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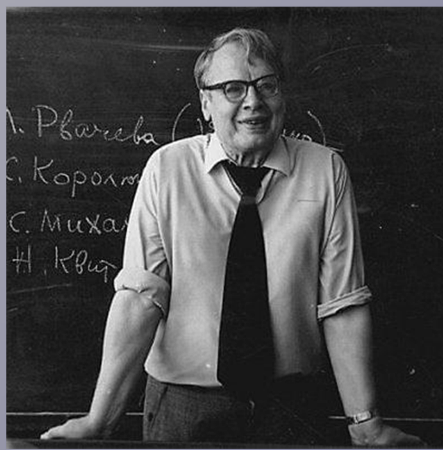

RELIABILITY:
THEORY&APPLICATIONS

RELIABILITY
RISK ANALYSIS
SAFETY
MAINTENANCE

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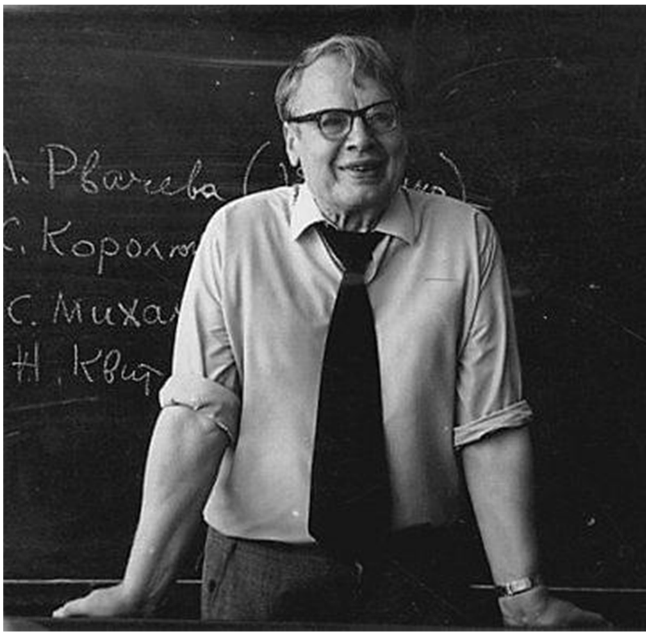

RELIABILITY:

THEORY & APPLICATIONS

Vol.7 No.4 (27),
December, 2012

Special Issue 5

100th anniversary of Boris Vladimirovich Gnedenko's birthday

San Diego
2012

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The basic stages of a design procedure of parameters of individual reliability the equipment and devices of electro power systems are considered. The recommended method illustrated on an example of parameters of reliability calculated as average arithmetic random variables. The method based on imitating modeling of random variables and the theory of check of statistical hypotheses.

In flow networks, a reliability model representing telecommunications networks is independent of topological information, but depends on traffic path attributes like delay, reliability and capacity etc.. The performance of such networks from quality of service point of view is the measure of its flow capacity which can satisfy the customers demand. To design a flow network which can meet the desired performance goal, a cardinality based approach for reliability redundancy optimization using composite performance measure integrating reliability and capacity has been proposed. The method utilizes cardinality based criteria to optimize main flow paths and backup paths on priority basis. The algorithm is reasonably efficient due to reduced computation work even for large telecommunication networks.

Stability of metallization of holes to thermomechanical pressure is provided with durability and plasticity of galvanic besieged copper. Distinctions in factors of thermal expansion of copper and the dielectric bases of printed-circuit boards create powerful thermomechanical factors of rupture of metallization of apertures, destructions of internal interconnections in multilayered structures of printed-circuit boards. Standard norms of requirements to a thickness of metallization of apertures, its durability and plasticity of copper were established in the course of manufacture of ordinary printed-circuit boards with reference to use of traditional technologies of the soldering by tin-lead solders. Return to consideration of a problem of plasticity of copper is caused first of all by transition on the Lead-free solders, initiated by the all-European Directive RoHS, rations different by a heat. More heats create the big deformations of metallization of holes those forces to reconsider requirements to plasticity of copper. At the same time, the tendency to reduction of diameter of the metallized holes, so also to reduction of the area of cross-section section of metallization is everywhere observed. Smaller sections have smaller resistance to rupture. Therefore, along with good plasticity, metallization of holes of printed-circuit boards should provide and higher breaking strength. In this connection deformation of metallization of holes at heating to soldering temperatures has been investigated. The purpose of researches - revision of norms on plasticity of copper in holes of printed-circuit boards. It is shown that plasticity copper deposition in holes of modern printed-circuit boards should not be less than 6 %. Modern cupper electrolytes allow to receive plasticity of copper of 12-18%.

Reliability of sensor networks system plays an important role in monitoring the operational health conditions of any critical engineering system. In this paper, a methodology is proposed to improve an initial optimal allocation of sensor system by considering fault acceptance degree (FAD), fault influence degree (FID), importance factor (α), and the actual operational data of the critical system with initial allocated sensors for a small initial period. The paper has also proposed a method for estimation of expected life of the sensor network based on the above factors. A hypothetical case of the sensor system of an electric motor is presented to illustrate the proposed approach.

The paper describes the calculation method of reliability and conservability products class of chemical current sources (CCS) in the design to the specific conditions in the electronic means. The technical specification for the indicators of CCS reliability are for specific operation modes and their use in the calculation gives a large error. In the U.S. (MIL-HDBK-217F, Telcordia (Bellcore) SR 332), French (CNET RDF-2000), English (British Telecom HRD5) and Chinese (GJB/z 299B) references to the reliability of electronic devices there is no information for the calculation of reliability and conservability CCS, no calculation models. The cumulative accounting model of physical factors affecting the calculated capacity of the chemical current sources, which is the main factor affecting the reliability is shown.

ON THE DEFINITION OF FUNCTIONAL RELIABILITY

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Abstract

The theory of reliability has been developed in order to ensure operability of technical objects (components and systems). However, no thorough explanation of the term "functional reliability" is given by now, although it is used with increasing frequency. We deduce a definition based on the terms property, quality and function.. In this connection, we draw attention on the principal differences between functional and structural reliability. We explain similarities between functional safety and functional reliability and show how they smoothly change one into the other.

Key words: functional reliability, reliability, safety, quality, function

1. Introduction

The theory of reliability has been evolved in order to ensure operability of technical objects (elements and systems). This assumes that the object is reliable, if it provides its intended function. This condition is absolutely necessary - however, is it also sufficient? Let us assume that an object is working and performs its intended function. Performance of the function is connected with output generated by the object. If there is no result - there is no realisation of the function. When considering the result, the latter cannot be considered abstractly – it should be characterized by its quality characteristics. For example, it needs to be defined when correctness, accuracy and efficiency of de delivered result of the function performance are acceptable.

If the object is intended for performance of two or more functions, then it is necessary to consider the reliability of each of the intended functions taking into account the functioning state of the said object. This assumes, firstly having the result of each function, and, secondly, knowing how much these results meet the given requirements.

Thus, functioning of the object is the necessary condition, but it is not sufficient for its reliable functioning. In the standard [1], an attempt has been undertaken to expand the interpretation of reliability by adding the aspect that an object is able to carry out the intended function, in addition to the traditional task of creating a working object. However, there is no explanation of the term "ability to carry out the intended functions" in the given standard and in the subsequent publications connected with it [8,9]. Moreover, it is not clear whether this term includes a judgement whether the function has been performed with sufficient quality. If this aspect is not included, the given definition is nothing else than the definition of the availability of the object. This is the same as the traditional understanding availability of the object as the property to perform any intended task..

If the introduction of the term is accompanied with a quality level with which each of the functions has to be carried out, the question arises, which quality characteristics have to be used und how they are defined? How can the space of working states of the objects be combined with the space of successful performed functions by the objects – and this taking into account possible different quality characteristics? Answers to these questions are extremely problematic as are attempts to analyse the reliability of execution of intended processes by the means of traditional reliability.

2. The tasks of structural and functional reliability

We introduced the term "Structural Reliability" for the definition of that part of the general reliability theory, which is focused on investigation of failure processes and renewal of technical objects, and on solving problems of ensuring their functioning. Earlier these problems were in the center of attention of the traditional reliability theory.

The concept of "Functional Reliability" is used increasingly, both in Russian as well as in the foreign literature [1, 8, 9]. However, we could not find a sufficiently deep interpretation of this concept. Since the role of the general reliability theory for solving tasks of ensuring reliability of modern complex technical systems is difficult to judge, we have decided to express our position in understanding of the subject, research target and problems in the field of functional reliability,

The research targets of structural and functional reliability are presented on fig.1

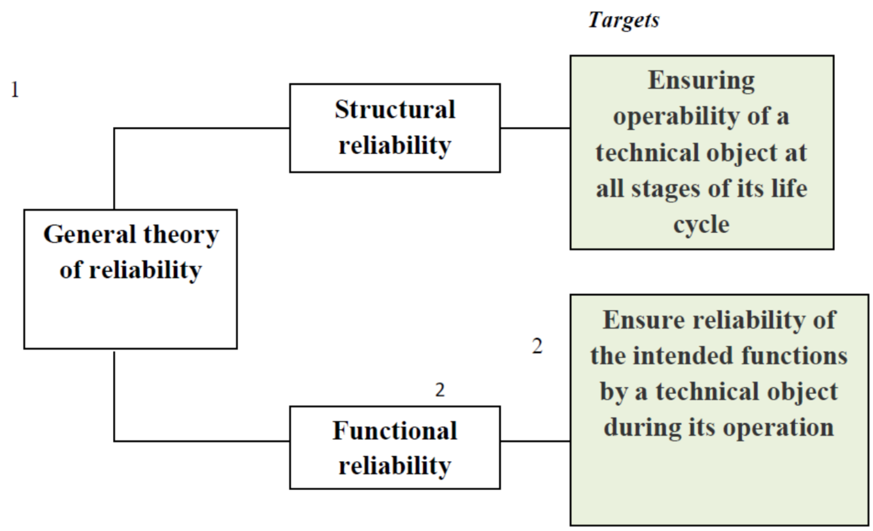

Figure 1. The targets of structural and functional reliability

Structural reliability covers a wide spectrum of tasks ensuring operability of technical objects (elements, systems). It includes the application of various kinds of redundancy, design of reasonable electric loads and climatic modes of operation during development and realisation of effective external and internal monitoring of the function of the object. Problems of maintenance optimisation (maintenance and repair), minimisation of life cycle cost, and prolongation of rated service life of an object (defined by the developer) are solved at operation stage.

The theory and practice of functional reliability are focused on studying functional failure processes, as well as renewal processes and restoration of information processes. A functional failure is a failure to fulfil a functional task caused by a deviation of an information process.

One of the main problems which cause faults of information processes (alongside with faults caused by software errors), operator errors, errors of input information is the problem of transitional failures of digital equipment and IC devices. The frequency of transitional failures exceeds the frequency of hardware failures by several orders of magnitude [2.] Transitional failures are caused by wrong performance of logic functions of discrete hardware components, which then progress into errors in performance of operations, procedures, programs, tasks.

The methodology of structural reliability is rather broad but it is not focused on calculations of reliability of information processes and their constituents - it is dedicated solely to failure processes and renewal of technical systems. Structural reliability does not deal with the sequence of tasks to be carried out. In addition to that, it does not take into account the influence of software errors, operator errors and errors in the input information on the results of the task algorithms. All these factors (threats) of unreliability serve as a study subject in the theory of functional reliability, as a component of the reliability theory. It is necessary to note, that operator errors are investigated in the theory of human reliability, which is a separate area not belonging to structural reliability.

3. Quality and properties

Each system possesses certain attributes describing it. ***Quality of a system*** is a set of properties defining the system for users according to its functional purpose and requirements. In addition, requirements can be understood in a very broad sense, which leads to many different definitions of the concept. Mostly, the used definition of quality is taken from the standard ISO 9001 [10] according to which “quality is the degree to which a set of inherent characteristics fulfils requirements”. System quality of is a relative concept, which makes sense only taking into account real application., Therefore the quality requirements are defined according to conditions and the specific area of application of the system.

According to the standard ISO 9126 properties are attributes defining properties of software which can be considered as quality characteristics. The standard ISO 9126 [3] presents a model of software quality. Quality models can be defined for all technical systems. They are in fact specifications of system properties. A well and logically written requirements specification is close to a quality model as it defines all essential system properties. The structure of system properties should correspond to the system structure. If the system consists of software and hardware, then properties are divided into the two groups for software and hardware, besides a third group of system properties.

For example, information protection can be an important system property. Information protection can be considered as software property, if corresponding measures are implemented in the software. Information protection can also be a hardware property, if corresponding measures are implemented in hardware.

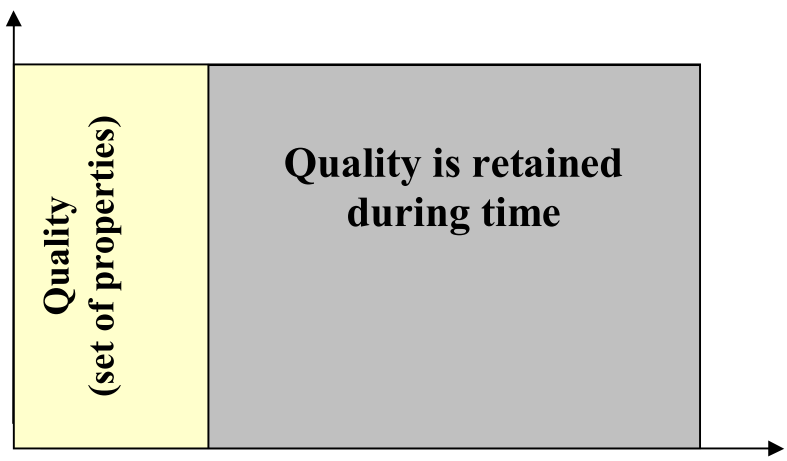

Figure 2. An explanation of functional reliability

It goes without saying, that total or partial loss of a certain property means decrease of system quality.

It is important, that properties are not only present in the object in the beginning of its life cycle, but it is also important that they were kept long enough during its lifetime. Retention of these properties (i.e. quality) is defined as reliability [4]. The second definition of reliability is given in the standard EN 50126 [5] as the probability that an object can perform the function in a certain time interval. Both the first and the second definitions directly relate to functional reliability.

A functional failure is the loss of the ability to perform the assigned function by the object. [2]. Failure can be caused by malfunction, i.e. functional deviation of a system component from the specification. Not each malfunction will lead to failure.

An intolerable deviation of the characteristics of the object [4] is called an error. Such deviations can be, for example, intolerable values, inadmissible precision, inadmissible time (late, early, too long) or others. The error is introduced into the system during its development, manufacturing, etc.

Failures are divided as systematic and random ones [5]. Systematic failures are often caused by errors. Software failures are always systematic by nature and they are caused by software (SW) errors. Usually, SW failures reveal themselves in a random manner since SW is subject to random influence of an environment, which activates an error and, hence, can lead to a fault or even to a system failure. Further, it is necessary to note, that failures can be partial in cases when only a part of a function fails.

4. Properties and hazards

We start from the obvious fact, that it is impossible to describe functional reliability by any single property. Only the entire set of properties (attributes) allows the system to perform the intended functionals, i.e. to provide qualitative service of inquiries of system users [2]. Properties and hazards regarding functional reliability of the system are shown on fig. 3.

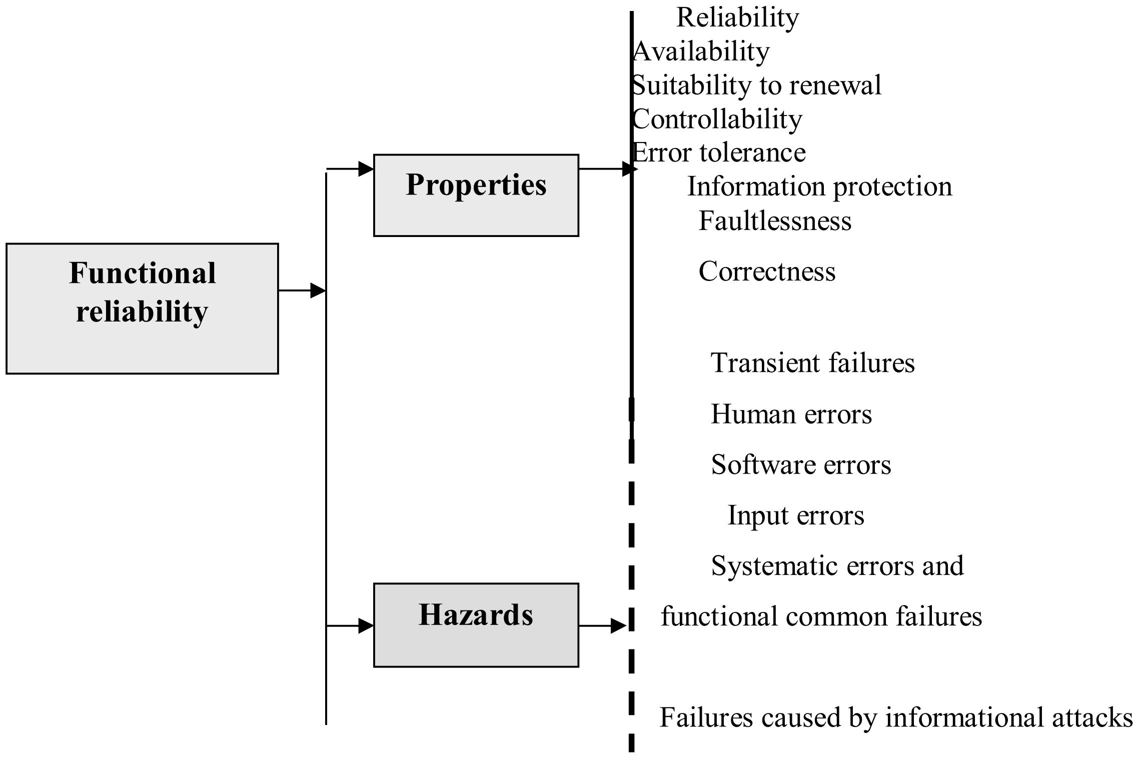

Figure3. Properties and hazards of system functional reliability

Besides the properties of reliability and availability which are typical for structural reliability, for objects studied by functional reliability, properties such as suitability for recovery of the function (analogue of maintainability), but also new properties: protection of information, monitoring of the quality of the functional performance, stability against errors, and last but not least faultlessness and correctness of the function. In the opinion of the authors, the latter is very important.

Let us consider the hazards connected with functional reliability. According to standard EN 50126 [5] "hazard is a physical situation with a potential for human injury". Certainly, hazard concerns not only harm for an individual, but also harm to material belongings or to the environment. Not each hazard always leads to a threat. For such a threat, it is necessary, that there was an initiating event. Then the chain of undesirable events can evolve from threat, which finally will lead to unfortunate event, i.e. to accident.

Typical groups of threats (hazards) for systems are the following (fig.3):

- *Transient failures* –results of disturbances of digital technical equipment leading to malfunction of the software performance, malfunction of information technologies and malfunction of information process as a whole. These effects reveal themselves in the form of distortions, distortion or loss of data, errors in intermediate and / or in output results.
- *Software errors* – these are essentially errors arising during information processing at various stages of the software process. They are grouped as follows: system errors, algorithmic errors, program errors.
- *Human errors.* Human error is defined as a failure to carry out a required task (or performance of a forbidden operation), which can lead to a deviation of the functional processes in the system or incorrect performance of at last one function.
- *Input message errors* [6] – *damage of the* integrity of the information flow (more or less messages were received, the order of messages was violated, delay of messages, message distortion or masquerade).
- *Systematic failures and common mode functional failures* - deviations of system functions caused by typical errors in designing components, operating modes, maintenance of the system, information coding, and also such factors, which cause simultaneous influence on several channels (in multichannel system), or several functions, or redundant devices or subsystems.
- *Errors due to information threats* - set of attacks of an intruder corrupting integrity and availability of the information.

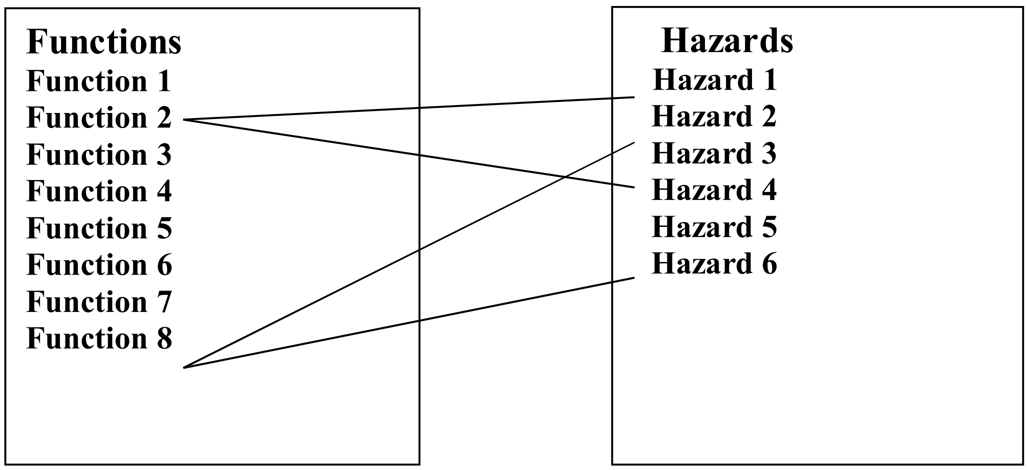

Figure 4. Connection of functions and threats

There is a connection between threats and system functions. This connection can be caused by two reasons:

a) The system function directly generates a threat or a hazard. For example, the function of high voltage generation implemented in a power supply system leads to high voltage, which represents a hazard (threat) and can lead to electrocution.
b) The system function itself is important for control or prevention of hazards. For example, a railway interlocking prevents collisions of trains.

Thus, there is an interrelation of functions and hazards (fig.4).

Not each failure, i.e. degradation or loss of a function, leads to hazard or threat. This depends on the specific properties of the failed function. And on the contrary, not each threat is caused by loss or degradation of some system function.

5. Functional failures

Let us now assume that the system can be described in a space of states. The dimensions of this space are the system properties. The system properties change in time. These changes can be caused both by external influence, and by internal processes, including system ageing. The space of states contains both allowed and not allowed states. If the system is in an inadmissible state, then there is a deviation from the intended functional behaviour, i.e. a partial or full functional failure has occurred.

The admissible part of the space of states is characterized by the fact that the admissible states satisfy the following requirements:

- Accuracy;
- Information output is provided (function);
- Logic admissibility of output data.

The allowed area of system states is shown on fig. 5.

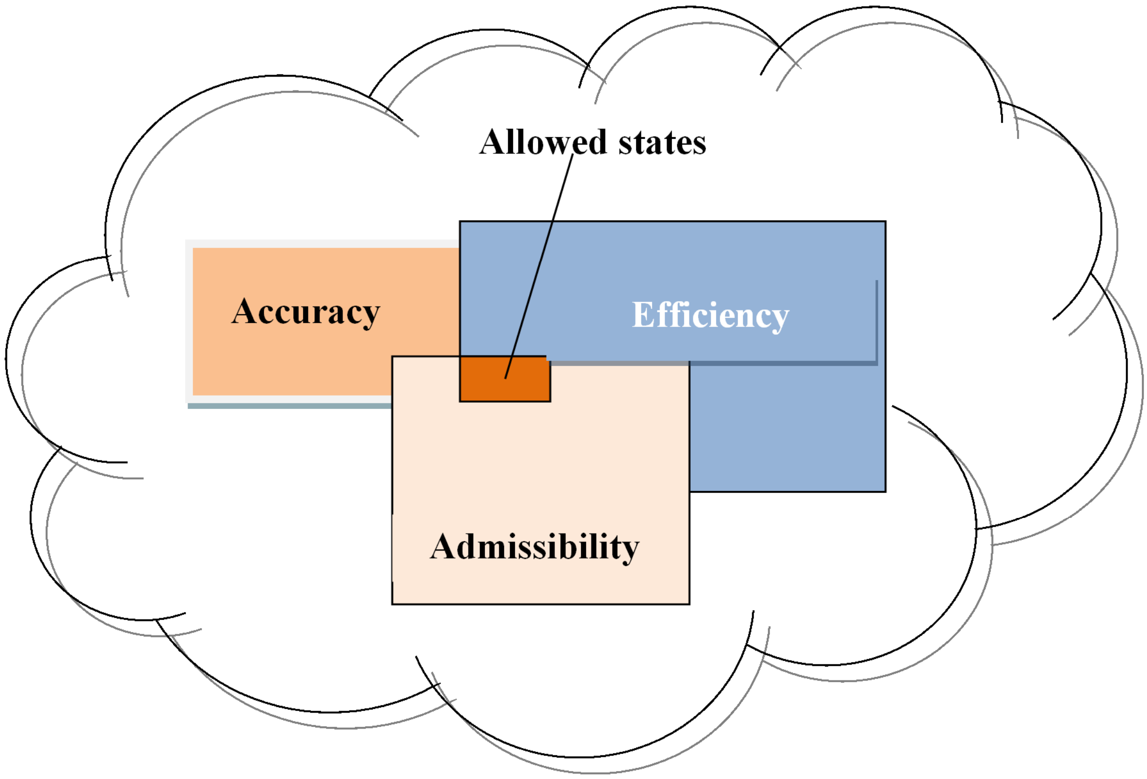

Figure 5. System states

Of course, all the three above mentioned requirements need to be satisfied for all intended functions.

