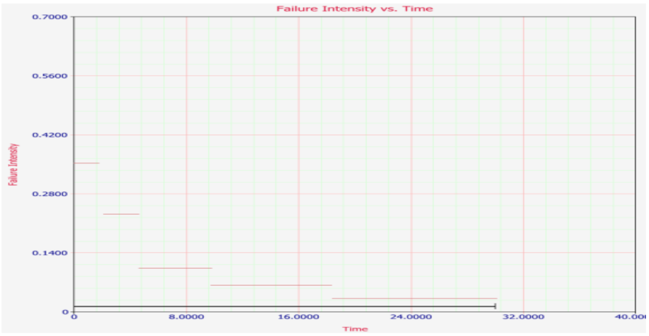

Fig.5. Failure intensity vs time plot using ERG II model for TT test

Table 5. IMTBF at different test times

Time (in min)	IMTBF (in min)
100	17.6617
200	24.1445
300	28.9899
400	33.0067
500	36.5018
600	39.6305
700	42.4842

The extrapolated data obtained using Duane mathematical forms [1] are as shown in table 5.

From table 5, approximately for 350 minutes the strain gauge should be conducted reliability growth test to achieve an instantaneous MTBF of 30 minutes.

∴ 320 additional minutes is required in Duane reliability growth model test to achieve an instantaneous MTBF of 30 minutes.

The extrapolated data obtained using Duane mathematical forms [2] are as shown in table 6.

Table 6. IMTBF at different test times

Time (in min)	**IMTBF (in min)**
100	18.8414
200	24.4319
300	28.4427
400	31.6816
500	34.4458
600	36.8824
700	39.0766

From table 6, approximately for 330 minutes the strain gauge should be conducted reliability growth test to achieve an IMTBF of 30 minutes.

∴ 300 additional minutes is required in AMSAA reliability growth model test to achieve an IMTBF of 30 minutes.

The extrapolated data obtained using Duane mathematical forms [4] are as shown in table 7.

Table 7. IMTBF at different test times

Time (in min)	**IMTBF (in min)**
100	19.8254
200	25.5132
300	29.8645
400	32.4430
500	35.6524
600	38.8960
700	41.1422

From table 7, approximately for 310 minutes the strain gauge should be conducted reliability growth test to achieve an instantaneous MTBF of 30 minutes.

∴ 280 additional minutes is required in ERG II reliability growth model test to achieve an IMTBF of 30 minutes.

b) Finally, from figures 3, 4, & 5 and tables 5, 6, and 7, we conclude that ERG II is the accurate model (due to step diagram as shown in fig.5, the value of failure intensity at any time instant is well defined) and AMSAA model better fits pressure sensor failure data compared to Duane model in time terminated reliability growth test.

Case study- II: Consider a failure terminated reliability growth test on a strain gauge of a pressure sensor. The test is conducted on it until four failures occurs. The strain gauge is subjected to as high pressures as in the actual propulsion system. Assuming (for illustrative purpose) that the data presented in the table 3 is that of failure terminated test, we need to evaluate the same parameters as in the previous case.

Solution:

a) Adopting the mathematical forms of Duane, AMSAA, and EG II models [1], [2], & eq.3 respectively the IMTBF obtained using above three models are as shown in table 8.

Table 8. IMTBF of strain gauge in FT reliability growth test

S.No	Model	IMTBF (in min)
1	Duane	8.2706
2	AMSAA	5.2157
3	ERG II	7.8853

The failure intensity versus time plot for three models in failure terminated test is as shown in fig.6, fig.7, & fig.8 respectively.

