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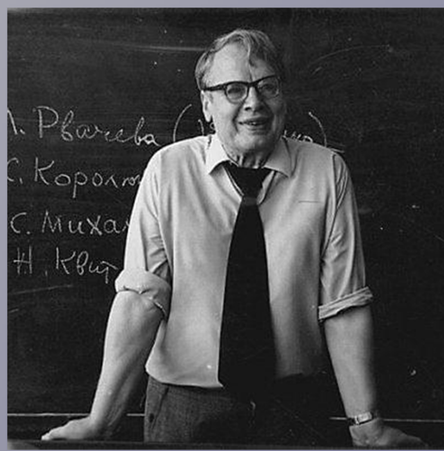

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RELIABILITY:
THEORY&APPLICATIONS

RELIABILITY
RISK ANALYSIS
MAINTENANCE
SAFETY

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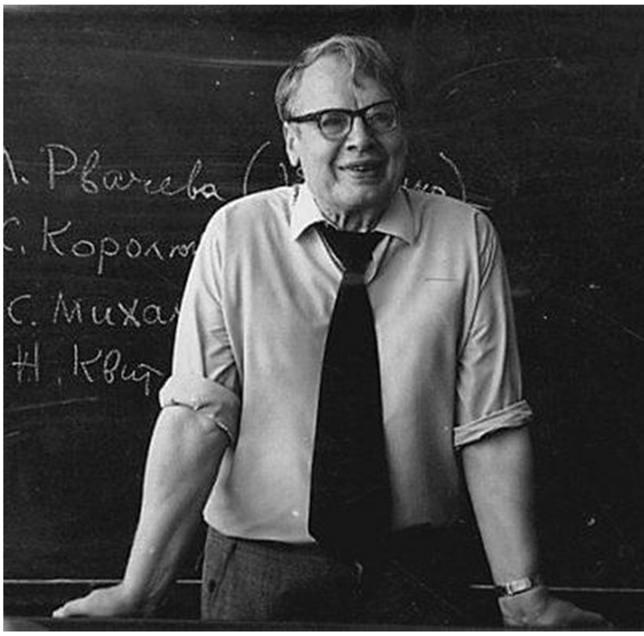

RELIABILITY:

THEORY & APPLICATIONS

Vol.1 No.1 (24),
March, 2012

Special Issue 2

100th anniversary of Boris Vladimirovich Gnedenko's birthday

San Diego
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Actually, this is not a review of past and recent works on reliability theory and adjoining areas. It is rather a selected bibliography with brief comments. Of course, any such selected bibliography or review reflects knowledge, experience and even scientific taste of the author. Nevertheless, I hope that, in general, the depicted picture of recent reliability theory state is more or less objective. Reliability Theory is alive but is it still developed? That's the question. "History teaches the continuity of the development of science. We know that every age has its own problems, which the following age either solves or casts aside as profitless and replaces by new ones." This is a citation from David Hilbert's Lecture "Mathematical Problems" delivered in 1900. Hilbert told about pure mathematics, however the same words are correct in respect to applied mathematics and, in particular, to reliability theory.

Fire PSA for all plant operational states is part of a state-of-the-art that a Level 1 PSA. Within a fire PSA not only the malfunction of systems and components has to be assessed but also all supply systems and cables have to be traced for a given component. In the past it was assumed that in the case of a fire in a compartment all components and corresponding cables in that compartment are destroyed. However, this is in many cases a very conservative approach which may lead to overestimated fire induced core damage frequencies. Therefore, a method is required to assess in a more realistic manner the effects of cables failures caused by fire. Such a procedure is based on a sound data base containing all relevant equipment, a list of cables and their properties as well as cable routing. Two methods which are currently developed and already partially applied are described in more detail. One of these methods is a cable failure mode and effect analysis which is easier to apply in practice.

External hazards can be safety significant contributors to the risk in case of operation of industrial plants. This has been strongly underlined by the nuclear accidents at Fukushima-Daiichi in March 2011.The paper concentrates on the procedure to assess external hazard explosion pressure waves within probabilistic safety assessment. This assessment starts with a screening procedure in order to determine scope and content of the analysis. The second step is to choose an appropriate approach in case that a full scope analysis has to be performed. Several methods can be applied to evaluate the probability of occurrence of an external explosion event at a plant. The presented results indicate that the probability of occurrence of external explosion pressure waves can be successfully assessed by means of the Monte Carlo simulation, in particular in difficult site-specific conditions.

This paper shows the evaluation of the data coming from the surgery operations and from the clinique of occupational and preventive medicine. Deals with the usage of logistic regression for predicting the classification of patients into one of the two groups. Our data come from patients who underwent Phadiatop test examinations and patients who underwent colectomy in the University Hospital of Ostrava. For Phadiatop test, as the predictor variables were chosen personal and family anamneses. Both of these anamneses were divided into four categories according to severity ranked by doctors. The model for Phadiatop test was tested

RELIABILITY THEORY: HISTORY & CURRENT STATE IN BIBLIOGRAPHIES

Igor Ushakov

List of contents

Introduction

Actually, this is not a review of past and recent works on reliability theory and adjoining areas. It is rather a selected bibliography with brief comments.

Of course, any such selected bibliography or review reflects knowledge, experience and even scientific taste of the author. Nevertheless, I hope that, in general, the depicted picture of recent reliability theory state is more or less objective.

Main results in Reliability Theory have been obtained in 1960-1970s. In this connection, it would be interesting to remember the speech at the closing banquet at the MMR-2004 Conference (Santa Fe, USA) made by one of the most prominent specialists on Reliability Theory – Nozer Singpurwalla, who is Professor of The George Washington University and Director of the Institute for Reliability and Risk Analysis. His speech title was: "IS RELIABILITY THEORY STILL ALIVE?"

Reliability engineering is like medicine. The difference is in the objects of application: systems in one case and human beings in another. Could you imagine that medicine could be exhausted? The same is with Reliability Theory! As Mark Twain told: "Rumors about my death are strongly exaggerated."

Probably, the question was formulated by Nozer Singpurwalla a bit incorrectly. Of course, Reliability Theory is alive but is it still developed? That's the question.

"History teaches the continuity of the development of science. We know that every age has its own problems, which the following age either solves or casts aside as profitless and replaces by new ones." This is a citation from David Hilbert's Lecture "Mathematical Problems" delivered in 1900. Hilbert told about pure mathematics, however the same words are correct in respect to applied mathematics and, in particular, to reliability theory.

Main Directions of Modern Reliability Theory

Historically, reliability field was divided into three main directions:

- Quality Control of Mass Production
- Reliability Engineering
- Reliability Testing
- Pure Theoretical Studies.

Of course, there are no strict borders between those directions. Actually, reliability testing takes the beginning in quality control as well mathematical modeling in reliability engineering is rooted in pure probabilistic investigations.

Basic fundamentals of the reliability theory that moved forward reliability engineering has been actually developed in 1960-s. Not in vain we see a lot of re-published works which first editions are date by 1960-1970-s. A good example is re-publishing the book by Igor Bazovsky "Reliability Theory and Practice" in 40 years after first edition!

Latest works mostly presented some developments and customizing of existent analytical methods, though it would be incorrect do not mention several outbursts of real fundamental publications that will be discussed later.

Classical "pure" Reliability Theory consists of the following main bodies:

◊ Structural models
◊ Functional models
◊ Maintenance models
◊ Computational methods (in particular, Monte Carlo)
◊ Testing
◊ Statistical inferences
◊ Optimization problems in reliability

We intentionally omitted problems of Quality Control because this old engineering problem (much older than reliability analysis!) is well developed and already almost "frozen".

It seems to us that chronological order of references will be more convenient for the reader: you can see "historical horizons" and process of reliability theory and applications development.

Everybody understands that any review of such kind bears the stamp of subjectivity and incompleteness. The author would be grateful for any comments, additions and corrections.

Books & Reviews

Modern market is full of different handbooks, textbooks, specialized monographs and reviews on Reliability Theory and its applications. Of course, it is practically impossible to compile a comprehensive review of all these materials. Even if one could undertake such a scientific adventure, it would be almost useless itself: it is better to have a list of publications in some thematic clusters than to read boring description of the book contents.

Keeping this in mind, we propose you very brief comments to sections and hope that the titles of books and reviews will say about their contents even more than trivial annotations.

Understanding of history of any subject is very important. It is timely to remember words from the famous anti-utopian George Orwell's novel "Nineteen Eighty-Four": "Who controls the past controls the future. Who controls the future controls the present". Only knowledge of the past allows us to move forward in a right direction.

To make historical horizons more clear, we give bibliographies in chronological-alphabetical order. Moreover, we decided to give up the total list of references: bibliography will be given by chapters, i.e. dividing it onto smaller lists related to concrete topic.

Handbooks

It seems that handbooks in each technical area gives the best understanding of the current state of the art of this particular area because namely accumulate accurate and practically useful results for engineering practice.

One of the first Handbooks on Reliability [1] for practical engineers was published in 1966. It reflected probabilistic methods of reliability analysis and synthesis of electronic devices and systems and statistical inferences of test and field data. Later this book was multiply revised [3, 5, 6] and translated into English, German and Check [2, 4, 5, 7, 10].

Then from 1991 handbooks began to consider some practical engineering methodology of design, not only methods of reliability evaluation. Among them we would like to distinguish the first handbooks on software reliability [8, 14].

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Textbooks

One of the first successful textbooks in reliability was written by Igor Bazovsky [1]. This book was simple, informative and instructive. Not in vain this book has been re-published [13] in almost half a century!

The next significant and deep book, written by David Lloyd and Myron Lipow [2], was full of interesting practical problems and original solutions. It does not lose its importance even now. Soon, the first monograph on reliability [3] was published in the former Soviet Union. Scientific competition between American and Soviet reliability schools began.

However, of course a real revolution was done by two excellent books: Richard Barlow and Frank Proschan [4] and Boris Gnedenko, Yuri Belyaev and Alexander Solovyev [5]. The role of those books is difficult to overestimate. The first one introduced new concepts of monotone systems, distributions with monotone increasing and decreasing failure rates and gave deep presentation of optimal maintenance and optimal redundancy problems. The second book contained many new results on repairable redundant systems (including first results on asymptotic analysis), specific inferences of reliability data and many solutions of interesting engineering problems.

One can say that these two books have laid a fundamental of the modern theory of reliability. They are real Bibles on Reliability Theory.

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Books

Monographs on reliability are dedicated to the entire spectrum of reliability problems. Among them a number of works on various statistical aspects of reliability [5, 14, 16-17, 19, 26, 42] including Bayesian methods [15, 51]. There are books on such important engineering problem as optimal maintenance [8, 36, 49] and optimal redundancy [2, 27, 39]. Do not leave without attention such important direction as reliability of mechanical systems [1, 7, 24, 31, 34, 52].

Many interesting new ideas can be found in Proceedings of Annual Reliability and Maintainability Symposium organized by IEEE.

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Reviews

The best image of current state of various aspects of Reliability Theory can be obtained from reviews. Some review are on general state of Reliability Theory [1-2, 5, 10, 19], and some of them cover special topics. We would like to mention reviews on a new direction in relatively – multi-state systems reliability analysis [3-4, 7, 20].

Very interesting and useful for understanding recent state and path of development of Reliability Theory one can find in such analytical papers as [12-13, 16-17, 19, 23, 26].

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Repairable Systems

Traditional mathematical tools for analyzing reliability of repairable systems are methods of the Queuing Theory. It is time to remember that this theory was originated in [1] by talented Danish mathematician, statistician and engineer Agner Erlang in the beginning of the last century.

A real burst of the Queuing Theory happened in early 1960s. One can mention that a number of problems in repairable systems reliability analysis are reduced to the queuing problems just by simple change of terms.

The next very powerful impact on repairable system analysis was done by a series of excellent works in the Renewal Theory. In the middle of 1950s Alfred Renyi [4] formulated an asymptotic theorem related to the "thinning" procedure, and approximately at the same time David Cox with Walter Smith [2] and Gennady Ososkov [3] proved an asymptotic theorem related to the superposition procedure for point stochastic processes. In the beginning of 1960-s Bronyus Grigelionis [7] generalized the theorem on point processes superposition. These theorems stated that random thinning of a point process or superposition of independent point processes asymptotically lead to the Poisson Process.

Boris Gnedenko [9, 10] was the first scientist who, in the beginning of 60-s, got asymptotical results for repairable systems reliability. He found asymptotic distributions of time to failure of such a system for the case when repair time is relatively small. This work was followed by a series of excellent works by Igor Kovalenko, Alexander Solovyev and others [14]. Now asymptotic methods in reliability take an important place in large-scale systems consisting of highly reliable units. One can find some review of strong and approximate models for highly available systems in [17].

In a sense, recent publications on the subject bring few new ideas; they are mostly "technological": main results were obtained about 30 years ago.

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Networks & Large Scale Systems

In the middle of 1950-s the Moore-Shannon model [1] was published. It opened a new direction – asymptotic analysis of network reliability. In late 1960s John Esary and Frank Proschan developed method of reliability bounds estimation for arbitrary two-pole networks with known structure. Later this direction was developed in [4-5, 7-8, 10]. All these works were based on counting minimal paths and cuts of a network rather than on enumeration of entire number of possible network states. Next step was implementation of Graph Theory for network reliability analysis [6, 11-13].

It was a beginning of a powerful direction in reliability analysis of large scale systems.

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OPTIMAL REDUNDANCY

First papers on optimal redundancy was published in the middle of 1950-s by F. Moskowitz and J. McLean [1]. Though now this paper might seem to be a little bit naïve, its role was significant. In brief terms, the problem of optimal redundancy is in finding such a redundant (or spare) unit allocation that deliver the required reliability under minimal cost or, in the inverse case, to get maximum reliability under certain constraints on the system cost. Later, methods of solution of the problem of optimal redundancy were developed by R. Bellman [2, 4], F. Proschan [3, 7] and J. Kettelle [5].

The first book on optimal redundancy [8] appeared only in the end of 1960s.

First papers and books described only traditional methods of optimization – dynamic programming and steepest descent method. Later some interesting approaches have been developed: Branch-and-Bound [19], Monte Carlo simulation [8-10, 17], genetic algorithm [32-33], evolutionary approach [15], “Ant colony method” [28] and others.

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Terrestrial Systems and Their Supply

First works on terrestrial systems concern such geographically dispersed telecommunication systems [3, 5-7, 9], energy systems [3] and military command system [1]. Afterwards papers on terrestrial supply systems appeared. Actually, the last ones were a generalization of optimal redundancy problems where supply system had a hierarchical structure and delivery of spare units took some fixed time from local stock to objects, from regional stocks to the local ones and from central stock to the regional ones [4, 8, 10-11, 13] .

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Reliability of multi-state sytems

First of all, we would like to mention a comprehensive bibliography "Reliability Analysis and Optimization of Multi-state Systems" compiled by A. Lisnianski, G. Levitin and E. Korczak. This bibliography can be found at

http://iew3.technion.ac.il/~levitin/MSS.html.

The main results on this theme can be found in the monograph by Anatoly Lisnianski and Gregory Levitin [41].

Multi-state systems reliability analysis

First work on multi-state systems with binary units [1] considered a situation when failures of system's units could lead to partial ability to perform required operations. As the reliability measure the author introduced mean operational effectiveness of the system. Later this idea was developed in [2-4, 6, 24, 30, 32]. Then in [7] a binary system with multi-state units was analyzed.

The most recent works on the theme relates to analysis of multi-state systems consisting of multi-state units. Papers on the subject appeared relatively rear until real burst in the beginning of 1980s.

The interest to this kind of systems is understandable: binary description of possible states of units and systems is far from reality. However, one should realize that such detailed description of a system needs more detailed statistical information that is not always accessible. Thus, a number of recent pure mathematical approaches present "games of keen brain", rather than working engineering tool. Nevertheless, there are many really constructive works in the area mostly belonged to the "scientific tandem" Levitin-Lisnianski.

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Universal Generating Function

This relatively new technique for multi-system analysis started after series of papers appeared in the late 1980s [1-5]. This new formalism actually based on a simple idea: a standard generating function deals with summation of powers of arguments and a generalized generating functions allows to perform, for instance, taking minimum or maximum or others operations.

We would like to distinguish [15] where all recent results are summarized and systemized.

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Continuous multi-state systems

In first works on multi-state systems, there were considered systems with discreet states. Later works on continuous and even fuzzy multi-state systems appears. It is, probably, time to mention that these works (by now) have a pure academic interest.

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Reliability optimization of multi-state systems

It was naturally that after developing methods of multi-state systems analysis, the methods of optimal synthesis of such systems were developed. A general methodology of optimal redundancy was kept though there are some specific due to new properties of multi-state systems.

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Reliability of Wearing Systems

In the end of 30-s Swedish engineer and mathematician Waloddi Weibull, analyzing ball bearing longevity, actually reduced the problem of assembly failure (he analyzed bearings) to the model of a "weakest link" [1, 4]. He suggested for description of the problem a simple and convenient mathematical model, which became known as Weibull distribution. Almost simultaneously and independently, outstanding Russian mathematician Boris Gnedenko found three classes of limit distributions [2-3], one of which corresponded to the Weibull distribution.

In the middle of 1960-s Richard Barlow and Frank Proschan [5-7] introduced classes of distributions with increasing and decreasing failure rates (IFR and DFR, respectively). That step was very significant because it opened the path for analyzing units and systems reliability invariantly to specific type of failure distributions.

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Software Reliability

Now we come to the most confusing area in reliability theory and practice – the so-called software reliability. This term is rooted in software engineering though it very much contradicts to traditional understanding of the term “reliability” in hardware engineering. It leads to erroneous attempts of applying probabilistic reliability concepts to this subject that led only to some disaster.

One of the most influenced reliability experts Nozer Singpurwalla [13] gave a good answer by his question ☺: “The failure rate of software: does it exist?”.

It is time to mention that one of the most brilliant specialists in software reliability engineering John Musa [1-2, 5-7, 15-16, 18, 22-23, 25-26] meant “reliability” in rather common sense. With the same success one can say about reliability of a person or reliability of an idea.

However, this discussion needs special time and place. One thing is clear: software reliability specialists should distinguish their reliability from hardware reliability, develop their own non-probabilistic and non-time dependent mathematical tools.

Let us just present recent works on software reliability without further discussion.

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STATISTICS

General methods

Above we mentioned various probabilistic approaches. However, Reliability Theory cannot be a real engineering tool without statistical methods. In this connection we have to mention names of two pioneers in statistical reliability – Benjamin Epstein and Mark Sobel [2-3, 6-8].

In the beginning of 1960 David Lloyd and Myron Lipow [7] find a heuristic solution for an interesting problem: estimation of system reliability on the basis of unit test data. Two years later Roald Mirny and Alexander Solovyev obtained first strong mathematical result in this direction (for no failure tests). Later Igor Pavlov got solution for a general problem [17].

Of course, during the period of time from then to now there were many improvements in methods of statistical estimate of systems reliability that the reader could see from the bibliography below.

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Accelerated Testing

Since testing in normal conditions (room temperature, no power overloading, no vibration, etc.) takes a too long time and requires usually a huge number of units, engineers invented method of accelerating tests. The problem arose how to extrapolate the results of such accelerated tests to a normal working conditions. There were developed several effective methods of getting needed data from results of accelerated tests. Among them the fundamental work by Wayne Nelson [3] has to be mentioned in the first place.

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Confidence Limits

In mathematical statistics from the very beginning there was developed method of confidence bounds. Actually, confidence bounds give us an understanding of the measure of possible deviation of “real” value from statistical estimate obtained on the basis of limited number of observations.

Specific of reliability problems led to developing new effective methods.

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Bayesian Metods

Last years a number of interesting publications appeared in Bayesian methods in reliability. Let us name Richard Barlow, Henry Martz and Nozer Singpurwalla whose numerous works made this branch of mathematical statistics a real working engineering tool. One can expect useful applications of these methods for aggregating field data and projecting reliability of new objects (especially, unique ones).

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Monte Carlo Simulation

In late 40-s John von Neuman invented the Monte Carlo simulation method for calculation of multi-dimensional integrals over some specific domains. (Actually, it was idea very close to Georges Buffon's Needle method.) Later it was developed into powerful calculation method with using modern computers.

Monte Carlo simulation is very effectively applied for various calculation problems for reliability evaluation. However, it should be noticed that there are few works where Monte Carlo is used for some optimization problems [3-4, 7].

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AREAS CLOSE TO RELIABILITY

Survivability

Reliability deals with random (mostly independent) unit failures that can appear during systems operation. However, sometimes we meet situations where we can only guess about possible impacts. These impacts can be unpredictable inner failures (usually due to operator errors) or environmental influences (earthquakes, floods, hurricanes). In this case one assumes that the impacts are directed to the most critical components of the system.

Survivability analysis is usually performed in minimax terms and reduced to "bottleneck analysis", or "minimum cut" searching. Usually survivability consideration is related to large terrestrial systems (telecommunication or power networks, transportation systems).

One of the first works on survivability was written by famous Russian naval architect academician Alexei Krylov [1]. From later works, one can distinguish [2-3] where concrete survivability analysis has been done for all-country power system.

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Counter-terrorism protection

Last decade is going under sign of inhuman terrorist attacks by Islamic terrorists. These hostile attacks directed mostly to ordinary people by terrorists who mimic as normal peaceful persons. It makes protection against such terrorist attacks very difficult. In this case one cannot consider probabilistic models: the impact is not random. Moreover, terrorists choose most vulnerable objects in sense of weak protection or huge loss in case of successful attack. In this case, the problem is close to the situations arising in the Game Theory. In this case the problem of minimizing of maximum possible damage arose [1-6].

Dealing with human enemies makes very important such actions as creating false targets [7, 10, 14, 19], preventive hits, misinformation of enemy, etc. In other words, this problem has its own specific.

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COMPREHENSIVE CABLE FAILURES ANALYSIS FOR PROBABILISTIC FIRE SAFETY ASSESSMENTS

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ABSTRACT

Fire PSA for all plant operational states is part of a state-of-the-art that a Level 1 PSA. Within a fire PSA not only the malfunction of systems and components has to be assessed but also all supply systems and cables have to be traced for a given component. In the past it was assumed that in the case of a fire in a compartment all components and corresponding cables in that compartment are destroyed. However, this is in many cases a very conservative approach which may lead to overestimated fire induced core damage frequencies. Therefore, a method is required to assess in a more realistic manner the effects of cables failures caused by fire. Such a procedure is based on a sound data base containing all relevant equipment, a list of cables and their properties as well as cable routing. Two methods which are currently developed and already partially applied are described in more detail. One of these methods is a cable failure mode and effect analysis which is easier to apply in practice.

1 INTRODUCTION

Fires have been recognized as one major contributor to the risk of nuclear power plants depending on the plant specific fire protection concept. Therefore, a state-of-the-art Level 1 probabilistic safety assessment (PSA) meanwhile includes fire PSA as part and supplement of the internal events PSA for full power as well as for low power and shutdown plant operational states (Berg & Röwekamp 2010, Röwekamp et al. 2011).

An overview of the main steps of an advanced fire PSA process is given in Figure 1. The task “fire PSA cable selection” is not (or not in detail) performed in current fire PSA.

One of the important parameters in a fire PSA is the conditional probability of a specific failure mode (e.g., loss of function, spurious actuation) of a selected component, given (assuming) that a postulated fire has damaged an electrical cable connected to that component.

In general, evaluation of this parameter can require the analysis of a number of cable failure scenarios, where each scenario involves a particular fire induced cable failure mode and the propagation of the effects of this failure through the associated electrical circuit.

The cable failures of interest cover the following conductor failure modes:

- Loss of continuity,
- Short-to-ground, and
- Conductor to conductor short.

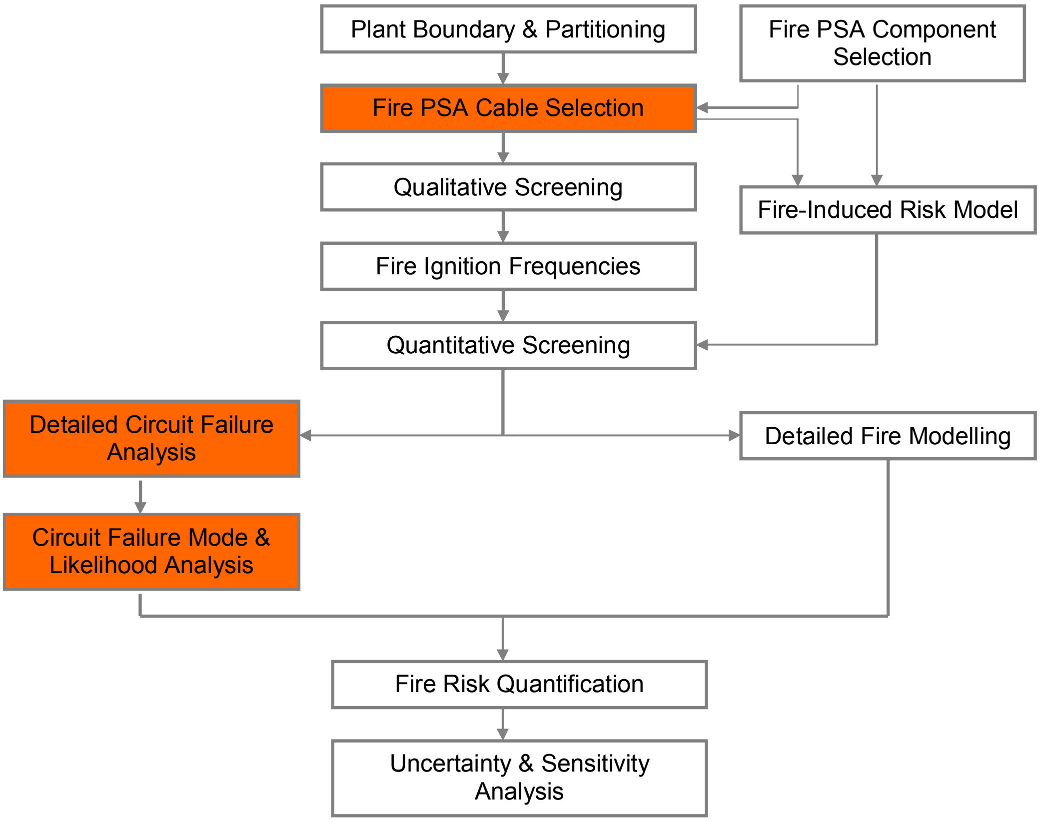

Figure 1. Overview of the main steps of an advanced fire PSA process.

There are three primary functional types of cables in a nuclear power plant: namely, power cables, instrumentation cables, and control cables as shown in Figure 2.

Cables can also be categorized by their physical configuration. The most common types are single conductor, multi-conductor, and triplex.

Cables are generally routed horizontally through the plant on raceways (in principle on cable trays or conduits) with vertical runs used as required between different elevations in the plant.

The cables are usually segregated by type as described above and illustrated in Figure 2. However, cables of various voltages and functions can be found together in the same raceway for some plants (in particular in nuclear power plants built to earlier standards).

While short-to-ground or open circuit failures may render a system unavailable, a hot short failure might lead to other types of circuit faults including spurious actuations, misleading or faulty signals, and unrecoverable losses of plant equipment.

These circuit failures, taken individually or in combination with other failures, may have unique and unanticipated impacts on plant safety systems and on plant safe shutdown capability being not always reflected in current fire PSA studies.

In most of the fire PSAs which have been performed to date, circuit failure analysis has been performed in a more simple manner and not in such a detailed manner as recommended in Figure 1.

Usually, the circuit failure analysis assumes that if any of the cables associated with a given circuit or system are damaged due to fire (i.e., the cables fail), then the circuit or system is rendered unavailable. This approach neglects the potential for spurious actuations entirely and may represent a too optimistic approach.

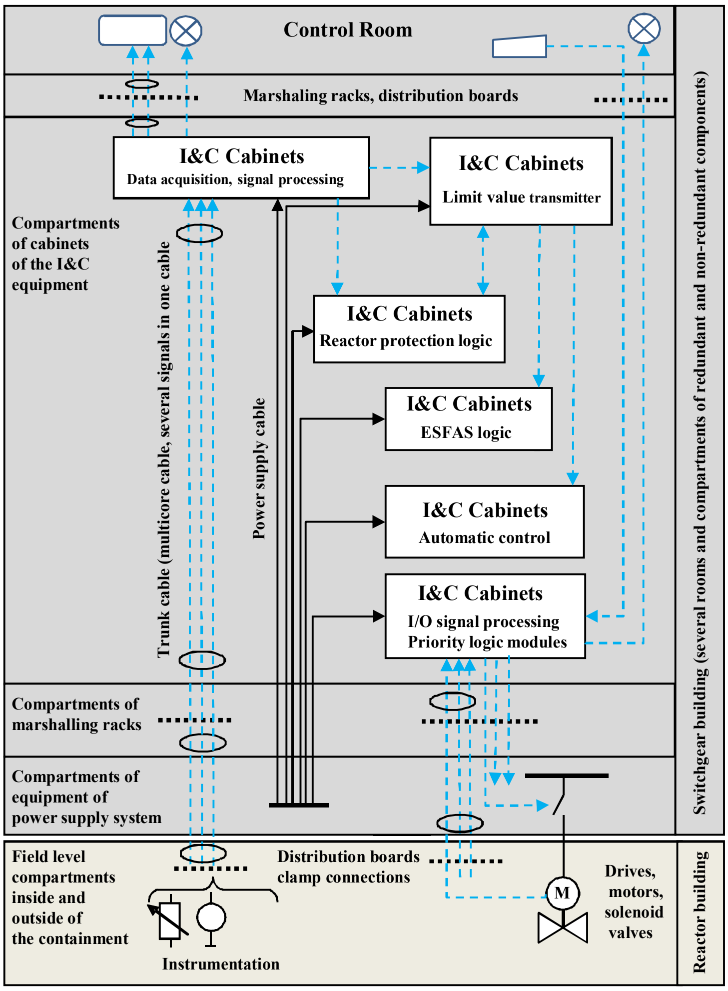

Figure 2. Schematic drawing of I & C (blue, dashed) and power cables (black, solid).

Most of the common approaches apply a single-valued damage threshold of temperature and/or heat flux to predict the onset of cable failure. When the cable reaches a predetermined temperature and/or the cable is exposed to a threshold heat flux, a worst case failure of the cables inside the respective fire compartment is assumed. The worst case failure modes have been deduced by expert judgment.

Simplified assumptions on the failure modes could lead to an overestimation of specific event sequences whereas other effects such as spurious actuation of not directly connected components were neglected.

On that background in the U.S. and in Germany two approaches have been developed. In both cases the success of the method strongly depends on the quality and form of the prerequisite information on the cables and their properties.

Therefore, several cable fire tests have been and are performed to gain the necessary data for the safety assessment.

For a more realistic picture of cable failure effects a cable failure mode and effect analysis (FMEA) methodology has been developed. It is intended to use this method as an integral part of Level 1 fire PSA in Germany in particular in the frame of periodic safety reviews, performed every ten years.

The main purpose of the methodology and its supporting tools is to improve the comprehensibility and completeness of cable failure analysis within the context of a fire PSA.

The computer aided methodology based on the principles of FMEA is supported by a plant specific database application named CaFEA (Cable Failures Effect Analysis) developed by GRS.

The database CaFEA comprises all relevant data of the cables, such as cable routing within the plant, cable type as well as data on the connected components. Availability of such information is a prerequisite for the implementation of a state-of-the-art FMEA methodology.

2 PROCEDURE OF RISK EVALUATION DEVELOPED IN THE U.S.

During the 1990s, both the Nuclear Regulatory Commission (NRC) Office of Nuclear Regulatory Research (RES) and the Electric Power Research Institute (EPRI) were active in the development of methods for fire risk analysis. U.S. NRC and EPRI initiated a collaborative project to document the state-of-the-art for conducting Fire PSA. The principal objective of the Fire Risk Study is to develop a technical basis and methodology that will clarify issues affecting application of fire risk methods.

The project was designed to culminate in a joint EPRI/RES publication of state-of-the-art fire PSA methodology. The report NUREG-CR-6850 (EPRI 2005) is a compendium of methods, data and tools to perform a fire probabilistic risk assessment and develop associated insights.

This report is intended to serve the needs of a fire PSA team by providing a structured framework for conduct and documentation of the analysis in four key areas:

- Fire analysis,
- General PSA and plant systems analysis,
- Human reliability analysis (HRA), and
- Electrical analysis.

One finding of the investigations outlined in the report was that the selection, routing, and failure analysis of cables and circuits have not been covered generally by past fire PSA methodology.

The issue of circuit analysis, including the spurious operation of components and systems, continues to be an area of significant technical challenge.

The approaches recommended in the report (EPRI 2005) provide a structured framework for the incorporation of fire-unique cable failure modes and effects in the fire PSA. The circuit analysis issue impacts fire PSA methods and practice broadly. Circuit analysis affects the following steps:

- Identification of fire PSA components and cables,
- Mapping of fire PSA components and cables to fire analysis compartments,
- Development of the plant post-fire safe shutdown response model,
- Incorporation of circuit failure modes in the quantitative screening analysis,
- Detailed analysis of cable failure modes and effects,
- Detailed analysis of circuit fault modes and effects, and
- Quantification of human actions in response to a fire.

A possible process for including circuit analysis into a fire PSA as proposed in U.S. NRC report (EPRI 2005) is shown in Figure 3.

Another report of the U.S. NRC (LaChance et al. 2003) presents a new methodology for the analysis of cable failure modes and effects as illustrated in Figure 4.

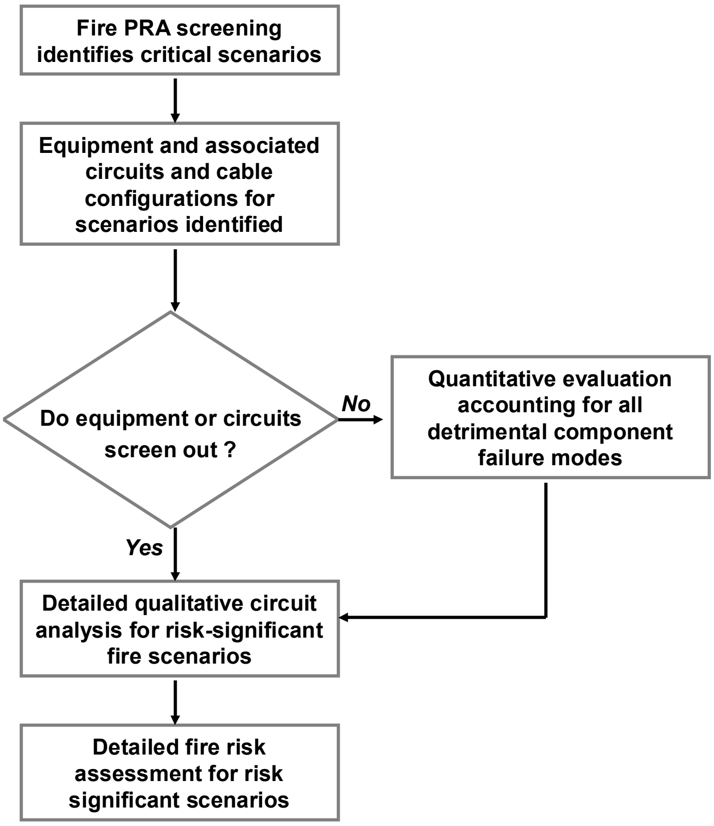

Figure 3. Circuit analysis for fire risk assessment.

Electrical analysis involves circuit failure modes and affects the analysis conducted for specific plant circuits, including the selection of circuits and systems, cable and component routing, development of the fire PSA database and quantification of failure mode likelihood values.

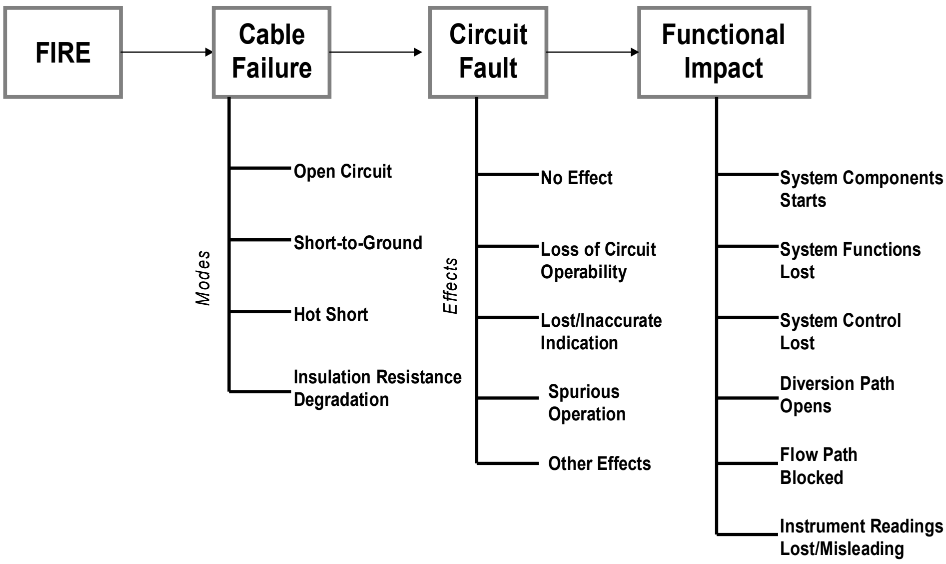

Figure 4. Circuit analysis process structure.

Based on experience with the demonstration studies and the collective experience of the authors of the report, at least 4000 engineering hours would be needed to perform a complete plant-wide fire PSA using the methods recommended in (LaChance et al. 2003). This estimate is predicated on a large number of positive factors in terms of the quality of the plant analyses and the level of sophistication desired in the fire PSA.

The low-end manpower estimate for the circuit and cable selection, tracing, and analysis efforts (600 hours) represents a case where the following three factors apply:

- The plant has a pre-existing state-of-the-art deterministic post-fire safe shutdown analysis.
- There is a pre-existing and well-documented electronic system for tracing cables and components-
- There is a pre-existing and well-documented fire PSA safe shutdown plant response model.

The upper end of the manpower estimates for the circuit and cable selection, tracing, and analysis efforts (6000 hours) represents a case where the following conditions apply:

- The plant has a pre-existing deterministic post-fire safe shutdown analysis that has not undergone significant review.
- The plant has merely a paper (non-electronic) cable and raceway system and/or database.
- The fire PSA model is intended to include at least all components that are credited in the internal events PSA.

The report (LaChance et al. 2003) also provides findings regarding cable fire performance testing in the U.S. over the past three decades. From the viewpoint of cable failure mode likelihood estimation, the available information in these reports is sparse. This is because the bulk of fire-related cable research has focused on one of two areas:

- Most large-scale cable tests were designed to examine the flammability and fire behaviour of cables. In a minority of these tests electrical performance of a small sample of cables was monitored, but this was rarely a primary test objective. Even in those cases where electrical function was monitored, only a small subset of these tests explicitly sought information on cable failure modes.
- A second class of cable tests has sought to determine the failure thresholds of the cables. These are typically small-scale tests where cables are exposed to simulated fire conditions (Wyant & Nowlen 2002). The time to failure for exposed cables is commonly monitored. The failure behaviour is commonly characterized based on the heat flux or atmospheric temperature in the test chamber and the time of exposure to these conditions.

A second potential source of information on fire-induced cable failure behaviour is actual fire experience. However, fire experience is relatively limited, and fire reports rarely focus on details of cable failures or the resulting circuit faults. The most significant exception to this observation is the 1975 Browns Ferry fire (Scott 1976). This fire damaged more than 1600 cables routed in 117 conduits and 26 cable trays. Various studies of that incident have noted that the fire resulted in spurious initiation of components, spurious control room annunciation, spurious indicator light behaviour, and loss of many safety related systems. Examples of the component and system behaviour observed during the fire are outlined in the U.S. NRC report (Collins et al. 1976).

A range of factors may affect the conditional probability that for a given a fire induced cable failure a particular mode of failure might be observed. Various factors may also affect the timing of potential faults being observed as well as the timing of fault mode transitions (e.g., hot short transition to a short-to-ground). The identified factors can be roughly categorized into one of four broad groups; namely, factors associated with the cable's physical properties and configuration, factors associated with the routing of the cable, factors associated with the electrical function of the circuit, and factors associated with the fire exposure conditions. The report (EPRI 2005) discusses each of the influence factors identified to date including the current evidence available regarding each of the factors from both experiments and actual experience.