System failure can lead to consequences of different nature, for example:

- Material losses due to non-availability of the system function,
- Material losses due to damages caused by the system to itself or to other systems,
- Material losses due to damages to the environment,
- Loss of life or injury, caused to people.

The consequence, which takes place, depends not only on the system itself, but also on the function, that is performed by system. Hence, it is meaningful to speak about functional reliability.

6. Definition of functional reliability

Based on reasoning presented in the previous chapters we are now ready to give definition of functional reliability.

In the given paper, *functional reliability of a system is defined as the set of its properties, which define the ability of the system to correctly perform the intended tasks with an acceptable level of precision with the output results being in admissible limits.*

Some positions of the definition require an explanation. First of all, the combination of attributes "precision" and "correctness", which have a different meaning, requires an explanation. The term "correctness" means, that information processes are realized according to the specified set of rules and instructions, i.e. in essence according to the algorithms for performance of information processes intended in a system. The term "correctness" of functional reliability is similar to the concept of "failure free function" in structural reliability. Any deviation from the specified rules and instructions leads to deviation *from correct* functioning of the information system. This means the following. If the algorithm for information processing is constructed correctly then it is possible to receive correct results provided program or transitional faults are absent during information processing. We say "it is possible to receive" keeping in mind, that correct results are not solely achieved due to a correctly constructed algorithms of information processing. Wrong results can be obtained caused by deviations in boundary values of data and / or results, boundary values of lengths of keys, admissible number of file records, admissible length of keys, admissible number of search criteria, etc. Transitional failures of digital technical equipment, for example, can cause wrong results, which lead to distortion of separate instructions of the algorithm or falsification of stored boundary values. Thus, the property *of correctness* is one of the important components of system *functional reliability* and it characterizes the credibility of results (for example, code combinations or results are in the allowed region).

Let us assume that the system correctly performs the intended tasks. Does it mean that the system is functionally reliable? The answer is no, because *correct function must be ensured, but it is not sufficient.* For example, an error caused by informational threats and violation of data integrity, intermediate and / or distrortion of output results in failure of control. In this case, the algorithm of the task being performed has been correct. As an example, we consider a graph describing part of an algorithm (fig. 6). The graph shows procedures (nodes) and connections between them (edges). All the defined task procedures (they can be considered as instructions) must be executed according to the defined rules (they can be considered as connection between the nodes). If in the given task all operations are performed strictly according to rules and instructions, then we can assume that information process used for implementation of the function has been realised correctly.

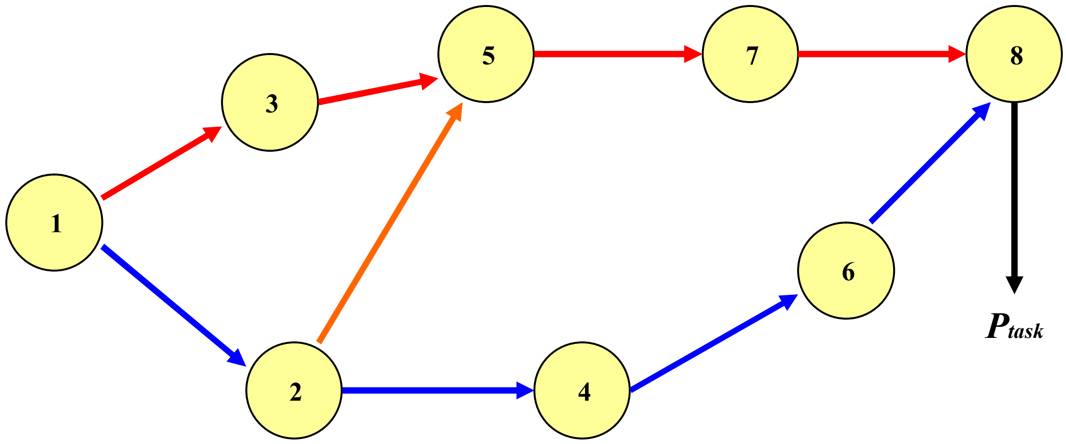

Figure.6. Explaining correctness and faultlessness of system functional reliability

Probabilities of faultless performance of tasks can be related to the nodes of the graph. The resulting probability of faultless performance of all tasks is calculated taking into account the connections between the tasks (edges). The property of faultlessness is a complex one. It is ensured both by faultless execution of each task taken separately and the correct execution of the sequence of the tasks according to the algorithm.

A definition of functional reliability of information system would be incomplete without taking into account the transformation of a possible output error into a functional failure of the information system. It is necessary to understand, that with the help of the intended functions in the system it is possible not only to exclude the progression of an error into a functional failure within the admissible limits, but also to neutralize that error. This is achieved under the following necessary conditions:

a) the system has a qualitatively high means for detection of errors,

b) the system has in its structure means for ensuring error tolerability

Note that, if there is some, even small time interval available for detection and reaction of the system, then there is always a certain probability to efficiently eliminate the detected error, even if the system is not equipped with intrinsic error tolerability. However, with the help of means for error tolerability, there are possibilities to considerably increase the probability of preservation of correct output results and, certainly, to enhance faultlessness of transformation of the initial data into results.

7. Safety

So far, we used the term “safety” without defining it. According to the standard EN 50126 [5], safety is defined as freedom from unacceptable risk. Risk is a combination of damage and probability of its occurrence [7]. The definition of risk given in EN 50126 is somewhat ambiguous. In EN 50126 risk is defined as “The probable rate of occurrence of a hazard causing harm and the degree of severity of the harm.”. One should talk about a specific combination meaning a specific operation, instead of a simple compilation. However, in the sequel the term risk is used correctly in the given standard

Depending on consequences, it is possible to consider separately:

a) functional reliability of the system if it performs its function (i.e. does not lose the defined properties) in a chain of all those systems contribute to functional performance;
b) functional safety if consequences do not lead to unacceptable risks.

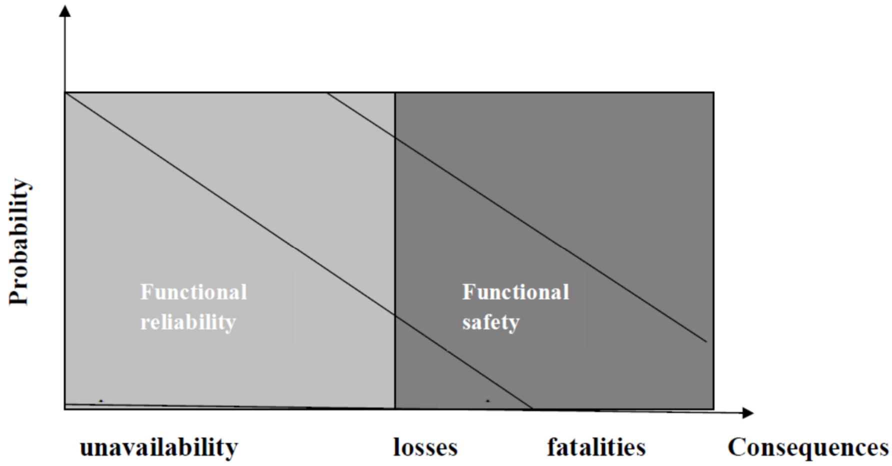

Figure 7. Functional reliability and functional safety.

Figure 7 shows, that as far as consequences are considered, functional reliability smoothly transforms into functional safety if the consequences of functional failures become more and more critical. From this fact it is evident, that systems, which are critical in the area of functional reliability (for example, with a rate of functional failures in the area $10^{-6}...10^{-7}$ 1/h), can handled using the concept of functional safety. Such systems are often called as critical systems.

Finally, from the above considerations we conclude that those methods which are used in functional safety for neutralization of threats, may be used in functional reliability in the same manner.

8. Combining reliability, safety and quality

Functional reliability and safety of a system in its specific application and environment, can be considered as complex properties. However, it is not always logical when recalling the definition of reliability definition.

Thus, we have the following picture (fig.8).

Thus, properties of functional reliability and functional safety are explained through other system properties, and they also transform smoothly into each other. This has already been shown on fig. 7.

It should be noted, that the quality model, i.e. the set of properties as described in figure 8, differs from that, which is presented in section 3. It is larger and cannot be used for an explanation of the terms functional reliability or safety.

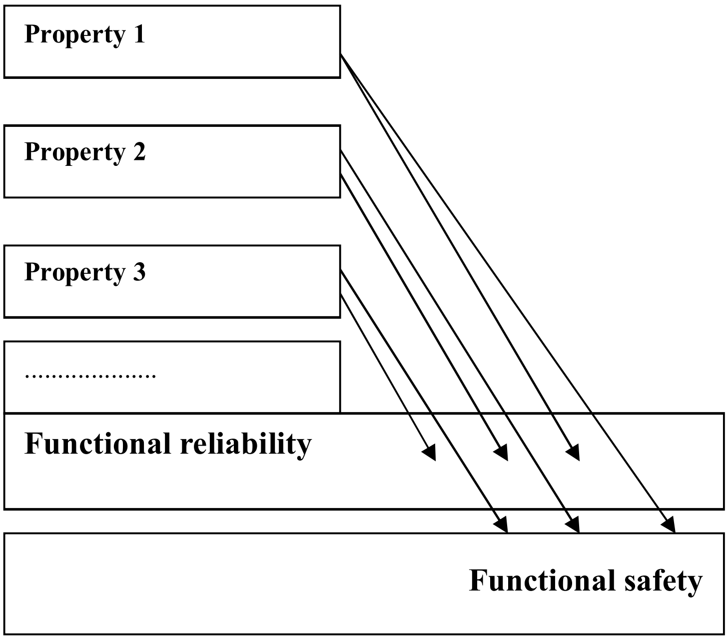

Figure 8. Functional reliability and functional safety as system properties.

9. Conclusions

In this paper, we have defined the concept of functional reliability. Much attention has been paid to the fact that the system is not so much important itself, but functions it performs. We have substantiated this statement considering system functions and the threats caused by them. Our discussion has shown how the concepts of functional reliability and functional safety are connected with each other. On the one hand, they both smoothly transform into each other, on the other hand, they cover a broader spectrum of properties and threats, than the terms “structural reliability” or "simple" safety.

We hope, that systematic interpretation and deeper understanding of functional reliability facilitates the solution of many urgent problems of reliability and safety.

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APPLICATION OF RELIABILITY GROWTH MODELS TO SENSOR SYSTEMS

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ABSTRACT

This paper presents the three reliability growth models namely Duane, AMSAA, and ERG II models briefly. The paper compares the three models for both time and failure terminated tests. Comparisons are carried out by simulating and conducting the statistical hypothesis t-test on twenty sets of failure data. An inference made from statistical hypothesis t-test is that AMSAA model is better choice for time terminated reliability growth test. Duane model is better choice for failure terminated reliability growth test. This is based on comparison with ERG II model which is expected to give best results. Case study on reliability growth tests of strain gauge of pressure sensor used in propulsion systems of satellites is also presented.

NOTATIONS

TT	Time terminated test
FT	Failure terminated test
t_o	$t_{observed}$
IMTBF	Instantaneous mean time between failures
ERG	Exponential reliability growth

KEYWORDS

Duane model; AMSAA model; ERG II model; time terminated test; failure terminated tes; reliability growth.

1 INTRODUCTION

Reliability of sensors used for monitoring various parameters of critical systems is very important for timely assessment of their health and to take appropriate measures for fault diagnosis at incipient stages in order to prevent any catastrophic failures. Reliability growth models help a sensor to undergo series of improvement stages till a final design is frozen for meeting the target reliability and MTBF requirements. This type of modelling enables sensors for successful operation in critical applications such as propulsion systems of a satellite, nuclear power plants, aircraft systems etc. Such systems require continuous reliable monitoring system to avoid unexpected failures which might result in huge economic losses apart from ill effects on environment, health & safety of human beings and other species. Instead of spending huge amounts on replacement/repair of industrial systems due to unreliable sensors it may be better to have high reliable sensors by conducting reliability growth tests. An attempt is made in this paper to compare the three reliability growth models namely Duane, AMSAA, and ERG II models. All these tests are performed for both time and failure terminated test on the strain gauge of a pressure sensor used in propulsion system of a satellite. This is explained in detail with case studies.

The rest of the paper is organized as follows:

In Section-2 the problem statement and reliability growth model approach for sensors are presented. In Section-3, drawback of Duane and AMSAA reliability growth models are presented. A method is proposed in section-4 for comparing reliability growth models for both time terminated as well as failure terminated test data. In section-5, case studies related to time and failure terminated reliability growth tests on a strain gauge of a pressure sensor which is used in propulsion system of a satellite are discussed. Results of this paper are presented in section-6. Conclusions are presented in section-7, followed by selected references.

2 RELIABILITY GROWTH TEST FOR SENSORS

A. Problem statement

Operational health of critical systems like aircrafts, propulsion systems of satellite, nuclear power plants etc, can be assessed by monitoring their critical parameters using sensors. Application of highly reliable sensors is essential for successful operation of the system till the end of mission time. For this purpose, sensor design and development must undergo a series of improvement stages till a final design is frozen for meeting the target reliability and MTBF requirements. A series of life tests need to be conducted after every design improvement. The results of this test will reveal whether the design has reached the target requirements. This procedure is known as reliability growth tests. Several types of reliability growth models such as Duane, AMSAA, and more recently ERG II models have been developed over the years for this purpose [1], [2], [3], [4].

This paper is focused on this important problem. Research objective is to fit the reliability growth model for sensor failure data using ERG II model [4] for both time and failure terminated tests.

Reliability growth model approach

The objective of reliability growth testing of sensors is to improve reliability over time through changes in product design, in manufacturing processes and procedures [5]. This is accomplished through test-fix-test-fix cycle. Reliability growth test is performed on prototypes. During test if any failure occurs due to critical failure mode [6], the failure mode is eliminated through redesign and the test cycle is repeated. Finally, the test continues till the required target MTBF is achieved. In this paper we discussed and compared Duane, AMSAA, and ERG II reliability growth models with case studies.

Redesign of a sensor’s component should be in such a way that obeys the following characteristics of a good design.

- Reliability
- Long useful life
- Low maintenance and noise level (if any)
- High accuracy
- Low cost
- Attractive appearance
- Trouble free
- simplicity

Reliability growth test eliminates the sensor components which obey the following characteristics:

- Poor accuracy
- High noise level and non adjustable
- Poor reliability
- Non-repairable and flimsy
- Wear out
- Rattles and rusts
- Cracks and corrodes
- High maintenance costs
- Short useful life

Design/redesign process of sensor components should include the following steps:

Step 1: Perform analysis requirements.
Step 2: Define the scope, objectives, and pertinent restraints with respect to the requirements; identify any significant problem, which has to be solved.
Step 3: Develop alternative design
Step 4: Perform feasibility analysis of alternative design
Step 5: Optimize the promising design
Step 6: Select the design for use
Step 7: Implement the design

3 DRAW BACK OF DUANE AND AMSAA RELIABILITY GROWTH MODELS

A. Sensor and Sensor System

Duane [1] and AMSAA [2] are frequently used reliability growth models. These models when applied to sensors, the failure intensity versus time of operation graph is continuous. So, the exact failure intensity of a sensor after fixing an occurred failure through redesign in not known. Exact test-fix-test-fix cycles are not known in these models. ERG II model [4], in which failure intensity versus time of operation plot for sensor is discontinuous (step wise). Exact failure intensity after fixing failure can be known in this model. Therefore, ERG II model is an accurate reliability growth model for sensors. This is further explained in detail in section 4 & 5 with case studies.

ERG II model is briefly discussed in this section. Sen et.al., proposed a constant-step failure-rate model called ERG II model [4], amounting to the assumption that inter failure times $X_i = T_i - T_{i-1}$, (i= 1, 2…) denoting the successive failure times are independent exponential random variables with hazard λi , (i= 1, 2….), the structure of λ_i is assumed to be of the form as shown below [4].

$$\lambda_i = \frac{\mu}{i^\delta - (i-1)^\delta}, \mu > 0, \delta \geq 1 \quad (1)$$

Where, ‘μ’ is scale parameter and ‘δ’ is shape parameter of ERG II model.

MTBF (estimated) in time terminated test is:

$$\theta(t^*) = \frac{n^\delta - (n-1)^\delta}{\mu} \quad (2)$$

MTBF (estimated) in failure terminated test is:

$$\theta(t_n) = \frac{n^\delta - (n-1)^\delta}{\mu} \quad (3)$$

For more details about this model readers may refer [4].

4 SELECTION OF RELIABILITY GROWTH MODEL

Error in estimation using any model is defined as the deviation of the estimated parameter from the desired parameter. Any natural random errors usually follow normal distribution [7], [8] if the sample size for estimation is large (at least 30). For smaller sample sizes t-test is used as an approximation of normal distribution for hypothesis testing. In this section, a method based on statistical sampling technique is used to find the better model among Duane and AMSAA models for time and failure terminated reliability growth tests by comparing with accurate model i.e. ERG II model. A t-test is applied for statistically concluding which is the better model for two test cases namely time terminated and failure terminated tests.

4.1. t-test

The t-test is a statistical method for testing the degree of difference between two means in small sample. It uses t-distribution theory to deduce the probability when difference happens, then judges whether the difference between two means is significant.

For further details on t-test refer [9].

4.2. Time terminated test

In time terminated test, we conduct reliability growth test on a sensor up to certain mission time and then we terminate the test. After termination we estimate the instantaneous MTBF from the test data obtained. In time terminated test, given n successive failure times $t_1 < t_2 < \cdots < t_n$ that occur prior to the accumulated test time or observed system time, T.

Twenty sets of failure data (assumed for hypotheses) are simulated for conducting the hypothesis test for comparison of reliability growth models. The failure data includes sets of 3, 4, 5, 6, 7, 8, 9, and 10 numbers of failures. The range of failure data is between 0 and 30 minutes i.e. the mission time for time terminated test. The failure data is simulated for evaluating IMTBF in time terminated Duane, AMSAA, and ERG II models. The results obtained after simulating twenty samples are tabulated as shown in table 1. A t-test is performed between ERG II and Duane models and the results are compared with a similar t-test between ERG II and AMSAA models for selecting a more appropriate model.

The $T_{critical}$ value with degree of freedom of 19, with 95% confidence interval is obtained as **2.093**.

From t-test the t_0 value obtained for Duane model in comparing with ERG II model is **-2.100**.

Similarly, from t-test, the t_0 value obtained for AMSAA model in comparing with ERG II model is **+1.769**.

Fig.1 shows the results of t-test between ERG II and Duane, ERG II and AMSAA models in time terminated test for twenty samples.

Table 1. IMTBF in TT reliability growth test for different simulations

Simulation Number	Number of failures	IMTBF (in min)		
		Duane	AMSAA	ERG II
1	3	9.4507	11.012	12.245
2	3	10.1418	10.357	10.335
3	3	11.9593	9.3678	8.6458
4	4	8.2706	11.997	12.425
5	4	17.5087	13.057	12.417
6	4	12.0615	9.1239	11.563
7	4	9.9432	12.799	11.548
8	5	8.9326	7.3740	6.8882
9	5	15.4799	9.6276	11.479
10	5	12.5313	8.7551	9.1577
11	6	11.5370	8.0476	9.4361
12	6	9.2731	7.3194	7.1354
13	6	6.7138	5.5993	5.8971
14	7	7.4591	5.7930	6.2416
15	7	8.3275	5.6956	6.5554
16	8	6.8532	5.6475	4.9845
17	8	6.9742	5.3884	5.7987
18	9	6.2077	4.6915	5.3578
19	9	7.8065	4.9501	5.4523
20	10	6.3343	4.8001	5.5487

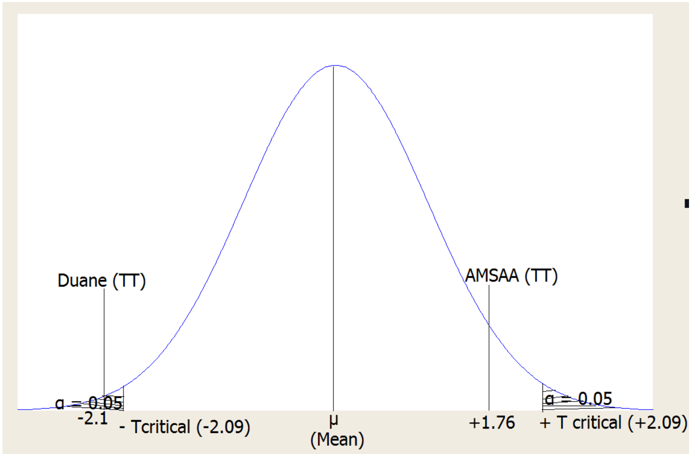

Fig.1. Critical values and t_0 values of T-test between ERG II and Duane, ERG II and AMSAA models in TT test.

From fig.1, the inferences are as following:

a) t-test has shown that the Duane model is falling beyond the critical value, and
b) AMSAA model is within the critical limits.

Therefore, statistically AMSAA model is better model than Duane model for time terminated reliability growth test.

This is further explained using a case study of reliability growth test on a strain gauge of a pressure sensor which is used in propulsion system of a satellite in section 5. The strain gauge is selected for this test due to its high failure rate compared to other components of a pressure sensor [10]. This strain gauge has to operate successfully in propulsion system of a satellite for 30 minutes, by sustaining huge amounts of pressure.

4.3. Failure terminated test

In failure terminated test, we conduct reliability growth test on a sensor up to certain number of failure occurs irrespective of time and then we terminate the test. After termination we estimate the IMTBF from the test data obtained. In failure terminated test, given n successive failure times $t_1 < t_2 < \cdots < t_n$ following accumulated test time or observed system time $T = t_N$.

Similar analysis as in section 4.2 has been carried out on failure terminated simulation data which is presented in table 2.

From t-test, the t_0 value obtained for Duane model in comparing with ERG II model is **+2.03**.

Similarly, from t-test, the t_0 value obtained for AMSAA models in comparing with ERG II model is **-4.12**.

Fig.2 shows the results of t-test between ERG II and Duane, ERG II and AMSAA models in time terminated test for twenty samples.

Table 2. IMTBF in FT reliability growth test for twenty different simulations

Simulation Number	Number of failures	IMTBF (in min)		
		Duane	AMSAA	ERG II
1	3	9.4507	5.2117	9.3179
2	3	10.1418	5.9038	10.283
3	3	11.9593	6.2871	11.472
4	4	9.9432	5.7126	8.4592
5	4	17.5087	8.7312	13.147
6	4	12.0615	6.9772	10.635
7	4	8.2706	5.2157	7.8853
8	5	8.9326	6.9316	9.1426
9	5	15.4799	9.3680	12.900
10	5	12.5313	8.5101	11.167
11	6	11.5370	7.8308	10.1502
12	6	9.2731	7.1148	10.133
13	6	6.7138	4.9040	7.0484
14	7	7.4591	5.7259	7.1179
15	7	8.3275	5.6292	10.250
16	8	6.8532	5.3363	6.5220
17	8	6.9742	5.3884	6.5592
18	9	6.2077	4.6378	5.7306
19	9	7.8065	4.9501	6.5423
20	10	6.3343	4.8001	5.8250

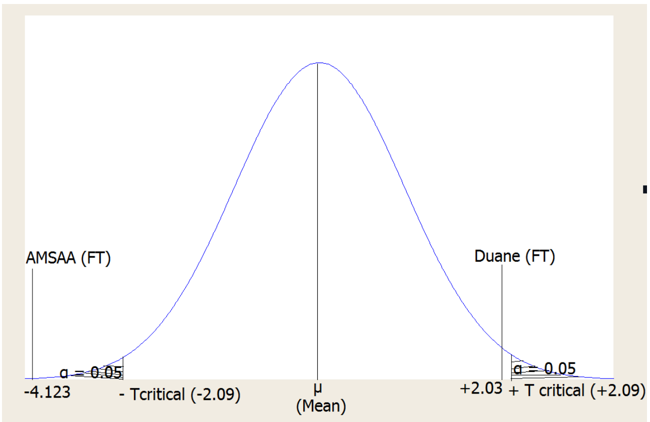

Fig.2. Critical values and t_o values of t-test between ERG II and Duane, ERG II and AMSAA models in FT test.

From fig.2, the inferences are as following:

a) t-test has been shown that the AMSAA model is falling beyond the critical value, and
b) Duane model is within the critical limits.

