ESTIMATION IN CONSTANT STRESS PARTIALLY ACCELERATED LIFE TESTS FOR RAYLEIGH DISTRIBUTION USING TYPE-I CENSORING

S. ZARRIN, M. KAMAL, S. SAXENA
Department of Statistics & Operations Research
Aligarh Muslim University
Aligarh, India
shaziazarrin@gmail.com

ABSTRACT

Partially Accelerated life tests are used when the data obtained from Accelerated life tests cannot be extrapolated to use conditions. This study deals with simple Constant Stress Partially Accelerated life tests using type-I censoring. The lifetime distribution of the test item is assumed to follow Rayleigh distribution. The maximum likelihood estimates are obtained for the distribution parameter and acceleration factor. In addition, asymptotic variance and covariance matrix of the estimators are given. Interval estimation that generates narrow intervals to the parameters of the distribution with high probability is obtained. Simulation procedure is used to illustrate the statistical properties of the parameters and the confidence bounds.

Key words: Acceleration factor; Maximum likelihood estimation; Reliability function; constant stress; Fisher Information matrix; generalized asymptotic variance; optimum test plans; time censoring

1. INTRODUCTION

The continuous improvement in manufacturing design creates a problem in obtaining information about lifetime of some products and materials with high reliability at the time of testing under normal conditions. Under such conditions the life testing becomes very expensive and time consuming. To obtain failures quickly, a sample of these materials is tested at more severe operating conditions than normal ones. These conditions are referred to as stresses, which may be in the form of temperature, voltage, force, humidity, pressure, vibrations, etc. This type of testing is called accelerated life testing (ALT), where products are run at higher-than-usual stress conditions, to induce early failures in a short time. The life data from the high stresses are used to estimate the life distribution at design condition, see Abdel-Hamid et al., (2009).

ALT is of mainly three types. The first is the constant stress ALT. It is used when the stress remain unchanged, that is, if the stress is weak, the stress has to run for a long time. The second is referred to as step-stress accelerated life test (SSALT) and the third is progressive-stress ALT. These three methods can reduce the testing time and save a lot of manpower, material and money, see Rao (1992). The main assumption in ALT is that the mathematical model relating the lifetime of the unit and the stress is known or can be assumed. In some cases, such life stress relationships are not known and cannot be assumed, i.e., the data obtained from ALT cannot be extrapolated to use condition. So, in such cases, another approach can be used, which is partially accelerated life tests (PALT). In PALT, test units are run at both usual and higher-than usual stress conditions see Abd-Elfattah et al., (2008).

The stress loading in a PALT can be applied in various ways. They include step-stress, constant-stress and random-stress. Nelson (1990) discussed their advantages and disadvantages. One way to accelerate failures is step-stress which increases the stress applied to test products in a specified discrete sequence. Generally, a test unit starts at a certain low stress. If the unit does not fail at a particular time, stress on it raised and held to a specified time. Stress is repeatedly increased until test unit fails or censoring time is reached. But in a constant-stress PALT each test item is run at either normal use condition or at accelerated condition only, i.e., each unit is run at a constant-stress level until the test is terminated.

For an overview of constant-stress PALT, there is amount of literature on designing PALT. Bai and Chung, 1992 discussed both, the problem of estimation and optimally designing PALT for test item having an exponential distribution. They also considered the problem of optimally designing constant-stress PALT that terminates at a pre-determined time. For items having lognormally distributed lives, PALT plans were developed by Bai et al., 1993a. Abdel-Ghaly et al. (2003) discussed the problem of parameter estimation for Pareto distribution using PALT in case of type-I censoring. More recently, Ismail (2004) used the maximum likelihood approach for estimating the acceleration factor and the parameters of Pareto distribution of the second kind. This work was conducted under constant-stress PALT in the case of type-I and type-II censored data. Also, the problem of optimal design was considered for this type of PALT. Since Abdel-Ghani (1998) considered only the estimation problem in constant-stress PALT for the Weibull distribution parameters, the present investigation extends this work in which statistically optimal PALT plan is developed under type-I censoring. This article is to focus on the maximum likelihood method for estimating the acceleration factor and the parameters of Rayleigh distribution. This work was conducted for constant-stress PALT under type I censored sample. The confidence intervals of the estimators will be obtained.

