ELECTRONIC JOURNAL
OF INTERNATIONAL GROUP
ON RELIABILITY

Gnedenko Forum Publications

JOURNAL IS REGISTERED
IN THE LIBRARY
OF THE U.S. CONGRESS

RELIABILITY:
THEORY&APPLICATIONS

ISSN 1932-2321

VOL.1 NO.2 (25)
JUNE, 2012

RELIABILITY
RISK ANALYSIS
MAINTENANCE
SAFETY

Vol.1
No.2 (25)
June
2012

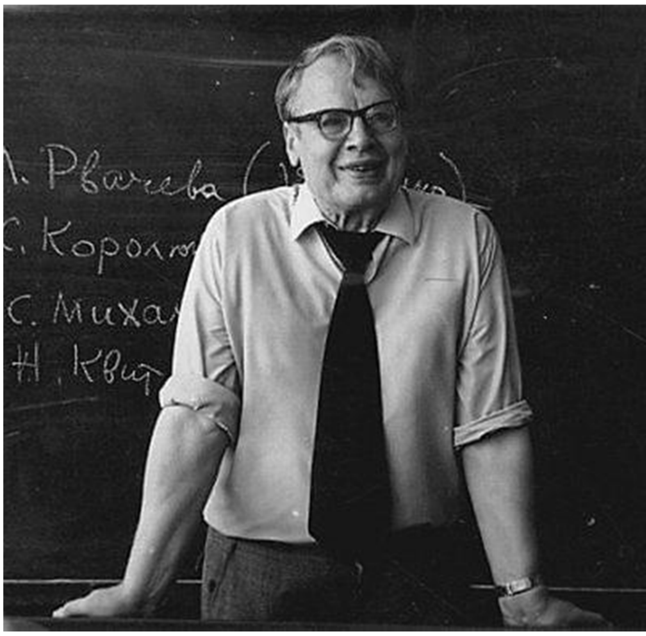
Special Issue 3

San Diego

ISSN 1932-2321

http://www.gnedenko-forum.org/Journal/index.htm

Journal of International Group on Reliability



RELIABILITY:
THEORY & APPLICATIONS

Vol.1 No.2 (25),
June, 2012

Special Issue 3

100th anniversary of Boris Vladimirovich
Gnedenko's birthday

San Diego
2011

Send your paper

e-Journal ***Reliability: Theory & Applications*** publishes papers, reviews, memoirs, and bibliographical materials on Reliability, Quality Control, Safety, Survivability and Maintenance.

Theoretical papers have to contain new problems, finger practical applications and should not be overloaded with clumsy formal solutions.

Priority is given to descriptions of case studies.

General requirements for presented papers

1. Papers have to be presented in English in MSWord format. (Times New Roman, 12 pt , 1.5 intervals).
2. The total volume of the paper (with illustrations) can be up to 15 pages.
3. A presented paper has to be spell-checked.
4. For those whose language is not English, we kindly recommend to use professional linguistic proofs before sending a paper to the journal.

* * *

The Editor has the right to change the paper title and make editorial corrections.

The authors keep all rights and after the publication can use their materials (re-publish it or present at conferences).

Publication in this e-Journal is equal to publication in other International scientific journals.

Papers directed by Members of the Editorial Boards are accepted without referring.

The Editor has the right to change the paper title and make editorial corrections.

The authors keep all rights and after the publication can use their materials (re-publish it or present at conferences).

Send your papers to

the Editor-in-Chief ,
Igor Ushakov
igusha22@gmail.com

or

the Deputy Editor,
Alexander Bochkov
a.bochkov@gmail.com

Table of Contents

Solving practical optimal redundancy problems, one can ponder: what is the sense of optimizing if input data are taken "from the ceiling"? Indeed, statistical data are so unreliable (especially in reliability problems that such doubts have a very good ground. Not found any sources after searching the answer for this question, the author decided to make some investigation of optimal solutions sensitivityunder influence of data scattering. Thus, one can notice that input parameters variation may influence enough significantly enough on probability of failure-free operation and the total system cost from run to run of Monte Carlo simulation though optimal solution remains more or less stable.

The paper offers the method for calculation of reliability parameters and functional safety of technical systems, differing from known methods by an opportunity of obtaining strict formula expressions of stationary parameters directly from a system state graph. The method is suitable for solution of both Markov, and semi-Markov models of reliability and safety. In addition the paper presents some examples of determining safety and availability factors, as well as time parameters of safety and reliability of the two-channel safety related device.

In this paper a sequence of parallel-serial connections is considered. In this sequence next connection is obtained by parallel or serial linking of new arc to obtained connection. Distributions of random numbers of connectivity components are analyzed. These distributions are considered intensively now. Central limit theorem is proved for these distributions and parameters (mean and variance) of normal limit distribution are calculated.

The Method of U-functions, or the Method of the Universal Generating Function (UGF), was introduced in [(1986) Ushakov, (1987) Ushakov] and later developed in [(1988) Ushakov; (1995) Gnedenkon& Ushakov]. Actually this is a generalization and "algebraic" formalization of the well-known Kettelle's Algorithm [(1962) Kettelle]. In turn, Kettelle's Algorithm, is a form of presentation of convolution of discrete random variables. The method of U-functions is very convenient for computerized calculations. Last years, this method was significantly developed by G. Levitin and A. Lisnianski

The paper considers the characteristics of the functioning and sustainability of the structurally complex techno-economic systems (SCTES) in terms of different types of risk. The validity of the application to describe the behavior of this class of systems of semi-empirical mathematical models, which are based on a vector description of the system states, using the criteria approach for assessing the quality of its functioning, is demonstrated. Under discussion is conceptual model for the interaction of the object and its environment, allowing to estimate the "optimal" allocation ratio between the productive system and its development potential. The concept of non-formalizable threats for the sustainable functioning of this systems class was introduced. Expert procedure to account non-formalizable threats in case of risk assessment was proposed. For the construction of indicators for assessing the status the methods of quantitative analysis based on the theory and multi-criteria utility was used. Multi-criteria utility as an indicator of sustainability in the form of dimensionless complex hierarchy of indicators was proposed. Computing for through the convolution of the primary indicators.

In this work we interest to study the beam behavior under monotonic loads in four point, to improve the mechanical properties of a concrete beam fiber and establish an identification card of the new concrete beams were comparing these beams witnesses.

Accuracy asymptotics for differences between prelimit and limit distributions of nodes powers in models of random growing networks are constructed. A rate of a convergence in these relations is power. Obtained formulas allow to ground the continuum approximations for considered models.

The Motorways of the Sea is rather a new concept and, thus, is still in the process of development by the European Commission and in the Baltic Sea Region. The Baltic Sea Region is one of the most dynamic growth areas. Due to this fact the new ideas and technologies are needed to optimize the sea transport system. In the paper the simulation model for the system safety state evaluation is presented. The simulation program can constitute a base for decision-support tool, on the level of safety management, especially to optimally plan the safety transport system.

In flow networks, from the quality and service management point of view, measurement of the transmission ability of a network to meet the customers demand is very important. To meet the ultra high reliable requirements of such networks, a heuristic method for reliability redundancy optimization of flow networks using composite performance measure (CPM) integrating reliability and capacity has been proposed. The method is based upon the selection of main flow paths and backup paths and then optimizing main paths on priority basis. Thus, the reduced computation work makes the proposed algorithm suitable for designing of large, reliable telecommunications networks.

This paper presents estimation of reliability $R = P(X \geq Y)$ of a system, for the cases when its strength (X) and stress (Y) follow exponential, normal or gamma distributions, using Monte Carlo simulation (MCS). First the parameters of strength and / stress are estimated and substituting them in the reliability expressions, in different cases, the estimates of reliability are obtained. Normal distribution is fitted to various sets of estimated reliability $\hat{R}$, generated by MCS. The goodness of fit is tested using Kolmogorov-Smirnov one sample test.

A key requirement in defining a multistate coherent system (MCS) is the relevance condition of its components. A new class of MCSs is introduced with a new component relevance condition. Also we introduce a more general relevance condition. They are compared with some existing component relevance conditions. Based on the two new relevance conditions, two component importance measures for MCSs are defined. They are most appropriate for comparing components when certain type of system improvement is sought. We introduce new joint importance measures for two or more components with respect to the proposed relevance conditions. The new MCS classes include several existing MCSs as special case. An illustrative example of the proposed MCSs is also provided.

SENSITIVITY ANALYSIS OF OPTIMAL REDUNDANCY SOLUTIONS

Igor Ushakov
•
San Diego, California, USA
e-mail: igusha22@gmail.com

Solving practical optimal redundancy problems, one can ponder: what is the sense of optimizing if input data are taken “from the ceiling”? Indeed, statistical data are so unreliable (especially in reliability problems) that such doubts have a very good ground.

Not found any sources after searching the answer for this question, the author decided to make some investigation of optimal solutions sensitivityunder influence of data scattering.

A simple series system of six units has been considered (see Figure 1). For reliability increasing, one uses a loaded redundancy, i.e. if a main unit k has xk redundant units, its reliability is found from

$$P_k(x_k) = 1 - (1 - p_k)^{x_k + 1}$$

where pk is a probability of failure free operation (PFFO) of a single unit *k*. And the total cost of x_k redundant units is equal to $c_k \cdot x_k$, where c_k is the cost of a single unit of type *k*.

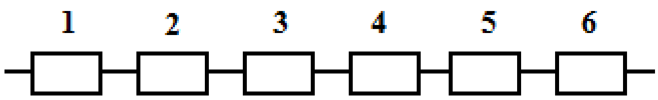


Figure 1. Series system underwent analysis

Units’ parameters are presented in Table 1.

Table 1. Input data

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
p_k	0.8	0.8	0.8	0.9	0.9	0.9
c_k	5	5	5	1	1	1

Assumed that units are mutually independent, i.e. system’s reliability is defined as

$$P_{System}(x_k, 1 \le k \le 6) = \prod_{1 \le k \le 6} P_k(x_k)$$

And the total system’s redundant units cost is:

$$C_{System}(x_k, 1 \le k \le 6) = \sum_{1 \le k \le 6} c_k x_k$$

Below presented solutions of both problems of optimal redundancy: direct:

$$\min_{1 \le x_k < \infty} \{C(x_k, 1 \le k \le 6) \mid P(x_k, 1 \le k \le 6) \ge P^*\}$$

and inverse:

$$\max_{1 \le x_k < \infty} \left\{ P(x_k, 1 \le k \le 6) \mid C(x_k, 1 \le k \le 6) \le C^* \right\}$$

For finding the optimal solutions, the Steepest Descent Method was applied. For this “base” system the solutions for several sets of parameters are presented for Direct Problem in Table 2 and for Inverse Problem in Table 3. (Numbers are given with high accuracy only for demonstration purposes; in practice, one has to use only significant positions after a row of nines.)

Table 2. Solution for Direct problem

P*	**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**x_6**	**Achieved P**	**System C**
0.95	3	3	3	3	2	2	0.9559520	52
0.99	4	4	3	3	3	3	0.991187	69
0.995	5	4	4	4	3	3	0.995229	75
0.999	6	5	5	4	4	4	0.999218	93

Table 3. Solution for Inverse problem

C*	x_1	x_2	x_3	x_4	x_5	x_6	**Achieved *C***	**System *P***
50	3	3	2	2	2	2	46	0.931676
75	4	4	3	3	3	3	75	0.995229
100	5	4	4	4	3	3	99.5	0.999602

The questions of interest are: how optimal solution will change if input data are changed? Two types of experiments have been performed: in the first series of experiments, different unit’s costs with fixed probabilities were considered (see Figure 2) and in another one different unit’s probabilities with fixed costs were considered (see Figure 3).

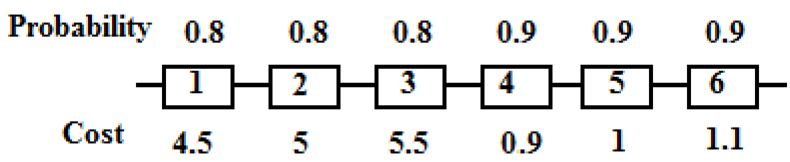


Figure 2. Input data for the first series of experiments

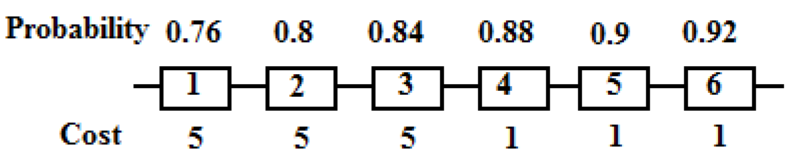


Figure 3. Input data for the second series of experiments

The results of calculations are as follows:

Table 4. Values of Probabilities of Failure-free operations

	0.999	0.995	0.99	0.95
Initial	0.999218	0.99566	0.9922	0.955952
Various *C*	0.998996	0.99566	0.9922	0.955952
Various *P*	0.999218	0.99566	0.9922	0.955952

In conclusion , there was performed a Monte Carlo simulation where parameter of the PFFO and cost were changed simultaneously. In this case, parameters of probabilities of each unit were calculated (in Excel) as:

p_k=0.8p_k+0.4p_k*RAND()
and
c_k= 0.8c_k+0.4*RAND(),

i.e. considered a random variation of the values within ±20% limits..

The final results for this case are presented in Tables 5 – 8.

Table 5. Results of Monte Carlo simulations for P*=0.999

No.	**P* = 0.999**							
	P	**C**	x_1	x_2	x_3	x_4	x_5	x_6
1	0.999352	100	6	6	6	4	4	4
2	0.999218	102	6	6	6	5	4	4
3	0.999313	102	6	6	6	4	4	4
4	0.999212	97	5	6	6	4	4	4
5	0.999182	102	6	6	6	4	4	4
6	0.999171	97	6	6	5	4	4	4
7	0.999171	103	6	6	6	4	5	4
8	0.999596	100	6	6	6	4	4	4
9	0.999526	100	6	6	6	4	4	4
10	0.999399	100	6	6	6	4	4	4

Table 6. Results of Monte Carlo simulations for P*=0.995

No.	**P* = 0.995**							
	P	**C**	x_1	x_2	x_3	x_4	x_5	x_6
1	0.995478	84	5	5	5	3	3	3
2	0.996755	85	5	4	4	4	3	3
3	0.995026	85	5	4	5	4	3	3
4	0.996777	79	4	5	5	3	3	3
5	0.996777	84	5	5	5	3	3	3
6	0.995525	79	5	5	4	3	3	3
7	0.996732	85	5	5	5	3	4	3
8	0.996732	85	5	5	5	3	4	3
9	0.995645	84	5	5	5	3	3	3
10	0.99567	84	5	5	5	3	3	3

Table 7. Results of Monte Carlo simulations for P*=0.99

No.	P* = 0.99							
	P	**C**	x_1	x_2	x_3	x_4	x_5	x_6
1	0.990147	69	4	4	4	3	3	3
2	0.990965	70	4	4	4	4	3	3
3	0.990229	70	4	4	4	4	3	3
4	0.99185	69	4	4	4	3	3	3
5	0.990389	71	4	4	4	4	4	3
6	0.99107	69	4	4	4	3	3	3
7	0.992185	74	5	4	4	3	3	3
8	0.990422	71	4	4	4	3	4	3
9	0.990893	71	5	4	4	3	3	3
10	0.990466	69	4	4	4	3	3	3

Table 7. Results of Monte Carlo simulations for P*=0.95

No.	P* = 0.95							
	P	**C**	x_1	x_2	x_3	x_4	x_5	x_6
1	0.950045	52	3	3	3	3	2	2
2	0.955842	52	3	3	3	3	2	2
3	0.951936	52	3	3	3	3	2	2
4	0.951711	54	3	3	3	2	2	2
5	0.957883	50	3	3	3	3	3	2
6	0.951908	51	3	3	3	2	2	2
7	0.962227	51	3	3	3	2	2	2
8	0.962227	51	3	3	3	3	3	2
9	0.95261	50	3	3	3	3	2	3
10	0.950393	52	3	3	3	3	2	2

Analysis of data presented in Tables 5-8 shows relatively significant difference in numerical results (see Figure 4).

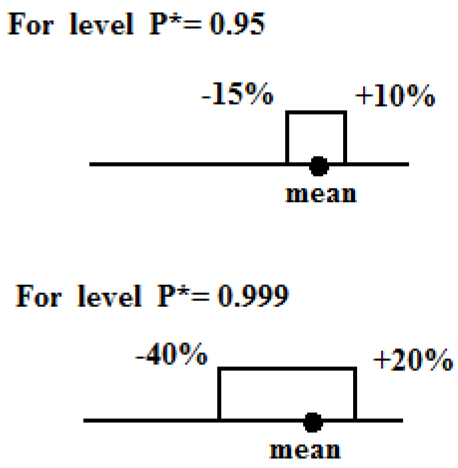


Figure 4. Deviation of maximum and minimum values of probability of failure-free operation in results of Monte Carlo simulation

However, the problem is not in coincidence of final values of PFFO or cost. The problem is: how change of parameters influences on the optimal values of x_1, x_2, … .

However, one can observe that even with a system of six units (redundant groups) a visual analysis of sets ($\boldsymbol{x_1}$, $\boldsymbol{x_2}$, …, $\boldsymbol{x_6}$) is extremely difficult and, at the same time, deductions based on some averages or deviations of various x_k are almost useless.

The author was forced to invent some kind of a special presentation of sets of x_k's. Since there is no official name for such kind of graphical presentation, it is called "multiple polygons" (in Russian "мульти-звездограммы»). On such multiple polygon there are numbers of "rays" corresponding to the number of redundant of units (groups). Each ray has several levels corecponded to the number of calculated redundant units for considered case (see Figure 5).

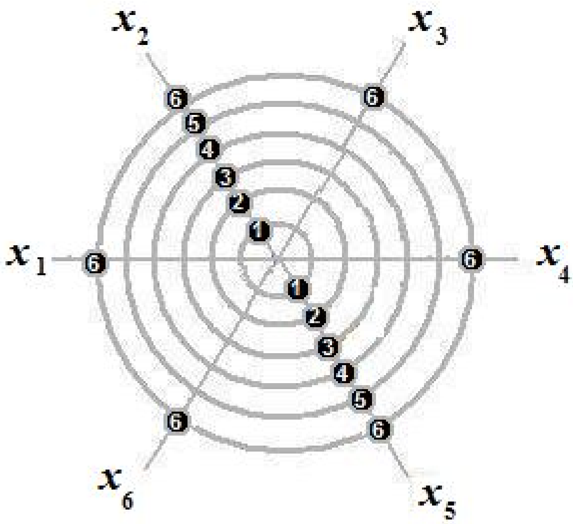


Figure 5. Multiple polygon axes with numbered levels

These multiple polygons give a perfect visualization of "close-to-optimal" solutions and characterize observed deviation of particular solutions. Such multiple polygons for considered example are given in Figure 6. (Here bold lines re used for connection xk obtained as optimal solution for units with parameters given in Table 1.)

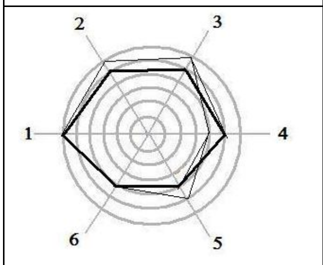
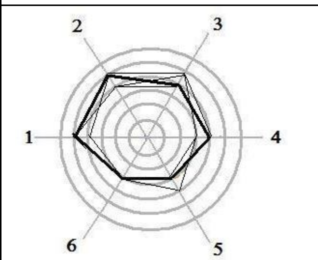
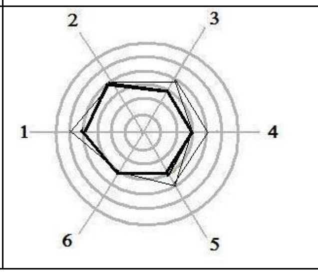
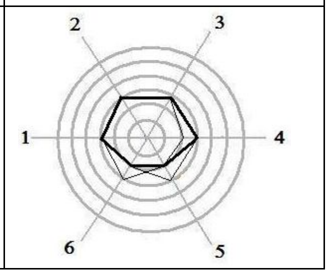
***P** = 0.999**	***P** = 0.999**	***P** = 0.999**	***P** = 0.999**
			

Figure 6. Deviations of optimal solutions for randomly variation of parameters from the optimal solution obtained for parameters given in Table 1

Thus, one can notice that input parameters variation may influence enough significantly enough on probability of failure-free operation and the total system cost from run to run of Monte Carlo simulation though optimal solution remains more or less stable.

TOPOLOGICAL SEMI-MARKOV METHOD FOR CALCULATION OF STATIONARY PARAMETERS OF RELIABILITY AND FUNCTIONAL SAFETY OF TECHNICAL SYSTEMS

Shubinsky Igor B.

●

Doctor of Technical Sciences, professor, director
of the company " IBtrans", The Public Corporation "RZD", Moscow,
Russian Federation
e-mail: igor-shubinsky@yandex.ru

Zamyshlyaev Alexey M.

●

Ph.D., The Head of Scientific-Technical Complex of JSC "NIIAS",
The Public Corporation "RZD", Moscow, Russian Federation
e-mail: A.Zamyshlyaev@gismps.ru

ABSTRACT

The paper offers the method for calculation of reliability parameters and functional safety of technical systems, differing from known methods by an opportunity of obtaining strict formula expressions of stationary parameters directly from a system state graph. The method is suitable for solution of both Markov, and semi-Markov models of reliability and safety. In addition the paper presents some examples of determining safety and availability factors, as well as time parameters of safety and reliability of the two-channel safety related device.

Keywords: reliability, functional safety, parameters of reliability and safety, Markov and semi-Markov models of reliability, loop weight, graph breakdown weight.

1. Introduction

When solving problems of reliability and functional safety of technical systems mathematical tools of Markov and semi-Markov random processes are widely applied. Development and solution of Markov and semi-Markov reliability models by traditional methods in general terms is brought into making up a system of the homogeneous differential equations describing behavior of the investigated system, their operational transformation, solving system of equations in the operational form, inverse transformation and finding the required reliability parameters. Such a way is always fraught with mathematical difficulties, especially when the number of equations exceeds ten and is problematic to execute correctly inverse transformations of solutions of system equations obtained in the operational form. Therefore in the majority research people and, especially practical workers, are compelled to introduce a lot of assumptions which radically simplify solution of reliability models and allow obtaining reliability parameters of considered systems in the analytical or numerical form. However these results are already far from true and there is a natural question: whether it is necessary to aspire to realization of the traditional plan of construction and solving reliability models of systems.

In many problems of reliability calculation it is enough to be limited by stationary reliability and availability factors (parameters). In these cases it is necessary to switch over from the model of differential equations to the model of algebraic equations describing system behavior in the steady-state mode, to solve them, find stationary probabilities of staying system in each of possible states.

Then based on system failure criteria with the help of the specified probabilities probabilistic system availability and unavailability should be found. Thereafter stationary parameters of system non-failure operation and maintainability should be defined. Such problems are not connected with necessity of operational calculus application for development and solution of reliability models. The required stationary reliability parameters are calculated sufficiently strictly. However, alongside with the fact that the given plan does not provide definition of a full list of reliability parameters, there is also the unsolved problem of the big dimension of algebraic equations' model. Therefore even at rather small number of states it is not possible in many cases to analytically describe required reliability parameters of the system. This circumstance does not depend on a degree of system graph model connectivity. Dimensions of a system matrixes for algebraic equations representing reliability of investigated technical system model do not vary both with weak connectivity and with strong connectivity.

At the same time, graph models of complex systems' reliability, as a rule, are poorly connected. This circumstance has stimulated us to switch from the traditional plan of solving linear algebraic equations by Kramer's rule to the scheme of breakdown initial graph to the constituent sub graphs which are not containing single out nodes (for example, disabled states of model or states which are being on the way from one node to another, or an initial system state). At application of such a scheme (plan) it has turned out sufficient for solving the system of algebraic equations. Moreover it has turned out sufficient to be limited by finding ways and loops on the graph, what is now well formalized.

2. Problem definition

Stationary parameters of technical systems' reliability are factors of availability $K_Г$ and unavailability $\overline{K}_Г$, mean time to failure T_{CP}, dispersion of mean time to failure D_{CP}, an average time between failures T, mean idle time average of a $T_{ПР}$. As functional dependence $K_Г = f\,(T, T_{ПР})$ is known methods of calculation of these three parameters are expedient for considering simultaneously. Similarly it is necessary to simultaneously consider methods for calculation of parameters T_{CP} and D_{CP}.

Stationary parameters of functional safety of safety related *systems* is a factor of safety $K_Б$, mean time to dangerous failure $T_{ОП}$, dispersion of time to dangerous failure $D_{ОП}$, mean time to protective failure $T_З$, dispersion of time to protective failure $D_З$, mean time to dangerous (hazardous) failure $T_П$.

Methods of calculation of corresponding groups of reliability and functional safety parameters are identical. Difference is only in the separation of initial system states on efficient and disabled (concerning reliability) subsets and nonhazardous, hazardous and protective subsets of states (relating to functional safety). So in the study [1] the following formula of calculation of system availability factor which behavior is described by semi-Markov random process is determined

$$K_Г = \frac{\sum_{i \in S_P} P_i T_i}{\sum_{j \in S} P_j T_j} (i, j \in S; S_P \subset S), \quad (1)$$

where S_P is a subset of efficient system' states , S is full set of system states; P_i is a final probability of staying Markov chain in *i-th* state; T_i is an expectancy of unconditional time of system staying in *i-th* state.

By turn, formulas of the calculation of mean time to failure (time between failures) and the average idle time of the system which behavior is described by semi-Markov random process, and determined according to the study [2] are the following:

$$T_0 = \frac{\sum_{i \in S_P} P_i T_i}{\sum_{i \in S_+} P_i \sum_{j \in \overline{S_P}} p_{ij}}; \qquad (2)$$

$$T_{\Pi P} = \frac{\sum_{j \in \overline{S_P}} P_j T_j}{\sum_{j \in S_-} P_j \sum_{i \in S_P} p_{ji}}, \qquad (3)$$

where it is implied, that transition from a subset S_P into a subset $\overline{S}_P$ can be carried out not from any working state capacity, but only from boundary conditions (subset) S_+. Similarly transition from $\overline{S}_P$ into S_P can be carried out from a subset belonging to the subset of boundary disabled states S_-.

Practical methods for calculation of functional safety parameters of recoverable safety related systems nowadays are poorly developed.

The purpose of this paper consists in development of practical methods of calculation listed above stationary parameters of reliability and functional safety of complex technical systems. It is a question of formalization of calculations on the basis of the graph theory.

3. Calculation of availability and safety factors

3.1. Topological concepts:

- *Path* – chain of consistently connected unidirectional arcs starting from a state *i* and ending in a state *j*, path weight $l^{ij} = \prod_{i,r,j \in S} p_{ir} p_{rj}$, where p_{ir} - probability of one step transition for *i* – th state in a state *r*;

- *Closed loop* is a chain of consistently connected unidirectional arcs in which the output of final vertex in the chain is connected to starting vertex of the chain;
- *Weight of j – th loop* $C_j = \prod_{i,j \in S} p_{ij} p_{ji}$; self-loop is a special case of the closed loop (entering and leaving arcs in self-loop merge into one arch), weight of a self-loop $C_j = p_{jj}$;
- *Graph decomposition* - a graph part which is not containing assigned vertices and arcs connected with them; weight of a decomposition ΔG^i is calculated taking into account the exclusion from the graph a vertex *i* and the arcs connected with it; the weight of a decomposition $\Delta G^i_{\overline{S_P}}$ is calculated taking into account the additional exclusion from the graph the vertices of a set of disabled states $\overline{S_P}$ and the arcs connected with them; the weight of a decomposition ΔG^i_k is calculated taking into account the exclusion from the graph the vertex *i,* as well as the vertices located on *k*-th path from starting vertex into a vertex *i* and arcs connected with them;
- *The decomposition weight (determinant)* is calculated under Mason's formula

$$\Delta G = 1 - \sum_j C_j + \sum_{rj} C_r C_j - \sum_{irj} C_i C_r C_j + \ldots \qquad (4)$$

Application of Mason's formula allows to considerably reducing labor input of calculations of minors on the rarefied matrixes, and matrix *G*, as a rule, is rarefied.

3.2. Topological formulas of calculation of availability and functional safety factors

The statement 1. If system reliability is modeled by means of the graph states and semi-Markov random process on this set of states, specified by transition probability matrix and a vector of unconditional expectances of staying time in each graph state the factor of system availability in the topological form is equal:

$$K_{\Gamma} = \frac{\sum_{i \in S_P} \Delta G^i T_i}{\sum_{j \in S} \Delta G^j T_j}, \tag{5}$$

where ΔG^i - decomposition weight of the graph without a state *i,* T_i - expectance of unconditional staying time of the system in states s_i.

Proof. The stationary probability of enclosed homogeneous Markov chain staying in a state i, is equal

$$P_i = \frac{D_i}{\sum_{j=1}^{n} D_j},$$

where n - number of states in initial set of states of the system S

$$D = \begin{array}{c} \\ 1 \\ 2 \\ \\ n \end{array} \begin{pmatrix} 1 & 2 & \dots & n \\ 1-p_{11} & -p_{12} & \dots & -p_{1n} \\ -p_{21} & 1-p_{22} & \dots & -p_{2n} \\ \dots & \dots & \dots & \dots \\ -p_{n1} & -p_{n2} & \dots & 1-p_{nn} \end{pmatrix}$$

and $D_i(D_j)$ - a minor obtained by deletion of *i (j)* line and of *i (j)* column in matrix D. In turn, both the determinant D and minors $D_i(D_j)$ can be strictly or with acceptable accuracy calculated under Mason's formula (4). Hence, the stationary probability of enclosed homogeneous Markov chain staying in a state i, is equal to the following

$$P_i = \frac{D_i}{\sum_{j=1}^{n} D_j} = \frac{\Delta G^i}{\sum_{j=1}^{n} \Delta G^j} = \frac{\Delta G^i}{\sum_{j \in S} \Delta G^j} \tag{6}$$

Substituting the formula (6) in the expression (1) we obtain the formula (5), as was to be shown.

The consequence 1. If $S_H \subset S$ - a subset of nonhazardous states of safety related system. The system safety factor is determined as the following

$$K_{Б} = \frac{\sum_{i \in S_1} \Delta G^i T_i}{\sum_{j \in S} \Delta G^j T_j} \quad (7)$$

The formula (7) is obtained by analogy to the formula (5) concerning set of nonhazardous states.

4. Topological formulas for calculation of stationary time parameters of reliability

Mean time to system failure

$$T_0 = \frac{\sum_{i \in S_P} \Delta G^i T_i}{\sum_{i \in S_+} \Delta G^i \sum_{j \in \overline{S}_P} p_{ij}} \quad (8)$$

Average idle time of a system

$$T_{ПР} = \frac{\sum_{j \in \overline{S}_P} \Delta G^j T_j}{\sum_{j \in S\_} \sum_{i \in S_P} p_{ji}} \quad (9)$$

Formulas (8) and (9) are obtained from formulas (2) and (3) by substitution in them the formula (6).

Mean time between hazardous failures

$$T = \frac{\sum_{i \in S_1} \Delta G^i T_i}{\sum_{i \in S_{1+}} \Delta G^i \sum_{j \in \overline{S}_1} p_{ij}}, \quad (10)$$

where S_1 - a subset of nonhazardous states, $\overline{S}_1$ - a subset of hazardous states $S_1 \cup \overline{S}_1 = S$, S_{1+} - a subset of boundary nonhazardous states ($S_{1+} \subset S_1$).

Mean time to system failure and dispersion of mean time to failure

With a view of development of the formalized engineering methods for determining these parameters we shall prove the following statement.