The advanced cable failure analysis should be able to predict when a cable failure occurs, the relative likelihood that specific modes of cable failure would occur given failure, how long a particular failure mode is likely to persist, and the overall occurrence frequency of each cable damage state or failure mode (including fire frequency, fire severity, mitigation by detection and suppression before damage, etc.).

The electrical circuit fault analysis determines how each circuit will respond to the various modes of cable failure that may be observed. The circuit fault analysis also feeds information back to the cable failure analysis task by means of specific cable failure modes that may be of particular interest to the PSA and provides occurrence frequency estimates for each of the circuit fault modes of potential interest to risk quantification.

One task is to estimate the probability of hot short cable failure modes of interest, which in turn can be correlated to specific component failure modes. The methods and techniques for deriving circuit failure mode probability estimates are based on limited data and experience. Consequently, this area of analysis is not yet a mature technology, and undoubtedly further advances and refinements will come with time.

The final task assesses the functional impact of the circuit faults on the potential for plant safe shutdown, i.e. it should provide a probabilistic assessment of the likelihood that a cable will experience one or more specific failure modes (e.g., short-to-ground, intra-cable conductor-to conductor short, inter-cable conductor-to-conductor short, etc.). The results of this assessment are entered into the fire PSA database, allowing generation of equipment failure reports, including the estimated likelihood of the failure modes of concern. This is needed for the quantification of the contribution for the postulated fire scenarios to the total core damage frequency. This task is in the domain of PSA plant systems modelling and event/fault tree analysis and quantification.

3 FAILURE MODE AND EFFECT ANALYSIS

A computer aided methodology based on the principles of FMEA provided in (LaChange et al. 2003) has been developed by GRS (Germany) to systematically assess the effects of cable failures caused by fire in a nuclear power plant.

The main objective of the approach of the GRS is the standardization of the FMEA for similar components of affected electrical circuits.

Cable FMEA (CaFEA) consists of two phases of analysis: In the first phase an analysis of generic cable failures of standardized electrical circuits of the nuclear power plant is performed. In the second phase, those generic failure modes are identified for each cable which could affect safety related components.

3.1 Generic FMEA

Based on the circuit type, the attached source and target component types and sub-types, the operating condition, and the transmitted signal, the generic FMEA is performed (see Figure 5).

All possible circuit failures have to be considered, because it is not necessarily known which cable failures have to be considered while performing the specific FMEA. The experiences gained while applying the computer aided cable FMEA to all cables within one fire compartment demonstrated that about 100 generic circuit types have to be investigated for a whole nuclear power plant.

In a first step, the FMEA expert has to screen the list of safety related components typically provided by a Level 1 PSA for full power operational plant states and to define the generic circuit types to be investigated.

Examples of circuit types may be power supply circuits, instrumentation circuits or control circuits.

In the next step, for each circuit type “source” and “target” component types have to be specified. Typical source component types are switchgear, electronic board, and relay. Examples of target component types are pumps, valves, motor drives, and measurement sensors.

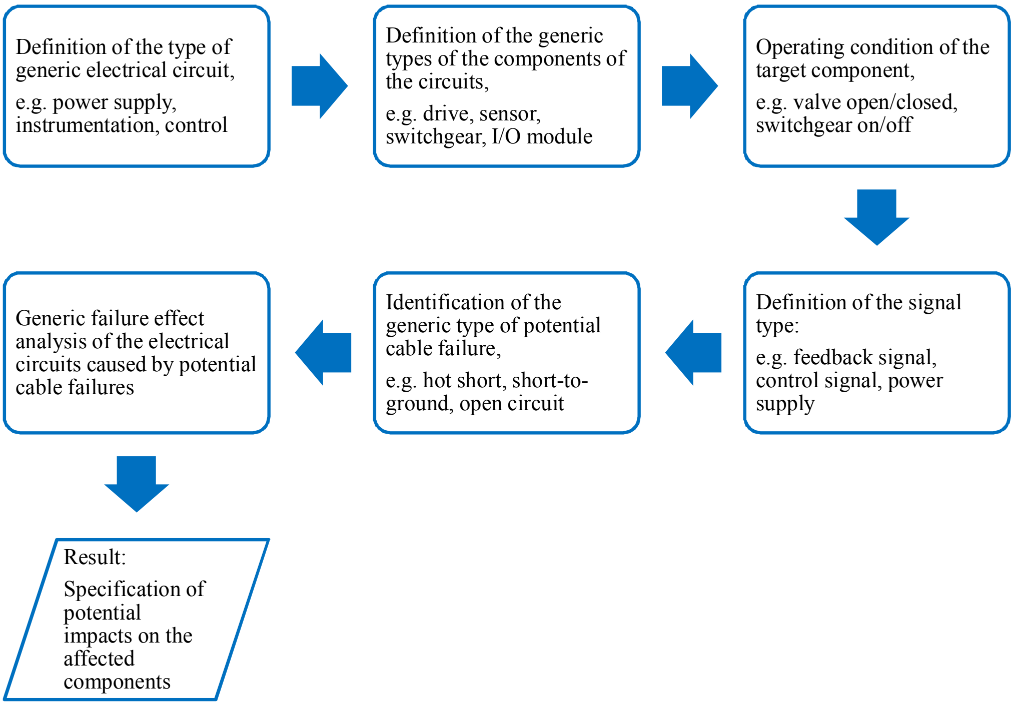

Figure 5. Generic phase of CaFEA.

For both, the source and the target components a sub-type or signalling type has to be additionally specified.

The sub-type is used to distinguish between different circuit types connected to one component (type). A valve might be attached to the circuit type "power supply" as well as to the circuit type "feedback signal". For the circuit type "power supply" the source component sub-type might be "power supply" and the target component sub-type "motor". For the circuit type "feedback signal" the source component sub-type might be "drive control module" and the target component sub-type "control head".

Examples of a generic FMEA are provided in Table 1 (see also Piljugin et al. 2011) for one combination of source and target (sub-)types.

The possible effects on the attached component depend on the operating condition of the target component type. Therefore, the generic FMEA has to be performed for all operating conditions of the generic circuit type. The effects also depend on the type of signal transmitted by the cable. Valid signal types could be, e.g., feedback signal of a valve or control signal for a motor.

Table 1. Examples of a generic FMEA

Description of the electrical circuit							Generic FMEA		
Source of signal (power) of process, electric or electronic components		Target of signal (power) of process, electric or electronic components		Description of the signal			Failure mode	Failure effect	Identification
Type	Subtype	Type	Subtype	State	Type	function			
I&C cabinet (Data aquistion sub-system)	Analog input module SAA (TXS)	Level transmitter	Differential pressure transmitter (4 lead / 0-20mA Loop)	Normal value	Level measurment	Power supply	Intrerruption of the circuit (broken conductor)	Interruption of the power supply of the transmitter	output Signal of transmitter I=0mA (Message: signal is out of the range)
I&C cabinet (Data aquistion sub-system)	Analog input module SAA (TXS)	Level transmitter	Differential Pressure Transmitter (4 lead / 0-20mA Loop)	Normal value	Level measurment	Power supply	Ground fault of the circuit	Interruption of the power supply of the transmitter	Message: signal is out of the range (output Signal of transmitter I=0mA)
I&C cabinet (Data aquistion sub-system)	Analog input module SAA (TXS)	Level transmitter	Differential pressure transmitter (4 lead / 0-20mA Loop)	Normal value	Level measurment	Measurement loop	Intrerruption of the circuit (broken conductor)	Signal is out of the range	Open circuit monitoring
I&C cabinet (Data aquistion sub-system)	Analog input module SAA (TXS)	Level transmitter	Differential pressure transmitter (4 lead / 0-20mA Loop)	Normal value	Level measurment	Measurement loop	Ground fault of the circuit	False value (higher or lower) of the output signal of transmitter	Signal range monitoring / redundant signal comparator
I&C cabinet (Data aquistion sub-system)	Analog input module SAA (TXS)	Level transmitter	Differential pressure transmitter (4 lead / 0-20mA Loop)	Normal value	Level measurment	Measurement loop	Hot-short fault of the circuit	False value (higher or lower) of the output signal of transmitter	Signal range monitoring / redundant signal comparator
I&C cabinet (drive control circuits)	Analog output module XPA92, Output C18	Contactor relais of the MOV	Contacts of the control circuit	open	Normally open circuit	control command CLOSE to coupling relay	Intrerruption of the circuit (broken conductor)	Loss of CLOSE function of the MOV	MOV remains in "OPEN" position by test
I&C cabinet (drive control circuits)	Analog output module XPA92, Output C18	Contactor relais of the MOV	Contacts of the control circuit	open	Normally open circuit	control command CLOSE to coupling relay	Ground fault of the circuit	spurious close of the MOV	Indication of the RUN and CLOSED functions of the MOV
I&C cabinet (drive control circuits)	Analog output module XPA92, Output C18	Contactor relais of the MOV	Contacts of the control circuit	open	Normally open circuit	control command CLOSE to coupling relay	Hot-short fault of the circuit	spurious close of the MOV	Indication of the RUN and CLOSED functions of the MOV
I&C cabinet (drive control circuits)	Analog output module XPA92, Output C18	Contactor relais of the MOV	Contacts of the control circuit	open	Normally open circuit	control command CLOSE to coupling relay	Hot-short fault (overvoltage) of the circuit	Destroying of the analog output module XPA92	Loss of the control of the MOV
Motor-operated valve (MOV)	Contacts of the position indication	I&C cabinet (drive control circuits)	Module XKU98, Input signal B03	Closed loop	position indication of the MOV	CLOSED indication of the MOV	Intrerruption of the circuit (broken conductor)	Loss of the indication of the position CLOSED of the MOV	Functional test
Motor-operated valve (MOV)	Contacts of the position indication	I&C cabinet (drive control circuits)	Module XKU98, Input signal B03	Closed loop	position indication of the MOV	CLOSED indication of the MOV	Ground fault of the circuit	Loss of the indication of the position CLOSED of the MOV	Functional test
Motor-operated valve (MOV)	Contacts of the position indication	I&C cabinet (drive control circuits)	Module XKU98, Input signal B03	Closed loop	position indication of the MOV	CLOSED indication of the MOV	Hot-short (shorts to power lead)	False indication „MOV contactor CLOSED" and "MOV run"	Inconsistency of MOV position indication (e.g. MCR, I&C cabinet, alarm system)
Motor-operated valve (MOV)	Contacts of the position indication	I&C cabinet (drive control circuits)	Module XKU98, Input signal B04	Closed loop	position indication of the MOV	OPEN indication of the MOV	Intrerruption of the circuit (broken conductor)	Loss of the indication of the position OPEN of the MOV	Functional test
Motor-operated valve (MOV)	Contacts of the position indication	I&C cabinet (drive control circuits)	Module XKU98, Input signal B04	Closed loop	position indication of the MOV	OPEN indication of the MOV	Ground fault of the circuit	Loss of the indication of the position OPEN of the MOV	Functional test
Motor-operated valve (MOV)	Contacts of the position indication	I&C cabinet (drive control circuits)	Module XKU98, Input signal B04	Closed loop	position indication of the MOV	OPEN indication of the MOV	Hot-short (shorts to power lead)	False indication „MOV contactor OPEN " and "MOV run"	MOV position indication (e.g. MCR, I&C cabinet, alarm system)

3.2 Component specific FMEA

In the second phase, those generic failure modes are identified for each cable which could affect safety related components in the respective compartment (see Figure 6). Based on the information on the cable type, the attached components and their types, as well as on their operational mode, all the possible cable failures have to be identified by the FMEA expert. The probable cable failures are a sub-set of the failure modes found in the generic FMEA. The specific effects identified in the second phase of the FMEA are mapped to basic events used as initiating events and/or component failures in the fire PSA.

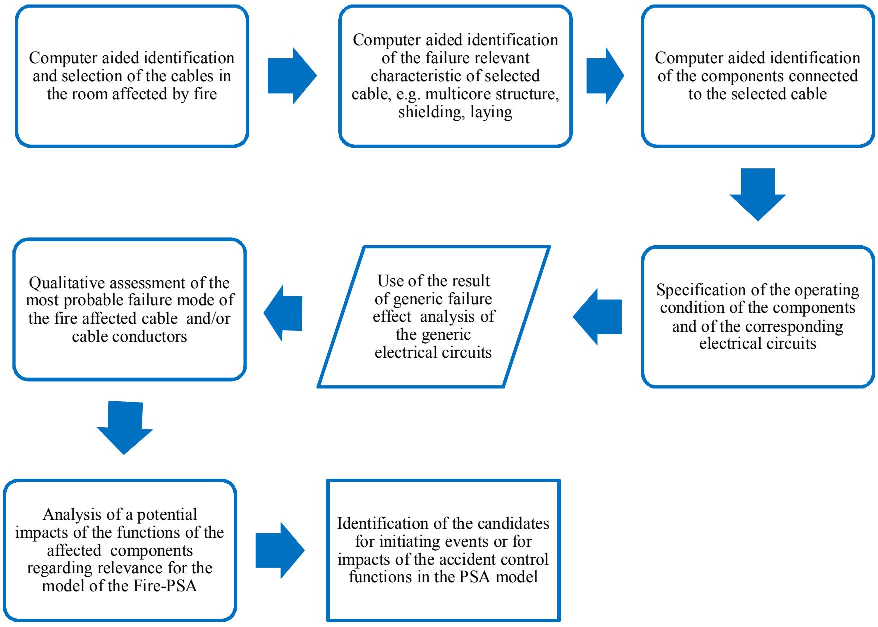

Figure 6. Component specific phase of CaFEA.

The failure conditions for the cables were specified on the basis of the results of fire tests carried out at the Technical University of Braunschweig, Institute for Building Materials, Concrete Construction and Fire Protection (iBMB) - see (Hosser et al. 2005) and (Riese et al. 2006) for typical cables used in nuclear power plant in Germany.

Comparable tests have also be conducted in other countries (see, e.g., EPRI 2002, Keski-Rahkonen et al. 1997 and Mangs et al. 1999), partially also with cables from Germany.

In the fire tests at iBMB, among other things, the fire induced functional failures of the cables were examined for both, energized as well as non-energized cables.

Based on the test results of the iBMB study (Riese et al. 2006), the following different types of cable failure modes were specified and are used in the cable FMEA:

- Short-to-ground via insulation material of the cable jacket or an earthed conductor inside or outside a cable or via earthed structures, e.g. a cable tray;
- Hot short to an energized conductor inside or outside a cable (e.g. high-voltage propagation, impacts of electric arcs);
- Short circuit fault to a de-energized conductor inside or outside a cable (high or low impedance failure);
- Interruption of the cable conductor (open circuit failure mode).

4 DATABASE APPLICATION

The database application consists of a user interface frontend and a database backend. With the aid of CaFEA, the data obtained in the FMEA for fires can be systematically evaluated for cable failures. The CaFEA database comprises the data from different sources, correlates them to each other and displays the correlation results to the FMEA expert who carries out the actual failure mode and effects analysis and stores the results in the database (Herb & Piljugin 2011). The database frontend can be used for data sets of different nuclear power plants.

The FMEA is specific for the plant operational state stored in the database. After opening the database application the user can choose if the generic or the specific FMEA shall be performed. For both tasks input forms are available.

For both generic and component specific FMEA results the database provides import and export functions to and from Microsoft® Excel®.

4.1 Generic FMEA

If (incomplete) specific FMEA results already exist in the database the user can create template data for the generic FMEA. The input form for the generic FMEA contains questions with respect to the following data:

- Type and sub-type of source component,
- Type and sub-type of target component,
- Operating condition of target component,
- Identification of the signal type (circuit type),
- Failure of the cable occurring in the electrical circuit affected by the fire,
- Effect on the target component,
- Optional comment on the determined component effect and its relevance for the PSA.

4.2 Component specific FMEA

The user interface for the component specific FMEA in the CaFEA application subdivides the different analytical steps into several sub-tasks:

- After selecting a compartment and a cable function (corresponding to one signal transmitted via the cable) the first sub-task consists in providing information about the components connected to the cable ("start" and "end" component) and the target component. For the target component the operating condition has also to be provided. The last step is supported by providing information from the plant operating manual and/or safety specifications included in the database.
- In the second sub-task, the FMEA expert has to specify all possible cable failure modes for the selected cable function. As the information about the cable type, routing, etc. has to be considered, it has to be provided by the FMEA expert and stored in the database.
- The third sub-task consists in the determination of the effect on the component by the cable failure mode. By means of a query in the database it is checked if a generic FMEA result provided by the FMEA expert in the previous steps is applicable to the specific case. If a generic FMEA result has been found, it is shown how the FMEA expert can take the decision if and how this generic result can be applied in the specific case.

5 FIRST EXEMPLARY APPLICATION OF THE CABLE FMEA

The analytical method and database tool CaFEA has been developed by GRS based on the available plant data (database with respect to components and compartments and cable routing in the reference nuclear power plant) and on a generic procedure for analyzing fire induced circuit failures in the cables concerned. The FMEA method was tested using data of a reference plant for a given compartment. 432 cables are routed through this compartment transmitting in total 932 signals because of some cables representing I&C cables with multiple conductors.

The qualitatively estimated probability (high, medium and low probability class) was assigned as conditional probability in case of fire to the corresponding effect on the component and the resulting PSA basic event or initiating event.

6 CONCLUSIONS AND OUTLOOK

This paper describes, in addition to the approach applied to some extent in the U.S., a second possible method to assess effects of cable failures.

Basis for this activity is a fire PSA cable list which is not simply a list of cables but establishes for each cable a link to the associated fire PSA component and to the cable routing and its location. These relationships provide the basis for identifying potential equipment functional failures at a fire area, fire compartment or raceway level.

During the pilot applications of the U.S. approach it was noticed that circuit analysts were basically assuming that many cables within a fire area could cause a spurious operation independently of the other cables affected by the same fire (EPRI 2010). However, under certain conditions, when the first cable is damaged (either from spurious operation or blowing the fuse in the circuit), the damage to the other cables does not affect the outcome, i.e., the likelihood of a spurious actuation of the component is not increased.

Therefore it is recommended that the “exclusive or” combinatorial approach for spurious actuation probabilities can only be applied in cases where multiple cables can cause the undesired component effect and the postulated cable failure modes and effects are found to be independent (EPRI 2010). In cases where the cables of concern are dependent, the likelihood of spurious actuation should be determined by the first cable failure only. If the spurious actuation probability is different for the different cables of concern (e.g., due to differences in the cable or routing configuration), the analysis can either determine which cable would likely fail first for the given scenario, or simply bound the individual cable values.

The computer aided methodology of the FMEA as another approach compared with the U.S process offers a good basis for performing a systematic and traceable analysis of the effects of fire induced cable failures in the frame of a fire PSA. The methodology was tested on the basis of data for a given compartment which have been provided by a reference nuclear power plant in Germany.

The major difference between the methodology proposed in (EPRI 2005) and (LaChange et al. 2003) and that one developed by GRS is that the computer aided methodology CaFEA allows to use a combination of generic and (component) specific tasks of the FMEA of the cable failures. This can reduce the specific FMEA of all circuits in the fire affected compartments of the nuclear power plant significantly. The database application of generic cable FMEA can be extended with regard to consideration of all typical electrical circuits in a generic nuclear power plant.

Up to now, the results of the FMEA provide only qualitative indications for those component effects which result in the unavailability of system functions or in new initiating events in the fire PSA.

In a next step, quantification of the failure mode probabilities and the corresponding effects on the affected components shall be included in the approach. The current database architecture of CaFEA allows an easy integration of this feature in the future. In general, two options are possible:

to use failure mode probability tables from literature or to perform explicit model calculations which involves to apply circuit failure mode probability estimation formulas. The second approach is currently under development within a new investigation project. Results including an application for an exemplary room in the reference plant will be available in 2013.

Future challenges of the CaFEA development are the consideration of failure modes of new (digital) technologies of signal transmission and processing, e.g. bus architectures of I&C systems, fibre optical cables, etc.

In principle, the FMEA methodology developed may be also applied for investigating cable failures in the frame of analyzing the effect of other plant internal or external hazards such as flooding and or structural damage by earthquakes.

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HOW TO ASSESS EXTERNAL EXPLOSION PRESSURE WAVES

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ABSTRACT

External hazards can be safety significant contributors to the risk in case of operation of industrial plants. This has been strongly underlined by the nuclear accidents at Fukushima-Daiichi in March 2011.The paper concentrates on the procedure to assess external hazard explosion pressure waves within probabilistic safety assessment. This assessment starts with a screening procedure in order to determine scope and content of the analysis. The second step is to choose an appropriate approach in case that a full scope analysis has to be performed. Several methods can be applied to evaluate the probability of occurrence of an external explosion event at a plant. The presented results indicate that the probability of occurrence of external explosion pressure waves can be successfully assessed by means of the Monte Carlo simulation, in particular in difficult site-specific conditions.

1 INTRODUCTION

External hazards (e.g. earthquake, flooding including tsunamis, external explosion) can be safety significant contributors to the risk in case of nuclear power plant operation because such hazards have the potential to trigger initiating events simultaneously and reduce the level of redundancy by damaging redundant systems or their supporting systems. This has been strongly underlined by the nuclear accidents at the Fukushima-Daiichi nuclear power plants in March 2011.

Therefore, comprehensive safety assessments have to be performed in advance with most actual site-specific data und current knowledge of new research results. Potential methods to analyse existing plants, in particular those built to earlier standards, systematically regarding the adequacy of their existing protection against different types of hazards are deterministic as well as probabilistic.

For all external hazards the first step is a screening process in order to determine scope and content of the assessment to be performed. The approach for these screening processes is different for each type of external hazard.

This paper deals with the assessment of external explosion pressure waves, resulting from accidents of transport means on the road, railway or river, and the calculation of their probabilities at the plant under consideration.

Although some part of this paper is correlated to the application with respect to nuclear power plants, the presented approach is independent from the type of installation and can also be applied to other industrial plants.

In contrast with almost all internal hazards, external hazards can simultaneously affect the whole plant, including back up safety systems and non-safety systems alike.

The assessment of external hazards requires detailed knowledge of natural processes, along with the plant itself and the whole site layout. In addition, the potential for widespread failures and hindrances to human intervention can occur. For multi-plant sites this makes the situation even more complex and it requires appropriate interface arrangements to deal with the potential effects on several facilities.

An explosion is a rapid and abrupt energy release, which produces a pressure wave and/or shock wave. A pressure wave has a certain pressure rise time, whereas a shock wave has zero pressure rise time.

Explosion is used broadly to mean any chemical reaction between solids, liquids, vapours or gases which may cause a substantial rise in pressure, possibly to impulse loads, fire or heat. An explosion can take the form of a deflagration or a detonation.

In deflagrations, the reaction zone travels through the explosive mass at subsonic speed, while the propagation mechanism is heat transfer (by conduction, radiation and convection). Reaction zone propagation velocities (flame speeds) of deflagrations may vary over a wide range and so do the corresponding explosion pressures. One example of a deflagration experiment is shown in Figure 1; in this case the deflagration was very short and lasted less than one second.

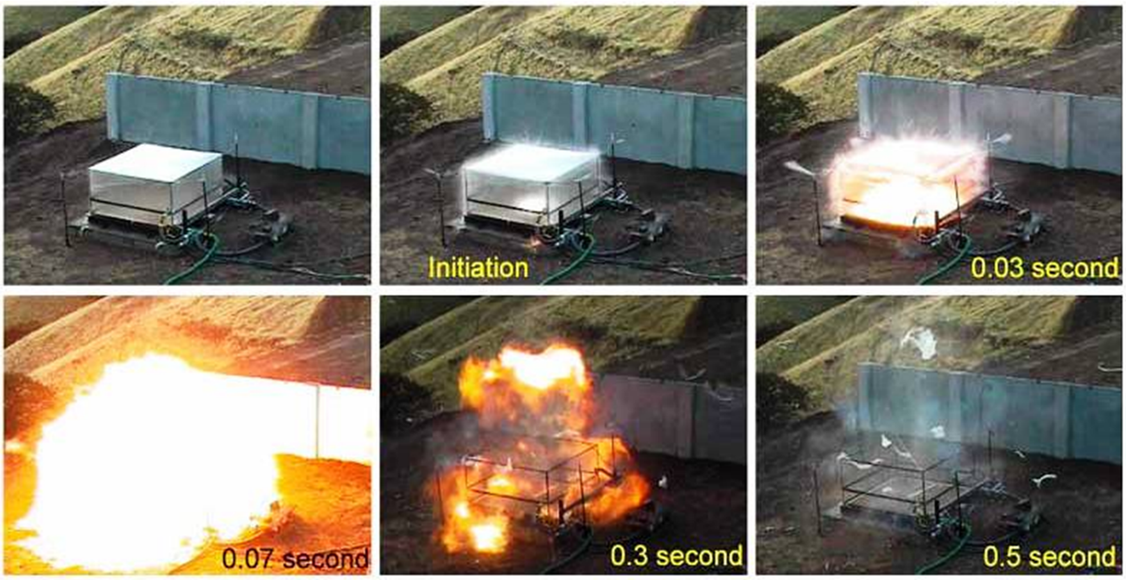

Figure 1. Experiment of a deflagration (Shepherd 2007)

The major characteristic of a detonation is its extremely high speed: the explosion zone moves at a supersonic speed. While, for deflagrations the flame speeds are low (typically one to several hundreds of metres per second), detonation flame speeds in air can easily reach one to two kilometres per second.

The propagation mechanism of a detonation is an extremely rapid and sharp compression occurring in a shock wave as one can see from Figure 2. In contrast to a reversible adiabatic compression, shock compression occurs irreversibly (non-isotropic), due to the extreme rapidity with which it occurs.

Both types of explosion pressure waves (caused by detonation of liquids or solid explosives or air-gas mixtures and such pressure waves caused by deflagrations of only air-gas mixtures) have to be taken into account in the safety assessment of the plant under consideration.

In order to determine scope and content of the assessment to be performed the first step of the assessment is a screening procedure.

The second step is to propose an appropriate approach for those cases where a full scope analysis has to be performed.

Figure 2. Detonation as the strongest type of explosion (Shepherd 2007)

In the latter case several methods can be applied to evaluate the probability of occurrence of an external explosion event, in particular fault tree analysis, event tree analysis and Monte Carlo simulation.

The results presented in the following illustrate that the probability of occurrence of external explosion pressure waves can be successfully assessed by means of the Monte Carlo simulation.

2 GUIDANCE ON ASSESSING EXTERNAL EVENTS

Since 2005, a revised guideline for a probabilistic safety assessment (BfS 2005) as well as revised and extended supporting technical documents (FAK PSA 2005a and 2005b) are issued in Germany which describe the methods and data to be used in performing probabilistic safety assessment in the frame of comprehensive safety reviews.

In these documents, probabilistic considerations of aircraft crash, external flooding, earthquake and explosion pressure waves are required. Also on international level, new recommendations regarding external hazards including explosions pressure waves are recently issued (see, e. g., IAEA 2003a, 2009, 2010).

For the site evaluation for nuclear installations which will be built in the future safety requirements have been developed (IAEA 2003b). In that context activities in the region that involve the handling, processing, transport and storage of chemicals having a potential for explosions or for the production of gas clouds capable of deflagration or detonation shall be identified.

Hazards associated with chemical explosions shall be expressed in terms of overpressure and toxicity (if applicable), with account taken of the effect of distance. A site shall be considered unsuitable if such activities take place in its vicinity and there are no practicable solutions available.

The safety assessment should demonstrate that threats from external hazards are either removed, minimised or tolerated. This may be done by showing that safety related plant buildings and equipment are designed to meet appropriate performance criteria against the postulated external hazard, and by the provision of safety systems which respond to mitigate the effects of fault sequences.

Explosion pressure waves with relevance to the site can be caused by shipping, fabrication, storage and reloading of explosive materials in closer distances to a nuclear power plant or any other industrial plant with a high hazard potential (e. g., process industry).

These different causes lead to two significant different types of risky situations for the site and the plant which have to be assessed within a probabilistic safety assessment:

1. The explosive material is available as a stationary source in the neighbourhood of the plant under consideration (e.g., a storage or a fabrication facility).
2. The explosive material is mobile, i.e. it is shipped in close distance to the plant on the road, by train or on ships along a river or the sea nearby.

In the latter case, the situation is not stable and changes with the varying distances. Moreover, the transport way could be a straight line or a bent which has to be addressed in the calculations - see (Hauschild & Andernacht 2010) for a straight road and (Berg & Hauschild 2010) for a bent river.

Usually, a uniformly distributed accident probability is assumed along the transport way. However, in reality the accident probability may increase in junctions or confluences and – in case of rivers and roads – in curves or strictures. Such on example is explained in section 5 in more detail.

Accidents with explosive material are not only theoretical considerations but happen in reality, sometimes with catastrophic consequences. One extremely severe transportation accident took place in June 2009 in Viareggio which resulted in comprehensive safety evaluations (Pontiggia et al. 2010). Although no industrial plant was damaged in this accident, the potential explosion severity is visible. The accident followed the derailment of a train carrying 14 tank cars of liquefied petroleum gas. The first tank car was punctured after the derailment releasing its entire content that ignited causing an extended and severe flash-fire that set on fire several houses and lead to 31 fatalities.

A more recent accident happened in January 2011 on the river Rhine in Germany, fortunately without any environmental consequences. However, a ship capsized and blocked for many weeks the river for other transportation but, in particular, had the potential to lead to an explosion because – in addition to 2400 tons mainly of sulphuric acid – one tank also contained water and hydrogen.

8 SCREENING PROCESS

In a first step, the important areas of the plant are divided into the three classes A, B and C for the analysis of explosion pressure waves to reflect the degree of protection against the impact by the explosion pressure waves. These classes are the same as for the consideration of aircraft crashes (Berg 2005).

Class A contains systems, where in case of their damages a hazard state directly arises or where an initiating event may occur which cannot be controlled by the emergency cooling system.

Class B contains systems where in case of their damages a hazard state not directly arises, but where an initiating event may occur which is controlled by the emergency cooling system.

Class C contains these safety systems needed for core cooling.

Typical examples of these different classes are (Berg 2010):

A: e.g. primary circuit,
B: e.g. turbine building,
C: separated emergency building.

Basic idea in case of explosion pressure waves is a prescribed check if the frequency of core damage states is less than 1E-07 per year for the plant under consideration. This is the case when

- the total occurrence frequency of the event "explosion pressure wave" (i.e. the sum of all contributions from detonation and deflagration) is determined to be less than 1E-05 per year,
- the building of classes A and C are designed against the load assumptions shown in Figure 3,

- the safety distances according to the BMI guideline (BMI 1976) are fulfilled, based on the formula (1):

$$R = 8m \cdot \sqrt[3]{\frac{L}{kg}} \tag{1}$$

with

R = safety distance (in m) of the place where the explosive gas is handled from to the respective plant which should be larger than 100 m, and
L = assumed mass of the explosive material (in kg).

It should be noticed that the total mass to be assumed depends on the type of explosive material.

For the case that the prerequisites of this prescribed check are met, no further probabilistic considerations are necessary.

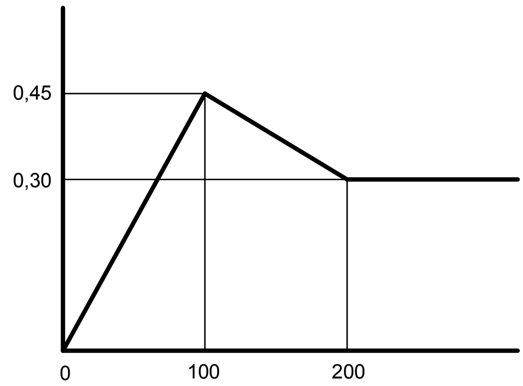

Figure 3. Pressure behaviour at the building for a single pressure wave according to (BMI 1976).

Otherwise the procedure has to be in accordance with the graded process of evidence regarding explosion pressure waves as presented in Table 1 (Berg & Hauschild 2011).

Table 1. The graded process of analysing explosion pressure waves

Criteria	**Extent of analysis**
Occurrence frequency <1E-05 per year Classes A and C are designed according to load assumptions and safety distances determined in length l_R according to (BMI 1976)	Verification using the prescribed check
Not fulfilled Fulfilled	Conservative estimation of occurrence frequency
Not fulfilled Not fulfilled	Detailed probabilistic safety analysis

4 METHODS AS RECOMMENDED IN THE GERMAN PSA DOCUMENT FOR NUCLEAR POWER PLANTS

4.1 Introduction

The German PSA document on methods (FAK PSA 2005a) describes the approaches to be used in the probabilistic safety assessment which have to be performed in the frame of comprehensive safety reviews of nuclear power plants.

One part of this approach is dedicated to the screening process already explained in section 2, the further parts of this document deal in more detail with the occurrence frequency of explosion pressure waves taking into account the site-specific situation, sources of possible explosion pressure waves in the surrounding of the plant, and the procedure for the calculation of occurrence frequencies of accidents during transportation of explosive material by ships, trains or trucks and of accidents of stationary plants near the plant under consideration.

4.2 Assessment

In case that the plant buildings classified as A and C are designed according to the BMI guideline (BMI 1976) and the safety margins regarding distance and mass of the explosive material are kept, it can be assumed that in the most unfavourable case of an explosion pressure wave event

- no event is initiated which directly leads to a hazard state,
- due to the event explosion pressure wave a system failure occurs in the class B and an initiating event is initiated which can be controlled by the emergency cooling system as designed,
- the emergency cooling system is protected against the effects of the event explosion pressure wave.

In the most unfavourable case, a loss of offsite power with destruction of the secondary plant parts (main heat sink, feed water supply) can be assumed, which occurs with the total occurrence frequency of the event explosion pressure wave. It is assumed for simplifying the analysis that together with the occurrence of this event those systems which are outside of the classes A and C fail.

For the calculation of the frequency of the hazard state, resulting from explosion pressure waves, this initiating event and the incident-controlling functions of the emergency cooling system (stochastic non-availabilities) are to be modelled and quantified in an event tree (or using another appropriate method).

The frequency of the event explosion pressure wave to be chosen is the sum of all contributions of the events detonation and deflagration, as far as they can lead to an hazardous state of the plant, resulting from accidents during transportation procedures or the operation of stationary plants in the surrounding of the plant under consideration.

The occurrence frequency of a detonation is several orders of magnitude lower compared with a deflagration (Federal Minister for Research and Development 1990). As far as the distance of the area where the deflagration started has a distance larger than 100 m from the plant under consideration (see safety margins in accordance with (BMI 1976), no endangerment of the plant buildings has to be assumed.

In case of accidents with materials with the potential of a detonation (in particular explosives, ammunition, gases exothermically disintegrating) the detonation is expected to occur at the accident location, i.e. at a transport route or a fixed industrial installation. Here the approach as provided in formula (2) is applied:

$$H_{E,SMZ} = H_{U,SMZ} \cdot W_Z \tag{2}$$

with

$H_{E,SMZ}$ Annual frequency of a explosion pressure wave by explosives, ammunition or gases exothermically disintegrating in the surroundings of the nuclear power plant,

$H_{U,SMZ}$ Annual frequency of accidents with explosives, ammunition or gases exothermically disintegrating in the surroundings of the nuclear power plant,

W_Z Conditional probability of the ignition in an accident.

The deflagration pressure of max. 10 bar drops over 100 m around a factor 1E04, so that within the power station pressure values within the range of the wind pressures are reached.

In case of explosive gas air mixtures (combustible gases with air; inflammable steams, e.g. also of liquid gas, with air) clouds can be appear and a drifting of these clouds from the place where the accident happened into the direction of the plant is possible.

In this situation the deflagration can take place in the area of the plant buildings. The approach applied for this case is described in the following equation (Federal Minister for Research and Development 1990):

$$H_{E,GLG} = H_{U,GLG} \cdot W_M \cdot W_D \cdot W_Z \tag{3}$$

with

$H_{E,GLG}$ Annual frequency of an explosion pressure wave by gas air mixtures in the surroundings of the nuclear power plant,

$H_{U,GLG}$ Annual frequency of accidents with combustible gas in the surroundings of the nuclear power plant,

W_M Conditional probability for the development of an explosive gas air mixture in case of an accident with combustible gas,

W_D Conditional probability for drifting the gas air mixture to the nuclear power plant (as a result of temporal averaging of the arising wind directions),

W_Z Conditional probability of the ignition at the area of the plant.

In a more detailed verification the assumptions introduced can be replaced by plant-specific proofs, considering the different effects of the determined explosion pressure waves.

In the case of a deviation from the BMI guideline (BMI 1976) partial results of the total occurrence frequency of the event arise which contribute directly to the frequency of the hazard states. These contributions are to be determined by a differentiated view of the assigned explosion pressure waves and their effects.

5 MONTE CARLO SIMULATION

5.1 Application

The following application is a case study that represents the evaluation of the probability of occurrence of an external explosion pressure wave that takes place near a plant. The probability of occurrence is assessed on the condition that an accident with combustible gas already occurred.

The application is not restricted to a special field of industry; plants of process industry might be in the focus as well as nuclear power plants. It is assumed that the external explosion pressure wave is initiated by an accident of a gas-tanker that carries explosive liquids on a river.

Although the application is described in a generalized way, it incorporates several elements that are typical in order to assess the impact of explosion pressure waves: accident, wind direction, wind speed and ignition.

It should be noticed that the events, boundary conditions and parameters given in Figure 4 to 7 and Tables 2 and 3 are only example values and do not represent conditions of any specific application.

5.1.1 Plant environment

The plant and its environment are depicted in Figure 4. The length l_S of the section of interest is 4800m and the width w_S is 1800m. The river is subdivided into 7 subsections; each subsection is characterised by an individual length, width and gas-tanker accident frequency.

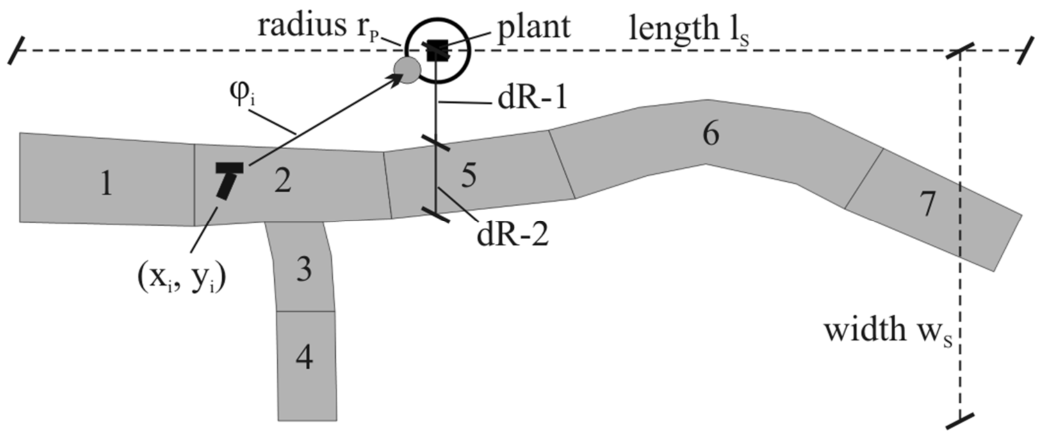

Figure 4. Plant environment and hazardous scenario.

The vertical distance between the plant and the river is between 440m (dR-1) and 780m (dR-2).

In the given application ships can reach every location at the river. An accident at the river-coordinate (x_i, y_i) may cause the development of explosive gas mixture.

Depending on the wind direction φ_i the cloud of gas mixture can drift to the plant. An ignition of the gas mixture close to the plant (within the radius r_P) is in the focus of this study.

All relevant application parameters of Figure 4 are given in Table 2.

Table 2. Relevant application parameters

Description	Parameters
length l_S	4800m
width w_S	1800m
distance [dR-1, dR-2]	[440m, 780m]
radius r_P	150m
plant	100m x 100m

5.1.2 Assumptions

The case study depends on the following assumptions:

- Empirical-distributed accident probability depending on the subsection of the river on condition that the accident already occurred. It is assumed, that the accident frequency is higher in sections with confluences or curves than in straight river-sections.
- Uniformly-distributed accident-coordinate (x_i, y_i) on condition that the accident occurred in the river-section i.
- The development of explosive gas mixture occurs with fixed probability w_G.
- Empirical-distributed wind direction.
- Empirical-distributed wind speed.
- Exponentially-distributed ignition probability depending on the time.
- An explosion within the radius r_P around the plant is in the focus of this study.

The parameters and distribution models are given in Figures 5 to 7 and Table 3.