2. THE MODEL

The lifetimes of the test items are assumed to follow a Rayleigh distribution. The probability density function (pdf) of the Rayleigh distribution is given by

$$f(t) = \frac{t}{\theta^2} \exp\left(-\frac{t^2}{2\theta^2}\right), \qquad 0 \le t < \infty,\ \theta > 0 \tag{1}$$

And the cumulative distribution function (cdf) is given by

$$F(t) = 1 - \exp\left(-\frac{t^2}{2\theta^2}\right), \qquad 0 \le t < \infty,\ \theta > 0 \tag{2}$$

where, θ is the scale parameter.

The reliability function of the Rayleigh distribution takes the form

$$R(t) = \exp\left(-\frac{t^2}{2\theta^2}\right), \tag{3}$$

and the corresponding hazard rate is given by

$$h(t) = \frac{t}{\theta^2};$$

The Rayleigh distribution has played an important role in modeling the lifetime of random phenomena. It arises in many areas of applications, including reliability, life testing and survival analysis. Rayleigh distribution is a special case of Weibull distribution (shape parameter=2). Rayleigh distribution is frequently used to model wave heights in oceanography, and in communication theory to describe hourly median and instantaneous peak power of received radio signals. It has been used to model the frequency of different wind speeds over a year and a wind turbine sites. The distance from one individual to it nearest neighbor when the spatial pattern is generated by Poisson distribution follows a Rayleigh distribution. In communication theory, Rayleigh distribution is used to model scattered signals that reach a receiver by multiple paths. Depending on the density of scatter, the signal will display different fading characteristics. Rayleigh distribution is used to model dense scatter.

In a constant-stress PALT, all of the n items are divided into two parts. nr items are randomly chosen among n items, which are allocated to accelerated conditions and the remaining $n(1-r)$ are allocated to normal use conditions, where r is proportion of sample units allocated to accelerated condition then each test item is run until the censoring time τ and the test condition is not changed. Some assumptions are also made in a constant-stress PALT.

- The lifetimes $T_i, i = 1,\ldots,n(1-r)$ and $X_j, i = 1,\ldots,nr$, of items allocated to normal and accelerated conditions, respectively, are i.i.d. random variables.
- The lifetimes T_i and X_j are mutually statistically independent.

3. MAXIMUM LIKELIHOOD ESTIMATION

The maximum likelihood estimation (MLE) is one of the most important and widely used methods in statistics. It is commonly used for the most theoretical model and kinds of censored data. The idea behind the maximum likelihood parameter estimation is to determine the estimates of the parameter that maximizes the likelihood of the sample data. Also the MLEs have the desirable properties of being consistent and asymptotically normal for large samples.

In a simple constant stress PALT, the test item is run at use condition or at accelerated condition only. A simple constant stress plan uses only two stresses and allocates the n sample units to them; see Miller and Nelson, 1983.
In this study, the lifetimes of test items are assumed to follow a Rayleigh distribution.
The pdf of an item tested at use condition is given by

$$f(t) = \frac{t}{\theta^2}\exp\left(-\frac{t^2}{2\theta^2}\right), \qquad t \ge 0$$

and for an item tested at accelerated condition, the pdf is given by

$$f(x) = \frac{\beta^2 x}{\theta^2}\exp\left(-\frac{(\beta x)^2}{2\theta^2}\right), \quad x \ge 0$$

where $X = \beta^{-1}T$, β is the acceleration factor which is the ratio of mean life at use condition to that at accelerated condition, usually $\beta > 1$.