The statement 2. If system reliability is modeled by means of the graph states and semi-Markov random process on this set of states then the confidence curve to system failure in Laplace transformations at *i-th* initial state is determined by the following expression

$$\widetilde{\Phi}_i(z) = \frac{\sum_{j \in \overline{S}_P} \sum_k \widetilde{l}_k^{ij}(z) \cdot \Delta \widetilde{G}_k^j(z)}{\Delta \widetilde{G}_{\overline{S}_P}(z)}, \quad (11)$$

where $\widetilde{l}_k^{ij}(z)$ - *k*-th path in Laplace transformations, leading from an efficient state of the graph $i \in S_P$ into the failure state $j \in \overline{S}_P$; $\Delta \widetilde{G}_k^j(z)$ - graph decomposition weight in Laplace transformations without *j-th* vertex and the graph vertices located on the *k-th* path; $\Delta \widetilde{G}_{\overline{S}_н}(z)$ - without vertices, graph decomposition weight of a set of failure states

Proof

In the study [1] it is shown, that the function of time distribution of system staying in a set of efficient conditions S_P in Laplace transformations can be obtained from the following equation

$$\tilde{\Phi}_i(S) - \sum_{j \in S_P} \tilde{Q}_{ij}(S)\ \tilde{\Phi}_j(S) = \sum_{l \in \bar{S}_P} \tilde{Q}_{il}(S).$$

Let's transform this equation to a matrix form, keeping in mind, that the right part of the equation is a vector-column of free terms of semi-Markov transitions' probabilities for one step from vertices $i, j, \ldots z \in S_P$ into the vertex $l \in \bar{S}_P$.

$$\tilde{\Phi}(S) - \tilde{Q}(S)\ \tilde{\Phi}(S) = \tilde{Q}^*(S),$$

where $\tilde{Q}(S) = (\tilde{Q}_{ij}(S))$ - is a matrix of semi-Markov probabilities; $\tilde{Q}^*(S) = (\tilde{Q}_{il}(S))$ - is a vector-column.

In system of the equations the unknown elements are those of the vector-column $\tilde{\Phi}(S)$. After their grouping in the left part we shall obtain

$$\tilde{\Phi}(S)\left[I - \tilde{Q}(S)\right] = \tilde{Q}^*(S).$$

Then by Kramer's rule we can find $\tilde{\Phi}_i(S) = \dfrac{\Delta_i(S)}{\Delta(S)}$, where $\Delta(S) = \left|I - \tilde{Q}(S)\right|$, and $\Delta_i(S)$ - the determinant, obtained by replacement of *i-th* column in the matrix $I - \tilde{Q}(S)$ on a vector-column of free terms $\tilde{Q}^*(S)$ provided that $\Delta_i(S)$ and $\Delta(S)$ are not equal to zero.

The determinant $\Delta_i(S)$ differs from the determinant $\Delta(S) = \Delta G_{\bar{S}_P}$ by the fact that in the column i the element $\tilde{p}_{ij}(S)$ is replaced with the element $\tilde{p}_{il}(S)$ where $i, j \in S_P$, and $l \in \bar{S}_P$. As a result we obtain the following

$$\Delta_i(S) = \Delta G^i_{\bar{S}_н} = \sum_{l \in \bar{S}_P} \sum_k \tilde{l}_k^{il}(S)\ \Delta G_k^l(S).$$

Hence,

$$\tilde{\Phi}_i(S) = \frac{\sum_{l \in \bar{S}_P} \sum_k \tilde{l}_k^{il}(S) \cdot \Delta G_k^l(S)}{\Delta \tilde{G}_{\bar{S}_н}}$$

and at replacement of an index l on j the required result is obtained. The statement is proved.

From the formula (11) follows

- Mean time to system failure at initial state i=1

$$T_{CP} = -\frac{\partial \tilde{\Phi}_1(z)}{\partial z}\Big|_{z=0}; \tag{12}$$

- Dispersion of the mean time to system failure at initial state i=1

$$D_{CP} = \frac{\partial^2 \tilde{\Phi}_1(z)}{\partial z^2}\Big|_{z=0} - \left[\frac{\partial \tilde{\Phi}_1(z)}{\partial z}\right]^2\Big|_{z=0}. \tag{13}$$

Consequence 2 The function of time distribution (confidence curve) to system hazardous failure in Laplace transformations at *i-th* initial state is determined by the following expression

where $\widetilde{l}_k^{ij}(z)$ - *k-th* path in Laplace transformations leading from a nonhazardous state of the graph $i \in S_H$ into hazardous failure state $j \in \overline{S}_H$;

$$\widetilde{\Phi}_{iO}(z) = \frac{\sum_{j \in S_H} \sum_k \widetilde{l}_k^{ij}(z) \cdot \Delta\widetilde{G}_k^j(z)}{\Delta\widetilde{G}_{\overline{S}_H}(z)}, \qquad (14)$$

From the formula (14) follows

- Mean time to system failure at an initial state i=1

$$T_{ОП} = -\frac{\partial\widetilde{\Phi}_{1O}(z)}{\partial z}\Big|_{z=0}; \qquad (15)$$

- A dispersion mean time to system failure at an initial state i=1

$$D_{ОП} = \frac{\partial^2\widetilde{\Phi}_{1O}(z)}{\partial z^2}\Big|_{z=0} - \left[\frac{\partial\widetilde{\Phi}_{1O}(z)}{\partial z}\right]^2\Big|_{z=0} . \qquad (16)$$

5. Examples

Example 1

The two-channel device is analyzed. It contains two identical and independent channels, as well as diagnostics tools which check with acceptable frequency for good safety the functioning state of each channel and compare their output results. Failures of channels are asymmetrical. When diagnostics tools are sound the fact of failure of any one channel is detected and then the device transition in a state of protective failure is carried out. In the case of diagnostics tools' failure only a nonhazardous failure of the device can occur. The subsequent behind this event failure of a channel leads to hazardous failure of the device.

Graph states of reliability and safety of the two-channel device with diagnostics tools without channels' restart is shown on fig. 1.

The description of states:

1 - Serviceable state;

2 - Diagnostics tool failure;

3 - protective failure of the device caused by detected failure of one of the channels; detection was carried by regular diagnostics tools with probability ν ;

4 - Not detected failure of one of the channels, owing to failure or insufficient efficiency of diagnostics tools (hazardous failure of the device).

For presentation of an illustration of opportunities of the offered method we assume, that failure and recovery flows, as well as a flow of detected failures of one channel are the simple flows with rates λ, $\lambda_д$, μ.. Restoration is carried out in the state of protective failure 2.

Graph edges on fig. 1 are marked by following parameters: $\lambda_д$ - failure rate of diagnostics tools; 2λ - failure rate of the two working channels; μ - recovery rate of failures by one repair team.

Transition from a hazardous state 3 into initial state 0 is shown. The edge 3-0 is marked by parameter $c\mu$ - recovery rate of hazardous failure of the device, where the factor $0 < c \le 1$. If for the elimination of hazardous failure there is no need to update the device then *c=1* and the rate of hazardous failure elimination is equal to recovery rate of the device. If it is required to update the device depending on duration of updating time τ the given factor will have the value $c = 1/\tau$ which is much less 1. The opportunity of failure of one more channel when the device is in the condition 3, is not considered, as hazardous failure has already taken place and either one channel or two channels are subjects to recovery.

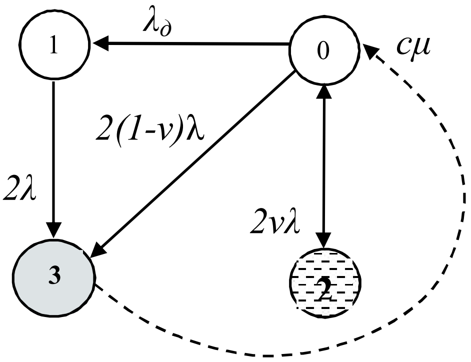


Figure 1. Graph of safety states of the two-channel device without channels' restart

The model of reliability and functional safety of the two-channel device on fig.1 provides the following logic of device operation: an initial state 0 (all elements of the device are serviceable). In case of diagnostics tools failure there is a transition into a state 1. If at serviceable diagnostics tools any one channel (a state 2) has failed, and the channel failure is detected in due time out with probability ν the device is transferred into a state of protective failure (the device does not function, the channel is under repair). At the latent failure of the channel probability $1-\nu$ or at failure of one channel after the failure of diagnostics tools (the path 0 - 1 - 3) there is a transition into transition a state 3 of hazardous failure.

Failure criterion: $S_p = \{0,1\}$ $\overline{S}_P = \{2,3\}$ $S_p \cup \overline{S}_P = S$.

Hazardous failure criterion: $S_H = \{0,1,2\}$ $\overline{S}_H = \{3\}$ $S_H \cup \overline{S}_H = S$.

It is required by means of formulas (5) and (7) to determine availability and functional safety factors of the two-channel device

Solution

- Initial parameters should be defined:

$$T_0 = \int_0^\infty \exp[-(2\lambda + \lambda_l)\cdot t]dt = \frac{1}{2\lambda + \lambda_l}; T_1 = \frac{1}{2\lambda}; T_2 = \frac{1}{\mu}; T_3 = \frac{1}{c\mu};$$

$$p_{01} = \int_0^\infty \lambda_l \exp[-(2\lambda + \lambda_l)\cdot t]dt = \frac{\lambda_l}{2\lambda + \lambda_l}; p_{02} = \frac{2\nu\lambda}{2\lambda + \lambda_l}; p_{03} = \frac{2(1-\nu)\lambda}{2\lambda + \lambda_l};$$

$$p_{13} = p_{20} = p_{30} = 1$$

$$\Delta G^0 = 1; \Delta G^1 = 1 - p_{02}p_{20} - p_{03}p_{30} = 1 - p_{02} - p_{03}; \Delta G^2 = 1 - p_{01} - p_{03};$$

$$\Delta G^3 = 1 - p_{02}$$

- Find availability factor

$$K_{Г} = \frac{\sum_{i \in S_P} \Delta G^i T_i}{\sum_{j \in S} \Delta G^j T_j} = \frac{\Delta G^0 T_0 + \Delta G^1 T_1}{\Delta G^0 T_0 + \Delta G^1 T_1 + \Delta G^2 T_2 + \Delta G^3 T_3} =$$

$$\frac{\frac{1}{2\lambda + \lambda_\partial} + \frac{\lambda_\partial}{2\lambda(2\lambda + \lambda_\partial)}}{\frac{1}{2\lambda + \lambda_\partial} + \frac{\lambda_\partial}{2\lambda(2\lambda + \lambda_\partial)} + \frac{2\nu\lambda}{\mu(2\lambda + \lambda_\partial)} + \frac{2(1-\nu)\lambda + \lambda_\partial}{c\mu(2\lambda + \lambda_\partial)}} =$$

$$\frac{(2\lambda + \lambda_\partial) \cdot c\mu}{c\mu(2\lambda + \lambda_\partial) + 4\lambda^2(1 - \nu(1-c)) + 2\lambda\lambda_\partial},$$

If for elimination of hazardous failure there is no need to update the device then the factor *c=1* and expression for availability factor of the device will be transformed in to the following form:

$$K_{Г} = \frac{\mu}{2\lambda + \mu}.$$

- Find safety factor of the device

$$K_{Б} = \frac{\sum_{i \in S_H} \Delta G^i T_i}{\sum_{j \in S} \Delta G^j T_j} = \frac{\Delta G^0 T_0 + \Delta G^1 T_1 + \Delta G^2 T_2}{\Delta G^0 T_0 + \Delta G^1 T_1 + \Delta G^2 T_2 + \Delta G^3 T_3} =$$

$$\frac{\frac{1}{2\lambda + \lambda_\partial} + \frac{\lambda_\partial}{2\lambda(2\lambda + \lambda_\partial)} + \frac{2\nu\lambda}{\mu(2\lambda + \lambda_\partial)}}{\frac{1}{2\lambda + \lambda_\partial} + \frac{\lambda_\partial}{2\lambda(2\lambda + \lambda_\partial)} + \frac{2\nu\lambda}{\mu(2\lambda + \lambda_\partial)} + \frac{2(1-\nu)\lambda + \lambda_\partial}{c\mu(2\lambda + \lambda_\partial)}}$$

At *c=1* the safety factor of the device is determined by means of the following expression:

$$K_{Б} = \frac{\mu}{2\lambda + \mu} + \frac{4\lambda^2 \nu}{(2\lambda + \mu) \cdot (2\lambda + \lambda_\partial)}.$$

If the two-channel system is inefficient (in extreme case $\nu = 0$) the safety factor of the device is equal, as one would expect, to its availability factor.

Example 2

In conditions of the example 1 it is required by means of formulas (11), (12), (14), (15) to determine time parameters of reliability and functional safety of the two-channel device

Solution

From the formula (11) follows, that functions of time distribution to system failure at an initial state 0 in operational transformations has the following form:

$$\widetilde{\Phi}_0(z) = \frac{\widetilde{p}_{01}(z)\widetilde{p}_{13}(z) + \widetilde{p}_{03}(z) + \widetilde{p}_{02}(z)}{1},$$

where $\widetilde{l}_1^{\,03}(z) = \widetilde{p}_{01}(z)\widetilde{p}_{13}(z); \widetilde{l}_2^{\,03} = \widetilde{p}_{03}(z); \Delta\widetilde{G}_1^3(z) = 1; \Delta\widetilde{G}_2^3 = 1; \Delta\widetilde{G}_{\bar{S}_P}(z) = 1.$

In Laplace transformations at exponential distributions of random variables

$$\tilde{p}_{01}(z) = \frac{\lambda_\partial}{2\lambda + \lambda_д}\int_0^\infty e^{-zt} dF_0(t) = \int_0^\infty e^{-zt} d[1 - e^{-(2\lambda+\lambda_д)\cdot t}] = \frac{\lambda_\partial}{2\lambda + \lambda_\partial + z};$$

$$\tilde{p}_{13}(z) = \int_0^\infty e^{-zt} dF_1(t) = \int_0^\infty e^{-zt} d[1 - e^{-2\lambda\cdot t}] = \frac{2\lambda}{2\lambda + z};$$

$$\tilde{p}_{03}(z) = \frac{2(1-\nu)\lambda}{2\lambda + \lambda_\partial}\int_0^\infty e^{-zt} dF_0(t) = \int_0^\infty e^{-zt} d[1 - e^{-(2\lambda+\lambda_\partial)\cdot t}] = \frac{2(1-\nu)\lambda}{2\lambda + \lambda_\partial + z};$$

$$\tilde{p}_{02}(z) = \frac{2\nu\lambda}{2\lambda + \lambda_\partial}\int_0^\infty e^{-zt} dF_0(t) = \int_0^\infty e^{-zt} d[1 - e^{-(2\lambda+\lambda_\partial)\cdot t}] = \frac{2\nu\lambda}{2\lambda + \lambda_\partial + z}.$$

$$\tilde{p}_{20}(z) = \int_0^\infty e^{-zt} dF_2(t) = \int_0^\infty e^{-zt} d[1 - e^{-\mu\cdot t}] = \frac{\mu}{\mu + z}.$$

Hence,

$$\tilde{\Phi}_0(z) = \frac{2\lambda}{2\lambda + z} \; T_{CP} = -\frac{\partial\tilde{\Phi}_0(z)}{\partial z}\Big|_{z=0} = \frac{1}{2\lambda}.$$

The formula of function of time distribution to system failure in Laplace transformations under conditions of the given example has the following form:

$$\tilde{\Phi}_{0O}(z) = \frac{\tilde{p}_{01}(z)\tilde{p}_{13}(z) + \tilde{p}_{03}(z)}{1 - \tilde{p}_{02}(z)\cdot\tilde{p}_{20}(z)},$$

as conditions 0,1 and 2 are non hazardous and $\Delta\tilde{G}_{\bar{S}_H} = 1 - \tilde{p}_{02}(z)\cdot\tilde{p}_{20}(z)$

Hence,

$$\tilde{\Phi}_{0O}(z) = \frac{2\lambda\cdot[\lambda_д + (1-\nu)(2\lambda + z)](\mu + z)}{[2\lambda\cdot(2\lambda + \lambda_д - \nu\mu) + (2\lambda + \lambda_д)z + z^2]\cdot(2\lambda + z)}.$$

From here

$$T_{ОП} = -\frac{\partial\tilde{\Phi}_{0O}(z)}{\partial z}\Big|_{z=0} = \frac{2\lambda\cdot(\mu + 2\lambda\nu) + \lambda_д\mu}{2\lambda\mu\cdot[2\lambda(1-\nu) + \lambda_д]}.$$

If to take into account, that $\lambda << \mu; \lambda_\partial << \lambda$, with an margin error less than the first infinitesimal order then the following expression is true

$$T_{ОП} \approx \frac{1}{2\lambda(1-\nu) + \lambda_\partial}.$$

At high efficiency of detection of hazardous failures on the basis of two-channel architecture of the device ($\nu = 1$) its safety depends only on the reliability of the built in diagnostics tools and the comparator (i.e. on failure rate λ_∂).

The conclusion

The offered topological semi-Markov method for calculation of reliability and safety parameters of technical systems allows determining directly on the states' graph the strict or approximates formula expressions of typical reliability and safety parameters of technical systems which behavior is described by both Markov, and semi-Markov random processes. Mathematical positions of the method are illustrated by examples which show simplicity and rigor of finding out the required reliability and safety parameters.

The literature

1. Korolyuk V.S., Turbins A.F. Semi-Markov processes and their applications. - Kiev: Naukova dumka, 1976, 179 p..
2. Rainshke K., Ushakov I.A. Estimation of system reliability with use of graphs /under under the editorship of. I.A.Ushakov.-M.: Radio and communication, p.p.1988.-209.

RECURRENT SEQUENCE OF PARALLEL-SERIAL CONNECTIONS

G. Tsitsiashvili*, M. Osipova**

•

* IAM FEB RAS, Vladivostok, Russia, ** FEFU, Vladivostok, Russia
* guram@iam.dvo.ru, ** mao1975@list.ru

ABSTRACT

In this paper a sequence of parallel-serial connections is considered. In this sequence next connection is obtained by parallel or serial linking of new arc to obtained connection. Distributions of random numbers of connectivity components are analyzed. These distributions are considered intensively now. Central limit theorem is proved for these distributions and parameters (mean and variance) of normal limit distribution are calculated.

1. INTRODUCTION

In the reliability theory parallel-serial connections play important role [1] – [6] etc. These connections are widely used in electrotechnics, in computer networks etc. A specific of these connections is a possibility to calculate their reliability by algorithms with linear complexity by a number of arcs.

Last years large interest is called to characteristics of networks sparseness. It means that powers of nodes (a number of incident arcs) is bounded by some positive number (see [7] and large bibliography in this article). Stochastic modeling and statistical processing of internet type networks data showed that nodes powers have distribution with heavy tails [8]. Last circumstance makes actual to consider parallel-serial connections which are free of this lack.

Last time a distribution of numbers of connectivity components in different random networks are analyzed intensively now [9] – [11]. In this paper numbers of connectivity components in recurrent sequence of connections obtained by parallel or serial linking of new arc is considered. For this sequence central limit theorem is proved and parameters of limit normal distribution are calculated.

A problem to calculate a mean and mainly a variance of limit normal distribution in this model is technically sufficiently complicated. In this paper it is based on central limit theorem for discrete Markov chains [12] and on a construction of special and sufficiently fast algorithm of such calculations.

2. MODEL DESCRIPTION

Consider the sequence $\mathcal{A}_n,\ n \geq 1$, of ports defined recursively by a sequential or parallel connection of new arc b_n to the port $\mathcal{A}_n$. Denote a type of connection by $||$ or $\rightarrow$ accordingly. Suppose that random variable ω_n characterizes a type of the arc b_n connection to the port $\mathcal{A}_n$ and put

$$\pi_{\rightarrow} = P(\omega_n = \rightarrow), \qquad \pi_{||} = P(\omega_n = ||) = 1 - \pi_{\rightarrow}, \qquad 0 < \pi_{\rightarrow} < 1.$$

Here random variable β_n characterizes a state of the arc b_n:

$$P(\beta_n = 1) = P(b_n \text{ in working state}) = p,\ \ P(\beta_n = 0) = 1 - p = q,\ \ 0 < p < 1.$$

The sequences of random variables $\{\omega_n,\ n \geq 1\}$, $\{\beta_n,\ n \geq 1\}$ are independent and each of them consists of independent and identically distributed random variables.

The port $\mathcal{A}_n$ with randomly working arcs is characterized by random vector (α_n, η_n) there α_n is an indicator of a connectivity between initial and final nodes of parallel-sequential connection $\mathcal{A}_n$ and η_n is a number of connectivity components in $\mathcal{A}_n$. Introduce auxiliary random variables

$$\vec{\alpha}_{n+1} = \alpha_n \wedge \beta_n,\ \vec{\eta}_{n+1} = \eta_n + 1 - \beta_n, \tag{1}$$

$$\overline{\overline{\alpha}}_{n+1} = \alpha_n \vee \beta_n,\ \overline{\overline{\eta}}_{n+1} = \eta_n - \beta_n + \alpha_n\beta_n, \tag{2}$$

then

$$(\alpha_{n+1}, \eta_{n+1}) = I(\omega_n = \to)(\overleftarrow{\alpha}_{n+1}, \overleftarrow{\eta}_{n+1}) + I(\omega_n = ||)(\overline{\overline{\alpha}}_{n+1}, \overline{\overline{\eta}}_{n+1}), \tag{3}$$

where $I(C)$ is an indicator of an event C.

3. LIMIT THEOREM FOR MARKOV CHAIN CHARACTERIZING CONNECTIVITY OF PARALLEL-SERIAL CONNECTIONS

Denote $\Delta_{n+1} = \eta_{n+1} - \eta_n$, then the sequence $X_k = (\alpha_k, \Delta_k),\ k \geq 1$, is Markov chain with the states set $\mathcal{X} = \{(i,j),\ i = 0,1,\ j = -1,0,1\}$ as follows

$$(\alpha_{n+1}, \Delta_{n+1}) = I(\omega_n = \to)(\alpha_n\beta_n, 1 - \beta_n) + I(\omega_n = ||)(\alpha_n \vee \beta_n, -\beta_n + \alpha_n\beta_n).$$

From the equalities (1) - (3) and the conditions $0 < p < 1, 0 < \pi_\to < 1$ we see that Markov chain $X_k, k \geq 1$, states are interconnected. Consequently from the central limit theorem for discrete Markov chains with finite states set [12, chapters V,VI} there are normally distributed random vector $N(0,\mathcal{B})$ with the dimension six and with zero mean and with covariance matrix $\mathcal{B}$ and real numbers $A(x),\ x \in \mathcal{X}$, which do not depend on initial state X_1 so that for any real $t(x),\ x \in \mathcal{X}$,

$$P\left(\left(\frac{N_n(x) - nA(x)}{\sqrt{n}}, x \in \mathcal{X}\right) > (t(x),\ x \in \mathcal{X})\right) \to P(N(0,\mathcal{B}) > (t(x),\ x \in \mathcal{X})),\ n \to \infty. \tag{4}$$

Here $N_n(x) = \sum_{k=1}^{n} I(X_k = x)$ and the inequalities are defined componentwise.

Introduce auxiliary numbers $a(x),\ x \in \mathcal{X}$:

$$a(i,0) = 0, \qquad a(i,1) = 1, \qquad a(i,-1) = -1, \qquad i = 0,1.$$

From the formula (4) it is simple to obtain that there is normally distributed random variable $N(0,B)$ with zero mean and with the covariance $B > 0$ so that for any real t

$$P\left(\frac{1}{\sqrt{n}}\sum_{x \in \mathcal{X}} a(x)(N_n(x) - nA(x)) > t\right) \to P(N(0,B) > t),\ n \to \infty. \tag{5}$$

Using obvious equality $\sum_{x \in \mathcal{X}} a(x)N_n(x) = \sum_{k=1}^{n} \Delta_k = \eta_n,\ n \geq 1$, rewrite the formula (5) as follows

$$P\left(\frac{\eta_n-nA}{\sqrt{n}}>t\right)\to P(N(0,B)>t),\ n\to\infty,\ A=\sum_{x\in\mathcal{X}}a(x)A(x). \tag{6}$$

Remark 1. A calculation of the vector $(A(x),\ x\in\mathcal{X}))$ and especially of covariance matrix $\mathcal{B}$ in the formula (4) is sufficiently complicated procedure [12, chapters V, VI}. So to define the mean A and the covariance B we use following limit formulas

$$A=\lim_{n\to\infty}\frac{M\eta_n}{n},\ B=\lim_{n\to\infty}\frac{D\eta_n}{n} \tag{7}$$

which are corollaries of the formula (6) with special initial distribution of X_1.

4. CALCULATION OF LIMIT NORMAL DISTRIBUTION PARAMETERS

Choose random vector $(\alpha_1,\Delta_1)=(\alpha_1,\eta_1)$ which does not depend on random sequences $\{\omega_n,\ n\geq 1\}$, $\{\beta_n,\ n\geq 1\}$ and satisfies the equalities

$$P((\alpha_1,\eta_1)=(1{,}1))=P=\frac{\pi_{||}p}{\pi_{||}p+\pi_{\to}q},\ P((\alpha_1,\eta_1)=(0{,}2))=Q=1-P \tag{8}$$

with $P(\alpha_n=1)\equiv P,\ P(\alpha_n=0)\equiv Q$. Random sequence $\alpha_n,\ \ n\geq 1$, is stationary Markov chain.

Theorem 1. The equalities

$$A=Q\pi_{\to}q, \tag{9}$$

$$B=\pi_{\to}qQ(1-\pi_{\to}qQ+2PQ)>0 \tag{10}$$

are true.

Proof. To define the constants $A,\ B$ from (7) we construct recurrent algorithm. Denote

$$M_n=M\eta_n,\ A_n=M(\eta_n\,|\alpha_n=1),\ B_n=M(\eta_n\,|\alpha_n=0),\ M_n=A_nP+B_nQ, \tag{11}$$

$$M_n'=M\eta_n^2,\ A_n'=M(\eta_n^2|\alpha_n=1),\ B_n'=M(\eta_n^2|\alpha_n=0),\ M_n'=A_n'P+B_n'Q \tag{12}$$

where

$$A_1=1,\ B_1=2,\ A_1'=1,\ B_1'=4.$$

Using the formulas (1) - (3), (11) obtain for $n\geq 1$:

$$A_{n+1}=\frac{A_nP\pi_{\to}p+A_nP\pi_{||}p+(B_n-1)Q\pi_{||}p+A_nP\pi_{||}q}{P},$$

$$B_{n+1}=\frac{B_nQ\pi_{\to}p+(A_n+1)P\pi_{\to}q+(B_n+1)Q\pi_{\to}q+B_nQ\pi_{||}q}{Q},$$

$$M_{n+1}=A_nP+B_nQ-Q\pi_{||}p+P\pi_{\to}q+Q\pi_{\to}q=M_n+Q\pi_{\to}q=M_1+nQ\pi_{\to}q,\qquad M_1=1+Q.$$

Then from (7) we obtain the equality (9).

And

$$A_{n+1}-B_{n+1}=(A_n-B_n)\lambda-\left(2\pi_{\to}q+\pi_{||}p\right),\ n\geq 1,\ \lambda=\pi_{||}q+\pi_{\to}p<1. \tag{13}$$

so

$$A_{n+1}-B_{n+1}=-\left[\lambda^n+\left(2\pi_{\to}q+\pi_{||}p\right)\frac{1-\lambda^n}{1-\lambda}\right]=\lambda^nQ-1-Q,\qquad A_{n+1}P+B_{n+1}Q=M_{n+1}$$

consequently

$$A_{n+1} = M_{n+1} + Q[\lambda^n Q - 1 - Q],\ B_{n+1} = M_{n+1} - P[\lambda^n Q - 1 - Q],\ n \geq 1. \qquad (14)$$

Begin now a calculation of M'_{n+1}. Using the formulas (1) - (3), (12) obtain for $n \geq 1$:

$$A'_{n+1} = \frac{A'_n P\pi_{\rightarrow} p + A'_n P\pi_{||} p + (B'_n - 2B_n + 1)Q\pi_{||} p + A'_n P\pi_{||} q}{P},$$

,

$$B'_{n+1} = \frac{B'_n Q\pi_{\rightarrow} p + (A'_n + 2A_n + 1)P\pi_{\rightarrow} q + (B'_n + 2B_n + 1)Q\pi_{\rightarrow} q + B'_n Q\pi_{||} q}{Q},$$

$$M'_{n+1} = M'_n + 2A_n Q\pi_{||} p + 2B_n Q(\pi_{\rightarrow} q - \pi_{||} p) + \pi_{\rightarrow} q(1 + P).$$

So from (14) we obtain

$$M'_{n+1} = M'_1 + 2Q\pi_{||} p \sum_{k=0}^{n-1} A_{k+1} + 2Q(\pi_{\rightarrow} q - \pi_{||} p) \sum_{k=0}^{n-1} B_{k+1} + n\pi_{\rightarrow} q(1 + P) =$$

$$= M'_1 + 2Q\pi_{\rightarrow} q \sum_{k=0}^{n-1} M_{k+1} - 2nQP(1 + Q)\pi_{||} p + n\pi_{\rightarrow} q(1 + P) + 2\pi_{\rightarrow} q P^2 Q \frac{1 - \lambda^n}{1 - \lambda} =$$

$$= M'_1 + 2Q\pi_{\rightarrow} q(n(1 + Q) + \pi_{\rightarrow} qQn(n - 1)/2) - 2nQP(1 + Q)\pi_{||} p +$$

$$+ n\pi_{\rightarrow} q(1 + P) + 2P^2 Q^2 (1 - \lambda^n), \qquad M'_1 = 1 + 3Q.$$

Consequently

$$D\eta_{n+1} = M'_{n+1} - M^2_{n+1} = n\pi_{\rightarrow} q[1 + P - Q^2 \pi_{\rightarrow} q - 2P^2(1 + Q)] + 2P^2 Q^2 (1 - \lambda^n) + QP.$$

Then from (7), (13) we have

$$B = \pi_{\rightarrow} q\big(1 + P - Q^2 \pi_{\rightarrow} q - 2P^2(1 + Q)\big) = \pi_{\rightarrow} qQ(1 - \pi_{\rightarrow} qQ + 2PQ) > 0.$$

Theorem is proved.

Remark 2. From Remark 1 is possible to replace the condition (8) by more natural suggestion

$$P\big((\alpha_1, \eta_1) = (1{,}1)\big) = p, \qquad P\big((\alpha_1, \eta_1) = (0{,}2)\big) = q$$

so that the equalities (6), (9), (10) are true also.

REFERENCES

1. Barlow R.E., Proschan F. 1965. *Mathematical Theory of Reliability*. London and New York: Wiley.
2. Ushakov I.A. et al. 1985. *Reliability of technical systems.* Handbook. Moscow: Radio and Communication (in Russian).
3. Riabinin I.A. 2007. *Reliability and safety of structural complicated systems.* Sankt-Petersberg: Edition of Sankt-Petersberg university (in Russian).

4. Solojentsev E.D. 2006. Specific of logic-probability risk theory with groups of incompatible events. *Automatics and remote control,* no. 7: 187-203 (in Russian).
5. Satyanarayana A., Wood R.K. 1985. A linear time algorithm for computing k-terminal reliability in series-parallel networks. *SIAM, J. Computing,* v. 14: 818-832.
6. Ball M.O., Colbourn C.J., Provan J.S. 1995. Network Reliability. Network Models. *Handbook of Operations Research and Management Science,* v. 7: 673-762.
7. Raigorodskiy A. 2010. Models of random graphs and their application. *Proceedings of MPhTI*, v. 2, no. 4: 130-140 (in Russian).
8. Kumar R., Raghavan P., Rajagopalan S., Sivakumar D., Tomkins A., Upfal E. 2000. Stochastic model for the web graph. *Proc. 41-st Symposium on Foundation of Computer Science.*
9. Sukach E., Ratobilskaia D., Demuskov A. 2011. Computer system of probability-algebraic modeling of systems with many states. *Mathematical mashines and systems,* no. 3: 32-39 (in Russian).
10. Sukach E., 2011. Approaches to decrease dimension of multi component and structurally complicated systems with many states in estimate of their reliability. *Problems of physics, mathematics and technique.* no. 3: 99-103 (in Russian).
11. Timashev A. 2011. Asymptotic expansions for distributions of numbers of connectivity components in random images and decompositions. *Discrete mathematics,* v. 23, no. 2: 66-75 (in Russian).
12. Romanovsky V. 1949. *Discrete Markov chains.* Moscow-Leningrad: State Publishing House of Technical-Theoretical Literature (in Russian).

UNIVERSAL GENERATING FUNCTION & OPTIMAL REDUNDANCY PROBLEMS

Igor Ushakov

•

San Diego, California, USA
e-mail: igusha22@gmail.com

1. Introduction

The Method of U-functions, or the Method of the Universal Generating Function (UGF), was introduced in [(1986) Ushakov, (1987) Ushakov] and later developed in [(1988) Ushakov; (1995) Gnedenkon& Ushakov]. Actually this is a generalization and "algebraic" formalization of the well-known Kettelle's Algorithm [(1962) Kettelle]. In turn, Kettelle's Algorithm, is a form of presentation of convolution of discrete random variables. The method of U-functions is very convenient for computerized calculations.