Therefore, statistically Duane model is better model than AMSAA model for failure terminated reliability growth test.

This is further explained using a case study of reliability growth test on a strain gauge of a pressure sensor which is used in propulsion system of a satellite in section 5.

5 CASE STUDY

Case study- I: Consider a time terminated reliability growth test on a strain gauge of a pressure sensor. Assume that the test is conducted on it for a mission time of 30 minutes. The strain gauge is subjected to as high pressures as in the actual propulsion system. For each failure occurred, failure cause is analysed & eliminated. It is redesigned and again the reliability growth test is conducted. The table 3 shows the assumed times at which the initial deigned strain gauge and redesigned same strain gauge failed until 30 minutes.

Table 3. Failure times of strain gauge for each redesign

S.No	**Design**	**Failure time (in min)**
1	D1 (initial design)	1.5
2	D2	4.6
3	D3	10.5
4	D4	18.6

We require evaluating the following parameters using Duane, AMSAA, & ERG II models.

a) IMTBF at the end of mission time, and additional test time required to achieve an IMTBF of 30 minutes.
b) Based on above results estimate & comment on the better fitting model between Duane and AMSAA models for a pressure sensor next to ERG II model.

Solution:

a) Adopting the mathematical forms of Duane, AMSAA, and EG II models from [1], [2], & eq. 2 respectively the IMTBF obtained using above three models are as shown in table 4.

Table 4. IMTBF of a strain gauge in TT reliability growth test

S.No	**Reliability growth model**	**IMTBF (in min)**
1	Duane	8.2706
2	AMSAA	11.9976
3	ERG II	12.4256

The failure intensity versus time plot for above three models is as shown in fig.3, fig.4, & fig.5 respectively.

Fig.3. Failure intensity vs time plot using Duane model for TT test

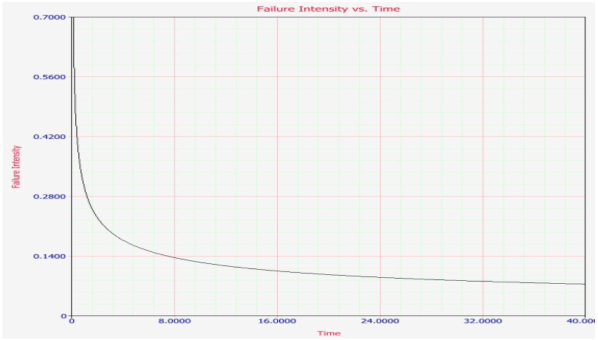

Fig.4. Failure intensity vs time plot using AMSAA model for TT test

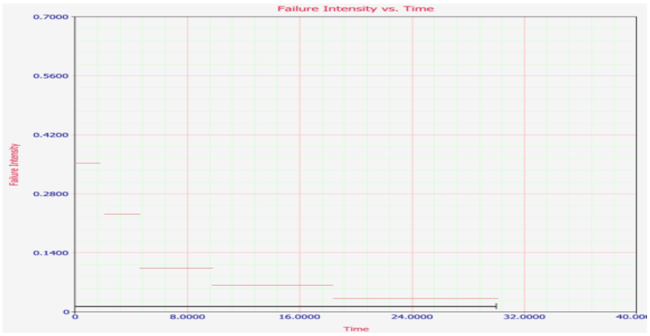

Fig.5. Failure intensity vs time plot using ERG II model for TT test

Table 5. IMTBF at different test times

Time (in min)	IMTBF (in min)
100	17.6617
200	24.1445
300	28.9899
400	33.0067
500	36.5018
600	39.6305
700	42.4842

The extrapolated data obtained using Duane mathematical forms [1] are as shown in table 5.

From table 5, approximately for 350 minutes the strain gauge should be conducted reliability growth test to achieve an instantaneous MTBF of 30 minutes.

∴ 320 additional minutes is required in Duane reliability growth model test to achieve an instantaneous MTBF of 30 minutes.

The extrapolated data obtained using Duane mathematical forms [2] are as shown in table 6.

Table 6. IMTBF at different test times

Time (in min)	IMTBF (in min)
100	18.8414
200	24.4319
300	28.4427
400	31.6816
500	34.4458
600	36.8824
700	39.0766

From table 6, approximately for 330 minutes the strain gauge should be conducted reliability growth test to achieve an IMTBF of 30 minutes.

∴ 300 additional minutes is required in AMSAA reliability growth model test to achieve an IMTBF of 30 minutes.

The extrapolated data obtained using Duane mathematical forms [4] are as shown in table 7.

Table 7. IMTBF at different test times

Time (in min)	IMTBF (in min)
100	19.8254
200	25.5132
300	29.8645
400	32.4430
500	35.6524
600	38.8960
700	41.1422

From table 7, approximately for 310 minutes the strain gauge should be conducted reliability growth test to achieve an instantaneous MTBF of 30 minutes.

∴ 280 additional minutes is required in ERG II reliability growth model test to achieve an IMTBF of 30 minutes.

b) Finally, from figures 3, 4, & 5 and tables 5, 6, and 7, we conclude that ERG II is the accurate model (due to step diagram as shown in fig.5, the value of failure intensity at any time instant is well defined) and AMSAA model better fits pressure sensor failure data compared to Duane model in time terminated reliability growth test.

Case study- II: Consider a failure terminated reliability growth test on a strain gauge of a pressure sensor. The test is conducted on it until four failures occurs. The strain gauge is subjected to as high pressures as in the actual propulsion system. Assuming (for illustrative purpose) that the data presented in the table 3 is that of failure terminated test, we need to evaluate the same parameters as in the previous case.

Solution:

a) Adopting the mathematical forms of Duane, AMSAA, and EG II models [1], [2], & eq.3 respectively the IMTBF obtained using above three models are as shown in table 8.

Table 8. IMTBF of strain gauge in FT reliability growth test

S.No	Model	IMTBF (in min)
1	Duane	8.2706
2	AMSAA	5.2157
3	ERG II	7.8853

The failure intensity versus time plot for three models in failure terminated test is as shown in fig.6, fig.7, & fig.8 respectively.

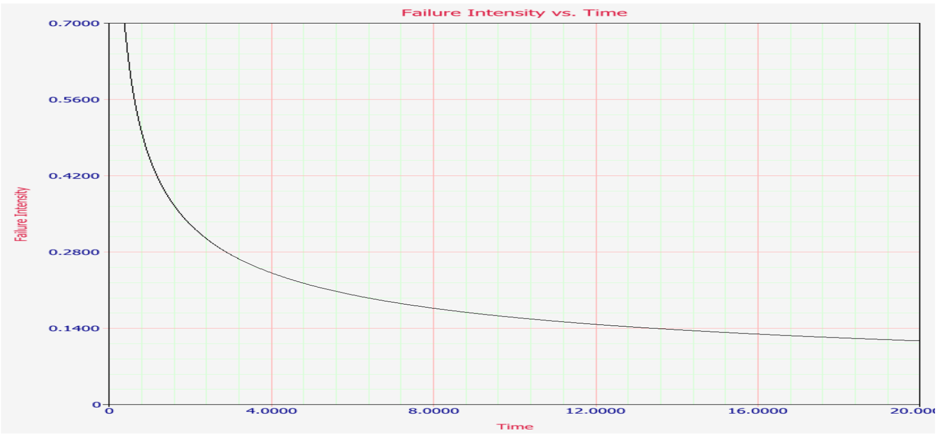

Fig.6. Failure intensity vs time plot using Duane model for FT test

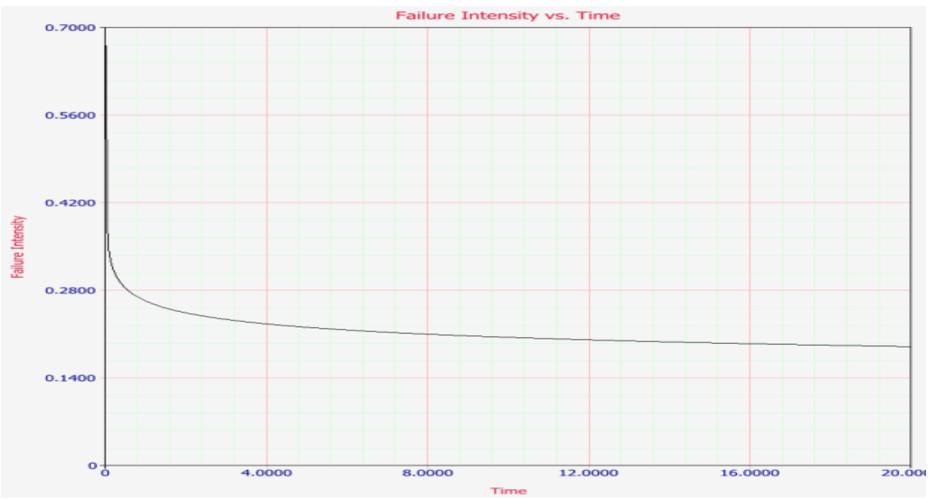

Fig.7. Failure intensity vs time plot using AMSAA model for FT test

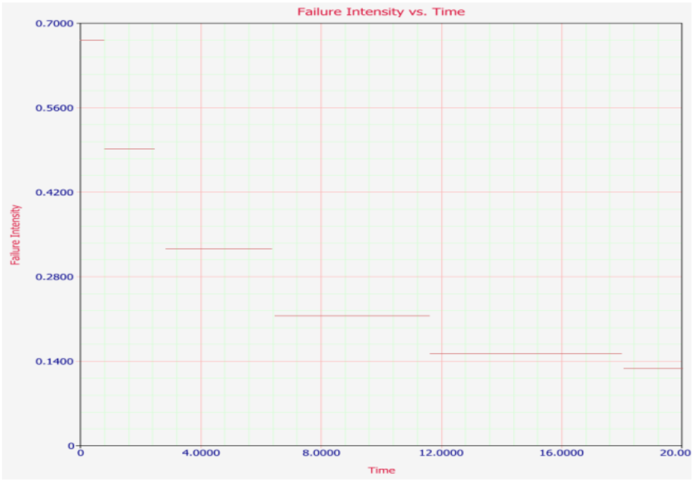

Fig.8. Failure intensity vs time plot using ERG II model for FT test

b) ERG II reliability growth model is the accurate model. Comparing the other two model's results with ERG II, we therefore conclude that pressure sensor failure data can be better fit with Duane model compared to AMSAA model for failure terminated reliability growth test.

6 RESULTS

The results obtained from section- 4 & 5 are tabulated in tables 9, & 10 as shown below:

From section-4, we statistically conclude the following:

i. AMSAA model is the better model compared to Duane for time terminated reliability growth test.

ii. Duane model is the better model compared to AMSAA for failure terminated reliability growth test.

Table 9. IMTBF & additional time required for strain gauge in TT test

S.No	Model	IMTBF (in min)	Additional time required (in min)
1	Duane	8.2706	320
2	AMSAA	11.9976	300
3	ERG II	12.4256	280

Table 10. Instantaneous MTBF of strain gauge in FT test

S.No	Model	IMTBF (in min)
1	Duane	8.2706
2	AMSAA	5.2157
3	ERG II	7.8853

7 CONCLUSIONS

In this paper, three reliability growth models namely Duane, AMSAA, and ERG II models are critically analysed using the reliability growth test data on strain gauge of a pressure sensor used in propulsion systems of satellites. Using simulated data on twenty samples and using the t-test, it is found that the AMSAA model is a better choice for reliability growth analysis of time terminated tests. It is also found that the Duane model is a better choice for reliability growth analysis of failure terminated tests. This is based on the comparison of the two models with the ERG II model which is expected to give the best results. The additional time required to conduct reliability growth test for achieving the target instantaneous MTBF are also evaluated for these models.

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FUNDAMENTALS OF SOFTWARE STABILITY THEORY

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ABSTRACT

The theoretical fundamentals of software stability were elaborated on the basis of software dynamic theory. The concepts of internal and external equilibrium have been introduced and the condition of reliability has been proved. The law of defect flow equilibrium has been formulated. The existence of unknown before mutual dependences among the defect flows in software has been revealed.

1 INTRODUCTION

Methods of mathematical modeling are widely used to determine the software systems (SS) reliability. The objective of software reliability modeling indexes is the assess of the amount of defects left in the system and forecasting of time and dynamics of their detection. This task is not new. But methods of its solving cannot be considered to be thoroughly studied. Thus, for example, the authors (Kharchenko 2004) note "It should be emphasized that so far the theory of software reliability cannot be regarded as an established science ….the existence of considerable discontinuity between theory (mathematical models and methods) and practice."

Theory of software systems dynamics (SSD), as it is shown in (Maevsky 2011) and (Maevsky at al. 2012) and confirmed in practice (Maevsky 2012), allows to eliminate this gap and make the first step to approach the science of software reliability to the definition "established science".

In the SSD theory the process of defect detection in SS and introduction of new secondary defects in it are regarded as the interaction process of two flows. The first outgoing flow removes the defects from the system; the second – incoming – brings the secondary defects in it.

Due to the existence of two oppositely directed defect flows in SS the detailed study of this influence and phenomena accompanying flows is vital. This article is thus devoted to the above study.

2 PHASE TRAJECTORIES OF DEFECTS. STATE OF EQUILIBRUM

As shown in (Maevsky at al. 2012), the behavior of SS from the point of view of appearance and development of defect flows is described by dynamic system (1):

$$\begin{cases} \dfrac{df_1}{dt} = -A_1 \cdot f_1 - A_2 \cdot f_2 \\ \dfrac{df_2}{dt} = -A_2 \cdot f_1 - A_1 \cdot f_2 \end{cases} . \tag{1}$$

To research the behavior and quality analysis of dynamic system of SS, specified by the equations (1), let's set up its phase trajectory (phase plane).

Definition 1. Let state vector of SS be vector

$$\vec{u} = \langle f_1, f_2 \rangle, \tag{2}$$

where f_1 and f_2 are state variables at a point in time t.

Definition 2. Let's consider the space state (phase space) to be the subset $X = U \subseteq R_+$ with coordinates (2), where $R_+ = \{N \in R : N \geq 0\}$.

Each phase trajectory corresponds to the definite particular solution of the system (1) on the definite initial conditions. In case of SS and defect flow research, the values f_1 and f_2 determining the number of defects of incoming and outgoing flows, should be chosen as the coordinates of phase space. To build up the phase trajectories of SS, it would be necessary to obtain the dependency $f_2(f_1)$. Differential equation, binding f_2 with f_1, can be received from the system (1), dividing the second equation by the first. We obtain:

$$\frac{df_2}{df_1} = \frac{A_2 \cdot f_1 + A_1 \cdot f_2}{A_1 \cdot f_1 + A_2 \cdot f_2}. \tag{3}$$

Herewith it is taken into account, that in accordance with (Maevsky at al. 2012),

$$\begin{aligned} f_1 &= F_0 \cdot e^{-A_1 t} \cdot ch(A_2 t) \\ f_2 &= -F_0 \cdot e^{-A_1 t} \cdot sh(A_2 t) \end{aligned}. \tag{4}$$

Taking into consideration these correlations, the equation (3) can be rewritten as following:

$$\frac{df_2}{df_1} = \frac{A_2 \cdot chA_2 t + A_1 \cdot shA_2 t}{A_1 \cdot chA_2 t + A_2 \cdot shA_2 t}.$$

By dividing nominator and denominator of this fraction by $A_1 \neq 0$ and introducing coefficient $k = A_2/A_1$ we obtain:

$$\frac{df_2}{df_1} = \frac{k \cdot chA_2 t + shA_2 t}{chA_2 t + k \cdot shA_2 t}. \tag{5}$$

The equation (5) represents the linear non-homogenous differential equation, which includes time t on its right side. After accomplishing the numerical solution we get phase plane of SS, see Figure 1.

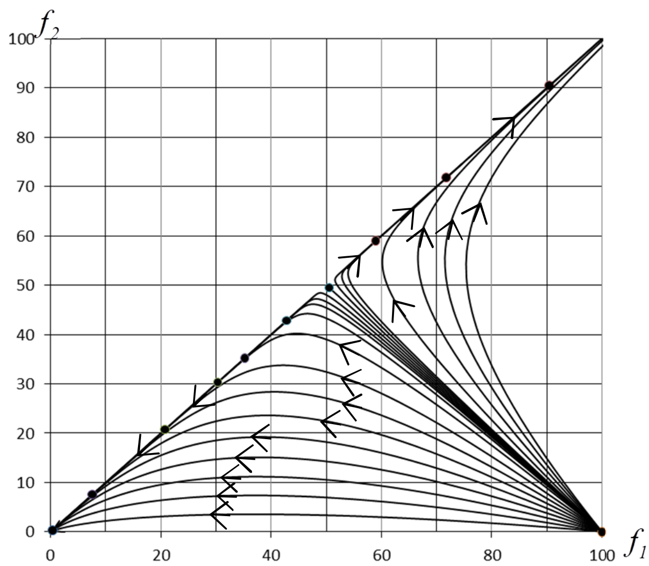

Figure 1. Phase trajectories of SS

Phase plane is built up for SS with the following characteristics: number of defects at $t=0$ – $F_0 = 100$, coefficient value $A_1 = 0,01day^{-1}$, coefficient k is changed from 0 to 1.1.

Let's clarify the formation of phase trajectories. The pair of values (f_1, f_2) corresponds to each point on the trajectory (generating point). Movement of generating point occurs from the initial state (with $t\to 0$) to its end state (with $t\to\infty$). In our case, the initial point on the X-axis corresponds to the initial state $f_1 = F_0, f_2 = 0$. The generating point moves from right to left (shown by arrows, see Figure 1).

It can be seen in the picture, that at $k < 1$ all trajectories converge to equilibrium whereby the speed of flow changes become equal to 0. To this state correspond the equilibrium of SS, when the defects are absolutely absent in it, what leads to the absence of flows. In Figure 1 the point $(0,0)$ corresponds to this state.

With $k = 1$, i.e. in case of intensity equilibrium of incoming and outgoing flows the generating point moves from right to left along the straight-line segment (f_1 is equal to f_2 in each of its points). The system reaches the equilibrium state and zero speed of flow changes at the finishing point of this straight line. This point has coordinates – $f_1=F_0/2, f_1=F_0/2$, i.e. at achieving this value by amount of introduced and eliminated defects the flows stop changing.

In case $k > 1$ the generating point moves along phase trajectory tending to infinity. However, thus, as we can see in Figure 1, first the point comes to the straight line $f_1 = f_2$. The number of the defects in the system will be growing continuously and the flow change speed will never achieve zero. At the same time analyzing Figure 1 we can draw even more important conceptual conclusions.

Conclusion 1. All phase trajectories tend to the strait line $f_1 = f_2$, i.e. with the time the number of defects brought into the system equalize with those taken out of the system. It can be said, that SS acquires ***internal equilibrium*** with time among the flows existing in it. After achieving equilibrium the two flows become the same both in amount of defects, forming the flow, and in the speed of flow changes in time. The appearance of equilibrium has been unknown before and needs though studying.

Conclusion 2. There are points in the phase plane of SS, where the software system itself as a whole achieves equilibrium with its external environment: the flows stop changing. Such points we will call the states of ***external equilibrium***. According to the theory of dynamic systems (Samoilenko 1989), these points are called stationary. The stationary point with $k < 1$ is the origin of coordinates. Herewith the steady state (recall the transient processes) of SS will be considered to be the achievement of its equilibrium with the surrounding medium (object area), i.e. absolute absence of defects in SS.

With $k = 1$ the stationary point, corresponding the intensity of direct and reversed flows, appears as well. Besides, as it has been said, this conclusion absolutely corresponds to the expected result. Unlike both examined cases when $k > 1$, i.e. with the exceeding of intensity of incoming flow over the outgoing one, the stationary points are not observed. Thus phase curves extend at infinity, what corresponds to the expected results. Nevertheless, let's turn our attention to the fact, that SS first achieve the state of equilibrium, and after that both flows simultaneously extend to infinity. It gives ground to suggest that ***achievement of internal equilibrium state is the necessary condition for all SS in any correlations between incoming and outgoing flows***. Let's prove this assumption.

3 INTERNAL EQUILIBRUM OF SOFTWARE SYSTEM

In the process of research of SS internal equilibrium phenomenon it is necessary to answer the following questions:

1. Will any SS always achieve the internal equilibrium state?
2. Is the internal equilibrium steady?

3. At what point of time the internal equilibrium will appear with the given SS parameters?

Before considering these questions let's give a formal definition to the internal equilibrium state.

Definition 3. The internal equilibrium of SS will be refered to as the establishment of equilibrium between incoming and outgoing flows in it, whereby their intensity and number of defects coinside.

While answering the first question we should determine whether the internal equilibrium phenomenon is random or it is common to all software systems. The existence of the internal equilibrium phenomenon for any SS is proved in the theorems 1 and 2.

Theorem 1. (First theorem of equilibrium). There exists such value of time, that for all $t > t'$ the condition $|f_1(t) - f_2(t)| \leq \partial$ holds for no matter how small ∂.

Proof of theorem 1. Taking into consideration that the remainder $f_1(t) - f_2(t)$ is taken modulo, we should pay attention to probable correlations between values $f_1(t)$ and $f_2(t)$. First it should be noted that in the expressions (4) the product A_2t is always positive. Therefore we can state, that $chA_2t \geq shA_2t$ wherefrom $f_1(t) \geq f_2(t)$. With such correlation modulus $|f_1(t) - f_2(t)|$, according to modulus property can be changed for the remainder $f_1(t) - f_2(t)$. On this basis let's form the equation:

$$f_1(t) - f_2(t) - \varepsilon = 0 \,. \tag{6}$$

The theorem will be proved, if the value $t = t'$ satisfying the equation is found. Using the expressions (4), the equation (6) can be rewritten as:

$$F_0 \cdot e^{-A_1 t} \cdot [ch(A_2 t) - sh(A_2 t)] - \varepsilon = 0 \,.$$

Disclosing the hyperbolic functions after transformations we get

$$F_0 \cdot e^{-(A_1 + A_2) \cdot t} = \varepsilon \,. \tag{7}$$

The equation (7) is solvable related to t with any positive ε. In fact, after taking the logarythm we obtain:

$$-(A_1 + A_2) \cdot t = ln \frac{\varepsilon}{F_{10}} \,.$$

Whence we have the value $t = t'$:

$$t' = -\frac{ln \frac{\varepsilon}{F_0}}{A_1 + A_2} \,. \tag{8}$$

The "minus" sign on the left part (8) results from the statement that whatever smallest value $\varepsilon < F_0$, therefore $ln(\varepsilon / F_0) < 0$. Thus the value $t = t'$ exists and with $t > t'$ the left part (8) will become less than ε.

The theorem 1 is proved.

To consider the intensity of flows let's analyze theorem 2.

Theorem 2. (The second theorem of equilibrium*)* There exists such value of time t'' that for all $t > t'$

$$\left| \frac{df_1}{dt} - \frac{df_2}{dt} \right| \leq \varepsilon$$

is fulfilled for any whatever smallest values ε.

Proof of the theorem 2. While proving theorem 2, taking into consideration the property of remainder modulus as well as in the previous theorem proof, all possible cases should be observed.

First, it should be noted that the equilibrium state between the flaws is possible only when their intensities (speed changes in time) have similar signs. The equilibrium is not possible, when one flow is increasing and the other one is decreasing.

Secondly, it is necessary to consider separately the possible correlations between speed changes of the flows, i.e. the case, when

$$\left|\frac{df_1}{dt}\right| > \left|\frac{df_2}{dt}\right|$$

or case when

$$\left|\frac{df_1}{dt}\right| < \left|\frac{df_2}{dt}\right|.$$

Therefore, let's consider this two cases.

Case 1. Herewith, as it is resulted from the modulus properties, remainder modulus can be changed by the common difference

$$\left|\frac{df_1}{dt} - \frac{df_2}{dt}\right| = \frac{df_1}{dt} - \frac{df_2}{dt}.$$

Taking into account (4) we'll obtain

$$\frac{df_1}{dt} - \frac{df_2}{dt} = F_{10} \cdot e^{-A_1 t} \cdot \begin{pmatrix} -A_1 \cdot chA_2 t - A_2 \cdot shA_2 t + \\ A_1 \cdot shA_2 t + A_2 \cdot chA_2 t \end{pmatrix} =$$

$$= \frac{F_{10}}{2} \cdot e^{-A_1 t} \cdot \begin{pmatrix} -A_1 e^{A_2 t} - A_1 e^{-A_2 t} - A_2 e^{A_2 t} + A_2 e^{-A_2 t} + \\ + A_1 e^{A_2 t} - A_1 e^{-A_2 t} + A_2 e^{A_2 t} + A_2 e^{-A_2 t} \end{pmatrix}$$

or:

$$\frac{df_1}{dt} - \frac{df_2}{dt} = \frac{F_{10}}{2} \cdot e^{-A_1 t} \cdot \left(2 \cdot A_2 e^{-A_2 t} - 2 \cdot A_1 e^{-A_2 t}\right).$$

From this we can obtain equation:

$$F_{10} \cdot (A_2 - A_1) \cdot e^{-(A_1 + A_2)t} = \varepsilon. \tag{9}$$

With $A_2 > A_1$ the value t satisfying this equation exists, with the increase of t the left part becomes smaller than ε. For the case 1 theorem 2 is proved.