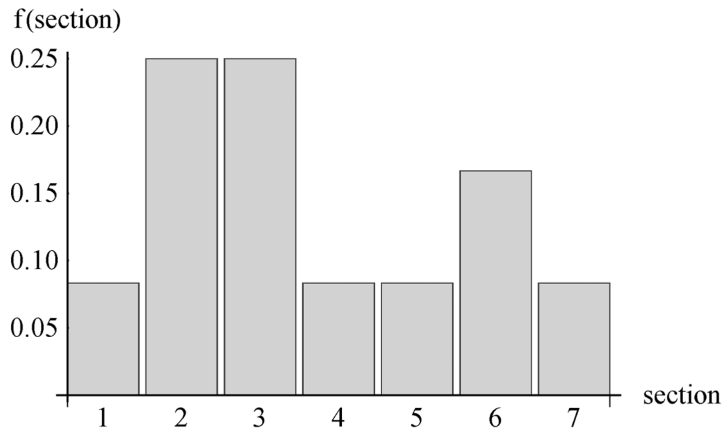

Figure 5. Empirical accident river-section frequencies.

Table 3. Parameters and distribution models

Description	Distribution	Parameters
accident river-section	empirical	-----
accident (x, y)-coordinate	U(a, b)	depending on river-section
development of explosive gas	fixed probability	0,3
mixture	empirical	-----
wind direction φ	empirical	-----
wind speed v_W	Exp(λ)	Exp(0,01 s^{-1})
time τ to ignition		

5.2 Basics

5.2.1 Monte Carlo Simulation

Detailed basics of the Monte Carlo simulation like random sampling, estimators and biasing techniques are specified for example in (Dubi 2000) and (Marseguerra & Zio 2002). In (Berg & Hauschild 2010), (Hauschild & Andernacht 2009) and (Hauschild & Andernacht 2010) the Monte Carlo simulation has been applied and verified successfully in order to estimate the probability of external explosion pressure waves.

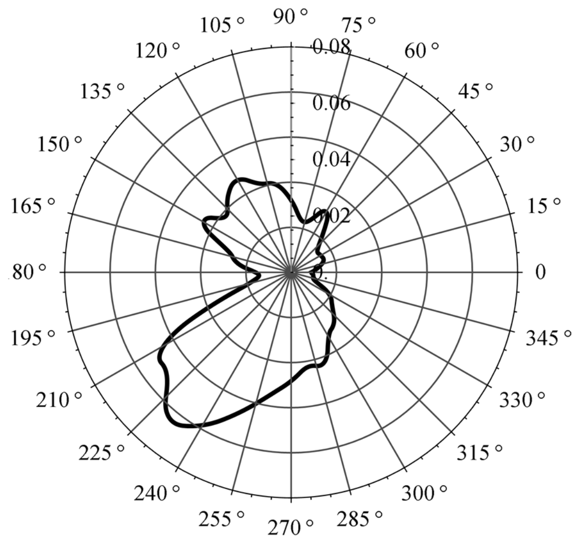

Figure 6. Empirical wind-direction frequencies.

5.2.2 Distribution Models in use

The pdf of the uniform distribution U(a, b) with the parameters a < b is given by

$$f(x) = \frac{1}{\mathrm{b - a}} \qquad \textit{for } \mathrm{a} \le \mathrm{x} \le \mathrm{b}. \tag{4}$$

The pdf of the exponential distribution exp(λ) with the parameter λ > 0 is given by

$$f(x) = \lambda \cdot \exp(-\lambda \cdot x) \qquad \textit{for } x \ge 0. \tag{5}$$

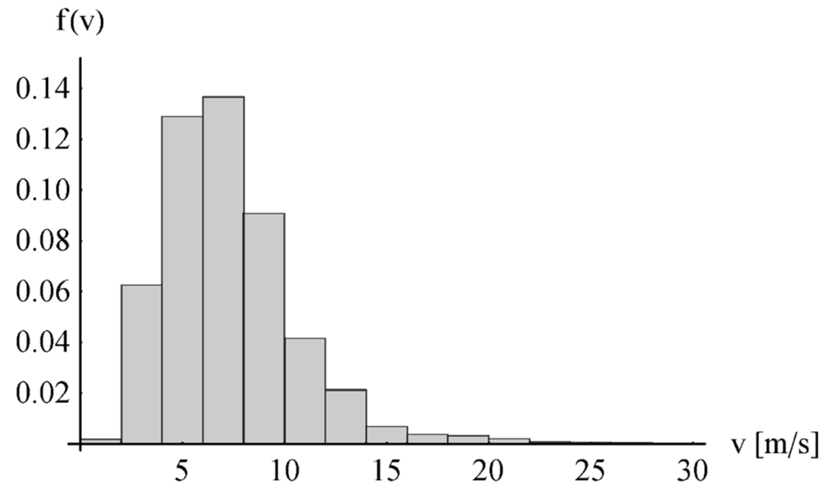

Figure 7. Empirical wind-speed frequencies.

5.2.3 Estimators in use

As the last event estimator (lee) (Marseguerra et al. 1998) is used to predict the probability of an event (e.g. an explosion event), the observed frequency of explosions within the radius r_P is determined. The sample mean probability is

$$\hat{P}_E = \frac{1}{N} \cdot \sum_{i=1}^{N} P_E(i) \tag{6}$$

where $P_E(i) \in \{0, 1\}$ and N = number of trials.

An alternative method is to compute the theoretical probability of an explosion event within the radius r_P in each scenario the wind direction will move the explosive gas mixture to the plant. The advantage over the lee is that each scenario gives a contribution to the probability of occurrence. By analogy with transport theory, this procedure is called free flight estimator (ffe) (Marseguerra et al. 1998). Depending on the accident coordinate (x_i, y_i), the wind direction φ_i and the wind speed v_{Wi} in trial i, the probability of an explosion event within the radius r_P is given by

$$\begin{aligned} P_E(x_i, \varphi_i) &= \exp(-\lambda \cdot 1/v_{Wi} \cdot d_1(x_i, \varphi_i)) \\ &\quad - \exp(-\lambda \cdot 1/v_{Wi} \cdot d_2(x_i, \varphi_i)) \end{aligned} \tag{7}$$

where $d_1(x, \varphi)$ and $d_2(x, \varphi)$ are the distances between the accident coordinate and the intersection of the wind direction and the plant area with radius r_P.

The intersection coordinates (x_I, y_I) of the wind direction φ_i and the plant area with radius r_P are determined by means of

$$x_I^2 + (y_i + \tan(\varphi_i) \cdot (x_I - x_i))^2 = r_P^2 \tag{8}$$

and

$$y_I = (y_i + \tan(\varphi_i) \cdot (x_I - x_i))^2 . \tag{9}$$

The sample mean probability is

$$\hat{P}_E = \frac{1}{N} \cdot \sum_{i=1}^{N} P_E(x_i, \varphi_i) \tag{10}$$

where N = number of trials.

5.3 Analysis

The Monte Carlo simulation is performed by means of the last event estimator and the free flight estimator.

The algorithm to model and solve the problem is based on the German PSA guideline (BfS 2005) and the supporting technical document on PSA methods (FAK PSA 2005a).

The Monte Carlo simulation depends on a sequence of single events:

- accident river-section: empirical-distributed (Figure 5),
- accident (x, y)-coordinate: uniformly-distributed on condition that the accident occurred in the river-section i,
- development of explosive gas mixture: fixed probability (0,3),
- wind-direction φ: empirical-distributed (see Figure 6),
- wind-speed v_W: empirical-distributed (see Figure 7),
- time τ to ignition: Exp(0,01 s^{-1})-distributed.

5.4 Results

The results of the Monte Carlo simulation are evaluated on the condition that the accident already occurred.

In order to assess the frequency of occurrence of an external explosion event the frequency of accidents with combustible gas has to be considered. It should be noticed, that the results for the frequency of occurrence of an external explosion event will be several magnitudes lower than the results for the conditional explosion event probability given in this paper.

Different ranges of conditional explosion-probability P_E are depicted in Figure 8 and 9.

Areas with higher gray-level intensity represent higher conditional explosion-probability. In order to compare the results to the conditional explosion event probability P_E close to the plant (within the radius r_P) the results in Figure 8 are normalised on the plant area $\pi \cdot r_P^2$.

The number of trials, the simulation time and the results like mean value and variance are listed in Table 4.

Figures 8 and 9 indicate that the conditional explosion event probability decreases as the distance to the river (place of the assumed accident) increases. This is due to the exponentially distributed ignition probability which depends on the time or the distance to the accident.

Close to the river-sections 2 and 3 the conditional explosion event probability increases, this is due to the higher accident frequency in these sections combined with the specific wind-direction frequencies.

As the different Monte Carlo methods given in Table 4 are compared it can be found out, that both solutions fit a mean about 1,2E-03 which verifies the results as well as the adopted different Monte Carlo algorithms.

If the variance is regarded, the Monte Carlo simulation in combination with the free flight estimator is the most efficient approach.

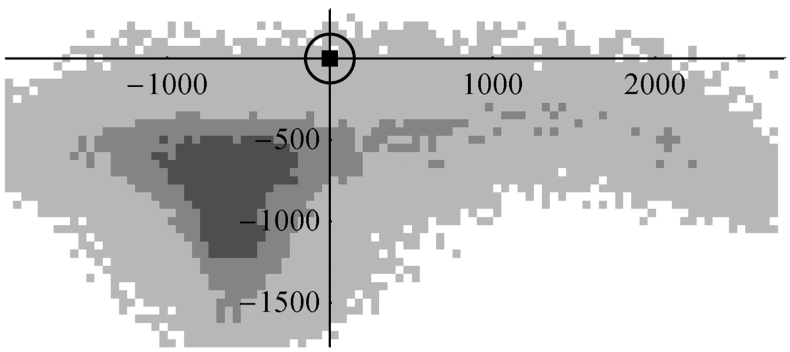

Figure 8. Ranges of conditional explosion event probability P_E – normalised on $1 m^2$.

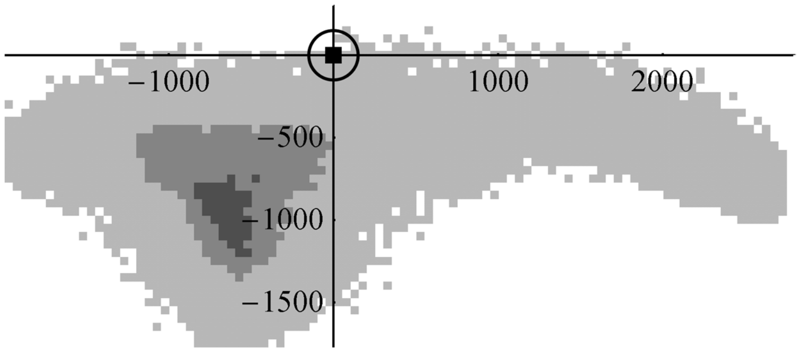

Figure 9. Ranges of conditional explosion event probability P_E – normalised on the plant area $\pi \cdot r_P^2$.

Table 4. Conditional probability of an explosion event within the plant area with radius r_P

Method	Trials	Time	Mean	Variance
analog MCS - lee	1E06	60.7s	1.21E-03	1.20E-03
analog MCS - ffe	1E06	65.1s	1.23E-03	1.13E-04

6 CONCLUDING REMARKS

6.1 Countermeasures to avoid or mitigate the adverse effects of external explosions

Knowledge of the explosion characteristics and the structural impact on buildings of the respective plant is necessary to determine the appropriate countermeasures in order to ensure a safe operation of the plant. However, fundamental changes of the plant under consideration are mainly possible only during the design and construction phase. In case of a plant already operating since several years, the implementation of effective countermeasures is much more difficult or even not possible.

On the one hand, comprehensive calculations can be performed to show that existing assumptions in the calculation provided for the licensing of the plant have been very conservative.

On the other hand, organizational and technical provisions can be taken to reduce the occurrence of an external explosion pressure wave at the plant.

One organizational possibility is to interdict the transport of explosive material, e.g. on a road, in the neighbourhood of the plant. Another solution is to close the road for transit traffic such that the road is only leading to the plant.

One technical countermeasure to reduce the explosion frequency on site is the installation of an automatic ignition system placed at a save distance from the site. An assessment has been performed for such an installation which showed that – if the igniters are correctly designed and installed – the shock wave impact after an ignition on the buildings will be limited and will not cause any structural damage.

6.2 Modelling of external explosions and potential for improvements of the methods

The evaluation of external hazards in relation to nuclear power plant design is traditionally considered as a two-step process. The detailed evaluation is preceded by a screening phase where potential scenarios are identified. Many scenarios are screened out on the basis of different criteria, such as distance from the site, probability of occurrence, expected consequence on the plant, or because their effects on the plant are expected to be enveloped by some others. Typically, explosion pressure waves are part of the probabilistic safety assessment as in case of comprehensive periodic safety reviews.

In the German safety guidance document on methods (FAK PSA 2005a) the screening process for the explosion events is explicitly described. The classes of buildings with respect to their protection are the same as for the aircraft crash assessments. Since the updated PSA guideline has been issued in 2005 also requiring the assessment of external events, first practical experience in performing and reviewing the external probabilistic safety assessments are available. One topic is the assessment of the conditional probability of the occurrence of external explosion pressure wave and the discussion of appropriate methods according to the state of art.

The procedure and methods applied are used for the evaluation of external explosion pressure waves with respect to nuclear power plants. However, they can also be applied to other types of industrial plants.

The presented case study and its results (Figures 7 to 8 and Table 4) in the second part of this paper indicate that the conditional probability of occurrence of external explosion pressure waves in consideration of realistic conditions (accident frequency depending on environmental conditions, wind direction and wind speed) can be successfully assessed by means of the Monte Carlo simulation.

As a next step the assessment of explosion events should be extended to include much more realistic boundary conditions:

- the extent of the hazard and the explosive gas mixture,
- ignition probability that depends on environmental conditions (Hauschild & Schalau 2011).

Different ignition models are discussed in (Drewitz et al. 2009). The applied model should be more realistic like the applied exponentially-distributed ignition model; moreover the applicability to integrate the new ignition model into Monte Carlo algorithm should be given.

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THE PROCESSING OF THE MEDICAL DATA WITH THE USE OF LOGISTIC REGRESSSION

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ABSTRACT

This paper shows the evaluation of the data coming from the surgery operations and from the clinique of occupational and preventive medicine. Deals with the usage of logistic regression for predicting the classification of patients into one of the two groups. Our data come from patients who underwent Phadiatop test examinations and patients who underwent colectomy in the University Hospital of Ostrava. For Phadiatop test, as the predictor variables were chosen personal and family anamneses. Both of these anamneses were divided into four categories according to severity ranked by doctors. The model for Phadiatop test was tested with the use of a medical database of 1027 clients. The developed models predict the right results with 75% probability for Phadiatop. For colectomy operation we were the physiological and operative scores as predictor variables. Scores for Colectomy operation were based on the POSSUM system (Copeland et al. 1991). The psychological score comprises 12 factors and the operative score comprises 6. Colectomy operation was tested upon a medical database of 364 clients. The developed models are successful with 70% probability for morbidity in surgery.

1 INTRODUCTION

Score system in surgery generally aims at quantification and consequently at objectification of risk of surgery patients. In particular, it means to determine the probability of complications occurrence, morbidity for an individual patient or for a group of patients. Applied on the groups of patients they enable meaningful analysis of achieved records of morbidity and the stratification of patients. At the same time they provide a tool for objective assessment of newly implementing techniques and methods. In this case we have an operation and a physiological score. The operation score contains 6 hazard factors of the surgical intervention. The physiological score contains 12 factors related to the physiological state of the patient before operation. Scores for morbidity were based on the POSSUM system (Copeland et al. 1991) [7].

The atopy rate of inhabitants of the Czech Republic is increasing. Atopy could be understood as a personal or family predisposition to become, mostly in childhood or adolescence, hyper sensible to normal exposure of allergens, usually proteins. These individuals are more sensitive to typical symptoms of asthma, eczema, etc. The Phadiatop test is used as a measure of atopy. Information obtained from personal and family anamneses were used for examining presence of asthma, allergic rhinitis, eczema or other forms of allergy (e.g. contact or food allergy). Family and personal anamneses of each patient were evaluated.

Unfortunately, the Phadiatop test is expensive, so we try to predict results of the test on the basis of a detailed family and personal anamneses. The knowledge of results of the Phadiatop test is very important especially for diagnosis of allergic dermatoses and also for the professional medical care for travellers [1]-[2].

Information about the appearance of postoperative complications or atopy is given by results of Phadiatop test or morbidity, where there is the value 0 for none or low form atopy or morbidity and value 1 for medium or high form atopy or morbidity.

2 LOGISTIC REGRESSION AS A TOOL FOR DISCRIMATION

A common problem is to classify objects into one of the two given groups. Each object is described by attributes. The aim of the task is to assign a new object into one of the groups so that the object belongs to one of the two groups (labelled as 0 and 1). The discriminatory problem will be solved on the basis of the logistic regression model.

Generally, we have n objects with p measured attributes. But in case of some of the objects, we do not know whether the object is a member of the group. The measured attributes are represented as p-dimensional random vectors $X_1, \dots, X_n$.

The classification of the i^{th} object is expressed by random variable Y_i, which has the value 0 or 1 depending on their membership in the group.

The logistic regression was not originally created for the purpose of discrimination, but it can be successfully applied for this kind of analysis [3]. A logistic regression model, which is modified for the purpose of discrimination, is defined as follows. Let $Y_1, \dots, Y_n$ be a sequence of independent random variables with alternative distributions, whose parameters satisfy:

$$P(Y_i = 1 | X_i = x_i) = \frac{e^{\beta_0 + \beta \mathrm{r} x}}{e^{\beta_0 + \beta \mathrm{r} x} + 1},$$
$$P(Y_i = 0 | X_i = x_i) = \frac{1}{e^{\beta_0 + \beta \mathrm{r} x} + 1}, \tag{1}$$

for $i = 1, \dots, n$, where $\beta \mathrm{r} = (\beta_1, \dots, \beta_p)^{\mathrm{r}}$, is unknown p-dimensional parameter and $X_1, \dots, X_n$ are ($p+1$)-dimensional random vectors $(\beta_0, \dots, \beta_p)$. This model can be called a learning phase, in which both values X_i and Y_i are known for each object (i.e. it is known to which group each object belongs to). Based on this knowledge, we try to predict parameters $\beta_0, \dots, \beta_p$ and thus we try to estimate function $\pi(x)$, where

$$\pi(x) = P(Y = 1 | X = x) = \frac{e^{\beta_0 + \beta \mathrm{r} x}}{e^{\beta_0 + \beta \mathrm{r} x} + 1} \tag{2}$$

Another subject, whose classification is unknown, is assigned to one of the two groups according to the value of decision function $\pi(x)$.

The object will be included in the first group if $\pi(x)>0.5$. Otherwise, the object will be included in the second group. The main advantage of this model is that it does not require conditions for distributions of random vectors $X_1, \dots, X_n$. However, the model assumes a very specific form of probability $P(Y = 1 | X = x)$ and we should verify the significance of the relationship (2) using appropriate statistical tests.

3 APPLICATIONN ON BIOMEDICAL DATA

3.1 Phadiatop test

The tested biomedical data are from the University Hospital of Ostrava, Department of Occupational and Preventive Medicine, the Czech Republic. The logistic regression is used in order to predict the results of the Phadiatop test. The medical database for predicted Phadiatop test contained information about 1027 patients.

Patients in Group 0 have the Phadiatop test results either 0 or I (no visible symptoms), so no treatment is necessary. The remaining patients with Phadiatop test II – VI are members of Group 1. For these patients a medical treatment is necessary.

We have one dependent variable *Y*, Phadiatop (*Ph*), which depends on two independent variable personal anamneses (*OA*) and family anamneses (*RA*).

Variable *Y* can be either 0 or 1, according to the membership of a patient in Group 0 or Group 1. Values of these independent factors were obtained from medical experts. The expert severity scores for personal and family anamneses are presented in Table 1. Here the category "Others" represents the score of various kinds of allergies (e.g. contact or food allergies). The independent variable of the logistic regression was obtained as a sum of these scores for each patient from the database [4]-[6].

Table 1. Expert severity scores for personal and family anamneses.

Factor	Score
Asthma	10
Allergic rhinitis	8
Eczema	6
Others	4

3.2 Colectomy operation

The tested biomedical data are from the University Hospital of Ostrava, Department of surgery, Ostrava, Czech Republic. The logistic regression is used in order to predict the results of the colectomy operation (morbidity). The medical database for predicted morbidity contained information about 364 patients.

The processed data come from the open and laparoscopic operations of the colon carrying out. Data file contains information about patients, such as their operational score *(OS)* and physiological score *(PS)* and the information about the appearance of postoperative complications. Operation score contains 6 hazard factors of the surgical intervention (severity of the surgery, blood loss, contamination of peritoneum, etc.). Physiological score contains 12 factors related to the physiological state of the patient before surgery (age, cardiac stress, blood pressure, etc.). We have the information about the appearance of postoperative complications, where there is the value 0 for none or low form morbidity *(R)* and value 1 for medium or high morbidity.

3.3 Regression model

In accordance with the Chapter 2, the logistic regression model has the form:

$$g(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2, \tag{3}$$

where $\beta_i \in \Re, i = 0,1,2$ are logistic regression coefficients and vector x has two components $x_1 = OA, x_2 = RA$ (respectively $x_1 = OS, x_2 = PS$). The dependence $\pi(x)$ on x has the form:

$$\pi(x) = \frac{e^{g(x)}}{1+e^{g(x)}}. \tag{4}$$

Here $\pi(x)$ denotes the probability of occurrence of Phadiatop group, i.e. $\pi(x) = Ph$, (respectively $\pi(x) = R$). Unknown coefficients β_i, where $i = 0, 1, 2$ are determined by the maximum likelihood method. In our case, the maximum likelihood for our logistic regression model can be expressed as

$$L(\beta) = \prod_{i=1}^{n} \pi(x_i)^{y_i} \cdot (1-\pi(x_i))^{1-y_i} , \tag{5}$$

where n is the number of patients. The vector $\beta = (\beta_0, \beta_1, \beta_2)$ denotes the unknown regression coefficients. If the i^{th} patient is identified as a member of Phadiatop or morbidity Group 1, then $y_i = 1$, otherwise $y_i = 0$. The vector x_i, where $i = 1, \dots, n$ has two components: $x_{i,1}=0A$ and $x_{i,2}=RA$ which represent personal and family anamneses, respectively $x_{i,1}=0S$ and $x_{i,2}=PS$, which represent operative and physiological score.

The estimation of regression parameters β is provided by maximization the logarithm of the maximum likelihood, which can be expressed as:

$$\ln L(\beta) = \sum_{i=1}^{n} [y_i \cdot \ln \pi(x_i) + (1-y_i) \cdot \ln(1-\pi(x_i))]. \tag{6}$$

Now the regression model can be used for predictions, whether the patient with given personal and family anamneses (respectively physiological and operative score) is a member of the selected Phadiatop group (respectively morbidity).

4 LOGISTIC MODEL AND PREDICTION RESULTS

4.1. Phadiatop test

In the first step, we try to create a regression model of all supplied data. We used the data corresponding to all 1027 patients. We obtained the following logistic model:

$$\ln\left(\frac{Ph}{1-Ph}\right) = -1.5435 + 0.2124 \cdot OA + 0.0146 \cdot RA. \tag{7}$$

Results of this model are summarized in Table 2, column Case A.

Prediction results of Phadiatop test were incorrect for 220 patients, which we could describe as error of prediction rate model: $\frac{220}{1027} = 0.2142$.

In the second step, we created a learning group as a random sample from 90% of database (926 patients). To verify the correctness of model assumptions, the logistic model was created using the learning group. We obtained the following updated model:

$$\ln\left(\frac{Ph}{1-Ph}\right) = -1.5667 + 0.2112 \cdot OA + 0.0199 \cdot RA. \tag{8}$$

For testing this updated model, we analysed the remaining data set, i.e. 10% of the database (102 patients), which were not assumed for the training phase. In this case we calculated prediction error of model, too: $\frac{24}{124} = 0.1935$. The results are summarized in Table 2, column Case B.

Table 2. Prediction results of regression models.

	Case A	Case B	Case C	Case D
Number of correctly classified patients	807	78	240	26
Number of incorrectly classified patients	220	24	124	10
Number of patients predicted for Group 1	233	21	88	2
Number of real patients in Group 1	331	33	78	12
Prediction error	21.4%	19.4%	33.7%	30.5%

In Table 3, there is a summary of the results for Phadiatop test Case A. The following table shows the results of the model, which presents the cases where the model works correctly and where not.

Table 3. Verification of the diagnosis correctness for Phadiatop test.

	Model 1	Model 0
Phadiatop 1	166	165
Phadiatop 0	55	641

In the first column there are results of the observation from the Faculty Hospital in Ostrava and in the first line there are results of our model, equation (7).

The Phadiatop value 1 and model 1 mean, that we measured well, and on the other hand the Phadiatop value 1 and model 0 show, that the programme predicted the value 0, but the patients are classified into the Phadiatop group 1 (165 patients were classified incorrectly). Nevertheless, it is obvious from the results that most of the cases work correctly and the mistake of the model we created is only about 21.4%.

4.2. Colectomy operation

In total we had 364 data from patients, who underwent surgery operations. In the first step, we created a model containing all data from years 2001-2006.

We received the following logistic model:

$$\ln\left(\frac{R}{1-R}\right) = -2.1997 + 0.0726 \cdot PS + 0.0549 \cdot OS. \tag{9}$$

The model of morbidity was incorrectly predicted for 124 patients, which give us the following error prediction rate of the model: $\frac{124}{368} = 0.337$. The outcomes of this model are written in Table 2, column Case C.

In the second step, we took the data from the group of 328 patients from years 2001-2005 and their records have been used for the creation of the new logistic regression model:

$$\ln\left(\frac{R}{1-R}\right) = -2.3339 + 0.0637 \cdot PS + 0.0702 \cdot OS. \tag{10}$$

We applied this model to the group of 36 patients, who underwent the surgery operations in year 2006. In general, results of morbidity were incorrectly predicted for 36 patients, which we could calculate as a prediction error of the model: $\frac{10}{36} = 0.3056$. The obtained results are summarized in Table 2, column Case D.

4.3. Verifications of the models

We made the test for models for Phadiatop test and colectomy operation. For Phadiatop test we tested a model created from 90% of data. For colectomy operation we tested a model created from data of years 2001-2005.

We also provided the analysis of variance for the models, see Table 4. Since the p-value is less than 0.01, there is a statistically significant relationship between variables at 99% confidence level.

We evaluated coefficients of *OA* and *RA* (respectively *OS* and *PS*) using the Pearson Chi-Square significance test, see Table 5. Variable *OA* (personal anamneses) is statistically significant at the 95% confidence level. On the other hand, p-value for *RA* variable (family anamneses) is greater than 0.05. Thus, *RA* variable is not statistically significant and may be excluded from the model. This result is maybe due to insufficient information on family anamneses in the database. This was caused by the fact, that the doctor did not manage to get the information on further family anamneses of the patients.

For coefficients of logistic regression *PS* and *OS* we can see (Table 5., lines Colectomy operation) that they are both statistically significant at the 95% confidence level. This result has shown, that both variables are important and we cannot exclude any of them.

Table 4. Analysis of variance.

	Source	Deviance	Df	P-Value
Phadiatop test	Model	197.312	2	0.0000
	Residual	966.168	923	0.1575
	Total(corr.)	1163.168	925	
Colectomy operation	Model	11.2321	2	0.0036
	Residual	432.099	324	0.0001
	Total (corr.)	443.332	326	

Table 5. Test of statistical significance.

	Factor	Chi-Square	Df	P-Value
Phadiatop test	OA	169.768	1	0.0000
	RA	2.12564	1	0.1448
Colectomy operation	OS	6.42149	1	0. 0113
	PS	5.37696	1	0.0204

5 CONCLUSION

The multidimensional logistic regression analysis has been applied to the actual medical data which describe the results of Phadiatop test and of the open surgeries of colon.

Model for Colectomy operation was designed and its good prediction qualities were demonstrated on the group of the patients from year 2006. Phadiatop test is a cost-expensive medical procedure. For this reason it would be very interesting to predict patient diagnosis by assuming personal and family anamneses, which can however be easily obtained. The data of patients include the results of the Phadiatop test with detailed description of personal illnesses, allergies and also family anamneses. It was statistically proved that the family anamnesis is not statistically significant, probably due to insufficient information from patients which caused incomplete database of the patients.

The statistical tests were made to verify the model quality.

Also the results of the analysis of variance and the likelihood ratio test of the significance of the regression coefficients were positive for this model. New prediction model for Phadiatop test which were developed using 90% of data and the updated model were successfully tested using remaining 10% of the data.

Models which were however created are suitable for predictions of the Phadiatop test with 75% probability of success and for Colectomy operation with 70% probability of success.

In the future research we would like to predict the results of atopy or morbidity in more groups according to the seriousness of illnesses. For Phadiatop test, we will deal in detail with cases, where the model does not work and we will try to examine it closely. Detailed analysis of the results will also be important for future search of biomedical relations, which are hidden in the given biomedical database.

Acknowledgement:
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THE REVERSE LOGISTICS MODEL WITH REUSING OF COMPONENTS OF SERIES SYSTEM PRODUCT

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ABSTRACT

The main goal of this paper is to create the reverse logistics model that uses reliability theory to describe reusability of product parts with assumption that recovered components are used in production process but they aren't as good as new ones. The model allows to estimate the potential profits of the reusing policy in a production and gives the base to optimize some of the process parameters: the threshold work time of returns or the warranty period for products containing reused elements.

1 INTRODUCTION

Until recently logistics systems supported only processes carried out in classical material flow from producer to final user. Recently it has been a remarkable growth of interest in optimizing logistics processes that supports recapturing value from used goods. The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal is called reverse logistics. Reverse logistics has become one of the logicians' key areas of interest. It enjoys ever-increasing interest of many industrial branches. Nowadays a growing number of companies realize the meaning of that field of logistics. Reuse of products or product parts can bring direct advantages to the company because it reduces costs associated with acquiring new components by using recycled materials or recovered components instead of expensive raw material. Literature survey that has been done around the theme of the reverse logistics area, allowed to set out this article aims and objectives.

In the reverse supply chain the issue of: timings, quantities and conditions of returned and reusable components make production planning difficult (Murayama, T. et al. 2006). The majority of models assume that demand for new products and returns quantity are independent Poisson random variables (Plewa, M. & Jodejko-Pietruczuk, A. 2011) and very few use the reliability theory to estimate the number of reusable products. Murayama et al. (Murayama, T. & Shu, L. H. 2001, Murayama, T. et al. 2005, Murayama, T. et al. 2006) propose the method to predict the number of quantities of returned products and reusable components at each time period by using series system reliability models. The condition aspect of returns in a reverse logistic system is usually omitted by using assumption that all the returns are reusable and usually "as good as new" (Kiesműller, G.P. 2003). Only few models use the reliability theory to diversify returned elements' reusability, but they don't give any guidelines for the way to optimize the threshold value of components' residual life (e.g. (Murayama, T. & Shu, L. H. 2001, Murayama, T. et al. 2005, Murayama, T. et al. 2006)).

In literature the field of reverse logistics is usually subdivided into three areas: inventory control, production, recovery and distribution planning. The model presented in this paper is a inventory model and hence the literature survey refers to this area. Basic inventory model in reverse logistics is built on following assumptions (Fleischmann, M. et al. 1997):

- the manufacturer meets the demand for the final products;
- the manufacturer receives and stores the products returned from the final user;
- demand for the new products can be ful-filled by the production or by the recovery of returned products;
- recovered products are as good as new ones;
- the goal of the model is to minimize the total costs.

Inventory models in reverse logistics can be divided into deterministic and stochastic models.

Deterministic models presented in the literature are mainly the modifications of the EOQ model (e.g. (Dobos, I. & Richter, K. 2000, Dobos, I. 2002, Mabini, M.C. et al. 1992, Richter, K. 1994, Richter, K. 1997, Richter, K. 1996a, b, Schrady, D.A. 1967, Teunter, R.H. 2001)). There are also some models based on dynamic programming which are the extensions of the classical Wagner Whitin model (e.g. (Kleber, R. et al. 2002, Konstantaras, I. & Papachristos, S. 2007, Richter, K. & Sombrutzki, M. 2000, Richter, K. & Weber, J. 2001)).

In the class of stochastic models there are two model groups:

- models, in which demand for the new product is s a consequence of the return;
- models, in which the demand for new products does not depend on the number of returns, but the number of returns may depend on the previous demand.

In first group there are models of repair systems. These are closed systems in which the number of elements remains constant. The main goal of such a system is to keep sufficient number of spare parts to provide the required level of availability of the technical system (e.g. (Muckstadt, J.A. 1973, Sherbrooke, C.C. 1971, Sherbrooke, C.C. 1968)). The second group of stochastic inventory models can be divided into continuous (e.g. (Fleischmann, M. et al. 2002, Fleischmann, M. & Kuik, R. 2003, Heyman, D.P. 1997, Korugan, A. & Gupta, S. M. 1998, Muckstadt, J.A. & Isaac. M.H. 1981, Van der Laan, E.A. et al. 1996, Van der Laan, E.A. et al. 1996, Van der Laan, E.A. & Salomon, M. 1997]) and periodic review models (e.g. (Inderfurth, K, Inderfurth, K. & van der Laan, E. 2001, Kelle, P. & Silver, E.A. 1989, Simpson, V.P. 1978)). A more detailed description of inventory models in reverse logistics can be found in (Plewa, M. & Jodejko-Pietruczuk, A. 2011).

Literature review allows to summarize the current state of knowledge and to define the main shortages of existing logistics models that deal with the reverse logistics problem. The majority of models assume that demand for new products and returns quantity are independent Poisson random variables (Plewa, M. & Jodejko-Pietruczuk, A. 2011). Few authors examine the relationship between the demand, and the number of returns but there are no inventory models in reverse logistics that use reliability theory to assess the number of returns and very few use the reliability theory to estimate the number of reusable products. Murayama et al. (Murayama, T. & Shu, L. H. 2001, Murayama, T. et al. 2005, Murayama, T. et al. 2006) propose the method to predict the number of quantities of returned products and reusable components at each time period by using series system reliability models. The condition aspect of returns in a reverse logistic system is usually omitted by using assumption that all the returns are reusable and usually "as good as new" (Kiesműller, G.P. 2003). Only few models use the reliability theory to diversify returned elements' reusability, but they don't give any guidelines for the way to optimize the threshold value of components' residual life (e.g. (Murayama, T. & Shu, L. H. 2001, Murayama, T. et al. 2005, Murayama, T. et al. 2006)). Most of created models assume single component product.

Main goal of this paper is to create the reverse logistics inventory model that uses the reliability theory to describe reusability of product parts with assumption that recovered components are used in a production process but they aren't as good as new ones. The model allows to estimate the potential profits of the reusing policy in production and inventory management. It gives the base to optimize some of the process parameters: the threshold work time of returns, the warranty period for products containing reused elements.

2 THE MODEL OF THE REUSING POLICY

The model that is presented in the paper is based on the following assumptions:

- A company produces the object composed of two elements (A and B). The product fails when one of components fails – series reliability structure.
- A failure of each component occurs independently on other components' failures.
- If the product fails during the warranty period, it is returned to the manufacturer and he has to pay some penalty cost (e.g. the cost of a new product).
- The products are returned as soon as their lives are ended and reusable B components are stored in a stock until new production batch running, when they may be reused.
- The component B of the product may be reused in a new production, if it was not the cause of a product failure and its total work time up to this moment is not greater than some acceptable - threshold time T (Fig.1).
- Neither failed elements B can be reused in a new production (not repairable) nor any A element. All A components are new in a new production.
- Demand for the products is determined and fixed.
- New products are manufactured and sold periodically in established moments.

The process of reusing of the component B, dependently on its threshold age T, is shown in (Fig. 1).

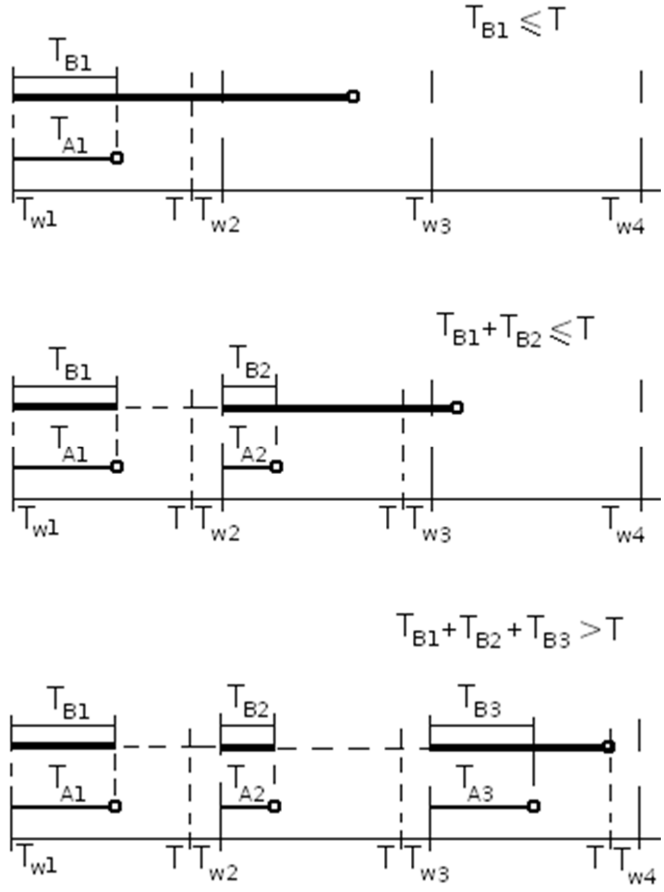

Figure 1. Process of element reusing in a new production.

Despite the fact that companies realize the potential of reusing products, the question "is it worth to do it", is not so simple to answer. Within main reasons for products reusing are: difficulties with raw material supplying, high cost of utilization of returned and damaged products or lower cost of reusing of products' components than buying new ones. The objective of the presented model is to estimate profitability of using returned and recovered elements in a new production, in the case when they are not as good as new.

According to the assumptions, before every production beginning, the manufacturer has to make the decision: which of returned elements B should be used in the new production. The usage of recovered components decreases production costs but also increases the risk that additional costs occur because of larger amount of returns during the warranty period.

The objective is to find the threshold work time T for returned element that equalizes potential cost and profits of the reusing policy:

$$E\left(C_{WO}\left(T_W, T\right)\right) - E\left(C_{WN}\left(T_W, T\right)\right) = C_B - C_R - C_M \quad (1)$$

$$E\left(C_{WO}\left(T_W, T\right)\right) = \left[1 - \frac{R_B\left(T_W + T\right)R_A\left(T_W\right)}{R_B\left(T\right)}\right]C_O \quad (2)$$

$$E\left(C_{WN}\left(T_W, T\right)\right) = \left[1 - R_B\left(T_W\right)R_A\left(T_W\right)\right]C_O \quad (3)$$

where C_{WO} = the cost of warranty services if an "old" element is used in a new production; T_W = the warranty period of the product; T = the threshold age of the element B, after that the further exploitation isn't continued; C_{WN} = the cost of warranty services if a "new" element is used in production; C_B = the purchase cost of a new element B; C_R = the total cost of all activities of: decomposition, cleaning, preparing of the returned B element to reusing in a production; C_O = penalty cost resulting from a product failure during warranty period (e.g. the total cost of production of a new object); $R_A(t)$ = reliability of the element A in t moment; $R_B(t)$ = reliability of the element B in t moment; n = production batch percent of reusable B elements, that return during warranty period; E(x) = the expected value of variable x.

The left side of the equation 1 specifies the increase in expected costs of product warranty services (during a single warranty period) caused by the reusing in a production element that is not as good as new. This part of the expression depends on: both elements' reliability (A and B), the length of warranty period and the length of the acceptable total work time T of the returned B element. The increase in expected cost of warranty services is calculated for the case, when all reused elements are in the age of T. The real age of reused elements is equal or lower than T and this way the left side of the expression 1 estimates the maximum possible growth in warranty cost when "old" elements are reused in the production. The direction of changes in the expected cost of reusing elements is not so obvious. In special cases (low value of time T and short warranty period) it can happen that reused product is more reliable than a new one.

The right side of the equation 1 determines the potential cost savings resulting from using cheaper, recovered components instead of new elements.