Last years, this method was significantly developed by G. Levitin and A. Lisnianski

2. Briefly about Generating Function

Everybody knows that Generating Function (GF) is very convenient mathematical tool for finding a convolution of discrete random variables.

Consider two non-negative discrete random variables X_1 and X_2 that are characterized by discrete distributions

$$P^{(1)}(x) = \begin{cases} P\{X^{(1)} = x_1^{(1)}\} = p_1^{(1)}, \\ P\{X^{(1)} = x_2^{(1)}\} = p_2^{(1)} \\ \dots \\ P\{X^{(1)} = x_{n(1)}^{(1)}\} = p_{n(1)}^{(1)}, \end{cases} \quad \text{and} \quad P^{(2)} = \begin{cases} P\{X^{(2)} = x_1^{(2)}\} = p_1^{(2)}, \\ P\{X^{(2)} = x_2^{(2)}\} = p_2^{(2)}, \\ \dots \\ P\{X^{(2)} = x_{n(2)}^{(2)}\} = p_{n(2)}^{(2)}, \end{cases} \tag{1}$$

correspondingly where $n(1)$ and $n(2)$ are numbers of discrete realizations of values of each type.

If we are interested in the distribution of r.v. $X = X_1 + X_2$, we perform product of generating functions, perform collecting terms, and get:

$$\varphi(z) = \varphi^{(1)}(z) \cdot \varphi^{(2)}(z) = \left(\left(\sum_{1 \le i \le n(1)} p_i^{(1)} z^{a_i^{(1)}} \right) \times \left(\sum_{1 \le j \le n(2)} p_j^{(2)} z^{a_j^{(2)}} \right) \right) = \sum_{1 \le k \le n} p^k z^{A_k} \tag{2}$$

where A_k is a convolution of two r.v.'s. – $a_i^{(1)}$ and $a_i^{(1)}$.

Thus, this transform suggests multiplication of polynomial coefficients and summation of polynomial powers. The method of U-functions suggests a transparent and convenient method of computerized solutions of various enumeration problems where variables are subjects to operations beyond multiplication and summation, for instance, finding distribution of minimum, maximum, geometrical summation, etc., depending on physical nature of the analyzed problem.

Table 1. Examples of object interaction depending on physical nature of unit parameters

Name	$\bigotimes_{SERIES} (\mu_1, \mu_2)$	$\bigotimes_{PARALLEL} (\mu_1, \mu_2)$
PFFO (for "hot" redundancy)	$\mu_1 \times \mu_2$	$(1-\mu_1)\times(1-\mu_2)$
Cost	$\mu_1 + \mu_2$	$\mu_1 + \mu_2$
Weight	$\mu_1 + \mu_2$	$\mu_1 + \mu_2$
el. Resistance	$\mu_1 + \mu_2$	$((\mu_1)^{-1} + (\mu_2)^{-1})^{-1}$
el. Capacity	$((\mu_1)^{-1} + (\mu_2)^{-1})^{-1}$	$\mu_1 + \mu_2$
el. Conductivity	$((\mu_1)^{-1} + (\mu_2)^{-1})^{-1}$	$\mu_1 + \mu_2$
pipeline capacity	$\min(\mu_1, \mu_2)$	$\mu_1 + \mu_2$
random time to failure	$\min(\mu_1, \mu_2)$	$\max(\mu_1, \mu_2)$
…	…	…
number of different redundant units	(μ_1, μ_2)	(μ_1, μ_2)

Here by symbol "$\otimes$" we denote interaction of parameters of various physical nature. In particular, the method of U-functions can be effectively applied to solving the optimal redundancy problem.

3. Method of U-functions

Let us consider GF from another viewpoint. Each *k*-th discrete distribution one can represented as a set of triplets:

$$S^{(k)} = \left\{ (p_1^{(k)}, c_1^{(k)}, 1), (p_2^{(k)}, c_1^{(k)}, 2), ..., (p_1^{(k)}, c_{n(k)}^{(k)}, n(k)) \right\} \tag{3}$$

where $p_j^{(k)}$ and $c_j^{(k)}$ are the probability of failure-free operation (PFFO) of unit *k* with *j*-th variant of redundancy and the cost of this variant, correspondingly. The third component is the number of redundant units of type *k* (or, in more general case, the ordering number of variant of unit *k*).

Indeed, product of two GF's is equivalent to "Descartes interaction" of two sets $S^{(1)}$ and $S^{(2)}$, i.e. each triplet of set $S^{(1)}$ interacts with all triplets of set sets of $S^{(2)}$. Interaction of two triplets can be conditionally written as follows:

$$(p_i^{(1)}, c_i^{(1)}, i^{(1)}) \otimes (p_j^{(2)}, c_j^{(2)} j^{(2)}) \tag{4}$$

In turn, interaction of triplets consists of interactions of its components that produce a new triplet

$$\left(p_i^{(1)} \otimes^{\Pi} p_j^{(2)}; c_i^{(1)} \otimes^{\Sigma} c_j^{(2)}; i^{(1)} \otimes^{\cup} j^{(2)} \right) = \left(p^{(2*)}, c^{(2*)}, j^{(2)}) \right) \tag{5}$$

Here interaction $\otimes^{\Pi}$ means product, operator $\otimes^{\Pi}$ does summation, and operator $\otimes^{\Pi}$ does union (a vector with corresponding components), i.e.

$$p^{(2*)} = p_i^{(1)} \otimes^{\Pi} p_j^{(2)} = p_i^{(1)} \times p_j^{(2)};$$
$$c^{(2*)} = c_i^{(1)} \otimes^{\Sigma} c_j^{(2)} = c_i^{(1)} + c_j^{(2)};$$
$$j^{(2*)} = i^{(1)} \otimes^{\cup} j^{(2)} = (i^{(1)},\ j^{(2)}) \qquad (6)$$

One can easily see that Descartes interaction of duplets that belongs sets $S^{(1)}$ and $S^{(2)}$ is completely equivalent to product of two generating functions $\varphi^{(1)}(z)$ and $\varphi^{(2)}(z)$.

Analogously with the product of GF's one has to collect terms for getting the final set

$$S = S_1 \otimes S_2.$$

Naturally, operator $\otimes$ possesses commutativity property, i.e.

$$\otimes\ (a, b) = \otimes\ (b, a) \qquad (7)$$

and associativity property, i.e.

$$\otimes\ (a, b, c) = \otimes\ (a \otimes (b, c)) = \otimes\ ((a \otimes b), c). \qquad (8)$$

4. Using U-function for solving of optimal redundancy problems

Let us consider a series system consisting of n units, each of which has PFFO equals p_k and costs c_k units. For increasing reliability of each unit, one can use redundancy of individual units. Each unit k is represented by set of triplets

$$S_k = \left[\{R_0^{(k)}; C_0^{(k)}; 0\}, \{R_1^{(k)}; C_1^{(k)}; 1\}, \{R_2^{(k)}; C_2^{(k)}; 2\}, ..., \{R_s^{(k)}; C_s^{(k)}; s\} ...\right] \qquad (9)$$

where s is the number of redundant units (any natural number); $C_s^{(k)}$ is the total cost of s redundant units (usually, a linear function of the number s); and $R_s^{(k)}$ is the PFFO of unit k with s redundant units. It is well-known that for loaded redundancy of group including one main and s identical redundant units:

$$R_s^{(k)} = 1 - (1 - p_k)^{s+1};$$

and for an unloaded redundant (spare) units:

$$R_s^{(k)} = \sum_{0 \le j \le s} \frac{(\lambda_k t)^j}{j!} \exp(-\lambda_k t);$$

Now consider a general procedure of optimal redundancy with the use of U-functions. First of all, take units 1 and 2 and arrange the Descartes interaction procedure between sets S_1 and S_2. In our case

$$R_i^{(1)} \otimes R_j^{(2)} = R_i^{(1)} \times R_j^{(2)} = R_K^{(2*)};$$
$$C_i^{(1)} \otimes C_j^{(2)} = C_i^{(1)} + C_j^{(2)} = C_K^{(2*)};$$
$$i \otimes j = (i,\ j) = \vec{K},$$

i.e. interaction between two numbers produces vector, containing numbers of redundant units of the 1st and 2nd types..

Here symbol "*" relating to the number means that this "aggregated" unit includes all previous units.

At the next step of sets S_1 and S_2 interaction one takes "aggregated unit 2* and unit 3:

$$R_K^{(2*)} \otimes R_k^{(3)} = R_K^{(2*)} \times R_k^{(3)} = (R_i^{(1)} \times R_j^{(2)}) \times R_k^{(3)} = R_L^{(3*)};$$
$$C_K^{(2*)} \otimes C_k^{(3)} = C_K^{(2*)} + C_k^{(3)} = C_i^{(1)} + C_j^{(2)} + C_k^{(3)} = C_L^{(3*)};$$
$$\vec{K} \otimes k = (\vec{K},\ k) = (i,\ j,\ k) = \vec{L}.$$

Vector $\vec{L}$ shows that in a series system of 3 units the 1st unit has i redundant ones, the 2nd unit has j redundant ones and the 3rd units has k redundant ones.

This procedure continues until necessary final triplets will have been generated. Instead of further abstract presentation of the procedure, let us turn to a simple illustrative numerical example.

The result of interaction is presented in the table below.

Example 1. Consider a series system of four units with parameters given in the table below. Assume that "hot" redundancy is used for the system reliability improvement.

Table 2. System unit parameters

	Unit-1	Unit-2	Unit-3	Unit-4
PFFO	0.6	0.6	0.7	0.7
Cost	3	5	3	5

In accordance with the description given above, the block diagram of the using U-functions in this particular case can be presented as follows (see Figure 1).

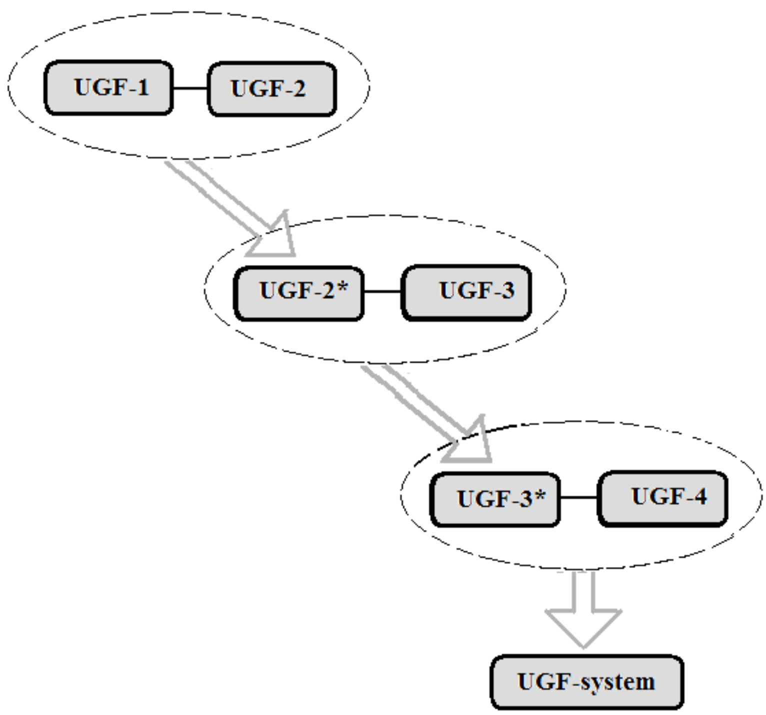


Figure 1. Block-diagram of the solution procedure for Example 1

Let us solve two problems of optimal redundancy:

(a) _Direct problem_: Find the optimal allocation of redundant units to reach required PFFO level of the system equals to 0.97;

(b) _Inverse problem_: Find the optimal allocation of redundant units to reach maximum possible PFO level under condition that the total cost of the system does not exceed 70 units of cost.

In this case, the UGF for each unit is defined by a set of triplets (Cost, PFFO, Number of redundant units). Solution for the first step of the solution (interaction of set 1 and set 2) can be presented in the form of the table below.

First of all, restrict ourselves with possible solutions for the Direct Problem: since the system PFFO has to be not less than 0.97, it means that PFFO of each of four redundant groups has to be not less than 0.97.

Since cost restriction equals 70 cost units, the total cost of redundant units in each redundant group has to be not larger than, say 20-30.

Keeping this in mind let us construct the table with triplets obtained in the result of interaction sets for Unit-1 and Unit -2.

Table 3. Result of interaction of UGF's for Unit-1 and Unit-2

		S_1					
		9 0.936 3	**12 0.9744 4**	**15 0.9898 5**	**18 0.9959 6**	**21 0.9984 7**	**24 0.9993 8**
	15 0.936 3	24 0.8761 (3; 3)	27 0.912 (4; 3)	30 0.9264 (5; 3)	33 0.9322 (6; 3)	36 0.9345 (7: 3)	39 0.9354 (7:3)
	20 0.9744 4	29 0.912 (3; 4)	32 0.9495 (4; 4)	35 0.9644 (5; 4)	38 0.9704 (6; 4)	41 0.9728 (7; 4)	44 0.9738 (8; 4)
S_2	**25 0.9898 5**	34 0.9264 (3; 5)	37 0.9644 (4; 5)	40 0.9796 (5; 5)	43 0.9857 (6; 5)	46 0.9881 (7; 5)	49 0.9891 (8; 5)
	30 0.9959 6	39 0.9322 (3; 6)	42 0.9704 (4; 6)	45 0.9857 (5; 6)	48 0.9918 (6; 6)	51 0.9943 (6; 7)	54 0.9953 (6;8)
	35 0.9984 7	44 0.9345 (3; 7)	47 0.9728 (4; 7)	50 0.9881 (5; 7)	53 0.9943 (6; 7)	56 0.9967 (7; 7)	59 0.9977 (7; 8)
	40 0.9993 8	49 0.9354 (3; 8)	52 0.9738 (4; 8)	55 0.9891 (5; 8)	58 0.9953 (6; 8)	61 0.9977 (7; 8)	64 0.9987 (8; 8)

In this table triplets that are dominated by others are marked with grey shadowing. One can observe that dominating sequence occupies an area around "diagonal of the table. This property can be successfully used for minimizing the calculations: as soon as a dominated triplet appears below this "diagonal area", the further calculation in cells located below this cell can be stopped. Analogously, if a dominated triplet appears upper this "diagonal area" , the further calculation in cells located to the right from this cell can be also stopped. We will use this property in further calculating.

Thus, the dominating sequence characterizing an "equivalent" Unit-2*is presented in non-shadowed area of table 1. On the basis of data for Unit-2*, we can construct an analogous table for "equivalent Unit-3* (see Table 4).

Table 4. Result of interaction of UGF's for Unit-2* and Unit-3

		***S*₃**					
		9 0.973 3	**12 0.9919 4**	**15 0.9976 5**	**18 0.9993 6**	**21 0.9998 7**	**24 0.9999 8**
	24 0.8761 (3; 3)	33 0.8524 (3; 3; 3)	36 0.869 (3; 3; 4)	xxx	xxx	xxx	xxx
	27 0.912 (4; 3)	36 0.8874 (4; 3; 3)	39 0.9046 (4; 3; 4)	42 0.9098 (4; 3; 5)	xxx	xxx	xxx
	30 0.9264 (5; 3)	39 0.9014 (5; 3; 3)	42 0.9189 (5; 3; 4)	45 0.9242 (5; 3; 5)	xxx	xxx	xxx
S*₂	**32 0.9495 (4; 4)**	41 0.9239 (4; 4; 3)	44 0.9418 (4; 4; 4)	47 0.9472 (4; 4; 5)	xxx	xxx	xxx
	35 0.9644 (5; 4)	44 0.9384 (5; 4; 3)	47 0.9566 (5; 4; 4)	50 0.9621 (5; 4; 5)	xxx	xxx	xxx
	38 0.9704 (6; 4)	0.9442 (6; 4; 3)	50 0.9625 (6; 4; 4)	53 0.9681 (6; 4; 5)	xxx	xxx	xxx
	40 0.9796 (5; 5)	0.9532 (5; 5; 3)	52 0.9717 (5; 5; 4)	55 0.9772 (5; 5; 5)	xxx	xxx	xxx
	43 0.9857 (6; 5)	0.9591 (6; 5; 3)	55 0.9777 (6; 5; 4)	58 0.9833 (6; 5; 5)	61 0.9850 (6; 5; 6)	xxx	
	46 0.9881 (7; 5)	xxx	58 0.9801 (7; 5; 4)	61 0.9857 (7; 5; 5)	64 0.9874 (7; 5; 6)	67 0.9879 (7; 5; 7)	xxx
	49 0.9891 (8; 5)	xxx	xxx	64 0.9867 (8; 5; 5)	67 0.9884 (8; 5; 6)	70 0.9889 (8; 5; 7)	73 0.9885 (8; 5; 8)

Table 5. Final result of calculating

		***S*₄**					
		15 0.973 3	**20 0.9919 4**	**25 0.9976 5**	**30 0.9993 6**	**35 0.9998 7**	**40 0.9999 8**
	41 0.9239 (4; 4; 3)	56 0.899 (4; 4; 3; 3)	61 0.9164 (4; 4; 3; 4)	xxx	xxx	xxx	xxx
	44 0.9418 (4; 4; 4)	59 0.9164 (4; 4; 4; 3)	64 0.9342 (4; 4; 4; 4)	69 0.9395 (4; 4; 4; 5)	xxx	xxx	xxx
	47 0.9566 (5; 4; 4)	62 0.9308 (5; 4; 4; 3)	67 0.9489 (5; 4; 4; 4)	72 0.9543 (5; 4; 4; 5)	xxx	xxx	xxx
S*₃	**50 0.9625**	65 0.9365	70 0.9547	75 0.9602	xxx	xxx	xxx

	(6; 4; 4)	(6; 4; 4; 3)	(6; 4; 4; 4)	(6; 4; 4; 5)			
	52 **0.9717** **(5; 5; 4)**	67 0.9455 (5; 5; 4; 3)	72 0.9638 (5; 5; 4; 4)	77 0.9694 (5; 5; 4; 5)	xxx	xxx	xxx
	55 **0.9777** **(6; 5; 4)**	70 0.9513 (6; 5; 4; 3)	75 0.9698 (6; 5; 4; 4)	80 0.9754 (6; 5; 4; 5)	85 0.9770 (6; 5; 4; 6)	xxx	xxx
	58 **0.9833** **(6; 5; 5)**	0.9568 (6; 5; 5; 3)	78 0.9753 (6; 5; 5; 4)	83 0.9809 (6; 5; 5; 5)	88 0.9826 (6; 5; 5; 6)	xxx	xxx
	61 **0.9857** **(7; 5; 5)**	0.9591 (7; 5; 5; 3)	81 0.9777 (7; 5; 5; 4)	86 0.9833 (7; 5; 5; 5)	91 0.985 (7; 5; 5; 6)	xxx	xxx
	64 **0.9874** **(7; 5; 6)**	xxx	84 0.9794 (7; 5; 6; 4)	89 0.985 (7; 5; 6; 5)	94 0.9867 (7; 5; 6; 6)	99 0.9872 (7; 5; 6; 7)	xxx
	67 **0.9884** **(8; 5; 6)**	xxx	87 0.9804 (8; 5; 6; 4)	92 0.9860 (8; 5; 6; 5)	97 0.9877 (8; 5; 6; 6)	102 0.9882 (8; 5; 6; 7)	107 0.9883 (8; 5; 6; 8)

All calculations have been done with a simple Excel program.

Solutions of the problems above can be easily found from the last table. First time PFFO exceed level of 0.97 when X=(6; 5; 5; 4) and the corresponding system cost is 78 cost units. The inverse problem solution for restriction on the cost equals 70 cost units reaches when X=(4,4,5,3) and corresponding PFFO is equal to 0.9547.

Notice that due to associativity property of U-functions it is possible to get the same solution using another order of units' interaction.

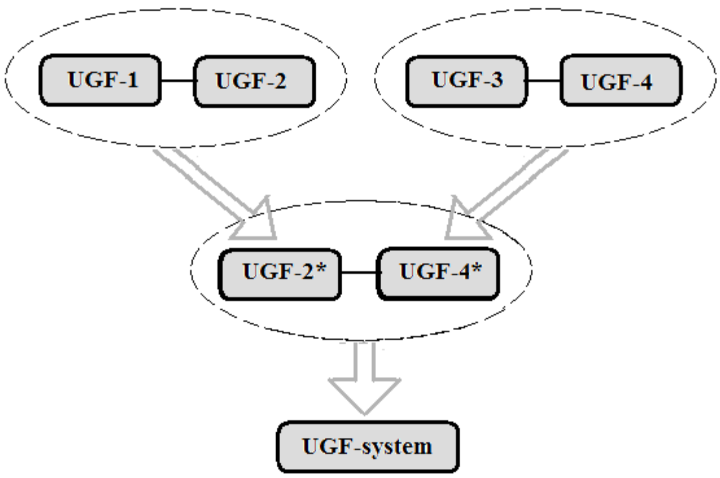


Figure 2. Second type of units' interaction procedure

Remark. By the way, this example shows with transparency that one can consider not only redundancy as a method of system reliability increase. For instance, one can consider a set of variants of the units with various reliability and cost. Actually, Unit-2* and Unit-4* can be considered as "black boxes" that are characterized by corresponding dominating sequences of triplets $\{Q_s^{(k)};C_s^{(k)};s\}$ where *s* is just a number of variants of considered Unit-*k*.

Bibliography in chronological order

(This bibliography on the subject is presented in chronological order to show the steps of the U-functions method development).

(1962) Kettele, J. D. , Jr. Least-coast allocation of reliability investment. *Operations Research*, No. 2.

(1986) Ushakov, I.A. Universal Generating Function (in Russian). Engineering Cybernetics, No. 5.

(1987) Ushakov, I.A. Universal generating function. Soviet Journal Computer Systems Science, No.3.

(1987) Ushakov, I.A. Optimal standby problem and a universal generating function. Soviet Journal Computer and System Science, No.4.

(1987) Ushakov, I.A. Solution of multi-criteria discrete optimization problems using a universal generating function. Soviet Journal of Computer and System Sciences , No. 5.

(1988) Ushakov, I.A. Solving of optimal redundancy problem by means of a generalized generating function. Elektronische Informationsverarbeitung und Kybernetik, No.4-5

(1995) Gnedenko, B.V., and Ushakov I.A. Probabilistic Reliability Engineering. John Wiley & Sons.

(1998) Levitin, G., Lisnianski, A., Ben Haim, H., and Elmakis, D. Redundancy optimization for series-parallel multi-state systems, IEEE Transactions on Reliability, , No. 2.

(1999) Levitin, G., and Lisnianski, A. Importance and sensitivity analysis of multi-state systems using universal generating functions method, Reliability Engineering & System Safety, No. 65.

(2000) Levitin, G., and Lisnianski, A. Optimal Replacement Scheduling in Multi-state Series-parallel Systems. Quality and Reliability Engineering International, , No. 16.

(2000) Ushakov, I.A. The method of generating sequences. European Journal of Operational Research, No. 2.

(2001) Levitin, G. Redundancy optimization for multi-state system with fixed resource-requirements and unreliable sources. IEEE Transactions on Reliability, No. 50.

(2001) Levitin, G., and Lisnianski, A. A new approach to solving problems of multi-state system reliability optimization, Quality and Reliability Engineering International, No. 47.

(2002) Levitin, G. Optimal allocation of multi-state elements in linear consecutively-connected systems with delays. International Journal of Reliability Quality and Safety Engineering, No. 9.

(2003) Levitin, G. Optimal allocation of multi-state elements in linear consecutively-connected systems. IEEE Transactions on Reliability, No. 2.

(2005) Levitin, G. The Universal Generating Function in Reliability Analysis and Optimization. Springer.

DYNAMIC MULTI-CRITERIA DECISION MAKING METHOD FOR SUSTAINABILITY RISK ANALYSIS OF STRUCTURALLY COMPLEX TECHNO-ECONOMIC SYSTEMS

Alexander V. Bochkov[1], Valery V. Lesnykh, Nikolay N. Zhigirev,

•

Science Research Institute of Economics and Management
in Gas Industry (LLC "NIIGAZECONOMIKA")
[1]Corresponding author e-mail: a.bochkov@gmail.com

Abstract

The paper considers the characteristics of the functioning and sustainability of the structurally complex techno-economic systems (SCTES) in terms of different types of risk. The validity of the application to describe the behavior of this class of systems of semi-empirical mathematical models, which are based on a vector description of the system states, using the criteria approach for assessing the quality of its functioning, is demonstrated.

Under discussion is conceptual model for the interaction of the object and its environment, allowing to estimate the "optimal" allocation ratio between the productive system and its development potential.

The concept of non-formalizable threats for the sustainable functioning of this systems class was introduced. Expert procedure to account non-formalizable threats in case of risk assessment was proposed. For the construction of indicators for assessing the status the methods of quantitative analysis based on the theory and multi-criteria utility was used. Multi-criteria utility as an indicator of sustainability in the form of dimensionless complex hierarchy of indicators was proposed. Computing for through the convolution of the primary indicators.

The hierarchical model proposed to calculate the integral index of multi-criteria preference of one embodiment of the system over the other. Some results of case study are discussed.

Key words: structurally complex techno-economic systems, risk analysis, sustainability, multi-criteria method

SCTES are characterized by distribution in space, big variety and interaction of objects types, non-uniform structure of processing chains, unique conditions of influence of risks of the various nature on objects of the subsystem and the system as a whole.

In the idea of situation management of SCTES principles of changeable (adaptive) behavior in terms of possible risks and uncertainties are initially put. Presence of such risks generated by different circumstances is capable to brake or change this or that way of movement, to force the system to live «under another scenario», different from all variety of plans shaped before.

If as sustainability of SCTES functioning to understand the plan performance of its development with admissible variation on volumes and terms of problems performance then situation safety management in this system is reduced to minimization of hazardous losses at extraordinary situations and to carrying out of actions for their prediction. The success of such tactics depends substantially on intuition and talent of management of the company, on its ability to expect the possibility of weakly formalized threats outgrowth into notable risks and losses.

Under weakly formalized threats we understand here the threat for criticality estimation of which the development of original algorithm of the decision depending on a concrete situation is required and for which uncertainty and dynamism of the initial data and knowledge can be characteristic.

However, in the absence in enough of adequately estimated information necessary for decision-making, tactics of adaptive management quite often turns to a continuous chain of the "emergency" scenarios leading to disruption of controllability of all system. Hence, company management should be engaged not only the current work bringing quite notable results which utility is measured in economic factors, but also to care of creation the company condition monitoring system and the world surrounding it, watching dynamics of internal and external threats to its growth and development.

That is optimal control of SCTES aimed at reception of profit on its activity, consists in ability to find balance of redistribution of the available resources (material, human, information) by proprietor of the company between «productive activity» and «maintenance of development potential ».

The elementary model illustrating the abovementioned and allowing to estimate "optimal" proportions of resources distribution between "useful" (production) system and its development potential is the model of interaction of developing object and its environment (**Klykov, 1970; Zhigirev, etc., 1983**).

Let's present the activity of some SCTES, consisting of two subsystems (fig. 1).

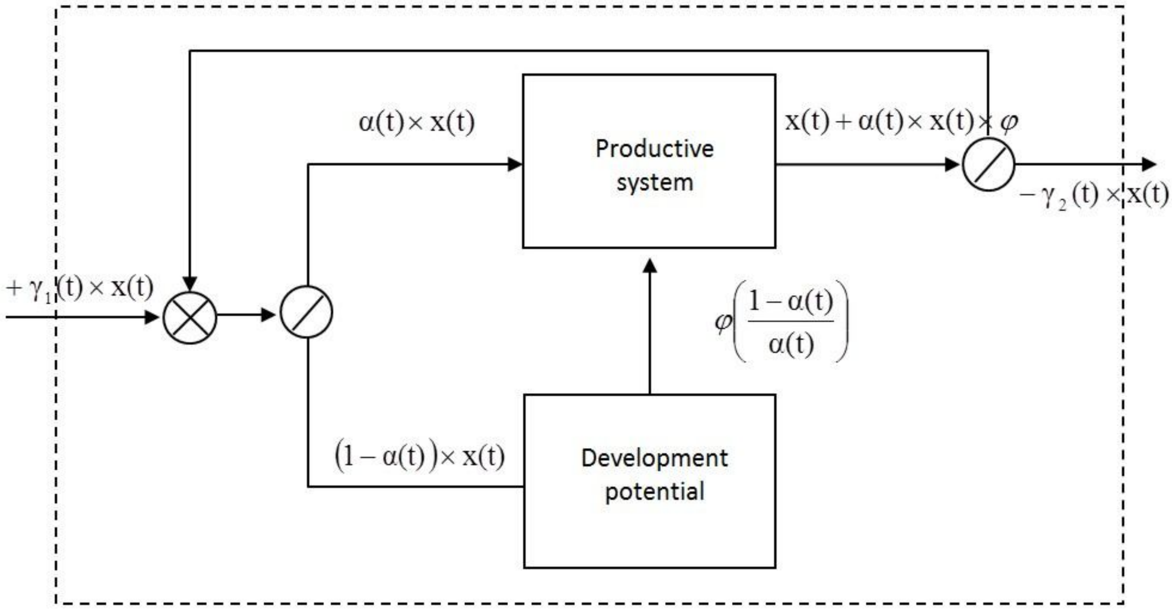


Figure 1. Activity of productive system

The first, «productive subsystem», brings profit, proportionally to quantity of received resources $(\alpha(t) \times x(t))$ with some positive resources increase speed coefficient $\varphi\left(\frac{1-\alpha(t)}{\alpha(t)}\right)$, available in the system.

The second subsystem - «development potential» plays the role of the accelerator (retarder) of resources reproduction speed in the system.

Herein $\alpha(t)$ is proportion of resources distribution between productive subsystem and its development potential, $\gamma_1(t), \gamma_2(t)$ - coefficient of resources exchange intensity between investigated system and some external in relation to it system in the process of coexistence.

Actually the difference $\gamma(t) = (\gamma_1(t) - \gamma_2(t))$ is the share of resources deduced from the cycle of reproduction in the form of losses of one kind or another, for example, of final consumption, taxes, etc.

In the elementary representation "potential" influence is the value of function $\varphi\left(\frac{1-\alpha(t)}{\alpha(t)}\right)$ and coefficient $\gamma(t)$ for large-scale systems we consider independent of times in an explicit form as constants. In this case system development is described by the homogeneous linear equation on a variable $x(t)$ at parameters α and γ

$$\frac{dx(t)}{dt} = \alpha(t) \times x(t) \times \varphi\left(\frac{1-\alpha(t)}{\alpha(t)}\right) + \gamma \times x(t) \tag{1}$$

The optimum proportion $\alpha^*(t)$ between productive system and its development potential is defined from the condition

$$\alpha^* \times \varphi\left(\frac{1-\alpha^*}{\alpha^*}\right) \to \max \tag{2}$$

At the natural assumption that $\varphi(\xi)$ is monotone function with saturation (fig. 2) there is a simple way of its optimum definition, as $\alpha = \left(\frac{1}{1+\xi}\right)$.