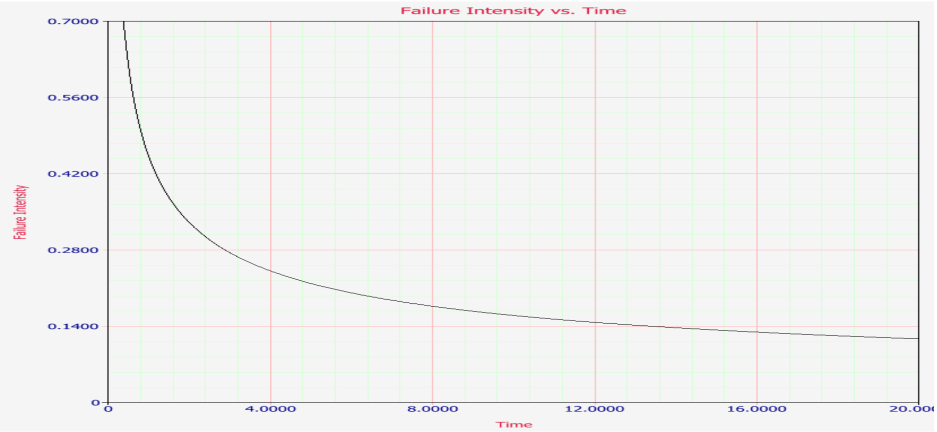

Fig.6. Failure intensity vs time plot using Duane model for FT test

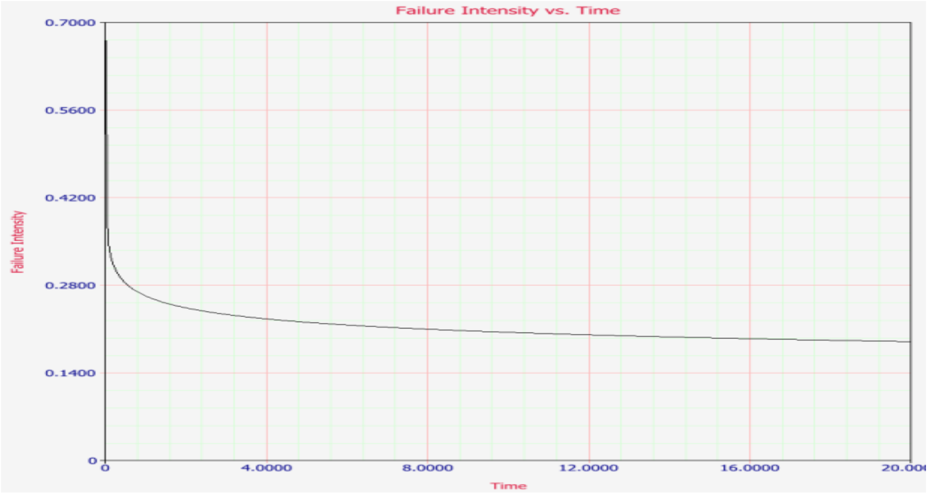

Fig.7. Failure intensity vs time plot using AMSAA model for FT test

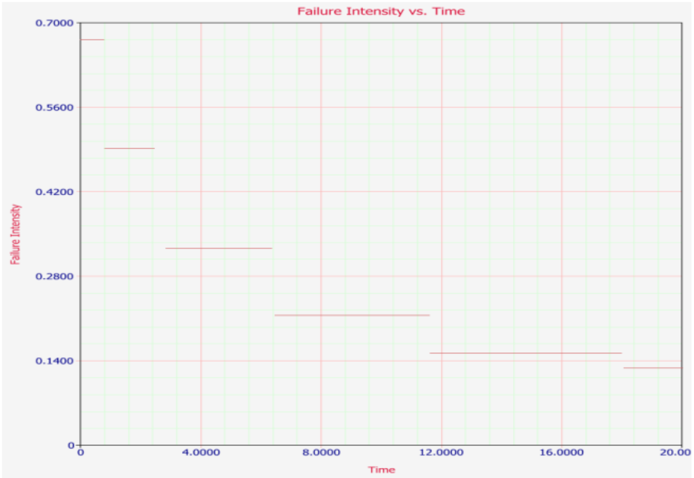

Fig.8. Failure intensity vs time plot using ERG II model for FT test

b) ERG II reliability growth model is the accurate model. Comparing the other two model's results with ERG II, we therefore conclude that pressure sensor failure data can be better fit with Duane model compared to AMSAA model for failure terminated reliability growth test.

6 RESULTS

The results obtained from section- 4 & 5 are tabulated in tables 9, & 10 as shown below:

From section-4, we statistically conclude the following:

i. AMSAA model is the better model compared to Duane for time terminated reliability growth test.
ii. Duane model is the better model compared to AMSAA for failure terminated reliability growth test.

Table 9. IMTBF & additional time required for strain gauge in TT test

S.No	Model	IMTBF (in min)	Additional time required (in min)
1	Duane	8.2706	320
2	AMSAA	11.9976	300
3	ERG II	12.4256	280

Table 10. Instantaneous MTBF of strain gauge in FT test

S.No	Model	IMTBF (in min)
1	Duane	8.2706
2	AMSAA	5.2157
3	ERG II	7.8853

7 CONCLUSIONS

In this paper, three reliability growth models namely Duane, AMSAA, and ERG II models are critically analysed using the reliability growth test data on strain gauge of a pressure sensor used in propulsion systems of satellites. Using simulated data on twenty samples and using the t-test, it is found that the AMSAA model is a better choice for reliability growth analysis of time terminated tests. It is also found that the Duane model is a better choice for reliability growth analysis of failure terminated tests. This is based on the comparison of the two models with the ERG II model which is expected to give the best results. The additional time required to conduct reliability growth test for achieving the target instantaneous MTBF are also evaluated for these models.

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