On the basis of this expression the threshold value T of total work time of the element B can be found, above which reusing the component is not economical. An analytical solution of the equations can't be achieved for the majority of probability distributions describing components' time to failure, but the value of T can be easily found by applying numerical calculations for given vector of T.

The practical application of the proposed model is limited because of the mentioned simplification that all reused elements are in the age of T. According to model assumptions, the demand for the products is determined and fixed. It means that a new production is usually mix of new and reused elements, dependently on their accessibility. The threshold work time T (for which savings from components' reusing are equal losses) can be specified on the base of Equation 1, but practical questions require more precise data: how many reusable elements will return before the new production batch, how many new components must be kept in a stock or what is the expected profit/cost when mix of new-old elements is used in the production?

To answer this questions, the model should consider the percent of returns used in the production. The number of returns that can return between two moments of a production beginning and may be reused depends on the number of the products that were sold earlier, the length of the period between two consecutive production batch, the length of the warranty period and threshold work time T. The number of returns (calculated as a percent of the production batch size) may be estimated as follows (Plewa, M. & Jodejko-Pietruczuk, A. 2011):

$$n(t_a, t_b, T_W, T) = n_1 + n_2 + n_3 \tag{4}$$

$$n_1 = \int_{\max(t_a - T_M, 0)}^{t_b - T_M} D(t) F(t_a - t, T_M) dt \tag{5}$$

$$n_2 = \int_{t_b - T_M}^{t_a} D(t) F(t_a - t, t_b - t) dt \tag{6}$$

$$n_3 = \int_{t_a}^{t_b} D(t) F(0, t_b - t) dt \tag{7}$$

$$T_M = \min(T_W, T) \tag{8}$$

$$F(t_1, t_2) = F(t_2) - F(t_1) \tag{9}$$

$$F(t) = (1 - R_A(t)) R_B(t) \tag{10}$$

$$F(t) = (1 - R_A(t)) R_B(t + T - T_M) \tag{11}$$

where n = production batch percent of reusable B elements, that return during warranty period; T_W = the warranty period of the product; T = the threshold age of the element B, after that the further exploitation isn't continued; T_M = minimal value of threshold time T and warranty period T_W; $D(t)$ – size of the production sold in t moment; $F(t_1,t_2)$ = the increase of product unreliability (caused by A component when B element is still working) in period between t_1 and t_2 moments; $R_A(t)$ = reliability of the element A in t moment; $R_B(t)$ = reliability of the element B in t moment; min, max (t_1,t_2) = minimum/maximum value of variables t_1 and t_2.

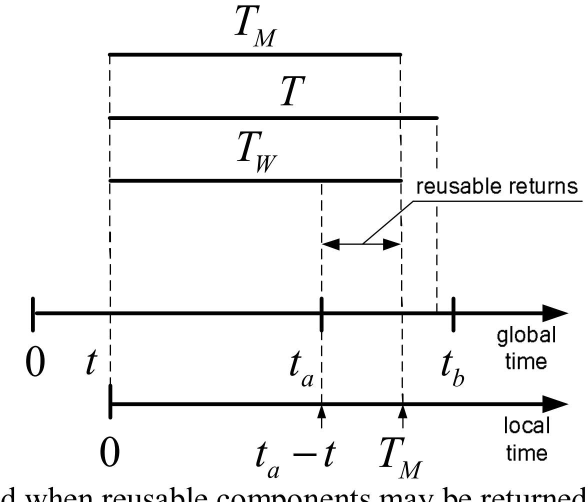

Figure 2. The period when reusable components may be returned to a producer according to equation 5.

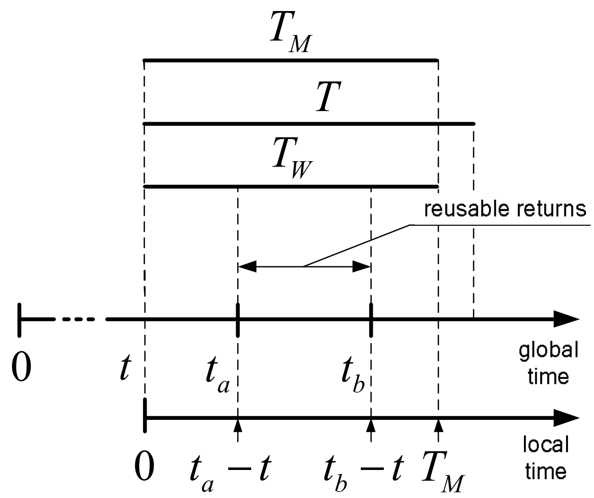

Figure 3. The period when reusable components may be returned to a producer according to equation 6.

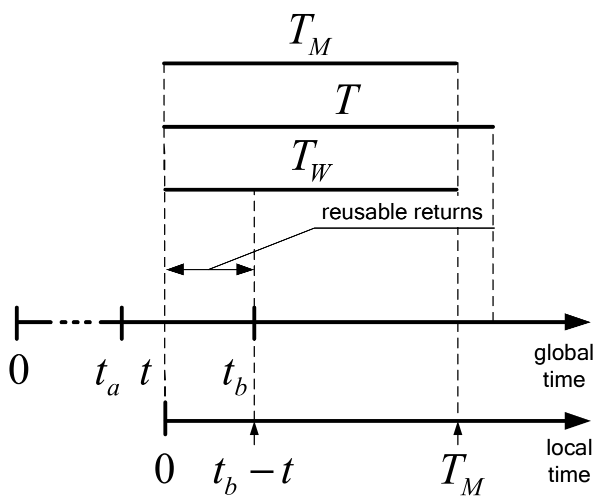

Figure 4. The period when reusable components may be returned to a producer according to equation 7.

The expression 4 allows to calculate the percent of B elements that can be returned to a manufacturer between two consecutive production batch and may be reused for the case when $t_2 - t_1 < T_M$. Value T_M is the minimal value of warranty period and threshold time lengths. It limits the possibility of returns: component B may be returned only during the product warranty period and reused only if it is not older then T. The expression may estimate maximal or minimal number of reusable B elements dependently on the form of the reliability function of B component. If the formula 10 is used, the maximum number of reusable elements is estimated according to the assumption that all products were as good as new when they were sold. The minimal number of reusable returns may be calculated when the expression 11 is considered in the equation 4 because it assumes that all components B in new products are in the maximal allowed age of $T - T_M$. Other cases (when $t_2 - t_1 > T_M$) are presented in detail in literature (Plewa, M. & Jodejko-Pietruczuk, A. 2011).

Real values of costs and savings coming from the reusing policy is proportional to the variable n:

$$[E(C_{WO}(T_W, T)) - E(C_{WN}(T_W))]n = (C_B - C_R)n - C_M \tag{12}$$

where C_{WO} = the cost of warranty services if "old" element is used in a new production; T_W = the warranty period of the product; T = the threshold age of the element B, after that the further exploitation isn't continued; C_{WN} = the cost of warranty services if "new" element is used in a production; n = production batch percent of reusable B elements, that return during warranty period; C_B = the purchase cost of a new element B; C_R = the total cost of all activities of preparing of returned B element to reusing in a production; C_O = penalty cost resulting from a product failure during warranty period; E(x) = the expected value of variable x.

The cost of elements' identification C_M is independent on the number of returns, because if a company decides to apply the reusing policy, all elements used in a production have to be identified.

3 RESEARCH RESULTS

Some example of analytical model results are presented in figures 5-17 (for process parameters: $C_O = 5$, $C_B = 1$, $C_R = C_M = 0$, $R_A = R_B = \exp(-(t/100))^2$.

Figure 5 presents the number of reusable returns assuming that all new products sold earlier:

- were as good as new (maximal number),
- include reused B elements (minimal number)

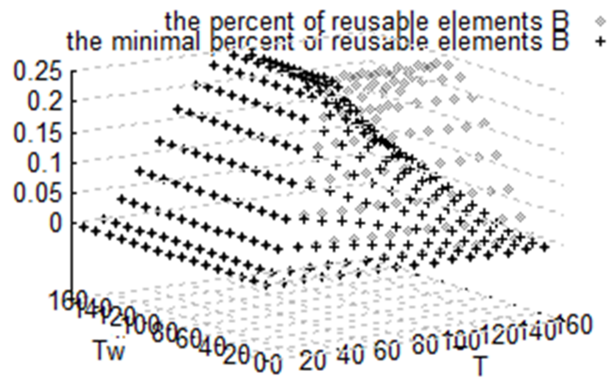

Figure 5. Maximal and minimal production batch percent of reusable B elements, that may be used in new production.

The minimal and maximal number of reusable elements are similar for low values of threshold time T and warranty period T_W. When an acceptable time T exceeds the average life time of the element B, the minimal number of reusable elements decreases very fast – second reusing of "old" elements is little probable.

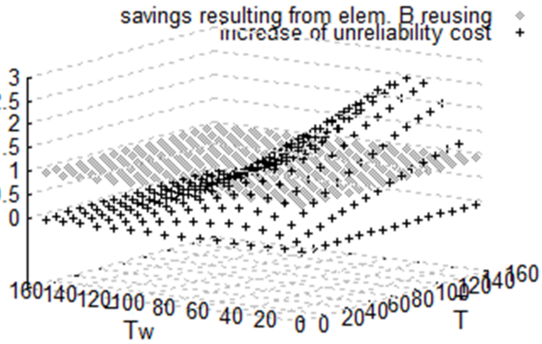

Figure 6. Increase of unreliability costs and savings resulting from the reusing policy according to equation 1.

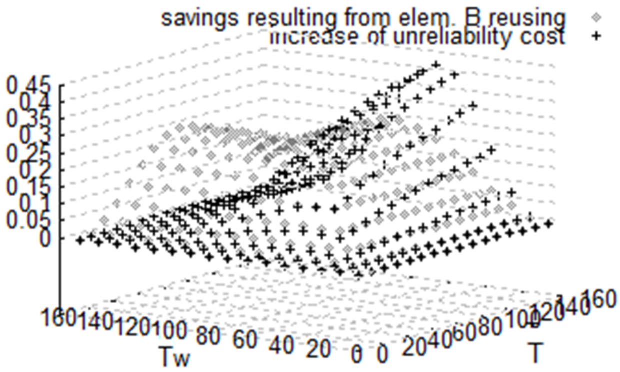

Figure 7. Increase of unreliability costs and savings resulting from the reusing policy according to equation 12 with maximal possible number of reusable returns.

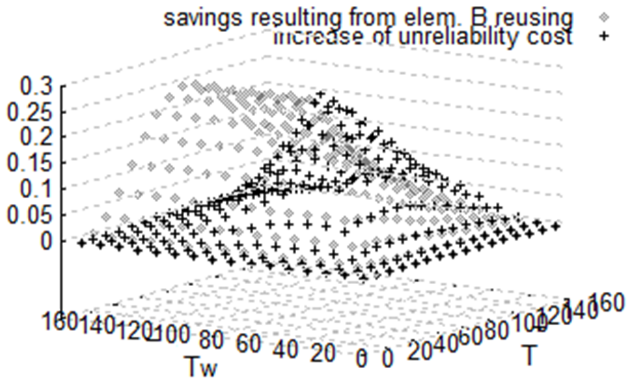

Figure 8. Increase of unreliability costs and savings resulting from the reusing policy according to equation 12 with minimal possible number of reusable returns.

The results in Figures 6-8 are obtained for the same process parameters, but are different because of the influence of returned elements amount, used in a new production batch. Savings coming from element reusing in Figure 6 (without considering the number of returns) are constant for all analysed values of T and T_W, but they have irregular shape in Figure 7,8. Considering two alternative values of the total work time T, sometimes it is more economical to get lower unit profit but coming from greater quantity of reused elements. Potential savings and costs are proportional to the number of reusable elements (Fig. 5). Their real values will take place between plates presented in Figure 9 and it is seen that incorrect estimation of T time (too long) causes great costs of the reusing policy. Irregular shape of reusing policy benefits and costs shows that the plane may have more than one local maximum or minimum. The research conducted for various process parameters haven't given any general rules where the optimal value of work time T may be found.

The estimation of threshold and optimal work time T for various warranty periods proves that they are the same for the minimal and maximal number of returns (Fig. 10,11).

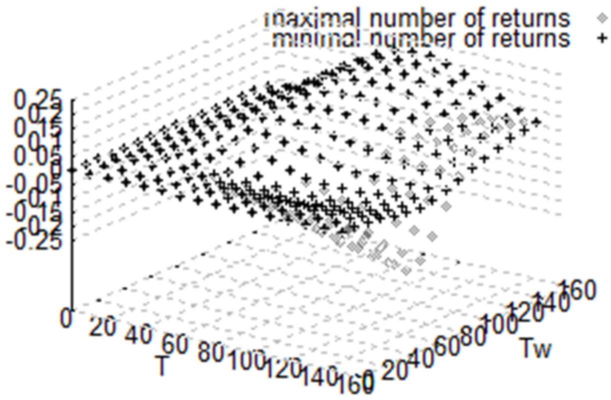

Figure 9. Summary profit resulting from reusing of elements B in new production.

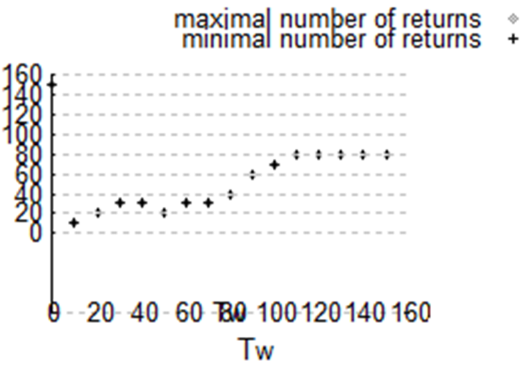

Figure 10. Optimum value of T (savings resulting from the reusing policy get maximum).

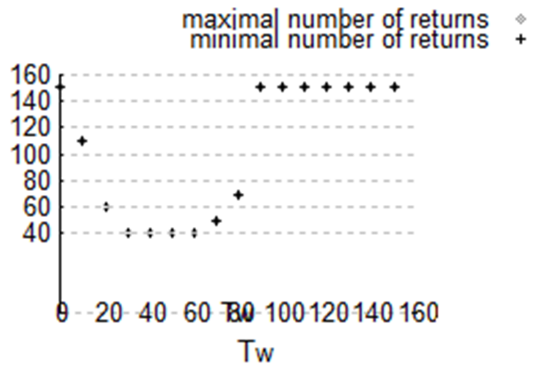

Figure 11. Threshold value of T (reusing is not economical above this value).

The optimum work time *T*, after that the further exploitation isn't continued, rises when a warranty period gets longer, but does not exceed the average lifetime of the component B.

The shape of threshold values of *T* time may also be interesting (Fig. 11). For very short warranty periods costs are usually equal to savings and are very close to zero. It is the effect of very low changes of reliability of the product in very short time. In such cases reusing even "very old" (high values of threshold time *T*) component is profitable.

In order to assess the influence of the reusing process parameters on its cost results, the sensitivity analysis of the proposed model was conducted. The parameters that was concerned as the most meaningful were tested and the results obtained during the research are shown in Figures 12 - 17.

When a returned component that was working some time in the past is used in the product offered as new for a customer, the element's intensity of failures is one of the determinant of possible reusing. For this reason the impact of the reusable component failure rate on production process costs and profit was tested.

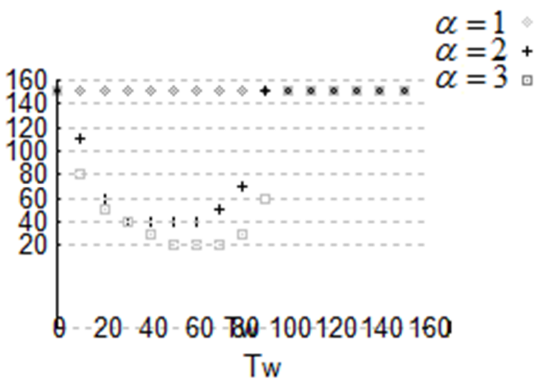

Figure 12. Threshold value of *T* for various failure intensity of the component B: R_B = exp(-(t/100))□.

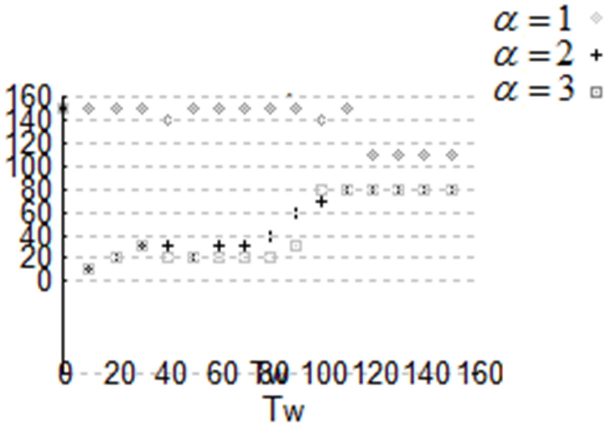

Figure 13. Optimal value of T for various failure intensity of the component B: R_B = exp(-(t/100))□.

Exponentially distributed time to failure has a constant failure rate and reusing of such elements always is profitable (Fig. 12,14). The optimal value of work time T, for which the profit is the highest, gets maximum possible value for all tested warranty periods (Fig. 13). The sudden decrease in optimal T value for warranty periods two times longer than co-component's (A) expected time to failure is the effect of lower number of possible returns of the product. Only in the case when A element fails during a warranty period and is returned to the manufacturer, reusing of B components is possible. The optimum T takes value of double A lifetime.

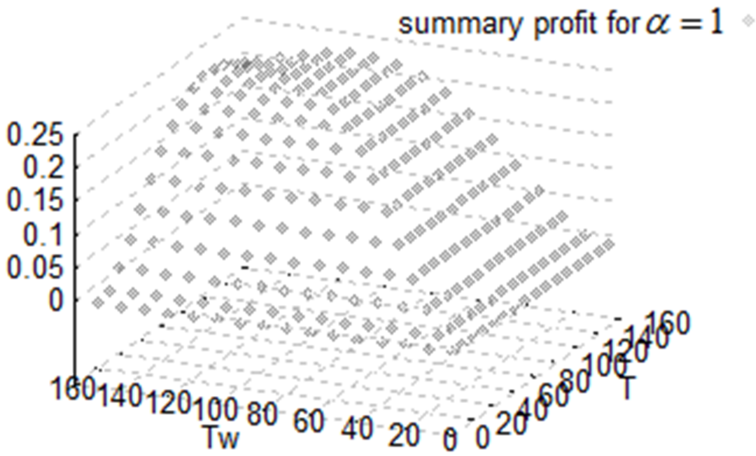

Figure 14. Summary profit resulting from reusing of elements B in new production for $R_B = \exp(-(t/100))^1$.

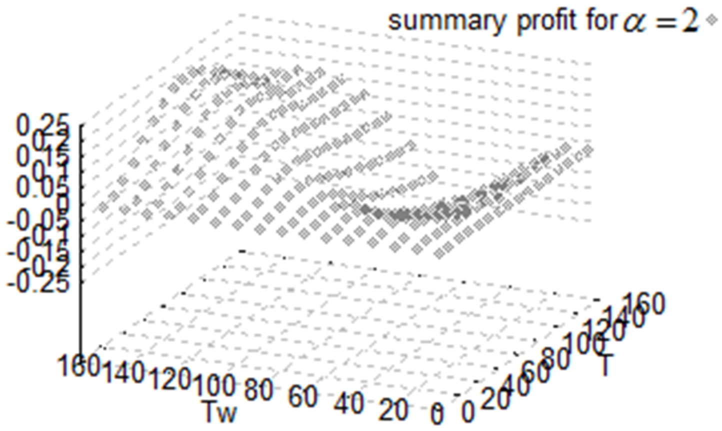

Figure 15. Summary profit resulting from reusing of elements B in new production for R_B = exp(-(t/100))^2.

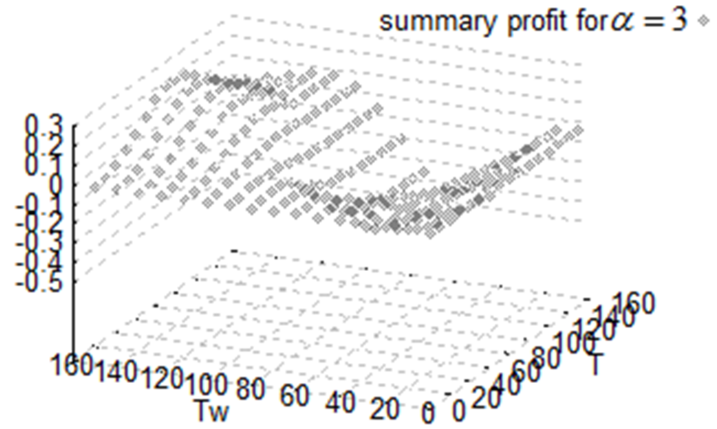

Figure 16. Summary profit resulting from reusing of elements B in new production for R_B = exp(-(t/100))^3.

Figures 15,16 present cost results of the reusing policy for the growing failure rate of the reusable component. The faster growth of this parameter shifts the threshold and the optimal *T* to lower values – reusing is less profitable (Fig. 12,13).

The second most meaningful parameter of the reusing process is the relationship between possible costs of warranty services and savings coming from the lower number of elements in the production process. The research results are presented in Figures 17-18.

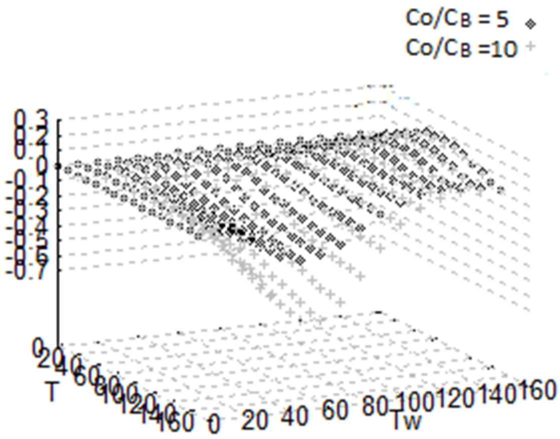

Figure 17. Summary profit resulting from reusing of elements B for c_O = 5 and c_O = 10, c_B = 1, c_R = c_M = 0.

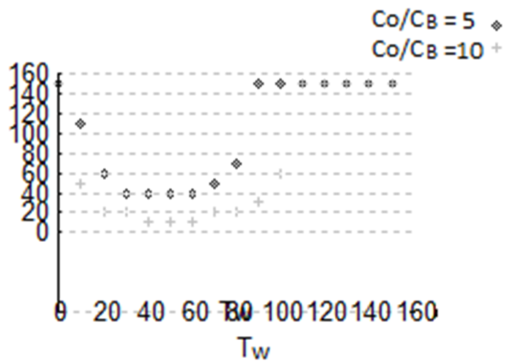

Figure 18. Threshold value of *T* for various relationship between possible costs ratios.

The results of the research are quite obvious – higher cost of a product failure during warranty period causes lower profitability of reusing policy.

4 CONCLUSIONS

The model presented in this paper is the continuation of wide researches in the reverse logistics area. The majority of models deal with single – element system or with the assumption that reused elements are as good as new. The proposed model develops the previous ones by releasing both assumptions and gives the base to determine some of reusing policy parameters such as: the threshold work time of returned element that can be used again, the warranty period for the product containing elements which have some history of work, the size of new elements' stock necessary to fulfil production planes. The model is presented and tested for two-element, series system but it is very simple to be developed it to the case of *x*-element, series system. From the point of view of possible product returns during a warranty period this assumption is usually real because only a failure of a one product component allows to reuse others.

Although the majority of presented expressions is difficult to solve analytically, their analytical form enables easy implementation to a numerical search of optimal process parameters.

Practical application of the model is possible in companies which are interested in the reusing policy because the decision must be supported by the calculation of possible cost and benefits. There are many possible extensions of the presented model by taking into account:

- parameters associated with the process of collection and transportation of returns. Particularly important may be the consideration of different return sources;
- longer than a period, time of recovery;
- random times of recovery;
- random lead time for external orders;
- ability to deliver returns in batches;
- non-deterministic demand for the final product;
- complex reliability structure of the technical objects.

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THE REVERSE LOGISTICS MODEL WITH REUSING OF PRODUCT PARTS

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ABSTRACT

Main goal of this paper is to create the reverse logistics model that uses reliability theory to describe reusability of product parts with assumption that recovered components are used in process of new products manufacturing. Authors assume that they aren't as good as new ones which is an important difference compared to the most models that were created before. The model allows to estimate the potential profits of the reusing policy in a production and gives the base to optimize some of the process parameters: the threshold work time of returns or the warranty period for products containing reused elements.

1 INTRODUCTION

Reverse logistics understood as the process of managing reverse flow of materials, in-process inventory, finished goods and related information has become one of the logicians' key areas of interest. It enjoys ever-increasing interest of many industrial branches. Nowadays a growing number of companies realize the meaning of that field of logistics. Reuse of products can bring direct advantages because company uses recycled materials or recovered components instead of expensive raw materials.

Literature survey that has been done around the theme of the reverse logistics area, allowed to set out this article aims and objectives. In the reverse supply chain the issue of: timings, quantities and conditions of returned and reusable components make production planning difficult (Murayama et al., 2006). The majority of models assume that demand for new products and returns quantity are independent Poisson random variables (Plewa & Jodejko-Pietruczuk, in prep.) and very few use the reliability theory to estimate the number of reusable products. Murayama et al. (Murayama et al., 2001, 2005, 2006) propose the method to predict the number of quantities of returned products and reusable components at each time period by using series system reliability models. The condition aspect of returns in a reverse logistic system is usually omitted by using assumption that all the returns are reusable and usually "as good as new" (e.g. Kiesműller, 2003). Only few models use the reliability theory to diversify returned elements' reusability, but they don't give any guidelines for the way to optimize the threshold value of components' residual life (e.g. Murayama et al., 2001, 2005, 2006).

Literature review that have been done so far, allowed to define the main shortages of existing logistics models that deal with the reverse logistics problem:

- most of the created models assume single-component product,
- they are based on the assumption that recovered products are as good as new,
- they don't optimize reusability of the returns.

Main goal of this paper is to create the reverse logistics model that uses the reliability theory to describe reusability of product parts with assumption that recovered components are used in a production process but they aren't as good as new ones. The model allows to estimate the potential profits of the reusing policy in a production and gives the base to optimize some of the process parameters: the threshold work time of returns or the warranty period for products containing reused elements.

2 THE MODEL OF THE REUSING POLICY

The model that is presented in the paper is based on the following assumptions:

- A company produces the object composed of two elements (A and B). The product fails when one of components fails – series reliability structure.
- A failure of each component occurs independently on other components' failures.
- If the product fails during the warranty period, it is returned to the manufacturer and he has to pay some penalty cost (e.g. the cost of a new product).
- The products are returned as soon as their lives are ended.
- The component B of the product may be reused in a new production, if it was not the cause of a product failure and its total work time up to this moment is not greater than some acceptable – threshold time *T* (Fig.1).
- Neither failed elements B can be reused in a new production (not repairable) nor any A element. All A components are new in a new production.
- Demand for the products is determined.
- New products are manufactured and sold in established moments (for simplicity – at the beginning of every warranty period).

The process of reusing of the component B, dependently on its threshold age *T*, is shown in Figure 1.

Despite the fact that companies realize the potential of reusing products, the question "is it worth to do it", is not so simple to answer. Within main reasons for products reusing are: difficulties with raw material supplying, high cost of utilization of returned and damaged products or lower cost of reusing of products' components than buying new ones. The objective of the presented model is to estimate profitability of using returned and recovered elements in a new production, in the case when they are not as good as new.

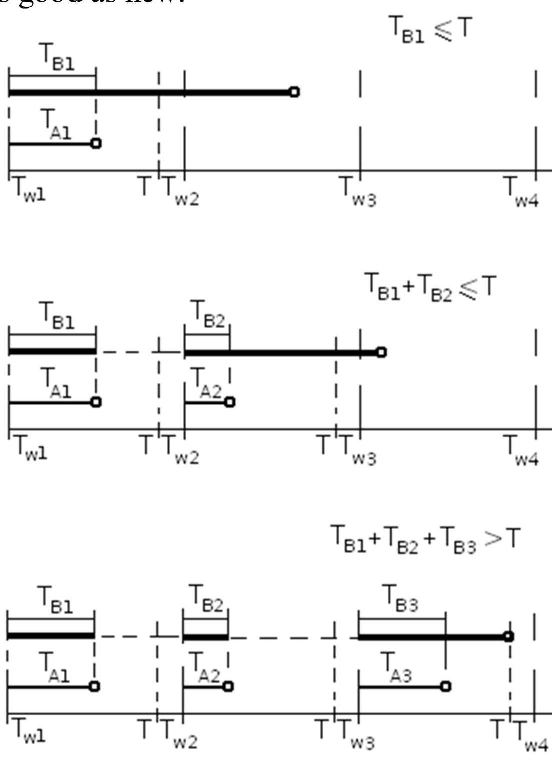

Figure 1. Process of element reusing in a new production.

According to above assumptions, before every production beginning, the manufacturer has to make the decision: which of returned elements B should be used in the new production. The usage of recovered components decreases production costs but also increases the risk that additional costs occur because of larger amount of returns during the warranty period. The objective is to find the threshold work time T for returned element that equalizes potential cost and profits of the reusing policy:

$$E(C_{WO}(T_W, T)) - E(C_{WN}(T_W, T)) = C_B - C_R - C_M \quad (1)$$

$$E(C_{WO}(T_W, T)) = \left[1 - \frac{R_B(T_W + T)R_A(T_W)}{R_B(T)}\right]C_O \quad (2)$$

$$E(C_{WN}(T_W, T)) = [1 - R_B(T_W)R_A(T_W)]C_O \quad (3)$$

where C_{WO} = the cost of warranty services if an "old" element is used in a new production; T_W = the warranty period of the product; T = the threshold age of the element B, after that the further exploitation isn't continued; C_{WN} = the cost of warranty services if a "new" element is used in production; C_B = the purchase cost of a new element B; C_R = the total cost of all activities of: decomposition, cleaning, preparing of the returned B element to reusing in a production; C_M = the cost of elements' identification; C_O = penalty cost resulting from a product failure during warranty period (e.g. the total cost of production of a new object); $R_A(t)$ = reliability of the element A in t moment; $R_B(t)$ = reliability of the element B in t moment; E(x) = the expected value of variable x.

The left side of the equation 1 specifies the increase in expected costs of product warranty services (during a single warranty period) caused by the reusing in a production element that is not as good as new. This part of the expression depends on: both elements' reliability (A and B), the length of warranty period and the length of the acceptable total work time T of the returned B element. The direction of changes in the expected cost of reusing elements is not so obvious. In special cases (low value of time T and short warranty period) it can happen that reused product is more reliable than a new one. The right side of the equation 1 determines the potential cost savings resulting from using cheaper, recovered components instead of new elements.

On the basis of this expression you can find the threshold value T of total work time of the element B, above which reusing the component is not economical. An analytical solution of the equations can't be achieved for the majority of probability distributions describing components' time to failure, but the value of T can be easily found by applying numerical calculations for given vector of T. The example of numerical analysis of the equation 1 is presented in Figure 2.

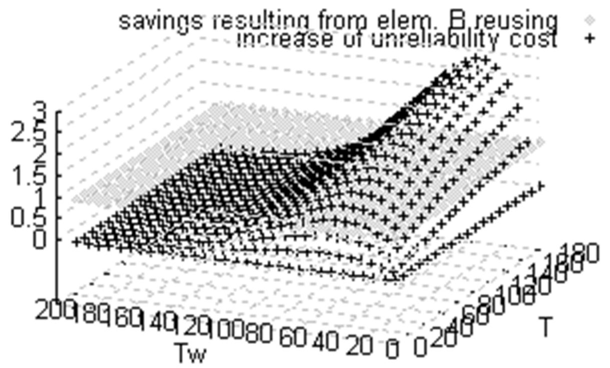

Figure 2. Increase of unreliability costs and savings resulting from the reusing policy for various values of threshold work time (T) and warranty periods (T_w) for process parameters: $c_O = 5$, $c_B = 1$, $c_R = c_M = 0$, $R_A = R_B = \exp(-(t/100))^2$.

This way, for given warranty period, you can find the threshold total work time *T*, for which using returned element B is economical (costs are lower than savings). It is interesting that for very long warranty periods (longer than double expected lifetime of the product) reusing elements is still profitable. It is an obvious effect of a very low product reliability in the whole warranty time and even using “old” components can't deteriorate it.

3 MODEL DEVELOPMENT

The practical application of the presented model is limited, because it calculates potential profits and costs assuming that all elements used in a new production are in the age of *T*. Monte Carlo simulation that was built to compare analytical and numerical results showed that this simplified assumption deforms real values of cost and benefits of components' reusing.

According to previous assumptions, at every moment of production beginning the producer has to make the decision: which of returned elements B should be used in the production. The threshold work time *T* (for which savings from components' reusing are equal losses) can be specified on the base of Equation 1, but practical questions require more precise data: how many reusable elements will return before the new production batch, how many new components must be kept in a stock or what is the expected profit/cost when mix of new-old elements will be used in the production.

To answer this questions, the model should consider the percent of returns used in the production:

$$n = \left(1 - R_A\left(\min\left(T, T_W\right)\right)\right) R_W\left(\min\left(T, T_W\right)\right) \qquad (4)$$

where n = production batch percent of reusable B elements, that return during warranty period; T_W = the warranty period of the product; T = the threshold age of the element B, after that the further exploitation isn’t continued; $R_A(t)$ = reliability of the element A in t moment; $R_B(t)$ = reliability of the element B in t moment; min(x,z) = minimum value of variables x and y.

Real values of costs and savings coming from the reusing policy is proportional to the variable n:

$$\left[E\left(C_{WO}\left(T_W, T\right)\right) - E\left(C_{WN}\left(T_W\right)\right)\right] n = \left(C_B - C_R\right) n - C_M \qquad (5)$$

where C_{WO} = the cost of warranty services if “old” element is used in a new production; T_W = the warranty period of the product; T = the threshold age of the element B, after that the further exploitation isn’t continued; C_{WN} = the cost of warranty services if “new” element is used in a production; n = production batch percent of reusable B elements, that return during warranty period; C_B = the purchase cost of a new element B; C_R = the total cost of all activities of preparing of returned B element to reusing in a production; C_O = penalty cost resulting from a product failure during warranty period; $E(x)$ = the expected value of variable x. C_M = the cost of elements' identification, it is independent on the number of returns, because if a company decides to apply the reusing policy, all elements used in a production have to be identified.

4 RESEARCH RESULTS

The process of planning, implementing, Some example of analytical model and simulation results are presented in figures 3-7 (for process parameters: $C_O = 5$, $C_B = 1$, $C_R = C_M = 0$, $R_A = R_B = \exp(-(t/100))^2$. The results are obtained for the same process parameters as in Figures 2, but are different than previous ones because of the influence of returned elements amount, used in a new production batch.

Savings coming from element reusing in Figure 2 are constant for all analysed values of T and T_W, but they have irregular shape in Figure 4. Considering two alternative values of the total work time T, sometimes it is more economical to get lower unit profit but coming from greater quantity of reused elements.

Figure 5 presents the summary profit resulting from reusing of elements in new production. The profit sometimes gets values lower than zero because losses of components' reusing are higher than potential savings. Irregular shape of reusing policy benefits and costs shows that plane may have more than one local maximum or minimum. The research conducted for various process parameters haven't given any general rules where the optimal value of work time T may be found.

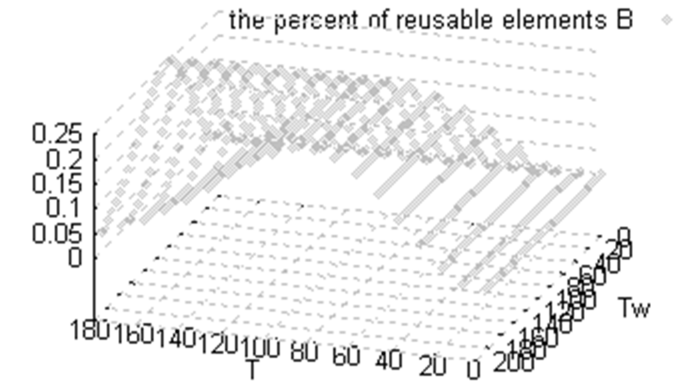

Figure 3. Production batch percent of reusable B elements, that may be used in new production.

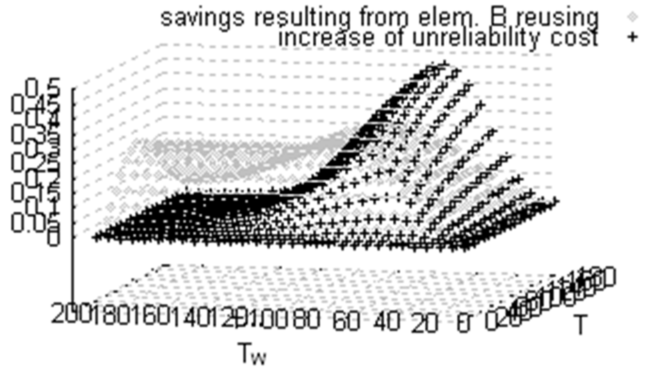

Figure 4. Savings and losses resulting from reusing of elements B in new production.

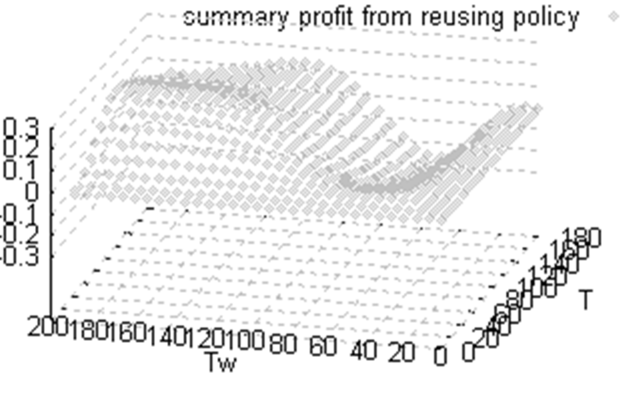

Figure 5. Summary profit resulting from reusing of elements B in new production.

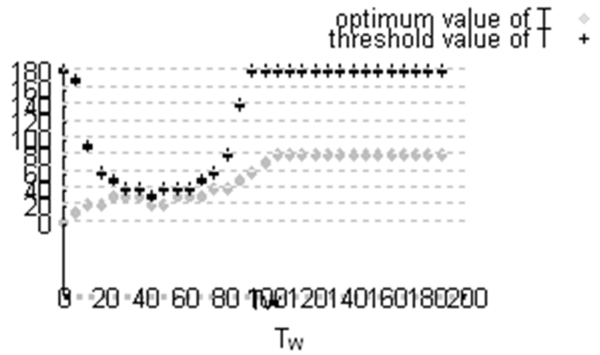

Figure 6. Optimum value of *T* (saving resulting from the reusing policy gets maximum) and threshold value of *T* (reusing is not economical above this value).

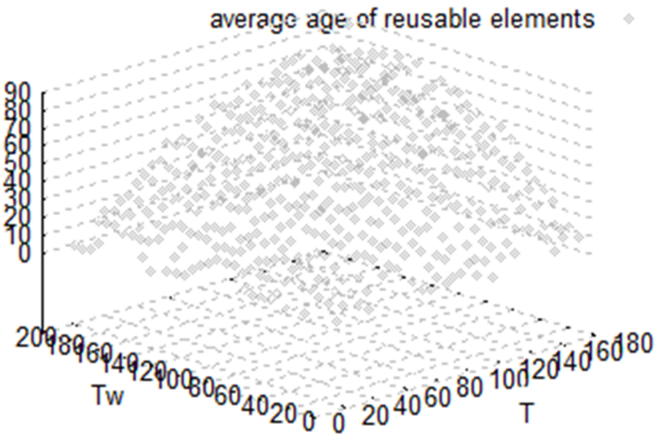

Figure 7. Average age of reusable elements

Figure 7 presents the average age of returned elements that can be reused in a new production, obtained during simulation experiments. The age usually is equal approximately 0,4 – 0,6 of the shorter value: warranty period or threshold work time *T*.

5 SENSITIVITY ANALYSIS

In order to assess the influence of the reusing process parameters on its cost results, the sensitivity analysis of the proposed model was conducted. The parameters that was concerned as the most meaningful were tested and the results obtained during the research are shown in Figures 8-15.