Case 2. For this case

$$\frac{df_1}{dt} > 0, \; \frac{df_2}{dt} > 0, \; \left|\frac{df_1}{dt}\right| < \left|\frac{df_2}{dt}\right|.$$

Herewith it follows from the modulus properties that

$$\left|\frac{df_1}{dt} - \frac{df_2}{dt}\right| = \frac{df_2}{dt} - \frac{df_1}{dt}.$$

Taking into consideration (4) we obtain:

$$\frac{df_2}{dt} - \frac{df_1}{dt} = F_{10} \cdot e^{-A_1 t} \cdot \begin{pmatrix} A_1 \cdot chA_2 t + A_2 \cdot shA_2 t - \\ A_1 \cdot shA_2 t - A_2 \cdot chA_2 t \end{pmatrix},$$

or after transformation of similar mentioned in case 1:

$$F_{10}\cdot\left(A_1-A_2\right)\cdot e^{-(A_1+A_2)t}=\varepsilon. \tag{10}$$

With $A_2 > A_1$ the value t satisfying this equation exists, with the increase of *t* the left part becomes smaller than *ε*. For the case 2 theorem 2 is proved.

Case 3. For this case:

$$\frac{df_1}{dt}<0,\ \frac{df_2}{dt}<0,\ \left|\frac{df_1}{dt}\right|>\left|\frac{df_2}{dt}\right|.$$

With such correlation the remainder modulus can be changed for

$$\left|\frac{df_1}{dt}-\frac{df_2}{dt}\right|=\left|\frac{df_1}{dt}\right|-\left|\frac{df_2}{dt}\right|.$$

Taking into consideration (4) we get:

$$\left|\frac{df_1}{dt}\right|-\left|\frac{df_2}{dt}\right|=F_{10}\cdot e^{-A_1t}\cdot\begin{pmatrix}A_1\cdot chA_2t+A_2\cdot shA_2t-\\ A_1\cdot shA_2t-A_2\cdot chA_2t\end{pmatrix},$$

or after transformations

$$F_{10}\cdot\left(A_1-A_2\right)\cdot e^{-(A_1+A_2)t}=\varepsilon. \tag{11}$$

The expression (11) is identical to that received in case 2, expression (10), but only with $A_1 > A_2$. It means that with any correlations between A_1 and A_2, the value *t*, satisfying this correlation exists; besides with the increase of *t* the left part becomes smaller than ε. For the case 3 theorem 2 is proved.

Case 4. For this case

$$\frac{df_1}{dt}<0,\ \frac{df_2}{dt}<0,\ \left|\frac{df_1}{dt}\right|<\left|\frac{df_2}{dt}\right|.$$

Therewith, proceeding form modulus properties, it can be rewritten as

$$\left|\frac{df_1}{dt}-\frac{df_2}{dt}\right|=\left|\frac{df_2}{dt}\right|-\left|\frac{df_1}{dt}\right|.$$

With the regard to (4) we obtain

$$\left|\frac{df_2}{dt}\right|-\left|\frac{df_1}{dt}\right|=F_{10}\cdot e^{-A_1t}\cdot\begin{pmatrix}A_2\cdot chA_2t+A_1\cdot shA_2t-\\ A_2\cdot shA_2t-A_1\cdot chA_2t\end{pmatrix},$$

or after transformations:

$$F_{10}\cdot\left(A_2-A_1\right)\cdot e^{-(A_1+A_2)t}=\varepsilon. \tag{12}$$

The expression (12) is identical to the obtained one in case 1, expression (8), but with $A_2 > A_1$.

It means that with any correlations between A_1 and A_2, the value *t* satisfying this equation exists, besides with the increase of *t* the left side becomes smaller than ε. For the case 4 theorem 2 is proved.

Thus, in all possible cases there exists such value *t,* which satisfy the equation

$$\left|\frac{df_1}{dt}-\frac{df_2}{dt}\right|=\varepsilon,$$

moreover with the increase of time *t* the left part becomes smaller than *ε*.

The theorem 2 is proved.

Let's consider the question of stability of internal equilibrium state. It should be found out whether SS can spontaneous come out of balance. If the answer is positive, it means that internal equilibrium is a temporal event. At a certain time it is reached, but subsequently the system comes out of the condition itself and the flows cease to be concerted. If the equilibrium state is stable, the system cannot come out of it itself and the flows constantly remain concerted. They either decrease to zero or increase infinitely synchronously.

Stability of internal equilibrium state is proved by the theorem 3.

Theorem 3. (Third theorem of equilibrium). Internal equilibrium phenomenon is stable with all influence coefficient meanings

Proof of the theorem 3. Assume that the internal equilibrium isn't stable. It means that after obtainment the same number of defects of incoming and outgoing flows at the moment of time t' with $t > t'$ the equilibrium will be disturbed, i.e. with $t > t'$ $f_1(t)>f_2(t)$. Inequality of defects number of both flows after achieving equilibrium is possible, only if the speed changes of incoming and outgoing flows differ. But it comes into conflict with theorem 2, which proves the speed equality of flow changes after achievement of internal equilibrium. Supposition is not true, therefore, *theorem 3 is proved.*

Thus, the stability of SS internal equilibrium state is proved. Let's define the time, at which the system achieves this condition. For this purpose we will use the results, received while proving theorems 1 and 2.

The time value, whereby the software system achieves the state of equilibrium of defects number is determined by the expression (8). In practice, taking into consideration the fact that the defect number can always be a whole number, we can assume that $\varepsilon = 1$, i.e. it corresponds the least distinguished number of defects. Than we obtain from (8):

$$t' = \frac{\ln F_0}{A_1 + A_2} . \qquad (13)$$

To determine the time at which the equlibrium reaches the flow speed, let's use the formulae (9) and (10). The formula (9) can be used with correlation of infuence coefficients $A_2 > A_1$. Here the time, necessary to achieve the speed equilibrium is defined as:

$$t'' = -\frac{\ln \dfrac{\varepsilon}{(A_2 - A_1)\cdot F_0}}{A_1 + A_2} . \qquad (14)$$

When analysing the equation (8) it was assumed that $\varepsilon = 1$, as the number of defects can only be a whole number. For analysis and interpretation (14) it is necessary to assess the possible value ε in this equation. For this purpose, let's refer to the system (1) and define the intensity of both flows with $t = 0$ for initial conditions $f_1(0)=F_0$ and $f_2(0)=0$

$$\left|\frac{df_1}{dt}\right| = A \cdot F_0 ; \ \left|\frac{df_2}{dt}\right| = A_2 \cdot F_0 .$$

Therefore, with the absolute error not more than $1/F_0$, we can take $\varepsilon = A_2 - A_1$.. With such meaning ε (14) can be rewritten as:

$$t'' = \frac{\ln F_0}{A_1 + A_2} . \qquad (15)$$

Comparing (13) and (15) we can state that $t'' = t'$ i.e. both conditions of internal equilibrium are achieved simultaneously. We can come to the same conclusion, analyzing the case $A_1 > A_2$..

Theorems 1 and 2 state that the inevitability of appearance of internal equilibrium is achieved simultaneously. It gives grounds to formulate the law of flows equilibrium in the software systems.

Law of flows equilibrium. In any software system input and output flows always achieve the internal equilibrium. The time of achievement the internal equilibrium is directly proportional to the initial number of defects and inversely related to sum of influence coefficient.

Thus, to summarize the obtained results, we can state that the condition of internal equilibrium is inherent to all software systems and is stable. Such conclusions of SS dynamics theory are new and require the detailed study and experimental validation.

4 EXTERNAL EQUILIBRUM AND STABILITY OF SOFTWARE SYSTEM

The phase trajectories of SS contain important information about their asymptotic conditions, on the basis of which we can deduce the concept of external equilibrium.

Definition 4. As external equilibrium position of SS (stationary points) we will take such points of phase space $u^*=\langle f_1^*, f_2^*\rangle$, that:

$$\begin{cases} A_1 \cdot f_1^* + A_2 \cdot f_2^* = 0 \\ A_2 \cdot f_1^* + A_1 \cdot f_1^* = 0 \end{cases}.$$

It is evident that u^* is the system (1) solution with

$$\frac{du^*}{dt} = 0 .$$

Definition 5. The external equilibrium position of SS can be defined as stable, if for any $\varepsilon > 0$ there exist such $\delta > 0$, that for any u_0 , $|u_0 - u^*| < \partial$ the inequality $|u(t,u_0) - u^*| < \delta$ is satisfied with all $t > 0$.

Definition 6. The external equilibrium position of SS can be defined as asymptotic stable if during fulfillment of conditions of the definition 5, the condition $|u(t,u_0) - u^*| \to 0$ is additionally satisfied with all $t \to \infty$.

Let's consider the external equilibrium position of SS, which dynamics is defined by the equation (1). The software system comes into the state of external equilibrium with its surrounding medium (object domain), when the defect flows stop varying in time. It's noteworthy, that the equivalence to zero of *only speed of flow changes* is mentioned here. The values f_1 and f_2 themselves in the general case can be other than zero. Only in the particular case, when $k < 1$, we obtain at the limit $f_1 = 0$ and $f_2 = 0$.

In the theory of dynamic systems the so-called stationary (exceptional points) correspond the external equilibrium. Therefore the terms "external equilibrium point" and "stationary point" will be further used as synonyms.

The conditions of appearance in the external equilibrium system are formulated in theorem 4.

Theorem 4. (The fourth theorem of equilibrium). Provided that $A_1 > A_2$, the software system acquires stable asymptotic state of external equilibrium.

Proof of the theorem 4. From the equations (1) for external equilibrium state let's note down:

$$\begin{cases} -A_1 \cdot f_1 - A_2 \cdot f_2 = 0 \\ -A_2 \cdot f_1 - A_1 \cdot f_2 = 0 \end{cases}. \quad (16)$$

With $A_1 > A_2$, determinant of this system is $det\ A = A_1^2 - A_2^2 > 0$. Thus the system (16) can have only one solution $f_1 = f_2 = 0$, which corresponds the equilibrium state.

Let's prove the asymptotic reliability of this solution. To do this we consider the matrix of the system (16)

$$\|A\| = \begin{Vmatrix} -A_1 & -A_2 \\ -A_2 & -A_1 \end{Vmatrix}$$

and find its eigenvalue from the correlation:

$$\lambda^2 + 2 \cdot A_1 \cdot \lambda + A_1^2 - A_2^2 = 0 .$$

Solving this equation we obtain eigenvalue

$$\lambda_1 = -A_1 + A_2 , \; \lambda_2 = -A_1 - A_2 .$$

According to Lyapunov theorem (Samoilenko 1989) a stationary point is asymptotically stable, if all eigenvalues have negative sign of real part. Root of λ_2 will be negative with any values of A_1 and A_2. The negative value of λ_1 is possible only if $A_1 > A_2$.

Theorem 4 is proved.

Theorem 5. (The fifth theorem of equilibrium). Provided that $A_1 = A_2$ the software system acquires the stable external equilibrium.

Proof of the theorem 5. With $A_1 = A_2$ determinant of the system (16) is $det\ A = A_1^2 - A_2^2 > 0$, thus this system can have infinite number of solutions. But taking into consideration the physical meaning, with equality A_1 and A_2, we obtain the solution to each equation only with $f_1 = -f_2$.

Theorem 5 is proved.

The minus sign in front of f_2, as we have seen, denotes the reverse direction of flows. At the same time, the total number of defects, contained in SS, as it was shown in (Maevsky at al. 2012), is always equal to $f_1 + f_2$. This fact allows to define the position of external equilibrium point with $A_1 = A_2$. Consider SS with $t = 0$. At this point in time $f_1 + f_2 = F_0$, whence it follows that $f_1 = |f_2| = F_0 / 2$. Thus, the external equilibrium with $A_1 = A_2$ is achieved at the level of half of initial number of defects in the software system. The state of equilibrium in this case is stable.

It is interesting to research the SS reliability with $A_1 < A_2$.With such correlation of influence coefficients the determinant of matrix system (16) $det\ A = A_1^2 - A_2^2 < 0$, that states for the existence of one equilibrium position. Eigenvalues of matrix $\|A\|$, in this case $\lambda_1 > 0$, $\lambda_2 < 0$ according to (Samoilenko 1989), corresponds to the unstable point of "saddle" type.

The possible types of software system equilibrium positions are presented in the table 1.

Table 1. Possible types of equilibrium positions of a software system.

Correlation A_1 and A_2	Position type
More defects are removed than inserted $A_1 > A_2$; $\lambda_1 < 0, \lambda_2 < 0$	Stable node
More defects are inserted than removed $A_1 < A_2$; $\lambda_1 > 0, \lambda_2 < 0$	Saddle
The same number of defects is removed and inserted $A_1 = A_2$; $\lambda_1 = 0, \lambda_2 < 0$	Stable node

5 CONCLUSIONS

In the article the SS phase plane is researched and the questions of reliability are considered. From the phase trajectories behaviour it was concluded about the existence of internal equilibrium in SS, when the same number of defects is removed and inserted and the speeds of their changes within the time are the same. The law of flows equilibrium is formulated. The possibility of existence of internal equilibrium state and necessity for its achievement by the system are proved by a number of theorems.

One of such regularities is formulated in the law of flow equilibrium. The other very important regularity can be seen in the SS phase plane (1). This regularity becomes apparent in the increase of the number of secondary defects in the system. Indeed, it results from Figure 1, that with $A_1 > A_2$, even with steady decrease of defects number of the outgoing flow, the number of secondary defects in the system increases attains a maximum and only then, starts to decrease. The SS testers should consider this fact.

The mentioned dependences allow to predict the time interval, where there is an increased risk of inserting the secondary defects, and to take appropriate measures for their reduction.

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ESTIMATION IN CONSTANT STRESS PARTIALLY ACCELERATED LIFE TESTS FOR RAYLEIGH DISTRIBUTION USING TYPE-I CENSORING

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ABSTRACT

Partially Accelerated life tests are used when the data obtained from Accelerated life tests cannot be extrapolated to use conditions. This study deals with simple Constant Stress Partially Accelerated life tests using type-I censoring. The lifetime distribution of the test item is assumed to follow Rayleigh distribution. The maximum likelihood estimates are obtained for the distribution parameter and acceleration factor. In addition, asymptotic variance and covariance matrix of the estimators are given. Interval estimation that generates narrow intervals to the parameters of the distribution with high probability is obtained. Simulation procedure is used to illustrate the statistical properties of the parameters and the confidence bounds.

Key words: Acceleration factor; Maximum likelihood estimation; Reliability function; constant stress; Fisher Information matrix; generalized asymptotic variance; optimum test plans; time censoring

1. INTRODUCTION

The continuous improvement in manufacturing design creates a problem in obtaining information about lifetime of some products and materials with high reliability at the time of testing under normal conditions. Under such conditions the life testing becomes very expensive and time consuming. To obtain failures quickly, a sample of these materials is tested at more severe operating conditions than normal ones. These conditions are referred to as stresses, which may be in the form of temperature, voltage, force, humidity, pressure, vibrations, etc. This type of testing is called accelerated life testing (ALT), where products are run at higher-than-usual stress conditions, to induce early failures in a short time. The life data from the high stresses are used to estimate the life distribution at design condition, see Abdel-Hamid et al., (2009).

ALT is of mainly three types. The first is the constant stress ALT. It is used when the stress remain unchanged, that is, if the stress is weak, the stress has to run for a long time. The second is referred to as step-stress accelerated life test (SSALT) and the third is progressive-stress ALT. These three methods can reduce the testing time and save a lot of manpower, material and money, see Rao (1992). The main assumption in ALT is that the mathematical model relating the lifetime of the unit and the stress is known or can be assumed. In some cases, such life stress relationships are not known and cannot be assumed, i.e., the data obtained from ALT cannot be extrapolated to use condition. So, in such cases, another approach can be used, which is partially accelerated life tests (PALT). In PALT, test units are run at both usual and higher-than usual stress conditions see Abd-Elfattah et al., (2008).

The stress loading in a PALT can be applied in various ways. They include step-stress, constant-stress and random-stress. Nelson (1990) discussed their advantages and disadvantages. One way to accelerate failures is step-stress which increases the stress applied to test products in a specified discrete sequence. Generally, a test unit starts at a certain low stress. If the unit does not fail at a particular time, stress on it raised and held to a specified time. Stress is repeatedly increased until test unit fails or censoring time is reached. But in a constant-stress PALT each test item is run at either normal use condition or at accelerated condition only, i.e., each unit is run at a constant-stress level until the test is terminated.

For an overview of constant-stress PALT, there is amount of literature on designing PALT. Bai and Chung, 1992 discussed both, the problem of estimation and optimally designing PALT for test item having an exponential distribution. They also considered the problem of optimally designing constant-stress PALT that terminates at a pre-determined time. For items having lognormally distributed lives, PALT plans were developed by Bai et al., 1993a. Abdel-Ghaly et al. (2003) discussed the problem of parameter estimation for Pareto distribution using PALT in case of type-I censoring. More recently, Ismail (2004) used the maximum likelihood approach for estimating the acceleration factor and the parameters of Pareto distribution of the second kind. This work was conducted under constant-stress PALT in the case of type-I and type-II censored data. Also, the problem of optimal design was considered for this type of PALT. Since Abdel-Ghani (1998) considered only the estimation problem in constant-stress PALT for the Weibull distribution parameters, the present investigation extends this work in which statistically optimal PALT plan is developed under type-I censoring. This article is to focus on the maximum likelihood method for estimating the acceleration factor and the parameters of Rayleigh distribution. This work was conducted for constant-stress PALT under type I censored sample. The confidence intervals of the estimators will be obtained.

2. THE MODEL

The lifetimes of the test items are assumed to follow a Rayleigh distribution. The probability density function (pdf) of the Rayleigh distribution is given by

$$f(t)=\frac{t}{\theta^2}\exp\left(-\frac{t^2}{2\theta^2}\right), \qquad 0\le t<\infty,\ \theta>0 \tag{1}$$

And the cumulative distribution function (cdf) is given by

$$F(t)=1-\exp\left(-\frac{t^2}{2\theta^2}\right), \qquad 0\le t<\infty,\ \theta>0 \tag{2}$$

where, θ is the scale parameter.

The reliability function of the Rayleigh distribution takes the form

$$R(t)=\exp\left(-\frac{t^2}{2\theta^2}\right), \tag{3}$$

and the corresponding hazard rate is given by

$$h(t)=\frac{t}{\theta^2};$$

The Rayleigh distribution has played an important role in modeling the lifetime of random phenomena. It arises in many areas of applications, including reliability, life testing and survival analysis. Rayleigh distribution is a special case of Weibull distribution (shape parameter=2). Rayleigh distribution is frequently used to model wave heights in oceanography, and in communication theory to describe hourly median and instantaneous peak power of received radio signals. It has been used to model the frequency of different wind speeds over a year and a wind turbine sites. The distance from one individual to it nearest neighbor when the spatial pattern is generated by Poisson distribution follows a Rayleigh distribution. In communication theory, Rayleigh distribution is used to model scattered signals that reach a receiver by multiple paths. Depending on the density of scatter, the signal will display different fading characteristics. Rayleigh distribution is used to model dense scatter.

In a constant-stress PALT, all of the n items are divided into two parts. nr items are randomly chosen among n items, which are allocated to accelerated conditions and the remaining $n(1-r)$ are allocated to normal use conditions, where r is proportion of sample units allocated to accelerated condition then each test item is run until the censoring time τ and the test condition is not changed. Some assumptions are also made in a constant-stress PALT.

- The lifetimes $T_i, i = 1,\ldots,n(1-r)$ and $X_j, i = 1,\ldots,nr$, of items allocated to normal and accelerated conditions, respectively, are i.i.d. random variables.
- The lifetimes T_i and X_j are mutually statistically independent.

3. MAXIMUM LIKELIHOOD ESTIMATION

The maximum likelihood estimation (MLE) is one of the most important and widely used methods in statistics. It is commonly used for the most theoretical model and kinds of censored data. The idea behind the maximum likelihood parameter estimation is to determine the estimates of the parameter that maximizes the likelihood of the sample data. Also the MLEs have the desirable properties of being consistent and asymptotically normal for large samples.

In a simple constant stress PALT, the test item is run at use condition or at accelerated condition only. A simple constant stress plan uses only two stresses and allocates the n sample units to them; see Miller and Nelson, 1983.
In this study, the lifetimes of test items are assumed to follow a Rayleigh distribution.
The pdf of an item tested at use condition is given by

$$f(t) = \frac{t}{\theta^2}\exp\left(-\frac{t^2}{2\theta^2}\right), \qquad t \geq 0$$

and for an item tested at accelerated condition, the pdf is given by

$$f(x) = \frac{\beta^2 x}{\theta^2}\exp\left(-\frac{(\beta x)^2}{2\theta^2}\right), \quad x \geq 0$$

where $X = \beta^{-1}T$, β is the acceleration factor which is the ratio of mean life at use condition to that at accelerated condition, usually $\beta > 1$.

Let δ_{ui} and δ_{aj} be the indicator functions, such that

$$\delta_{ui} = \begin{cases} 1 & t_i \leq \tau \\ 0 & otherwise \end{cases} \qquad i = 1,2,\ldots,n(1-r)$$

and

$$\delta_{aj} = \begin{cases} 1 & x_j \leq \tau \\ 0 & otherwise \end{cases} \qquad j = 1,2,\ldots,nr$$

The likelihood functions for (t_i, δ_{ui}) and (x_j, δ_{aj}) are respectively given by

$$L_{ui}(t_i, \delta_{ui} \mid \theta) = \prod_{i=1}^{n(1-r)} \left\{ \frac{t_i}{\theta^2} \exp\left(-\frac{t_i^{\,2}}{2\theta^2} \right) \right\}^{\delta_{ui}} \left\{ \exp\left(-\frac{\tau^2}{2\theta^2} \right) \right\}^{\bar{\delta}_{ui}} \tag{4}$$

$$L_{aj}(x_j, \delta_{aj} \mid \beta, \theta) = \prod_{j=1}^{nr} \left\{ \frac{\beta^2 x_j}{\theta^2} \exp\left(-\frac{(\beta x_j)^2}{2\theta^2} \right) \right\}^{\delta_{aj}} \left\{ \exp\left(-\frac{(\beta \tau)^2}{2\theta^2} \right) \right\}^{\bar{\delta}_{aj}} \tag{5}$$

where, $\bar{\delta}_{ui} = 1 - \delta_{ui}$ and

$\bar{\delta}_{aj} = 1 - \delta_{aj}$.

The total likelihood function for
$(t_1; \delta_{u1}, \ldots, t_{n(1-r)}; \delta_{u\,n(1-r)}, x_1; \delta_{a1}, \ldots, x_{nr}; \delta_{a\,nr})$ is given by

$$L(t, x \mid \beta, \theta) = \prod_{i=1}^{n(1-r)} \left\{ \frac{t_i}{\theta^2} \exp\left(-\frac{t_i^{\,2}}{2\theta^2} \right) \right\}^{\delta_{ui}} \left\{ \exp\left(-\frac{\tau^2}{2\theta^2} \right) \right\}^{\bar{\delta}_{ui}}$$
$$\prod_{j=1}^{nr} \left\{ \frac{\beta^2 x_j}{\theta^2} \exp\left(-\frac{(\beta x_j)^2}{2\theta^2} \right) \right\}^{\delta_{aj}} \left\{ \exp\left(-\frac{(\beta \tau)^2}{2\theta^2} \right) \right\}^{\bar{\delta}_{aj}} \tag{6}$$

It is usually easier to maximize the natural logarithm of the likelihood function rather than the likelihood function itself. Therefore, the logarithm of (6) is

$$\ln L = \sum_{i=1}^{n(1-r)} \delta_{ui} \left[\ln t_i - 2\ln\theta - \frac{t_i^2}{2\theta^2} \right] - \frac{\tau^2}{2\theta^2} \sum_{i=1}^{n(1-r)} (1 - \delta_{ui})$$
$$+ \sum_{j=1}^{nr} \delta_{aj} \left[\ln x_j + 2\ln\beta - 2\ln\theta - \frac{\beta^2 x_j^2}{2\theta^2} \right] - \frac{\beta^2 \tau^2}{2\theta^2} \sum_{j=1}^{nr} (1 - \delta_{aj}) \tag{7}$$

MLEs of β and θ are solutions to the system of equations obtained by letting the first partial derivatives of the total log likelihood be zero with respect to β and θ, respectively. Therefore, the system of equations is as follows:

$$\frac{\partial \ln L}{\partial \theta} = -\frac{2}{\theta}(n_u - n_a) + \frac{1}{\theta^3} \sum_{i=1}^{n(1-r)} \delta_{ui}\, t_i^2 + \frac{\tau^2}{\theta^3}\{n(1-r) - n_u\} + \frac{\beta^2}{\theta^3} \sum_{j=1}^{r} \delta_{aj} x_j^2$$
$$+ \frac{\beta^2 \tau^2}{\theta^3}(nr - n_a) = 0 \tag{8}$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{2n_a}{\beta} - \frac{\beta}{\theta^2} \sum_{j=1}^{nr} \delta_{aj} x_j^2 - \frac{\beta \tau^2}{\theta^2}(nr - n_a) = 0 \tag{9}$$

where n_u and n_a are the number of items failed at normal and accelerated conditions respectively.