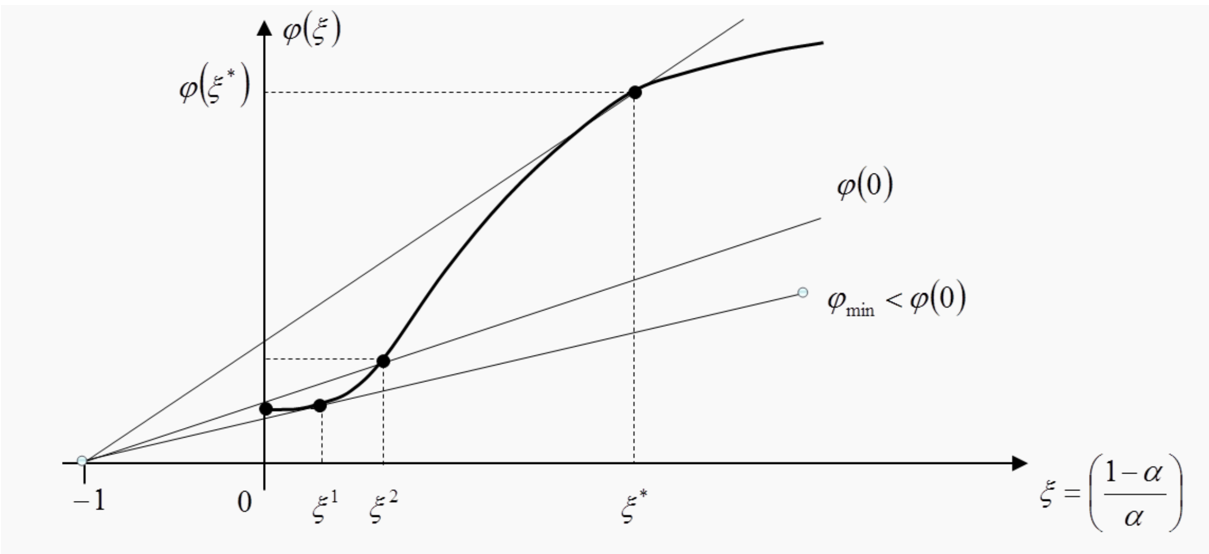


Figure 2.

According to fig. 2 it is clear that this optimum is reached in some point ξ^*, having quite certain sense. So, if resources for development potential are allocated «excessively much» $(\xi \succ \xi^*)$, then means $(\xi - \xi^*)$ are incorrectly withdrawn from the current reproduction and there is a situation when efforts to studying and counteraction to numerous risks which the developing system can never come across are spent.

The point $(\xi = 0)$ corresponds to the situation when all resources are spent exclusively for growth of productive system. The potential of similar system is low because of constant losses which it is possible to avoid if there is a potential for prediction of arising risks and struggle against them.

The segment $\xi = \left(0, \xi^1\right)$ shows that if the means allocated for studying and counteraction to threats and risks are small, then return from similar researches and done actions less than the resources allocated for them. Information gathering, research of internal and external threats on a low level doesn't allow to receive an adequate estimation for improvement of decision-making quality in most cases anyhow developing circumstances.

On the segment $\xi = \left(\xi^1, \xi^*\right)$ the contribution to development potential starts to give positive return, however only in the point ξ^2 the level of "self-support" of expenses for development of "potential" of system will be reached $\left(\varphi\left(\xi^2\right) = \varphi(0)\right)$.

Therefore it is expedient to consider this point as the point of "critical" position.

Decrease of potential $(\varphi(\xi))$ to the level $\left(\varphi\left(\xi^2\right)\right)$ threatens that "in accordance with the circumstances" economically expediently there will be «strategy of survival» – strategy of full refusal of expenses for the decision of problems of prediction and anticipation of threats and risks and reproduction maintenance only at the expense of escalating inefficient capacities in productive subsystem $\xi \to 0$.

In spite of the fact that the conceptual model stated above is schematical, it, nevertheless, provides guidance that threats and risks as a matter of fact are "antipotentials" of development, that is they are retarders of speed of all system reproduction.

Since SCTES, as a rule, are non-uniform, they are subject to risks various by the nature and on influence levels. The received expert estimations of optimum proportions, certainly, need updating if to consider balanced development of the system consisting from many productively and territorially connected subsystems.

The logic of optimum proportional development in this case also remains. Received estimations should be considered only as "reference points" for the further researches, otherwise struggle for escalating of development potential will be carried out only in those territories and only in those productive-technological chains for which it by theoretical estimations is "economically expedient" that will lead to destruction of integrity of the system (connectivity loss), withdrawal from unified state and branch standards.

Let's suppose that the exit of investigated system runway from admissible corridor (a component of the vector of functioning efficiency indicators) can be caused to four reasons:

a) owing to increase of importance of the problems put before the system to such level that default of these problems at occurrence of extraordinary situation (and furthermore in a normal mode of functioning) appears critical for system existence, up to necessity of its re-structuring as a whole;

b) owing to system simplification or destruction at which locally arising extraordinary situations are really capable to outgrow in events with large-scale losses under scenarios of cascade type;

c) owing to dramatic or long deterioration of operating conditions of objects and subsystems in one or several territorial formations formed, including, as a result of non-formalizable threats increase;

d) owing to decrease in level of industrial and fire safety and (or) physical protection for technological blocks and objects of various type.

It is offered for an estimation of extraordinary situation threat level in SCTES to use the following hierarchical multi-criteria model (**Russman, 1991**).

Integrated risk of extraordinary situation $R(r_1, \ldots r_i, \ldots, r_n)$ represents function of risks of private extraordinary situations occurrence $r_i \; (i = 1, \ldots, n)$. The kind of dependence R on the arguments gets out proceeding from conditions:

$$0 \le R(r_1, \ldots, r_i, \ldots, r_n) \le 1 ; \qquad (3)$$

$$R(0, \ldots, 0, \ldots, 0) = 0 ; \qquad (4)$$

$$R(0,\ldots,r_i,\ldots,0) = r_i; \tag{5}$$

$0 \le R(r_1,\ldots,1,\ldots,r_n) = 1$ for $\forall\ r_i = 1$ irrespective of values of other arguments. (6)

Continuous function $R(r_1,\ldots r_i,\ldots,r_n)$, meeting (3)-(6), has the following general view

$$R(r_1,\ldots r_i,\ldots,r_n) = 1 - \left\{\prod_{i=1}^{n}(1 - r_i)\right\} \times g(r_1,\ldots r_i,\ldots,r_n), \tag{7}$$

where $g(0,\ldots,r_i,\ldots,0) = 1$.

If in special case $g(r_1,\ldots r_i,\ldots,r_n) \equiv 1$, then formula (7) is of the form

$$R(r_1,\ldots r_i,\ldots,r_n) = 1 - \left\{\prod_{i=1}^{n}(1 - r_i)\right\} \tag{8}$$

states the underestimated estimation of integrated risk from calculation that the stream of extraordinary situations represents a mix of ordinary events taken from homogeneous, but differing with values $r_i\ (i = 1,\ldots,n)$ samples.

But for real systems risks, as a rule, are dependent.

Then we have

$$g(r_1,\ldots r_i,\ldots,r_n) = 1 - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} C_{ij} \times [r_i]^{\alpha_{ij}} \times [r_j]^{\beta_{ij}} \tag{9}$$

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} C_{ij} \le 1 \quad C_{ij} \ge 0$$
$$\alpha_{ij} > 0 \tag{10}$$
$$\beta_{ij} > 0$$

where C_{ij} - coefficients of risks connection of i and j extraordinary situation; α_{ij} and β_{ij} - positive coefficients of elasticity of replacement of corresponding risks, allow to consider the facts of risks replacement, caused mainly by that simultaneously effective actions for decrease in all risks can't be done owing to limitation of time and resources.

The current risks values $r_i\ (i = 1,\ldots,n)$, entering integral risks factor R are values changed in time with various speeds (for example depending on the seasonal factor priorities of solved technological problems essentially change).

Thereof classical calculation of risks equation leads to problems of combinatory complexity on the initial data having objectively casual, uncertain, often qualitative (semiquantitative) nature.

The decision of problems of risks analysis becomes complicated also because non-formalizable threats can play considerable role.

For the account of these factors it is offered to form values r_i as product of four components:

$$r_i = r_i^{(a)} r_i^{(b)} r_i^{(c)} r_i^{(d)}. \tag{11}$$

The component $r_i^{(a)}$ in (11) is estimated through categorizing of problems the performance of which is cancelled or delayed owing to the arisen extraordinary situation (for example, in gas supply systems categorizing can be defined through percentage distribution of categories of power users, affected in case of extraordinary situation because of the termination of gas supply).

The component $r_i^{(b)}$ is estimated through maximum permissible losses (MPL) in extraordinary situations at existing technological levels and materials (subjectively established) calculations of such losses.

Before achievement the level of MPL $r_i^{(b)}$ can be considered as linear function

$$r_i^{(b)} = \frac{\mathrm{L}_i}{\mathrm{MPL}}, \tag{12}$$

where L_i - current level of losses.

At exceeding of the level of MPL $r_i^{(b)}$ fixed with value 1.

The multiplier $r_i^{(c)}$ is estimated as dimensionless value, is calculated under empirically picked up statistical data about characteristics of objects in a binding to their territorial placing and has the meaning of indicator of absolute vulnerability of object on which the scenario of i extraordinary situation is initiated.

In general $r_i^{(c)}$ can be considered as the indicator of environment aggression in which the object functions.

For each territory owing to geographical factors, features of productive structure, sociocultural, ethnic and other differences the construction of unique models of calculation significantly found on personal assessment of the experts familiar with this specificity is required.

And a last, valuation of $r_i^{(d)}$ is in terms of ranging of objects types. It reflects quality of relative "susceptibility" of objects of the set type on a wide range of external changes of the factors defining $r_i^{(c)}$. Values $r_i^{(d)}$ are used so that to result estimations of risks of extraordinary situations initiated by events on objects of various types to a uniform scale.

The offered scheme of calculation of integrated index R is mainly intended for the preliminary analysis of variants of system development on the basis of hierarchy of the indicators characterizing all aspects of extraordinary situations including both estimations of consequences $r_i^{(a)}$, $r_i^{(b)}$, and estimations of causes $r_i^{(c)}$, $r_i^{(d)}$.

Lines of levels of R values allow to build borders of reaction zones to changes of all spectrum of risks: $R \geq R_{кр}$ (a "red" zone demanding change of the existing mode of functioning or additional means for decrease of risks $r_i^{(c)}$ and $r_i^{(d)}$), $R_{кр} \geq R \geq R_{op}$ (an «orange» zone demanding balancing of contract activity, carrying out of diagnostic and other actions), $R \leq R_{op}$ (a "green" zone in which pertinently to speak about growth and the further development of the system, introduction of new capacities and new risks connected with their occurrence).

It is obvious that between developing object what is any of SCTES and its development potential, one of the components of which is the subsystem of safety (risk) management there should be a balance. High-yielding system with the underestimated risk is doomed to inefficient functioning owing to losses and on the contrary, the excessive safety concern leads to withdrawal of resources from a cycle of reproduction.

Thus, for SCTES, having diversified multiphasic production and difficult space-territorial topology of estimations having only economic character are unacceptable. Expediently complex development of development potential control system which can be realized, for example, within the limits of audit formly occurred crisis and precritical situations, estimation and generalization of experience, development of system of indicators of early detection of threats to steady functioning of objects (groups of objects).

Let's notice that during development of such indicators system for realization of situational approach to management of the company, it is necessary to accept a number of "reconciliatory agreements».

The first main agreement consists that preservation of integrity of system and knowledge (information) about it is more significant, rather than economic success of separately taken productive element or productive-territorial formation.

The second agreement: threats and risks are considered as factors, braking development potential and, accordingly, use of the device of an estimation of the analysis and risks management without taking charge for risks corresponding to their competence by regulatory bodies is impossible. Concealment of risks or their revaluation'll become a subject of economic auction, inappropriate in the conditions of approaching threats.

The third agreement: pure «one-criteria» approach when to every disovered risk (social, economic, foreign policy) "cost" estimation of consequences of its realizations and (or) prevention of scenarios of their expansion is offered, isn't universal.

Activity of any person separately, groups of people, labor collective of the company as a whole many-sided and various, the use of multi-criteria approach with elements of "indistinct" logic, with use (as far as data permits) detection device of the latent laws in conjunction with and mutual strengthening of numerous factors therefore is the most pertinent.

The methodological approach taken as a principle offered method has advantage in comparison with the cost approaches, expressed that multi-criteria utility "absorbs" in itself in "share" participation all factors, but not just having cost expression ones (that, however, doesn't allow to remove all uncertainties).

Multi-criteria utility is formed on the indicators having in the basis different dimensions, units and scales of measurements which are easily arranged in the presence of computing resources to specific features of investigated objects and risks generated by numerous factors at different circumstances of place and time.

Classical schemes of multi-criteria analysis, based on linear and multiplicative convolutions are successfully enough used in design and predesign analysis, at the decision of some problems of situational management in marketing, at risks estimation of continuation or termination of research and developmental works but as SCTES is dynamic system, it is offered to use more developed model of dynamic multi-criteria analysis which will allow to include the situations generated so-called non-formalizable threats and risks-factors into consideration.

References

Klykov, Yu. I. (1974) Big systems situational management. «Energiya», Moscow – 130 p.

Zhigirev, N.N. (1983), Vorobiev, E.I., Kuzmin, I.I., Reznichenko, V.Yu. Model of interaction of developing object with environment. // Preprint Institute of Atomic Energy 3799/16. - 69 p.

Russman, I.B. (1991), Kaplinskiy, A.I., Umyvakin, V.M. Modeling and algorithmization non-formalizable problems of the best variants of system choice. Voronezh: VSU - 168 p.

BEAM BEHAVIOUR UNDER MONOTONIC LOADS

Fedol Ghazi
•
Bechar University B. P. 417, 08000 Bechar, Algeria
E-mail: gfedol@yahoo.fr

Pr: Hamouine Abdelmadjid
•
Bechar University B. P. 417, 08000 Bechar, Algeria
E-mail: ahamouine@gmail.com

Dr: Zouaoui Chikr el Mezouar
•
Bechar University B. P. 417, 08000 Bechar, Algeria
E-mail: *chikrtime@yahoo.fr*
Tel: 213 49 81 55 81/91; Fax: 213 49 81 52 44

Abstract

In this work we interest to study the beam behavior under monotonic loads in four point, to improve the mechanical properties of a concrete beam fiber and establish an identification card of the new concrete beams were comparing these beams witnesses.

Keywords: matrix materials, reinforcements, fiber characterization, charge arrow monotonous beam, ductility.

1. Introduction

From earlier the research were based on how to reinforce materials with fiber plantin order to increase there mechanical resistance and improve there stability. In the past they used the ‘’***Torchis ‘’ was*** of clay reinforced with straw put in place by compression.

(Kriker et al 2005) used the date palm fibers as building blocks in cement matrix composites. They showed that the increase in length and percentage of fibers improve the flexural strength and post-elastic hardness of the composite, but decrease the compressive strength. The work presented here is an analysis of the behavior of fiber-reinforced beams armed as bellow:

- Study the effect of the incorporation of fibers in a cement matrix
- Study the recovery of fibers as reinforcements
- Determine the increase in flexural strength under monotonic loading increasing
- Observe the mode of cracking

The first set of standardized test used to determine the compressive strength and tensile strength in bending.

The second set concerns the 4-points bending beams (15x10x60 cm) that will be subjected to a monotonically increasing load until failure.

2. Presentation of the materials tested

2.1 Materials in base

For any current use of concrete fiber in the building was used aggregate crushers in the region with a dosage of 350 kg/m3 cement

2.1.1 Cement

The cement used is a type of cement CEMII 42.5 NA 442, physical properties are given in the table below

Table 1: The results of standard tests carried out on this cement are given in Table 2

Test	Chatelier expansion	specific surface (Blaine)mm^2/g	consistency normal
start End 2h50mn 4h06mn	hot cold 2.90 1.65	3891	26.91

Table 2: Mechanical resistance cement (bar)

tests	Age		
	2days	7days	28 days
Compression (b)	143.2	266.4	433.1
Flexion (b)	35.7	58.3	77.9

2.1.2 Mixing water

The water used in mixing is the tap (dam Djorf ettorba), the results of physicochemical analysis are as follows:

Table 3: Results of analysis of water-chemical physic

PH	Matter in suspension	Chlorides Mg/l	Sulfates Mg/l	Residue sec 105 C^0	Conductivity 25 C^0 µs/cm
8,13	Null	234,3	123,02	800,00	0,93

2.1.3 Aggregates

The gravels are Petro graphically micritic limestone partly dolomitize sandstone, sand consists mainly of these proportions: Silica and limestone following rigorous testing of these materials are characterized as follows:

Table 4: Particle size analysis has given us the following composition for a dosage of 350Kg/M3

Designation Class d/D	Product Sand 0/3	 Gravel 3/8	 Gravel 8/15
Mass volumique Absolute Apparent	2,5 t/m^3 1,85 t/m^3	 1,53 t/m^3	2,66 t/m^3 1,41 t/m^3
Surface properties%		1,0 %	0,80 %
Equivalent sable %	67 %		

Coefficient LA	21 %≤ 40 %
Coefficient M.D.E	17 %≤ 35 %
Nature	calcareous silico Sandy dolomitic limestone
Fineness modulus	1,95

Table 5 Composition of 1 m3 of concrete

	%	Volume absolute	Mass volumique absolut [t/m^3]	Mass [Kg]	Mass volumique Apparent [t/m^3]
Ciment	13,85	112,9	3,1	350	1
Sand (0/4)	37,15	302,7	2,5	756,7	1,85
Gravel(3/8)	16	130,4	2,66	346,9	1,53
Gravel(8/15)	33	268,ç	2,66	715,2	1,41
Concrete sec	100	815	/	2168,8	/
Water	/	185	1	185	1

According to this composition were obtained concrete firm for a 1cm subsidence ratio (E / C = 0.53), the collapse is obtained for a de6cm (E / C = 0.60), for a Mix = 1.5%; Mix in means the ratio of fiber relative to the aggregates,. Here is the composition of the concrete practice

Table 6: Composition of Concrete Practice Mix for 1 m3 of 1.5%

	%	Volume absolute	Mass volumique absolut [t/m^3]	Mass [Kg]	Mass volumique Apparent [t/m^3]
Ciment	14,29	112,9	3,1	350	1
Sand (0/4)	36,71	290	2,5	725	1,85
Gravel(3/8)	16	126,4	2,66	336,2	1,53
Gravel(8/15)	33	260,7	2,66	693,46	1,41
Concrete sec	100	790	/	2104,66	/
Water	/	210	1	210	1

The identifications of the various physical and mechanical aggregates showed conformance to specifications of standard NF P18 301 (Georges 1990), Also, aggregates (G1, G2 and S) shows no abnormality in their grading curves, and Based on these results in these fractions fall into classes 0 / 3, 3 / 8 and 8/15selon NF P18 560.

2.1.4 Fibre Plant

We use the leaves of palms date palms of the type of Taghit Oisis "Fegousse" to the saturated state antecedent research have shown a clear difference between concrete and full of dry fiber Fiber identification:

A) Property-mechanical: test is used for axial tractions a sample of 20 fiber dimensions 0.35 mm thick, with a length of 30mm and a width of 6 mm,

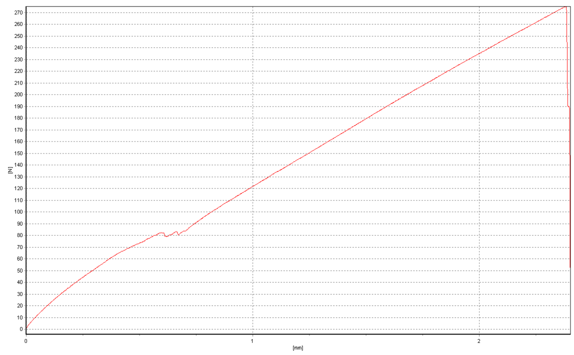


Figure 1: Tensile force in function of the elongation

The results are given in the following table:

Table 7: characteristics Mechanical of the fiber

Resistance to traction (Mpa)	The longer %	Coeff of absorption %	Modulus of elasticity (Mpa)
114	6,5	132	17,58

B) Adhesion Fibre – Concrete

The Essay adhesion pull-out test To evaluate the bond stress fiber matrix, we use the direct test method (pull-out), so we anchor the fiber in a cement matrix and then applying a tearing force on the fiber during the test is fixed fiber dimensions (length 150mm, width 7 mm, thickness 0.55mm), the only variable is the anchorage length we take the length and 3 cm respectively 1.5,2,2.5 are used specimens

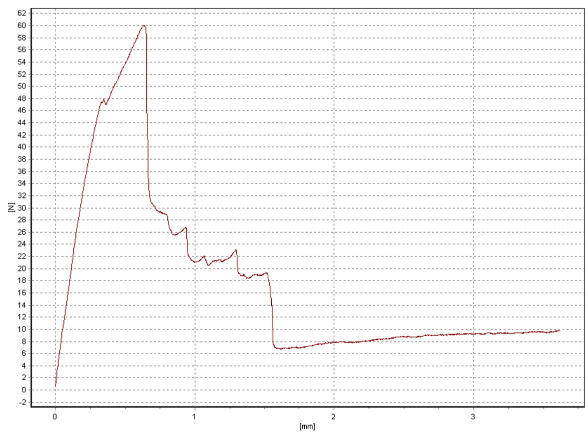


Figure 2: Breakout force versus slip

Analysis of the pull-out curve:

Pcr: the end of the elastic behavior, it is a critical shift Δcr (elastic zone)
Pmax is the maximum force before detachment, which corresponds to a shift Δmax
$\Delta 0$: the shift corresponding to the total detachment
After several direct axial pullout tests in the following figure summarizes the adhesion Force based on the anchorage length

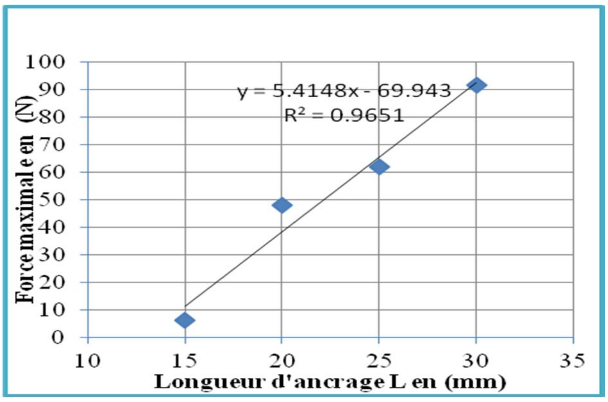


Figure 3-curve of the maximum force based on the anchorage length

3. Experimental Method

3.1 Compression test: Compression tests are carried out on cubes of 10x10x10 cm 3 after 28 days

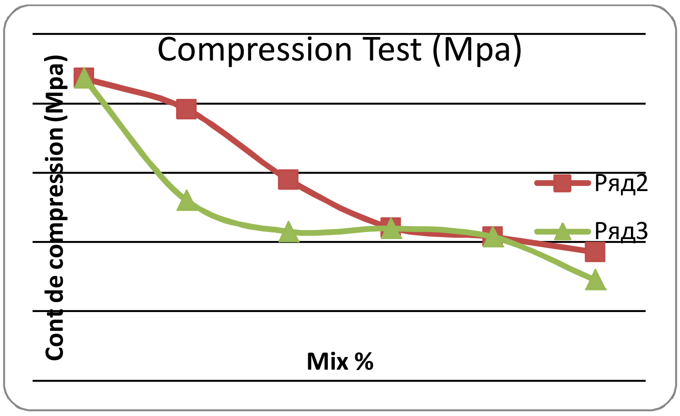


Figure 4: test on cubes of 10x10x10 cm 3 after 28 days

3.2 Tensile test 4-point bending

They summers 7X7X28 performed on samples after 28 days

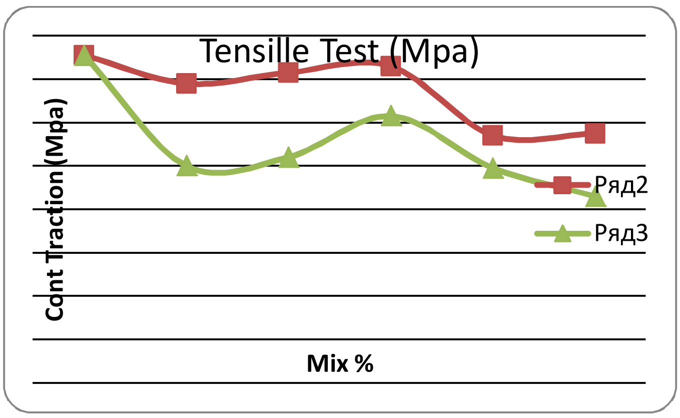


Figure 5: 7X7X28 performed on samples after 28 days

After compression tests and tensile Note that there is a considerable drop constraint (over 50%) are increasing the fiber content (Mix) as is consistent with studies Similar [2]

Reinforcement

For the longitudinal reinforcement is used under the terms of 4T10 non fragile (Section A.4.2 of BAEL)

For transverse reinforcement conditions of shear imposes Φ6 spacing of 7 cm at the supports.

We chose the minimum percentage to see the behavior of concrete in tension and compression.

4. Results of static tests:

4.1 Charge-arrow diagram

Analysis by the Mix

The results obtained can be classified our sample into 2 categories, the first consisting of the control concrete, and those of Mix 0.5%, 1.0%, the second category that of1, 5%, 2.0% and 3.0%. In the first category we find that the curves of the three components are combined in a first zone (zone without degradation) is the elastic zone and an area with a slight shift of the beginning of cracking (phase elasto-plstique) and finally a plastic phase that ends in failure. The tensile strength in bending of fiber-reinforced beams is 1.21% times more than the control concrete, for against the influence of fiber length appears in the arrow registered; it to an arrow of 2.46 mm for a fiber length of 6 cm and 2.54 mm for the fiber to 4cm Mix 0.5% in the second category there is clearly brought on beam ductility (an arrow of 3.8 mm for the Mix 3% to 6 cm fiber) against it by a break for a load of 55 KN of course this is a break due to compression

4.2 Module of elasticity

The modulus of elasticity is a constant mechanical stress of materials, is given by the slope of the first part of the diagram (σ, ε%) is an instantaneous modulus calculated first threshold. We note that for Mix 0.5% fiber is 4 cm in the same behavior materials by the cons there is a drop in module for the other

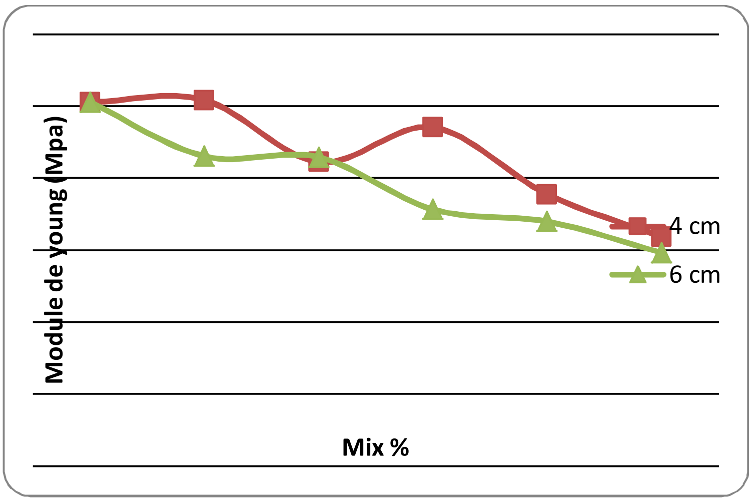


Figure 6 : Module of elasticity

4.3 Ductility Index

Among the important advantages of fiber concretes that improve the ductility of materials, which plays an important role in seismic areas (avoids sudden destruction of a building) is an important research in the future, so we see that the contribution of fibers to dramatically improve its ductility than the control concrete (Mix for 0.5%, 1% and 1.5%)

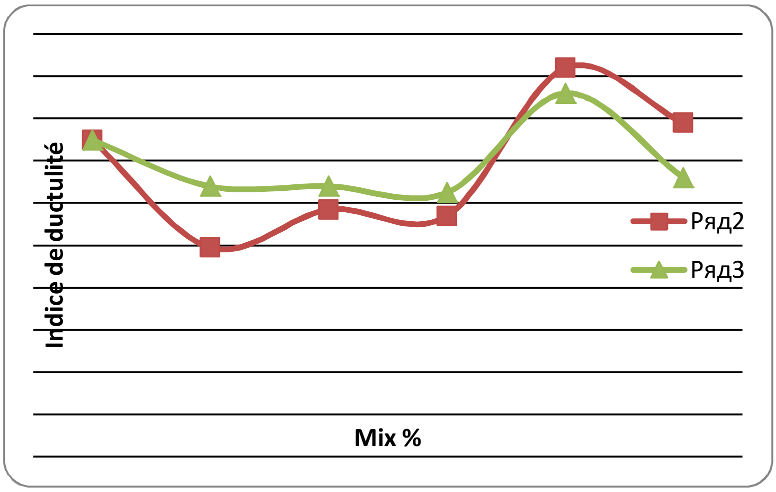
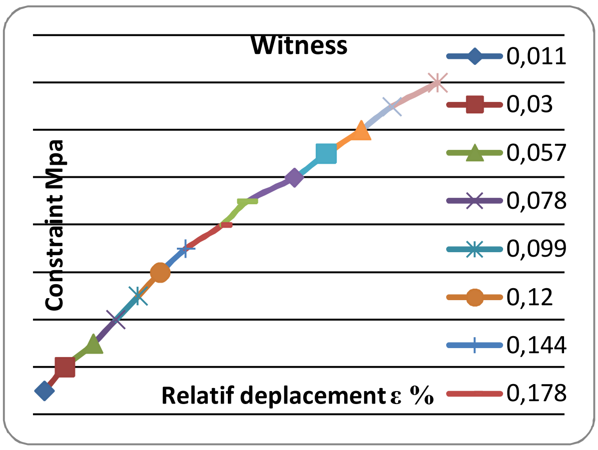


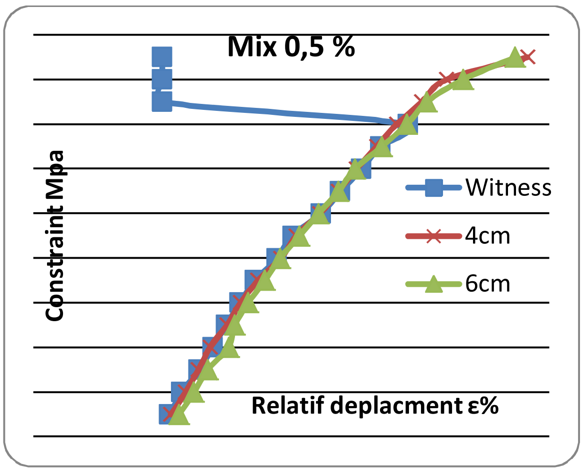
Figure 7 : Ductility Index

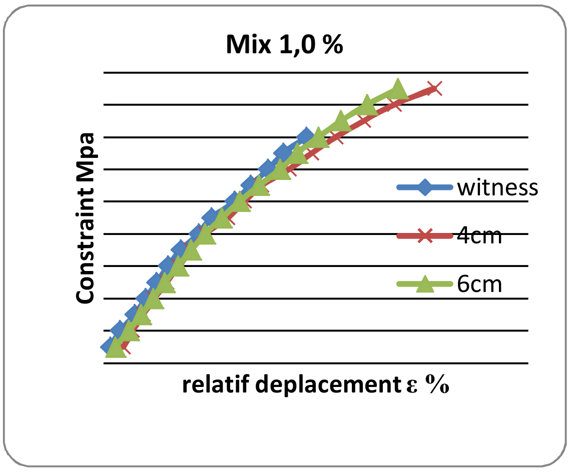
5. Conclusion

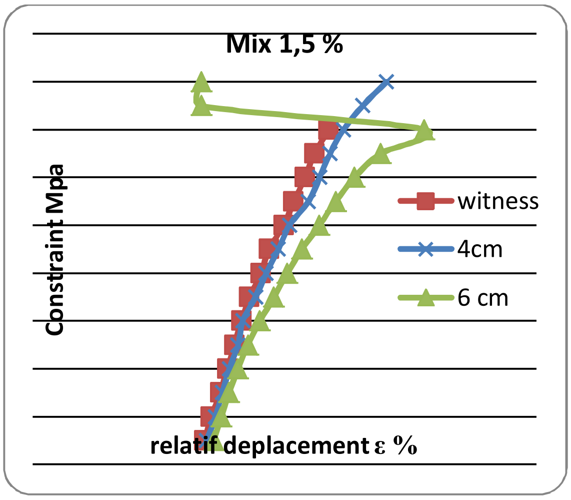
The incorporation of the fibers of date palms wet brings a significant improvement in the behavior of beams in four point bending ductility and its bearing capacity, noting that actual performance summers were obtained without requiring a particular choice of cement (CPJ 45) commonly used with a dosage of 350 kg/m3 or natural aggregates after the crusher, recalling that the compressive stress of the control concrete was 21.5 MPa, our goal was to explore the possibility of using fiber in the current building, after analyzing the different steps it is clear that the Mix 0.5% of the fibers of 4 cm gives the best results, the tensile strength was 1.21% higher than the control concrete by cons ductility was lower than the control concrete, the increase in fiber actually increased its plastic deformation for the Mix 0.5% (4 cm) is the strain at break is greater than that of control concrete by 50% but in a sharp decrease its compressive stress and tensile

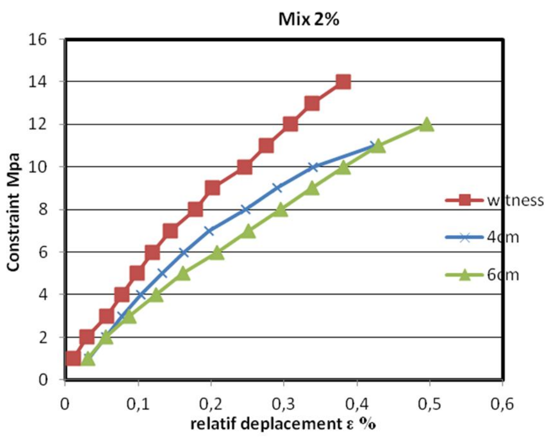
Results of tests:

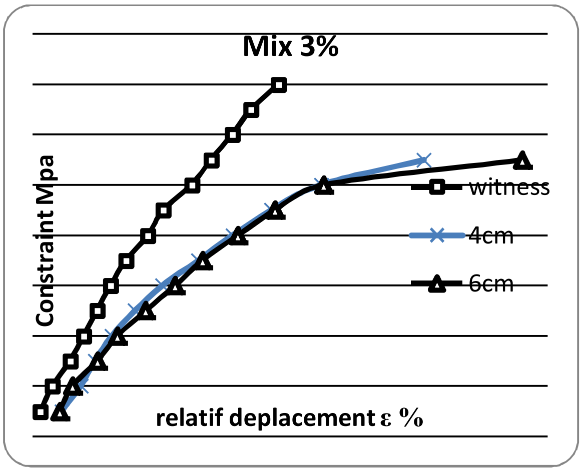












References:

[1] Abbani Said. «Caractérisation et durabilité des bétons renforcés de fibres végétales de palmier dattiers».2003.Mémoire de magister, Université d'Ouargla.