Let δ_{ui} and δ_{aj} be the indicator functions, such that

$$\delta_{ui} = \begin{cases} 1 & t_i \le \tau \\ 0 & otherwise \end{cases} \qquad i = 1,2,\ldots,n(1-r)$$

and

$$\delta_{aj} = \begin{cases} 1 & x_j \le \tau \\ 0 & otherwise \end{cases} \qquad j = 1,2,\ldots,nr$$

The likelihood functions for (t_i, δ_{ui}) and (x_j, δ_{aj}) are respectively given by

$$L_{ui}(t_i,\delta_{ui}\mid\theta)=\prod_{i=1}^{n(1-r)}\left\{\frac{t_i}{\theta^2}\exp\left(-\frac{t_i^{\,2}}{2\theta^2}\right)\right\}^{\delta_{ui}}\left\{\exp\left(-\frac{\tau^2}{2\theta^2}\right)\right\}^{\bar{\delta}_{ui}} \tag{4}$$

$$L_{aj}(x_j,\delta_{aj}\mid\beta,\theta)=\prod_{j=1}^{nr}\left\{\frac{\beta^2 x_j}{\theta^2}\exp\left(-\frac{(\beta x_j)^2}{2\theta^2}\right)\right\}^{\delta_{aj}}\left\{\exp\left(-\frac{(\beta\tau)^2}{2\theta^2}\right)\right\}^{\bar{\delta}_{aj}} \tag{5}$$

where, $\bar{\delta}_{ui}=1-\delta_{ui}$ and
$\bar{\delta}_{aj}=1-\delta_{aj}$.

The total likelihood function for
$(t_1;\delta_{u1},\ldots,t_{n(1-r)};\delta_{u\,n(1-r)},x_1;\delta_{a1},\ldots,x_{nr};\delta_{a\,nr})$ is given by

$$L(t,x\mid\beta,\theta)=\prod_{i=1}^{n(1-r)}\left\{\frac{t_i}{\theta^2}\exp\left(-\frac{t_i^{\,2}}{2\theta^2}\right)\right\}^{\delta_{ui}}\left\{\exp\left(-\frac{\tau^2}{2\theta^2}\right)\right\}^{\bar{\delta}_{ui}}$$
$$\prod_{j=1}^{nr}\left\{\frac{\beta^2 x_j}{\theta^2}\exp\left(-\frac{(\beta x_j)^2}{2\theta^2}\right)\right\}^{\delta_{aj}}\left\{\exp\left(-\frac{(\beta\tau)^2}{2\theta^2}\right)\right\}^{\bar{\delta}_{aj}} \tag{6}$$

It is usually easier to maximize the natural logarithm of the likelihood function rather than the likelihood function itself. Therefore, the logarithm of (6) is

$$\ln L=\sum_{i=1}^{n(1-r)}\delta_{ui}\left[\ln t_i-2\ln\theta-\frac{t_i^2}{2\theta^2}\right]-\frac{\tau^2}{2\theta^2}\sum_{i=1}^{n(1-r)}(1-\delta_{ui})$$
$$+\sum_{j=1}^{nr}\delta_{aj}\left[\ln x_j+2\ln\beta-2\ln\theta-\frac{\beta^2x_j^2}{2\theta^2}\right]-\frac{\beta^2\tau^2}{2\theta^2}\sum_{j=1}^{nr}(1-\delta_{aj}) \tag{7}$$

MLEs of β and θ are solutions to the system of equations obtained by letting the first partial derivatives of the total log likelihood be zero with respect to β and θ, respectively. Therefore, the system of equations is as follows:

$$\frac{\partial\ln L}{\partial\theta}=-\frac{2}{\theta}(n_u-n_a)+\frac{1}{\theta^3}\sum_{i=1}^{n(1-r)}\delta_{ui}\,t_i^2+\frac{\tau^2}{\theta^3}\{n(1-r)-n_u\}+\frac{\beta^2}{\theta^3}\sum_{j=1}^{r}\delta_{aj}x_j^2$$
$$+\frac{\beta^2\tau^2}{\theta^3}(nr-n_a)=0 \tag{8}$$

$$\frac{\partial\ln L}{\partial\beta}=\frac{2n_a}{\beta}-\frac{\beta}{\theta^2}\sum_{j=1}^{nr}\delta_{aj}x_j^2-\frac{\beta\tau^2}{\theta^2}(nr-n_a)=0 \tag{9}$$

where n_u and n_a are the number of items failed at normal and accelerated conditions respectively.