From (8) and (9), the MLEs of β and θ can be expressed as

$$\tilde{\theta} = \left[\frac{\sum_{i=1}^{n(1-r)} \delta_{ui} t_i^2 + \tau^2 \{n(1-r) - n_u\} + \beta^2 \sum_{j=1}^{nr} \delta_{aj} x_j^2 + \beta^2 \tau^2 (nr - n_a)}{2(n_u - n_a)} \right]^{\frac{1}{2}} \tag{10}$$

and,

$$\tilde{\beta} = \left[\frac{2n_a}{\frac{1}{\theta^2} \sum_{j=1}^{nr} \delta_{aj} x_j^2 + \frac{\tau^2}{\theta}(nr - n_a)} \right]^{\frac{1}{2}} \tag{11}$$

From (11), $\tilde{\beta}$ can be calculated easily, and once the value of $\tilde{\beta}$ is obtained then $\tilde{\theta}$ can also be estimated by substituting the value of $\tilde{\beta}$ in (10). The asymptotic variance-covariance matrix of β and θ is obtained by numerically inverting the Fisher-information matrix composed of the negative second derivatives of the natural logarithm of the likelihood function evaluated at the ML estimates. The asymptotic Fisher-information matrix can be written as:

$$F = \begin{bmatrix} -\frac{\partial^2 \ln L}{\partial \theta^2} & -\frac{\partial^2 \ln L}{\partial \beta \partial \theta} \\ -\frac{\partial^2 \ln L}{\partial \theta \partial \beta} & -\frac{\partial^2 \ln L}{\partial \beta^2} \end{bmatrix}$$

The elements of the above information matrix can be expressed by the following equations:

$$\frac{\partial^2 \ln L}{\partial \theta^2} = \frac{2}{\theta^2}(n_u - n_a) - \frac{1}{\theta^4}\left[\sum_{i=1}^{n(1-r)} \delta_{ui} t_i^2 + \tau^2 \{n(1-r) - n_u\} + \beta^2 \sum_{j=1}^{nr} \delta_{aj} x_j^2 + \beta^2 \tau^2 (nr - n_a) \right]$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -\frac{2n_a}{\beta^2} - \frac{1}{\theta^2} \sum_{j=1}^{nr} \delta_{aj} x_j^2 - \frac{\tau^2}{\theta^2}(nr - n_a)$$

$$\frac{\partial^2 \ln L}{\partial \theta \partial \beta} = \frac{2\beta}{\theta^3}\left[\sum_{j=1}^{nr} \delta_{aj} x_j^2 + \tau^2 (nr - n_a) \right]$$

$$\frac{\partial^2 \ln L}{\partial \beta \partial \theta} = \frac{2\beta}{\theta^3}\left[\sum_{j=1}^{nr} \delta_{aj} x_j^2 + \tau^2 (nr - n_a) \right]$$

Consequently, the maximum likelihood estimators of β and θ have an asymptotic variance-covariance matrix defined by inverting the Fisher information matrix given above.

4. INTERVAL ESTIMATES

If $L_\lambda = L_\lambda(y_1,\ldots,y_n)$ and $U_\lambda = U_\lambda(y_1,\ldots,y_n)$ are functions of the sample data $y_1,\ldots,y_n$, then a confidence interval for a population parameter λ is given by

$$p[L_\lambda \le \lambda \le U_\lambda] = \gamma \tag{12}$$

where, L_λ and U_λ are the lower and upper confidence limits which enclose λ with probability γ. The interval $[L_\lambda, U_\lambda]$ is called a two sided $100\gamma\%$ confidence interval for λ.
For large sample size, the MLEs, under appropriate regularity conditions, are consistent and asymptotically normally distributed.

Therefore, the two sided approximate $100\gamma\%$ confidence limits for the MLE $\tilde{\lambda}$ of a population parameter λ can be constructed, such that

$$p[-z \le \frac{\tilde{\lambda}-\lambda}{\sigma(\tilde{\lambda})} \le z] = \gamma \tag{13}$$

where, z is the $\left[\frac{100(1-\gamma)}{2}\right]$ standard normal percentile. Therefore, the two sided approximate $100\gamma\%$ confidence limits for a population parameter λ can be obtained such that

$$p[\lambda - z\sigma(\tilde{\lambda}) \le \tilde{\lambda} \le \lambda + z\sigma(\tilde{\lambda})] \cong \gamma \tag{14}$$

Then, the two sided approximate confidence limits for β and θ will be constructed using (14) with confidence levels 95% and 99%.

5. OPTIMUM SIMPLE CONSTANT-STRESS TEST PLANS

This section deals with the problem of optimally designing a simple constant-stress PALT, which terminates at a pre-specified time. Optimum test plans for products having a Rayleigh distribution is developed. Here, the aim is to obtain the optimal proportion of sample units r^*, allocated to accelerated conditions based on the outputs of the stage of the parameter estimation that are in the same time considered inputs to the optimal design stage of the test. The proportion of sample units r, allocated to accelerated condition is pre-specified for the stage of parameter estimation. But for the optimal design stage of the test, r is considered a division parameter that has to be optimally determined according to a certain optimality criterion. The optimality criterion (Abdel-Ghaley et al., 2003) is to find the optimal proportion of sample units r^* allocated to accelerated condition such that the Generalized Asymptotic Variance (GAV) of the MLE of the model parameters at normal use condition is minimized.

Most of the test plans allocate the same number of test units at each stress i.e. they are equally-spaced test stresses. Such test plans are usually inefficient for estimating the mean life at design stress (Yang, 1994). To decide the optimal sample proportion allocated to each stress, statistically optimum test plans are developed. Therefore, to determine the optimal sample proportion r^* allocated to accelerated condition, r is chosen such that the GAV of the ML estimators of the model parameters is minimized. The GAV of the ML estimators of the model parameters as an optimality criterion is defined as the reciprocal of the determinant of the Fisher-Information matrix F (Bai et.al., 1993b). That is,

$$GAV(\tilde{\theta}, \tilde{\beta}) = \frac{1}{|F|} \tag{15}$$

The minimization of the GAV over τ solves the following equation

$$\frac{\partial GAV}{\partial r} = 0 \tag{16}$$

In general, the solution to (16) is not a closed form, so the Newton-Raphson method is applied to obtain r^* which minimizes the GAV. Accordingly, the corresponding expected number of items failed at use and accelerated conditions can be obtained, respectively, as follows

$$n_u^* = n(1 - r^*)P_u$$

and,

$$n_a^* = nr^* P_a$$

where,

P_u = Probability that an item tested only use condition fails by τ

P_a = Probability that an item tested only accelerated condition fails by τ

6. SIMULATION STUDIES

Abd-Elfattah et al., (2008), performed the simulation study for a two parameter Burr Type XII distribution using MathCAD (2001) for illustrating the theoretical results of estimation problem. The performance of the resulting estimators of the acceleration factor and two shape parameters has been considered in terms of their absolute relative bias (RABias), mean square error (MSE), and relative error (RE). Furthermore, the asymptotic variance and covariance matrix and two-sided confidence intervals of the acceleration factor and two shape parameters were obtained. Ismail and Aly (2009), considered the case of two stress levels under the failure-step stress partially accelerated life testing assuming type-II censoring for a two parameter Weibull distribution. The MLEs were studied together with some further properties. They also obtained an optimum Failure step-stress PALT numerically using the D-optimality via a simulation study. It was noted via the optimal value of proportion of test units to be observed at two stresses, that the PALT model is more appropriate model. That is, testing at both normal and accelerated conditions. Ismail (2009) performed the simulation study for the Weibull Failure distribution and results of simulation studies provide insight into the sampling behavior of the estimators. The numerical results indicated that the ML estimates approximate the true values of the parameters as the sample size increases and the asymptotic variances of the estimators are decreasing as the sample size is getting to be large.

Ismail (2011) considered the problem of optimally designing a simple time-step-stress PALT which terminates after a pre-specified number of failures and developed optimum test plans for products having a two-parameter Gompertz lifetime distribution. The main objective of his simulation study was to make a numerical investigation for illustrating the theoretical results of both estimation and optimal design problems under consideration.
The data in Table 1 and Table 2 gives the MSE, RBais, RE and variance of the estimators for two sets of parameters $(\theta = 4.60, \beta = 0.20)$ and $(\theta = 4.40, \beta = 0.80)$ respectively. While Table 3 and Table

4 presents the approximated two sided confidence limits at 95% and 99% level of significance for the scale parameter and the acceleration factor.

Table 1: The MSE, RBais, RE and Variances of the Parameters $(\theta = 4.60, \beta = 0.20)$ under Type-I Censoring

Size n	Parameters	MSE	RBias	RE	Variance
50	θ	0.0062	0.0009	0.0179	0.0062
	β	0.0306	0.0221	0.2187	0.0303
100	θ	0.1055	0.0006	0.0738	0.1055
	β	0.0047	0.0228	0.0857	0.0044
150	θ	0.0152	0.0014	0.0280	0.0152
	β	0.0024	0.0163	0.0612	0.0022
200	θ	0.0131	0.0016	0.0260	0.0131
	β	0.0209	0.0110	0.1807	0.0208
250	θ	0.0150	0.0033	0.0278	0.0148
	β	0.0028	0.0048	0.0661	0.0028
300	θ	0.0169	0.0191	0.0295	0.0099
	β	0.0016	0.0090	0.0500	0.0015
350	θ	0.0118	0.0204	0.0245	0.0037
	β	0.0197	0.0120	0.1754	0.0196
400	θ	0.0181	0.0219	0.0306	0.0088
	β	0.0029	0.0106	0.0673	0.0028
450s	θ	0.0108	0.0065	0.0236	0.0100
	β	0.0014	0.0080	0.0468	0.0014

Table 2: The MSE, RBais, RE and Variances of the Parameters $(\theta = 4.40, \beta = 0.80)$ under Type-I Censoring

Size n	Parameters	MSE	RBias	RE	Variance
50	θ	0.0227	0.0053	0.0328	0.0221
	β	0.0002	0.0165	0.0707	0.0002
100	θ	0.0399	0.0120	0.0434	0.0369
	β	0.0004	0.0275	0.1000	0.0004
150	θ	0.0914	0.0001	0.0657	0.0914
	β	0.0020	0.0040	0.2236	0.0020
200	θ	0.0162	0.0070	0.0277	0.0152
	β	0.0002	0.0360	0.0707	0.0001
250	θ	0.0266	0.0039	0.0355	0.0263
	β	0.0003	0.0380	0.0066	0.0002
300	θ	0.0698	0.0014	0.0574	0.0698
	β	0.0015	0.0010	0.1973	0.0015
350	θ	0.0107	0.0058	0.0225	0.0100
	β	0.0001	0.0095	0.0500	0.0001

Size n	Parameters	MSE	RBias	RE	Variance
400	θ	0.0014	0.0032	0.0081	0.0012
	β	0.0002	0.0140	0.0707	0.0002
450	θ	0.0166	0.0034	0.0280	0.0163
	β	0.0002	0.0540	0.0707	0.0001

Table 3: Confidence Bounds of the Estimates at Confidence Level at 0.95 and 0.99 $(\theta = 4.60, \beta = 0.20)$

n	Parameters	95%		99%	
		LCL	UCL	LCL	UCL
50	θ	4.2842	4.8669	4.3318	4.8194
	β	0.1689	0.2244	0.1735	0.2199
100	θ	4.2789	5.0319	4.3404	4.9704
	β	0.1663	0.2447	0.1727	0.2383
150	θ	4.0079	5.1931	4.1047	5.0963
	β	0.1115	0.2869	0.1259	0.2725
200	θ	4.3263	4.8094	4.3656	4.7617
	β	0.1876	0.2268	0.1908	0.2236
250	θ	4.3003	4.9361	4.3522	4.8842
	β	0.1647	0.2201	0.1191	0.2657
300	θ	4.0759	5.1115	4.1604	5.0269
	β	0.1239	0.2757	0.1363	0.2633
350	θ	4.3774	4.7694	4.4094	4.7374
	β	0.1979	0.1983	0.1817	0.2144
400	θ	4.5174	4.6532	4.5285	4.6421
	β	0.1695	0.2249	0.1740	0.2204
450	θ	4.3655	4.8659	4.4063	4.8251
	β	0.1676	0.2068	0.1708	0.2036

Table 4: Confidence Bounds of the Estimates at Confidence Level at 0.95 and 0.99 $(\theta = 4.40, \beta = 0.80)$

n	Parameters	95%		99%	
		LCL	UCL	LCL	UCL
50	θ	4.2498	4.5584	4.2749	4.5332
	β	0.4411	1.1235	0.4968	1.0678
100	θ	3.7659	5.0391	3.8698	4.9352
	β	0.6882	0.9482	0.7094	0.9269
150	θ	3.6299	5.1581	4.1918	4.5962
	β	0.6951	0.8789	0.7101	0.8639
200	θ	4.1749	4.6237	4.2116	4.5870
	β	0.5261	1.0915	0.5723	1.0453

n	Parameters	95%		99%	
		LCL	UCL	LCL	UCL
250	θ	4.1472	4.6240	4.1861	4.5851
	β	0.7000	0.9075	0.7170	0.8906
300	θ	4.1212	4.5112	4.1530	4.4794
	β	0.7169	0.8687	0.7293	0.8563
350	θ	4.1909	4.4294	4.2104	4.4099
	β	0.5352	1.0840	0.5800	1.0392
400	θ	4.1199	4.4877	4.1499	4.4576
	β	0.6878	0.8952	0.7047	0.8783
450	θ	4.1755	4.5675	4.2075	4.5355
	β	0.7203	0.8669	0.7322	0.8549

From these tables it is concluded that for the first set of parameters $(\theta = 4.60, \beta = 0.20)$, the ML estimates have good statistical properties than the second set off parameters $(\theta = 4.40, \beta = 0.80)$ for all sample sizes. Also as the acceleration factor increases the estimates have smaller MSE and RE. As the sample size increases the RBais and MSEs of the estimates parameters decreases. This indicates that the ML estimates provide asymptotically normally distributed and consistent estimators for the scale parameter and the acceleration factor.

When the sample size increases, the interval of the estimators decreases. Also the intervals of the estimators at $\gamma = 0.95$ is smaller than the interval of estimators at $\gamma = 0.99$.

The results of this simulation study suggests that PALT is a suitable model which enables to save time and money considerably without using a high stress to all test units. The optimal test plans improves the quality of the inference and increase the level of precision in parameter estimation. So, statistically, optimum plans are needed, and the experimenters are advised to use them for estimating the life distribution at design stress. These plans can also serve as a criterion for comparison with other designs. Further work can be extended by applying the results to other distributions and other censored schemes such as progressive censoring.

7. CONCLUSIONS

For products having high reliability, ALT or PALT results in shorter lives than would be observed under normal operating conditions. In ALT products are tested at higher then usual level of stress to induce early failures. In PALT, test units are run at both usual and higher-than usual stress conditions. In a constant stress PALT each test item is run at either normal use condition or at accelerated condition only.

This study has presented a constant stress PALT for Rayleigh distribution using type-I censoring. The MSE, RBais and RE of the estimators are obtained for two sets of parameters. For the first set of parameters, the ML estimates have good statistical properties. Regarding the confidence interval of estimators, it can be observed that the interval of the estimates at $\gamma = 0.99$ is greater than the corresponding interval at $\gamma = 0.95$.

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QUANTITATIVE ESTIMATION OF INDIVIDUAL RELIABILITY OF THE EQUIPMENT AND DEVICES OF THE POWER SUPPLY SYSTEM

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ABSTRACT

The basic stages of a design procedure of parameters of individual reliability the equipment and devices of electro power systems are considered. The recommended method illustrated on an example of parameters of reliability calculated as average arithmetic random variables. The method based on imitating modeling of random variables and the theory of check of statistical hypotheses.

1. Statement of a problem and some definitions

The objective estimation of parameters of reliability (PR) the equipment and devices of electro power systems (EPS) always was and remains to one of priority problems which decision directed on decrease in expenses at designing and operation of electro installations [1]. In addition, despite of urgency of this problem, calculations PR, traditionally, spent for assumptions rather far from the validity. The basic assumption is the opportunity of representation of statistical data of operation by representative sample of general set of these data, i.e. these data represented homogeneous. Calculated PR thus carries, naturally, average character. At the same time dependence PR on those or other factors as a class of a voltage, type of the equipment, duration and conditions of operation, the system of service and so forth, that already contradicts this assumption is marked.

Set of the statistical data describing reliability of the equipment, actually represents so-called final set of multivariate data (MD) [2]. MD essentially differs from sample of general set. First, MD are set not only set of the random variables describing reliability of objects of research, but also set of versions of attributes (VA), describing each random variable. Practically, these data formed in the so-called empirical table which lines allocate objects, and columns: a serial number of objects, the attributes describing object, realizations of random variables and casual events. The set of objects is limited to frameworks of a solved problem and shown in set VA. But distinction not only in it.

It is known, that about reduction of number of random variables of sample of general set accuracy of estimations PR decreases (width of a confidential interval increases) [2]. At classification MD on set significant VA, decrease in number of realizations of a random variable in sample accompanied by decrease in disorder of possible values of a random variable, i.e. accuracy of estimations PR increases.

As an example, consider duration of restoration of deterioration at emergency repair (τ_{em}) witches of a power supply system. It is known, that with increase in a class of a voltage of switches average value of a random variable τ_{em} also increases. Thus, if values τ_{em} for air switches in final set of multivariate data change in an interval (6-105) hr., for air switches with nominal voltage (15-20)кV this interval appears essentially less and is equal (6-25) hr.

The account of these features allows passing from calculation of average values PR to calculation of parameters of individual reliability, i.e. PR for set VA. It is necessary to have in view of, that parameters of individual reliability, in fact, also are average. However, averaging here spent on "rustling" VA. Difficulty of an estimation of parameters of individual reliability in many respects caused by necessity of ranging set VA on their importance.

The problem of an estimation of parameters of individual reliability of the equipment from the methodical point of view is a special case of a problem of classification of park of objects on groups for which calculated PR differs not casually.

Significant interest at the decision of some operational problems caused with laws of change of parameters of individual reliability in function VA. The scale of these attributes can be not only in a quantitative kind (for example, service life, an interval of time after scheduled repair, etc.), but in serial (for example, a class of a voltage, capacity, etc.) and in nominal (for example, units of object, its importance, etc.). Difficulty of the decision of noted problems increases also because at the constant approach algorithms of estimation various PR are various.

2. Algorithm of comparison statistical functions of distribution final set MD and not casual sample of these MD.

Classification of statistical data on set VA, first, assumes an opportunity of an estimation of its expediency. One of ways of the characteristic of expediency of classification of data is the estimation of character of a divergence of statistical functions of distribution (s.f.d.) final set MD and sample of these MD on set VA. The approach to such comparison we shall consider on example PR, calculated as an average arithmetic X. Let us specify initial data:

- in the empirical table some final set MD of a random variable of X. Numerical value X is set depends from «n» considered attributes. Each of «n» attributes is presented to one of r_i VA with i=1,n.
  $F^*_\Sigma(X)$ - s.f.d., and $M^*_\Sigma(X)$ - average value of final set MD (index Σ carries parameters and characteristics of reliability to final set MD);
- certain combination VA sets object, PR that in the form of estimation $M^*_V(X)$ should estimated. Directly to estimate $M^*_V(X)$ it is impossible, since data about X at this object practically are absent. And without taking into account these VA, about any individuality to speak it is not necessary;
- not casual sample of values of random variable X, as result of classification MD on one VA is set. S.f.d. this sample we shall designate, as $F^*_V(X)$. To compare $F^*_\Sigma(X)$ and $F^*_V(X)$, we spend following sequence of calculations:

2.1. We count the greatest empirical deviation Δ.

For this purpose:

- for each value X_j from set $\{X\}_V$ samples it is defined absolute size of a deviation s.f.d. $F^*_\Sigma(X)$ from s.f.d. $F^*_V(X)$ under the formula

$$\Delta(X_j) = \left|F^*_\Sigma(X_j) - F^*_V(X_j)\right| \tag{1}$$

  with j=1,m, where m- number of realizations of random variable X in sample;
- define the greatest value among m realizations Δ(X) under the formula

$$\Delta_E = \max\left\{\Delta(X_1); \Delta(X_2); \ldots; \Delta(X_j); \ldots \Delta(X_m)\right\} \tag{2}$$

As distributions $F^*_\Sigma(X)$ and $F^*_V(X)$ constructed on statistically given operation, size Δ_E there is the greatest empirical deviation.

It is necessary to note, as statistics of criterion of a divergence can be chosen not only size Δ, but also average value of the greatest deviation Δ_{AV}, average quadratic value Δ_{AQ}, average geometrical value and a number of others. However, as shown in [4], statistics Δ has at the fixed value of a error of I type, the greatest capacity of criterion.

2.2. Modeling of distribution $F^*[\Delta(H_1)]$

S.f.d. $F^*[\Delta(H_1)] = P[\Delta < \Delta(H_1)]$ - distribution of realization of absolute size of the greatest deviation of modeled realizations s.f.d. $F_V^*(X)$ from s.f.d. $F_\Sigma^*(X)$ for assumption H_1 (divergences of realizations s.f.d. $F_\Sigma^*(X)$ also $F_V^*(X)$ has casual character)

Modeling $F^*[\Delta(H_1)]$ spent in following sequence:

- on distribution, $F_\Sigma^*(X)$ it is modeled m random variables X. According to [3] calculation of realization of random variable X it is carried out under the formula

$$X = X_i + (X_{i+1} - X_i)[(n+1)\xi - i] \quad (3)$$

  where ξ- proggrammatic a modeled pseudo-random variable with uniform distribution in an interval [0,1]. Let's designate this set of values X as $\{X\}_V^*$
- according to $\{X\}_V^*$ is under construction s.f.d. $F_V^{**}(X)$;
- transformation of final set MD is spent. For this purpose:
  - from set of values X of final set MD are withdrawn m the values describing $\{X\}_V$;
  - instead of $\{X\}_V$ values $\{X\}_V^*$ are entered;
  - pays off s.f.d. on transformed final set MD. Designate it is $F_\Sigma^{**}(X)$;
- for realizations of random variables X_j with j=1,m samples are calculated absolute deviations s.f.d. $F_\Sigma^{**}(X_j)$ and s.f.d. $F_V^{**}(X_j)$ under the formula:

$$\Delta(X_j) = \left|F_\Sigma^{**}(X_j) - F_V^{**}(X_j)\right|; \quad (4)$$

- the greatest deviation s.f.d. defined $F_\Sigma^{**}(X)$ from s.f.d. $F_V^{**}(X)$ under the formula

$$\Delta(H_1) = \max\{\Delta(X_1); \Delta(X_2); ...; \Delta(X_j); ...\Delta(X_m)\}; \quad (5)$$

- it is modeled N realizations of a random variable $\Delta(H_1)$;
- N realizations $\Delta(H_1)$ placed in ascending order. Further to each value $\Delta(H_1)$ the probability $F^*[\Delta_i(H_1)] = i/N$, where i–serial number of realizations of set of values is compared $\Delta(H_1)$. Calculations $F^*[\Delta(H_1)]$ come to the end with that.

2.3. Modeling of distribution $F^*[\Delta(H_2)]$.

S.f.d. $F^*[\Delta(H_2)] = P[\Delta < \Delta(H_2)]$- distribution of realizations of absolute size of deviations s.f.d. $F_V^*(X)$ from s.f.d. $F_\Sigma^*(X)$ for assumption H_2 (the divergence $F_\Sigma^*(X)$ and $F_V^*(X)$ is not casual). The algorithm of modeling $F^*[\Delta(H_2)]$ is similar to algorithm of modeling of distribution $F^*[\Delta(H_1)]$ with that essential difference, that modeling of sample from m values of random variable X is spent not on s.f.d. final set MD $F_\Sigma^*(X)$, and on s.f.d. $F_V^*(X)$.