[2] Khenfer.M.M, MorlierP. 'Effet de la longueur des fibres sur les propriétés mécaniques des ciments renforcés de fibres cellulosiques'. *Materials and structures* (24)(1991)185-190.

[3] A. Kriker, G. Debicki, A. Bali, M. M. Khenfer, M. Chabannet 'Mechanical properties of date palm fibres and concrete reinforced with date palm fibres in hot-dry climate *Cement and Concrete Composites, Volume 27 (2005) 554-564*

[4] A.Kriker, A.Bali, G. Debicki, M. Bouziane, M. Chabannet 'Durability of date palm fibres and their use as reinforcement in hot dry climates *Cement and Concrete Composites' Volume 30 (2008) 639-648.*

[5] Nadia Benmansour « Etude des performances de produits renouvelables et locaux adaptés aux applications de l'isolation thermique dans le bâtiment »-Thèse de Magister Université Hadj Lakhdar, Batna (2011)

[6] G.Dreux et J.Festa « Nouveau guide du béton et de ses constituants » 8^{iem} édition Eyrolles 1998

[7] Cesar Juarez, Gerardo Fajardo « Caracterisation microstructurale des fibres naturelles pour des materiaux composites a base de ciment » Canadian Journal of Civil Engineering March 1, 2009.

[8] «Guide de bonnes pratiques pour l'utilisation des fibres dans le béton »-Réalisé par l'association béton Québec en collaboration avec les manufacturiers de fibres métalliques et de fibres synthétiques-janvier 2005.

[9] Hachani Imane et Khelif Rima « Etude de l'effet de la géométrie des fibres synthétiques sur les propriétés physico synthétiques sur les propriétés physico-mécaniques des mortiers à sable calcaire » mémoire d'ingénieur d'état (2009) Université Ammar Telidji à Laghouat.

[10] Mourad Chikhi « Métrologie et Modélisation des Transferts dans les Composites Naturels à Faible Coût pour l'Isolation Thermique dans le Bâtiment en Algérie » Thèse de Magister Université Hadj Lakhdar, Batna (2010)

[11] Pascal Casanova « Béton renforces de fibres » thèse de Doctorat l'Ecole Nationale des Ponts et Chaussées 26 juin 1995.

[12] Ali Kaflou « Etude du comportement des interfaces et des interphases dans les composites a fibres et a matrices céramiques » thèse de Doctorat L'Institut National des Sciences Appliquées de Lyon, 20 Mars 2006.

[13] J.M.Alwan , A.E.Naaman et W.Hansen « Pull-out Work of Steel Fibers From Cementitious Composites :Analyse Investigation » Department of Civil Engineering, 2340 G.G. Brown Building, College of Engineering, University of Michigan, Ann Arbor, MI 48109-2125, USA, 24 may 1991.

[14] H.Houari. « Contribution à l'étude du comportement du béton renforcé de fibres métalliques soumis à l'action des charges maintenues et cycliques. »1993. INSA Lyon.243 p.

[15] Valade. T. 'Comportement des bétons armés de fibres' *; Mémoire de D.E.A, L.C.P.C,* (Décembre 1987).

[16] Amina Djoudi, Mohammed Mouldi Khenfer, Abderrahim Bali «Etude d'un nouveau composite en platre renforce avec les fibres végétale du palmier »-1st International Conf erence on sustainable Built Environment Infrastructures in Developing Countries ENSET Oran (Algeria) – Octobre 12-14, 2009.

[17] Richane SACI « Prise en compte de l'orientation des fibres dans une poutre continue à matrice de béton » -Thèse de Magister Université Mohamed Boudiaf M'sila (juillet2004).

BASIS OF CONTINUUM APPROXIMATION IN MODELS OF GROWING RANDOM NETWORKS

G. Tsitsiashvili*, M. Osipova**

* IAM FEB RAS, Vladivostok, Russia, e-mail: guram@iam.dvo.ru
** FEFU, Vladivostok, Russia, e-mail: mao1975@list.ru

ABSTRACT

Accuracy asymptotics for differences between prelimit and limit distributions of nodes powers in models of random growing networks are constructed. A rate of a convergence in these relations is power. Obtained formulas allow to ground the continuum approximations for considered models.

1. INTRODUCTION

Main aim of this paper is to estimate a rate of a convergence to limit distributions in models of growing random networks [1]. One of the most convenient methods to define limit distribution of node exponent is the continuum approximation [2], [3]. It is based on asymptotic behavior of considered distribution when a number of steps tends to the infinity. In [4, p. 124] it is marked that ``a problem of strict formal description of statistical ensemble of random networks with fixed distribution of nodes exponents is not yet solved``. An absence of correct mathematical base of the continuum approximation makes susceptible results obtained using this approximation in spite of its calculation convenience.

In this paper a ground of the continuum approximation in a calculation of limit distributions for main models of growing random networks is made. Here the model of growing exponential network, the model of Barabasi and the model of Dorogovtsev are analyzed. Exact asymptotic of a difference between prelimit and limit distributions when a number of steps tends to the infinity is obtained and this asymptotic is power. Such results are based on a construction of recurrent relations for prelimit distributions of nodes exponents distributions and on asymptotic series in these relations.

2. EXPONENTIAL NETWORKS

Consider a model of growing exponential network in which new node is connected with each existing node with equal probabilities. Denote $p(k,s,t)$ the probability that on the step $t \ge 1$ the node $s, 1 \le s \le t,$ is connected with k arcs of non oriented graph of exponential network. Then k is called the power of the node s. In [2] the following relations are obtained (here δ_{ij} is the Kroneker index):

$$p(k,t,t) = \delta_{k1},\ \ p(k,s,t+1) = \frac{p(k-1,s,t)+(t-1)p(k,s,t)}{t},\ \ k \ge 1.$$

Designate

$$P(k,t) = \frac{1}{t}\sum_{s=1}^{t} p(k,s,t) \tag{1}$$

then the recurrent relation

$$(t+1)P(k,t+1)=P(k-1,t)+(t-1)P(k,t)+\delta_{k1},\ k>1,\ t\geq 1,$$

is true and $P(1,1)=1$, $P(k,t)=0$, $k>t\geq 1$. So for $k=1$

$$P(1,t+1)=\frac{t+t(t-1)P(1,t)}{(t+1)t}=\frac{t+(t-1)+(t-1)(t-2)P(1,t-1)}{(t+1)t}=\ldots=$$

$$=\frac{1+\ldots+t}{(t+1)t}=\frac{1}{2},\ t\geq 1, \tag{2}$$

and for $k\geq 2$

$$P(k,t+1)=\frac{t\,P(k-1,t)+t(t-1)P(k,t)}{(t+1)t}=$$

$$=\frac{t\,P(k-1,t)+(t-1)P(k-1,t-1)+(t-1)(t-2)P(k,t-1)}{(t+1)t}=\ldots=$$

$$=\frac{1}{(t+1)t}\sum_{j=1}^{t} jP(k-1,j). \tag{3}$$

From the last formula

$$P(2,t+1)=\frac{1+\ldots+t}{2t(t+1)}=\frac{1}{4},\ t\geq 1, \tag{4}$$

$$P(3,t+1)=\frac{2+\ldots+t}{4t(t+1)}=\frac{1}{8}-\frac{1}{4t(t+1)},\ t\geq 1. \tag{5}$$

Denote $f_k(t)=P(k,t+1)-2^{-k}$, $k\geq 1$, then from the formulas (2) – (5) we have

$$f_1(t)\equiv f_2(t)\equiv 0,\ f_3(t)\sim -\frac{1}{4t^2},\ t\to\infty. \tag{6}$$

Theorem 1. For $t\to\infty$ the relations

$$f_k(t)\sim -\frac{C_k \ln^{k-3} t}{t^2},\ C_k=\frac{1}{4(k-3)!},\ k\geq 3, \tag{7}$$

take place.

Proof. For $k=3$ the formula (7) is a corollary of the formula (6) then by an induction we obtain from (3) that

$$f_{k+1}(t)=P(k+1,t+1)-\frac{1}{2^{k+1}}=\frac{1}{t(t+1)}\sum_{j=1}^{t} jP(k,j)-\frac{1}{2^{k+1}}=$$

$$=\frac{1}{t(t+1)}\sum_{j=1}^{t} j\left(\frac{1}{2^k}+f_k(j-1)\right)-\frac{1}{2^{k+1}}=\frac{1}{t(t+1)}\sum_{j=1}^{t} jf_k(j-1)\sim -\frac{C_k \ln^{k-2} t}{(k-2)t^2},\ t\to\infty.$$

The formula (7) is proved.

Remark 1. In all sections of this paper the continuum approximation is based on the limit

$$t\big(P(k+1,t)-P(k,t)\big)=t\big(f_k(t)-f_k(t-1)\big)\to 0,\ t\to\infty. \tag{8}$$

In this section the formula (8) is a corollary of the formula (7).

3. MODEL OF BARABASI-ALBERT

Consider Barabasi-Albert model of growing network [1] in which new node is connected with each existing node with probability proportional to a power of existing node. Denote $p(k,s,t)$ the probability that on the step $t \geq 1$ the node s, $1 \leq s \leq t$, is connected with k arcs of non oriented graph of Barabasi-Albert network. In [2] the following relations are obtained

$$p(k,t,t) = \delta_{k1},\ p(k,s,t+1) = \frac{k-1}{2t}p(k-1,s,t) + \left(1 - \frac{k}{2t}\right)p(k,s,t),\ k \geq 1.$$

From the formula (1) we have

$$(t+1)P(k,t+1) = \frac{k-1}{2}P(k-1,t) + \left(t - \frac{k}{2}\right)P(k,t) + \delta_{k1},\ k > 1,\ t \geq 1,$$

$$P(1,1) = 1,\ P(k,t) = 0,\ k > t \geq 1,\ P(0,t) \equiv 0, t \geq 1.$$

Analogously with (2), (3) it is not difficult to obtain

$$P(1,t+1) = \frac{1}{t+1}\left[1 + \sum_{j=1}^{t}\prod_{s=j}^{t}\left(1 - \frac{1}{2s}\right)\right],\ t \geq 1, \quad (9)$$

$$P(k,t+1) = \frac{k-1}{2(t+1)}\sum_{j=1}^{t}P(k-1,j)\prod_{s=j+1}^{t}\left(1 - \frac{k}{2s}\right),\ k \geq 2,\ t \geq 1, \prod_{t+1}^{t} = 1. \quad (10)$$

Lemma 1. For $A > 0$ the equalities

$$\sum_{j=1}^{t}\prod_{s=j}^{t}\left(1 - \frac{A}{s}\right) = \frac{t-A}{1+A} + \frac{\psi(t)}{(1+A)^2\Gamma(-1-A)},\ \psi(t) = \frac{\Gamma(1-A+t)}{\Gamma(t+1)}, \quad (11)$$

$$\sum_{j=1}^{t}\prod_{s=j+1}^{t}\left(1 - \frac{A}{s}\right) = \frac{t+1}{1+A} - \frac{\psi(t)}{(1+A)\Gamma(1-A)} \quad (12)$$

are true. Here $\Gamma(z)$ is the gamma function.

Proof. Denote $S(t) = \sum_{j=1}^{t}\prod_{s=j}^{t}\left(1 - \frac{A}{s}\right)$ then

$$S(1) = 1 - A = \frac{1-A}{1+A} + \frac{\psi(1)}{(1+A)^2\Gamma(-1-A)},$$

consequently the formula (11) for $t = 1$ is proved. Suppose that this formula is true for t and prove it for $t+1$

$$S(t+1) = \sum_{j=1}^{t+1}\prod_{s=j}^{t+1}\left(1 - \frac{A}{s}\right) = \frac{t+1-A}{t+1}(S(t)+1) =$$

$$= \frac{t+1-A}{t+1}\left(\frac{t-A}{1+A} + \frac{\psi(t)}{(1+A)^2\Gamma(-1-A)}\right) = \frac{t+1-A}{t+1} + \frac{\psi(t+1)}{(1+A)^2\Gamma(-1-A)}.$$

The relation (11) is proved for all natural t. The formula (12) may be proved similar.

Remark 2. In left sides of the formulas (11), (12) we have functions defined for $A > 0$, $t > 0$ and on right sides - the gamma functions which may be non defined for some $A > 0$, $t > 0$. But ratios of these gamma functions in such points may be redefined using limit transition to these points.

Designate

$$f_k(t) = P(k, t+1) - \Pi(k),\ \Pi(k) = \frac{4}{k(k+1)(k+2)},\ k \geq 1. \qquad (13)$$

Theorem 2. For $t \to \infty$ and $k \geq 1$ the following relations are true

$$f_k(t) \sim \frac{t^{-3/2}}{3\sqrt{\pi}}. \qquad (14)$$

Proof. It is known [5, subsection 1.18] that for $A > 0$

$$\psi(t) = \frac{\Gamma(1-A+t)}{\Gamma(t+1)} \sim t^{-A}, t \to \infty. \qquad (15)$$

Assume that $k=1$ then the relation (14) is a corollary of the formulas (9), (10), (15) for $A = 1/2$

$$f_1(t) = P(1, t+1) - \Pi(1) = \frac{1}{t+1}\left(1 + \frac{t-1/2}{3/2} + \frac{\psi(t)}{(3/2)^2\,\Gamma(-3/2)}\right) - \frac{2}{3} \sim$$

$$\sim \frac{4t^{-1/2}}{9\Gamma(-3/2)t} = \frac{t^{-3/2}}{3\sqrt{\pi}},\ t \to \infty.$$

Suppose that the formula (14) is true for $k-1$ and prove it for k, $k > 1$. Represent $f_k(t)$ in the form $f_k(t) = a_k(t) + b_k(t)$ where

$$a_k(t) = \frac{k-1}{2(t+1)} \sum_{j=1}^{t} \Pi(k-1) \prod_{s=j+1}^{t}\left(1 - \frac{k}{2s}\right) - \Pi(k),$$

$$b_k(t) = \frac{k-1}{2(t+1)} \sum_{j=1}^{t} f_{k-1}(j-1) \prod_{s=j+1}^{t}\left(1 - \frac{k}{2s}\right).$$

Consequently from the formulas (12), (13) for $A = k/2$ we have

$$a_k(t) = \frac{4}{k(k+1)}\left[\frac{1}{2(t+1)} \sum_{j=1}^{t} \prod_{s=j+1}^{t}\left(1 - \frac{k}{2s}\right) - \frac{1}{k+2}\right] =$$

$$= \frac{4}{k(k+1)}\left[\frac{1}{2(t+1)}\left(\frac{t+1}{1+k/2} + \frac{\psi(t)}{(1+k/2)\Gamma(1-k/2)}\right) - \frac{1}{k+2}\right] =$$

$$= \frac{4\psi(t)}{2(t+1)k(k+1)(1+k/2)\Gamma(1-k/2)} \sim \frac{\Pi(k)t^{-1-k/2}}{\Gamma(1-k/2)},\ t \to \infty.$$

From the induction assumption $f_{k-1}(t) \sim \frac{t^{-3/2}}{3\sqrt{\pi}}$, $t \to \infty$, and from the gamma function properties and from the formula (15) for $A = k/2$ we obtain

$$b_k(t) = \frac{k-1}{2(t+1)} \sum_{j=1}^{t} f_{k-1}(j-1) \frac{\psi(t)}{\psi(j)} \sim \frac{k-1}{2(t+1)} \int_1^t \frac{j^{k/2}}{3\sqrt{\pi} j^{3/2} t^{k/2}} dj \sim \frac{t^{-3/2}}{3\sqrt{\pi}},\ t \to \infty.$$

Then asymptotic relation (14) is true for arbitrary natural k.

4. MODEL OF DOROGOVTSEV

Consider Dorogovtsev model of growing network in which new node is connected with each existing node with the probability proportional to a sum of its power (a number of arcs incoming to existing node) and some constant $a>0$. Here $a>0$ is model parameter. Denote $p(k,s,t)$ the probability that on the step $t\ge 1$ the node $s,\ 1\le s\le t,$ has the power k. In [2] we obtain the relations

$$p(k,s,t+1)=\frac{k-1}{t(1+a)}p(k-1,s,t)+\left(1-\frac{k+a}{t(1+a)}\right)p(k,s,t),\ p(k,t,t)=\delta_{k0},\ k\ge 0.$$

Designate

$$P(k,t)=\frac{1}{t}\sum_{s=1}^{t}p(k,s,t)$$

then

$$P(k,t+1)=\frac{1}{t(a+1)}\Big[P(k,t)\big((a+1)(t-1)-k-a\big)+P(k-1,t)(k-1+a)+(1+a)\delta_{k0}\Big],$$

$P(0,1)=1,\ P(k,t)=0,\ k\ge t,\ P(-1,t)\equiv 0,\ t\ge 0.$

Analogously with the formulas (9), (10) it is not difficult to obtain

$$P(0,t+1)=\frac{1}{t}\left[1-\frac{a}{a+1}\prod_{j=1}^{t-1}\left(1-\frac{a}{j(a+1)}\right)+\sum_{s=1}^{t-1}\prod_{j=s}^{t-1}\left(1-\frac{a}{j(a+1)}\right)\right], \tag{16}$$

$$P(k,t+1)=\frac{k-1+a}{t(a+1)}\sum_{s=1}^{t}P(k-1,s)\prod_{j=s}^{t-1}\left(1-\frac{k+a}{(a+1)j}\right),\ k>0. \tag{17}$$

Denote $A_k=(k+a)/(a+1),\ k\ge 0,$

$$\Pi(k)=(1+a)\frac{\Gamma(1+2a)\Gamma(k+a)}{\Gamma(a)\Gamma(k+2+2a)},\ f_k(t)=P(k,t+1)-\Pi(k),\ k\ge 0.$$

Theorem 3. The formulas

$f_k(t)\sim C_k t^{-1-A_0},\ t\to\infty,\ k\ge 0,$

$$C_0=\frac{1}{(1+A_0)^2\,\Gamma(-1-A_0)}-\frac{A_0}{\Gamma(1-A_0)},\ C_k=\frac{C_{k-1}(k-1+a)}{(a+1)(A_k-A_0)},\ k>0, \tag{18}$$

are true.

Proof. From the formulas (11), (15), (16) for $A=A_0$ we have

$$f_0(t)=P(0,t+1)-\Pi(0)=\frac{1}{t}\left[1-\frac{A_0\psi(t-1)}{\Gamma(1-A_0)}+\frac{t-(1+A_0)}{1+A_0}+\frac{\psi(t-1)}{\Gamma(-1-A_0)(1+A_0)^2}\right]-$$

$$-\frac{1+a}{1+2a}=\frac{\psi(t-1)C_0}{t}\sim C_0t^{-1-A_0},\ t\to\infty.$$

For $k>0$ we seek $f_k(t)=P(k,t+1)-\Pi(k)$ in the form $f_k(t)=a_k(t)+b_k(t)$,

$$a_k(t)=\frac{k-1+a}{t(a+1)}\sum_{s=1}^{t}\Pi(k-1)\prod_{j=s}^{t-1}\left(1-\frac{A_k}{j}\right)-\Pi(k),$$

$$b_k(t)=\frac{k-1+a}{t(a+1)}\sum_{s=1}^{t}f_{k-1}(s-1)\prod_{j=s}^{t-1}\left(1-\frac{A_k}{j}\right).$$

Then from the formula (15) for $t \to \infty$ and $A = A_k$ we have

$$a_k(t) = \frac{(k-1+a)\Pi(k-1)}{t(a+1)}\left[\sum_{s=1}^{t-1}\prod_{j=s}^{t-1}\left(1-\frac{A_k}{j}\right)+1\right]-\Pi(k)=$$

$$=\frac{(k-1+a)\Pi(k-1)\psi(t-1)}{t(a+1)\Gamma(-1-A_k)(1+A_k)^2}\sim\frac{(k-1+a)\Pi(k-1)t^{-A_k-1}}{(1+A_k)^2(a+1)\Gamma(-1-A_k)}.$$

From the induction assumption $f_{k-1}(t)\sim C_{k-1}t^{-1-A_0}$ and from the formula $A_k > A_0$ and from the formula (15) for $A = A_k$ we obtain

$$b_k(t)=\frac{k-1+a}{t(a+1)}\sum_{s=1}^{t} f_{k-1}(s-1)\prod_{j=s}^{t-1}\left(1-\frac{A_k}{j}\right)=\frac{k-1+a}{t(a+1)}\sum_{s=1}^{t} f_{k-1}(s-1)\frac{\psi(t-1)}{\psi(s-1)}\sim$$

$$\sim\frac{C_{k-1}(k-1+a)}{t^{1+A_0}(a+1)(A_k-A_0)}=C_k t^{-1-A_0},\ t\to\infty.$$

Consequently asymptotic relation (18) is proved for arbitrary natural k.

Remark 3. A consideration of Dorogovtsev model [2] is connected with its wide application to modern models of growing random networks. For small a this model gives sufficiently simple and convenient description of the Internet network with power distribution network nodes exponents

$$\Pi(k)\sim Dk^{-2-a},\ D=\frac{(1+a)\Gamma(1+2a)}{\Gamma(a)},\ k\to\infty$$

with the parameter close to two [6].

REFERENCES

1. Barabasi L.A., Albert. 1999. Emergence of scaling in random networks. *Science,* v. 286. 509-512.

2. Dorogovtsev S.N., Mendes J.F.F. 2002. Evolution of Networks. *Adv. Phys.,* v. 51, no 4. 1079-1187.

3. Ginzburg S.L., Nakin A.V., Savitskaya N.E. 2009. Influence of complex network structure on properties of its dynamical processes. *Letters to JETPh,* v. 90, no 12. 873-878. (In Russian).

4. Evin I. 2010. Introduction to theory of complicated networks. *Computer researchs and modeling,* v. 2, no 2. 121-141 (In Russian).

5. Beitman G., Erdeii A. 1965. *Supreme transcendental functions.* Moscow: Science. (In Russian).

6. Raigorodskiy A. 2010. Models of random graphs and their application. *Proceedings of MPhTI*, v. 2, no. 4. 130-140 (in Russian).

COLLISION RISK ESTIMATION FOR MOTORWAYS OF THE SEA

A. Blokus-Roszkowska, L. Smolarek
•
Gdynia Maritime University, Gdynia, Poland
e-mail: ablokus@am.gdynia.pl

ABSTRACT

The Motorways of the Sea is rather a new concept and, thus, is still in the process of development by the European Commission and in the Baltic Sea Region. The Baltic Sea Region is one of the most dynamic growth areas. Due to this fact the new ideas and technologies are needed to optimize the sea transport system. In the paper the simulation model for the system safety state evaluation is presented. The simulation program can constitute a base for decision-support tool, on the level of safety management, especially to optimally plan the safety transport system.

1 INTRODUCTION

The Motorways of the Sea is (MoS) still in the process of development by the European Commission in the Baltic Sea Region. Cargo transported in containers and trailers have increased rapidly, also oil tanker traffic has seen a noticeable increase in the Baltic. In the Motorways of the Sea the focus should be put on the safety of shipping. Sea motorways main elements can be point out as a part of the European transport corridors network, see Paulauskas & Bentzen (2007).

The safety of MoS routes and quality assurance of the main fairway system is the most important problem. Traditional risk analysis approach calculates the possibility of ship collision with historical data, mathematical models and opinions of experts, which evaluates the present risk level. However, there are not any historical data of MoS crossing that can be used for the analyses to describe the future of the studied area.

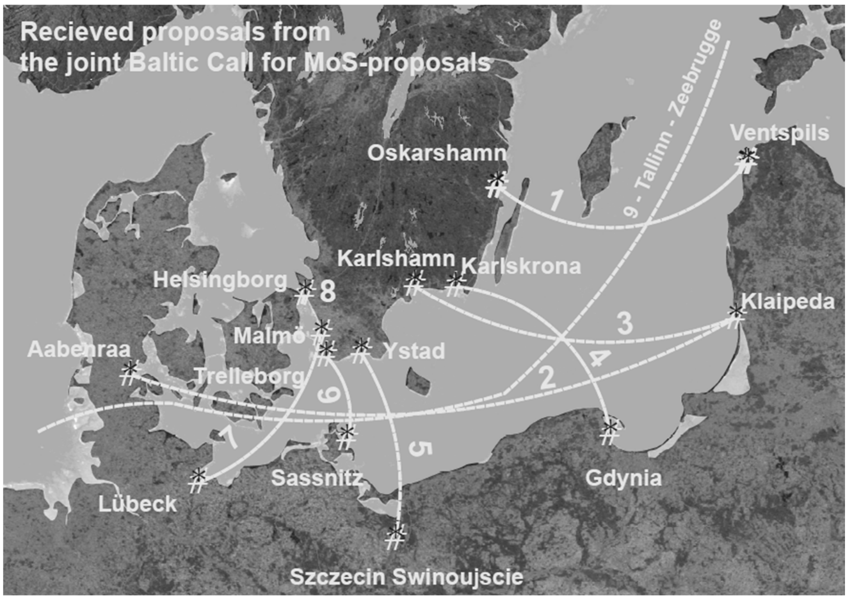


Figure 1. Proposals from the joint Baltic Call for MoS, www.pisil.pl

So it is necessary to do a risk analysis based on a mathematical model, with a combination of forecasting and simulating system, which should be verified by a concrete example.

2 STOCHASTIC MODEL

The model can be used to protect safety of navigation in the congested areas. This model presents an approach for modeling both spatial interactions and detailed succession dynamics in the MoS crossing by placing the semi-Markov processes within a class of stochastic process called piecewise deterministic Markov (PDM) processes.

A piecewise-deterministic Markov process is a stochastic process that evolves deterministically until a random time when the process jumps to a new (random) state. PDM processes were introduced by Davis (1984). These processes are readily amenable to computer simulations. PDM processes have been used in a variety of settings, including storage processes, capacity expansion problems, and financial investment models.

To define a PDM process we have to define four basic components such as:

–the state space,

–a description of the deterministic motion between jumps,

–the rate at which jumps occur,

–the distribution of the state after a jump has occurred.

Each of these components is described below within the context of ships dynamics. The deterministic motion of the PDM processes given here is only used to keep track of the times that ships have been in their current states. In fact, the processes used here are piece-wise linear. A MoS intersection is divided into several cells (Fig. 2). The area within a cell is assumed to be environmentally homogeneous – belongs to only one MoS or crossing place.

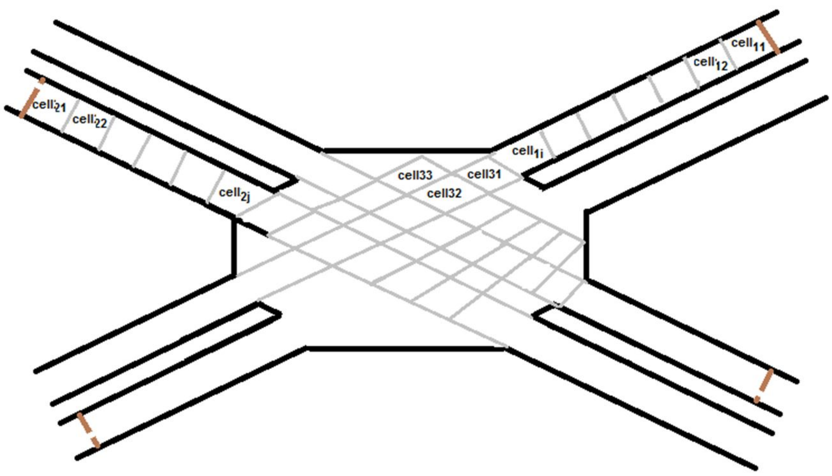


Figure 2. An example of a MoS intersections' grid of three types of cells

The state of a MoS intersection is determined by the distribution of ships within it and the respective functional roles of those ships. This paper focuses on modeling the dynamics of cells. Assume that a cell may be in any of 5 states and denote the set of possible states by *S*. The Table 1 specifies five status types or states defined by a ship appearance.

Table 1. Status types and state numbers

State Number	Status Type
1	Gap
2	A ship entering a cell
3	A ship in a cell
4	A ship living a cell
5	Collision alert

The state space of the MoS crossing is defined to be, (Monticino et al. 2002)

$$E = (S \times [0, \infty))^N . \tag{1}$$

The state of the MoS crossing at time t is

$$X_t = ((s_1(t), \tau_1(t)), \ldots, (s_N(t), \tau_N(t)))$$

where N = number of cells; $s_i(t)$ = is the state of cell i at time t; $\tau_i(t)$ = is the time that the plot has been in state $s_i(t)$ since the last time it changed states.

The deterministic portion of the PDM processes is used here only to keep track of how long cells have been in their current states. Thus, between jump times, the state of the each cell remains constant, while the time in that state evolves at unit rate.

3 COLLISION FREQUENCY MODELS

There is one major difference between two collision types (Fig. 3). At X-shaped intersection the traces of two ships always intersect, whereas they on average only intersect in one out of two cases at Y-shaped intersection. This means that the geometrical collision probability needs to be corrected by a factor of 0.5 in case of a Y-shaped intersection.

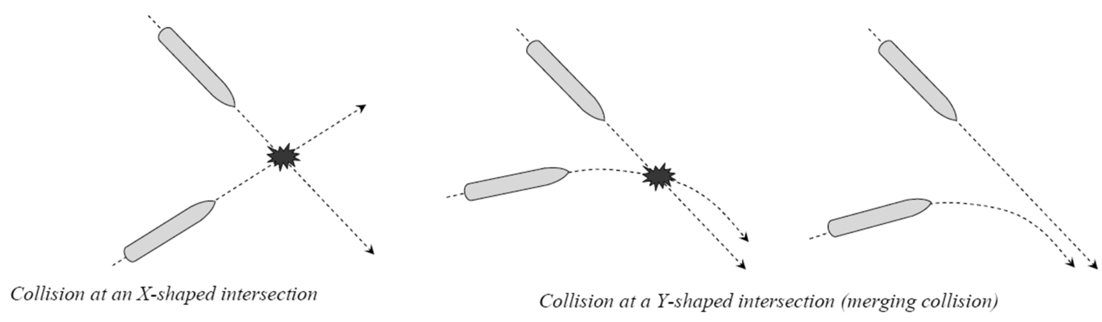


Figure 3. The two collision type, www.sofartsstyrelsen.dk/SiteCollectionDocuments/CMR/Sejladssikkerhed,%20GMDSS%20og%20SAR/Bassy/BASSY%20Evaluation.pdf

The Pedersen's model considers the crossing of two waterways, (Pedersen 1995). Ships are grouped by their type and length in order to utilize the different characteristics of vessel groups like the average speed or manoeuvrability which varies significantly from one ship group to another.