From (8) and (9), the MLEs of β and θ can be expressed as

$$\tilde{\theta}=\left[\frac{\sum_{i=1}^{n(1-r)}\delta_{ui}\,t_i^2+\tau^2\{n(1-r)-n_u\}+\beta^2\sum_{j=1}^{nr}\delta_{aj}x_j^2+\beta^2\tau^2(nr-n_a)}{2(n_u-n_a)}\right]^{\frac{1}{2}} \tag{10}$$

and,

$$\tilde{\beta}=\left[\frac{2n_a}{\frac{1}{\theta^2}\sum_{j=1}^{nr}\delta_{aj}x_j^2+\frac{\tau^2}{\theta}(nr-n_a)}\right]^{\frac{1}{2}} \tag{11}$$

From (11), $\tilde{\beta}$ can be calculated easily, and once the value of $\tilde{\beta}$ is obtained then $\tilde{\theta}$ can also be estimated by substituting the value of $\tilde{\beta}$ in (10). The asymptotic variance-covariance matrix of β and θ is obtained by numerically inverting the Fisher-information matrix composed of the negative second derivatives of the natural logarithm of the likelihood function evaluated at the ML estimates. The asymptotic Fisher-information matrix can be written as:

$$F=\begin{bmatrix}-\frac{\partial^2\ln L}{\partial\theta^2} & -\frac{\partial^2\ln L}{\partial\beta\partial\theta}\\ -\frac{\partial^2\ln L}{\partial\theta\partial\beta} & -\frac{\partial^2\ln L}{\partial\beta^2}\end{bmatrix}$$

The elements of the above information matrix can be expressed by the following equations:

$$\frac{\partial^2\ln L}{\partial\theta^2}=\frac{2}{\theta^2}(n_u-n_a)-\frac{1}{\theta^4}\left[\sum_{i=1}^{n(1-r)}\delta_{ui}\,t_i^2+\tau^2\{n(1-r)-n_u\}+\beta^2\sum_{j=1}^{nr}\delta_{aj}x_j^2+\beta^2\tau^2(nr-n_a)\right]$$

$$\frac{\partial^2\ln L}{\partial\beta^2}=-\frac{2n_a}{\beta^2}-\frac{1}{\theta^2}\sum_{j=1}^{nr}\delta_{aj}x_j^2-\frac{\tau^2}{\theta^2}(nr-n_a)$$

$$\frac{\partial^2\ln L}{\partial\theta\partial\beta}=\frac{2\beta}{\theta^3}\left[\sum_{j=1}^{nr}\delta_{aj}x_j^2+\tau^2(nr-n_a)\right]$$

$$\frac{\partial^2\ln L}{\partial\beta\partial\theta}=\frac{2\beta}{\theta^3}\left[\sum_{j=1}^{nr}\delta_{aj}x_j^2+\tau^2(nr-n_a)\right]$$

Consequently, the maximum likelihood estimators of β and θ have an asymptotic variance-covariance matrix defined by inverting the Fisher information matrix given above.

4. INTERVAL ESTIMATES

If $L_\lambda = L_\lambda(y_1,\dots,y_n)$ and $U_\lambda = U_\lambda(y_1,\dots,y_n)$ are functions of the sample data $y_1,\dots,y_n$, then a confidence interval for a population parameter λ is given by

$$p[L_\lambda \le \lambda \le U_\lambda] = \gamma \tag{12}$$

where, L_λ and U_λ are the lower and upper confidence limits which enclose λ with probability γ. The interval $[L_\lambda, U_\lambda]$ is called a two sided $100\gamma\%$ confidence interval for λ.
For large sample size, the MLEs, under appropriate regularity conditions, are consistent and asymptotically normally distributed.