2.4. Decision-making

To make a decision on character of a divergence $F_{\Sigma}^{*}(X)$ and $F_{V}^{*}(X)$, i.e. to choose one of two assumptions (H_1 or H_2) and by that to estimate expediency of classification of statistical data to the set attribute, it is necessary:

1. To define average value N of realizations $\Delta(H_1)$ under the formula
$$M^{*}[\Delta(H_1)] = \sum_{i=1}^{N} \Delta_i(H_1) \Big/ N$$
2. To define average value N of realizations $\Delta(H_2)$ under the formula
$$M^{*}[\Delta(H_2)] = \sum_{i=1}^{N} \Delta_i(H_2) \Big/ N$$
3. To construct s.f.d., describing error of I $\alpha^*(\Delta)$ and the II $\beta^*(\Delta)$ types
   3.1. If $M^{*}[\Delta(H_1)] < M^{*}[\Delta(H_2)]$, that
$$\alpha^{*}[\Delta(H_1)] = 1 - F^{*}[\Delta(H_1)] \qquad \beta^{*}[\Delta(H_2)] = F^{*}[\Delta(H_2)]$$
   3.2. If $M^{*}[\Delta(H_1)] > M^{*}[\Delta(H_2)]$, that
$$\alpha^{*}[\Delta(H_2)] = 1 - F^{*}[\Delta(H_2)] \qquad \beta^{*}[\Delta(H_1)] = F^{*}[\Delta(H_1)]$$
4. On s.f.d. $\alpha^*(\Delta)$ and $\beta^*(\Delta)$ to define critical values of absolute size of the greatest deviation Δ_{cr}.. Size $\Delta_{cr.}$ in practice are calculated for the set significance values $\alpha_{cr.}$ and $\beta_{cr.}$, usually accepted equal $\alpha_{cr.}=\beta_{cr}=0.05$ (0.1). As actually distributions $\alpha^*(\Delta)$ and $\beta^*(\Delta)$ have discrete character, and among discrete values s.f.d., as a rule, there are no probabilities α_{cr} and β_{cr}, equal 0,05 or 0,1, recommended to accept as an admissible error of I type the nearest to α_{cr} smaller value among set of discrete values s.f.d. $\alpha^*(\Delta)$, and as an admissible error of II type - the nearest to β_{cr}, smaller value among set of discrete values s.f.d. $\beta^*(\Delta)$. The valid boundary values of these errors at $M^{*}[\Delta(H_1)] < M^{*}[\Delta(H_2)]$ designate accordingly: for a error of I type - through sh1[$\Delta(H_1)$], and for a error of II type – through sh2[$\Delta(H_2)$]. Corresponding sh1[$\Delta(H_1)$] critical value of the greatest deviation will be Δ_{cr}[sh1(H_1)], and for sh2[$\Delta(H_2)$] - will be Δ_{cr} [sh2(H_2)]. If $M^{*}[\Delta(H_1)] >$ $M^{*}[\Delta(H_2)]$ boundary values of these errors accordingly will be sh1[$\Delta(H_2)$] and sh2[$\Delta(H_1)$]. Corresponding mistakes of the first and second sort sh1[$\Delta(H_2)$] and sh2 [$\Delta(H_1)$] critical values of the greatest deviation will be Δ_{cr}[sh1(H_2)] and Δ_{cr}[sh2(H_1)]
5. To compare with an empirical deviation Δ_E with critical values of mistakes of the first and second sort. Thus

5.1. If $M^{*}[\Delta(H_1)] < M^{*}[\Delta(H_2)]$ and $\Delta_E \geq \Delta_{cr}$[sh1($H_1$)], with a significance value sh1[$\Delta(H_1)$] assumption H_2 is accepted. If $M^{*}[\Delta(H_1)] >$ $M^{*}[\Delta(H_2)]$ and $\Delta_E \geq \Delta_{cr}$[sh2($H_2$)], with a significance value sh1[$\Delta(H_2)$] assumption H_1 is accepted

5.2. If $M^{*}[\Delta(H_1)] < M^{*}[\Delta(H_2)]$, and $\Delta_E < \Delta_{cr}$sh1(H_1)] and $\Delta_E \leq \Delta_{cr}$[sh2($H_2$)], with a significance value sh2[$\Delta(H_2)$] assumption H_1 is accepted. If $M^{*}[\Delta(H_1)] > M^{*}[\Delta(H_2)]$, and $\Delta_E < \Delta_{cr}$[sh1(H_2)] and $\Delta_E \leq \Delta_{cr}$[sh2 ($H_1$)], with a significance value sh2[$\Delta(H_1)$] assumption H_2 is accepted

The total risk of the erroneous decision pays off under the formula:
$$Ri(\Delta) = A \cdot F[\Delta(H_1)] + B \cdot F[\Delta(H_2)] = Ri[\Delta(H_1)] + Ri[\Delta(H_2)]$$
where A and B – factors of the importance of errors of I and II types; A+B=1. If the information on consequences of possible errors of I and II types is absent, is accepted A=B=0.5, and Ri(Δ) calculated as an average arithmetic errors of I and II types.

Choice of one of two assumptions is spent on following conditions:

$$\left.\begin{array}{l} \text{If } Ri[\Delta_E(H_1)] >> \mathrm{Ri}[\Delta_E(H_2)], \text{ that H=H}_1 \\ \text{If } \mathrm{Ri}[\Delta_E(H_2)] >> Ri[\Delta_E(H_1)], \text{ that H=H}_2 \\ \text{If } \mathrm{Ri}[\Delta_E(H_1)] \cong Ri[\Delta_E(H_2)], \text{ that H=H}_1 \end{array}\right\} \quad (6)$$

As an example in table 1 initial data are cited: final set MD $\{X\}_\Sigma$ and sample of these MD $\{X\}_V$, s.f.d. $F^*_\Sigma[X]$ and $F^*_V[X]$ and results of calculation Δ_E.

In table 2 results of calculations of critical values of the greatest deviation and risk of the erroneous decision are resulted. From this table it is evidently visible, that errors of I and II types should be calculated not proceeding from corresponding assumptions (H_1 and H_2), and proceeding from a parity of average values of a random variable of sets $\{X\}_\Sigma$ and $\{X\}_V$. Not the account this parity leads to essential decrease in significance values (errors of I and II types). Data of tables 1 and 2 testify to inexpediency of classification MD, i.e. $H \Rightarrow H_1$.

In table 3 results of calculation for a case, when $H \Rightarrow H_2$

Table 1

Illustration of calculation of the greatest empirical deviation

N	$\{X\}_\Sigma$	$F^*_\Sigma(X)$	$\{X\}_v$	$F^*_V(X)$	$\|\Delta(X)\|$
1	105.8	0.059			
2	109.6	0.118			
3	109.9	0.176			
4	110.3	0.235			
5	111.3	0.294			
6	111.7	0.353	111.7	0.333	0.02
7	112.7	0.412			
8	113.7	0.471	113.7	0.667	0.196
9	113.9	0.529			
10	114.7	0.588			
11	115.2	0.647			
12	115.5	0.706			
13	117.2	0.765			
14	117.4	0.824			
15	117.7	0.882	117.7	1.000	0.118
16	119.2	0.941			
17	119.6	1.0			

Note: Δ_E=0,196

3. Algorithm of a choice of the most significant VA

It would seem algorithm of a choice it is simple enough:

- it is necessary to receive not casual sample of data of final set MD on everyone VA;
- to compare $F^*_\Sigma(X)$ and $F^*_V(X)$;
- to define risk of the erroneous decision;
- to define VA with the minimal risk of the erroneous decision

However simplicity of algorithm is deceptive, since it is required to lead generally n! calculations, that on time exceeds comprehensible opportunities of computer facilities. The problem consists in comparison not casual выборок data on everyone VA. It is necessary to allocate sample, s.f.d. This to the greatest degree would differ from s.f.d. final set MD. The preference is given sample, numerical characteristics s.f.d. this to the greatest degree differed from numerical characteristics s.f.d. $F^*_\Sigma(X)$ in comparison with the others s.f.d. In particular considered:

Table 2

Illustration of calculation of critical values of the greatest deviation and risk of the erroneous decision

N	$\Delta(H_1)$	$F^*[\Delta(H_1)]$	$\Delta(H_2)$	$F^*[\Delta(H_2)]$	$\{1-F^*[\Delta(H_2)]\}$	$Ri^*(\Delta)$	Results of calculation
1	0.000	0.000	0.000	0.000	1.000	0.500	$M^*[\Delta(H_1)] = 0,271$
2	0.020	0.003					$M^*[\Delta(H_2)] = 0,245$
3	0.039	0.006					$Sh1[\Delta(H_2)] = 0,023$
4	0.059	0.012					$\Delta_{cr}[sh1(H_2)] = 0,471$
5	0.078	0.031					$Sh2[\Delta(H_1)] = 0,045$
6	0.098	0.045					$\Delta_{cr}[sh2(H_1)] = 0,098$
7	0.118	0.097	0.118	0.150	0.850	0.474	
8	0.137	0.140	0.137	0.206	0.794	0.467	$Ri^*(\Delta) = 0,433$
9	0.157	0.214	0.157	0.332	0.668	0.441	$H \Rightarrow H_1$
10	0.176	0.256	0.176	0.390	0.610	0.433	
11	0.196	0.355	0.196	0.475	0.525	0.440	
12	0.216	0.412					
13	0.235	0.460	0.235	0.568	0.432	0.446	
14	0.255	0.545	0.255	0.641	0.359	0.452	
15	0.275	0.622					
16	0.294	0.679	0.294	0.759	0.241	0.460	
17	0.314	0.728	0.314	0.798	0.202	0.465	
18	0.353	0.772	0.353	0.838	0.162	0.467	
19	0.373	0.806					
20	0.412	0.847	0.412	0.911	0.089	0.468	
21	0.431	0.876					
22	0.471	0.941	0.471	0.977	0.023	0.482	
23	0.490	0.966					
24	0.529	0.995	0.529	1.000	0.000	0.498	
25	0.549	1.000			0.000	0.500	

Table 3

Results of calculations of expediency of classification final set MD

N_Σ	$\{X\}_\Sigma$	N_V	$\{X\}_V$	Results of calculations
1	100	1	100	$M^*[\Delta_{(H1)}] = 0,188$
2	103	2	104,6	$M^*[\Delta_{(H2)}] = 0,269$
3	104,6	3	105,9	$\Delta_{Э=}0,265$
4	105,9	4	113,5	$Sh1[\Delta(H_1)] = 0,029$
5	107,3	5	116	$\Delta_{cr}[sh1(H_1)] = 0,265$
6	108,3	6	139,6	$Sh2[\Delta(H_2)] = 0,030$
7	109,7			$\Delta_{cr}[sh2(H_2)] = 0,147$
8	113,5			$Ri^*(\Delta) = 0,245$
9	114			$H \Rightarrow H_2$
10	116			
11	116,7			
12	118,9			
13	122,1			
14	132			
15	134,7			
16	139,6			
17	140,5			

– the greatest value of absolute size of distinction of average values of random variables of final set MD and samples of this set on each of set VA, calculated under the formula:

$$\Delta[M^*(X)] = \max\{\Delta_1[M^*(X)];\Delta_2[M^*(X)];...;\Delta_i[M^*(X)];...;\Delta_n[M^*(X)]\} \quad (7)$$

where: $\Delta_i[M^*(X)] = \left|M^*_\Sigma(X) - M^*_{V,i}(X)\right|$; i=1,n;

$$M^*_\Sigma(X) = \sum_{i=1}^{L} X_i \Big/ L; \qquad M^*_{V,i}(X) = \sum_{j=1}^{m_i} X_j \Big/ m_i$$

– the greatest value of absolute size of distinction of average quadratic deviations of random variables of final set MD and samples of this set on each of set VA, calculated under the formula:

$$\Delta[G^*(X)] = \max\{\Delta_1[G^*(X)];\Delta_2[G^*(X)];...;\Delta_i[G^*(X)];...;\Delta_n[G^*(X)]\} \quad (8)$$

where: $\Delta_i[G(X)] = \left|G^*_\Sigma(X) - G^*_{V,i}(X)\right|$; i=1,n;

$$G^*_\Sigma(X) = \left\{\frac{\sum_{j=1}^{L}\left[M^*_\Sigma(X) - X_j\right]^2}{L-1}\right\}^{1/2}; \qquad G^*_{V,i}(X) = \left\{\frac{\sum_{j=1}^{m_i}\left[M^*_{V,i}(X) - X_j\right]^2}{m_i-1}\right\}^{1/2};$$

L – Number of random variables of final set MD; m_i – number of random variables of i-th sample

– the greatest value of absolute size of distinction of estimations of factors of a variation of final set MD and samples of this set on each of set VA, calculated under the formula:

$$\Delta[K^*(X)] = \max\{\Delta[K^*_1(X)];\Delta[K^*_2(X)];...;\Delta_i[K^*_i(X)];...;\Delta[K^*_n(X)]\} \quad (9)$$

where: $\Delta[K_i(X)] = \left|K^*_\Sigma(X) - K^*_{V,i}(X)\right|$; i=1,n;

$$K^*_\Sigma(X) = G^*_\Sigma(X) \Big/ M^*_\Sigma(X); \qquad K^*_{V,i}(X) = G^*_{V,i}(X) \Big/ M^*_{V,i}(X).$$

– the least relative value of disorder of random variables samples, calculated under the formula:

$$\delta(X) = \min\{\delta_1(X);\delta_2(X);........,\delta_n(X)\}$$

(10)

where: i=1,n; $\delta_i(X) = (X_{\max,V,i} - X_{\min,V,i}) / (X_{\max,\Sigma} - X_{\min,\Sigma})$;

$$X_{\max,V,i} = \max\{X_{V,i,1}; X_{V,i,2}; .....X_{V,i,m_i}\};$$
$$X_{\min,V,i} = \min\{X_{V,i,1}; X_{V,i,2}; .....X_{V,i,m_i}\};$$
$$X_{\max,\Sigma} = \max\{X_{\Sigma,1}; X_{\Sigma,2}; .....X_{\Sigma,L}\};$$
$$X_{\min,\Sigma} = \min\{X_{\Sigma,1}; X_{\Sigma,2}; .....X_{\Sigma,L}\}.$$

Thus it was supposed, that if s.f.d. samples with extreme values of distinction of numerical characteristics casually differs from s.f.d. $F^*_\Sigma(X)$, casually differs from $F^*_\Sigma(X)$ and s.f.d. all others samples. How much this assumption is true? Experiences of calculations have allowed drawing following conclusions:

- by comparison $F^*_\Sigma(X)$ and $F^*_V(X)$ in a kind of small number of realizations of sample, there is an uncertainty of the decision caused by classification that testifies to inexpediency of classification. I.e. the preference given H_1. Here finds the reflection influence not so much numbers of random variables, how many sizes of their disorder concerning disorder of final set MD. It is established, that the importance VA above, the concerning the average value random variables of sample more concentrate on axes of set of values of final set MD. This conclusion explains cases of erroneous decisions on a condition (7)
- the average quadratic deviation characterizes disorder of random variables concerning their average value. Consequently, it should seem to carry out a choice of the most significant VA more authentically. But it has appeared that the decision depends on average value of sample and number of random variables. Than it is less $M^*_\Sigma(X)$ and more $M^*_{V,i}(X)$, and the number of random variables of sample is more, the reliability of the decision of a choice between H_1 and H_2 is more;
- the condition (9) in which basis there is a comparison of change of factors of a variation, eliminates dependence of estimations of an average quadratic deviation on average value of sample of random variables. Reliability of the decision in comparison with a condition (8) has increased. The condition (9) has eliminated the errors caused by influence of average values $M^*_\Sigma(X)$ and $M^*_{V,i}(X)$ with i=1,n, but has kept their dependence on number of random variables that is shown already at number of attributes i>3
- the condition (10), reflecting physical essence of importance VA, has appeared the most sensitive and authentic. It precisely proves to be true graphically by comparison s.f.d. $F^*_\Sigma(X)$ and $F^*_{V,i}(X)$ with i=1,n.

Thus, algorithm of definition of the most significant VA and consequently working sample, at each stage of classification MD it reduced to following sequence of calculations:

- formation (n+1-i) samples from final set MD of realizations of random variables X for set VA, where i- number simultaneously considered VA, i=1,n is spent;
- the interval of change of random variables X for final set MD and (n+1-i) samples is defined;
- under the formula (10) relative values of an interval of change of random variables in (n+1-i) samples are calculated and defined sample with the minimal value of relative value of an interval is δ(X);
- constructions under s.f.d. this sample $F^*_V(X)$ and final set MD $F^*_\Sigma(X)$

According to the algorithm stated in p.2 comparison s.f.d. is spent. $F^*_\Sigma(X)$ and $F^*_V(X)$.

4. The integrated algorithm of definition of parameters of individual reliability

The essence of algorithm reduced to following sequence of calculations:

- at the first stage most significant of VA defined. The methodology of the decision of this problem considered us in section 3. Designate the sample corresponding most significant VA as $\{X(i,j)\}_V$, where i=1,n. j=1,r_i, and a serial number of it VA - (i, j);
- check of assumptions of character of a divergence $F^*_\Sigma(X)$ is spent and $F^*_{V,P}(X)$. If assumption H_2 of difference $F^*_\Sigma(X)$ also $F^*_{V,P}(X)$ is rejected (theoretically it probably) and data do not contradict assumption H_1 of casual character of difference significant VA are absent, classification of final set MD is inexpedient, and PR are calculated on final set MD. If with the set significance value assumption H_1 is rejected and data do not contradict assumption H_2 we pass to the second stage of calculations;
- at the second stage calculations similar to calculations at the first stage with that essential difference carried out, that as final set MD sample $\{X(i,j)\}_V$ undertakes. From this final set samples on all set VA except for VA with a serial number (i,j) undertake. Among (n-1) samples there is a sample relative size of an interval of which changes of random variables the least. S.f.d. the sample it compared with s.f.d. final set MD.

Thus, naturally, there is a question on to what distribution to compare s.f.d. samples on two VA – with initial s.f.d. final set MD $F^*_\Sigma(X)$ or with s.f.d. the final set MD received to the most significant attribute $F^*_\Sigma(X) = F^*_{V,1}(X)$?

At the decision of this problem, it is necessary to start with following three axiomatic positions:

P 1. If $F^*_\Sigma(X)$ and $F^*_{V,1}(X)$ differ not casually
and if $F^*_{V,1}(X)$ and $F^*_{V,2}(X)$ differ not casually
that $F^*_\Sigma(X)$ and $F^*_{V,2}(X)$ also differ not casually (11)

P 2. If $F^*_\Sigma(X)$ and $F^*_{V,1}(X)$ differ casually
and if $F^*_{V,1}(X)$ and $F^*_{V,2}(X)$ differ not casually
that $F^*_\Sigma(X)$ and $F^*_{V,2}(X)$ also differ not casually (12)

In other words, the neglect casual character of a divergence $F^*_\Sigma(X)$ and $F^*_{V,1}(X)$ conducts to artificial distortion of size and understating of accuracy of an estimation of parameters of individual reliability, decrease in number of stages of classification of data, to the erroneous list significant VA

P 3. If $F^*_\Sigma(X)$ and $F^*_{V,1}(X)$ differ not casually
and if $F^*_{V,1}(X)$ and $F^*_{V,2}(X)$ differ casually
that $F^*_\Sigma(X)$ and $F^*_{V,2}(X)$ also differ not casually (13)

Positions (P1-P3) testify that at each i- th stage of classification distributions $F^*_{V,(i-1)}(X)$ should be compared and $F^*_{V,i}(X)$. At all subsequent stages of classification MD the calculations similar to the above-stated are spent, and come to the end provided that distinction s.f.d. $F^*_{V,(i-1)}(X)$ and $F^*_{V,i}(X)$ becomes casual

CONCLUSIONS

As a result of the lead researches methodical bases are developed:

- quantitative estimation of parameters of individual reliability of the equipment and devices of power supply systems;
- classifications set VA on significant and insignificant;
- ranging of significant attributes in ascending order the importance;
- transition of the decision of operational problems on the basis of ranging reliability of the equipment and devices at an intuitive level, to the decision on the basis of comparison of quantitative estimations of parameters of their individual reliability

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CARDINALITY BASED APPROACH FOR RELIABILITY REDUNDANCY OPTIMIZATION OF FLOW NETWORKS

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ABSTRACT

In flow networks, a reliability model representing telecommunications networks is independent of topological information, but depends on traffic path attributes like delay, reliability and capacity etc.. The performance of such networks from quality of service point of view is the measure of its flow capacity which can satisfy the customers demand. To design a flow network which can meet the desired performance goal, a cardinality based approach for reliability redundancy optimization using composite performance measure integrating reliability and capacity has been proposed. The method utilizes cardinality based criteria to optimize main flow paths and backup paths on priority basis. The algorithm is reasonably efficient due to reduced computation work even for large telecommunication networks.

Keywords: flow networks; capacity; telecommunication networks; heuristics.

1 INTRODUCTION

In networks where nodes and links associate reliability or probabilities of failure are called reliability or probability network models. In such models, reliability optimization of networks is a common topic. Many researchers have emphasized that such evaluations are useful in designing reliable telecommunications networks (Abraham 1979, Theologou & Carlier 1991, Kuo et al. 2007, Schneeweiss 1989). However, this might not be true because constrained reliability optimization of networks has generally been studied with reliability as connectivity measure only. But the practical systems such as computer networks, telecommunication networks, transportation systems, electrical power transmission networks, internet etc. are mostly linked with the performance. The performance of such networks is not only associated with network reliability but also depends on load carrying capacity of each node and link of the network and are termed as flow networks. Therefore, some researchers (Nagamochi & Ibaraki 1992, Varshney et al. 1994, Chan et al. 1997, Soh & Rai 2005) have proposed improved models (termed as capacity related reliability models) to represent performance degradation. These give additional capacities to nodes & links and define the performance for telecommunications networks as the maximum flow determined using standard graph theory. However, it is not true in modern telecommunication networks as the selection of paths to transport flow are decided by routing mechanism and logical links assigned in physical layer. Therefore, all paths of network are not active to carry flow from source to destination. The selection of specific routing paths out of various possibilities is based on certain considerations like reliability, cost and quality. Capacity related reliability (CRR) graph models ignore such actual conditions of telecommunication network design (Hayashi 2008).

Many workers (Wang 2004, Hwang 2005, Ha 2006, Ramirez 2005, 2006) have applied different hierarchical importance criterion such as cutsets and pathsets criticality, Birnbaum importance, component importance, optimal assignment, structural importance and cardinality of pathsets, cutsets and subsystems etc. for solving reliability redundancy optimization problems of general systems. These criterions are used to devise heuristics for optimal assignments such as more important component gets priority over the less important component for applying redundancy. However, a reliability model of flow networks focuses on traffic path attributes like delay, reliability and capacity etc.. Therefore, CRR model must be modified incorporating attributes of routing paths and logical links assigned in physical layer. In the following sections a novel approach considering the above attributes and also combining the hierarchical importance criterions such as cardinality of pathsets and cutsets, disjoint paths and the cardinality of subsystems for reliability redundancy optimization using composite performance measure (CPM) integrating reliability and capacity has been proposed. The proposed method is capable of addressing the ultrahigh reliability requirements of flow networks efficiently even for large telecommunication networks.

2 COMPOSITE PERFORMANCE MEASURE

A path is a sequence of arcs and nodes connecting a source to a sink. All the arcs and nodes of network have its own attributes like delay, reliability and capacity etc.. From the quality and service management point of view, measurement of the transmission ability of a network to meet the customers demand is very important (Lin 2006). When a given amount of flow is required to be transmitted through a flow network, it is desirable to optimize the network reliability to carry the desired flow. The capacity of each arc (the maximum flow passing the arc per unit time) has two levels, 0 and/or a positive integer value. The system reliability is the probability that the maximum flow through the network between the source and the sink is not less than the demand (Pahuja 2004, Lin 2006, 2007a, b). The presumption that in network any amount of flow can pass through any node or path, is neither valid nor justifiable for real life systems as links and nodes can carry only limited amount of flow. Reliability under flow constraint is a more realistic performance measure for flow networks. A concept of weighted reliability was introduced by Pahuja (2004), which requires that all the successful states qualifying connectivity measure of the network be enumerated and the probability of each success state is evaluated and multiplied by the normalized weight to find out the composite performance of flow networks.

2.1 Notation

$a_l(X)$	Sensitivity factor of l^{th} minimal path set
$b_i(x_i)$	Subsystem selection factor for i^{th} subsystem with x_i components
C_j	Total amount of resource j available
$g^j_i(x_i)$	Amount of resources consumed for j^{th} constraint in subsystem-i with x_i components
$c_{ji}(x_i)$	Cost of subsystem i for j^{th} constraint with x_i components
cg	Number of different cardinality groups.
$cg_a(x_i)$	a^{th} cardinality group, a = 1, 2,..., d.
$h(.)$	Function yielding system reliability; dependent on number of subsystems (n) and configuration of subsystems
k	Number of constraints, j = 1, 2,…, k
$L(x)$	$(L_{x_1}, L_{x_2}, ..., L_{x_n})$, Lower limit of each subsystem i,
m	Number of main minimal path sets, l = 1, 2,., m
n	Number of subsystems, i = 1, 2,…, n
P_l	l^{th} minimal path set of the system
Ps	$(l^1, l^2, ..., l^{min})$: priority vector s.t. l^1 and l^{min} are the number of minimal path sets arranged in decreasing order of path selection parameter $a_l(X)$.