Model of Fowler and Sørgård suggest that the frequency of critical situations is calculated assuming that traffic movements are uncorrelated, (Jutta 2010). A critical situation denotes that two ships are crossing within half a nautical mile from each other. Encounter frequency is estimated by a pair-wise summation across all shipping lanes at the considered location. They do not present a practical procedure to calculate the number of critical situations.

Macduff's Model is build on molecular collision theory, (MacDuff 1974). Ships on a shipping lane are regarded as a homogenous group: they are navigating at the same speed and they have similar dimensions.

Cowi Crossing Collision Model defines crossing collision as a collision that includes ships sailing along different waterways. Two ships can theoretically collide if their traces intersect. The possibility of a collision between two ships navigating at intersecting routes can be expressed by critical time interval.

4 SIMULATION APPROACH

The necessary time to make a decision by navigator usually amounts 3 to 6 min and this decision time can be also considered in the collision checking range (Xue et al. 2009). However as we consider only the crossing situation, in which may be involved more than two ships in close proximity, the average checking range on the waterway should be smaller. For example, nowadays, for security reasons it is recommended that Automatic Identification System AIS, thereby controlling system, allows for transmitting minimum 2000 messages per minute.

First we have to find the cell size, as we will determine the simulation time step with the use of it. We assume, similarly as in the Nagel-Schreckenberg model presented in (Nagel & Schreckenberg 1992, Wahle et al. 2001), that the waterway is divided into cells with a length of $CS = \rho_{cross}^{-1}$. We determine the total crossing density ρ_{cross} according to the following formula:

$$\rho_{cross} = \max\{\rho_{12}, \rho_{13}\}, \text{ where } \rho_{12} = \sqrt{\rho_1^{\;2} + \rho_2^{\;2}},\ \ \rho_{13} = \sqrt{\rho_1^{\;2} + \rho_3^{\;2}} \qquad (2)$$
$$\text{and } \rho_1 = \frac{LOA_1}{T_1 \cdot V_1 + LOA_1},\ \ \rho_2 = \frac{LOA_2}{T_2 \cdot V_2 + LOA_2},\ \ \rho_3 = \frac{LOA_3}{T_3 \cdot V_3 + LOA_3},$$

LOA_1 = a length of a ship on the main waterway; LOA_2, LOA_3 = a length of ships on the lateral waterways 2 and 3; T_1, T_2, T_3 = mean times between ships' starting for waterways no 1, 2 and 3.

The article deals with different levels of risk depending on the mutual distance of vessels that are on collision courses. Distance is measured in taxicab metric, according to the sequence of grid cells. We denote a size of a cell by *CS*. To determine the threshold values of safety distances defining the states of collision risk we have to consider following results. A ship domain is the area around the vessel that should be avoided by other vessels and an overlap of two vessels' domains is concerned with a very high risk of collision. A length of a ship domain is assumed to be equal 4*LOA* (Fujii & Tanaka 1971) and we consider this values as a critical distance of high risk of collision to define the first risk level. To define the second risk level we assume from references a distance of passing clear *CPA* (*Closest Point of Approach*) equal to 1 *nm* (1852*m*). This distance of passing clear will be considered in the simulation as a critical value between high and low risk of collision. Further we take a distance of 2 *nm* as a safety distance corresponding to the low risk of collision.

Then we define following risk levels:

high risk of collision – both ships are entering a cell and number blank cells between two ships is equal at least d_1-2, one ship is entering a cell and second ship is in a cell or leaving a cell and number blank cells between two ships is equal at least d_1-1, both ships are in a cell or leaving a cell and number blank cells between two ships is equal at least d_1; if none of these conditions if fulfilled we define this situation as a collision alert; the distance d_1 is determined from the equation:

$$d_1 = \left\lceil \frac{4LOA_1 + 4LOA_{\max}}{CS} \right\rceil; \tag{3}$$

where $\lceil d \rceil$ denotes the integer part of number d plus 1; LOA_1 = a length of a ship on the main waterway; LOA_2, LOA_3 = a length of ships on the lateral waterways 2 and 3; $LOA_{\max} = \max\{LOA_2, LOA_3\}$; CS = size of a cell;

low risk of collision – both ships are entering a cell and number blank cells between two ships is equal at least d_2-2, one ship is entering a cell and second ship is in a cell or leaving a cell and number blank cells between two ships is equal at least d_2-1, both ships are in a cell or leaving a cell and number blank cells between two ships is equal at least d_2, were distance d_2 is determined from the equation:

$$d_2 = \left\lceil \frac{1852}{CS} \right\rceil; \tag{4}$$

negligible risk of collision – both ships are entering a cell and number blank cells between two ships is equal at least d_3-2, one ship is entering a cell and second ship is in a cell or leaving a cell and number blank cells between two ships is equal at least d_3-1, both ships are in a cell or leaving a cell and number blank cells between two ships is equal at least d_3, were distance d_3 is determined from the equation:

$$d_3 = \left\lceil \frac{2 \cdot 1852}{CS} \right\rceil. \tag{5}$$

We assume that the simulation time step corresponds to time of vessel moving with the largest velocity from one cell to the next. Thus the simulation step time, denoted by Δt, is determined from the formula:

$$\Delta t = \frac{CS}{V_{\max}}, \text{ where } V_{\max} = \max\{V_1, V_2, V_3\} \tag{6}$$

and size of a cell CS was described with use of crossing density given in (2). Velocities of vessels on each waterway V_1, V_2, V_3 in knots in the program corresponds to the speed measured in cells per time step.

4.1 Simulation language and environment

The computer program is written in Java language using SSJ V2.1.3 library with support of stochastic simulations. The documentation of SSJ can be found in Simard (2012). The Java platform is the object-oriented programming language that provide several standard packages. To perform the simulation we use a javaSimulation package that is devoted to process-based discrete event simulation. The package is a Java implementation of the simulation facilities provided by the programming language SIMULA. The javaSimulation package provides three different approaches to discrete event simulation: event-based, activity-based and process-based. The description in details along with appendices containing Java source code and documentation are given in the report (McNab 1996).

Java-based simulation tools are very popular because it is the only object-oriented programming environment that effectively supports standardized components (Kilgore et al. 1998). For example the book (Garrido 2001) concentrates on object-oriented modeling of simulation using Java and practical simulation techniques.

4.2 Simulation model

Discrete-event system is a system completely determined by random event times and by the changes in state taking place at these moments. Basic approaches for constructing a discrete-event simulation model are event scheduling, activity scanning and process interaction approach. The event scheduling approach focuses on event, i.e. the moment in time when state changes occur, while process interaction focuses on processes, i.e. the flow of each entity through the system. In the activity scanning approach in each cycle of simulation there are independently checked conditions of all events occurring. For a comprehensive description of basic methods and techniques related to computer simulation of discrete event systems, a reader is referred to Tyszner (1990). In the paper we will concentrate on simulation of discrete-event systems by event scheduling approach.

The event oriented simulation concentrates on handling and sending events. The activity following each event is implemented as an event routine and the event routine may schedule new events and re-schedule existing event. In this approach we have to define states, events, rules telling what will happen when an event occurs and some parameters.

In the simulation model there are considered three sea motorways with four points of collision marked at the scheme (Fig. 4). Collision points number 1 and 3 are X-shaped intersection while collision points number 2 and 4 are the type of Y-shaped intersection. These two types of collision are described in Section 3. The main flow is on the sea motorway 1. The collision problem includes the characteristics of the ships and their motion before, during and after collision.

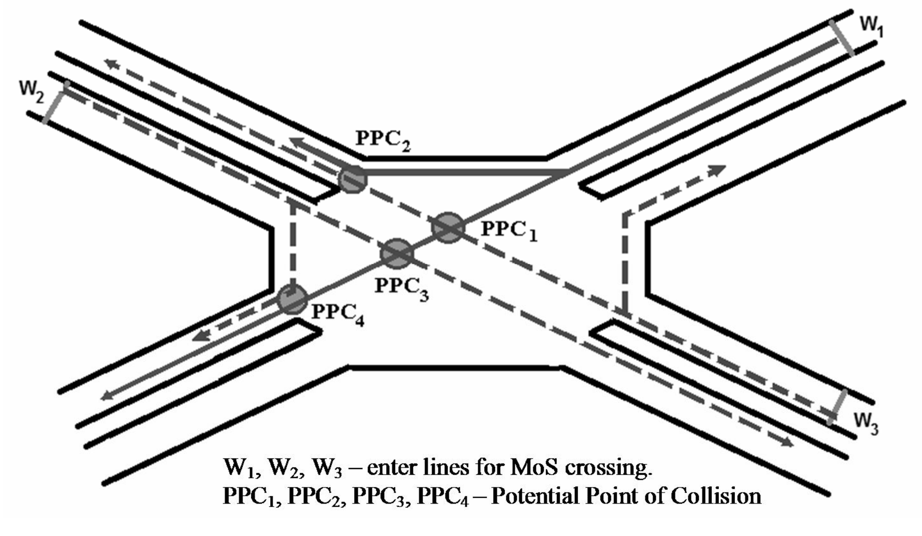


Figure 4. The scheme of the collision problem

4.3 Computer program

We consider following major ship types: a tanker, a container carrier, a passenger ship, a RoPax, a general cargo ship and fast ferry. The mean velocity and length for different types of vessels at each waterway is given by default from empirical data for maritime traffic in Gulf of Finland presented in (Montewka et a. 2010). The user can change manually, given by default while computer starting, values of the vessels' velocity and length (Fig. 5).

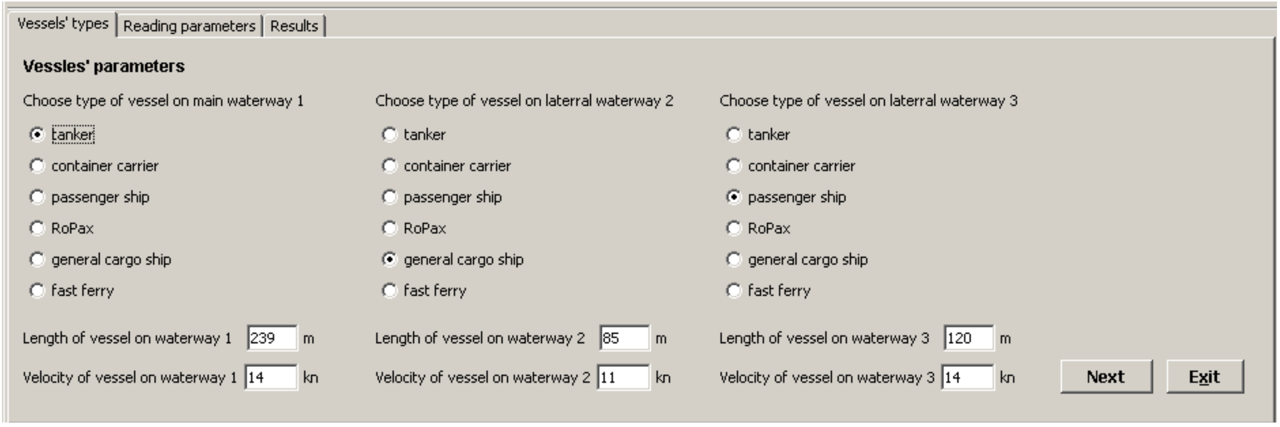


Figure 5. The starting window of the simulation program

In the simulation the ships' starting times can follow different distribution. We can choose these distribution on the main and lateral waterways from the following list: deterministic, uniform, exponential, Erlang, normal, log-normal, Beta, gamma and triangular. Thus using the program there is possible to simulate also non Poisson streams. In Fig. 6 there is presented a program window that is showing the moment of reading data and choosing the distributions of ships' starting times. The mean waiting time between ships' departing for each sea motorway is given by the user. Given by default the percentage of the starboard ships at each waterway can be modified by the user. In the simulation the user can also change the distances to collision points. In the case the flow of vessels into waterway is uniform there is also necessary the minimal accepted distance between vessels, that is given by default in the computer program depending on the vessels length. It is assumed by default that the intervals times between successive notifications at the sea motorways 2 and 3 have exponential distribution and at the sea motorway 1 an uniform distribution.

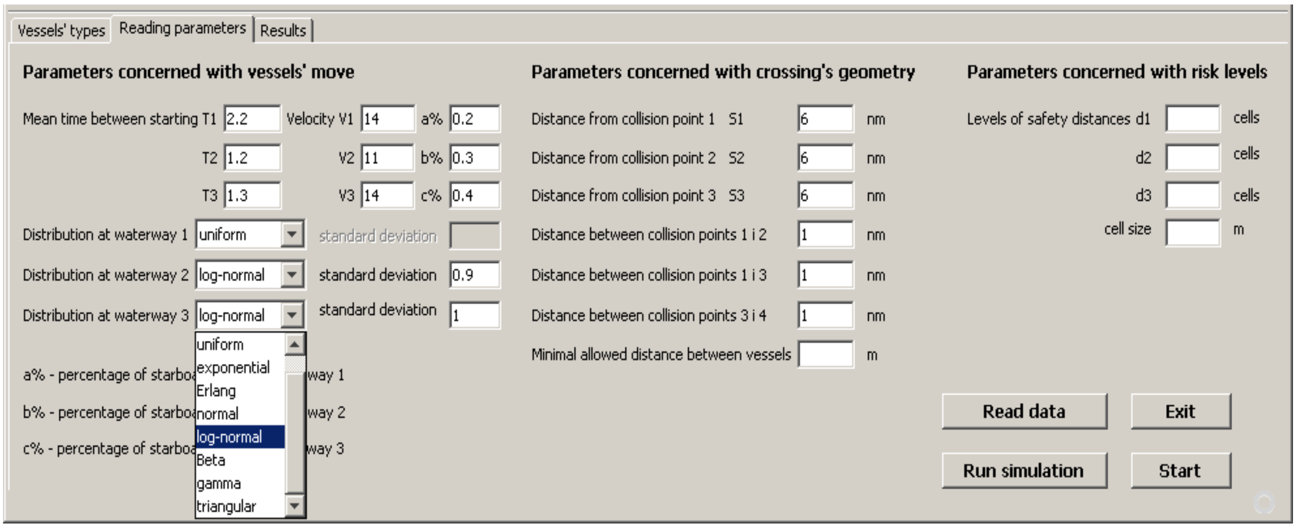


Figure 6. The program window for reading parameters concerned with vessels' move

The scheme of the computer program used for simulation is presented in Fig. 7 and the rules applied in the computer simulation can be described as follows:

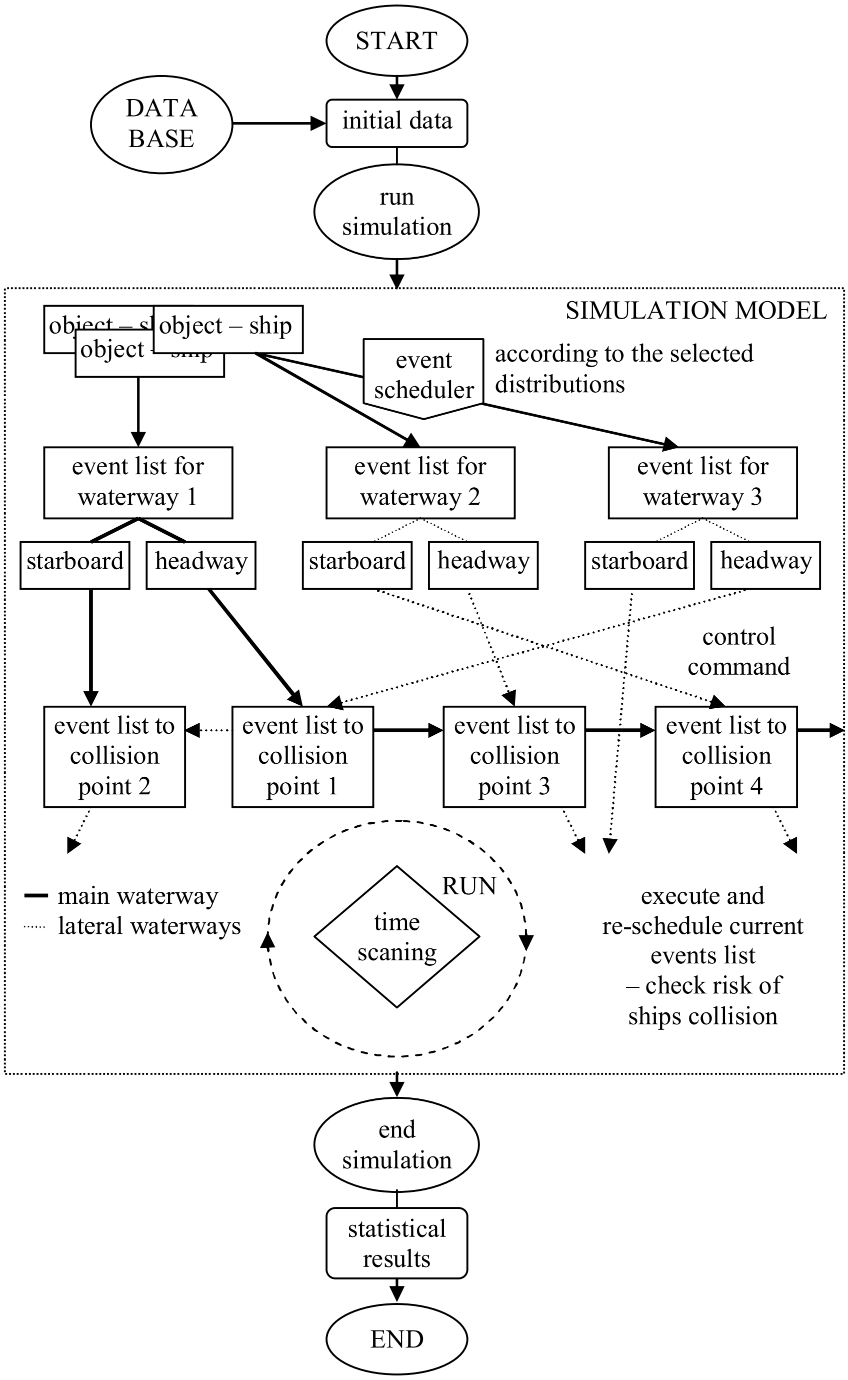


Figure 7. The scheme of the simulation program

−the system saves data and gives results from the point of view vessels being on the main waterway 1; we assume that vessels on the main waterway 1 are stand-on vessels, while vessels on the waterways 2 and 3 are give-way vessels,

−at each time step the system examines the ship closest to the crossing on each waterway and if the safety state of a ship on the waterway 1 has changed the system saves this transition and the time during which the ship was in the previous safety state,

–if the ship on the main waterway 1 is not fully safe the system also examines the next ships on this waterway,

–in the considered crossing situation the starboard ships on the waterway 3 are safe as they do not cross any other waterway,

–in the vessel on the waterway 3 follows headway this waterway then the system checks situation at the collision point 1 and 2 and the safety state of the vessel, if there is on the main waterway 1; after passing the collision point 1 if there is a ship on the waterway 1 going starboard the system checks the situation at the collision point 2, if there is no starboard ship on the waterway 1 the vessel on the waterway 3 is safe and it is not controlled by the system,

–the starboard ships on waterway 1 are examined at the collision point 2, while the headway ships are first checking at the collision point 1, next at the collision point 3 and 4; after passing the collision point 4, there is assumed the negligible risk of collision,

–the starboard ships on the waterway 2 are examined at the collision point 4, while the headway ships are examined at the collision point 3,

–the process is repeated throughout the entire simulation time.

In the computer program, according to the accepted before risk levels and their critical distances defined by (3)-(5), there are assumed following states:

state 0 – collision alert;
state 1 – high risk of collision;
state 2 – low risk of collision;
state 3 – negligible risk of collision.

We denote by $p(i)$, i = 0,1,2,3, probability of system being in the safety state i.

4.4 Results of the program

As a result of the program we obtain following data: the matrix of the system transitions' number between the states and the realizations of the conditional sojourn times at the state until the transition to the other state. From these results there are also determined the matrix of probabilities of the system's transitions between the states and the vector of probabilities of the system's being in the particular states during the simulation time. The obtained from the simulation results can be used for further identification and safety analysis of the system (Blokus-Roszkowska et al. 2011).

The proposed simulation model is sensitive to its changing parameters, that is depicted at the graph (Fig. 8). For this reason the input data of the program must be properly selected. The obtain from the simulation results show the dependency of probabilities of system being in the safety states during the simulation time on the mean time between vessels' departure i.e. on the intensity, at the sea motorway 1. In the presented example we assume that the departure time of a ship on the main waterway follows an uniform distribution and the interval times between ships' departure on the crossing waterways are log-normal. We assume that the mean time between ships' departure on the sea motorway 2 equals 1.2 h with standard deviation 0.9 h and on the sea motorway 3 equals 1.3 h with standard deviation 1 h, equivalently.

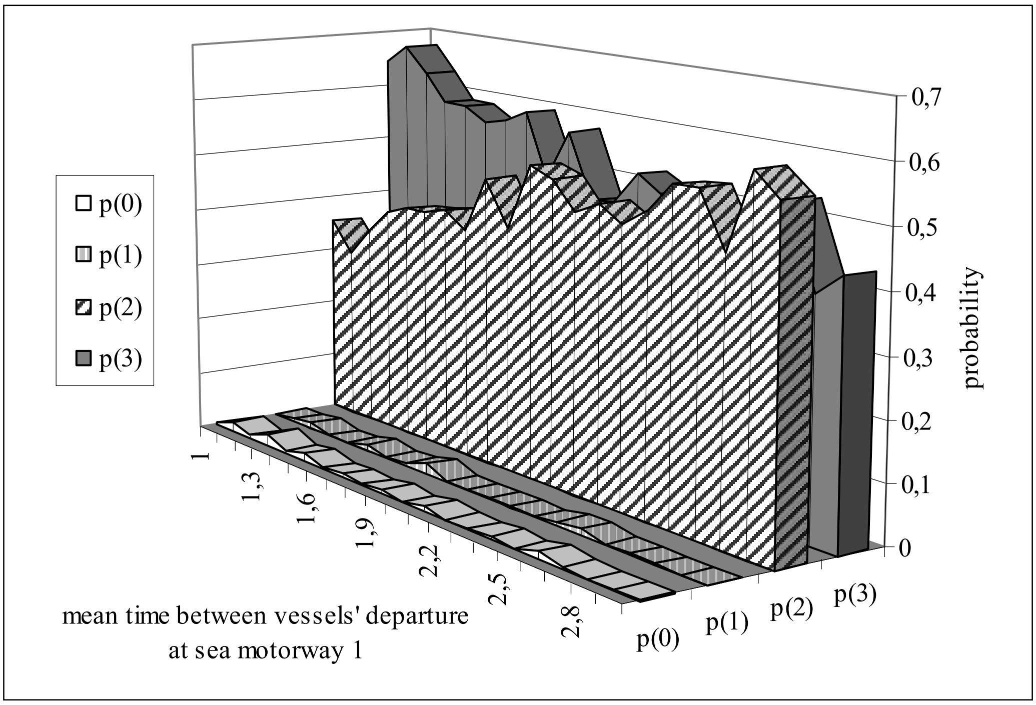


Figure 8. Probability of being in the safety states depending on the mean time between vessels' departure at the sea motorway 1

5 CONCLUSIONS

Java can expose the benefit of computer simulation to a larger audience of problem-solvers, decision-makers and trainers. Java-based simulation components could be easily distributed, executed and modified throughout the word over the internet. The main advantage of Java is cross-platform compatibility that can eliminate the need to maintain different versions of the software. Considering this Java-based programs can provide solutions or assistance in safety transportation system planning and support cooperation or exchange of information between ships, ports and terminals. The presented program can serve a base to create a new service that supports the development of the concept of sea motorways. The simulation programs can optimize the logistical transportation system with integration of sea motorways.

The idea of the paper was to develop the simulation environment to test the features of computer-controlled sea motorways. The proposed simulation model for safety state evaluation can be helpful on the level of safety management. The model allows to predict safety state depending on changing traffic strength at sea motorways. With the concept of safety management such simulation models could face the problem of sea transportation development and provide new solutions for operational optimization and safety transportation system planning.

REFERENCES

Blokus-Roszkowska, A., Kołowrocki, K. & Fu X. 2011. Integrated software tools supporting decision making on operation, identification, prediction and optimization of complex technical systems reliability and safety, Part 2. *Summer Safety & Reliability Seminars Journal of Polish Safety and Reliability Association* (5) 2: 279-288.

Davis, H.H.A. 1984. Piecewise deterministic Markov processes: a general class of non-diffusion stochastic models. *Journal of the Royal Statistical Society Series B*, 46(3): 353-388.

Fujii, Y., Tanaka, K. 1971. Traffic capacity. *Journal of Navigation* 24: 543-552.

Garrido, J.M. 2001. *Object-Oriented Discrete-Event Simulation with Java*. New York: Kluwer Academic/Plenum Publisher.

Jutta Y. 2010. Modelling Marine Accident Frequency. Master's thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Technology in the Degree Programme in Engineering Physics.

Kilgore, R., Healy, K. & Kleindorfer, G. 1998. The future of Java-based simulation. Proceedings of the 1998 Winter Simulation Conference, *IEEE Computer Society Press*: 1707-1712.

MacDuff, T. 1974. The probability of vessel collisions. *Ocean Industry*: 144-148.

McNab, R. 1996. SimJava: a discrete event simulation library for Java. University of Newcastle upon Tyne. Available from http://www.javaSim.ncl.ac.uk

Montewka, J., Goerlandt, F., Kujala, P. 2012. Determination of collision criteria and causation factors appropriate to a model for estimating the probability of maritime accidents. *Ocean Engineering* 40: 50-61.

Montewka, J., Hinz T., Kujala, P., Matusiak, J. 2010. Probability modelling of vessel collisions. *Reliability Engineering and System Safety* 95: 573-589.

Monticino, M.G., Cogdill, T. & Acevedo, M.F. 2002. Cell Interaction in Semi-Markov Forest Landscape Models, Integrated Assessment and Decision Support, *Proceedings of the First Biennial Meeting of the International Environmental Modelling and Software Society* iEMSs 1: 334- 339.

Nagel, K., Schreckenberg, M. 1992. A cellular automaton model for freeway traffic. *Journal de Physique I France* 2(12): 2221-2229.

Paulauskas, V. & Bentzen, K. 2007. *Scenario Analysis of New Maritime Corridors*, BaSIM-Baltic Sea Information Motorways, Aalborg 27.

Pedersen, P. T. 1995. Collision and Grounding Mechanics. *Proceedings WEMT 1995*, 1: 125-157.

Pietrzykowski, Z. & Uriasz, J. 2009. The Ship Domain – A Criterion of Navigational Safety Assessment in an Open Sea Area. *Journal of Navigation*, 62(01): 93–108.

Simard R. 2012. SSJ: Stochastic Simulation in Java, Document available online at: http://www.iro.umontreal.ca/~simardr/ssj/indexe.html.

Smalko Z. & Smolarek L. 2010. Modelling a ship safety according to collision threat for ship routes crossing. *Scientific Journals Maritime University of Szczecin*, 20(92): 120–127.

Tyszner, J. 1990. *Digital simulation* (*in Polish*). Wydawnictwo Naukowo-Techniczne, Warsaw.

Wahle, J., Neubert, L., Esser, J., Schreckenberg, M. 2001. A cellular automaton traffic flow model for online simulation of traffic. *Parallel Computing* 27: 719-735.

Xue, Y., Lee, B.S., Han, D. 2009. Automatic collision avoidance of ships. Proceedings of the Institution of Mechanical Engineers, Part M: *Journal of Engineering for the Maritime Environment* 233(1): 33-46.

A HEURISTIC METHOD FOR RELIABILITY REDUNDANCY OPTIMIZATION OF FLOW NETWORKS

Kumar Pardeep * and Chaturvedi D.K. **

•

Department of Instrumentation,
Kurukshetra University, Kurukshetra, – 136 119, India
e-mail: pardeep_kuk@rediffmail.com*, dhiraj_chaturvedi@yahoo.co.in**
*Corresponding author

Pahuja G.L.

•

Department of Electrical Engineering,
National Institute of Technology, Kurukshetra – 136 119, India
e-mail: pahuja_gl@yahoo.co.in

ABSTRACT

In flow networks, from the quality and service management point of view, measurement of the transmission ability of a network to meet the customers demand is very important. To meet the ultra high reliable requirements of such networks, a heuristic method for reliability redundancy optimization of flow networks using composite performance measure (CPM) integrating reliability and capacity has been proposed. The method is based upon the selection of main flow paths and backup paths and then optimizing main paths on priority basis. Thus, the reduced computation work makes the proposed algorithm suitable for designing of large, reliable telecommunications networks.

Keywords: flow networks; capacity related reliability; constrained redundancy optimization; heuristic algorithm.

1 INTRODUCTION

Constrained reliability redundancy optimization of networks has generally been studied with reliability as connectivity measure. The practical systems such as computer networks, telecommunication networks, transportation systems, electrical power transmission networks, internet etc. can only transport limited amount of flow therefore these are termed as flow limited networks. To meet the ultra high reliability requirements of such networks, a heuristic method for reliability redundancy optimization of flow networks has been proposed.

Literature is enriched with reliability redundancy optimization of networks with connectivity only as a measure of performance (Sharma & Venkateswaran 1971, Aggarwal 1976, Golden & Magnanti 1977, Gopal et al. 1978, Lee 1980, Bodin et al. 1982, Xue 1985, Dinghua 1987, Fredman & Tarjan 1987, Kim & Yum 1993, Shen 1995, Martins & Santos 1997, Schrijver 1998, Ahuja 1998, Kuo & Prasad 2000, Kuo et al. 2001, Park et al. 2004, Pascoal et al. 2005, Kumar et al. 2009, 2010a, b, 2011). However, reliability redundancy optimization of flow networks has rarely been studied. In present days context existing methods do not fulfil the requirements of management of quality of service. The reliability redundancy optimization techniques discussed in Gopal et al. (1978), Dinghua (1987), Kim & Yum (1993), Shen (1995), Park et al. (2004), Kumar et al. (2009, 2010a, b, 2011) are not suited to flow networks. This paper presents a technique for reliability redundancy optimization of flow networks using combined performance measure named capacity related reliability (CRR).

A path is a sequence of arcs and nodes connecting a source to a sink. All the arcs and nodes of network have its own attributes like delay, reliability and capacity etc.. From the quality and service management point of view, measurement of the transmission ability of a network to meet the customers demand is very important (Lin 2006). When a given amount of flow is required to be transmitted through a flow network, it is desirable to optimize the network reliability to carry the desired flow. In such cases, the system reliability is the measure of quality of the system capability to transmit desired flow. The capacity of each arc (the maximum flow passing the arc per unit time) has two levels, 0 and/or a positive integer. The system reliability is the probability that the maximum flow through the network between the source and the sink is not less than the demand (Golden & Magnanti 1977, Lee 1980, Bodin et al. 1982, Fredman & Tarjan 1987, Ahuja 1998, Lin 2003, 2004, Pahuja 2004, Lin 2006, 2007a, b). For determining the reliability it is generally assumed that network is capable of transmitting any required amount of flow between source (s) and terminal (t) nodes of the network. This presumption is neither valid nor justifiable for real life systems as links and nodes can carry only limited amount of flow. In 1980 Lee did the pioneer work of integrated both capacity and reliability & named it combined performance measure as capacity related reliability (CRR) and also termed such networks as flow networks. Max-Flow-Min-Cut theorem has been used to determine the capacity of the network (Sharma & Venkateswaran 1971, Xue 1985, Shen 1995, Martins & Santos 1997, Schrijver 1998, Lin 2003, 2004, Park et al. 2004, Lin 2006, 2007a, b).

2 COMPOSITE PERFORMANCE MEASURE

Reliability under flow constraint is a more realistic performance measure for flow networks. The concept of weighted reliability introduced by Aggarwal (1976) requires that all the successful states qualifying connectivity measure of the network be enumerated. The probability of each success state is evaluated and is multiplied by the normalized weight (Aggarwal 1985). Rushdi (1988) evaluated the same performance index as evaluated by Aggarwal 1985 using decomposition approach. Methods given by Aggarwal (1985), Rushdi (1988), and Shakti (1995) generate both cancelling (failed) and non-cancelling (success) terms.