Therefore, the two sided approximate $100\gamma\%$ confidence limits for the MLE $\tilde{\lambda}$ of a population parameter λ can be constructed, such that

$$p[-z \le \frac{\tilde{\lambda} - \lambda}{\sigma(\tilde{\lambda})} \le z] = \gamma \tag{13}$$

where, z is the $\left[\frac{100(1-\gamma)}{2}\right]$ standard normal percentile. Therefore, the two sided approximate $100\gamma\%$ confidence limits for a population parameter λ can be obtained such that

$$p[\lambda - z\sigma(\tilde{\lambda}) \le \tilde{\lambda} \le \lambda + z\sigma(\tilde{\lambda})] \cong \gamma \tag{14}$$

Then, the two sided approximate confidence limits for β and θ will be constructed using (14) with confidence levels 95% and 99%.

5. OPTIMUM SIMPLE CONSTANT-STRESS TEST PLANS

This section deals with the problem of optimally designing a simple constant-stress PALT, which terminates at a pre-specified time. Optimum test plans for products having a Rayleigh distribution is developed. Here, the aim is to obtain the optimal proportion of sample units r^*, allocated to accelerated conditions based on the outputs of the stage of the parameter estimation that are in the same time considered inputs to the optimal design stage of the test. The proportion of sample units r, allocated to accelerated condition is pre-specified for the stage of parameter estimation. But for the optimal design stage of the test, r is considered a division parameter that has to be optimally determined according to a certain optimality criterion. The optimality criterion (Abdel-Ghaley et al., 2003) is to find the optimal proportion of sample units r^* allocated to accelerated condition such that the Generalized Asymptotic Variance (GAV) of the MLE of the model parameters at normal use condition is minimized.

Most of the test plans allocate the same number of test units at each stress i.e. they are equally-spaced test stresses. Such test plans are usually inefficient for estimating the mean life at design stress (Yang, 1994). To decide the optimal sample proportion allocated to each stress, statistically optimum test plans are developed. Therefore, to determine the optimal sample proportion r^* allocated to accelerated condition, r is chosen such that the GAV of the ML estimators of the model parameters is minimized. The GAV of the ML estimators of the model parameters as an optimality criterion is defined as the reciprocal of the determinant of the Fisher-Information matrix F (Bai et.al., 1993b). That is,

$$GAV(\tilde{\theta},\tilde{\beta})=\frac{1}{|F|} \tag{15}$$

The minimization of the GAV over τ solves the following equation

$$\frac{\partial GAV}{\partial r}=0 \tag{16}$$

In general, the solution to (16) is not a closed form, so the Newton-Raphson method is applied to obtain r^* which minimizes the GAV. Accordingly, the corresponding expected number of items failed at use and accelerated conditions can be obtained, respectively, as follows

$$n_u^*=n(1-r^*)P_u$$

and,

$$n_a^*=nr^*P_a$$

where,

P_u = Probability that an item tested only use condition fails by τ

P_a = Probability that an item tested only accelerated condition fails by τ

6. SIMULATION STUDIES

Abd-Elfattah et al., (2008), performed the simulation study for a two parameter Burr Type XII distribution using MathCAD (2001) for illustrating the theoretical results of estimation problem. The performance of the resulting estimators of the acceleration factor and two shape parameters has been considered in terms of their absolute relative bias (RABias), mean square error (MSE), and relative error (RE). Furthermore, the asymptotic variance and covariance matrix and two-sided confidence intervals of the acceleration factor and two shape parameters were obtained. Ismail and Aly (2009), considered the case of two stress levels under the failure-step stress partially accelerated life testing assuming type-II censoring for a two parameter Weibull distribution. The MLEs were studied together with some further properties. They also obtained an optimum Failure step-stress PALT numerically using the D-optimality via a simulation study. It was noted via the optimal value of proportion of test units to be observed at two stresses, that the PALT model is more appropriate model. That is, testing at both normal and accelerated conditions. Ismail (2009) performed the simulation study for the Weibull Failure distribution and results of simulation studies provide insight into the sampling behavior of the estimators. The numerical results indicated that the ML estimates approximate the true values of the parameters as the sample size increases and the asymptotic variances of the estimators are decreasing as the sample size is getting to be large.