$Q_i\,(x_i)$	Unreliability of subsystem i with x_i components.
r_i	Reliability of a component at subsystem i.
$R_i\,(x_i)$	Reliability of subsystem i with x_i components.
R_r	Residual resources [total resource available (C_j) - resources consumed ($\sum g_i^j x_i$)]
R_s (X)	System reliability
$S\,(x)$	Set of variables that have been used as key-elements in a given decomposed expressions
$U\,(x)$	$(U_{x_1}, U_{x_2}, ..., U_{x_n})$, Upper limit of each of subsystem i,
x^*	Optimal solution
x_i	Number of components in subsystem i; i = 1,2,….n
X	A vector $(x_1, \ldots\ldots\ldots x_n)$
Y	Finite set of traffic paths
Z	Finite set of cuts of the network
ΔR_i	Increment in i^{th} stage reliability when a unit is added in parallel to the i^{th} stage

2.2 Assumptions

Following are the assumptions for the rest of the sections:

1. The system and all its subsystems are coherent.
2. Subsystem structures (other than coherence) are not restricted.
3. The networks are modelled with the help of graphs, the paths (ordered pair of arcs and the members of the ordered pair are reliability and capacity respectively) where in are assigned as the weight of each link.
4. Each link can have only two stages up and down.
5. The network nodes are perfect. If the nodes are not perfect, the method needs to be modified to deal with nodes failures.
6. All component states are mutually and statistically independent.
7. All constraints are separable and additive among components.
8. Each constraint is an increasing function of x_i for each subsystem.
9. Redundant components cannot cross subsystem boundaries.

2.3 Composite Performance Measure (CPM)

The weighted reliability measure i.e. composite performance measure (CPM), integrating both capacity and reliability may be stated as by:

$$\text{CPM} = \sum_{i \in S(x)} \omega t_i \, R_i \tag{1}$$

Where ωt_i is the normalized weight and is defined as:

$$\omega t_i = Cap_i \,/\, Cap_{max}$$

i.e. the ratio of capacity in the i^{th} state to the maximum capacity (Cap_{max}) of the system and R_i probability of the system being in state S_i and is computed as:

$$R_i = P_r\{S_i\} = \prod_{j/S_{ij}=1} p_j \times \prod_{k/S_{ik}=0} q_k \tag{2}$$

2.4 Capacity Functions of Networks

The capacity function of different arcs connected in parallel is (Ramirez et al. 2005):

$$C(X)_{Par} = \sum_{i \in x} Cap_i \tag{3}$$

and the capacity function of different arcs connected in series is:

$$C(X)_{Ser} = \min\{Cap_i\} \tag{4}$$

The rules for connecting series and parallel arcs to integrate capacity and reliability to give composite performance measure are expressed as:

$$CR(X)_{Ser} = \{\min_{i \in x} Cap_i\} \prod_{i=1}^{n} r_i \tag{5}$$

$$CR(X)_{Par} = \sum_{i=1}^{n} Cap_i \, . \bigcup_{i=1}^{n} r_i \tag{6}$$

CPM for series and parallel networks can be defined as:

$$\text{CPM}_{Par} = CR(X)_{Par} / Cap_{max} \tag{7}$$

and

$$\text{CPM}_{Ser} = CR(X)_{Ser} / Cap_{max} \tag{8}$$

3 PROBLEM FORMULATION AND HEURISTIC METHOD

3.1 Problem Formulation

The general constrained redundancy optimization problem in complex systems can be reduced to the following integer programming problem (Kuo et al. 2001):

Maximise

$$R_s(X) = h(R_1(x_1), ..., R_n(x_n)), \tag{9}$$

subject to

$$\sum_{i=1}^{n} g_i^j(x_i) \le C_j, \quad j = 1, 2, \ldots, k \tag{10}$$

and

$$1 \le x_i \le U_{xi}, \qquad i = 1, 2, \ldots, n.$$

3.2 Proposed Heuristic Method

In real life systems all the arcs are not simultaneously connected to carry flow from source to sink as the selection of paths to transport flow are decided by routing mechanism and logical links assigned in physical layer. Thus in practical systems the entire pathsets are never utilized for transfer of information (Hayashi & Abe 2008). The flow is transmitted through the main path(s) only and in case of failure of main path(s), backup path(s) takes over the task of main path(s). As discussed above the main and backup paths of flow networked are decided by the routing mechanism hence it is presumed that these are known. The proposed algorithm first optimizes the main path(s) and then back up path(s) using cardinality approach for redundancy optimization. The cardinality is defined as number of elements in a mathematical set. On the basis of this definition the cardinality of a subsystem is defined as its frequency of occurrence in all pathsets and cutsets of the network whereas, the cardinality of a pathset is the number of subsystems contained in the pathset. The proposed algorithm first combine the cardinality of different pathsets and cutsets, disjoints paths to form different groups of subsystems to be optimized on priority basis using three phases. Unlike existing heuristics, a switching criterion has been applied to switch from CRR optimization of one priority group to another priority group. Using this approach network designer can utilize generally limited resources more efficiently (Kumar et al. 2009, 2010a, b, 2011, 2012). The three phases of the proposed method for optimization are:

(i) In the first phase, the path sets having minimum cardinality are given highest priority and the path sets having maximum cardinality are given least priority then all the subsystems having maximum cardinality are found. All highest priority minimal pathsets containing the highest cardinality subsystems form first group. If highest priority minimal path sets are different than that of containing highest cardinality subsystem(s), these path sets along with the path sets containing highest cardinality subsystems are grouped together in the first group. Using this criterion all subsystems of the network are arranged in different groups with decreasing priority importance.

(ii) In the second phase highest selection-factor $b_i(x_i)$ is computed for the chosen priority group using

$$b_i(x_i) = \frac{\Delta R_i}{\sum_{j=1}^{k} (g_i^j(x_i)/k\,C_j)}, \quad \text{for each } i \in cg_a(x_i) \tag{11}$$

where

$$\Delta R_i = R_i(x_i) - R_i(x_i - 1) \tag{12}$$

d is the total number of cardinality groups formed and
$cg_a(x_i)$ is the a^{th} cardinality group such that a = 1, 2,..., d.

(iii) In the third phase, a redundant parallel subsystem is added to the unsaturated subsystem belonging to the chosen $cg_a(x_i)$ with highest selection factor. The three phases are repeated till optimal solution is reached.

After obtaining the optimal solution for the network; calculate the composite performance measure (CPM) for each subsystem of the network. Then evaluate the system reliability using the CPM of the each subsystem. Novelty of the method is that unlike other existing heuristic for complex systems it requires only one selection factor instead two. To determine the total capacity, if system is working normally then capacity for each primary path is ensured otherwise the flow capacity of the primary path is the minimum, and is the summation of the capacities of reserved backup paths that are working. Finally, total capacity is computed by summing the ensured capacities.

3.3 Steps of the Proposed Method

Step1: Find all path sets and cut sets for the network then using cardinality approach:

i) The path sets having minimum cardinality are given highest priority and the path sets having maximum cardinality are given least priority then all the subsystems having maximum cardinality are found.

ii) All highest priority minimal path sets containing the highest cardinality subsystems form first group. If highest priority minimal path sets are different than that of containing highest cardinality subsystem(s), these path sets along with the path sets containing highest cardinality subsystems are grouped together in the first group.

iii) Using this criterion all subsystems of the network are arranged in different d groups with decreasing priority importance.

Step2: Let a = 1; from a = 1, 2,…, d.

Step3: Let x_i = 1 for all i; i = 1, 2,…, n.

Step4: Compute $b_i(x_i)$ using (4.1) for each subsystem belonging to selected cardinality group cg_a, find $i^* \in cg_a(x_i)$ such that $b_{i^*}(x_i) = \max[b_i(x_i)]$.

Step5: Check, if by adding one redundant subsystem to unsaturated subsystem i^*:

i) no constraints are violated and reliability of the subsystem satisfies the stopping criterion and also the capacity of the subsystem is >= flow capacity of path, add one redundant subsystem to unsaturated subsystem i^* by replacing x_{i^*} with $x_{i^*}+1$, and go to step 4.

ii) if at least one constraint is exactly satisfied and other are not violated, also and reliability of the subsystem satisfies the stopping criterion and also the capacity of the subsystem is >= flow capacity of path, then add one redundant subsystem to unsaturated subsystem i^* by replacing x_{i^*} with $x_{i^*}+1$. The $x^* = X$ is the optimal solution. Go to step 6.

iii) if at least one constraint is violated, then remove subsystem *i** from further consideration and consider the next subsystem having maximum $b_{i^*}(x_i)$ value and go to step 5.

iv) if all $i^* \in cg_a(x_i)$ have now been exhausted, check if $a < d$; then $a = a+1$ and go to step 4;

v) if $a \geq d$ then $x^* = X$ is the optimal solution, go to step 6.

Step6: Evaluate the composite performance measure (CPM) for each subsystem of the network.

Step7: Evaluate the system reliability using the CPM of the each subsystem.

4 COMPUTATION AND RESULTS

To illustrate the performance of the proposed algorithm a network having six arcs $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ and five minimal path sets $\{y_1, y_2, y_3, y_4, y_5\}$ as shown in the Figure 1 is considered and solved for capacity related redundancy reliability optimization using CPM (7 and 8). System reliability is determined using Bayes method. The network shown in Figure 1 is a bench mark problem, considered by Hayashi & Abe (2008).

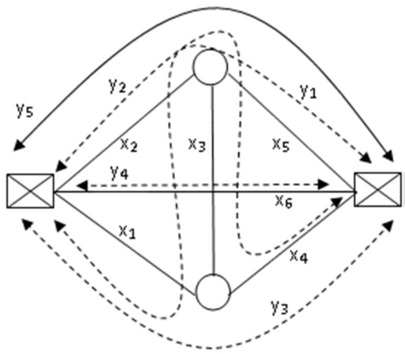

Figure 1 Illustration Network

Using Baye's method, the Reliability of the system can be expressed as:

$$R_s(X) = R_3\,[1- Q_6\,\{1-(1-Q_1Q_2)(1-Q_4Q_5)\}] + Q_3[1-(1-R_2R_5)(1-R_1R_4)]*Q6 \quad (13)$$

The problem is solved for data given in Table 1. For this first determine all the simple minimal pathsets and cutsets of the Network:

$Y = \{y_1, y_2, y_3, y_4, y_5\}$ where $y_1 = \{1, 3, 5\}$, $y_2 = \{2, 3, 4\}$, $y_3 = \{1, 4\}$, $y_4 = \{6\}$, $y_5 = \{2, 5\}$
$Z = \{z_1, z_2, z_3, z_4\}$ where $z_1 = \{1, 2, 6\}$, $z_2 = \{4, 5, 6\}$, $z_3 = \{2, 3, 4, 6\}$, $z_4 = \{1, 3, 5, 6\}$

and then as discussed in Section-3.2 above, on the basis of cardinality of pathsets and cutsets of the subsystems, all the subsystems of the network are arranged in two groups $cg_1(x_i) = \{6\}$ having cardinality 5 and $cg_2(x_i) = \{1, 2, 3, 4, 5\}$ of cardinality 4. The general problem of constrained reliability redundancy allocation has been solved using the steps discussed in Section 3.3 above. The problem is solved by considering that every flow path has a capacity of 100. The flow network is solved by considering paths y_2, y_3, and y_4 as main paths and y_1, y_5 as backup paths. The total flow through network at any time should not exceed above 200 in any case. The proposed algorithm gives the optimal solution (2, 2, 2, 2, 1, 3) with system reliability R_s = 0.9578, the optimized subsystem reliability probability R_i and unreliability probabilities Q_i are shown in Table 2.

Table 1 Data for Fig. 1

i	1	2	3	4	5	6
r_i	0.70	0.75	0.8	0.85	0.70	0.90
c_{1i}	2	3	2	3	1	3
C_1	30					

Table 2 Optimized subsystem reliability/unreliability for Fig. 1

i	x_1	x_2	x_3	x_4	x_5	x_6
X^*	2	2	2	2	1	3
R_i	0.9100	0.9375	0.9600	0.9775	0.7000	0.9990
Qi	0.090	0.0625	0.0400	0.0225	0.3000	0.0009

The capacity of each subsystem of the flow path is taken as 100 and the capacity of flow paths of the network is determined using (3) of proposed approach as:

$$\left.\begin{aligned} Cap\{y_1\} &= \min Cap\,\{1, 3, 5\} = \min Cap\{2*100, 2*100, 100\} = 100 \\ Cap\{y_2\} &= \min Cap\,\{2, 3, 4\} = \min Cap\{2*100, 2*100, 2*100\} = 200 \\ Cap\{y_3\} &= \min Cap\,\{1, 4\} = \min Cap\{2*100, 2*100\} = 200 \\ Cap\{y_4\} &= \min Cap\,\{6\} = \min Cap\{3*100\} = 300 \\ Cap\{y_5\} &= \min Cap\,\{2, 5\} = \min Cap\{2*100, 100\} = 100 \end{aligned}\right\} \quad (14)$$

Next the CPM expression (15-20) are derived using (7 and 8) and the value for CPM for an assumed flow of 200 is suppose to pass through the flow path and it comes out to be 1.0000 .

$$\text{CPM}_{y1} = \frac{\min Cap_i}{Cap_{\max}}[R_1 R_3 R_5] \quad (15)$$
$$= (100/200) \; * 0.91 \; * .96 \; * .7 = .3058$$

$$\text{CPM}_{y2} = \frac{\min Cap_i}{Cap_{\max}}[R_2 R_3 R_4] \quad (16)$$
$$= (200/200) \; * 0.9375 \; * .96 \; * .9775 = 0.8797$$

$$\text{CPM}_{y3} = \frac{\min Cap_i}{Cap_{\max}}[R_1 R_4] \quad (17)$$
$$= (200/200) \; * 0.91 \; * .9775 = 0.8895$$

$$\text{CPM}_{y4} = \frac{\min Cap_i}{Cap_{\max}}[R_6] \quad (18)$$
$$= (300/200) \; * 0.9990$$

$$= (1.5) * 0.9990 \quad \text{as } 0 \le (\min Cap_i / Cap_{\max}) \le 1$$

$$\text{so} \quad = 1 * 0.9990 = 0.9990$$

$$\text{CPM}_{y5} = \frac{\min Cap_i}{Cap_{\max}} [R_2 R_5] \tag{19}$$

$$= (100/200) * 0.90 * .70 = 0.3281$$

Composite performance measure integrating the reliability with capacity is calculated as:

$$\text{CPM}_{\text{Network}} = 1 - (1 - \text{CPM}_{y1}) * (1 - \text{CPM}_{y2}) * (1 - \text{CPM}_{y3}) * (1 - \text{CPM}_{y4}) * (1 - \text{CPM}_{y5}) \tag{20}$$

$$= 1 - (1 - 0.3058)(1 - 0.8787)(1 - 0.8895)(1 - 0.9990)(1 - 0.3281)$$

$$\cong 1.0000$$

The above result shows that proposed method is capable of optimizing the flow network to transport the desired capacity through the network with highest reliability. However, the selection of main paths and backup paths will affect the quality of composite performance measure. Hence the proper choice of these paths may be done using cardinality criteria (Kumar et al. 2010b) or any other hierarchical measures of importance.

5 CONCLUSIONS

This paper presented a new model for designing reliable flow networks capable of transmitting required flow. The proposed algorithm utilizes the concept of main and backup flow paths. The choice of backup and flow paths is application specific and paths with minimum cardinality may be selected as main path and disjoint paths can be the backup paths. The numerical example demonstrates that the proposed algorithm is fast for designing large, reliable telecommunications networks because the task of optimization is reduced, as only few paths are selected as main paths.

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PRINTED-CIRCUIT BOARDS. RELIABILITY OF INTERCONNECTIONS

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ABSTRACT

Stability of metallization of holes to thermomechanical pressure is provided with durability and plasticity of galvanic besieged copper.

Distinctions in factors of thermal expansion of copper and the dielectric bases of printed-circuit boards create powerful thermomechanical factors of rupture of metallization of apertures, destructions of internal interconnections in multilayered structures of printed-circuit boards. Standard norms of requirements to a thickness of metallization of apertures, its durability and plasticity of copper were established in the course of manufacture of ordinary printed-circuit boards with reference to use of traditional technologies of the soldering by tin-lead solders. Return to consideration of a problem of plasticity of copper is caused first of all by transition on the Lead-free solders, initiated by the all-European Directive RoHS [1], rations different by a heat. More heats create the big deformations of metallization of holes those forces to reconsider requirements to plasticity of copper. At the same time, the tendency to reduction of diameter of the metallized holes, so also to reduction of the area of cross-section section of metallization is everywhere observed. Smaller sections have smaller resistance to rupture. Therefore, along with good plasticity, metallization of holes of printed-circuit boards should provide and higher breaking strength. In this connection deformation of metallization of holes at heating to soldering temperatures has been investigated. The purpose of researches - revision of norms on plasticity of copper in holes of printed-circuit boards. It is shown that plasticity copper deposition in holes of modern printed-circuit boards should not be less than 6 % [2]. Modern cupper electrolytes allow to receive plasticity of copper of 12-18 % [3].

Keywords: PCB, Interconnection, Reliability, Plasticity of copper

Essence of problem

The elements of interconnections exposed to thermal loads in the process of manufacturing, assembling and cyclic changes in temperature during operation of the equipment. Differences in temperature coefficient of linear expansion (TKLR) conductive structures and dielectric in electrical connections are the thermomechanical tension of various intensity. In longitudinal reinforced fiberglass patches, differences in TKLR are so small that they do not affect the strength of the connections of the longitudinal structure.

In the transversal direction perpendicular to the plane of the reinforcement, the differences are so significant in linear expansion ($17 \cdot 10^{-6}$ for copper (100…400) ·10 -6 for dielectric basic) that occur when thermal loadings are able to destroy the thermomechanical tension interlayer connection.

Es's know that resistance of metallized holes to the thermal-mechanical loads are thick and plasticity of metallization. Standard requirements for metallization on these quality criteria have been established during the years of practice manufacturing and operation of electronic devices with printed with a thickness of boards to the diameter of the hole from 1: 1 to 3: 1. When the amount of through holes less than 0.3 mm is the ratio can be as high as 10:1 current. 20:1. In such constructions for multilayer printed circuit boards (MLB) metallization aperture ratio results sections and the surrounding material of the bottom board is not in favor of metallization in conditions of thermal effects increases the deformation of metallization of vias (fig. 1). This phenomenon is exacerbated by the decline in copper metallization of plasticity with increasing temperature.

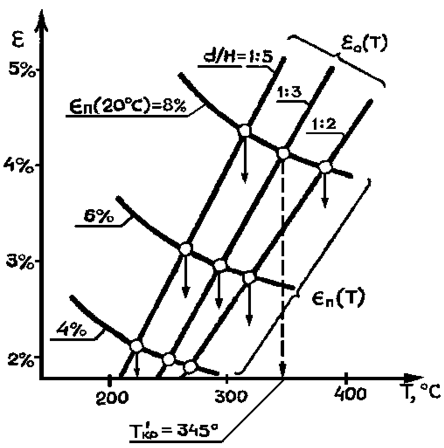

Figure 1. Ehe toughening of requirements to the plasticity of metal as the diameter of the hole.

Statistics show that especially big bounce stream observed in interlayer connections, systematically exposed to cyclic changes in temperature (Thermo-cycles). According to the long-term operation of aviation systems printed circuit failures are distributed as follows: metallized holes-24%, inner joins-72%, printed conductors internal layers-0.1%,-2%, 2.5%, breakages of solder wire-0.3%,-0.6%. Comparing the number of refusals of MLB in the stationary equipment operators in relative constancy of temperature, and airborne show the difference in nearly three orders of magnitude, that has convinced us that, if the level of variable tensile exceeds a certain limit of thermo-mechanical, is in the process of gradual accumulation of damage, which concludes with a fatigue destruction.

Model of Thermo-mechanical stressing

Thermomechanical stress during heating cause the stretching along the axis of the hole metallization (axial) and flex contact pads, with the largest concentration of which focuses on the junction with metal cylinder bores (tensile drop). A typical distortion of holes form when heated schematically shown in figure 2 and photos of microsection in figure 3.

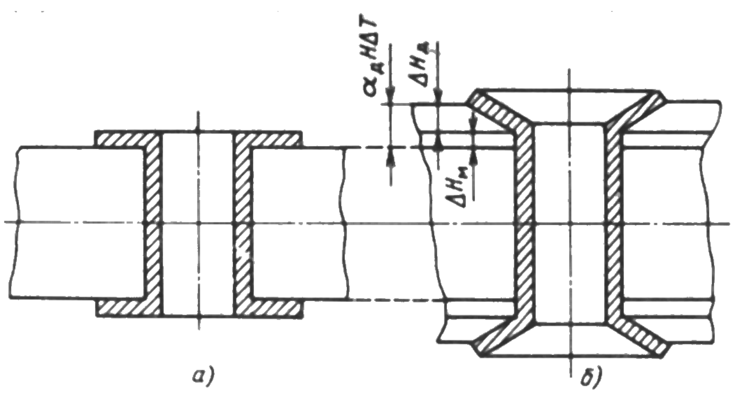

Figure 2. Distortion of metallized holes when heated

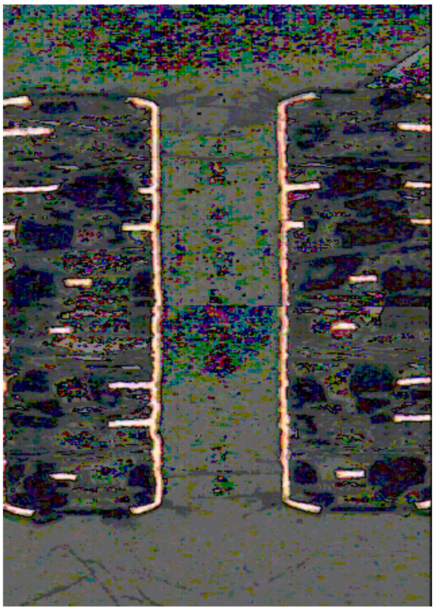

Figure 3. Microsection of metallized holes after thermo shock

In general, the relative deformation at temperature influences σ_Z plating can be represented as the sum of the elastic ε_Y and of the thermal deformations ε_T. Elastic deformation of the $\varepsilon_Y = \sigma E$ (E- elastic modulus). Thermal deformation of $\varepsilon_T = \alpha(T\text{-}T_0)$. Hence the thermomechanical stress $\sigma = E\ [(\sigma_Z - \alpha(T\text{-}T_0)]$. Thermomechanical efforts in each of the elements of metallized holes:

$$F = E[(\sigma_Z - \alpha(T\text{-}T_0)]\ h\ dZ.$$

To determine the characteristics of Thermo-mechanical deformation equilibrium equation we write metallization of the condition that the sum of all thermo-mechanical effort, resulting in components of "metallization-wall holes ", must be zero (Figure 4):

$$\int_0^{h_M} E_M[(\varepsilon_Z - \alpha_M(T - T_0)]hdZ + \int_{h_Д}^{0} E_Д[(\varepsilon_Z - \alpha_Д(T - T_0)]hdZ = 0$$

After integration and transformation it can be shown that deformation of copper in transversal Z-direction is:

$$\varepsilon_Z = (\alpha_Д - \alpha_M)\ (T - T_0)\ (1 + J_M/J_Д)^{-1} \qquad (1)$$

Here $\alpha_Д$ and α_M - thermal expansion coefficient, J_M and $J_Д$ -conventional hardness of copper and dielectric.

If ε_Z exceeds the limit of ductility of copper sludge in the hole (or $\sigma_P > \sigma_{ПЧ}$) annular gap metallization.

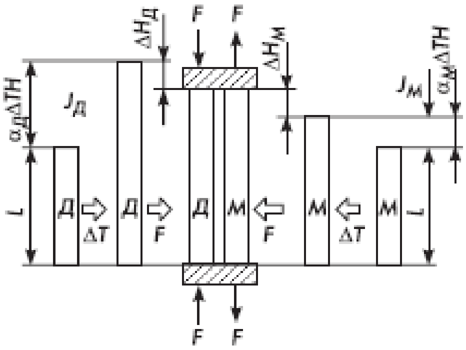

Figure 4. Thermo-mechanical stress analysis model of axial

If the forces of adhesion plating machines with small holes can be realized in the shear fracture of inner joins. Shear stress, obviously, should increase with increasing distance from the neutral axis interface 0- -0 (figure 5). The nudge distance, if it occurs, you can determine, on the basis of public views. But if the coupling forces hold metalizing on joints, holes developing temperature increases shear stress equals $\sigma_{Cдв} = G(\alpha_Д - \alpha_M)\Delta T$. Value destructive shear stress is determined based on the experimental values of hole metallization breaking efforts. In Figure 6 shows a picture of destruction resulting from an inner join on the walls of the hole metallization shift.