The following section presents a heuristic algorithm for reliability redundancy optimization of flow networks using composite performance measure (CPM) and the method has been utilized to determine the capacity related reliability performance index**.**

2.1 Notation

$a_l(X)$ Sensitivity factor of l^{th} minimal path set

$b_i(x_i)$ Subsystem selection factor for i^{th} subsystem with x_i components

C_j Total amount of resource j available

$g_i^j(x_i)$/ $c_{ji}(x_i)$ Amount of resources-j consumed in subsystem-i with x_i components / Cost of subsystem i for j^{th} constraint.

$h(.)$ Function yielding system reliability; dependent on number of subsystems (n) and configuration of subsystems

k Number of constraints, $j = 1, 2,\ldots, k$

$L(x)$ $(L_{x_1}, L_{x_2},..., L_{x_n})$, Lower limit of each subsystem i,

m Number of main minimal path sets, $l = 1, 2,., m$

n Number of subsystems, $i = 1, 2,\ldots, n$

P_l l^{th} minimal path set of the system

Ps $(l^1,l^2,...,l^{min})$: priority vector s.t. l^1 and l^{min} are the number of minimal path sets arranged in decreasing order of path selection parameter $a_l(X)$.

$Q_i(x_i)$ Unreliability of subsystem i with x_i components.

r_i Reliability of a component at subsystem i.

$R_i(x_i)$ Reliability of subsystem i with x_i components.

R_r Residual resources [total resource available (C_j) - resources consumed ($\sum g_i^j x_i$)]

$R_s(X)$ System reliability

$S(x)$ Set of variables that have been used as key-elements in a given decomposed expressions

$U(x)$ $(U_{x_1}, U_{x_2}, ..., U_{x_n})$, Upper limit of each of subsystem i,

x^* Optimal solution

x_i Number of components in subsystem i; i = 1,2,….n

X A vector $(x_1, \dots\dots x_n)$

ΔR_i Increment in i^{th} stage reliability when a unit is added in parallel to the i^{th} stage

2.2 Assumptions

Following are the assumptions for the rest of the sections:

1. The system and all its subsystems are coherent.
2. Subsystem structures (other than coherence) are not restricted.
3. The networks are modelled with the help of graphs, the paths (ordered pair of arcs and the members of the ordered pair are reliability and capacity respectively) where in are assigned as the weight of each link.
4. Each link can have only two stages up and down.
5. The network nodes are perfect. If the nodes are not perfect, the method needs to be modified to deal with nodes failures.
6. All component states are mutually and statistically independent.
7. All constraints are separable and additive among components.
8. Each constraint is an increasing function of x_i for each subsystem.
9. Redundant components cannot cross subsystem boundaries.

2.3 Composite Performance Measure (CPM)

The weighted reliability measure i.e. composite performance measure (CPM), integrating both capacity and reliability may be stated as by **[27, 28]**:

$$\text{CPM} = \sum_{i \in S(x)} \omega t_i R_i \tag{1}$$

Where ωt_i is the normalized weight and is defined as:

$$\omega t_i = Cap_i / Cap_{max}$$

i.e. the ratio of capacity in the i^{th} state to the maximum capacity (Cap_{max}) of the system and R_i probability of the system being in state S_i and is computed as:

$$R_i = P_r\{S_i\} = \prod_{j/S_{ij}=1} p_j \times \prod_{k/S_{ik}=0} q_k \tag{2}$$

2.4 Capacity Functions of Networks

The capacity function of different arcs connected in parallel is **[25]**:

$$C(X)_{Par} = \sum_{i \in x} Cap_i \tag{3}$$

and the capacity function of different arcs connected in series is:

$$C(X)_{Ser} = \min\{Cap_i\} \tag{4}$$

The rules for connecting series and parallel arcs to integrate capacity and reliability to give composite performance measure are expressed as:

$$CR(X)_{Ser} = \{\min_{i \in x} Cap_i\} \prod_{i=1}^{n} r_i \tag{5}$$

$$CR(X)_{Par} = \sum_{i=1}^{n} Cap_i \cdot \bigcup_{i=1}^{n} r_i \tag{6}$$

CPM for series and parallel networks can be defined as:

$$\text{CPM}_{Par} = CR(X)_{Par} / Cap_{max} \tag{7}$$

and

$$\text{CPM}_{Ser} = CR(X)_{Ser} / Cap_{max} \tag{8}$$

3 PROBLEM FORMULATION AND HEURISTIC METHOD

3.1 Problem Formulation

The general constrained redundancy optimization problem in complex systems can be reduced to the following integer programming problem (Kuo et al. 2001):

Maximise

$$R_s(X) = h(R_1(x_1),..., R_n(x_n)), \tag{9}$$

subject to $\sum_{i=1}^{n} g_i^j(x_i) \le C_j, \quad j = 1, 2, \ldots, k$ (10)

and $1 \le x_i \le U_{xi}, \quad i = 1, 2, \ldots, n.$

3.2 Proposed Heuristic Method

In real life systems all the arcs are not simultaneously connected to carry flow from source to sink. Hence a flow path set is the arcs and nodes that actually carry traffic. In these practical systems all the path sets are not utilized for transfer of information (Hayashi & Abe 2008). The flow is transmitted through the main path(s) and in case of failure of this path(s), a backup path completes the task of main path. The backup path(s) come in operation only when the main paths fail thus, enhancing the reliability of the network, it is presumed that the main path(s) and backup path(s) for the network are known. The proposed algorithm first optimizes the main path(s) and then back up path(s) using redundancy optimization technique. This leads to more efficient use of resources which are generally limited. Unlike existing heuristics, a stopping criterion has been applied to switch from reliability redundancy optimization of main path(s) to reliability redundancy optimization of backup paths. The algorithm considers sensitivity factor as the criteria for selecting the main path from the list of given main paths and then a subsystem for applying redundancy within the chosen flow path.

3.3 Steps of the Proposed Method

Step1: Firstly select the main path sets *m* and back up flow paths of the flow Network.

Step2: Let $x_i = 1$ for all *i; i* = 1, 2,…, *n*.

Step3: Calculate $a_l(X)$, *l* = 1, 2,…, *m* for each minimal path set and find *l** such that $a_{l*}(X) = max\,[\,a_l(X)\,]$ and

$$a_l(X) = \frac{\prod_{i \in P_l} R_i(x_i)}{\sum_{i \in P_l} \sum_{j=1}^{k} (g_i^j(x_i) / kC_j)},$$

$$l = 1,2,3........m.$$

Step4: For the chosen minimal path set – *l** find i^* such that $b_{i^*}(x_{i^*}) = max\,[\,b_i(x_i)\,]$ and

$$b_i(x_i) = \frac{\Delta R_i}{\sum_{j=1}^{k} (g_i^j(x_i)/kC_j)}, \quad \text{for each } i \in P_l$$

where $\Delta R_i = \left(1 - Q^2{}_i(x_i)\right) - R_i(x_i)$

Step5: Check, by adding one redundant subsystem to unsaturated subsystem i^* :

i) if no constraints are violated, add one redundant subsystem to unsaturated subsystem i^* by replacing x_{i^*} with $x_{i^*} + 1$, and go to step 3.

ii) if at least one constraint is exactly satisfied and others are not violated, then add one redundant subsystem to unsaturated subsystem i^* by replacing x_{i^*} with $x_{i^*} + 1$. The $x^* = X$ is the optimal solution. Go to step 6.

iii) if at least one constraint is violated, then remove minimal path set *l** from further consideration and consider the next path having maximum $a_{l*}(X)$ value and go to step 4.

iv) if all minimal path sets are now excluded from further consideration, then $x^* = X$ is the optimal solution; else go to step 3

Step6: In case any resources are still available optimize the backup paths as discussed in Step2.

Step4: Evaluate the composite performance measure (CPM) for each subsystem of the network.

Step5: Evaluate the system reliability using the CPM of the each subsystem.

4 COMPUTATION AND RESULTS

To illustrate the performance of the proposed algorithm a network having six arcs $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ and five minimal path sets $\{y_1, y_2, y_3, y_4, y_5\}$ as shown in the Figure 1 is considered and solved for capacity related redundancy reliability optimization using CPM (7 and 8). System reliability is determined using Bayes method. The network shown in Figure 1 is a bench mark problem, considered by Hayashi & Abe (2008).

Using Baye's method, the Reliability of the above system can be expressed as:

$$R_s(X) = R_3\,[1- Q_6\,\{1-(1-Q_1Q_2)(1-Q_4Q_5)\}] + Q_3[1-(1-R_2R_5)(1-R_1R_4)]*Q6 \quad (11)$$

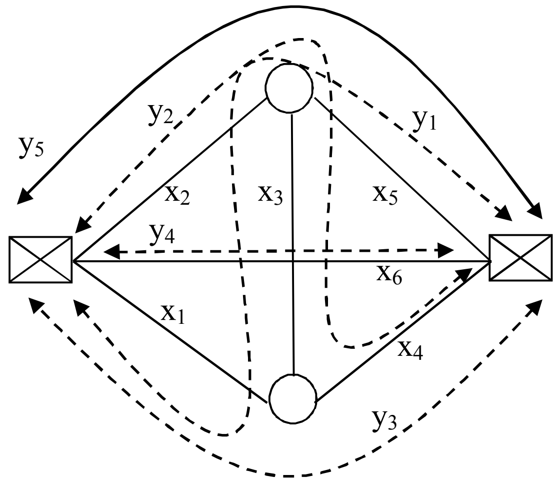


Figure 1 Illustration Network

The simple minimal path sets of the Network are

P1 = {1, 3, 5}, **P2** = {2, 3, 4}, **P3** = {1, 4}, **P4** = {6}, **P5** = {2, 5}

The problem is solved for data given in Table 1. Using this initial data the general problem of constrained reliability redundancy allocation has been solved using the steps discussed in Section 3.3 above. The problem is solved by considering the flow path sets P1, P2, P3 and P4 as main path sets and P5 as backup path. The proposed algorithm gives the optimal solution (3, 1, 1, 2, 1, 4) with system reliability R_s = 0.999998458, the optimized subsystem reliability probability R_i and unreliability probabilities Q_i are shown in Table 2.

Table 1 Data for Fig. 1

i	1	2	3	4	5	6
r_i	0.70	0.75	0.8	0.85	0.7	0.9
g_i^1 / c_{1i}	2	3	2	3	1	3
C_1	30					

Table 2 Optimized subsystem reliability/unreliability for Fig. 1

i	x_1	x_2	x_3	x_4	x_5	x_6
X^*	3	1	1	2	1	4
R_i	0.973	0.75	0.8	0.9775	0.7	0.9999
Qi	0.027	0.25	0.2	0.0225	0.3	0.0001

The capacity of each subsystem of the flow path is taken as 100 and the capacity of flow paths of the network is determined using proposed approach as:

$$\left.\begin{array}{l} P1 = \min\{3*100, 100, 100\} = 100 \\ P2 = \min\{100, 100, 100\} = 100 \\ P3 = \min\{3*100, 2*100\} = 200 \\ P4 = \min\{4*100\} = 400 \end{array}\right\} \quad (12)$$

Next the composite performance measure CPM expression (13) is derived using (7 and 8) and the value for CPM for an assumed flow of 200 is suppose to pass through the flow path and it comes out to be 1.0000 .

$$CPM_{P1} = \frac{\min Cap_i}{Cap_{\max}} [R_1 R_3 R_5] \quad (13)$$
$$= (100/200) \text{ x } 0.973 \text{ x } 0.8 \text{ x } 0.7 = 0.27244$$
$$CPM_{P2} = \frac{\min Cap_i}{Cap_{\max}} [R_2 R_3 R_4] \quad (14)$$
$$= (100/200) \text{ x } 0.75 \text{ x } 0.8 \text{ x } 0.9775 = 0.29325$$
$$CPM_{P3} = \frac{\min Cap_i}{Cap_{\max}} [R_1 R_4] \quad (15)$$
$$= (200/200) \text{ x } 0.973 \text{ x } 0.9775 = 0.9511075$$
$$CPM_{P4} = \frac{\min Cap_i}{Cap_{\max}} [R_6] \quad (16)$$
$$= (400/200) \text{ x } 0.9999$$
$$= (2) \text{ x } 0.9999 \quad \text{as } 0 \le (\min Cap_i / Cap_{max}) \le 1$$
$$\text{so} \quad = 1\text{x } 0.9999 = 0.9999$$

Composite performance measure integrating the reliability with capacity is calculated as:

$$CPM_{Network} = 1 - (1-CPM_{P1})(1-CPM_{P2})(1-CPM_{P3})(1-CPM_{P4})$$
$$= 1 - (1-0.27244) \text{ x } (1-0.29325) \text{ x } (1-0.9511) \text{ x } (1-0.9999)$$
$$= 1.0000$$

The above result shows that proposed method is capable of optimizing the flow network to transport the desired capacity through the network with highest reliability. However, the selection of main paths and backup paths will affect the quality of composite performance measure. Hence the proper choice of these paths may be done using cardinality criteria (Kumar et al. 2010b) or any other hierarchical measures of importance.

5 CONCLUSIONS

This paper has presented a new model for designing reliable flow networks capable of transmitting required flow. The proposed algorithm utilizes the concept of main and backup flow paths. The choice of backup and flow paths is application specific and paths with minimum cardinality may be selected as main path and disjoint paths can be the backup paths. The numerical example demonstrates that the proposed algorithm is fast for designing large, reliable telecommunications networks because the task of optimization is reduced, as only few paths are selected as main paths.

REFERENCES

Sharma J. and **Venkateswaran K. V. 1971,** "A direct method for maximizing the system reliability," *IEEE Trans. Reliability*, vol. R-20, no. 1, pp. 256–259.

Aggarwal K. K. 1976, "Redundancy optimization in general system IEEE Trans. Reliability, vol R-25, pp. 330-332.

Golden B. L. and **Magnanti T. L. 1977**, "Deterministic network optimization: A bibliography," *Networks*, vol. 7, no. 2, pp. 149–183.

Gopal K., Aggarwal K. K., and **Gupta J. S. 1978**, "An improved algorithm for reliability optimization," *IEEE Trans. Reliability*, vol. R-27, no. 5, pp.325–328.

Lee S. H. 1980, "*Reliability Evaluation of Flow Networks*", IEEE Tr. Rel. Vol. R-29, pp. 24-26.

Bodin L. D., Golden B. L., Assad A. A., and **Ball M. O. 1982**, "Routing and scheduling of vehicles and crews: the state of the art," *Computers and Operations Research*, vol. 10, no. 2, pp. 63–211.

Aggarwal K. K. 1985, "Integration of Reliability and Capacity in Performance Measure of Telecommunication Network", *IEEE Trans. Reliability,* vol. R-34, pp. 184-186.

Xue J. 1985, "On multistate system analysis," *IEEE Trans. Reliability*, vol. 34, no. 4, pp. 329–337.

Dinghua S. 1987, "A new heuristic algorithm for constrained redundancy-optimization in complex systems," *IEEE Trans. Reliability*, vol. R-36, no.5, pp. 621–623.

Fredman M. L. and **Tarjan R. E. 1987,** "Fibonacci heaps and their uses in improved network optimization algorithms," *J. ACM*, vol. 34, no. 3, pp. 596–615.

Rushdi M. 1988, "Performance Indexes of Telecommunication Networks", *IEEE Trans. Reliability,* vol. R-37, pp. 57-64.

Rai S., and **Soh S. 1991**, "A computer Approach for Reliability Evaluation of Telecommunication networks with heterogeneous Link capacities", IEEE trans. On Reliability, Vol. 40, pp 441-451.

Kim J. H. and **Yum B. J. 1993,** "A heuristic method for solving redundancy optimization problems in complex systems," *IEEE Trans. Reliability*, vol. 42, no. 4, pp. 572–578.

Varshney, Joshi, and **Chang A. R. 1994**, "Reliability modeling and performance evaluation of variable link capacity networks", *IEEE Trans. Reliability*, vol. 43(3), pp. 378-382.

Shen Y. 1995, "A new simple algorithm for enumerating all minimal paths and cuts of a graph," *Microelectronics and Reliability*, vol. 35, no. 6, pp. 973–976.

Shakti K. 1995, "*Some Aspects of Reliability Related Performance Evaluations of Computer Communication Networks*", Ph.D. Thesis, Kurukshetra University, Kurukshetra.

Martins E. D. Q. V. and **Santos J. L. E. D. 1997**, "An algorithm for the quickest path problem," *Operations Research Letters*, vol. 20, no. 4, pp. 195–198.

Schrijver 1998*, Theory of Linear and Integer Programming*. NY: John Wiley & sons, pp. 155–156.

Ahuja R. K. 1998, "Minimum cost-reliability ratio problem," *Computers and Operations Research*, vol. 16, no. 1, pp. 83–89.

Kuo W. and **Prasad V. R. 2000,** "An Annotated Overview of System-Reliability Optimization", *IEEE Trans. Reliability,* vol. 49(2), pp.176-187.

Kuo, W. Prasad, R., Tillman, F.A. and **Hwang, C.L. 2001** *Optimal Reliability Design Fundamentals and Applications*, Cambridge University Press.

Lin Y. K. 2003, "Extend the quickest path problem to the system reliability evaluation for a stochastic-flow network," *Computers and Operations Research*, vol. 30, no. 4, pp. 567–575.

Park C. K., Lee S., and **Park S. 2004**, "A label-setting algorithm for finding a quickest path," *Computers and Operations Research*, vol. 31, no. 14, pp. 2405–2418.

Lin Y. K. 2004, "Reliability of a stochastic-flow network with unreliable branches & nodes under budget constraints," *IEEE Trans. Reliability*, vol. 53, no. 3, pp. 381–387.

Pahuja G. L. 2004, "*Reliability Evaluation and Optimization Recent and New Approaches",* Ph.D. thesis, Kurukshetra University, Kurukshetra.

Pascoal M. M. B., Captivo M. E. V., and **Clímaco J. C. N. 2005**, "An algorithm for ranking quickest simple paths," *Computers and Operations Research*, vol. 32, no. 3, pp. 509–520.

Lin Y. K. 2006, "Reliability of a computer network in case capacity weight varying with arcs, nodes and types of commodity", *Reliability Engineering and System Safety*, vol. 2(5), pp. 1-7.

Lin Y. K. 2007,

a. "Reliability evaluation for an information network with node failure under cost constraint," *IEEE Trans. Systems, Man and Cybernetics-Part A: Systems and Humans*, vol. 37, no. 2, pp. 180–188.

b. "On a multi-commodity stochastic-flow network with unreliable nodes subject to budget constraint," *European Journal of Operational Research*, vol. 176, no. 1, pp. 347–360.

Hayashi M. and **Abe T. 2008**, "Evaluating Reliability of Telecommunications Networks Using Traffic Path Information", *IEEE Trans. Reliability*, vol. 57(2), pp. 283-294, 2008. pp. 283-294.

Kumar P., Chaturvedi D. K. and **Pahuja G. L. 2009,** Heuristic algorithm for constrained redundancy reliability optimization and performance evaluation. *Proc. IMechE, Part O: J. Risk and Reliability*, vol. **223** (4), 381-386.

Kumar P., Chaturvedi D. K. and **Pahuja G. L. 2010,**

a. "Heuristic Methods for Solving Redundancy Allocation in Complex Systems", *Int. J. of Reliability and Safety,* vol. 4, No. 2/3, pp. 285-298.

b. "An efficient heuristic algorithm for determining optimal redundancy allocation of complex networks", *Reliability Theory and Applications*, vol. 3(18), pp.15-28.

Kumar P., Chaturvedi D. K. and **Pahuja G. L. 2011,** "Constrained Reliability Redundancy Optimization of Complex Systems using Genetic Algorithm", *MIT Int. J of Electrical and Instrumentation Engineering,* vol. 1, No. 1, pp. 41-48.

ESTIMATION OF RELIABILITY IN INTERFERENCE MODELS USING MONTE CARLO SIMULATION

A.N. Patowary, J. Hazarika and G. L. Sriwastav

•

Department of Statistics, Dibrugarh University, India
e-mail: a_patowary@yahoo.com, jitenhazarika@rediffmail.com,
wastav2@yahoo.com

Abstract

This paper presents estimation of reliability $R = P(X \geq Y)$ of a system, for the cases when its strength (X) and stress (Y) follow exponential, normal or gamma distributions, using Monte Carlo simulation (MCS). First the parameters of strength and / stress are estimated and substituting them in the reliability expressions, in different cases, the estimates of reliability are obtained. Normal distribution is fitted to various sets of estimated reliability $\hat{R}$, generated by MCS. The goodness of fit is tested using Kolmogorov-Smirnov one sample test.

Keywords: Stress-Strength; Monte-Carlo Simulation; Kolmogorov-Smirnov one sample test.

1. Introduction

In interference theory of reliability, reliability and other reliability characteristics of a system can be expressed as some functions of the parameters of the distributions of the random variables (r.v.'s), strength (X) and stress (Y) associated with the functioning of the system. We estimate these parameters and substitute these values in the expressions for reliability and other characteristics to get their estimates. The estimates of parameters used here are maximum likelihood estimators and as such from the invariance property of MLE's, the corresponding estimators of reliability are also MLE's. In absence of hard data the numerical values of the estimators can be obtained from simulation. There exists extensive literature for estimation of reliability analytically for single component systems e.g. Mazumder [12], Church and Harris [4] etc. But the reliability expressions for multi-component systems are not simple enough to facilitate analytical estimation of reliability and its other characteristics. Also due to lack of hard data, one way out is simulation, in particular Monte Carlo simulation.

With simulation technique it is possible to estimate reliability (or probability of failure) or other reliability characteristics without going into the analytical techniques. The availability of personal computer and software makes the process comparatively simple. In fact, to evaluate the accuracy of the sophisticated analytical techniques or to verify a new technique, simulation is routinely used to independently evaluate the underlying probability distributions.

1.1 Monte Carlo Simulation:

The Monte Carlo Simulation method is an artificial sampling method which may be used for solving complicated problems in analytic formulation and for simulating purely statistical problems. In the simplest form of simulation, each r.v. in a problem is sampled several times to represent its real distribution. Each realization of r.v.'s in the problem produces a set of numbers that indicates one realization of the problem itself. Solving the problem deterministically for each realization is known as a simulation cycle, a trial or a run. Using many simulation cycles we get the overall probabilistic characteristics of the problem, particularly when the number of cycles N is sufficiently large. Simulation, using a computer, is an inexpensive way (compared to laboratory testing) to study the uncertainty in a problem.

The primary components of a Monte Carlo Simulation include the followings:

(i) Probability distribution function (or probability density function): The physical (or mathematical) system must be described by a set of probability distribution functions.

(ii) Random number generator: A source of random numbers uniformly distributed on the unit interval must be available.

(iii) Sampling rule: A prescription of sampling from specified distribution function, assuming the availability of random numbers on the unit interval.

(iv) Scoring (or tallying): The outcome must be accumulated into overall tallies or scores for the quantities of input.

In this paper except exponential distribution we have not used uniformly distributed random numbers, rather obtained random numbers following particular distribution directly from MATLAB.

In Section 2, we have estimated reliability of an n-standby system (n=1, 2, 3), through Monte Carlo Simulation technique. Simulation is performed for exponential stress-strength, normal stress-strength and gamma stress-strength. In Section-3 we have considered fitting of normal distribution to estimated reliability in each case, for different true values of the parameters. The goodness of fit is tested by K-S one sample test (Seigel [18]). Since we have taken a small sample, 20, only, when using χ^2 test, the number of classes becomes too few, due to pooling. We have considered the fitting of normal distribution to check whether normal approximation is good enough for a small of 20.

Some literatures on the topic which we have come across are:

Kamat and Riley [8] presented MCS for a complex system for time to failure (TTF) models.

Some of the others studies of reliability estimation using MCS for TTF models includes Pulido et.al. [15], Goel [5], Hong and Lind [6], Landis et.al. [10], Tunak et.al. [23], Naess et.al. [13], Wu et.al. [24] etc.

Stancampiano [21] applied simulation to interference models. Manders et.al. [11], Aldrisi [1], Stumpf and Schwartz [22], Zhang et.al. [25] have simulated stress-strength. Paul and Borhanuddin [14], Rezaei et.al. [17] estimated reliability of stress-strength model, using MCS. Ahmad et.al.[2] obtain Bayes estimates of P (Y< X) using MCS. Borhanuddin et.al. [3] estimated reliability for multicomponent system using MCS. Rao et al [14] compared reliability estimates for multicomponent systems evaluated by different methods such as method of moments, modified ML method and Best Linear Unbiased Estimator through MCS technique.

Kakati and Sriwastav [7] and Sriwastav [20], used simple simulation by taking random exponential numbers to represent stress-strength. They considered very small samples. From these samples they first estimate the parameters and substituting these in the expressions of reliability they get estimated reliability.

2 Reliability Estimation through Monte Carlo Simulation:

Let us consider an n-standby system. Let $X_1, X_2,\ldots,X_n$ be the strengths of the n components in the system arranged in the order of activation. Let $Y_1, Y_2,\ldots,Y_n$ be the stresses faced, respectively, by $1^{st}, 2^{nd},\ldots,n^{th}$ component, when they are activated; X_i's and Y_i's are all independent. For a detailed description of such a system one may refer (Sriwastav and Kakati, [19]).

The reliability R_n of an n-standby system for a single impact of stress is given by,

$$R_n = R(1) + R(2) + \quad \ldots \quad + R(n), \tag{2.1}$$

where R(i) is the increment in the system reliability due to the i^{th} component, defined as

$$R(i) = P[X_1 < Y_1, X_2 < Y_2, ..., X_{i-1} < Y_{i-1}, X_i \geq Y_i] \quad (2.2)$$

Here, we have assumed that all the components are having the same strength distributions and are working under the same environment (stress), i.e. all X_i's and Y_i's are i.i.d. with probability density functions (pdf's) f(x) and g(y), respectively.

In this section, we use MCS to estimate reliability. The programs are developed in MATLAB, separately for exponential, normal and gamma. First a set of 5000 values of the particular r.v. viz. (exponential, normal or gamma) are generated for a particular value of the parameter(s). Using these values an estimate of the parameters involved is obtained. Substituting this estimate(s) in the expression of reliability we get an estimate of the reliability. This process is repeated j times to give j estimates of the parameter(s) and subsequently j estimates of reliability. The whole process is repeated for different true values of the parameters; for a particular true value of the parameter(s) j is the sample size.

2.1 (a) Exponential Stress-Strength:

Let us assume that the component's strength follows exponential distribution with mean λ and the stress on it follows exponential distribution with mean strength unity, without loss of generality. Then the marginal reliability expression due to the n th component is, (ibid)

$$R(n) = R(n) = \frac{\lambda}{1+\lambda}\left(\frac{1}{1+\lambda}\right)^{n-1} \quad (2.3)$$

So, $$R_1 = \frac{\lambda}{1+\lambda} \quad (2.4)$$

$$R_2 = R_1 + (1 - R_1) R_1, \quad (2.5)$$

$$R_3 = R_1 + (1 - R_1) R_1 + (1 - R_1)^2 R_1. \quad (2.6)$$

For MCS, let U be the uniform r.v. over (0, 1). Then by following inverse transformation we can generate exponential random variable with mean λ as:

Let $U = F(x) = 1 - \exp(-x/\lambda)$

$\Rightarrow X = -\lambda \log(1 - U)$

Now if U is uniform over (0, 1), (1 – U) is also uniform over (0, 1). So

$$X = -\lambda \log(U) \quad (2.7)$$

From uniform r.v. U we can generate exponential r.v. with parameter λ using the above transformation (2.7). We generate 5000 of U. Then from (2.7), for each U, – log U gives a value of the exponential r.v. X with mean unity. Thus we get 5000 values of X. Multiplying each of these 5000 values of X by $\lambda(=0.5, 2, 3)$ we get 5000 values of exponential r.v. (say X_1) with mean λ. The mean of these 5000 values give an estimate of λ for a particular true value of λ. Substituting these estimates in (2.4), (2.5) and (2.6) we get an estimate of R_1, R_2 and R_3, respectively. For each true value of λ the whole process is repeated j times there by giving j estimates of λ and R's for a particular true value of λ. Here, we have taken j = 20.

(b) Normal Stress-Strength: In case of normal stress-strength let $X \sim N(\mu, \sigma^2)$ and stress $Y \sim N$ (0, 1) by without loss of generality. The reliability expressions are (ibid)

$$R(n) = [1 - \Phi\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)]^{n-1} \Phi\left(\frac{\mu}{1+\sigma^2}\right). \quad (2.8)$$

$$R_1 = \Phi\left(\frac{\mu}{1+\sigma^2}\right), \quad (2.9)$$

and R_2 and R_3 are given by (2.5) and (2.6), respectively.

For generating normal random numbers for particular true values of μ and σ^2, we first generate standardized normal numbers (Z) by the MATLAB. Next by the following transformation we generate normal random numbers particular true values of μ and σ^2.

$$X = \mu + Z\sigma \text{ where } Z \sim N(0, 1).$$

We estimate $\hat{\mu}$ and $\hat{\sigma}^2$ from the sample of X of size 5,000. Substituting these estimates in R_1, R_2 and R_3, given above, we get estimates of the system reliabilities R's. This process is repeated j times for a particular set of set of true values of μ and σ^2. We have taken (μ, σ) = (-1, .5), (0, .5), (1, .5), (2, .5), (-1,1), (0, 1), (1, 1), (2, 1), (-1, 2), (0, 2), (1, 2), (2, 2) and j = 20.

(c) Gamma Stress-Strength: We assume that the strength $X \sim \Gamma(1, m)$ and stress $Y \sim \Gamma(1, k)$.
Then if m and/ or k is an integer (ibid)

$$R(n) = \left(1 - \sum_{i=0}^{m-1} \frac{\Gamma(m+k-i-1)}{\Gamma k(m-i-1)! 2^{m+k-i-1}}\right)^{n-1} \sum_{i=0}^{m-1} \frac{\Gamma(m+k-i-1)}{\Gamma k(m-i-1)! 2^{m+k-i-1}} \quad (2.10)$$

So, $$R_1 = \sum_{i=0}^{m-1} \frac{\Gamma(m+k-i-1)}{\Gamma k(m-i-1)! 2^{m+k-i-1}} \quad (2.11)$$

Here also R_2 and R_3 are given by (2.5) and (2.6).

If m and k are not necessarily integers (Kapur and Lamberson, [9])

$$R(n) = [1 - \text{Beta}(m,k)\text{Beta}_{1/2}(m,k)]^{n-1}\, \text{Beta}(m,k)\text{Beta}_{1/2}(m,k) \quad (2.12)$$

where Beta (. , .) is Beta function and $\text{Beta}_{1/2}$(. , .) is incomplete beta function, with parameters m and k.

So, $$R_1 = \text{Beta}(m,k)\text{Beta}_{1/2}(m,k) \quad (2.13)$$

and then R_2 and R_3 are given by (2.5) and (2.6).

Here also, by MATLAB, we directly generate gamma random numbers X for strength population and Y for stress population, of sizes 5000 each with different true values of the parameters m and k. Substituting these estimates in the above expressions of R's we get a reliability estimate of the systems. This process is repeated j times for a particular set of true values of m and k. Then for each set of true values of m and k the above process is repeated. We have taken (m, k) = (1,1), (1,2), (2,1), (2,2) and j = 20.

3. Fitting of Normal Distribution to Systems Reliability:

From different expressions of reliability in Section-2 we have obtained the estimates of reliability substituting the estimated values of the parameters in respective cases. To these estimated values of reliability for different cases we have fitted normal distribution and tested the goodness of fit by one sample K-S test. The tabulated value of D for sample size 20 at 5% level of significance is 0.294 (see Seigel [18]).

Let us first consider the case of exponential stress-strength. For each $\lambda(= 0.5, 2, 3)$ j =20, the values of $\hat{R}_1, \hat{R}_2$ and $\hat{R}_3$ are obtained by substituting the corresponding values of $\hat{\lambda}$ obtained in Section-2, in the expressions (2.4), (2.5) and (2.6). Then for each value of true λ, mean and s.d. of $\hat{R}_1, \hat{R}_2$ and $\hat{R}_3$ are calculated. In each case normal distribution is fitted and the goodness of fit is tested by K-S test. The values are tabulated in Table-3.1. True values of R_1, R_2, and R_3 are also given in the same table for comparison.