When a returned component that was working some time in the past is used in the product offered as new for a customer, the element's intensity of failures is one of the determinant of possible reusing. For this reason the impact of the reusable component failure rate on production process costs and profit was tested.

Exponentially distributed time to failure has a constant failure rate and reusing of such elements always is profitable (Fig. 8,11). The optimal value of work time *T*, for which the profit is the highest, gets maximum possible value for all tested warranty periods (Fig. 12). The sudden decrease in optimal *T* value for warranty periods two times longer than co-component's (A) expected time to failure is the effect of lower number of possible returns of the product. Only in the case when A element fails during a warranty period and is returned to the manufacturer, reusing of B components is possible. The optimum *T* takes value of double A lifetime.

Figures 9,10 present cost results of the reusing policy for the growing failure rate of the reusable component. The faster growth of this parameter shifts the threshold and the optimal T to lower values – reusing is less profitable (Fig. 11,12).

The second most meaningful parameter of the reusing process is the relationship between possible costs of warranty services and savings coming from the lower number of elements in the production process. The research results are presented in Figures 13-15. They are quite obvious – higher cost of a product failure during warranty period causes lower profitability of reusing policy.

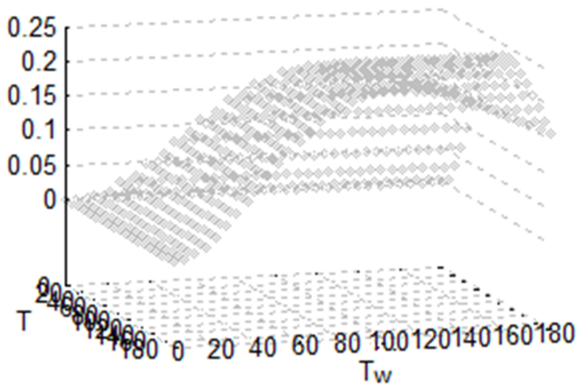

Figure 8. Summary profit resulting from reusing of elements B in new production for $C_O = 5$, $C_B = 1$, $C_R = C_M = 0$, $R_A = \exp(-(t/100))^2$, $R_B = \exp(-(t/100))^1$.

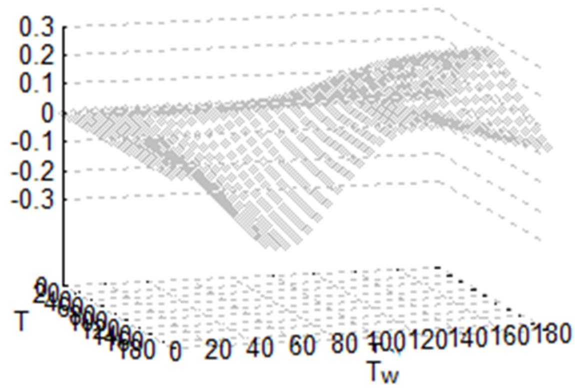

Figure 9. Summary profit resulting from reusing of elements B in new production for $C_O = 5$, $C_B = 1$, $C_R = C_M = 0$, $R_A = \exp(-(t/100))^2$, $R_B = \exp(-(t/100))^2$.

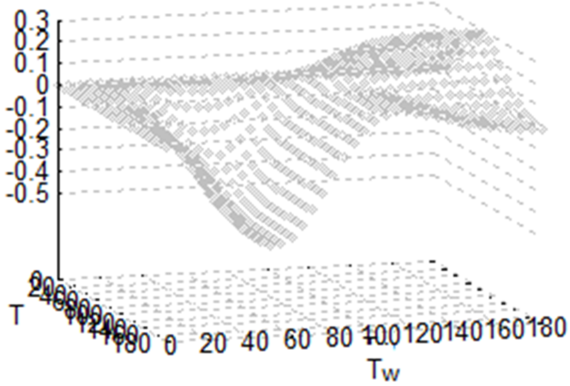

Figure 10. Summary profit resulting from reusing of elements B in new production for $C_O = 5$, $C_B = 1$, $C_R = C_M = 0$, $R_A = \exp(-(t/100))^2$, $R_B = \exp(-(t/100))^3$.

The shape of threshold values of T time may also be interesting (e.g. Fig.11,14). For very short warranty periods the costs are usually equal to the saving and are very close to zero. It is the effect of very low changes of reliability of the product in very short time. In such cases reusing even "very old" (high values of threshold time T) component is profitable.

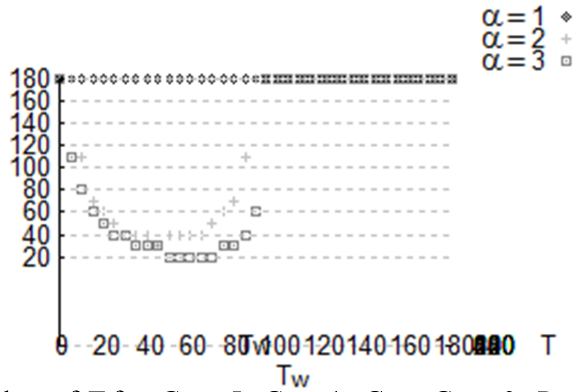

Figure 11. Threshold value of T for $C_O = 5$, $C_B = 1$, $C_R = C_M = 0$, $R_A = \exp(-(t/100))^2$, $R_B = \exp(-(t/100))^{1,2,3}$.

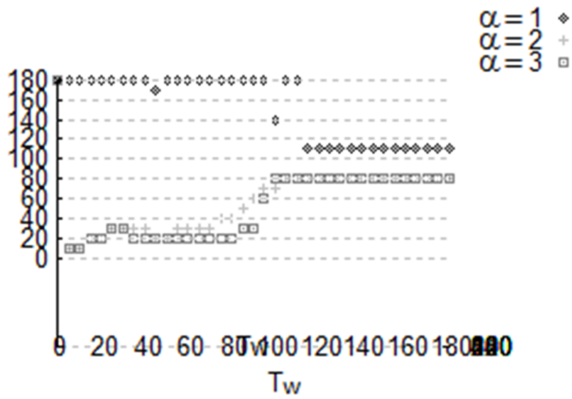

Figure 12. Optimum value of T for $C_O = 5$, $C_B = 1$, $C_R = C_M = 0$, $R_A = \exp(-(t/100))^2$, $R_B = \exp(-(t/100))^{1,2,3}$.

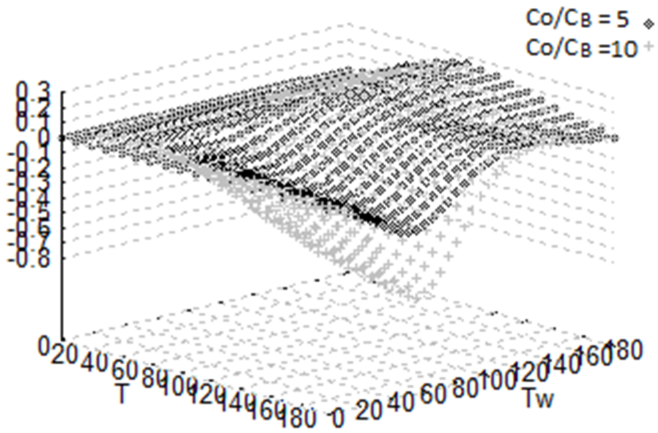

Figure 13. Summary profit resulting from reusing of elements B in new production for $C_O = 5$ and $C_O = 10$, $C_B = 1$, $C_R = C_M = 0$, $R_A = \exp(-(t/100))^2$, $R_B = \exp(-(t/100))^2$.

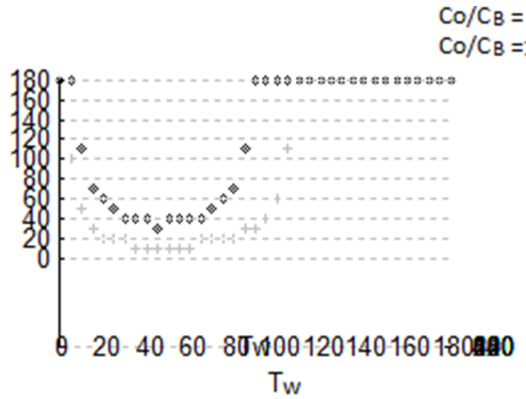

Figure 14. Threshold value of T for $C_O = 5$ and $C_O = 10$, $C_B = 1$, $C_R = C_M = 0$, $R_A = \exp(-(t/100))^2$, $R_B = \exp(-(t/100))^2$.

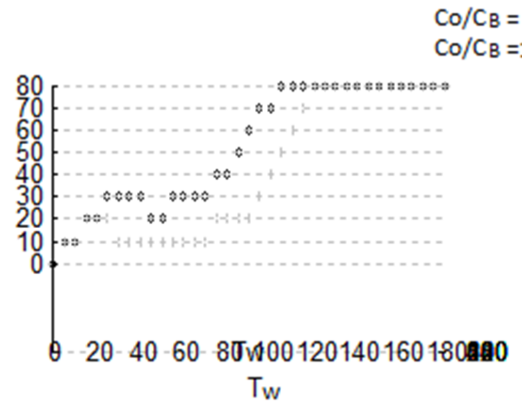

Figure 15. Optimum value of T for $C_O = 5$ and $C_O = 10$, $C_B = 1$, $C_R = C_M = 0$, $R_A = \exp(-(t/100))^2$, $R_B = \exp(-(t/100))^2$.

6 CONCLUSIONS

The model presented in this paper is the continuation of wide researches in the reverse logistics area. The majority of models deal with single – element system or with the assumption that reused elements are as good as new. The proposed model develops the previous ones by releasing both assumptions and gives the base to determine some of reusing policy parameters such as: the threshold work time of re-turned element that can be used again, the warranty period for the product containing elements which have some history of work, the size of new elements' stock necessary to fulfill production planes. The model is presented and tested for two-element, series system but it is very simple to be developed it to the case of x-element, series system. From the point of view of possible product returns during a warranty period this assumption is usually real because only a failure of a one product component allows to reuse others.

Although the majority of presented expressions is difficult to solve analytically, their analytical form enables easy implementation to a numerical search of optimal process parameters.

Effects of the analytical calculations of the presented model were confirmed and fulfilled by simulation results but it is obvious that the model should be verified in practice.

Practical application of the model is possible in companies which are interested in the reusing policy because the decision must be supported by the calculation of possible cost and benefits. Further development of the model should release the assumptions about deterministic demand for the product and constant moments of production.

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THE REVERSE LOGISTICS FORECASTING MODEL WITH WHOLE PRODUCT RECOVERY

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ABSTRACT

Classical logistics systems supports only processes carried out in material flow from manufacturer to final user. Recently it has been a striking growth of interest in optimizing logistics processes that supports recapturing value from used goods. The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal is called reverse logistics. The main goal of this paper is to create the forecasting model to predict returns quantity for reverse logistics system that supports processes of recapturing value from used products that has been decommissioned (because of failure) before a certain time called an allowable working time before failure.

1 INTRODUCTION

Until recently logistics systems supported only processes carried out in classical material flow from manufacturer to final user. Recently it has been a remarkable growth of interest in optimizing logistics processes that supports recapturing value from used goods. The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal is called reverse logistics. In reverse logistics systems demand can be partially satisfied with new items manufacture or procurement and returned products recovery. Reuse of products can bring direct advantages because company uses recycled materials instead of expensive raw materials. The first part of the article contains literature survey, which is a summary of work that has been done around the theme of research.

The main goal of this paper is to create the model of logistics system that supports processes of recapturing value from used products that has been decommissioned (because of failure) before a certain time called an allowable working time before failure. Article contains forecasting model which explains the process of moving goods from maintenance system to recovery system. In presented model there is an assumption that there is a relationship between demand for new products and the number of returns. There is also an assumption that every product that fail before acceptable time to failure can be recovered and used again. After recovery process products are as good as new ones. Demand for new products can be fulfilled by production or by recovery of used (failed) products. This article deals with single-item products.

This paper objectives are achieved by creating the mathematical model that describe analyzed processes. Some objectives can’t be achieved without creating a simulation model, which make possible to describe analyzed processes with various probability distributions. The next step is a sensitivity analysis of created model and verification process. The article ends with conclusions and directions for further research.

2 LITERATURE SURVEY

In literature the field of reverse logistics is usually subdivided into three areas: inventory control, production, recovery and distribution planning. The model presented in this paper is a inventory control model and hence the literature survey refers to this area.

Basic inventory control model in reverse logistics is built on following assumptions (Fleischmann et al. 1997):

- he manufacturer receives and stores the products returned from the final user;
- requirement for the new products is addressed to the final products storage;
- demand for the new products can be fulfilled by the production of the new ones or by the recovery of returned products;
- recovered products are as good as new ones;
- the goal of the model is to minimize the total costs.

Inventory control models in reverse logistics can be divided into deterministic and stochastic models.

2.1 Deterministic models

Deterministic models presented in the literature are mainly the modifications of the classical EOQ model.

First models in this category was created by Schrady (Fleischmann et al. 1997, Schrady 1967, Teunter 2001). In his models demand and return rates are constant. Return rate is described as a part of demand rate. Returns are put into recoverable inventory. Recovered products are as good as new ones and they are stored with new products in serviceable inventory. There is no disposal option, all returns are recovered and used to fulfill the demand. There are fixed lead-times for new products procurement and returns recovery. The goal is to set up optimal lot sizes for procurement and recovery. Schrady considers fixed setup costs for procurement and recovery and holding costs for recoverable and serviceable inventory.

Schradys work was continued by Steven Nahmias & Henry Rivera (Nahmias & Rivera 1979). In contrast to Schrady they assume limited capacity of the recovery process and zero lead times. Schradys work was also continued by Teunter. In his model there are multiple lots for procurement and recovery. In Schradys model there was only one procurement lot. Teunter assume zero lead times but he allows disposal. Teunter considers linear costs for recovery, production, disposal and inventory holding. He also considers fixed setup costs for production and recovery. (Teunter 2001, 2002, 2003, 2004).

Model presented in (Teunter 2004) was extended in (Konstantaras & Papachristos 2006, 2008). In (Konstantaras & Papachristos 2008) authors assume that the parameters determining the number of production and recovery runs are integers. In (Konstantaras & Papachristos 2006) authors add the possibility of occurrence of backorders.

Applicability of the EOQ model in reverse logistics was also studied by Marilyn C. Mabini, Liliane M. Pintelon & Gelders Ludo F. These authors extend model formulated by Schrady. They consider both single and multi-item systems. In single item model in contrast with model proposed by Schrady, Mabini et al. consider backorders. In multi-item model they assume limited capacity of recovery process which must be allocated among multiple products. New products are ordered when recovery capacity is exhausted. They consider the possibility of selling excess of recoverable inventory (Mabini et al. 1992). Another EOQ model in reverse logistics was created by Knut Richter. Richter describes two workshops. In the first manufacturing and recovery processes are carried out and serviceable products are stored. In the second workshop products are used and recoverable items are stored. Recoverable items are periodically transferred to the first workshop. (Richter 1994, 1996a, b, 1997, 1999).

Economic lot size problem was also analyzed by Shie-Gheun Koh, Hark Hwang, Kwon-Ik Sohn and Chang-Seong Ko. They present model similar to the models described by Schrady (1967), Mabini (1992) & Richter (1996). In contrast with previous authors, they take into account the duration of the recovery process. They assume that at least one of the variables describing the number of external orders and the number of starts of the recovery process is equal to one. (Koh et al. 2002).

Production and recovery rates are also taken into account by Knut Richter & Imre Dobos. These authors extend the model formulated previously by Richter by taking into account the quality factor in recovery management. (Dobos 2002, Dobos & Richter 2000, 2003, 2004, 2006, Richter & Dobos 1999).

Model presented by Richter in (Richter 1996a, b) was extended by Ahmed M.A. El Saadany & Mohammad'a Y. Jaber. Ahmed & Jaber believe that in the first period, only the production should be car-ried out. They also consider fixed switching costs. (El Saadany & Jaber 2008, Jabber & Rosen 2008). In another work (Jaber & El Saadany 2009) the same authors assume that the recovered products differ from new ones. They consider the different demand rates for new products and products of recovery process.

The possibility of applying EOQ models in reverse logistics was also studied by Yong Hui Oh & Hark Hwang. These authors analyze the system in which after recovery returns are treated as a raw material in the manufacturing process (Hwang & Oh 2006).

Particularly noteworthy articles are (Inderfurth et al. 2005, 2006). In these papers authors analyze the impact of recoverable inventory holding time on recovery process.

Equally noteworthy is article (Mitra 2009), in which author presents the analysis of the two echelon inventory model.

In the analyzed literature, a separate category are models based on dynamic programming. In most papers, presented models are the extensions of the classical Wagner Whitin model. Applicability of dynamic programming in reverse logistics is discussed in (Inderfurth et al. 2006, Kiesműller 2003, Konstantaras & Papachristos 2007, Minner & Kleber 2001, Richter & Sombrutzki 2000, Richter & Weber 2001, Van Eijl & van Hoesel 1997). In models described above authors assume one recovery option. Multiple recovery options are considered in (Corbacıoğlu & van der Laan 2007, Kleber et al. 2002).

2.2 Stochastic models

In the class of stochastic models there are two model groups:

- models, in which demand for the new product is s a consequence of the return;
- models, in which the demand for new products does not depend on the number of returns, but the number of returns may depend on the previous demand.

The first group are models of repair systems. These are closed systems in which the number of elements remains constant. The main goal of such a system is to keep sufficient number of spare parts to provide the required level of availability of the technical system.

There are many literature reviews on repair systems. The most significant papers are (Diaz & Fu 2005, Guide & Srivastava 1997, Kennedy et al. 2002, Mabini & Gelders 1991, Pierskalla & Voelker 2006, Valdez-Flores & Feldman 1989, Wang 2002). The well-known inventory control model in repair systems is METRIC which was firstly presented in (Sherbrooke 1968). Modifications of METRIC model can be found in many subsequent works (e.g. Cheung et al. 1993, Muckstadt 1973, Pyke 1990, Sherbrooke 1971, Sleptchenko et al. 2002, Wang et al. 2000, Zijm & Avsar 2003).

The second group of stochastic inventory control models can be divided into continuous and periodical review models.

2.2.1 Continuous review models

First model in this category was created by Hayman. He built the model, in which demand for the new products and returns quantity are independent Poisson random variables. In Heyman model there is only recoverable inventory, new products are not stored because Heyman assumes zero lead times for external orders and recovery process. Recovered products are as good as new ones. Heyman doesn't consider setup costs for recovery and procurement. Heyman defines disposal level. Newly returned products are disposed of if recoverable inventory level exceed disposal level. Heyman is looking for an optimal solution using the theory of mass service (Heyman 1997).

The work of Hayman was continued by John A. Muckstadt & Michael H. Isaac. These authors con-sider single and two echelon models. In single echelon model, as in the work of Heyman, the demand for final products and product returns are independent Poisson random variables. In contrast to Heyman, Muckstadt & Isaac consider lead times for recovery and procurement. Procurement lead time is fixed and recovery lead time is a random variable. Muckstadt and Isaac allow backorders. Recovered products are as good as new ones.

In two echelon model, on the upper echelon, there is a central depot where recovery is carried out and new products are delivered. Lower echelon contains the group of sellers who collect returns from customers and send them to the central depot (Muck-stadt & Isaac 1981).

Multi echelon reverse logistics model was also created by Aybek Korugan & Surendra M. Gupta. In contrast with previous model Korugan and Gupta consider disposal option but they do not allow backorders (Korugan & Gupta 1998). The work of Korugan & Gupta has been pursued by Mitra S. in (Mitra 2009) by considering fixed setup costs.

The model similar to the single echelon model presented in (Muckstadt & Isaac 1981) has been proposed by Ervin van der Laan, Rommert Dekker, Marc Salomon a& Ad Ridder (Van der Laan et al. 1996b). Van der Laan, Dekker & Salomon present also the comparison of alternative inventory control strategies. They compare their models with those presented by Muckstadt et al. & Heyman (der Laan et al. 1996a). The comparison of alternative control strategies is also presented in (Bayindir et al. 2004). Van der Laan & Salomon also compared PUSH and PULL strategies in reverse logistics. They show that PULL strategy is more profitable only if holding costs in recoverable inventory are significantly lower than in serviceable inventory (Van der Laan & Sa-lomon 1997).

The theory of inventory management in reverse logistics was also developed by Moritz Fleischmann, Reolof Kuik & Rommert Dekker. They built model similar to the one presented by John A. Muckstadt & Michael H. Isaac. They investigate a case in which returned products can by directly reused. They allow backorders and consider fixed setup costs for procurement and linear holding and backorders costs (Fleischmann et al. 2002). Fleischman & Kuik consider direct reuse of returns also in (Fleischmann & Kuik 2003, Fleischmann et al. 2003).

Particularly noteworthy is an article (Ouyang & Zhu 2006) in which authors present the model that describe the final stage of product life, when the number of returned products can significantly exceed the demand for these products.

2.2.1 Periodical review models

First model in this category was created by V. P. Simpson (1978). He assumes that demand for the new products and returns quantity are independent random variables. Simpson considers separate inventories for new products and returns. He assumes zero lead times for a procurement and recovery. Simpson allows disposal. He considers linear costs for external orders, recovery, inventory holding and unfulfilled demand and does not take into account disposal cost (Simpson 1978).

Model based on similar assumptions was developed by Karl Inderfurth. Unlike his predecessor, he takes into account recovery process and procurement lead time expressed in the number of periods. Unfulfilled demand takes the form of pending orders. Inderfurth investigates two cases: in first one he doesn't allow for returns storage, in the second he eliminates that constraint. Inderfurth notes that the difference between recovery and procurement lead limes is a factor that has significant impact on the model's level of complications (Inderfurth 1997, Inderfurth & van der Laan 2001).

Model presented in (Inderfurth 1997, Sobczyk 2001) is investigated by Kiesmüller & Scherer in (Kiesműller & Scherer 2003). They use computer simulation to find optimal solution of the model.

G. P. Kiesmüller & S, Miner are the following authors dealing with inventory control in reverse logistics. In their works (Kiesműller 2003, Kiesműller & Minner 2003) the authors assume that returns don't depend on demand. Kiesmüller & Minner don't con-sider disposal. They assume that all the returns are recoverable. They model a production system in which serviceable inventory is replenished with production and recovery. Authors take into consideration production and recovery lead times. They assume that review takes place at the beginning of each period.

Production system is also modelled by Mahadevan, Pyke & Fleischmann. They investigated the system in which returns and demand are described by Poisson distribution. The system consists recoverable and serviceable inventory. The authors take into account inventory holding and backorders costs. In modelled system a review is performed always after certain number of periods (Mahadevan 2003, Pyke 2001).

Peter Kelle & Edward A. Silver focus on a periodic review inventory control system in their work as well. They develop an optimal policy form new products purchase using the example of reusable packaging. They assume that all the returns undergo recovery process. The authors don't consider the disposal. Unfulfilled demand takes the form of pending orders and is satisfied during the further periods. The authors don't take into account the cost related with backorders. They substitute it with a customer service level. The authors reduce the stochastic model to a deterministic model which is analytically solved (Kelle & Silver 1989).

D. J. Buchanan & P. L. Abad describe reusable packaging inventory management system as well. The authors create a model similar to that of Kelle & Silver. Buchanan & Abad analyze single- and multi-period model. As for multi-period model, the authors assume that the number of returns at each period is a fraction of all the products found on the market at the beginning of this period. At each period some part of products found on the market isn't suitable for reuse. The authors take into consideration the cost connected with products that were not sold at the end of analyzed planning horizon. The authors assume that the time of product presence on the market is described by the exponential distribution (Buchanan & Abad 1998).

The models presented above are based on the assumptions of a classical periodic review model. Subsequent authors R. H. Teunter & D. Vlachos developed a model similar to continuous review model that had been earlier introduced by Ervin Van der Laan & Marc Salomon. Teunter & Vlachos analyse the model using a computer simulation (Teunter & Vlachos 2002).

Worth mentioning is model developed by Inderfurth. In (Inderfurth 2004) he describes the model in which recovered products differ from new ones and are stored separately. Model presented by Inderfurth is a single period inventory model. Inderfurth assumes that the demand for new products and the demand for recovered returns are independent random variables. Interesting is the assumption, which allows the use of excess inventory of new products to meet unfulfilled demand for recovered products.

Single period inventory model is also presented in (Bhattacharya et al. 2006, Mostard 2005, Toktay 2000, Vlachos & Dekker 2003).

2.3 Summary

Presented review allows to summarize the current state of knowledge. Most authors assume that demand and returns are independent Poisson random variables. Few authors examine the relationship between the demand, and the number of returns but there are no inventory models in reverse logistics that use reliability theory to assess the number of returns. There are few models that use the reliability to assess the reusability of components. In real reverse logistics systems products that fail in a short time after the purchase by the final user are withdrawn from the market.

In most cases these products are returned to the manufacturer. Recovery of such products or components is not labour-intensive, because undamaged components have not been exposed to the aging process and are suitable for direct reuse.

There is a lack of models that consider different recovery options and different return reasons. There are no models which support decision making process about recovery options.

The goal of this paper is to create an inventory control model in which products are recoverable if they are withdrawn because of failure before a specified time from the start of the operation process. The model takes into account the relationship between the size of demand for the new products, and the number of returned objects.

3 MODEL DESCRIPTION

In modelled system demand for the new products at the particular periods D_T is an independent random variable. The demand occurs at the beginning of each period. We assume that failed products are returned to the manufacturer and can be recovered if their operating time was shorter than τ_{dop}. We call it allowable working time before failure. Demand can be fulfill by production or recovery. Recovered products are as good as new ones. Failed products are returned at the beginning of each period. The number of products returned at the beginning of each period is the following:

$$Z_T = PN_{[t_{T-1},t_T)} \tag{1}$$

where $PN_{[tT-1,tT)}$ = the number of products withdrawn because of failure at period $[t_{T-1},t_T)$; t_{T-1} = the beginning of period *T-1*; t_T = the beginning of period *T*.

Returned products are stored in recoverable inventory and new products are stored with recovered ones in serviceable inventory. In presented model a review is performed every T_{prz} periods. We assume that the first review occur at the beginning of the first period. The set of review periods can be presented in the following way:

$$V_{T_{prz}} = \left\{T_{prz,1}, T_{prz,2}, T_{prz,3}, \ldots, T_{prz,n}, \ldots\right\} \tag{2}$$

where $T_{prz,n}=1+(n-1)\cdot T_{prz}$; $T_{prz,n} \leq H$ for n=1,2,3,…,; n = review number; H = planning horizon.

Inventory position can change as a result of new items production and returns recovery or disposal. Recovery process starts if the serviceable inventory position is equal or lower than s and there is enough products in recoverable inventory to fulfill fixed recovery batch. Otherwise the inventory is replenished owing to production. We assume fixed and equal to one, lead times for production and recovery. We allow the disposal of excess inventory of recoverable items if in the review period the level of recoverable inventory is higher than s_u. We don't allow backorders. Unfulfilled demand is lost.

Framework for this situation is depicted in Figure 1.

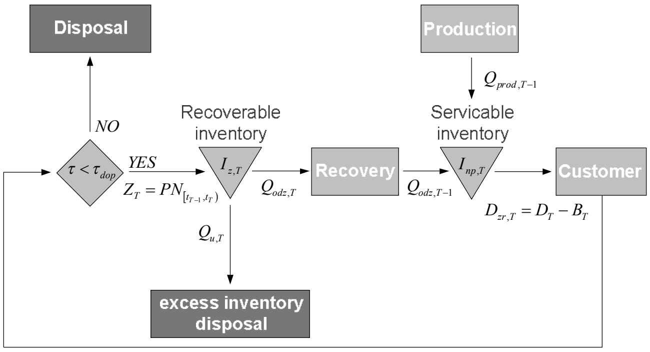

Figure 1. Framework inventory management

The serviceable inventory position in each period T may be presented as follows:

$$I_{np,T} = \max\left\{I_{np,T-1} + Q_{odz,T-1} + Q_{prod,T-1} - D_T, 0\right\} \tag{3}$$

where $I_{np,T-1}$ = on hand serviceable inventory at the end of period T-1; $Q_{odz,T-1}$ = recovery batch size launched in period T-1; $Q_{prod,T-1}$ = production batch size launched in period T-1; D_T = demand for the new products in period T.

Unfulfilled demand in period T can be presented in the following way:

$$B_T = \left|\min\left\{I_{np,T-1} + Q_{odz,T-1} + Q_{prod,T-1} - D_T, 0\right\}\right| \tag{4}$$

We don't allow the disposal of products in serviceable inventory. The recoverable inventory level at the end of period T is the following:

$$I_{z,T} = I_{z,T-1} + PN_{[t_{T-1},t_T)} - Q_{odz,T} - Q_{u,T} \tag{5}$$

where $I_{z,T-1}$ = on hand recoverable inventory at the end of period T-1; $Q_{odz,T}$ = recovery batch size launched in period T; $Q_{u,T}$ = the number of products disposed of in period T.

Recovery batch size launched in period T is the following:

$$Q_{odz,T} = Q_{odz} \cdot X_T \tag{6}$$

where

$$X_T = \begin{cases} 1 \; if \begin{cases} I_{np,T} \le s \\ and \\ I_{z,T-1} + PN_{[t_{T-1},t_T)} \ge Q_{odz} \\ and \\ T \in V_{T_{prz}} \end{cases} \\ 0 \; if \begin{cases} I_{np,T} > s \\ or \\ \left(I_{np,T} \le s \; and \; I_{z,T-1} + PN_{[t_{T-1},t_T)} < Q_{odz}\right) \\ or \\ T \notin V_{T_{prz}} \end{cases} \end{cases} \tag{7}$$

Production batch size launched in period T is the following:

$$Q_{prod,T} = Q_{prod} \cdot Y_T \tag{8}$$

where

$$Y_T = \begin{cases} 1\ if \begin{cases} I_{np,T} \le s\ and\ I_{z,T-1} + PN_{[t_{T-1},t_T)} < Q_{odz} \\ and \\ T \in V_{T_{prz}} \end{cases} \\ 0\ if \begin{cases} \left(I_{np,T} \le s\ and\ I_{z,T-1} + PN_{[t_{T-1},t_T)} \ge Q_{odz} \right) \\ or \\ I_{np,T} > s\ or\ T \notin V_{T_{prz}} \end{cases} \end{cases} \tag{9}$$

The number of products disposed of in period T is the following:

$$Q_{u,T} = \left(I_{z,T-1} + PN_{[t_{T-1},t_T)} - Q_{odz,T} - s_u \right) \cdot U_T \tag{10}$$

where

$$U_T = \begin{cases} 1\ if \begin{cases} \left(I_{z,T-1} + PN_{[t_{T-1},t_T)} - Q_{odz,T} \right) > s_u \\ and \\ T \in V_{T_{prz}} \end{cases} \\ 0\ if \begin{cases} \left(I_{z,T-1} + PN_{[t_{T-1},t_T)} - Q_{odz,T} \right) \le s_u \\ or \\ T \notin V_{T_{prz}} \end{cases} \end{cases} \tag{11}$$

Behavior of the system for $T_{prz}=1$ is depicted in Figure 2.

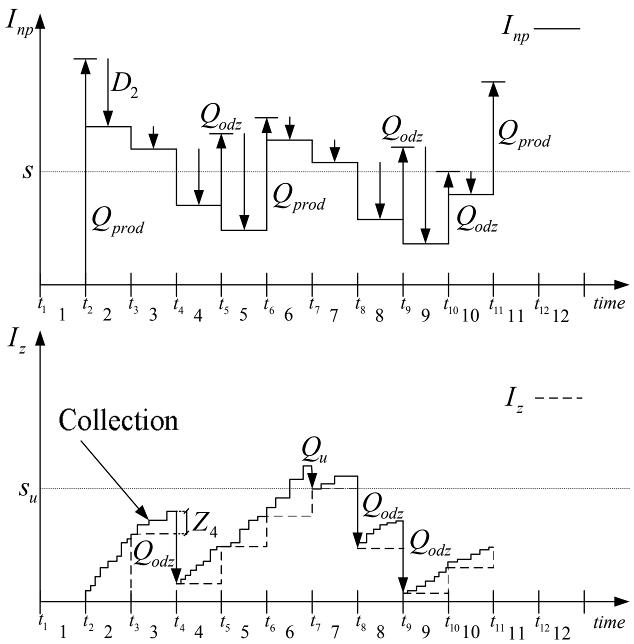

Figure 2. System behavior

3.1 The method of forecasting the number of objects returned to the system

Presented model of reverse logistics system required to develop methods for forecasting the number of objects returned to the system, within a specified period of time, at a fixed time limit τ_{dop}. Proposed forecasting model is a extension of model presented by Murayama in the (Murayama & Shu 2001, Murayama et al. 2004, 2005, 2006). The value of a τ_{dop} depends on aging process and total costs. Therefore, whether the object at the time t is located in a recovery system depends on:

- random variable $D_{zr,te}$ specifying the number of objects placed on the operating system at the moment t_e,
- random variable determining the probability of a technical object failure until τ_{dop}.

The probability of the object failure in the time interval from x_1 to x_2 can be written as follows (Migdalski 1982):

$$F\left(x_1, x_2\right) = P\left(x_1 \le \tau < x_2\right) \tag{12}$$

where τ = random variable that describes the time to object failure.

For t_b-t_a<τ_{dop} the number of objects $PN_{[ta,tb)}$, returned to the recovery system in the time period $[t_a, t_b)$ is the sum of three cases:

$$PN_{[t_a,t_b)} = PN_{1,[t_a,t_b)} + PN_{2,[t_a,t_b)} + PN_{3,[t_a,t_b)} \tag{13}$$

The first two are associated with objects, for which $t_e \le t_a$. The third one describes the situation in which objects are returned in interval (t_a, t_b).

In the first case (Fig. 3): $x_1=t_a-t_e$ and $x_2=\tau_{dop}$. The variable t_e can be in the range from $\max\left(t_a - \lfloor \tau_{dop} \rfloor, 0\right)$ to $t_b - \lceil \tau_{dop} \rceil$ and the number of retuned objects is the following:

$$PN_{1,[t_a,t_b)} = \sum_{t_e = \max\left(t_a - \lfloor \tau_{dop} \rfloor, 0\right)}^{t_b - \lceil \tau_{dop} \rceil} \left(D_{zr,t_e} \cdot F\left(t_a - t_e, \tau_{dop}\right)\right) \tag{14}$$

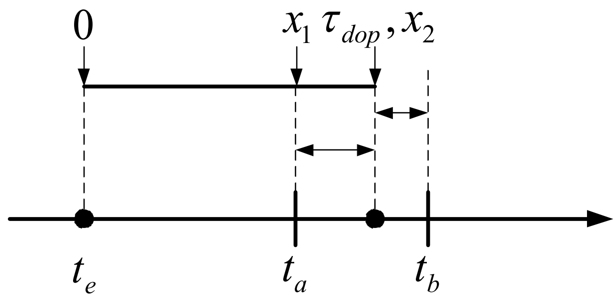

Figure 3. Predicted quantity of returned products between t_a and t_b described by Equation 14

In the second case (Fig. 4): $x_1=t_a-t_e$ and $x_2=t_b-t_e$. The variable t_e can be in the range from $t_b - \lceil \tau_{dop} \rceil + 1$ to t_a and the number of retuned objects is the following:

$$PN_{2,[t_a,t_b)} = \sum_{t_e = t_b - \lceil \tau_{dop} \rceil + 1}^{t_a} \left(D_{zr,t_e} \cdot F\left(t_a - t_e, t_b - t_e\right)\right) \tag{15}$$

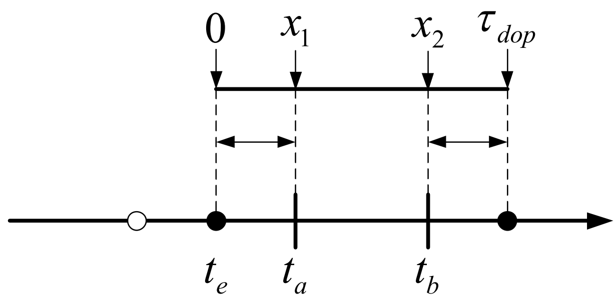

Figure 4. Predicted quantity of returned products between ta and tb described by Equation 15

In the third case (Fig. 5): x_1=0 and x_2=t_b-t_e. The variable t_e can be in the range from t_a+1 to t_b and the number of retuned objects is the following:

$$PN_{3,[t_a,t_b)} = \sum_{t_e=t_a+1}^{t_b} \left(D_{zr,t_e} \cdot F\left(0, t_b - t_e\right) \right) \tag{16}$$

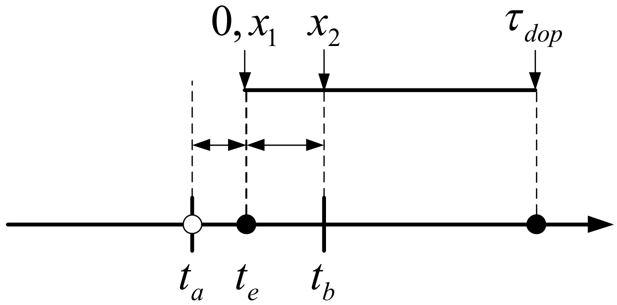

Figure 5. Predicted quantity of returned products between t_a and t_b described by Equation 16

The joint cost function in any period T takes the following form:

$$TC_T = \sum_{i=1}^{T} \begin{pmatrix} k_{skz} \cdot I_{z,i} + k_{sknp} \cdot I_{np,i} + \\ +k_{odz} \cdot Q_{odz,i} + k_{prod} \cdot Q_{prod,i} + \\ +k_b \cdot B_i + k_u \cdot Q_{u,i} \end{pmatrix} \tag{17}$$

where: k_{skz} = recoverable inventory holding cost; k_{sknp} = serviceable inventory holding cost; k_{odz} = recovery cost; k_{prod} = production cost; k_b = lack-of-inventory cost; k_u = excess inventory disposal cost.

In presented model we don’t consider production and recovery fixed costs and don’t allow backorders.

4 SIMULATION AND SENSITIVITY ANALYSIS

In presented model we don’t consider production and recovery fixed costs and don’t allow backorders In simulation we assume that demand D_T is a Poisson random variable with parameter λ_D and the time to failure is described with Weibull distribution with scale parameter α_r and shape parameter β_r. In simulation we use Monte Carlo methods with random number generators implemented in Matlab software. Table 1 shows the range of variation of parameters of the simulation model.

Table 1. Range of variation of parameters of the simulation model

Parameter		Range of variation		
	input	min	charge	Max
λ_D	1000	1000	-	1000
α_r	40	1.0	4.0	117
β_r	1.5	0.5	0.5	15
τ_{dop}	25	1.0	3.0	88
Q_{prod}	3000	250	250	7500
Q_{odz}	2000	100	100	3000
s	0.0	0.0	100	2900
s_u	2000	0.0	250	7250
T_{prz}	1.0	1.0	1.0	30
k_{sknp}	2.0	0.0	1.0	29
k_{skz}	1.0	0.0	1.0	29
k_{prod}	20	0.0	1.0	29
k_{odz}	10	0.0	1.0	29
k_u	0.5	0.0	1.0	29
k_b	5.0	0.0	1.0	29

Figures 6-9 show exemplary results of sensitivity analysis. Chosen example presents the impact of changes in the size of production batch on the output model parameters. Increasing batch leads to raise of the average level of serviceable inventory. Which reduces the amount of unfulfilled demand and thus increases the level of customer service. The increase of fulfilled demand leads to an increase in the number of returns to the system. Increasing number of returns initially increase the number of starts of the recovery process. However, a substantial extension of the production cycle results in reduction of recovery and thus raise the recoverable inventory level. This leads to an increase in disposal of products.