Ismail (2011) considered the problem of optimally designing a simple time-step-stress PALT which terminates after a pre-specified number of failures and developed optimum test plans for products having a two-parameter Gompertz lifetime distribution. The main objective of his simulation study was to make a numerical investigation for illustrating the theoretical results of both estimation and optimal design problems under consideration.
The data in Table 1 and Table 2 gives the MSE, RBais, RE and variance of the estimators for two sets of parameters $(\theta=4.60,\beta=0.20)$ and $(\theta=4.40,\beta=0.80)$ respectively. While Table 3 and Table

4 presents the approximated two sided confidence limits at 95% and 99% level of significance for the scale parameter and the acceleration factor.

Table 1: The MSE, RBais, RE and Variances of the Parameters $(\theta = 4.60, \beta = 0.20)$ under Type-I Censoring

Size n	Parameters	MSE	RBias	RE	Variance
50	θ	0.0062	0.0009	0.0179	0.0062
	β	0.0306	0.0221	0.2187	0.0303
100	θ	0.1055	0.0006	0.0738	0.1055
	β	0.0047	0.0228	0.0857	0.0044
150	θ	0.0152	0.0014	0.0280	0.0152
	β	0.0024	0.0163	0.0612	0.0022
200	θ	0.0131	0.0016	0.0260	0.0131
	β	0.0209	0.0110	0.1807	0.0208
250	θ	0.0150	0.0033	0.0278	0.0148
	β	0.0028	0.0048	0.0661	0.0028
300	θ	0.0169	0.0191	0.0295	0.0099
	β	0.0016	0.0090	0.0500	0.0015
350	θ	0.0118	0.0204	0.0245	0.0037
	β	0.0197	0.0120	0.1754	0.0196
400	θ	0.0181	0.0219	0.0306	0.0088
	β	0.0029	0.0106	0.0673	0.0028
450s	θ	0.0108	0.0065	0.0236	0.0100
	β	0.0014	0.0080	0.0468	0.0014

Table 2: The MSE, RBais, RE and Variances of the Parameters $(\theta = 4.40, \beta = 0.80)$ under Type-I Censoring

Size n	Parameters	MSE	RBias	RE	Variance
50	θ	0.0227	0.0053	0.0328	0.0221
	β	0.0002	0.0165	0.0707	0.0002
100	θ	0.0399	0.0120	0.0434	0.0369
	β	0.0004	0.0275	0.1000	0.0004
150	θ	0.0914	0.0001	0.0657	0.0914
	β	0.0020	0.0040	0.2236	0.0020
200	θ	0.0162	0.0070	0.0277	0.0152
	β	0.0002	0.0360	0.0707	0.0001
250	θ	0.0266	0.0039	0.0355	0.0263
	β	0.0003	0.0380	0.0066	0.0002
300	θ	0.0698	0.0014	0.0574	0.0698
	β	0.0015	0.0010	0.1973	0.0015
350	θ	0.0107	0.0058	0.0225	0.0100
	β	0.0001	0.0095	0.0500	0.0001

Size n	Parameters	MSE	RBias	RE	Variance
400	θ	0.0014	0.0032	0.0081	0.0012
	β	0.0002	0.0140	0.0707	0.0002
450	θ	0.0166	0.0034	0.0280	0.0163
	β	0.0002	0.0540	0.0707	0.0001