For K-S test, if calculated value is of D < 0.294, the fit is good. From Table- 3.1, columns 5, 9, 13 it is clear that normal distribution gives good fit to the values of $\hat{R}_1, \hat{R}_2$ and $\hat{R}_3$.

Table -3.1: Exponential Stress-Strength

True λ	True R_1	Mean $\hat{R}_1$	SD $\hat{R}_1$	D for $\hat{R}_1$	True R_2	Mean $\hat{R}_2$	SD $\hat{R}_2$	D for $\hat{R}_2$	True R_3	Mean $\hat{R}_3$	SD $\hat{R}_3$	D for $\hat{R}_3$
.5	.333	.333	.003	.090	.555	.556	.004	.067	.704	.704	.004	.116
1	.500	.500	.003	.072	.750	.750	.003	.072	.875	.875	.002	.075
2	.667	.667	.003	.076	.889	.889	.002	.063	.926	.963	.001	.063
3	.750	.750	.003	.071	.938	.948	.001	.088	.953	.984	.000	.023

N.B.: The entry .000 in the SD column indicates that the SD is very small. This is the situation for all the tables.

Next let us consider the case of normal stress-strength. The above procedure is repeated for different set of (μ, σ^2) and their corresponding estimated values from Section-2 are used in expressions (2.9), (2.5) and (2.6). The results are tabulated in Table- 3.2. From values of D (see Seigel [18]) in column 6, 10, 14 we see that normal distribution gives good fits to the distributions of $\hat{R}_1, \hat{R}_2$ and $\hat{R}_3$.

Table -3.2: Normal Stress-Strength

True σ	True μ	True R_1	Mean $\hat{R}_1$	SD $\hat{R}_1$	D for $\hat{R}_1$	True R_2	Mean $\hat{R}_2$	SD $\hat{R}_2$	D for $\hat{R}_2$	True R_3	Mean $\hat{R}_3$	SD $\hat{R}_3$	D for $\hat{R}_3$
.5	-1	.212	.186	.001	.073	.379	.337	.002	.117	.510	.460	.003	.119
	0	.500	.500	.003	.800	.750	.750	.003	.080	.875	.875	.002	.091
	1	.788	.814	.002	.083	.955	.966	.000	.073	.991	.994	.000	.149
	2	.945	.963	.001	.037	.997	.999	.000	.021	.999	.999	.000	.058
1	-1	.308	.240	.003	.141	.522	.424	.004	.115	.669	.561	.004	.205
	0	.500	.501	.005	.064	.750	.751	.005	.064	.875	.876	.003	.077
	1	.692	.760	.004	.047	.905	.942	.002	.043	.971	.986	.000	.026
	2	.841	.922	.002	.123	.975	.994	.000	.105	.996	.999	.000	.129
2	-1	.421	.327	.004	.077	.665	.547	.006	.044	.805	.695	.006	.088
	0	.500	.501	.005	.061	.750	.751	.005	.039	.875	.876	.004	.055
	1	.579	.672	.004	.107	.823	.892	.003	.078	.926	.965	.001	.129
	2	.655	.813	.005	.075	.899	.965	.002	.082	.977	.993	.001	.097

Here we would like to point out that for $\mu = 0$ and $\sigma = 0.5$, the fit is not good.

Similarly for gamma stress-strength, for different sets of true values of stress-strength parameters (m, k) and taking their corresponding estimates from Section-2 and using this in expressions (2.11), (2.5) and (2.6) we obtain estimates of R_1, R_2, and R_3 in different situations and calculate D statistics in each case. All these values are tabulated in Table- 3.3. Comparing the values of D in columns 6, 10 and 14 with the tabulated values (ibid) we see that normal distribution gives good fit to reliabilities of systems for gamma stress-strength also.

Table-3.3: Gamma Stress-Strength (m and/ or k are Integer)

True m	True k	True R_1	Mean $\hat{R}_1$	SD $\hat{R}_1$	D for $\hat{R}_1$	True R_2	Mean $\hat{R}_2$	SD $\hat{R}_2$	D for $\hat{R}_2$	True R_3	Mean $\hat{R}_3$	SD $\hat{R}_3$	D for $\hat{R}_3$
1	1	.500	.500	.007	.088	.750	.750	.007	.063	.875	.875	.005	.085
	2	.250	.250	.003	.118	.438	.438	.005	.126	.579	.579	.006	.123
2	1	.750	.745	.012	.103	.938	.935	.006	.087	.984	.983	.002	.119
	2	.500	.500	.004	.092	.750	.750	.004	.092	.875	.875	.003	.102

When neither m nor k is an integer then the corresponding estimates of R_1, R_2, and R_3 are obtained by substituting the values of $\hat{m}$ and $\hat{k}$ from Section-2 in the expressions (2.13) etc. and the corresponding values are tabulated in Table-3.4.

Table- 3.4: Gamma Stress-Strength (m and K are not necessarily Integer)

True m	True k	True R_1	Mean $\hat{R}_1$	SD $\hat{R}_1$	D for $\hat{R}_1$	True R_2	Mean $\hat{R}_2$	SD $\hat{R}_2$	D for $\hat{R}_2$	True R_3	Mean $\hat{R}_3$	SD $\hat{R}_3$	D for $\hat{R}_3$
1	1	.500	.499	.007	.090	.750	.749	.007	.090	.875	.874	.006	.076
	2	.250	.250	.005	.092	.438	.438	.007	.005	.579	.579	.008	.065
2	1	.750	.750	.005	.069	.938	.938	.002	.060	.984	.984	.001	.071
	2	.500	.500	.006	.095	.750	.750	.006	.070	.875	.875	.004	.096

Conclusion: In this paper, we have estimated the reliability through MCS. We have seen that normal distribution is fitted to the data sets of estimated reliability obtained by MCS. Once we know the distribution it is easy for us to obtain the other characteristics of the reliability data sets.

Acknowledgment: This paper was supported by UGC (India) under Major Research Project (Sc.).

References:

[1] Aldrisi, M. M., (1987): '*A Simulation Approach for Computing System Reliability',* Microelectronics Reliability, Vol. 27, Issue: 3, pp. 463-467.
[2] Ahmad, K.E., Fakhry, M.E. and Jaheen, Z.F., (1997): *'Empirical Bayes Estimation of P(X>Y) and characterizations of Burr-type X model',* Journal of Statistical Planning and Inference, Vol. 64, Issue: 2. pp. 297-308.
[3] Borhan Uddin, Md., Pandey, M., Ferdous, J. and Bhuiyan, M.R., (1993): *'Estimation of Reliability in a Multicomponent Stress-Strength Model',* Microelectronics and Reliability, Vol.33, Issue: 13, pp.2043-2046.
[4] Church, J.D. and Harris, V., (1970): *'The Estimation of Reliability from Stress-Strength Relationship'*, Technometrics, Vol.12, No.1, pp.49-54.
[5] Goel, L., (2000): *'Monte Carlo Simulation Based Reliability Studies of a Distribution Test System',* Electric Power System Research, Vol. 54, Issue: 1, pp.55-65.
[6] Hong, H.P. and Lind, N.C., (1996): *'Approximate Reliability Analysis using Normal Polynomial and Simulation Results',* Vol.18, Issue:4, pp.329-339.

[7] Kakati, M.C. and Sriwastav, G.L., (1981): *'Accelerated Life-Testing under Stress-Strength Model',* IAPQR Trans. Vol.11, No.1-2, pp.51-56.

[8] Kamat, S.J. and Riley, M.W., (1975): *'Determination of Reliability Using Event-Based Monte Carlo Simulation',* IEEE Trans. on Reliab., Vol. R-24, No.1, pp.73-75.

[9] Kapur, K.C. and Lamberson, L.R., (1977): *'Reliability in Engineering Design',* John Wiley and Sons, New York.

[10] Landis, C.M., Beyerlein, I.J. and McMeeking, R.M., (2000): *'Micromechanical Simulation of the Failure of Fiber Reinforced Composites',* Journal of the Mechanics and Physics of Solids, Vol.48, Issue:3, pp.621-648.

[11] Manders, W.P., Bader, M.G. and Chou, T.W., (1982): *'Monte Carlo Simulation of the Strength of Composite Fiber Bundles',* Fiber Science and Technology, Vol. 17, Issue:3, pp.183 –204.

[12] Mazumder, M.,(1970): *'Some Estimates of Reliability Using Interference Theory',* NRLQ, Vol.17, pp.159-165.

[13] Naess, A., Leira, B.J. and Batsevych, O., (2009): *'System Reliability Analysis by Enhanced Monte Carlo Simulation',* Structural Safety, Vol.31, Issue: 5, pp.349 – 355.

[14] Paul, R.M. and Borhar Uddin, MD., (1997): '*Estimation of Reliability of Stress-Strength Model with Non-identical Component Strengths',* Microelectronics and Reliability, Vol. 37, Issue: 6, pp.923-927.

[15] Pulido, J.E., Jacobs, T.L. and Parters de Lima, E.C., (1992): *'Structural Reliability Using Monte- Carlo Simulation with Variance Reduction Techniques on Elastic-Plastic Structures',* Computers and Structures, Vol. 43, Issue: 3, pp.419-430.

[16] Rao, G.S. and Kantam, RRL, (2010): *'Estimation of Reliability in Multicomponent Stress-Strength Model: Log Logistic Distribution'* Eletron. J. App. Stat. Annal., Vol.3, Issue: 2 pp. 75-84.

[17] Rezaei, S., Tahmasbi, R. and Mahmoodi, M., (2010): *'Estimation of* $P\ [\ Y < X\ ]$ *for Generalized Pareto Distribution',* Journal of Statistical Planning and Inference, Vol. 140, Issue: 2, pp. 480-494.

[18] Seigel, S. (1956): *'Non-Parametric Statistics for Behavioural Sciences',* McGraw Hill Book. Co. Ltd., N.Y.

[19] Sriwastav, G.L. and Kakati, M.C., (1981): *'A Stress-Strength Model with Redundancy',* IAPQR Trans. Vol.6, No.1, pp.21-27.

[20] Sriwastav, G.L., (1988): *'Reliability Estimation in Stress-Strength Model with Ancillary Information',* Assam Statistical Review., Vol. 2, No.1-2, pp.16-20.

[21] Stancampiano, P.A., (1978): *'Simulation Approaches to Probabilistic Structural Design at Component Level',* Clear Engineering, Vol. 50, Issue:1, pp.123-129.

[22] Stumpf, H. and Schwartz, P., (1993): *'A Monte-Carlo Simulation of the Stress-Rupture of Seven-Fiber Microcomposites',* Vol.49, Issue:3, pp.251-263.

[23] Tunak, M., Linka, A. and Volf, P., (2009): *'Load-Sharing and Monte Carlo Models of Defects in a Bundle fibres',* Composites Science and Technology, Vol.69, Issue: 9, pp. 1417-1421.

[24] Wu, L., Yi, L., Xue-qin, G. and Deng-hai, W., (2009): *'Monte-Carlo Simulation for the Reliability Analysis of Multi-Status Network System Based on Breadth First Search',* Information and Computing Science, pp.280-283.

[25] Zhang, H., Chandrangsu, T. and Ramussen, K.J.R., (2010): *'Probabilistic Study of the Strength of Steel Scaffold System',* Structural Safety, Vol.32, Issue: 6, pp.393-401.

MULTISTATE COHERENT SYSTEMS WITH MULTIPLE STATE TRANSITION AT A TIME

V. M. Chacko
Department of Statistics, University of Calicut,
Kerala -673635, India
chackovm@gmail.com
•
M. Manoharan
Department of Statistics, University of Calicut,
Kerala-673635, India

Abstract

A key requirement in defining a multistate coherent system (MCS) is the relevance condition of its components. A new class of MCSs is introduced with a new component relevance condition. Also we introduce a more general relevance condition. They are compared with some existing component relevance conditions. Based on the two new relevance conditions, two component importance measures for MCSs are defined. They are most appropriate for comparing components when certain type of system improvement is sought. We introduce new joint importance measures for two or more components with respect to the proposed relevance conditions. The new MCS classes include several existing MCSs as special case. An illustrative example of the proposed MCSs is also provided.

Keywords: Reliability, MCS, relevance condition, component importance, joint importance.

1. Introduction

Let us consider a coherent system with n components C={1,2,…,n}. Furthermore, suppose that each component can be in one of M+1 states, {0,1,2,…,M}, where '0' is the failed state and 'M' is the maximal or "perfect" state. To describe such a multistate system (MSS), a general theory has been developed in the literature.[5,9,11] A binary state system (BSS) of n components can be described by a structure function $\phi:\{0,1\}^n \rightarrow \{0,1\}$, which presents the state of system as a function of states of its n components.[4] A binary system is statistically coherent if it satisfies the following conditions;[4]

(i) $\phi(\bar{x})$ is non-decreasing in each argument, where $\bar{x}=(x_1,x_2,...,x_n)$ with $x_i \in \{0,1\}$, and

(ii) for each i, there exist a vector $(._i,\bar{x})$, such that $\phi(1_i,\bar{x}) > \phi(0_i,\bar{x})$, where $(._i,\bar{x})=(x_1,x_2,...,x_{i-1},._i,x_{i+1},...,x_n)$.

Note that the condition (i) and (ii) gives, $\phi(\bar{j})=j,\ j=0,1$ where $\bar{j}=(j,j,...,j)$.

In practice, a system and its components often have more than two states of performance.[5] The structure function of the MSS is $\phi:S^n \rightarrow S,\ S=\{0,1,...,M\}$, which relates the level of performance of system to level of performance of each of its components. There are various approaches which extends the structure function from the binary case to the multistate case.[5,9,10,11] The effort resulted in, extension of the requirement of non-decreasing binary structure function to MSS structure function. Also note the condition $\phi(\bar{j})=j,\ j=0,1$ of the binary coherent system (BCS) is extended to the MSS requiring $\phi(\bar{j}) = j,\ j \in \{0,1,...,M\}$.[11] The condition (ii) of relevancy in BCS is extended in various different ways. Some extensions can be seen in Refs. 1, 2 and 12.

In this paper, we extend the relevance condition to the MSS case in a general way, which includes several existing relevance conditions as special cases. Section 2 introduces the new class of multistate coherent system(MCS)s and its generalization by introducing a reasonable component relevance condition. The two new classes are compared with the some existing classes. Section 3 introduces two new component importance and joint importance measures for the proposed MCSs. Section 4 provides an example of an offshore electrical power generation system. Discussion and conclusion are given in section 5.

2. Component relevancy and the new classes of MCSs

In this section we discuss the new relevance condition and its generalization on which two new classes of MCSs are defined. Consider the following component relevance conditions.

NAT[13] : For every component *i* and level *j*>0, there exist $(\cdot_i, \bar{x})$ such that $\phi(j_i, \bar{x}) \geq j$ and $\phi((j-1)_i, \bar{x}) < j$.

GRI.1[11] : For every component *i* and level *j*>0, there exist $(\cdot_i, \bar{x})$ such that $\phi(j_i, \bar{x}) > \phi((j-1)_i, \bar{x})$.

GRI.2[11] : For every component *i*, there exist $(\cdot_i, \bar{x})$ such that $\phi(0_i, \bar{x}) < \phi(M_i, \bar{x})$.

and **EP**[9] **:** For every component *i* and level *j*≥1, there exist $(\cdot_i, \bar{x})$ such that $\phi(j_i, \bar{x}) > \phi(0_i, \bar{x})$.

NAT and **GRI.1** indicate degree of relevance of each component to every level of performance; while **GRI.2** merely states that ϕ is not a constant in any of its arguments.

Now consider a situation in which some component is not relevant to every level of performances, i.e., the system degrades from state *j* to *j-1* or *j-2* etc when the component degrades only from state *j* to *j-2* or *j-3* etc. In order to degrade the system, component must degrade more than one level of performance. For example,[14] let $S = \{0,1,2,3,4\}$, and the component can take 0, 2, and 4 when the system can take 0, 1, 2, 3 and 4. Consider the structure function ϕ_2 having 5 components in Ref.14. From the minimal path vectors of ϕ_2, we have, $\phi_2(4_1,4_2,2_3,4_4,2_5) = 4 > \phi_2(4_1,4_2,2_3,2_4,2_5) = 3$, when the 4[th] component degrades from state 4 to state 2, the system degrades from state 4 to state 3. Now consider the structure function ϕ_1 with three components in Ref.14. We have, $\phi_1(4_1,0_2,4_3) = 4 > \phi_1(4_1,0_2,2_3) = 2$, when the third component degrades from state 4 to state 2, the system degrades from state 4 to state 2. Here fourth component must degrade from state 4 to state 2 for the system to degrade from state 4 to state 3 with respect to ϕ_2. The third component must degrade from state 4 to state 2 for the system to degrade from state 4 with respect to ϕ_1.

We define a new component relevance condition as, degrading a component from state *j* to state *j-2* can cause system failure or degradation while degradation of the component from state *j* to *j*-1 cannot cause system failure or degradation.

Now the new class of MCSs can be defined as follows.

Definition.1.: A multistate system of *n* components with structure function ϕ belonging to class **CM.1** if ϕ is non-decreasing, $\phi(\bar{j}) = j$, and for each component, there exist $(\cdot_i, \bar{x})$ such that $\phi(j_i, \bar{x}) > ((j-2)_i, \bar{x})$.

Now consider the generalization of the new relevance condition, one or more than one level of degradation of the component can cause the system degradation, i.e., when the component 'i' degrades from state *j* to state *j'*∈*{j-1, j-2, j-3,…., 1, 0},* the system degrades from state *j* to any lower state. Thus we define the generalized class of MCSs with this relevance condition.

Definition 2.: A multistate system of *n* components with structure function ϕ belonging to class **CM.2** if ϕ is non-decreasing, $\phi(\bar{j}) = j$, and for each component, there exist $(\cdot_i, \bar{x})$ such that $\phi(j_i, \bar{x}) > (j'_i, \bar{x})$ where *j'*∈*{j-1, j-2,j-3,….,1,0}*.

In the following section we introduce the component importance and joint importance measures to the new classes of MCSs.

3. Component importance and Joint importance measures

We consider the problem of measuring the reliability importance and structural importance of individual components, and the joint reliability importance and joint structural importance of two or more components in the new classes of the MCSs. The main advantage of defining a new relevance condition is to obtain the importance measures.[6] At the reliability design phase, the joint importance can improve system designer's understanding of the relationship between the components and the system, and among the components,[3] which is quite desirable**.** Birnubaum measure provides the importance of a component in the BSS.[6] It is further extended to the MSS.[7,15] Now we consider $X = (X_1, X_2, ..., X_n)$ as a random vector with component states X_i as random variables and $p_{ij} = \Pr\{X_i = j\}$ where $j \in S = \{0,1,...,M\}$. For the BSS with structure function ϕ, the Birnubaum reliability importance[6] of component i is

$$I_i(B) = P(\phi(1_i, \bar{x}) - \phi(0_i, \bar{x}) = 1) = h(1_i, \bar{p}) - h(0_i, \bar{p}).$$

where $h(\bar{p})$, $\bar{p} = (p_1, p_2, ..., p_n)$ and $\forall i \; p_i = (x_i = 1)$, is the reliability function of the BCS,

$$h(\bar{p}) = p_i h(1_i, \bar{p}) + (1 - p_i) h(0_i, \bar{p}) = p_i I_i(B) + h(0_i, \bar{p}).$$

Therefore,

$$\frac{\partial h(\bar{p})}{\partial p_i} = I_i(B).$$

We propose the following component importance measures for the two classes of new MCSs.

1. $I_i(CM.1) = P(\phi(j_i, \bar{x}) > \phi((j-2)_i, \bar{x}))$.
2. $I_i(CM.2) = P(\phi(j_i, \bar{x}) > \phi(j'_i, \bar{x})), \quad j' \in \{j-1, j-2, ..., 1, 0\}$.

Let the distribution of X_i be described by $p_i = (p_{i0}, p_{i1}, ..., p_{iM})$. The reliability function of the MCS with minimum satisfactory system level j, is $P(\phi(\bar{x}) \geq j) = \sum_{j \in S} P(\phi(j_i, \bar{x}) \geq j)$, since $p_{i0} + p_{i1} + p_{i2} + ... + p_{iM} = 1$. Now we prove the following theorems.

Theorem 1. For the **CM.1** class, $I_i(CM.1)$ is the rate of improvement of $P(\phi(\bar{x}) \geq j)$ with respect to p_{ij}.

Proof. Clearly,

$$P(\phi(\bar{x}) \geq j) = \sum_{S \backslash \{j-2\}} p_{ij}[P(\phi(j_i, \bar{x} \geq j) - P(\phi((j-2)_i, \bar{x}) \geq j)] + P(\phi((j-2)_i, \bar{x}) \geq j),$$

since $1 - p_{ij-2} = p_{i0} + p_{i1} + ... + p_{ij-3} + p_{ij-1} + ... + p_{iM}$. Differentiating $P(\phi(\bar{x}) \geq j)$ partially with respect to p_{ij}, we get

$$I_i(CM.1) = \frac{\partial P(\phi(\bar{x}) \geq j)}{\partial p_{ij}} = P(\phi(j_i, \bar{x}) \geq j) - P(\phi((j-2)_i, \bar{x}) \geq j) = P(\phi((j-2)_i, \bar{x}) < \phi(j_i, \bar{x})).$$

Theorem 2. For the **CM.2** class, $I_i(CM.2)$ is the rate of improvement of $P(\phi(\bar{x}) \geq j)$ with respect to p_{ij}.

Proof. Clearly,

$$P(\phi(\bar{x}) \geq j) = \sum_{S \setminus \{j'\}} p_{ij}[P(\phi(j_i, \bar{x}) \geq j) - P(\phi(j'_i, \bar{x}) \geq j)] + P(\phi(j'_i, \bar{x}) \geq j),$$

where $1 - p_{ij'} = p_{i0} + p_{i1} + ... + p_{ij'-1} + p_{ij'+1} + ... + p_{iM}$ and differentiating the $P(\phi(\bar{x}) \geq j)$ partially with respect to p_{ij} we get,

$$I_i(CM.2) = \frac{\partial P(\phi(\bar{x}) \geq j)}{\partial p_{ij}} = P(\phi(j'_i, \bar{x}) < \phi(j_i, \bar{x})), \quad j' \in \{j-1, j-2, ..., 1, 0\}.$$

Now define the structural definition of the component importance (when reliabilities of components are not given) with respect to the new relevance conditions.

Consider $\phi(\bar{x}) = 1 \;\; if \;\; \phi(\bar{x}) \geq j$ and 0 otherwise. We define the structural importance of a component as follows.

Definition.3.: Let $\phi : S^n \to S$ be the MCS structure function in **CM.1** class. Then ϕ is said to have the following measures of structural importance for the level j of component i:

$$I_{ij}(CM.1) = \frac{1}{(M+1)^{n-1}} \sum_{\{\bar{x}: x_i = j\}} Max\{0, \phi(j_i, \bar{x}) - \phi((j-2)_i, \bar{x})\}.$$

Definition.4.: Let $\phi : S^n \to S$ be the MCS structure function in **CM.2** class. Then ϕ is said to have the following measures of structural importance for the level j of component i:

$$I_{ij}(CM.2) = \frac{1}{(M+1)^{n-1}} \sum_{\{\bar{x}: x_i = j\}} Max\{0, \phi(j_i, \bar{x}) - \phi(j'_i, \bar{x})\}, \quad j' \in \{j-1, j-2, ..., 1, 0\}.$$

In order to define the joint importance measures for two or more components in the new classes of MCSs, we recall the joint structural importance measure(*JSIM*)s [8] and joint reliability importance measure(*JRIM*)s [8] for the MSS with relevance condition in **GRI.1**. The *JSIM* (*i,j*) for two components *i* and *j* with the new relevance conditions can be obtained by replacing $\hat{m}$ with *m-2* or $m' \in \{m-1, m-2, ..., 2, 1, 0\}$ in,

$$JSIM(i,l) = \sum_{m=1}^{M} \sum_{k=1}^{M} \{SIM(i,l;m,k) - SIM(i,l;m,\hat{k})\}$$

where $SIM(i,l;m,k) = \frac{\sum_{\vec{x}_{il}} \sum_{q=1}^{j} \chi(\phi(m_i, k_l, \vec{x}_{il}) = j, \phi(\hat{m}_i, k_l, \vec{x}_{il}) = j - q)}{(M+1)^{n-2}}$.

Here χ(true)=1 and χ(false)=0, and $\phi(m_i, k_l, \bar{x}_{il}) = j, \phi(\hat{m}_i, k_l, \bar{x}_{il}) = j - q$ where $\bar{x}_{ij} = (x_1, ..., m_i, ..., k_l, ..., x_n)$ determines the critical path vector to the level j with state *m* of component *i*. The *JSIM* (*i,j,k*) for three components can be obtained as, for $\hat{k} = n-2$ or $\hat{k} \in \{n-1, n-2, ..., 2, 1, 0\}$,

$$JSIM(i,l,r) = \sum_{k=1}^{M} \sum_{n=1}^{M} \sum_{m=1}^{M} \{JSIM(i,l,r;m,k,n) - JSIM(i,l,r;m,k,\hat{n})\},$$

where $JSIM(i,l,r;m,k,n) = JSIM(i,l,r;m,k,n) - JSIM(i,l,r;m,\hat{k},n)$.

Thus we can find *JSIM* of any number of components w. r. t. both relevance conditions in the MCS classes **CM.1** and **CM.2**. Thus *JSIM* [8] holds with new relevance conditions by replacing $\hat{m}$ with suitable m-*2* or $m' \in \{m-1, m-2, ..., 2, 1, 0\}$.

Now we consider the *JRIM* for *k* components in the MSS. [8] The *JRIM* of state b_1 of component a_1, state b_2 of component a_2,..., state b_k of the component a_k $(k \leq n)$ of the MSS is

$$JRIM(a_1,...,a_k;b_1,...,b_k) = \frac{\partial^k E_s}{\partial R_{a_1} b_1 \partial R_{a_2} b_2 ... \partial R_{a_k} b_k}, k = 2,...,n,$$

where $E_s = \sum_{j=0}^{M} P(\phi(x) \geq j)$ is the expected system performance and $R_{a_i} b_i$ is the reliability function with respect to level b_i of component a_i .

Here $\tilde{m}$=*m-1* in the expansion of E_s.[8] The results also holds true with the other values of $\tilde{m}$=*m'*, *m'*$\in$ {*m-1,m-2,m-3,...,0*}. Hence the *JRIM* with the new MCSs can be obtained with appropriate *m'* values for $\tilde{m}$, i.e., *m'=m-2* or $m' \in \{m-1, m-2,...,2,1,0\}$.

Now consider some implications based on the new relevancy definitions. In fact one can easily prove the following implications.

Theorem 3. CM.1=>EP, CM.2=>NAT=>GRI.1=>GRI.2=>EP, CM.2=>GRI.1, CM.2=>GRI.2 and **CM.2=>EP**.

It is clear that all the existing relevance conditions are special cases of **CM.2** (or **CM.1)** relevance condition. Hence the existing MCSs are special cases of the proposed MCSs.

4. Example

Ref.14 considered an offshore electrical power generation system, which supply two nearby oilrings with electrical power. Both oilrings have their own main generation, represented by equivalent generators A_1 and A_3 each having capacity of 50MW. In addition the oilrings has a standby generator A_2 that is switched into the network in case of outage of A_1 or A_3, or may be used in extreme load situations in either of the oilrings. The A_2 also has capacity 50MW. The control unit, U, continuously supervises the supply from each of the generators with automatic control of the switches. If for instance the supply from A_3 to oilring 2 is not sufficient, whereas the supply from A_1 to oilring 1 is sufficient, *U* can activate A_2 to supply oilring 2 with electrical power through the subsea cables L. The components have states {0, 2, 4} and the system have states {0, 1, 2, 3, 4}, where 0, 1, 2, 3 and 4 represents the states of the system at capacities 0MW, 12.5MW, 25MW, 37.5MW, and 50MW respectively.

Table I .Minimal path vectors of ϕ_1

Levels	U	A_1	A_2
2	2	2	0
2	4	0	2
4	2	4	0
4	4	0	4
4	4	2	2

Table II .Minimal path vectors of ϕ_2

Levels	U	A_1	L	A_2	A_3
1	4	4	2	2	0
1,2	2	0	0	0	2
2	4	4	2	4	0
2	4	4	4	2	0
3	4	4	2	2	2
3,4	2	0	0	0	4

4	4	4	2	4	2
4	4	4	4	2	2
4	4	4	4	4	0

The minimal path vectors to the levels are given in table 1 and table II, of the structure functions,

$\phi_1(U, A_1, A_2) = I(U > 0)\min(A_1 + A_2 I(U = 4), 4),$ the amount of power that can be supplied to platform 1, and

$\phi_2(U, A_1, L, A_2, A_3) = I(U > 0)\min(A_3 + A_2 I(U = 4) I(A_1 = 4) L / 4, 4),$ the amount of power that can be supplied to platform 2, when $I(.)$ is the indicator function. One may easily verify from the tables that the new relevance condition of **CM.1** and **CM.2** are found to be holding good w. r. t. the structure functions considered in the example. As done for *JSIM* and *JRIM* for two components[11] and *JSIM* for three components,[8] we can compute the concerned joint importance measures for any number of components in the power generation system.

5. Discussion and Conclusion

The theory of MSS reliability models has been developed to cope with many real-life situations. The present paper introduced two classes of MCSs with a new relevance condition and its generalization. It is shown that many MCSs introduced earlier in the literature are included in the new classes. Structural definitions of importance and joint structural importance measures are given, and new reliability importance and joint reliability importance measures are introduced. In system engineering, a practical and difficult problem is the identification of those groups of components that mostly influence the system behavior with respect to safety and reliability. In this respect, the main advantage of our importance and joint importance measure with respect to the new MCS models is the information provided by them for the reliability theoreticians and design analysts. It gives useful information for safe and efficient operation of the system, where existing importance measures gives information about individual component importance and joint importance of components with some limited number of relevance conditions.

Acknowledgments

Authors are thankful to the financial support from CSIR[1] and UGC[2], India.

References

1. A. M. Abouammoh, and Al-kadi, On measures of importance for components in multistate coherent system, *Microel. Reliab.,* 31(2) (1991) 109-122.
2. A. M. Abouammoh, and Al-kadi, Multistate coherent system of order k, *Microel. Reliab.,* 35(11) (1995) 1415-1421.
3. M. J. Armstrong, Joint reliability importance of components, *IEEE Trans. Reliab.,* 44(3) (1995) 408-412.
4. R. E. Barlow, and F. Proschan, *Statistical theory of reliability and Life testing*, (Holt, Rinheart & Winston, NewYork, 1975).
5. R. E. Barlow, and A. Wu, Coherent system with multistate components, *Math. Oper. Res.,* 3 (1978) 275-281.
6. Z. W. Birnbaum, On the importance of different components in a multi-component system, in *Multivariate analysis II*, ed. P. R. Krishnaiah (NewYork, Academic Press, 1969), p. 591-592.
7. V. C. Bueno, On the importance of components for multistate monotone systems, *Stat. Prob. Lett*., 7 (1989) 51-59.

8. V. M. Chacko and M. Manoharan, Joint importance measures for the multistate system, *Advances in Performance and Safety of Complex systems*, eds. A. K., Verma, P. K, Kapur, S. G., Ghadge, (Macmillan, New Delhi, 2008), p. 308-314.

9. E. El-Neweihi and F. Proschan, Degradable systems; a survey of multistate system theory, *Commu. Statist. Theory and Methods*, 13 (1984) 405-432.

10. E. El-Neweihi, F. Proschan, and J. Sethuraman, Multistate coherent systems, *J. Appl. Prob*., 15 (1978) 675–688.

11. W. S. Griffith, Multistate reliability models, *J. Appl. Prob*., 17(1980) 735–744.

12. F. C. Meng, Component relevancy and characterization results in multistate systems, *IEEE Trans. Reliab*., 42 (1993) 478-483.

13. B. Natvig, Two suggestions of how to define a multistate coherent system, *Adv. Appl. Prob*., 14 (1982) 434-455.

14. B. Natvig, S. Sormo, A. T. Holen, and G. Hogasen, Multistate reliability theory -a case study, *Adv. Appl. Prob*, 18(1986) 921- 932.

15. S. Wu, Joint importance of multistate systems, *Comp. Indust. Eng.,* 49(2005) 63–75.

ISSN 1932-2321