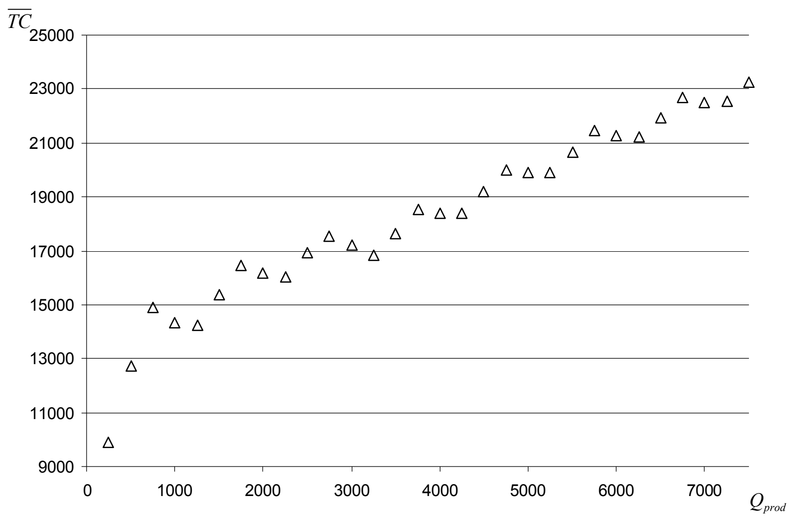

Figure 6. Variation of the average size of the total cost of the system, as a function of changes in production batch

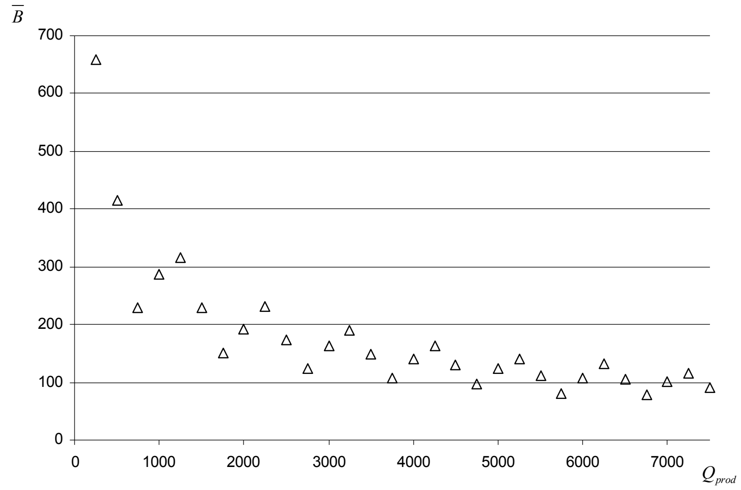

Figure 7. Variation of the average size of unfulfilled demand, as a function of changes in production batch

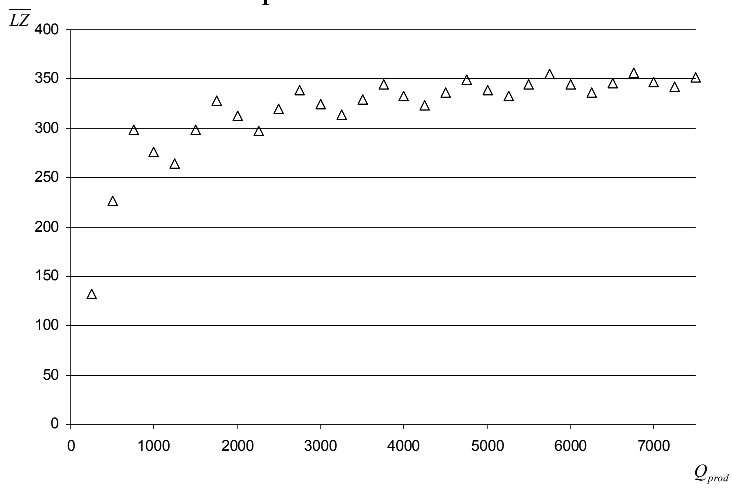

Figure 8. Variation of the average number of returns, as a function of changes in production batch

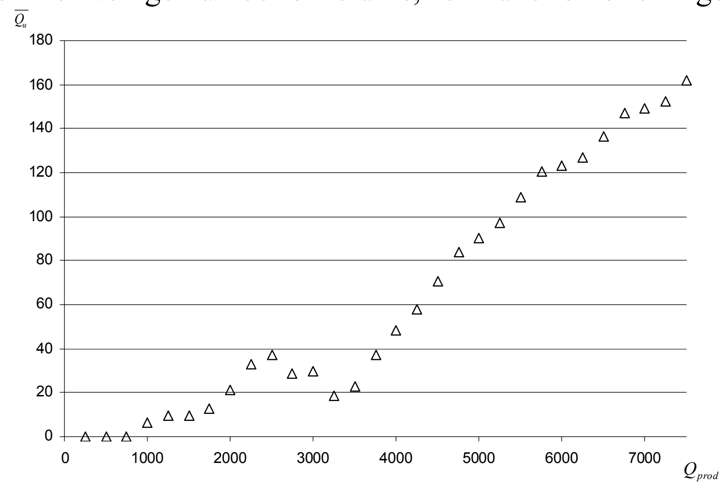

Figure 9. Variation of the average number of disposed products, as a function of changes in production batch

5 SIMULATION AND SENSITIVITY ANALYSIS

The analysis shows that the selection of appropriate parameter values has a significant impact on the returns management. Determining the appropriate value of the recovery and production batch helps to achieve a high level of reuse of recoverable products, increased service levels and low operating costs of the system.

Sensitivity analysis has shown that with the assumed values of input parameters, the level of the total cost of the system depends primarily on the level of the unit cost of storage of recoverable items. A much smaller impact on economic performance of the analyzed model was observed for the unit cost of storage of serviceable products and the unit cost of production. The smallest impact on the overall costs of the system has the change of the unit cost of disposal. Slightly more important impact proved to have a unit cost of the process of recovery and the cost of penalties associated with the occurrence of the lack of final products.

There are many possible extensions of the presented model by taking into account:

- parameters associated with the process of collection and transportation of returns. Particularly important may be the consideration of different return sources;
- ability to deliver returns in batches;
- longer than a period, time of recovery and production;
- random times of recovery;
- complex reliability structure of the technical objects.

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A FRAMEWORK FOR SELECTION OF TEST METHOD AND TEST INTERVAL FOR SAFETY CRITICAL VALVES IN SITUATIONS WITH LIMITED DATA

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ABSTRACT

In this paper we present a practical framework that can support the decision on test methods and test intervals for safety critical valves in situations where limited relevant historical data is available. The framework is based upon a systematic review of the valve functions and associated failure modes, as well as properties of the environment that the valve is located in, and evaluation of this knowledge in qualitative expert workshops. The main application area for the framework is valves located upstream or downstream of large gas transport pipeline segments. The framework is however general and can be applied to smaller hydrocarbon segments as well.

1 INTRODUCTION

This paper focuses on periodically tested items as part of passive safety critical systems. For such systems a failure can only be revealed during test or if the safety system is activated. There are many examples of such systems, but for the purpose of this paper, we consider safety critical valves. These valves are supposed to close and sectionalize the main process in case of an emergency situation at a process plant. It is vital that the safety critical valves are functioning on demand, as the consequences of failure could be significant, for example with reference to economy, production assurance, safety and emissions to the environment.

The paper is inspired by studying large land-based valves on the Norwegian gas distribution network. Many of these valves are unique in terms that there are no, or only a few, other valves that can be considered 'similar'. In addition to this, these valves were originally not designed to be tested at all, and have therefore only occasionally been tested. As a result, there is very little relevant test information that can be used to determine test methods and test intervals for some of the above-mentioned valves. But how can we in an appropriate way decide upon test methods and test intervals for safety critical valves when just limited relevant data is available? This question is the starting point for this paper.

Traditionally, decisions upon test methods have not been well-documented. It is, however, our impression that the decision on test method has primarily been made, with respect to the operational disadvantages of performing the test, for example in terms of shut-down time, rather than on a systematic review of the pros and cons of each alternative. It is also a challenge that tests are performed without considering the functions of the valve, i.e. the reason that they are installed. As discussed in Røed et. al. (2008), this may for example result in testing for potential leakage in one direction, while the most critical valve function is that the valve should not leak in the opposite direction.

For determination of test intervals a number of methods within reliability theory exist. See for example Rausand and Høyland (2004) and Aven (1991). Such methods are all based upon the availability of historical data considered relevant for the future performance of the system in focus. For most systems considered by reliability engineers, relevant data is available, making the above-mentioned methods relevant. But for the safety critical valves which are the starting point for this paper, little or limited historical data is available. The question may then be asked: 'How can test intervals be decided upon when no relevant data is available?'. This is certainly a challenge, but decisions on test intervals are also made in situations with no or limited data. Some key questions are then: What kind of information should be used to determine test intervals when no or limited data is available? And how should we take this information into consideration to ensure that decisions are made with the best knowledge available?

It may be argued that due to lack of test data we should implement short test intervals to be on the safe side. This is, however, not necessarily a good strategy since most of the test methods have operational disadvantages when the test is performed. This is not limited to economic loss in terms of reduced production assurance only. We see that some test methods also imply flaring/venting of large amounts of greenhouse gases. Serious accidents when performing a test may also occur. All in all, the above information means that there are many aspects that have to be taken into consideration when decisions are made for both test methods and test intervals.

This paper proposes a framework that takes the above-mentioned aspects into consideration, to suggest a maintenance regime for safety critical valves. The framework is based upon performing several steps, where initially the most appropriate test method is selected, and thereafter the test interval is decided upon for the selected test method. The framework is relevant in situations where little or no relevant historical data is available, but there is still some qualitative knowledge of the system in focus.

2 A FRAMEWORK FOR SELECTION OF TEST METHOD AND TEST INTERVAL FOR SAFETY CRITICAL VALVES

The proposed framework consists of three main steps: A, B and C, as illustrated in Table 1. In the first step, the activities to be carried out are prepared, including system understanding, identification of relevant consequence dimensions, identification of the functions that the valve is installed to achieve and potential failure modes. In the second step the most critical failure mode is identified. This is important as no test method can control all the failure modes of the valve. The third step of the framework is carried out in order to select a test interval for the test method selected in Step B.

Table 1. Framework for selection of test method and test interval

STEP A – Initial planning activities
Clarify boundaries of the system and corresponding segments
Identify relevant consequence dimensions
Identify the valve's functions and corresponding failure modes
Collect relevant historical test results
STEP B – Selection of test method
Identify the most critical failure mode
Select test method for the most critical failure mode
STEP C – Selection of test interval
Identify potential causes of each of the failure modes
Describe the valve's operating conditions
Evaluate the test interval

We suggest that Step B and parts of Step C are carried out in a multidisciplinary group. In the following we explain in more detail each step of this procedure. Each of the steps is explained by use of a realistic, although simplified, case study. The steps A, B and C are presented in Sections 2.1, 2.2 and 2.3, respectively.

2.1 Step A: Initial planning activities

In order to support decisions it is important that the boundaries of the analysis system are defined. It is also important to achieve an understanding of the boundaries of the corresponding hydrocarbon inventories, since this information is important when the consequences of a failure are going to be described.

To illustrate our ideas we will use the system as shown in Figure 1, including four valves and two pressurized hydrocarbon inventories. In this example, attention will be given to deciding upon the test method for valve 1 (V1) in this system.

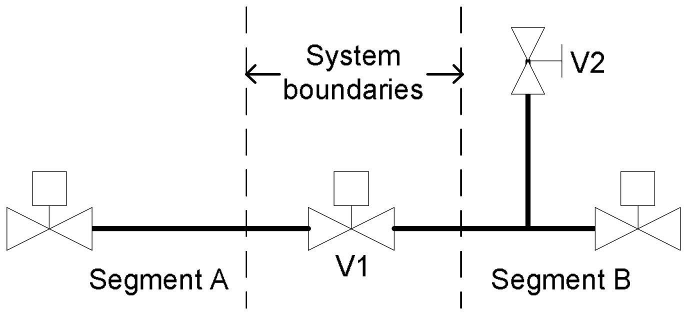

Figure 1: System description (simplified).

The test method decision will depend on several consequence dimensions such as production assurance, safety of personnel, costs and environmental aspects. The framework described in this paper is based on an analysis of each consequence dimension separately. Then the problem of weighting the consequences against each other, as part of the analysis, is avoided. We believe that it is important to leave the weighing of the consequence dimensions for the decision maker, and have not included this in the analysis. Arguments for our view can be found for example in Aven (2003).
As part of the planning activity the functions of the valve in focus should also be determined. A function may be seen as a reason for the valve to be installed. Examples of valve functions are:

- F1: Isolate in case of leakage in segment A
- F2: Isolate in case of leakage in segment B
- F3: Isolate in case of maintenance of valve 2

It is important to ensure that the valve function formulations reflect the 'real' reasons that the valve is installed, since this may affect subsequent parts of the assessment process. An example: Consider F1 in the list above. What does 'isolate in case of leakage in segment A' mean? Does the valve have to isolate the segment completely, or is it sufficient that a large leak through the valve is prevented? Perhaps the valve function should be re-phrased to 'Reduce leakage through valve in closed position to maximum X kg/s'? The way the valve function is formulated is important, since in the first case, the method may end up with the recommendation of a differential pressure test, while in the latter case, a partial stroke test may be sufficient. To ensure that the valve functions are correctly phrased, we could use control questions, for example whether complete isolation is needed or whether achievement of the function within a longer period of time is sufficient.

Next, the failure modes of the valve should be determined. Examples of failure modes are:

- FM1: The valve does not close on demand
- FM2: Leakage through closed valve from segment A to segment B
- FM3: Leakage through closed valve from segment B to segment A

The failure modes are closely related to the valve functions since, if a failure mode occurs, one or several valve functions are lost. The relationship between the valve functions and the failure mode should be described. An example of such a relationship is shown in Table 2.

Table 2. Relationship between valve functions and failure modes.

Failure modes / Functions	FM1: Does not close on demand	FM2: Leakage through closed valve from segment A to segment B	FM3: Leakage through closed valve from segment B to segment A
F1: Isolate in case of leakage in segment A	X	X	
F2: Isolate in case of leakage in segment B	X		X
F3: Isolate in case of maintenance of V2	X		X

Additionally, relevant information about the valve in focus should be obtained and described. For example, dependencies between the valve in focus and other systems should be determined. Examples of such dependencies include: Does failure to open or close the valve result in shut down of other process plants? To what extent may testing of the valve be carried out in periods that corresponding systems are shut down? The above may largely affect production assurance and economic loss, in the first case if a valve failure occurs, and in the latter case during testing.

Finally, historical test results for the valve in focus and similar valves should be collected. As presented in the introduction to the paper, our focus is on situations where no or limited relevant data is available. In the case of relevant data being available, a data-driven approach would be a natural starting point.

2.2 Step B: Selection of test method

Most safety critical valves have several failure modes, and the most critical failure mode should be identified in order to select a suitable test method. The most critical failure mode can be identified by using a traditional FMECA (Failure Modes, Effects and Criticality Analysis). But there is a need for clarification (a guideline) for how such an analysis should be carried out in order to identify the most critical failure mode for a safety critical valve. Such a guideline is presented in Section 2.2.1. In Section 2.2.2 we describe how to decide upon test method based on the information received from the analysis in Section 2.2.1.

2.2.1 Identification of the most critical FM

In the proposed framework the criticality for each failure mode is based on an evaluation of what the consequences may be if a failure mode occurs, and an evaluation of the probability of a failure mode occurring.

A description of the activities in the second part of the framework (Step B) is given in the following.

Evaluation of potential consequences

In this step of the approach we evaluate the potential consequences of each failure mode identified in Step A. Each failure mode may result in a number of scenarios with different consequences. Due to this, the assignment process has to be based upon a systematic approach. In the suggested approach the potential consequences of a failure mode are not assigned directly. Instead, attention is paid to the potential consequences when the *valve functions* are lost. Thereafter, the evaluations can be transferred to failure modes by using the relation between valve functions and failure modes obtained in Step A.

In the suggested procedure a qualitative approach, based on a multidisciplinary meeting and expert judgment, is used to assign the consequences of losing each valve function. Attention is paid to the consequences if a best-case scenario occurs, the consequences if a worst-case scenario occurs and the expected consequences. Evaluation of consequences in best and worst cases is important, and of special interest, taking into consideration that the consequences of losing a valve function can be very different in many situations. For example, in the case of an external leak from a pipeline to the atmosphere, followed by loss of the isolation function (F1 or F2), many consequences are possible, ranging from an unignited leak to a large explosion escalating to other equipment.

The expert group decides whether or not the consequences of losing a valve function are high, medium or low, based on an overall evaluation of the consequences for worst- and best-case scenarios as well as the expected consequences. An evaluation of the probability of being in the best and worst state is certainly important when deciding upon the consequence category. The categorization process should be based on some guidelines or criteria to ensure consistency. For each of the consequence dimensions (safety, production, environment, etc.) it should be defined what is meant by low, medium and high consequences.

A simplified version of a questionnaire that can be used in the evaluation of consequences is given in Table 3.

Table 3. Questionnaire used in the evaluation of potential consequences of losing valve functions.

	SAFETY						PRODUCTION						ETC.
	Cons. given loss of funct. on dem						Cons. given loss of funct. on dem						
Functions	**Worst case**	**Best case**	**Ex-pected**	**L**	**M**	**H**	**Worst case**	**Best case**	**Ex-pected**	**L**	**M**	**H**	
F1: Isolate in case of leakage in segment A	Text	Text	Text	X			Text	Text	Text			X	
F2: Isolate in case of leakage in segment B	Text	Text	Text		X		Text	Text	Text		X		
F3: Isolate in case of maintenance of V2	Text	Text	Text	X			Text	Text	Text		X		

The next step is to transfer the evaluations to failure modes by using the relation between valve functions and failure modes obtained in Step A. This is done by combining the information in Table 2 and Table 3. For example, from Table 2 we see that all valve functions (F1-F3) are lost if the valve does not close on demand (FM1 occurs). The safety consequences of losing F1, F2 and F3 are, as seen from Table 3, low, medium and low, respectively. We then consider the safety consequences if the valve does not close on demand equal to the worst consequence category for the relevant valve functions. This means that the safety consequences for failure mode 1 are within the medium category. All the results of this process are given in Table 4.

Table 4. Consequence assignment category for each failure mode.

	SAFE-TY			PROD-UCT.			ETC.
Failure modes	**L**	**M**	**H**	**L**	**M**	**H**	
FM1: Does not close on demand		X				X	
FM2: Leakage through closed valve from A to B	X					X	
FM3: Leakage through closed valve from B to A		X			X		

Evaluation of the probability of a failure mode occurring

The next step is to assign the probability of each failure mode occurring. Since the starting point for the paper is that little or no historical relevant data is available, we suggest that the probability of a failure mode occurring is revealed in a multidisciplinary meeting by use of expert facilitation. This can be carried out in the same meeting as the one in which the consequences are assigned. The procedure proposed uses a classification scheme with three categories: high, medium and low. The categorization should be based on some criteria to ensure consistency.

In some situations it is required to understand the potential causes of each of the failure modes in order to express the probability of the failure mode occurring. This information is revealed during Step C of the method. In case this information is required to assign the probability category, we suggest that parts of Step C are carried out before Step B is finalized.

In the evaluation of the probability of a failure mode occurring, there is a need for taking the uncertainties into consideration. We may for example consider the probability of a failure mode occurring within the category medium, but because of high uncertainties we say that the probability is considered to be within the high probability category. This procedure is justified in Abrahamsen and Røed (2010). The point is that the evaluations on the probabilities are based on some background information. In mathematical terms you may look at the probability of failure mode occurring as P(failure mode occurring |K) where K is the background information and knowledge. The background knowledge covers historical system performance data, system performance characteristics and knowledge about the phenomena in question. Assumptions and presuppositions are an important part of this information and knowledge. The background knowledge can be viewed as frame conditions of the analysis, and the assigned probabilities must always be seen in relation to these conditions. Thus, different analysts could come up with different values, depending on assumptions and presuppositions. The differences could be very large. Hence, uncertainty needs to be considered, beyond the assigned probability number (Aven, 2008). Similar ideas are found underpinning approaches such as the risk governance framework (Renn, 2008) and the risk framework used by the UK Cabinet Office (Cabinet Office, 2002).

Criticality evaluation

Deciding on the criticality for each failure mode is based on the information in the consequence assessment process and on the assigned probability of a failure mode occurring. The categorization of the criticality should be based on some guidelines to ensure consistency. One possible way to categorize the criticality of the failure modes is given in Table 5.

Table 5. Criticality categories of failure modes.

ASSIGNED CATEGORIES		RESULT
Consequence given that the failure mode occurs on demand	**Probability of failure mode occurring**	**Criticality category**
Medium	High	
High	Medium	**High**
High	High	
Low	Low	
Low	Medium	**Medium**
Medium	Low	
Low	High	
Medium	Medium	**Low**
High	Low	

The idea is to allocate the failure modes into the three criticality categories, based on the probability and consequence assignments. The most critical failure mode should be brought forward to Step C of the framework. Since the assignment process is carried out for the consequence dimensions separately, the failure mode considered the most critical may differ from one consequence dimension to another. In other words, the failure mode considered the most critical with regard to production assurance may not be the same as the one considered the most critical with regard to safety of personnel. We will come back to this in the discussion; cf. Section 3. In some cases, by following the suggested procedure, several failure modes will have equal criticality categorization. We will revisit this situation as well in the discussion; cf. Section 3.

2.2.2 Selection of test method for the most critical FM

Selection of test method is based upon identification of alternative test methods, evaluation of potential consequences, evaluation of the reliability of each test method and evaluation of the overall performance of each test method. These activities are presented in the following.

Identification of alternative test methods

All the possible ways the most critical failure mode can be tested should be listed. Examples of test methods are:

- Pressure differential across the valve
- Cavity test
- Partial stroke test
- …

Note that only those test methods able to test the failure mode classified as the most critical in the first part of the proposed framework should be included.

Evaluation of the potential consequences of each test method

Before a test is carried out, we do not know what the consequences of performing the test will be for the different consequence dimensions. We may predict that performing the test will result in a number of hours'/days' loss of production, a certain amount of gas flaring (resulting in CO_2 emission to the environment) and no harm to the personnel performing the test, but we should also describe other potential outcomes. Some test consequences depend on factors outside the analysis object. For example, in the case where the test can be performed when the facility at the other end of the pipeline is shut down, the consequences in terms of costs may be reduced. In many cases, the outcome of performing a test is associated with uncertainty. The test may for example take a longer or shorter time, or it may, in the worst case, result in an ignited external gas leak exposing the test personnel to fire or explosion.

In the suggested procedure a qualitative approach, based on a multidisciplinary meeting and expert judgment, is used to assign the potential consequences of performing each of the test methods. The consequence category is based on an overall evaluation of the consequences for worst- and best-case scenarios and the expected consequences in the same way as mentioned during identification of the most critical failure mode. These evaluations can be carried out in the same meeting as mentioned above. Table 6 presents a simplified version of the questionnaire.

Table 6. Questionnaire used to assign the potential consequences of performing each of the

	SAFETY						PRODUCTION						ETC.
	Consequence of performing test						Consequence of performing test						
Test methods	Worst case	Best case	Ex-pected	L	M	H	Worst case	Best case	Ex-pected	L	M	H	
T1: Pressure diff. across the valve	Text	Text	Text			X	Text	Text	Text			X	
T2: Cavity test	Text	Text	Text		X		Text	Text	Text		X		
T3: Partial stroke test	Text	Text	Text	X			Text	Text	Text	X			

alternative tests.

Evaluation of the reliability of each test method

Another important aspect to take into consideration is the reliability of each test method. In many cases there are great differences between test methods in how much the test results can be trusted. While a positive outcome of one test method gives high trust that the valve will work as intended on demand, a positive outcome of another test method only gives an indication that this is the case. For example, if a test including pressure differential across the valve tells us that there is no leak, this result may be trusted more than the result from a partial stroke test telling us that the valve can be turned. The latter test method gives no confidence that the valve is not leaking in the closed position. We suggest that three qualitative categories are used for the reliability assignment in line with the former assignments: high, medium and low reliability; ref. Table 7 below.

Table 7. Categorization of test reliability.

	Test reliability			
Test methods	Descr-iption	L	M	H
T1: Pressure diff. across the valve	Text			X
T2: Cavity test	Text		X	
T3: Partial stroke test	Text	X		

Evaluation of the overall performance of each test method

The overall performance of each test method is made based on the reliability and consequence assignments. The categorization should be based on some guidelines to ensure consistency. One possible way to categorize the overall performance of each test method is given in Table 8.

Table 8. The overall performance/quality of test methods.

ASSIGNED CATEGORIES		**RESULT**
Reliability	**Consequence of performing the test**	**Overall per-formance**
H	L	**High**
H	M	**Medium**
H	H	
M	L	
M	H	
L	L	
M	H	**Low**
L	M	
L	H	

2.3 Step C: Selection of test interval

In Step C a decision about test interval for the test method in Step B is made. This is determined by first considering the potential causes of each of the failure modes revealed in Step A. Next, the valve's operating conditions are described. A test interval suggestion is then revealed by combining the above-mentioned information. In practice, this is done by expert facilitation in the previously mentioned multidisciplinary group consisting of personnel with thorough understanding of the valve/system in focus. The different activities in Step C are presented in the following.

Identify potential causes of each of the failure modes

The expert group should systematically go through each of the failure modes revealed in Step A, and identify potential causes in terms of failures within the valve. To carry out this activity, detailed knowledge of the valve in focus is needed. As mentioned in the introduction to the paper, many of the large safety critical valves are unique, having no, or just a few, comparable items. The differences are primarily related to technical details within the valves, and for each valve in focus it is vital to understand and describe the design of the valve. To reveal this information, it may be necessary to study in detail information from the vendor delivering the valve, and this means that parts of the study may be carried out in the planning phase (as part of Step A). We will not in this paper go into detail on valve design, since the results will only be valid for one particular valve, and since the framework is the key focus of the paper. We do, however, emphasize that for most large safety critical valves, each failure mode may have a number of potential causes.

Each of the potential causes should be described qualitatively, based on discussions in the group. In the case of there being redundancy within the design, e.g. double seals, the extent to which this redundancy can be tested should be discussed. In the case where it is not possible to test that redundant components are working as they should - a situation that is common for safety critical valves - it is recommended not to take the redundancy into consideration when deciding upon the test intervals. In the opposite case, potential dependencies and common cause errors should be discussed. In most cases there will be a list of potential causes of each of the failure modes. For the failure mode 'leakage through the valve in closed position', potential causes can for example be 'seats in wrong position', 'damaged seals' and 'leak behind the valve's seats', etc.

Describe the valve's operating conditions

The valve's operating conditions should be described by use of a brainstorming process. It is important that all aspects that may affect the valve's performance are revealed. We suggest that check lists with guide words are used as part of the brainstorming process in order to reveal as many relevant operating conditions as possible. Examples of guide words are 'internal', 'external', 'temperature' and 'corresponding equipment'. Examples of relevant operating conditions that may be revealed by use of expert facilitation in combination with guide words are harsh weather, vibration, sand in the HC flow, hydrate build-up and regular pigging.

Evaluate the test interval

In this activity, the relationship between operating conditions and potential causes of each of the failure modes should be revealed and discussed, and this information should be used to evaluate the test interval. We suggest that the activity is carried out by performing the following steps:

- Reveal the relationship between operating conditions and potential causes of valve failure
- Discuss time to occurrence for the relationships revealed in the step above
- Evaluate the test interval based on the above information

These activities are discussed below:

Firstly, the relationship between the operating conditions and the potential causes of valve failure should be revealed. In most cases this can be carried out in the work group. A simple way to reveal the relationship is to consider the lists of operating conditions and potential causes, and draw lines between the elements on the lists. In this way, a simple influence diagram is made showing which of the operating conditions may cause a valve error.

The next step is to discuss time to occurrence for the relationships revealed in the step above. In order to cover this in a proper manner, in-depth knowledge of the system in focus in required. In some cases, the work group will have sufficient information to assign approximate time to failure. In other cases additional knowledge will be required, for example technical design information and other information from vendors.

In most cases, assigning time to failure is not a simple process, due to lack of complete understanding of design, degradation mechanisms and how such mechanisms are affected by the operating conditions. Due to this, there are, in most cases, considerable uncertainties. These uncertainties should somehow be included in the discussion during the assignment process. This can be taken into consideration by assigning the expected time to failure as well as best-case and worst-case values.

The last step is to evaluate the test interval based on the above information. In principle, the shortest of the above assigned times to failure should be chosen. Due to the uncertainties there is, however, a need to make an overall decision taking all the above-mentioned aspects into consideration.

In cases where a test interval less frequent than once a year is decided upon, we recommend that an annual review is carried out in order to reveal if there is any new information that may affect the test method and test interval evaluations. For example, suppose we have revealed that a valve may have been damaged during a pigging operation during the last year. Then the test method and test interval evaluations should be updated, taking the new information into consideration. In most cases, however, the annual review will conclude that no relevant new information is gained, and that the maintenance programme can be followed as scheduled. The annual review will not be particularly time consuming since it can be carried out by a few persons, and in many cases can be carried out for several valves at a facility at the same time.

3 DISCUSSION AND CONCLUSION

The qualitative framework suggested in this paper is particularly relevant in situations where the safety critical items are unique and limited historical test data is available. In such situations, the ability to learn from the past through relevant test experience is limited, and alternative sources of knowledge should be considered. The framework shows how expert knowledge can serve as an alternative source of information. The framework is based on a structured approach where key evaluations can be carried out in a multidisciplinary work group during one or a few days. Due to the limited need for resources, the framework can work as an alternative to more arbitrary - and in some cases unstructured - approaches. Since the evaluations carried out are documented in work sheets, the framework also provides documentation of test method decisions subsequent to the work group meeting. This documentation can also serve as transfer of knowledge from the valve experts, having key knowledge of the systems in focus, to other staff.

A key aspect of the framework is, however, that additional knowledge is gained each time a test is carried out. Since this knowledge is taken into consideration in the framework, the test method considered the best in one case may not be considered the best next time the framework is applied. Suppose, for example, that the framework is applied to a safety critical valve resulting in a differential pressure test being considered the best. Thereafter, this test is carried out. Then, next time the framework is applied, the test results and other information gained during the test are taken into consideration. The result may be that another test method is considered as the best next time. This dynamic characteristic of the framework makes it possible over time to build insight on the properties and performance of the valve in focus.

As mentioned in Section 2, there may be situations where several failure modes are considered equally critical. In such cases, all test methods testing one or several of the critical failure modes revealed in the initial part of Step B should be considered when the test method is decided upon. In such situations, the relationship between the failure modes and the test methods obtained in the initial part of Step B should be taken into consideration in the overall evaluation of test method performance in the last part of Step B.

One aspect of the framework is that the consequence dimensions such as, for example, production assurance, safety, environmental impact and economy are treated separately. This increases the number of evaluations that will have to be carried out during the multidisciplinary meeting. Certainly, it would also be possible to combine the consequence dimensions, and thus, to obtain a less comprehensive procedure. We do, however, argue that separating these dimensions is necessary to provide sufficient decision support: when a result is obtained, we need to know whether the test method suggested is the 'best' with regard to production assurance or safety. The above means that the method may result in several test methods being recommended, regarding different consequence dimensions. This is not a problem since there should always be a step from decision support on the one hand, and the decision and implementation, on the other hand.

Some readers would perhaps claim that the framework is not sufficiently founded on reliability theory. To this, the answer is that alternative methods founded on reliability theory have a common need for sufficient relevant test data. We do indeed agree that in the case of such test data being available, traditional reliability methods can and should be applied. But the purpose of the framework presented in this paper is to obtain decision support in situations where no or only limited data is available. And for such situations, the framework presented can provide additional decision support, and can serve as an alternative to more arbitrary - and in many cases unstructured - approaches. After all, decisions on test methods and test intervals are also made in situations with limited data available.

As argued in Abrahamsen and Røed (2010), decision processes regarding safety related items should not be mechanistic. There is always a need for broad evaluations taking all relevant aspects into consideration before decisions are made. This is also the case for decision situations regarding choice of test method and test interval: Although the framework could serve as a tool to decide upon a test method and a test interval in a mechanistic manner, it should not be used in such a way. There is always a need for broader reflections before a decision is made. This emphasizes that all assumptions made during the evaluations should be provided to the decision maker. And in the case of there being something about the result that can be questioned, further evaluations should be carried out to obtain further decision support.

The method presented in this paper concludes on one test method and one corresponding test interval for one individual valve. This is of course a simplification, since a complete maintenance programme may consist of several test methods with one test interval for each test method. It is also a challenge that only one individual valve is considered, since to ensure that the maintenance programme is manageable in practice, it may be argued that the valves should be put into groups with one dedicated combination of test methods/ intervals for each group. It is believed that the method presented can be further developed in the future to take these aspects into consideration. For the time being, the focus has, however, been on a simplified challenge considering one individual valve, one test method and one associated test interval. By gaining experience with such a simplification, we will be able to acquire information that can be used to develop more sophisticated methods in the future.

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RELIABILITY, RISK AND AVAILABILITY ANLYSIS OF A CONTAINER GANTRY CRANE

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ABSTRACT

The joint model of the system operation process and the system multi-state reliability is applied to the reliability, risk and availability evaluation of the container gantry crane. The container gantry crane is described and the mean values of the container gantry crane operation process unconditional sojourn times in particular operation states are found and applied to determining this process transient probabilities in these states. The container gantry crane different reliability structures in various its operation states are fixed and their conditional reliability functions on the basis of data coming from experts are approximately determined. Finally, after applying earlier estimated transient probabilities and system conditional reliability functions in particular operation states the unconditional reliability function, the mean values and standard deviations of the container gantry crane lifetimes in particular reliability states, risk function and the moment when the risk exceeds a critical value are found. Next the renewal and availability characteristics for the considered gantry crane are determined.

1 INTRODUCTION

Most real technical systems are very complex and it is difficult to analyze their reliability, availability and safety. Large numbers of components and subsystems and their operating complexity cause that the identification, evaluation, prediction and optimization of their reliability, availability and safety are complicated. The complexity of the systems' operation processes and their influence on changing in time the systems' structures and their components' reliability characteristics are very often met in real practice.

Taking into account the importance of the safety and operating process effectiveness of such systems it seems reasonable to expand the two-state approach to multistate approach (Aven 1985, Kolowrocki 2004, Kołowrocki, Soszyńska-Budny 2011) in their reliability analysis. The assumption that the systems are composed of multistate components with reliability states degrading in time gives the possibility for more precise analysis of their reliability and operation processes' effectiveness. This assumption allows us to distinguish a system reliability critical state (Kolowrocki 2004, Kołowrocki, Soszyńska 2010, Kołowrocki, Soszyńska-Budny 2011, Soszyńska 2010) to exceed which is either dangerous for the environment or does not assure the necessary level of its operation process effectiveness. Then, an important system reliability characteristic is the time to the moment of exceeding the system reliability critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multistate reliability function that is basic characteristics of the multi-state system.

The complexity of the systems' operation processes and these processes influence on changing in time the systems' structures and their components' reliability characteristics (Kołowrocki, Soszyńska 2010, Kołowrocki, Soszyńska-Budny 2010, Kołowrocki, Soszyńska-Budny 2011, Soszynska 2010) is often very difficult to fix and to analyse. A convenient tool for solving this problem is semi-Markov (Grabski 2002, Limnios 2001) modelling of the systems operation processes which is proposed in the paper. Using the joint model of the system multi-state reliability and the system semi-Markov operation process (Kołowrocki, Soszyńska 2010, Kołowrocki, Soszyńska-Budny 2011, Soszyńska 2010) it is possible to point out the variability of system components reliability characteristics by introducing the components' conditional multi-state reliability functions determined by the system operation states. Consequently it is possible to find the system conditional reliability function dependent on the operation states and next it is possible to find its unconditional reliability function and renewal and availability characteristics.

This way the obtained results concerned with multi-state systems in its varying in time operation states can be applied to the reliability, risk and availability evaluation of the container gantry crane.

2 CONTAINER GANTRY CRANE SYSTEM ANALYSIS

We analyse the reliability of the container gantry crane that is operating at the container terminal placed at the seashore (Kołowrocki, Soszyńska-Budny 2010, Kołowrocki, Soszyńska-Budny 2011). The considered container terminal is engaged in trans-shipment of containers. The loading of containers is carried out by using the gantry cranes called Ship-To-Shore (STS).

We consider the STS container gantry crane that is composed of 5 basic subsystems S_1, S_2, S_3, S_4 and S_5 having an essential influence on its reliability. Those subsystems are as follows:

S_1 - the power supply subsystem,
S_2 - the control and monitoring subsystem,
S_3 - the arm getting up and getting down subsystem,
S_4 - the transferring subsystem,
S_5 - the loading and unloading subsystem.

The gantry crane power supply subsystem S_1 consists of:

- a high voltage cable delivering the energy from the substation to the gantry crane $E_1^{(1)}$,
- a drum allowing the cable unreeling during the crane transferring $E_2^{(1)}$,
- an inner crane power supply cable $E_3^{(1)}$,
- a device transmitting the energy from the high voltage cable to the inner crane cable $E_4^{(1)}$,
- main and supporting voltage transformers $E_5^{(1)}$,
- a low voltage power supply cable $E_6^{(1)}$,
- relaying and protective electrical components $E_7^{(1)}$.

The gantry crane control and monitoring subsystem S_2 consists of:

- a crane software controller precisely analyzing the situation and takes suitable actions in order to assure correct work of the crane $E_1^{(2)}$,
- a measuring and diagnostic device sending signals about the crane state to the software controller $E_2^{(2)}$,
- a transmitter of signals from the controller to elements executing the set of commands $E_3^{(2)}$,
- devices carrying out the controller's orders (a permission to work, a blockade of work, etc.) $E_4^{(2)}$,
- control panels (an engine room, an operator's cabin, a crane arm cabin) $E_5^{(2)}$,
- control and steering cables' connections $E_6^{(2)}$.

The gantry crane arm getting up and getting down subsystem S_3 consists of:

- a propulsion unit (an engine, a rope drum, a transmission gear, a clutch, breaks, a rope) $E_1^{(3)}$,
- a set of rollers and multi-wheels $E_2^{(3)}$,
- a crane arm (joints, hooks fastening the arm) $E_3^{(3)}$.

The gantry crane transferring subsystem S_4 consists of:

- a driving unit (an engine, a clutch, breaks, a transmission gear, gantry crane wheels) $E_1^{(4)}$.

The gantry crane loading and unloading subsystem S_5 consists of the winch unit $E_1^{(5)}$ composed of:
- a propulsion unit (an engine, a clutch, breaks, a transmission gear, ropes),
- a winch head (which a container grab is connected to),
- a container's grab,
- a container's grab stabilizing unit

and the cart unit $E_2^{(5)}$ composed of:
- a propulsion unit (an engine, a clutch, breaks, a transmission gear, cart wheels, ropes),
- rails which cart is moving on during the operation,
- a crane cart.

The subsystems S_1, S_2, S_3, S_4, S_5 are forming a general series gantry crane reliability structure presented in Figure 1.

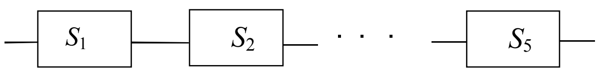

Figure 1. General scheme of gantry crane reliability structure

3 CONTAINER GANTRY CRANE OPERATION PROCESS CHARACTERISTICS PREDICTION

The container gantry crane reliability structure and the subsystems and components reliability depend on its changing in time operation states.

Taking into account expert opinions on the varying in time operation process of the considered container gantry crane we fix the number of the system operation process states $v = 6$ and we distinguish the following as its six operation states:

- an operation state $z_1 -$ the crane standby with the power supply on and the control system off,
- an operation state $z_2 -$ the crane prepared either to starting or finishing the work with the crane arm angle position of 90°,
- an operation state $z_3 -$ the crane prepared either to starting or finishing the work with the crane arm angle position of 0°,
- an operation state $z_4 -$ the crane transferring either to or from the loading and unloading area with the crane arm angle position of 90°,
- an operation state $z_5 -$ the crane transferring either to or from the loading and unloading area with the crane arm angle position of 0°,
- an operation state $z_6 -$ the containers' loading and unloading with the crane arm angle position of 0°.

Moreover, we fix that there are possible the transitions between all system operation states.

To identify all parameters of the container gantry crane operation process the statistical data about this process was needed. All statistical data are collected in (Kołowrocki, Soszyńska-Budny 2010). On the basis of statistical data from (Kołowrocki, Soszyńska-Budny 2010) the following matrix

$$[p_{bl}] = \begin{bmatrix} 0 & 0.648 & 0.336 & 0.008 & 0 & 0.008 \\ 0.525 & 0 & 0.373 & 0.093 & 0 & 0.009 \\ 0.105 & 0.111 & 0 & 0 & 0.118 & 0.666 \\ 0.417 & 0.583 & 0 & 0 & 0 & 0 \\ 0.005 & 0 & 0.220 & 0 & 0 & 0.775 \\ 0.012 & 0 & 0.628 & 0 & 0.360 & 0 \end{bmatrix},$$

of the probabilities p_{bl}, $b,l = 1,2,...,6,$ of the container gantry crane operation process transitions from the operation state z_b to the operation state z_l has been fixed.