Table 3: Confidence Bounds of the Estimates at Confidence Level at 0.95 and 0.99 $(\theta = 4.60, \beta = 0.20)$

n	Parameters	95%		99%	
		LCL	UCL	LCL	UCL
50	θ	4.2842	4.8669	4.3318	4.8194
	β	0.1689	0.2244	0.1735	0.2199
100	θ	4.2789	5.0319	4.3404	4.9704
	β	0.1663	0.2447	0.1727	0.2383
150	θ	4.0079	5.1931	4.1047	5.0963
	β	0.1115	0.2869	0.1259	0.2725
200	θ	4.3263	4.8094	4.3656	4.7617
	β	0.1876	0.2268	0.1908	0.2236
250	θ	4.3003	4.9361	4.3522	4.8842
	β	0.1647	0.2201	0.1191	0.2657
300	θ	4.0759	5.1115	4.1604	5.0269
	β	0.1239	0.2757	0.1363	0.2633
350	θ	4.3774	4.7694	4.4094	4.7374
	β	0.1979	0.1983	0.1817	0.2144
400	θ	4.5174	4.6532	4.5285	4.6421
	β	0.1695	0.2249	0.1740	0.2204
450	θ	4.3655	4.8659	4.4063	4.8251
	β	0.1676	0.2068	0.1708	0.2036

Table 4: Confidence Bounds of the Estimates at Confidence Level at 0.95 and 0.99 $(\theta = 4.40, \beta = 0.80)$

n	Parameters	95%		99%	
		LCL	UCL	LCL	UCL
50	θ	4.2498	4.5584	4.2749	4.5332
	β	0.4411	1.1235	0.4968	1.0678
100	θ	3.7659	5.0391	3.8698	4.9352
	β	0.6882	0.9482	0.7094	0.9269
150	θ	3.6299	5.1581	4.1918	4.5962
	β	0.6951	0.8789	0.7101	0.8639
200	θ	4.1749	4.6237	4.2116	4.5870
	β	0.5261	1.0915	0.5723	1.0453

n	Parameters	95%		99%	
		LCL	UCL	LCL	UCL
250	θ	4.1472	4.6240	4.1861	4.5851
	β	0.7000	0.9075	0.7170	0.8906
300	θ	4.1212	4.5112	4.1530	4.4794
	β	0.7169	0.8687	0.7293	0.8563
350	θ	4.1909	4.4294	4.2104	4.4099
	β	0.5352	1.0840	0.5800	1.0392
400	θ	4.1199	4.4877	4.1499	4.4576
	β	0.6878	0.8952	0.7047	0.8783
450	θ	4.1755	4.5675	4.2075	4.5355
	β	0.7203	0.8669	0.7322	0.8549

From these tables it is concluded that for the first set of parameters $(\theta = 4.60, \beta = 0.20)$, the ML estimates have good statistical properties than the second set off parameters $(\theta = 4.40, \beta = 0.80)$ for all sample sizes. Also as the acceleration factor increases the estimates have smaller MSE and RE. As the sample size increases the RBais and MSEs of the estimates parameters decreases. This indicates that the ML estimates provide asymptotically normally distributed and consistent estimators for the scale parameter and the acceleration factor.

When the sample size increases, the interval of the estimators decreases. Also the intervals of the estimators at $\gamma = 0.95$ is smaller than the interval of estimators at $\gamma = 0.99$.

The results of this simulation study suggests that PALT is a suitable model which enables to save time and money considerably without using a high stress to all test units. The optimal test plans improves the quality of the inference and increase the level of precision in parameter estimation. So, statistically, optimum plans are needed, and the experimenters are advised to use them for estimating the life distribution at design stress. These plans can also serve as a criterion for comparison with other designs. Further work can be extended by applying the results to other distributions and other censored schemes such as progressive censoring.

7. CONCLUSIONS

For products having high reliability, ALT or PALT results in shorter lives than would be observed under normal operating conditions. In ALT products are tested at higher then usual level of stress to induce early failures. In PALT, test units are run at both usual and higher-than usual stress conditions. In a constant stress PALT each test item is run at either normal use condition or at accelerated condition only.

This study has presented a constant stress PALT for Rayleigh distribution using type-I censoring. The MSE, RBais and RE of the estimators are obtained for two sets of parameters. For the first set of parameters, the ML estimates have good statistical properties. Regarding the confidence interval of estimators, it can be observed that the interval of the estimates at $\gamma = 0.99$ is greater than the corresponding interval at $\gamma = 0.95$.

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