The mean values $M_{bl} = E[\theta_{bl}],\ b,l = 1,2,...,6,\ b \neq l,$ of the container gantry crane operation process conditional sojourn times at the particular operation states according to (2.12) (Kołowrocki, Soszyńska-Budny 2011) are as follows:

$$M_{12} = 456.98,\ M_{13} = 36.86,\ M_{14} = 50,\ M_{16} = 3,\ M_{21} = 7.89,\ M_{23} = 9.12,\ M_{24} = 1.55,\ M_{26} = 16,$$

$$M_{31} = 5.50,\ M_{32} = 4.34,\ M_{35} = 6.82,\ M_{36} = 7.86,\ M_{41} = 2,\ M_{42} = 2.14,\ M_{51} = 10,\ M_{53} = 2.90,$$

$$M_{56} = 24.68,\ M_{61} = 22.60.\ M_{63} = 23.12,\ M_{65} = 20.51. \quad (1)$$

After considering the results (1) and applying the formula (2.21) from (Kołowrocki, Soszyńska-Budny 2011), the unconditional mean sojourn times of the container gantry crane operation process at the particular operation states are given by:

$$M_1 = E[\theta_1] = 0.648 \cdot 456.98 + 0.336 \cdot 36.86 + 0.008 \cdot 50 + 0.008 \cdot 3 \cong 308.93,$$

$$M_2 = E[\theta_2] = 0.525 \cdot 7.89 + 0.373 \cdot 9.12 + 0.093 \cdot 1.55 + 0.009 \cdot 16 \cong 7.83,$$

$$M_3 = E[\theta_3] = 0.105 \cdot 5.50 + 0.111 \cdot 4.34 + 0.118 \cdot 6.82 + 0.666 \cdot 7.86 \cong 7.09,$$

$$M_4 = E[\theta_4] = 0.417 \cdot 2 + 0.583 \cdot 2.14 \cong 2.08,$$

$$M_5 = E[\theta_5] = 0.005 \cdot 10 + 0.220 \cdot 2.90 + 0.775 \cdot 24.68 \cong 19.82,$$

$$M_6 = E[\theta_6] = 0.012 \cdot 22.60 + 0.628 \cdot 23.12 + 0.360 \cdot 20.51 \cong 22.17.$$

Since, according to (2.23) (Kołowrocki, Soszyńska-Budny 2011), from the system of equations

$$\begin{cases} [\pi_1,\pi_2,\pi_3,\pi_4,\pi_5,\pi_6] = [\pi_1,\pi_2,\pi_3,\pi_4,\pi_5,\pi_6][p_{bl}]_{6x6} \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 = 1, \end{cases}$$

we get

$$\pi_1 = 0.0951,\ \pi_2 = 0.1020,\ \pi_3 = 0.3100,\ \pi_4 = 0.0102,\ \pi_5 = 0.1547,\ \pi_6 = 0.3280.$$

Then, the limit values of the transient probabilities $p_b(t)$ of the gantry crane operation process at the operation states z_b, according to (2.22) in (Kołowrocki, Soszyńska-Budny 2011), are given by

$$p_1 = 0.6874,\ p_2 = 0.0187,\ p_3 = 0.0515,\ p_4 = 0.0005,\ p_5 = 0.0717,\ p_6 = 0.1702. \qquad (2)$$

4 CONTAINER GANTRY CRANE IN VARIABLE OPERATION CONDITION RELIABILITY AND RISK EVALUATION

After discussion with experts, taking into account the effectiveness of the operation of the container gantry crane, we fix that the system and its components have four reliability states 0, 1, 2, 3, i.e. $z = 3$. And consequently, at all operation states z_b, $b = 1,2,...,6$, we distinguish the following reliability states of the system and its components:

- a reliability state 3 – the gantry operation is fully effective,
- a reliability state 2 – the gantry operation is less effective because of ageing,
- a reliability state 1 – the gantry operation is less effective because of ageing and more dangerous,
- a reliability state 0 – the gantry is destroyed.

We assume that there are possible the transitions between the components reliability states only from better to worse ones and we fix that the system and components critical reliability state is $r = 2$.

Consequently, we assume that the gantry crane subsystems S_υ, $\upsilon = 1,2,...,5$ are composed of four-state components, i.e. z =3, with the multi-state reliability functions

$$[R_i^{(\upsilon)}(t,\cdot)]^{(b)} = [1,\ [R_i^{(\upsilon)}(t,1)]^{(b)}, [R_i^{(\upsilon)}(t,2)]^{(b)}, [R_i^{(\upsilon)}(t,3)]^{(b)}\ ],\ b = 1,2,...,6,$$

with exponential co-ordinates $[R_i^{(\upsilon)}(t,1)]^{(b)}$, $[R_i^{(\upsilon)}(t,2)]^{(b)}$, $[R_i^{(\upsilon)}(t,3)]^{(b)}$ different in various operation states z_b, $b = 1,2,...,6$.

In (Kołowrocki, Soszyńska-Budny 2010), on the basis of expert opinions, the reliability functions of the gantry crane components in different operation states are approximately determined. We will use them in our further system reliability analysis and evaluation.

At the system operation state z_1, the container gantry crane is composed of the subsystem S_1 which is a series system composed of $n = 7$ components $E_i^{(1)}$, $i = 1,2,..,7$ (subsystems) with the structure showed in Figure 2.

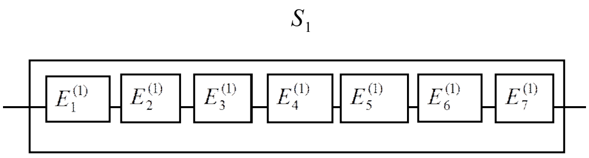

Figure 2. The scheme of the container gantry crane at operation state z_1

Thus, at the system operation state z_1, the container gantry crane is identical with subsystem S_1, that is a four-state series system with its structure shape parameter $n = 7$ and according to (1.22)-(1.23) (Kołowrocki, Soszyńska-Budny 2011), its four-state reliability function is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(1)} = [1, [\boldsymbol{R}(t,1)]^{(1)}, [\boldsymbol{R}(t,2)]^{(1)}, [\boldsymbol{R}(t,3)]^{(1)}], \quad t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(1)} = \exp[-0.020t]\exp[-0.040t]\exp[-0.040t]\exp[-0.020t]\exp[-0.020t]$$

$$\exp[-0.018t]\exp[-0.033t] = \exp[-0.191t], \quad (3)$$

$$[\boldsymbol{R}(t,2)]^{(1)} = \exp[-0.033t]\exp[-0.050t]\exp[-0.050t]\exp[-0.033t]\exp[-0.030t]$$

$$\exp[-0.028t]\exp[-0.040t] = \exp[-0.264t], \quad (4)$$

$$[\boldsymbol{R}(t,3)]^{(1)} = \exp[-0.050t]\exp[-0.066t]\exp[-0.066t]\exp[-0.050t]\exp[-0.040t]$$

$$\exp[-0.040t]\exp[-0.050t] = \exp[-0.362t]. \quad (5)$$

The expected values of the container gantry crane conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_1, calculated from the results given by (3)-(5), according to (3.8) (Kołowrocki, Soszyńska-Budny 2011), respectively are:

$$\mu_1(1) \cong 5.24, \ \mu_1(2) \cong 3.79, \ \mu_1(3) \cong 2.76 \text{ years.}$$

(6)

At the system operation states z_2 and z_3, the container gantry crane is composed of the subsystems S_1, S_2 and S_3 forming a series structure shown in Figure 3. The subsystem S_1 is a series system composed of $n = 7$ components $E_i^{(1)}$, $i = 1,2,..,7$, the subsystem S_2 is a series system composed of $n = 6$ components $E_i^{(2)}$, $i = 1,2,..,6$, and the subsystem S_3 is a series system composed of $n = 3$ components $E_i^{(3)}$, $i = 1,2,3$.

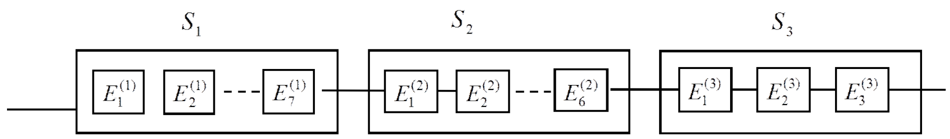

Figure 3. The scheme of the container gantry crane at operation states z_2 and z_3

Thus, at the system operation state z_2, the container gantry crane is composed of the subsystems S_1, S_2 and S_3 forming a series structure.
At this operation state, the subsystem S_1 is a four-state series system with its structure shape parameter $n = 7$ and according to (1.22)-(1.23) (Kołowrocki, Soszyńska-Budny 2011), its four-state reliability function is given by the vector

$$[\boldsymbol{R}^{(1)}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}^{(1)}(t,1)]^{(2)}, [\boldsymbol{R}^{(1)}(t,2)]^{(2)}, [\boldsymbol{R}^{(1)}(t,3)]^{(2)}], \ t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}^{(1)}(t,1)]^{(2)} = \exp[-0.020t]\exp[-0.040t]\exp[-0.040t]\exp[-0.020t]$$

$$\exp[-0.022t]\exp[-0.018t]\exp[-0.033t] = \exp[-0.193t], \quad (7)$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(2)} = \exp[-0.033t]\exp[-0.050t]\exp[-0.050t]\exp[-0.033t]$$

$$\exp[-0.027t]\exp[-0.028t]\exp[-0.040t] = \exp[-0.261t], \quad (8)$$

$$[\boldsymbol{R}^{(1)}(t,3)]^{(2)} = \exp[-0.050t]\exp[-0.066t]\exp[-0.066t]\exp[-0.050t]$$

$$\exp[-0.048t]\exp[-0.040t]\exp[-0.050t] = \exp[-0.370t]. \quad (9)$$

The subsystem S_2 at the operation state z_2, is a four-state series system with its structure shape parameter $n = 6$ and according to (1.22)-(1.23) (Kołowrocki, Soszyńska-Budny 2011), its four-state reliability function is given by the vector

$$[\boldsymbol{R}^{(2)}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}^{(2)}(t,1)]^{(2)}, [\boldsymbol{R}^{(2)}(t,2)]^{(2)}, [\boldsymbol{R}^{(2)}(t,3)]^{(2)}], \ t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}^{(2)}(t,1)]^{(2)} = \exp[-0.053t]\exp[-0.048t]\exp[-0.048t]\exp[-0.048t]$$

$$\exp[-0.020t]\exp[-0.018t] = \exp[-0.235t], \quad (10)$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(2)} = \exp[-0.059t]\exp[-0.053t]\exp[-0.053t]\exp[-0.053t]$$

$$\exp[-0.025t]\exp[-0.029t] = \exp[-0.272t], \quad (11)$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(2)} = \exp[-0.066t]\exp[-0.059t]\exp[-0.059t]\exp[-0.059t]$$

$$\exp[-0.033t]\exp[-0.040t] = \exp[-0.316t]. \quad (12)$$

The subsystem S_3 at the operation state z_2, is a four-state series system with its structure shape parameter $n = 3$ and according to (1.22)-(1.23) (Kołowrocki, Soszyńska-Budny 2011), its four-state reliability function is given by the vector

$$[\boldsymbol{R}^{(3)}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}^{(3)}(t,1)]^{(2)}, [\boldsymbol{R}^{(3)}(t,2)]^{(2)}, [\boldsymbol{R}^{(3)}(t,3)]^{(2)}], \ t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}^{(3)}(t,1)]^{(2)} = \exp[-0.025t]\exp[-0.033t]\exp[-0.033t] = \exp[-0.091t], \quad (13)$$

$$[\boldsymbol{R}^{(3)}(t,2)]^{(2)} = \exp[-0.040t]\exp[-0.040t]\exp[-0.040t] = \exp[-0.120t], \quad (14)$$

$$[\boldsymbol{R}^{(3)}(t,3)]^{(2)} = \exp[-0.066t] \exp[-0.066t] \exp[-0.066t] = \exp[-0.198t]. \quad (15)$$

Considering that the container gantry crane at the operation state z_2 is a four-state series system composed of subsystems S_1, S_2 and S_3, after applying (1.22)–(1.23) (Kołowrocki, Soszyńska-Budny 2011), its conditional four-state reliability function is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}(t,1)]^{(2)}, [\boldsymbol{R}(t,2)]^{(2)}, [\boldsymbol{R}(t,3)]^{(2)}], \ t \ge 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(2)} = \exp[-0.193t] \exp[-0.235t] \exp[-0.091t] = \exp[-0.519t], \quad (16)$$

$$[\boldsymbol{R}(t,2)]^{(2)} = \exp[-0.261t] \exp[-0.272t] \exp[-0.120t] = \exp[-0.653t], \quad (17)$$

$$[\boldsymbol{R}(t,3)]^{(2)} = \exp[-0.370t] \exp[-0.316t] \exp[-0.198t] = \exp[-0.884t]. \quad (18)$$

The expected values of the container gantry crane conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_2, calculated from the results given by (16)-(18), according to (3.8) (Kołowrocki, Soszyńska-Budny 2011), respectively are:

$$\mu_2(1) \cong 1.93, \ \mu_2(2) \cong 1.53, \mu_2(3) \cong 1.13 \text{ year}. \quad (19)$$

After proceeding in the analogous way in the system reliability analysis and evaluation at the remaining operation states z_3, z_4, z_5 and z_6, we may determine the system conditional reliability function that are presented below.

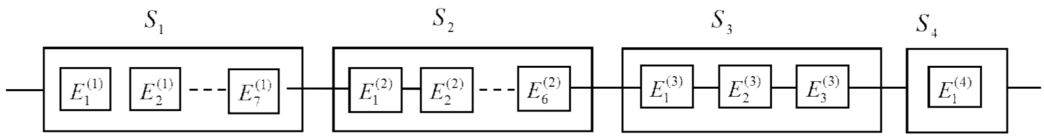

Figure 4. The scheme of the container gantry crane at operation states z_4 and z_5

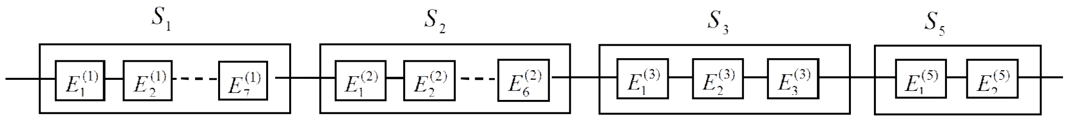

Figure 5. The scheme of the container gantry crane at operation state z_6

At the operation state z_3, the container gantry crane conditional reliability function of the system is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}(t,1)]^{(3)}, [\boldsymbol{R}(t,2)]^{(3)}, [\boldsymbol{R}(t,3)]^{(3)}], \ t \ge 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(3)} = \exp[-0.196t]\exp[-0.235t]\ \exp[-0.091t] = \exp[-0.522t], \quad (20)$$

$$[\boldsymbol{R}(t,2)]^{(3)} = \exp[-0.264t]\ \exp[-0.272t]\exp[-0.120t] = \exp[-0.656t], \quad (21)$$

$$[\boldsymbol{R}(t,3)]^{(3)} = \exp[-0.375t]\ \exp[-0.316t]\exp[-0.198t] = \exp[-0.889t]. \quad (22)$$

The expected values of the container gantry crane conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_3, calculated from the results given by (20)-(22), according to (3.8) (Kołowrocki, Soszyńska-Budny 2011), respectively are:

$$\mu_3(1) \cong 1.91,\ \mu_3(2) \cong 1.52,\ \mu_3(3) \cong 1.12 \text{ year}. \quad (23)$$

At the system operation states z_4 and z_5, the container gantry crane is composed of the subsystems S_1, S_2, S_3 and S_4 forming a series structure shown in Figure 4. The subsystem S_1 is a series system composed of $n = 7$ components $E_i^{(1)}$, $i = 1,2,..,7$, the subsystem S_2 is a series system composed of $n = 6$ components $E_i^{(2)}$, $i = 1,2,..,6$, the subsystem S_3 is a series system composed of $n = 3$ components $E_i^{(3)}$, $i = 1,2,3$, and the subsystem S_4 consists of a component $E_1^{(4)}$.
Thus, at the operation state z_4, the container gantry crane conditional reliability function of the system is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(4)} = [1, [\boldsymbol{R}(t,1)]^{(4)}, [\boldsymbol{R}(t,2)]^{(4)}, [\boldsymbol{R}(t,3)]^{(4)}],\ t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(4)} = \exp[-0.216t]\ \exp[-0.241t]\ \exp[-0.061t]\ \exp[-0.029t] = \exp[-0.547t], \quad (24)$$

$$[\boldsymbol{R}(t,2)]^{(4)} = \exp[-0.289t]\ \exp[-0.278t]\ \exp[-0.091t]\ \exp[-0.04t] = \exp[-0.698t], \quad (25)$$

$$[\boldsymbol{R}(t,3)]^{(4)} = \exp[-0.428t]\ \exp[-0.328t]\ \exp[-0.133t]\ \exp[-0.066t] = \exp[-0.955t]. \quad (26)$$

The expected values of the container gantry crane conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_4, calculated from the results given by (24)-(26), according to (3.8) (Kołowrocki, Soszyńska-Budny 2011), respectively are:

$$\mu_4(1) \cong 1.83,\ \mu_4(2) \cong 1.43,\ \mu_4(3) \cong 1.05 \text{ year}. \quad (27)$$

At the operation state z_5, the container gantry crane conditional reliability function of the system is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(5)} = [1, [\boldsymbol{R}(t,1)]^{(5)}, [\boldsymbol{R}(t,2)]^{(5)}, [\boldsymbol{R}(t,3)]^{(5)}],\ t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(5)} = \exp[-0.216t]\exp[-0.241t]\exp[-0.061t]\exp[-0.025t] = \exp[-0.543t], \quad (28)$$

$$[\boldsymbol{R}(t,2)]^{(5)} = \exp[-0.289t]\exp[-0.278t]\exp[-0.091t]\exp[-0.029t] = \exp[-0.687t], \quad (29)$$

$$[\boldsymbol{R}(t,3)]^{(5)} = \exp[-0.428t]\exp[-0.328t]\exp[-0.133t]\exp[-0.050t] = \exp[-0.939t]. \quad (30)$$

The expected values of the container gantry crane conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_5, calculated from the results given by (28)-(30), according to (3.8) (Kołowrocki, Soszyńska-Budny 2011), respectively are:

$$\mu_5(1) \cong 1.84, \ \mu_5(2) \cong 1.46, \ \mu_5(3) \cong 1.06 \text{ year.}$$

(31)

At the system operation state z_6, the container gantry crane is composed of the subsystems S_1, S_2, S_3 and S_5 forming a series structure shown in Figure 5. The subsystem S_1 is a series system composed of $n = 7$ components $E_i^{(1)}$, $i = 1,2,..,7$, the subsystem S_2 is a series system composed of $n = 6$ components $E_i^{(2)}$, $i = 1,2,..,6$, the subsystem S_3 is a series system composed of $n = 3$ components $E_i^{(3)}$, $i = 1,2,3$ and the subsystem S_5 is a series system composed of $n = 2$ components $E_i^{(5)}$, $i = 1,2$.

Thus, at the operation state z_6, the container gantry crane conditional reliability function of the system is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(6)} = [1, [\boldsymbol{R}(t,1)]^{(6)}, [\boldsymbol{R}(t,2)]^{(6)}, [\boldsymbol{R}(t,3)]^{(6)}], \ t \ge 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(6)} = \exp[-0.201t]\exp[-0.250t]\exp[-0.087t]\exp[-0.080t] = \exp[-0.618t], \quad (32)$$

$$[\boldsymbol{R}(t,2)]^{(6)} = \exp[-0.273t]\exp[-0.29t]\exp[-0.12t]\exp[-0.1t] = \exp[-0.783t], \quad (33)$$

$$[\boldsymbol{R}(t,3)]^{(6)} = \exp[-0.396t]\exp[-0.337t]\exp[-0.16t]\exp[-0.132t] = \exp[-1.025t]. \quad (34)$$

The expected values of the container gantry crane conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_6, calculated from the results (32)-(34), according to (3.8) (Kołowrocki, Soszyńska-Budny 2011), respectively are:

$$\mu_6(1) \cong 1.62, \ \mu_6(2) \cong 1.28, \ \mu_6(3) \cong 0.98 \text{ year.}$$

(35)

In the case when the operation time is large enough the unconditional four-state reliability function of the container gantry crane is given by the vector

$$\boldsymbol{R}(t, \cdot) = [1, \boldsymbol{R}(t,1), \ \boldsymbol{R}(t,2), \ \boldsymbol{R}(t,3)], t \ge 0, \quad (36)$$

where according to (3.5)-(3.6) (Kołowrocki, Soszyńska-Budny 2011) and considering (2), the vector coordinates are given respectively by

$$\boldsymbol{R}(t,1) = 0.6874\cdot[\boldsymbol{R}(t,1)]^{(1)} + 0.0187\cdot[\boldsymbol{R}(t,1)]^{(2)} + 0.0515\cdot[\boldsymbol{R}(t,1)]^{(3)} + 0.0005\cdot[\boldsymbol{R}(t,1)]^{(4)}$$

$$+ 0.0717\cdot[\boldsymbol{R}(t,1)]^{(5)} + 0.1702\cdot[\boldsymbol{R}(t,1)]^{(6)} \text{ for } t \ge 0,$$

(37)

$$\boldsymbol{R}(t,2) = 0.6874\cdot[\boldsymbol{R}(t,2)]^{(1)} + 0.0187\cdot[\boldsymbol{R}(t,2)]^{(2)} + 0.0515\cdot[\boldsymbol{R}(t,2)]^{(3)} + 0.0005\cdot[\boldsymbol{R}(t,2)]^{(4)}$$

$$+ 0.0717\cdot[\boldsymbol{R}(t,2)]^{(5)} + 0.1702\cdot[\boldsymbol{R}(t,2)]^{(6)} \text{ for } t \ge 0, \quad (38)$$

$$\boldsymbol{R}(t,3) = 0.6874\cdot[\boldsymbol{R}(t,3)]^{(1)} + 0.0187\cdot[\boldsymbol{R}(t,3)]^{(2)} + 0.0515\cdot[\boldsymbol{R}(t,3)]^{(3)} + 0.0005\cdot[\boldsymbol{R}(t,3)]^{(4)}$$

$$+ 0.0717\cdot[\boldsymbol{R}(t,3)]^{(5)} + 0.1702\cdot[\boldsymbol{R}(t,3)]^{(6)} \text{ for } t \ge 0, \quad (39)$$

and the coordinates $[\boldsymbol{R}(t,u)]^{(b)}$, $b = 1,2,...,6$, $u = 1,2,3$, are given by (3)-(5), (16)-(18), (20)-(22), (24)-(26), (28)-(30), (32)-(34). The graphs of the coordinates of the container gantry crane reliability function are presented in Figure 6.

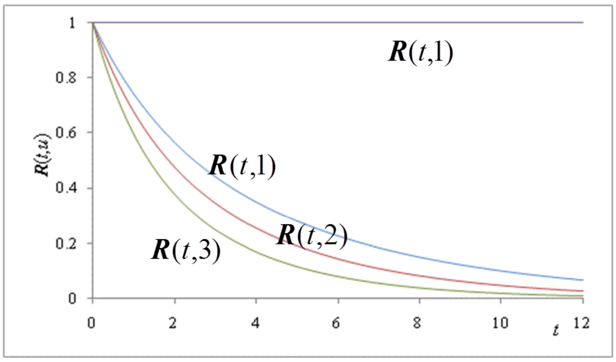

Figure 6. The graph of the container gantry crane reliability function $\boldsymbol{R}(t,\cdot)$ coordinates

The expected values and standard deviations of the container gantry crane unconditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ calculated from the results given by (37)-(39), according to (3.7)-(3.9) (Kołowrocki, Soszyńska-Budny 2011) and considering (2), (6), (19), (23), (27), (31), (35), respectively are:

$$\mu(1) = 0.6874\cdot 5.24 + 0.0187\cdot 1.93 + 0.0515\cdot 1.91 + 0.0005\cdot 1.83 + 0.0717\cdot 1.84$$

$$+ 0.1702\cdot 1.62 \cong 4.14 \text{ years}, \quad (40)$$

$$\sigma(1) \cong 4.71 \text{ years},$$

$$\mu(2) = 0.6874\cdot 3.79 + 0.0187\cdot 1.53 + 0.0515\cdot 1.52 + 0.0005\cdot 1.43 + 0.0717\cdot 1.46$$

$$+ 0.1702 \cdot 1.28 \cong 3.04 \text{ years,} \tag{41}$$

$$\sigma(2) \cong 3.43 \text{ years,} \tag{42}$$

$$\mu(3) = 0.6874 \cdot 2.76 + 0.0187 \cdot 1.13 + 0.0515 \cdot 1.12 + 0.0005 \cdot 1.05 + 0.0717 \cdot 1.06$$

$$+ 0.1702 \cdot 0.98 \cong 2.22 \text{ years,} \tag{43}$$

$$\sigma(3) \cong 2.50 \text{ years,}$$

Further, considering (3.10) from (Kołowrocki, Soszyńska-Budny 2011) and (40), (41), (43), the mean values of the unconditional lifetimes in the particular reliability states 1, 2, 3 respectively are:

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 1.10, \ \bar{\mu}(2) = \mu(2) - \mu(3) = 0.82, \ \bar{\mu}(3) = \mu(3) = 2.22 \text{ years.} \tag{44}$$

Since the critical reliability state is $r = 2$, then the system risk function, according to (3.11) (Kołowrocki, Soszyńska-Budny 2011), is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t, 2), \tag{45}$$

where $\boldsymbol{R}(t, 2)$ is given by (38).
Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (3.12) (Kołowrocki, Soszyńska-Budny 2011), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.126 \text{ year.}$$
(46)

The graph of the risk function $\boldsymbol{r}(t)$ of the container gantry crane operating at the variable conditions is given in Figure 7.

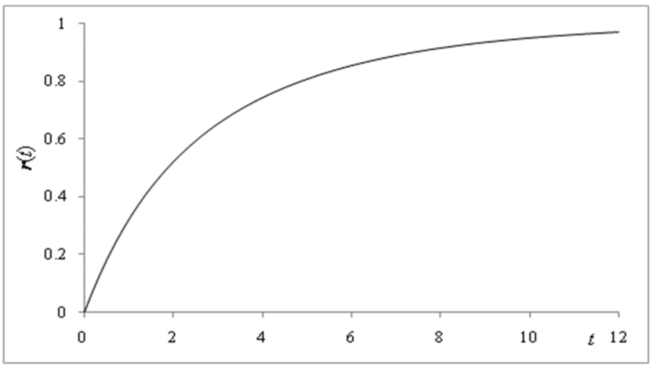

Figure 7. The graph of the container gantry crane risk function $\boldsymbol{r}(t)$

5 CONTAINER GANTRY CRANE AVAILABILITY PREDICTION

Using the results of the container gantry crane reliability prediction given by (41)-(42) and the results of the classical renew theory presented in (Kołowrocki, Soszyńska-Budny 2011), we may predict the renewal and availability characteristics of this system in the case when it is repairable and its time of renovation is either ignored or non-ignored.

First, assuming that the container gantry crane is repaired after the exceeding its reliability critical state $r = 2$ and that the time of the system renovation is ignored we obtain the following results:

a) the time $S_N(2)$ until the *Nth* exceeding by the system the reliability critical state $r = 2$, for sufficiently large *N*, has approximately normal distribution $N(3.04N, 3.43\sqrt{N})$, i.e.,

$$F^{(N)}(t,2) = P(S_N(2) < t) \cong F_{N(0,1)}(\frac{t - 3.04N}{3.43\sqrt{N}}), \ t \in (-\infty, \infty);$$

b) the expected value and the variance of the time $S_N(2)$ until the *Nth* exceeding by the system the reliability critical state $r = 2$ are respectively given by

$$E[S_N(2)] \cong 3.04N, \ D[S_N(2)] \cong 11.76N;$$

c) the number $N(t,2)$ of exceeding by the system the reliability critical state $r = 2$ up to the moment $t, t \geq 0,$ for sufficiently large *t*, approximately has the distribution of the form

$$P(N(t,2) = N) \cong F_{N(0,1)}(\frac{3.04(N+1) - t}{1.967\sqrt{t}}) - F_{N(0,1)}(\frac{3.04N - t}{1.967\sqrt{t}}), \ N = 0,1,...;$$

d) the expected value and the variance of the number $N(t,2)$ of exceeding by the system the reliability critical state $r = 2$ up to the moment $t, t \geq 0,$ for sufficiently large *t*, approximately are respectively given by

$$H(t,2) = 0.3289t, \ D(t,2) = 0.419t.$$

Further, assuming that the container gantry crane is repaired after the exceeding its reliability critical state $r = 2$ and that the time of the system renovation is not ignored and it has the mean value $\mu_0(2) = 0.0027$ and the standard deviation $\sigma_0(2) = 0.0014$, we obtain the following results:

a) the time $\overline{S}_N(2)$ until the *Nth* exceeding by the system the reliability critical state $r = 2$, for sufficiently large *N*, has approximately normal distribution $N(3.04N + 0.0027(N-1), \sqrt{11.76N - 0.0000019(N-1)})$, i.e.,

$$\overline{F}^{(N)}(t,2) = P(\overline{S}_N(2) < t) \cong F_{N(0,1)}(\frac{t - 3.0427N + 0.0027}{\sqrt{11.760002N - 0.0000019}}), \ t \in (-\infty, \infty);$$

b) the expected value and the variance of the time $\overline{S}_N(2)$ until the *Nth* exceeding by the system the reliability critical state $r = 2$, for sufficiently large *N*, are respectively given by

$$E[\overline{S}_N(2)] \cong 3.04N + 0.0027(N-1), \ D[\overline{S}_N(2)] \cong 11.76N + 0.0000019(N-1);$$

c) the number $\overline{N}(t,2)$ of exceeding by the system the reliability critical state $r = 2$ up to the moment $t, t \geq 0,$ for sufficiently large *t*, has approximately distribution of the form

$$P(\overline{N}(t,2)=N) \cong F_{N(0,1)}(\frac{3.0427(N+1)-t-0.0027}{1.966\sqrt{t+0.0027}})$$

$$-F_{N(0,1)}(\frac{3.0427N-t-0.0027}{1.966\sqrt{t+0.005}}),\ \ N=0,1,...;$$

d) the expected value and the variance of the number $\overline{N}(t,2)$ of exceeding by the system the reliability critical state *r* = 2 up to the moment $t, t \geq 0,$ for sufficiently large *t*, are respectively given by

$$\overline{H}(t,2) \cong 0.329(t+0.0027)\ ,\ \overline{D}(t,2) \cong 0.417(t+0.0027);$$

e) the time $\overline{\overline{S}}_N(2)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution $N(3.0427N, 3.429\sqrt{N})$, i.e.,

$$\overline{\overline{F}}^{(N)}(t,2) = P(\overline{\overline{S}}_N(2) < t) \cong F_{N(0,1)}(\frac{t-3.0427N)}{3.429\sqrt{N}}),\ \ t \in (-\infty, \infty);$$

f) the expected value and the variance of the time $\overline{\overline{S}}_N(2)$ until the *Nth* system's renovation, for sufficiently large *N*, are respectively given by

$$E[\overline{\overline{S}}_N(2)] \cong 3.0427N\ ,\ D[\overline{\overline{S}}_N(2)] \cong 11.76002N\ ;$$

g) the number $\overline{\overline{N}}(t,2)$ of the system's renovations up to the moment $t, t \geq 0,$ for sufficiently large *t*, has approximately distribution of the form

$$P(\overline{\overline{N}}(t,2)=N) \cong F_{N(0,1)}(\frac{3.0427(N+1)-t}{1.966\sqrt{t}})\ -F_{N(0,1)}(\frac{3.0427N-t}{1.966\sqrt{t}}),\ N=0,1,...;$$

h) the expected value and the variance of the number $\overline{\overline{N}}(t,2)$ of system's renovations up to the moment $t, t \geq 0,$ for sufficiently large *t*, are respectively given by

$$\overline{\overline{H}}(t,2) \cong 0.3286t,\ \ \overline{\overline{D}}(t,2) \cong 0.417t;$$

i) the steady availability coefficient of the system at the moment $t, t \geq 0,$ for sufficiently large *t*, is given by

$$A(t,2) \cong 0.9989,\ \ \ t \geq 0;$$

j) the steady availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ for sufficiently large *t*, is given by

$$A(t,\tau,2) \cong 0.329\int_{\tau}^{\infty} \boldsymbol{R}(t,2)dt, \quad t \geq 0, \quad \tau > 0,$$

where $\boldsymbol{R}(t,2)$ is given by (7.97).

6 CONCLUSION

In the paper the multi-state approach to the analysis and evaluation of systems' reliability and risk has been practically applied. The container gantry crane has been considered at varying in time operation conditions. The system reliability structure and its components reliability functions were changing at variable operation conditions. The paper proposed an approach to the solution of practically very important problem of linking the systems' reliability and their operation processes. To involve the interactions between the systems' operation processes and their varying in time reliability structures a semi-markov model of the systems' operation processes and the multi-state system reliability functions were applied. This approach gives practically important in everyday usage tool for reliability evaluation of the systems with changing reliability structures and components reliability characteristics during their operation processes what exemplary was illustrated in its application to the gantry crane.

The characteristics of the gantry crane operation process are of high quality because of the very good statistical data necessary for their estimation. Unfortunately, the reliability characteristics of the gantry crane components are evaluated on non sufficiently exact data coming from experts and concerned with the mean values of the components lifetimes only that because of the complete lack of statistical data about their failures are strongly lowered. Also, the system and its components reliability states are defined on a high level of generality and should be described more precisely. All these inaccuracies causes that the evaluation of the gantry crane reliability, risk and availability characteristics should be consider as an illustration of the proposed approach application.

Acknowledgements

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SAFETY ANALYSIS AND ASSESSMENT IN THE WATER SUPPLY SECTOR

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ABSTRACT

The paper presents a framework for the analysis of performance risk in water supply system (WSS) that can be applied to the entire system or to individual subsystems. It provided a background for the rules of management formulation. The aim of such management is to prepare resources and society for the case of an undesirable event occurrence which causes threat to health, property, environment and infrastructure. The risk elements, which always accompany crisis, have been presented.

1 INTRODUCTION

Safety of the WSS means the ability of the system safely execute its functions in given environment. The measure of WSS safety is risk. The notion of risk was introduced to European law by virtue of the instruction 89/392/EWG from 1989 on the adaptation of the state members regulations concerning machines.

The important problem is the exploitation of existing water supply systems (WSS) which should take into account the minimization of water losses, operational and safety reliability. Water supply providers seek to provide their customer with high-quality drinking water at all times. However this can sometimes be challenging because of changing raw water quality or problems with treatment and distribution. Opinions on WSS safety change along with the progress of science and technology.

The WSS safety management is an operator managerial activity to establish the aims (counteraction against lack of water or its bad quality, threatening health of municipal water pipe users) and to supervise their accomplishment using processes, information resources in the given operating conditions, in compliance with the valid law and with economic justification (Tchórzewska-Cieślak 2009). The transition to an explicit risk management philosophy within the water utility sector is reflected in recent revisions to the World Health Organization's (WHO) Guidelines for Drinking Water Quality (WHO 2003).

Nowadays safety of the technical and environmental systems functioning becomes a worldwide scientific tendency. In Poland, a ministerial document of National Frame Program has been issued and one of its strategic research areas is "safety". The priority directions of scientific research are, among others, crisis management, early warning systems in crisis situations and so on. In Europe, the program called GMES (Global Monitoring for the Environmental and Security) works. Also the sixth frame European Union program introduces those subjects in the priority "Information Society Technologies: 2.5.12 IST for Environmental Risk Management".

If the undesirable events have violent character and lead to some negative consequences connected with a serious failure, tragedy, catastrophe or disaster (e.g., floods, earthquakes, hurricanes, fires, terrorist attacks), then the relevant risk is the so-called "hard risk" type. The other type of risk is the so-called "soft risk", which is accompanied by slow and often combined actions of undesirable events (e.g., bad condition of municipal water purity, air pollution, noise), that, after some time, lead to irreversible changes in human health.

For purpose of this paper failure is defined as the event in which the system fails to functions with respect to its desired objectives. Safety of the WSS means the ability of the system safely execute its functions in given environment. The measure of WSS safety is risk (Rak 2009, Tchórzewska-Cieślak 2010).

The main objective of this paper is to present the issue of risk management in the water supply sector. The paper explores the basic concepts related to water supply safety and presents a new method for risk analysis.

2 SAFETY AND RISK MANAGEMENT

Under their current philosophy drinking water infrastructure decision-makers attempt to manage the risk of systems failure through deterministic trial and error approaches that provide inefficient solutions (Pollard et al. 2008). Decision-makers and engineers are increasingly using modeling software to determine the effect of human activities on water quality. There are many surface water quality modeling and algorithms software in the public and private domain.

A special case of the WSS safety management is system management in a crisis situation. The methodology to determine the crisis management time in the WSS, connected with a shortage of drinking water was shown in table 1 (Rak & Pietrucha 2008).

Table 1. Supplying of water in a crisis situation

Needs are fulfilled	Tolerable water shortage	Non-tolerable water shortage	Disaster
100% Q	~ 70% ≤ Q < 100%	~ 30% ≤ Q < 70%	0% ≤ Q < 30%
Normal operating time	Response time	Crisis management time	

where Q is water system capacity.

A crisis situation is characterized by the occurrence of undesirable events and processes, as well as their accumulation, which finally lead to a threat that the system will not be able to work autonomously or are not favourable for the system development. The undesirable phenomena (events, processes) which are a cause for the crisis are divided into internal (a source is inside the system) and external (a source is in the system surroundings).

From decision-making point of view the features of a crisis situation are a relatively short decision time, a low degree of predictability since the undesirable events often occur in an unexpected way (surprise), a high risk level, and fear resulting from the extreme acting conditions (panic, fright).

From crisis management point of view the character of a crisis situation lies in a permanent disturbance in working, a real or colourable loss of control, threat to the execution of the basic purposes, threat of a serious failure or disaster.

Taking into account the anti-crisis strategies related to a phase of crisis situation we can distinguish active actions (anticipated and preventive) and reactive actions (repulsive and liquidation). Considering the crisis extent and intensity and the possibility of overcoming it, we can distinguish:

- a potential crisis – the first symptoms of crisis situation can be seen,
- a hidden crisis – problems in system operation can be seen, it is impossible to identify their reasons,
- a hot crisis – consequences of intensified difficulties in system operation are noticed, system safety is threatened, there is still a possibility of keeping control of crisis situation,
- a burning crisis – an accumulation of threats occurs and progress of destructive phenomena is out of control, loss of system reliability and safety, system environment is not under control any more, there is no possibility of getting crisis situation under control.

A division according to the crisis consequences extent is as follows: global consequences, affect the whole system, and local consequences, affect objects or subsystems.

The aim of water consumers threat identification is to show the type of substance existing in drinking water, however the evaluation of threat level should be based on showing its harmful impact on human health and classifying the substances on the basis of all the available data. The impact of the particular substances on human health is determined by appropriate experts (doctors, chemists, biochemists, and microbiologist) on the basis laboratory and clinical studies, as well as from their experience (Hrudey 2001, Johanson 2008). Decisions on managing risk, if they are to be effective, need to be active rather than reactive and well structured. Risk management frameworks set out the relationship between the processes of risk identification, evaluation and management. They can be regarded as 'route maps' for decision makers (Ansell 1994).

Among the most important components of sustainable management strategies for WSS is the ability to integrate risk analysis and asset management decision-support systems, as well as the ability to incorporate in the analysis financial and socio-political parameters that are associated with the networks in study (Demotier et al. 2002, Ezell et al. 2000, Mac Gillivray et al. 2007, Quimpo & Wu 1997, Pollard et al. 2008).

Risk management in waterworks responsible for right water-pipe network operating is a formal program containing internal procedures which main purpose is to protect water consumers, environment, as well as waterworks interests (financial and personal). The water industry is undergoing a significant shift in its approach to risk management to one that is increasingly explicit and better integrated with other business processes. Risk management strategies and techniques traditionally applied to occupational health and safety and public health protection are now seeing broader application for asset management, watershed protection and network operation (Garnderr 2008, Rogers et al. 2008, Sadig et al. 2006, Shinstine 2002, Tanyimboh 1999).

It is very important for waterworks to identify risk correctly and to divide it into consumer risk and water producer risk. It allows choosing the right method for calculating different types of risks. The correct WSS risk management process should contain suitable organizational procedures within the framework of regular waterworks activity, the WSS operation technical control and supervisory system, a system of automatic transfer and data processing about WSS elements operation. The key role in this process is played by a system operator, whose main purpose is:

- to implement the reliability and safety management system,
- to operate the WSS according to valid regulations and in a way which ensures its long and reliable operation,
- to execute a program of undesirable events prevention,
- to develop failure scenarios for water supply in crisis situations,
- to develop a complex system of information about the possible threats for water consumers.

Such type of WSS risk management optimises an operation of particular WSS devices (e.g. parameters of operation of water pipe pumping stations which cooperate with network tanks), and the work of the whole system.

3 RISK ANALYSIS METHODOLOGY

3.1 Failures in the WSS

A failure in the WSS is a complex problem, every time it occurs, the primary reasons behind it must be analyzed carefully. Failure can be grouped into either structural failure or performance failure. The failures of the WSS which occur most often are the following (Craun & Calderon 2001, Franks 1999, Hastak & Baim 2001, Tchórzewska-Cieślak 2010):

- incidental contamination of water intakes, eg. chemical, biological contamination,

- failures in water treatment stations, eg. disturbances in the technological process of water treatment,
- failures in transit, main and distributional pipelines, which can result in the secondary water contamination in water-pipe network, as well as breaks or lack of water supply to the receivers, or the drop of water pressure in the network,
- deterioration in water quality in water-pipe network as a result of unfavourable hydraulic conditions (low speed of water flow, pipelines technical conditions),
- failures in power supply, which can cause a lack of the possibility to operate the particular subsystems and elements of the WSS and even the whole system.

The factors which form the probability that the negative consequences occur are, among others, the following:

- the probability that the undesirable event occurs,
- frequency and a degree of exposure,
- the possibility of avoidance or minimization of the negative consequences.

Risk assessment is a process consisting of a number of the systematic steps, in which the study of different kinds of threats connected with the WSS operating is carried out. The basic purpose of this kind of activities is to collect the information necessary to estimate the safety of the system. Risk assessment should contain:

- establishment of a ranking of the undesirable events,
- determination of the level (value) of risk,
- proposal of the activities aimed at risk minimisation,
- establishment of time after which the risk can obtain its critical value as a result of different processes, eg. materials ageing.

Risk assessment includes the so called risk analysis, which is the process aimed at the determination of the consequences of failures (undesirable events) in the WSS, their extend, sources of their occurrence and the assessment of the risk levels (Aven 2010, Kaplana & Garrick 1981, Zio 2009). Haimes (1998, 2009) suggests that risk assessment concerns its reasons, as well as its likelihood and consequences. Hastak and Baim (2001) define infrastructure risk as a product of the probability (likelihood) of system failure (p) and costs associated with its repair (economic-value) (C).

Drinking water infrastructure system uncertainty or risk is defined as the likelihood or probability that the drinking water service fails to provide water on-demand to its customers (Tchórzewska-Cieślak 2010).

The purpose of this paper is to present the risk analysis method for drinking water infrastructure.

Risk (r) is a function of three parameters (Rak 2009, Tchórzewska-Cieślak 2010): the probability P_{Si} that *i* representative emergency scenario S_i occurs, the magnitude of losses C_{Si} caused by *i* representative emergency scenario S_i and the consumers protection O_{Si} against *i* representative emergency scenario S_i, r =f (P_{Si}, C_{Si}, O_{Si}). In this way risk can be calculated from the equation (1):

$$r = \frac{P_{Si} \cdot C_{Si}}{O_{Si}} \quad (1)$$

where P_{Si} is the probability of S_i, C_{Si} is the degree, or point weight, of consequences connected with S_i for water consumers, O_{Si} is the level, or point weight, of protection of water consumers against S_i.

For every situation, a score is assigned to the parameters P_{Si}, C_{Si} and O_{Si}, according to the following point scale:

- low (L)=1,
- medium (M)=2,
- high (H)=3.

In this way, we obtain risk matrix and a point scale to measure risk: tolerable, controlled and unacceptable, in a numerical form, within the range [0.33÷9], according equation 1.

Failures in the WSS can be a consequence of errors made during design:

– errors in water-pipe network layout (ground conditions wrongly examined, an incorrect route for the water pipeline, the economic activity of a third party was not taken into account), wrong conception of water-pipe network geometry and structure,
– errors in network hydraulic calculations (an incorrect water-pipe diameter, incorrect pressure in network, wrong layout of water-pipe tanks), errors in a conception of the whole WSS control.

Errors made during construction:
– deviations from the design and the rules of correct construction, according to valid regulations, as concerns the technology of pipe laying, connections of the individual pipe sections; covering pipes for the passages going under and through the obstacles are not installed, improper anticorrosion protection (passive and active), badly performed pressure test and other procedures,

Errors made during operation:
– incorrect operating procedures, a lack of water pipeline operation monitoring,
– the scenarios for the emergency water supply were not taken into account,
– incoherent protecting and warning system for water quality,
– lack of programme to classify the network segment requiring the repair, lack of programme to obtain, process and storing the data on failures, their causes and consequences and records of data about failures.

3.2 The risk of design

The two-parameter matrix for risk assessment was proposed. The risk of design (r_d) can be calculated from the modified equation (1), we obtain equation (2) (Tchórzewska-Cieślak et al. 2011):

$$r_d = P_d \cdot C_d \qquad (2)$$

where P_d – point weight related to the probability of design error, C_d – point weight related to the size of possible losses.

Point weights associated with P_d are the following:
– L = 1 – a renowned design office with a quality certificate, having completed projects in the list of reference, a design is made by means of tested computer programs,
– M = 2 – a design office having the required license to design and the list of references,
– H = 3– a person with experience in designing segments of water pipe network.

Point weights associated with C_d are the following:
– L = 1 – financial loss up to 10^4 EUR,
– M = 2 – financial loss from 10^4 EUR to 10^5 EUR,
– H = 3 – financial loss above 10^5 EUR.

In table 2 the two-parameter risk matrix was presented.

Table 2. The two-parameter risk matrix at the stage of water-pipe network design.

C_d	P_d		
	L = 1	M = 2	H = 3
L = 1	1	2	3
M = 2	2	4	6
H = 3	3	6	9

The individual risk categories are the following:
– tolerable [1 ÷ 2],
– controlled [3 ÷ 4],
– unacceptable [6÷ 9].

3.3 The risk of construction

The three-parameter matrix for risk assessment was proposed. The risk of construction (r_c) can be calculated from the modified equation (1), we obtain equation (3) (Tchórzewska-Cieślak et al. 2011):

$$r_C = \frac{P_C \cdot C_C}{O_C} \tag{3}$$

where P_c – a point weight related to the probability of error made at construction, O_c – a point weight related to the probability of the detection of error, C_c – a point weight related to the size of possible losses.

Point weights associated with P_c are the following:

- L = 1 – a building company is certified ISO 9000 and has completed investments in the list of reference, procedures associated with the receipt of investment are obeyed, laying pipes according to the best available technology,
- M = 2 – a building company has completed investments in the list of reference, verification of the specification of materials and procedures for the receipt are performed,
- H = 3 – a building company enters the market of water-pipe network construction, lack of experience in this field.

Point weights associated with O_c are the following:

- H = 3 – procedures for pressure tests are scrupulously obeyed with the use of modern equipment, there are no derogations in relation to implementing the project, execution is supervised by an investor,
- M = 2 – procedures for the receipt of investment are implemented,
- L = 1 – questionable quality of the trials connected with the receipt of investment, frequent derogations from the design assumptions.

Point weights associated with C_c are the following:

- L = 1 – a financial loss up to 104 EUR,
- M = 2 – a financial loss from 104 EUR to 105 EUR,
- H = 3 – a financial loss above 105 EUR.

In table 3 the three-parameter risk matrix was presented, according to equation 3. The weighs of individual parameters presented above were established on the basis of works (Rak 2009, Rak & Tchórzewska 2006).

Table 3. The three-parameter risk matrix at the stage of construction

C_c	P_c								
	L =1			M=2			H=3		
	O_c								
	H 3	M 2	L 1	H 3	M 2	L 1	H 3	M 2	L 1
L = 1	0.33	0.5	1	0.67	1	2	1	1.5	3
M = 2	0.67	1	2	1.33	2	4	2	3	6
H = 3	1	1.5	3	2	3	6	3	4.5	9

The individual risk categories are the following:

- tolerable [0.33 ÷ 2],
- controlled [3 ÷ 4],
- unacceptable [4.5÷ 9].

3.4 The risk of operation

The four-parameter matrix for risk assessment was proposed. The risk of operation (r_o) can be calculated from the modified equation (1), we obtain equation (4):

$$r_O = \frac{S_O \cdot I_O \cdot U_O}{O_O} \tag{4}$$

where S_o – a point weight associated with a type of water-pipe network, I_o – a point weight associated with the failure rate λ [failure/km year], U_o – a point weight associated with the difficulty to repair damages, O_o – a point weight related to protection of water-pipe network operation.

Point weights associated with S_o are the following:

- L = 1 – household connections,
- M = 2 – distributional network,
- H = 3 – main network.

Point weights associated with I_o are the following:

- L = 1 – the failure rate $\lambda < 0.5$ failure./km year,
- M = 2 – 0.5 failure/ km year$\leq \lambda \leq$ 1.0 failure/ km year,
- H = 3 - $\lambda > 1.0$ failure/ km year

Point weights associated with U_o are the following:

- L = 1 – failure in the pipeline in not urbanized area, repair brigades are organized and equipped appropriately and they are in full readiness for 24 hours,
- M = 2 – failure in the pipeline in the pedestrian lane, basic equipment to repair a failure, one shift work.
- H = 3 – failure in the pipeline in the vehicles lane (streets), lack of mechanized equipment to repair a failure.

Point weights associated with O_o are the following:

- H = 3 – special, above standard, full monitoring of water pipe network by measuring the water pressure and flow rate of water, possession of a specialized apparatus to detect water leaks by acoustic methods, unrestricted communication with the public through the phone line active 24 hours, monitoring of water quality in water network by means of protection and warning system. The network is fully inventoried, an exploiter has numerical maps of water-pipe network,
- M = 2 – standard, simplified monitoring of water- pipe network with the use of pressure measurement, inability to respond to small water leaks, water quality tests in water- pipe network are conducted,
- L = 3 – none, lack of monitoring of water-pipe network and water quality. There are no current inventory of water-pipe network.

In table 4 the four-parameter risk matrix was presented. The individual risk categories are the following:

- tolerable [0.33 ÷ 3],
- controlled [4 ÷ 8],
- unacceptable [9÷ 27].

Table 4. The four-parameter risk in matrix at the stage of water-pipe network operation

U_o	Type of water-pipe network – $S_o = 1$								
	Household connections – $L_o = 1$								
	Failure rate – I_o								
	L = 1			M = 2			H = 3		
	Protection – O_o								
	H = 3	M = 2	L = 1	H = 3	M = 2	L = 1	H = 3	M = 2	L = 1
L=1	LLLH 0.33	LLLM 0.5	LLLL 1	LMLH 0.66	LMLM 1	LMLL 2	LHLH 1	LHLM 1.5	LHLL 3
M=2	LLMH 0.66	LLMM 1	LLML 2	LMMH 1.33	LMMM 2	LMML 4	LHMH 2	LHMM 3	LHML 6
H=3	LLHH 1.5	LLHM 1.5	LLHM 3	LMHH 2	LMHM 3	LMHL 6	LHHH 3	LHHM 4.5	LHHL 9
U_o	Type of water-pipe network – $S_o = 2$								
	Distribution – $M_o = 2$								
	Failure rate – I_o								
	L = 1			M = 2			H = 3		
	Protection – O_o								
	H = 3	M = 2	L = 1	H = 3	M = 2	L = 1	H = 3	M = 2	L = 1
L=1	MLLH 0.66	MLLM 1	MLLL 2	MMLH 1.33	MMLM 2	MMLL 4	MHLH 2	MHLM 3	MHLL 6
M=2	MLMH 1.33	MLMM 2	MLML 4	MMMH 2.66	MMMM 4	MMML 8	MHMH 4	MHMM 6	MHML 12
H=3	MLHH 2	MLHM 3	MLHL 6	MMHH 4	MMHM 6	MMHL 12	MHHH 6	MHHM 9	MHHL 18
U_o	Type of water-pipe network – $S_o = 3$								
	Main – $H_o = 3$								
	Failure rate – I_o								
	L = 1			M = 2			H = 3		
	Protection – O_o								
	H = 3	M = 2	L = 1	H = 3	M = 2	L = 1	H = 3	M = 2	L = 1
L=1	HLLH 1	HLLM 1.5	HLLL 3	HMLH 2	HMLM 3	HMLL 6	HHLH 3	HHLM 4.5	HHLL 9
M=2	HLMH 2	HLMM 3	HLML 6	HMMH 4	HMMM 6	HMML 12	HHMH 6	HHMM 9	HHM 18
H=3	HLHH 3	HLHM 4.5	HLHL 9	HMHH 6	HMHM 9	HMHL 18	HHHH 9	HHHM 13.5	HHHL 27

The integrated risk is a sum of the risks at the stages of design r_d, construction r_c and operation r_o. To get the individual risks compatible with each other we should multiply them by the weights W_i, whose values are shown in table 5.

The integrated risk is determined from the modified equation (5).

$$r = W_i \cdot r_d + W_j \cdot r_c + W_k \cdot r_o \tag{5}$$

It is included in the range [1.0 ÷ 81].

The individual categories of integrated risk are the following:

- tolerable [1.0 ÷ 9.0],

- controlled [12.0 ÷ 24],
- unacceptable [27 ÷ 81].

Table 5.Values of weights

Risk	r_d			r_c			r_o		
	W_i			W_j			W_k		
	low	high	medium	low	high	medium	low	high	medium
tolerable	0.33	1.5	0.9	1.0	1.5	1.25	1.0	1.0	1.0
controlled	1.33	2.0	1.67	1.33	2.0	1.67	1.0	1.0	1.0
unacceptable	1.5	3.0	2.25	2.0	3.0	2.5	1.0	1.0	1.0

4 DISCUSSIONS

The presented work is a result of the five-year cooperation with water authorities as part of research grants. At present authors are processing a development research project, a result of which will be a program of the risk management in the waterworks company. Suggested methods, will constitute the basis for so-called Water Safety Plans of (recommended by WHO) which will be compulsory in waterworks practice. The risk analysis in Water Safety Plans is the basis of ensuring water consumers safety.

Data received from water authorities are derived from exploiters of the water supply system. Criteria of the risk assessing suggested in the work were based on the mentioned above information. Water Safety Plans, and methods of the risk management in water supply system are included in works: (Craun & Calderon 2001, Demotier et al. 2002, Ezell et al. 2000, Hipel et al. 2003, Johanson 2008, Mac Gillivray 2007, Rak 2009, Tanyimboh 1999, WHO 2002, 2003).

An important challenge is to define the tolerable risk level, the so-called ALARP (As Low As is Reasonably Practicable), which means that risk level should be as low as it is reasonably practicable. The ALARP principle was first introduced in Great Britain, where the unacceptable (impermissible) value of risk of death for the individual worker was determined to be r=0.001 and the risk of death for the public was determined to be r=0.0001. Risk reducing process should take into account a cost benefit analysis. Such risk level should be determined at which costs of its further lowering are disproportional high. Health and Safety Executive, directives introduce a notion “the cost for preventing a fatality” which is estimated, according to the mentioned above directives, at about 1mln GBP (HSE book 2001).

Danger and hazard are the factors that determine the magnitude of the risk. Danger is considered a cause of loss. It is characterized by some kind of arranged time sequence of successive phases. In the first phase threat appears, which creates danger (e.g. an incidental water pollution in a source). In the second phase danger becomes real (e.g. polluted water appears in the distribution subsystem). In the third phase the effects of real danger are revealed (e.g. water consumers’ gastric problems). Hazard is identified as a set of conditions and factors that have a direct impact on the second phase of danger. The scales of parameters that describe risk on the different levels of its occurrence should be simple, which allows risk assessment and classification for every discussed case. The method has an expert character and is used to pre-estimate the risk associated with the WSS operation. In relation to specialist expertise made by experts, describing the identified water-pipe failures, which are superior, this method should be regarded as preliminary material. The detailed analysis of the risk associated with different stages of the WSS operation is important. Determination of the size of the risk associated with the design, construction and operation and its sum allows the appropriate reaction at each stage, and consequently contributes to reducing the risk of the WSS operation. A lot of experience gained from the analysis of risk associated with the WSS operation can be already generalized at the level of research and passed in the form of publication. The knowledge about risk does not have to be achieved by means of individual trial and error method. Risk management requires its identification, is directly associated with the control of quality and reliability of technical systems. The latter includes all actions which result is a product (article, object, subsystem, system) of the required quality and reliability. We still deal with the mistaken stereotype that the technical control in execution phase will ensure the required quality and reliability. Modern and perspective becomes a trend that the quality control and reliability control from the design phase, through construction, to operation of technical systems lead to a reduction of risk associated with their operation (Hipel et al. 2003).

In table 6 the quantitative and qualitative categories of the consequences connected with the three level risk gradation are presented.

Table 6. The quantitative and qualitative limits of risk connected with poor drinking water quality in public supply systems, related to 1 year

Consequence category	Description of consequences	Tolerable risk	Controlled risk	Unacceptable risk
Insignificant	Incidental difficulties that are not a threat to health, lack of consumers complaints	$<10^{-3}$	$10^{-1} \div 10^{-3}$	$>10^{-1}$
Marginal	Perceptible organoleptic changes, individual consumer complaints	$<10^{-4}$	$10^{-2} \div 10^{-4}$	$>10^{-2}$
Significant	organoleptic changes are significant, numerous consumers complaints, reports in local media, water can be used after 10 minutes boiling	$<10^{-5}$	$10^{-3} \div 10^{-5}$	$>10^{-3}$
Serious	mass gastric problems, relevant sanitary inspector turns off water pipe, toxic effects in pollution indicators, large number of reports in local media, general information in national media	$<10^{-6}$	$10^{-4} \div 10^{-6}$	$>10^{-4}$
Catastrophic	mass hospitalisation as a result of health complications, deaths, front news in national media	$<10^{-7}$	$10^{-5} \div 10^{-7}$	$>10^{-5}$

In crisis situation drinking water supplied to water pipes should be taken, if possible, from the underground water intakes. The other intakes become the reserve intakes. Water pipe should have the possibility to cut off water intakes with the operational possibility and to use the whole system or its fragments, e.g. water pipe network, water intake, transit water pipes, activate alternative water treatment technology (e.g. periodical dosage of active carbon in a powdery form), increase dosage of disinfecting agent, supply water bypassing Water Treatment Plant. If water pipe is inactivated and in the areas without water pipe network, water is supplied from emergency wells. When a number of the emergency wells is too law or their layout is unfavorable one should predict water delivery by tanks or water-carts. Water pipes and emergency wells should be prepared to get energy from generators, they should be equipped with generators which can start pumps and water supply during the limited deliveries. Fuel reserve should be enough for 400 hrs, however for not less than 200 hrs of generating sets operating. Water requirement in crisis situation should be established for all the municipal water pipes and for villages without water pipe network. It should be assured from water pipes and emergency wells, and also from industrial intakes, if necessary.

One can distinguish two kinds of water requirements in crisis situation (Rak 2009):

- necessary water quantity (for a few weeks time): population - 15 dm^3/person, day,
- minimum water quantity (for a few days time): population - 7.5 dm^3/person, day.

5 CONCLUSIONS

- To ensure WSS safety operating it is required to use the newest theoretical solutions, the basic category of which, nowadays, becomes the term risk. It comprises the evaluation of the relation between the occurring threats and the safety and protective barriers.
- It can be seen that there is a trend in legislation to ensure system safety through the implementation of the standard risk analysis and evaluation for the technological systems operation.
- The directions for further studies are determined by Frame Program which includes project V - safety. The following directions for operations, among others, are named: crisis management, information systems safety. This program is an extension of the Sixth Frame Program (FP6), the main aim of which is to manage risk situations through early warning and threats monitoring.

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COMPLETE CALCULATION OF DISCONNECTION PROBABILITY IN PLANAR GRAPHS

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ABSTRACT

In this paper complete asymptotic formulas for an disconnection probability in random planar graphs with high reliable arcs are obtained. A definition of coefficients in these formulas have geometric complexity by a number of arcs. But a consideration of planar graphs and dual graphs allow to solve this problem with no more than cubic complexity by a number of graph faces.

1. INTRODUCTION

A problem of a calculation of a connectivity probability in random graphs with unreliable arcs is considered in manifold articles and monographs devoted to the reliability theory [1] - [4] etc. It occurs in an analysis of electro technical devices, computer networks and has manifold applications to a research of honeycomb structures [5], [6], and nanosystems [7] – [9].

In [10] – [12] upper and low estimates of the connectivity probability are constructed for general type networks on a base of maximal systems of disjoint frames. For small numbers of arcs in [13] accelerated algorithms of a calculation of reliability polynomial coefficients are constructed. These algorithms showed good results in a comparison with direct calculations. In [14] this problem is solved using the Monte-Carlo method with some combinatory formulas. To calculate the connectivity probability in rectangle lattices the transfer matrix method is used [15]. But an increasing of arcs number leads to large complexity and so it is worthy to develop asymptotic methods.

In this paper an analog of the Burtin-Pittel asymptotic formula [16] for disconnection probability of random graph with high reliable arcs is constructed. Its parameters are the minimal number D of arcs in cross sections and the number C of cross sections with volume D . A definition of D for a random port demands to find a maximal flow and has cubic complexity [17]. But a definition of C has geometric complexity.

So we consider widely used planar graphs for which we prove that a definition of coefficients D, C has no more than cubic complexity by a number of faces. And there is a lot of graphs [18, Ch. IV] for which this complexity is linear and smaller. These results are based on a consideration of dual graphs [19, [20], in which cross sections generate cycles [21], [22]. Numerical experiment confirms an accuracy and a performance of suggested method.

2. ASYMPTOTIC FORMULAS

Consider non oriented connected graph G with finite sets of nodes U and of arcs W . Suppose that each pair of nodes in G may be connected with no more than single arc and there are not loops. Denote $\mathcal{L}(u,v)$ the set of all cross sections in G which divide nodes $u,v\in U,\ u\neq v,$ and define the set $\mathcal{L}=\bigcup_{u\neq v}\mathcal{L}(u,v)$ of all cross sections in G . Graph cross section is such set of arcs which deletion makes the graph non connected. Put $d(L)$ a number of arcs in the cross section L and

$$D(u,v)=\min\left(d\left(L\right):L\in \mathcal{L}\left(u,v\right)\right),\ D=\min_{u\neq v}D\left(u,v\right),\ \mathcal{L}_{*}=\left\{L\in \mathcal{L}:d\left(L\right)=D\right\},$$

C - is a number of cross sections in the set $\mathcal{L}_{*}$. Suppose that graph arcs work independently with probabilities $p(w), w\in W$.

Theorem 1. If $\overline{p}(w)=1-p\left(w\right)=h,\ w\in W$, then graph disconnection probability

$$\overline{P}\sim Ch^{D},\ h\to 0. \qquad (1)$$

Theorem 2. If $\overline{p}(w)\sim c_{w}h,\ h\to 0,\ w\in W$, then

$$\overline{P}\sim \sum_{L\in \mathcal{L}_{*}} h^{D}\prod_{w\in L} c_{w},\ w\in W,\ h\to 0.$$

Theorems 1, 2 are generalizations of the Burtin-Pittel asymptotic formula [16].

3. CALCULATION OF CONSTANTS *C, D*

Theorem 3. The set of arcs which do not belong to any cycle coincides with the set of cross sections $\mathcal{L}_{*}$ and $D=1$.

Assume that the graph G is planar and its each arc belongs to some cycle. Arcs of planar graph divide a plane into faces [19,Ch. 1]. }. Confront the graph G its dual graph G^{*}. Each face z in G accords the node z^{*} in G^{*}, each arc w in G belonging faces z_1, z_2 accords an arc w^{*} connecting nodes z_1^{*}, z_2^{*} in G.

A set of arcs $\{w_1,...,w_d\}$ in G accords some subgraph R^{*} in G^{*}. For its definition each arc w_i, $1\le i\le d$, accords a pair of faces which contain this arc. Then this pair of faces accords a pair of nodes in R^{*} connected by the arc w^{*}. Say that the graph R^{*} is generated by the set of arcs $\{w_1,...,w_d\}$.

Theorem 4. The set of cross sections $\mathcal{L}_{*}$ consists of all sets of arcs $\{w_1,...,w_d\}$ which generate cycles with minimal length D^{*} in the dual graph G^{*} and $D=D^{*}\le 5$.

This statement is a corollary of the Whithney theorem and the Euler formula [19, Theorem 1.5, Corollary of Theorem 1.6], [20]. In fig. 1 there are examples of planar graphs arranged on a sphere with $D=4,\ D=5$.

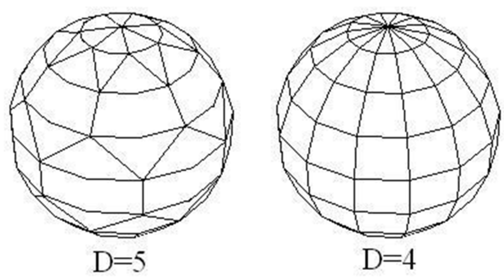

Fig. 1.

Suppose that elements a_{ij} of the matrix *A* define a number of arcs which belong to $z_i\bigcap z_j$, $i\neq j$, $a_{ii}=0$, in the planar graph G with *n* faces and *m* arcs and without loops and multiple arcs.

Corollary 1. If $\max_{1\le i,j\le n} a_{ij} > 1$, then

$$D = 2\,,\ C = \frac{1}{4}\sum_{1\le i,j\le n} a_{ij}\left(a_{ij} - 1\right) \tag{2}$$

and a complexity of constants D, C calculation by the formula (2) is squared by *n*. If for $i < j$ $a_{ij} > 1$ only for $j = n$ then this complexity is linear.

Define c_i the number of cycles with length i, $i = 3,4,5,$ in G^*. Assume that all cycles $u_1 \to u_2 \to \ldots \to u_k \to u_1$, consist of same set of nodes $\{u_1,\ldots,u_k\}$ and differ by an initial node u_1 and by a direction of a bypass coincide. Elements of a power A^l, $l > 1,$ of a matrix *A* denote by $a_{ij}^{(l)}$.

Corollary 2. If $\max_{1\le i,j\le n} a_{ij} = 1$, then

$$D = \min\left(i : c_i > 0, i = 3,4,5\right),\ C = c_D\ , \tag{3}$$

$$c_3 = \frac{1}{6} trA^3\ ,\ c_4 = \frac{1}{8}\left(trA^4 - 2m - 2\sum_{1\le i\ne j\le n} a_{ij}^{(2)}\right),$$

$$c_5 = \frac{1}{8}\left(trA^5 - 5trA^3 - 5\sum_{i=1}^{n}\left(\sum_{j=1}^{n} a_{ij} - 2\right)a_{ij}^{(3)}\right).$$

Complexity of the constants D, C calculation using the formula (3) is cubic by n. The formulas of c_3, c_4, c_5 calculation are obtained in [21], see also [22, Formulas (16), (17)].

Consider a connected graph G' which consists of plane faces in three dimensional space. Suppose that each pair of faces has not joint points or has joint node or has joint arc and each arc belongs at least to two faces. Take a set of arcs $\{w_i,\ 1\le i\le d\}$ from G' and confront each arc w_i a pair of faces z_i, z^i, which contain this arc. Then the set of arcs $\{w_1,\ldots,w_d\}$ accords some (non unique) graph Γ_d with the nodes $z_i, z^i, 1\le i\le d,$ and arcs $\{w_1,\ldots,w_d\}$ which connect these nodes.

Theorem 5. If the graph Γ_d is acyclic then the set of arcs $\{w_1,\ldots,w_d\}$ which generates it is not cross section in G'.

Corollary 3. Suppose that the set $\mathcal{L}'$ of arcs sets which generate cycles with minimal length D^* and which are cross sections in G' is not empty. Then $D = D^*$, $\mathcal{L}_* = \mathcal{L}'$.

4. EXAMPLES

Results of the number D definition and an enumeration of cross sections with minimal volume are based on listed theorems and simple geometric constructions.

Example 1. On fig. 2 there are examples of planar graphs with representatives of cross sections from the set $\mathcal{L}_*$:

1) an integer rectangle with the length *M* an integer rectangle with the length *N* ($\mathcal{L}_*$ consists of arcs pairs connected with angle nodes),

2 a honeycomb structure ($\mathcal{L}_*$ consists of all possible pairs of arcs which belong to internal and external faces simultaneously),

3) a tube which is constructed by a gluing of opposite sides (with a length M) of integer rectangle ($\mathcal{L}_*$ consists of arcs triplets which have common butt node), if N>3.

Fig. 2. Planar graphs with cross sections dedicated by bold type.

Example 2. On fig. 3 there are graphs with examples of their cross sections from the set $\mathcal{L}_*$:

1) a graph constructed from integer rectangle by a gluing of pairs of its opposite sides ($\mathcal{L}_*$ consists of arcs quads which have common node),
2) a graph constructed from unit cubes with integer coordinates of their nodes ($\mathcal{L}_*$ consists of arcs triplets which contain a cube node, in this node the cube does not intersect or has only common node with another cube).

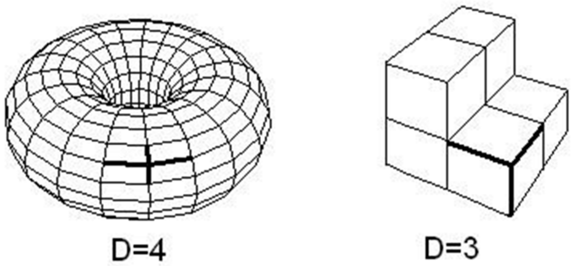

Fig. 3. Graph G' with dedicated cross sections.

5. NUMERICAL EXPERIMENT

Calculate the disconnection probability of honeycomb structure (fig. 1, in center) using Theorem 1 and Corollary 1 and by the Monte-Carlo method with 10^6 realizations. Failure probability of each arc is 0.005. Results of calculations are represented in the table. Time of calculations by asymptotic method is few seconds and by the Monte-Carlo method is some hours.

Size structure	Asymptotic method	Monte-Carlo method	Relative error
2×2	0.00045	0.000439	2.4 %
3×3	0.00055	0.000526	4.3 %
3×4	0.00060	0.000579	3.5 %
3×5	0.00065	0.000621	4.5 %
4×4	0.00065	0.000619	4.8 %
5×5	0.00075	0.000732	2.4 %

The author thanks A.S. Losev for numerical experiment realization.

6. PROOFS OF MAIN STSTEMENTS

Proof of Theorem 1. Suppose that V_L is a random event that all arcs in cross section L fail. Then

$$\overline{P} = P\left(\left(\bigcup_{L\in\mathcal{L}_*} V_L\right)\bigcup\left(\bigcup_{L\in\mathcal{L}\setminus\mathcal{L}_*} V_L\right)\right) \sim P\left(\bigcup_{L\in\mathcal{L}_*} V_L\right), h\to 0.$$

As $P(V_L)=o(h^D)$, $L\in\mathcal{L}\setminus\mathcal{L}_*, h\to 0$, so

$$P\left(\bigcup_{L\in\mathcal{L}_*} V_L\right)\sim Ch^D, h\to 0.$$

Proof of Theorem 5. Suppose that arcs set $\{w_1,...,w_d\}$ from the graph G generates acyclic graph R^*. Prove that each arc $w_i, 1\le i\le d,$ may by bypassed in G by a way which does not contain arcs of this set.

The subgraph R^* consists of trees $S_i^*,...,S_m^*$ which do not connect with each other. Arrange each tree S_i^*, $1\le i\le m,$ on a plane so that in each node z^* arcs connected with this node follow each other as their pre images on the face z if we bypass this face in some direction. Confront each tree S_i^* closed way which bypasses once all its arcs from both sides, $1\le i\le m$ (fig. 4).

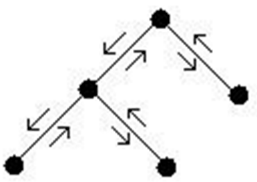

Fig. 4. Bypass of tree arcs.

Accord the way Γ_i^* bypassing tree S_i^* arcs a closed way Γ_i which passes in the graph G through all nodes of arcs $\{w_1,...,w_d\}$, which generate the tree S_i^* (fig. 5). The way Γ_i has not arcs from the set $\{w_1,...,w_d\}$. Consequently each arc from $\{w_1,...,w_d\}$ may be bypassed in G by a way which does not contain arcs from this set. So the set $\{w_1,...,w_d\}$ from the graph G, $d\le D^*$, which does not generate a cycle in G^*, does not belong to the set of cross sections $\mathcal{L}_*$.

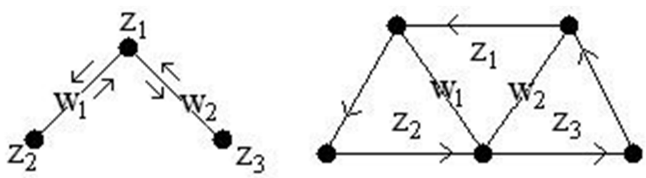

Fig. 5. Bypassing of arcs in a tree and in G .

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THE AMENDMENT TO AMDAHL ` S THE LAW

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INTRODUCTION

Law Amdahl, sometimes also law Amdahl-Uer, shows restriction of growth of productivity of the computing system with increase of quantity of calculators. It also is applicable to collective of people solving a problem, admitting parallels decisions between its members. John Amdahl has formulated the law in 1967, having found out idle time in essence, but insuperable restriction under the maintenance on growth of productivity at parallels calculations: «In a case when the problem is divided on some parts, total time of its performance for parallel system can not be less time of performance of the longest fragment ». According to this law, acceleration of performance of the program for the account parallels its instructions on set of calculators is limited to time necessary for performance of its consecutive instructions.

If to assume, that it is necessary to solve some computing problem with computing algorithm which α share from total amount of calculations can be received only by consecutive calculations, and the share $1-\alpha$ can parallels be ideal (that is time of calculation will be in inverse proportion to number of the involved calculators p) then acceleration on the computing system from p processors, in comparison with the uniprocessor decision will not exceed size

$$S_p = \frac{1}{\alpha + \frac{1-\alpha}{p}} . \qquad (1)$$

Law Amdahl shows, that the gain of efficiency of calculations depends on algorithm of a problem and is limited from above for any problem with $\alpha \neq 0$. Not for any problem escalating number of processors in the computing system is meaningful. Moreover, if to take into account time necessary for data transmission between processors of the computing system dependence of time of calculations on number of processors will have a maximum. It imposes restriction on we scaleing the computing system that means, that from the certain moment addition of new processors in system will increase time of the decision of a problem.

As analogue of law Amdahl law Gustavson-Barsis serves. It agrees to it an estimation of maximum achievable acceleration of performance of the parallel program depending on quantity of simultaneously carried out streams of calculations (processors) and shares of consecutive calculations of the program it is defined by the formula:

$$S_p = \alpha + (1-\alpha)p = p + (1-p)\alpha . \qquad (2)$$

The given estimation of acceleration name acceleration of scaling (scaled speedup) as this characteristic shows as far as parallel calculations can be effectively organized at increase of complexity of decided(solved) problems.

The proof (2) uses the attitude $S_p = \frac{T_1}{T_p}$, in which T_1, T_p – time of the decision of a problem for one and p processors. The size of a share of consecutive calculations $\alpha = \frac{\tau(n)}{\tau(n) + \pi(n)/p}$, where

$\tau(n)$ time of a consecutive part of the program, and $\pi(n)$ time of a part of the program which can be распараллелена is entered. Then $S_p = \frac{T_1}{T_p} = \frac{(\tau(n)+\pi(n)/p)(\alpha+(1-\alpha)p}{\tau(n)+\pi(n)/p}$, the formula (2) whence follows.

It is necessary to notice, that formulas (1) and (2) are not equivalent. The values calculated on them coincide only at $\alpha = 0$ and $\alpha = 1$. In a range of values $0 \langle \alpha \langle 1$ formula (1) in comparison with the formula (2) gives higher result of acceleration.

The purpose of present article is corrective action in law Amdahl represented by the formula (1).

THE AMENDMENT TO LAW AMDAHL

We shall take into account the remark that « if to take into account time necessary for data transmission between processors of the computing system dependence of time of calculations on number of processors will have a maximum. It imposes restriction on we scaleing the computing system that means, that from the certain moment addition of new processors in system will increase time of the decision of a problem ».

Let's make the following assumption: the share of time of the parallel decision of a problem on p processors will develop of two components. The first component corresponds to valid or real time of the parallel decision of a problem to each of p processors. She is less, than the specified size in law Amdahl $\frac{1-\alpha}{p}$ on size $\Delta(\alpha, p)\frac{1-\alpha}{p}$. This, second component, defines total expected expenses of time for definition of an opportunity paralleling algorithm of a problem and the organization paralleling with transfer of the necessary information to each processor of system for the subsequent performance by all processors of their independent parts of full algorithm of a problem. It is obvious, that with increase of quantity of processors in system the share of an expense of time for performance of parallel work of processors should be increased. It is finally possible to write down the following expression for acceleration of the computing system:

$$S_p = \frac{1}{\alpha + (1-\alpha)(\frac{1}{p} + kp^n)} \quad . \tag{3}$$

In the formula (3) k, n uncertain constants, which sizes depend on a kind of the program which is necessary for realizing in the computing system. Definition of values of these factors in given article is not considered it is considered. It represents a separate independent problem. The decision of this problem, in our opinion, should to be based on experimental data or analytical researches with subsequent use of necessary model numerical estimating.

The optimum number of processors at the found values of sizes k, n is defined under the formula $p_0 = 1 + trunc(\sqrt[n+1]{\frac{1}{kn}})$, where $trunc(x)$ the greatest whole part of number x.

EXAMPLE OF CALCULATION OF ACCELERATION

Let at research of some program it is established, that $\alpha = 0{,}2;\ k = 0{,}01;\ n = 3$. The result of calculation of size of acceleration is shown in figure 1.

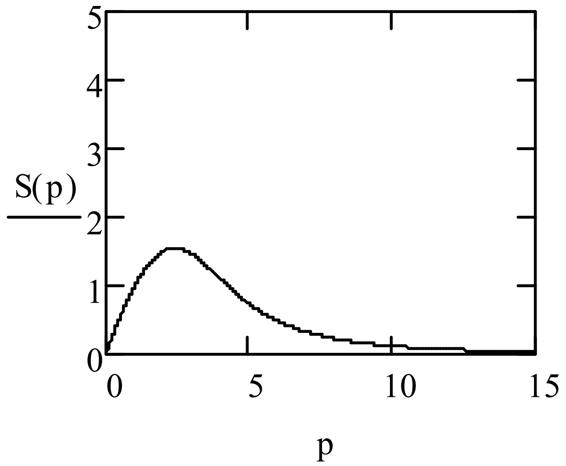

Fig. 1.

From him follows, that the maximal value $S_p = 1{,}465$. It is achieved at the number of processors equal $p_0 = 3$. Thus $\frac{1}{p_0} = 0{,}333;\ kp_0{}^n = 0{,}272$ and shares of expenses of time for actually parallel calculations and their organization will make accordingly $(1-\alpha)/p_0 = 0{,}267;\ (1-\alpha)kp_0^n = 0{,}216$.

CONCLUSION

Law Amdahl is known, allowing to establish the maximal size of acceleration of calculations of the system consisting of given quantity of processors, at the certain parity of a consecutive and parallel part of calculations of the program.

The given law does not allow to take into account system expenses of time for decision making and manufacture of parallel calculations in system.

The amendment to law Amdahl is offered, allowing in a quantitative kind to take into account the specified expenses of time. At presence of the given amendment acceleration of calculations of system reaches the maximal value at some optimum number of processors. The further increase of their quantity does not result in increase of acceleration of calculations.

Use of law Amdahl with the entered amendment allows to define optimum number of processors of the computing system for the organization in it of parallel calculations. Results of article can be used and in other systems with the parallel organization of work of their parts.

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