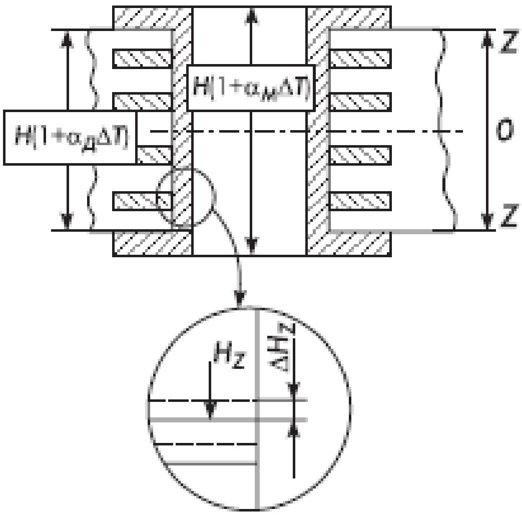

Figure 5. Shift metallization with the edges of contact pads inside layers

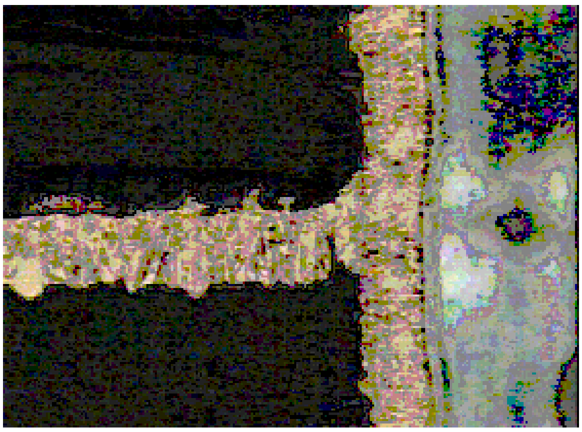

Figure 6. The photograph of the ruined inner join microsection

In Figure 7 shows chart of the temperature deformation freely expanding cylinder of copper, polymeric dielectrics and the resultant temperature deformation together, as in Figure 8 deformation-strain. Thermal expansion curve has a fracture at the base of the dielectric glass transition temperature *Tg.* zone of elastic deformation of copper is limited to the value of the ε_Y

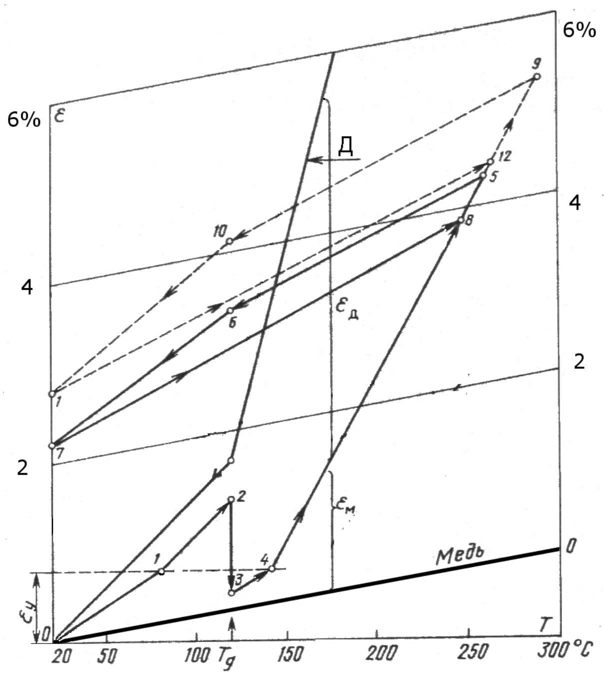

Figure 7. Graph temperature deformation of metallization aperture received grapho-analytical method

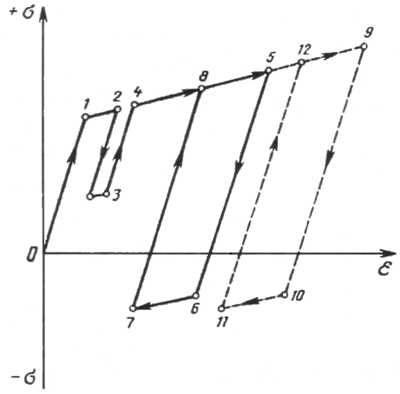

Figure 8. Figure deformation-strain

On the 0-1 the shear modulus of copper is part of elasticity, i.e. G_M, is the shear modulus of the dielectric is $G_Д$. Distribution of deformation of dielectric and copper is the ratio: $\alpha_Д/\alpha_M = G_M S_M / G_Д S_Д$, here S_M and $S_Д$ -loading cross sectional area of the copper cylinder and dielectric around hole wall. When at the point 1 deformation of copper goes to a plot point (1-2), the module is reduced, so the hole metallization deformed almost following the free expansion of the glass transition temperature dielectric. *Tg* dielectric substrate loses its stiffness through the copper cylinder is unloaded. The deformation is a value that corresponds to the point 3. When you move the glass transition temperature *Tg* dielectric begins to grow intensively. However, initially, this does not result in a large increase of copper, while its deformation does not exceed the limits of elasticity (section 3-4). Correlation of deformation of dielectric and copper on the plots:

$$\alpha_Д/\alpha_M = G_M' S_M / G_Д' S_Д \quad (2)$$

Curves of 5-6-7-8-5 and 9-10-11-12-9 show the changes of the linear dimensions of the metallized holes on cooling and heating-cooling cycle again for soldering temperatures 260° and

290° c, respectively. The presence of hysteresis in thermal deformation diagram reveals a certain percentage of plastic deformation of copper-a harbinger of fatigue damage under cyclic temperature stress.

Methodology of experimental research

Es' know the basic principles of studies stresses in the metallization of through-holes using the micrometrical sensor of motion, registering growth of thickness of dielectric and metal cylinder through openings as the heating of the MLB. Increasing the accuracy of measurements in a wide temperature range by using a quartz sample holders and rods passing movements. There were attempts to use overhead strain micro-sensors for measuring small elongations (extensometer) for the study of deformation of metallization of through holes during soldering. Comparison of measurement results of thermal expansions of the two methods obtained by different authors, demonstrates their ambiguity due to the uncertainty of the reference database in the first case and low sensitivity of tensometry to small samples, what are the openings of the MLB in the second case.

The author has used its own methodology to the study of Thermo-mechanical stresses that it be analyzed hole is used as a load cell to measure its temperature deformations. For this they proceeded from the following prerequisites. Link changes with deformation: the resistance $\Delta R/R = k\varepsilon$, where k is the tensosensitivity of element (in this case, the metallized holes) Since $R = \rho H/S$ differential form of expression $\Delta R/R$ has the appearance of $dR/R = d\rho/\rho + dH/H\text{-}dS/S.$ (here ρ - the electrical resistivity of metallic coating, H - the thickness of the Board (length of metallized cylinder hole), S-cross-sectional area of cross-section of holes in perpendicular to its axis. At low relative lengthening $d\varepsilon = dH/H$ the relative change in cross-section $dS/S = -2\mu(dH/H)$. So $dR/R = d\rho/\rho + \varepsilon + 2\varepsilon$ (here μ –Poisson coefficient). Then the tensosensitivity of metallization element - the metallization of holes:

$$k = (dR/R)\,\varepsilon^{-1} = (1 + 2\mu) + (d\rho/\rho)^{-1} \quad (3)$$

The expression (3) consists of two parts: the geometry of the dependent by ρ and displaying electrical resistance change due to changes in the size of the metal cylinder due to its longitudinal deformation and physical part connected with the change of resistivity of metallic coating when extending: $d\rho/\rho\ dV = B/V$ and reflecting the linear dependence between the change in resistivity and the relative change in volume dV/V-coefficient of Bridgman. In the case of uniaxial loading hole metallization generated during heating,

$$d\rho/\rho = B\,(1 - 2\,\mu)\,\varepsilon \quad (4)$$

Combining (3) and (4), we get:

$$k = 1 + 2\mu + B(1 - 2\mu) \quad (5)$$

The direct effect of temperature on resistance of the metallic coating is taken into account, based on known relationships: $\Delta R/R = (+\ 234)^{-1}$. For pure copper $B = 1$, at least for the temperature range from 0 to 300° c. From here on (5) numeric expression tensosensitivity of metallization of vias is 2. I.e. elongation of metallization on the 1% change of resistance of metallization holes on 2%. The research of deformation within the 6% from 0.1% in where the required accuracy of measurement of resistances to almost four-probe method with precision class instrument. For contacting the four probes wire to pin bonded cold Ga-soldering sites that after the formation of solid solutions can withstand without fracture temperature up to 800° c (Figure 9).

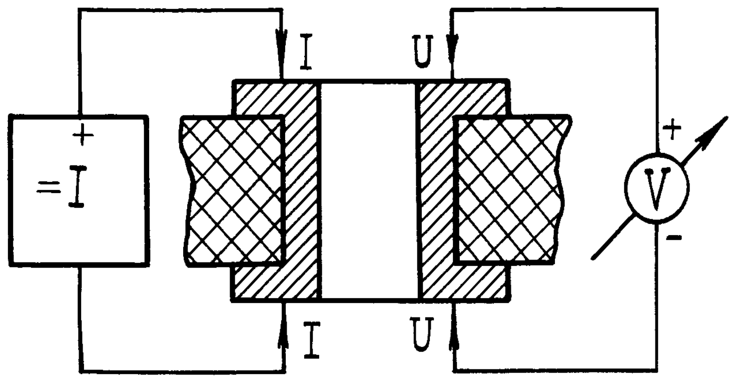

Figure 9. Circuit of resistance measurement metallization by four-probe method

The results of experimental research of deformation

Measurement of deformation of metallized holes with a diameter of 0.8 mm and 1.6 mm thick, MLB is shown in Figure 10, give good agreement with results of graphic-analytical analysis based on Nonlinear model of Thermo-mechanical deformation of through metal holes.

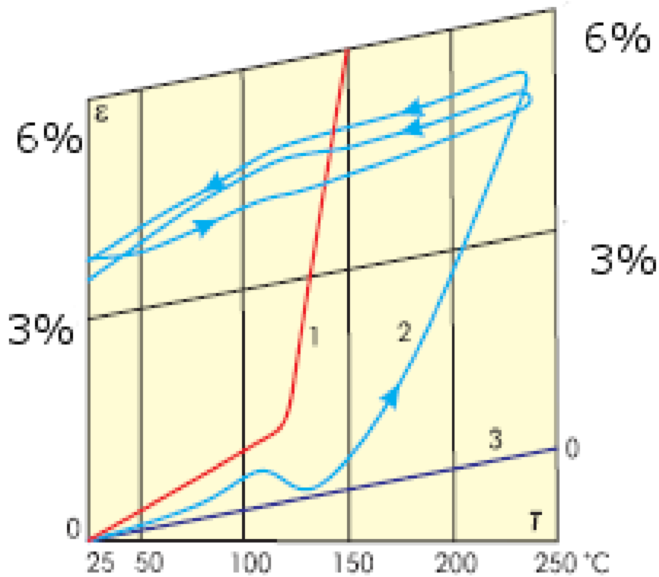

Figure 10. Experimentally obtained diagrams of temperature deformation of metallized holes: 1 and 3 – chart of free expansion of dielectric and copper; 2-experimental deformation diagram metallized holes.

The combination of large deformations of the metallization of vias in thermal loads and reduced ductility of copper may in certain circumstances lead to rupture hole metallization of metallization or the shift of the walls of the holes, if not to take measures to increase the plasticity of galvanic deposition at temperatures relevant to possible overheating of the MLB. Table 1 shows the threshold temperature value destruction interconnections in MLB.

Table 1. Threshold temperature of the beginning of destruction

The ratio of thickness of MLB to the size of the holes, the H/d	2:1	3:1	5:1	10:1	20:1
Plasticity of metallization, %	Threshold temperature, °C				
4	290	250	220	210	190
6	320	290	260	240	220
8	380	350	320	280	260

In high temperature deformations, plasticity of metallization and shaky grip of metallization with walls through holes, MLB may destroy internal connections. To identify such defect enough after thermal shock (reflow) provoke oxidation (humidity + heat) surfaces of physical contact metallization of vias with ends of internal contact pads and internal resistance measurement of compounds to diagnose reliability of MLB.

Fatigue fracture low-cycle damages are only possible when moving in the area of plastic deformation. And the deeper the temperature deformation in the area of plastic deformation, the earlier start connection failures during operation. The proposed means of monitoring the status of connections in the MLB started the plastic deformation is detected as the emergence of hysteresis in low-resistance of the circuit. Studies to quantify the influence of thickness of metallization of through holes at the temperature corresponding to the start of plastic deformation (table 2)

Table 2. Start of plastic deformation during heating

The thickness of metallization of the MLB in the hole, μ	MLB thickness ratio to diameter through-holes metallized (*H/d*)		
	2:1	3:1	5:1
	The temperature beginning of the plastic deformation, °C		
10	75	60	50
15	85	73	55
20	95	80	60
25	100	85	65
30	110	90	70

Local defects, particularly in the form of thinning ring, significantly reduce the stability of the metallization of vias to cyclic temperatures.

These studies demonstrate the futility of thermal cycling for grading and installation of products by identifying the weak elements of the cyclic load connections: destroy defective cells and create fatigue weaken the connections, close to the border of the differences of quality or defective items. This is due to the fact that the boundary of quality between defective and qualitative elements of blurred. Between them there are always intermediate states that characterize the opportunity to bounce connections due to fatigue phenomena.

Conclusion

Reliability of interconnections in modern electronic equipment technology is provided by the high level of plasticity of metallization of PCB resistant to by low-cycles fatigue destruction provoked group heat when soldering and manual many-stage soldering by repair print sites.

OPTIMAL SENSOR NETWORKS SYSTEM RELIABILITY ALLOCATION USING IMPROVED AGREE METHOD

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ABSTRACT

Reliability of sensor networks system plays an important role in monitoring the operational health conditions of any critical engineering system. In this paper, a methodology is proposed to improve an initial optimal allocation of sensor system by considering fault acceptance degree (FAD), fault influence degree (FID), importance factor (α), and the actual operational data of the critical system with initial allocated sensors for a small initial period. The paper has also proposed a method for estimation of expected life of the sensor network based on the above factors. A hypothetical case of the sensor system of an electric motor is presented to illustrate the proposed approach.

KEYWORDS: sensor reliability; fault acceptance degree (FAD); fault influence degree (FID); importance factor; optimal allocation.

1 INTRODUCTION

Reliability of sensor used for monitoring various parameters of critical systems is very important for timely assessment of their health and to take appropriate measures for fault diagnosis at incipient stages in order to prevent any catastrophic failures. This paper focuses on modelling of reliability of sensor under multiple load conditions and optimization of sensor networks system reliability. This type of modelling can be useful in applications such as: propulsion systems of a satellite, nuclear power plants, and aircraft systems etc. Such systems require continuous reliable monitoring system to avoid unexpected failures which might result in huge economic loses apart from ill effects on environment, health & safety of human beings and other species. Instead of spending huge amounts on replacement/repair of industrial systems due to unreliable sensors it may be better to have a highly reliable sensor networks system with adequate redundancies. An attempt is made in this paper to optimally allocate sensor networks system reliabilities and update the same by utilizing additional information on factors such as fault acceptance degree (FAD), fault influence degree (FID), importance factor, and the actual operational data of the critical systems. A method is also proposed for estimation of expected life of the sensor networks. This can be useful for taking proactive maintenance and replacement measures for achieving high sensor reliability over long operational period of the critical systems.

The rest of the paper is organized as follows:

In Section-2, the objectives of this paper are presented. In Section-3, proposed methodology to improve an initial optimal allocation of sensor networks system by considering a fault acceptance degree (FAD), fault influence degree (FID), and importance factor of the critical systems is presented. Mathematical models for the expected mean life & reliability of sensor system are presented in section 4. Redesign modelling approach for sensor networks system is presented in section 5. In section 6, an illustrative example for the proposed methodology is presented. Discussions on the results obtained in section 6 are presented in section 7. Conclusions are presented in section 8 followed by selected references in section 9.

2 OBJECTIVE

Swajeeth et al. [1] developed a model for evaluation of sensor reliability under multiple load conditions. An algorithm is also proposed for optimal allocation of number of sensors of different types such as current, temperature, and vibration sensors for achieving specified sensor networks system reliability with minimum cost. See [6, 7, and 10] for reliability optimization algorithms. Reliability of these sensors used for monitoring various parameters of critical systems is very important for timely assessment of their health and to take appropriate measures for fault diagnosis at incipient stages in order to prevent any catastrophic failures. The reliability of a sensor network decreases as time of operation progress continuously. Therefore, the actual reliability of sensor network after 'T' hours may be low/ very low compared to the designed system target reliability.

The objective of the present paper is to effectively remodel the sensor network configuration [1] to provide improved reliability at 'T' hours compared to initial allocation. This paper also deals in meeting the objectives of decision making in ambiguity situations arising from sensor networks used in [1], to improve the expected mean life of each sensor network. To fulfil the above mentioned objectives, it is essential to evaluate the following three critical values of sensor networks of a system.

a) Importance factor (α) of each sensor network in contributing to damage of operational system.
b) Expected mean life ('θ' in hours) of each sensor network.
c) Reliability of each sensor network ($R_i(T)$) after 'T' hours of operation.

Following methodology is used to evaluate the critical factors.

3 METHODOLOGY

Sensors are electronic elements. So, the failure distributions of sensors follow exponential distribution [4], [5], [9]. Refer [1, 8] for distribution used in sensor modelling. The sensor networks system presented in [1] used for operational health monitoring of an electric motor is as shown in fig.1. It is clear that the three sensors are in series configuration. The mathematical models suggested by Wang et al. [2] are used for evaluating the critical values mentioned in section 2 to fulfil the objectives. Suitably, Improved AGREE method [2] (including new parameter-FAD) is used in this work to meet the objectives mentioned in section 2.

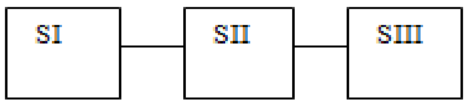

Fig.1. Functional wise series configuration of three sensors for motor monitoring

SI: current sensor
SII: Temperature sensor
SIII: Vibration sensor

4 EVALUATION OF CRITICAL VALUES OF A SENSOR NETWORK

4.1 Defining sensor network failure

A sensor network is considered as failed if it gives false alarms about the operational health condition of the main system.

4.2 Improved AGREE method for sensor network

AGREE (Advisory Group on Reliability of Electronic Equipment) method [2], allocates reliability to the components and possibly subcomponents in a manner that will support the system reliability goals defined. This method assumes that the components are in series, independent, and have constant failure rates.

4.3 Introduction to parameters:

4.3.1 Fault acceptance degree (FAD)

We define a term known as fault acceptance degree which is the degree of acceptance of faults in a sensor network based on severity of damages caused to the operational system. It is denoted by $\beta_{i\ (j)}$. The fault acceptance degree lies between 0 and 1. Generally, when a sensor network failure causes catastrophic, critical, marginal, and negligible damages to the operational system we assign acceptance degrees of 0.1, 0.5, 0.8, and 1 respectively. FAD is discussed in detail with an illustrative example in section 4.

4.3.2 Fault influence degree (FID)

Fault influence degree [2] is a probability that the failure of a sensor network will lead to the operational system failure. It is often marked by $\omega_{i\ (j)}$, which denotes the probability that the fault of sensor network 'i' (number of failure is 'j') will lead the system to fail. Sensors are very crucial in critical systems like rocket propulsion systems, aircrafts, nuclear power plants etc. So, we consider fault influence degree as unity in all sensor applications.

4.4 Calculation of Importance factor (α)

The Fault acceptance degree of sensor network 'i' can be expressed by row vector (β_{i1}, β_{i2},…..β_{in}), while the fault influence degree can be expressed by columns vector (ω_{i1}, ω_{i2},…. ω_{in}), Then the importance factor of a sensor network 'i' can be denoted as follows:

$$\alpha_i = \frac{1}{n}[\beta_{i1}, \beta_{i2}, ,\ldots, \beta_{in}]\ [\omega_{i1}, \omega_{i2}, \ldots\ \omega_{in}]^T \qquad (1)$$

Sensor network with high importance factor should be given first priority in decision making during ambiguity situations.

4.5 Calculation of mean life of sensor network

Notations

N : Total number of sensors in a system

n_i : Number of sensors in a network 'i' of a system
R_s : Sensor networks system reliability
T_i : Time (in Hours) for which a sensor network 'i' is operated continuously.
R_i : Reliability of a sensor network 'i'

The expected mean life of a sensor network 'i' can be expressed as [2]:

$$\theta_i^* = \frac{N\,\alpha_i\,T_i}{-n_i\,\ln(R_S(T))} \tag{2}$$

Note: For static reliability $R_s(T) = R_s$. As if all the sensor networks are operated continuously for 'T' hours, then $T_i = T$.

4.6 Calculation of a sensor network reliability at 'T' hours

Reliability of each sensor network follows exponential distribution while operating for 'T' hours continuously. The reliability at 'T' hours can be expressed as [3], [4]:

$$R_i(T) = e^{\frac{-T}{\theta_i^*}} \tag{3}$$

The reliability logic diagram (RLD) of a sensor networks used in monitoring operational health of an electric motor [1] is as shown in fig.2.

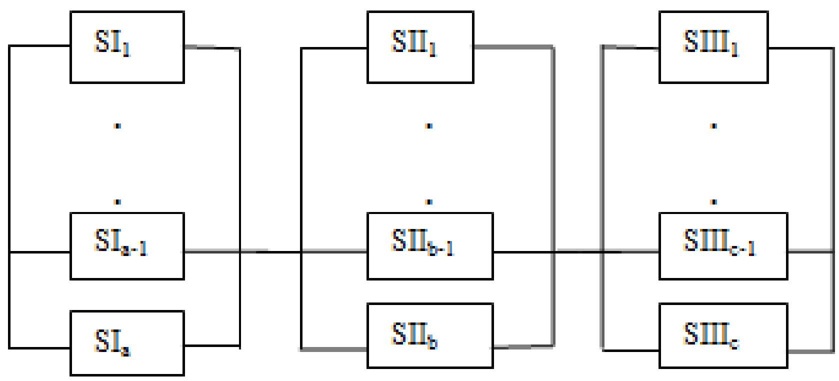

Fig.2. RLD of sensor networks monitoring operational health of an electric motor

a, b, & c are allocations for current, temperature, and vibration sensor networks.
For the sensor networks system shown in fig.2, the reliability is expressed as follows:

$$R_S(T) = e^{\frac{-T}{\sum_{i=1}^{3}\theta_i^*}} \tag{4}$$

The assessment of motor's operating health condition [1] is done based on the feedback from all three types of sensor networks shown in fig.2. From equation (4), it is clear that the reliability of senor networks system shown in fig.2, decreases exponentially with time of operation. Therefore, at time 'T' which is less than actual mission time, the reliability of system may be low/very low compared to designed target reliability. Many times this may leads to ambiguity in sensor network's feedback. All possible cases of ambiguities are shown in table I.

Table 1. Possible ambiguity cases in sensor networks under consideration

SI Network	SII Network	SIII Network
H	H	U
H	U	H
.	.	.
.	.	.
U	U	H

H: sensor network showing motor is in healthy condition.
U: sensor network showing motor is in unhealthy condition.

Suppose, consider the first row of table I i.e., H, H, and U. Here, current and temperature sensor networks are showing motor is in healthy condition. However, vibration sensor network is showing motor operating in unhealthy condition. In this situation, the user monitoring the sensor networks cannot make a proper decision. Therefore, in this ambiguous case, the decision from a sensor network with highest importance factor should be considered by the user for confident decision making.

In the following section, a redesign modelling approach steps which improves the reliability of sensor networks system at ‘T’ hours compared to initial allocation are presented.

5 REDESIGN MODELLING APPROACH FOR SENSOR NETWORKS

Steps

i. Obtain initial design configuration for sensor network to meet given target reliability with minimum cost using the procedure followed in [1].
ii. Evaluate importance factor and expected mean life of each sensor network shown in fig.2. using Eqs. (1) & (2) respectively.
iii. Evaluate reliability of each sensor network and the total sensor networks system reliability using Eqs. (3) & (4) respectively.
iv. Based on current sensor’s network reliability value obtained, evaluate each current sensor’s reliability present in the network.
v. Repeat step iv for temperature and vibration sensor networks.
vi. Put the new reliability values of current, temperature, and vibration sensors in the algorithm [1] with the same initial costs and obtain the new allocation to achieve the initially assigned sensor networks system target reliability.
vii. Update the number of redundancies for each type of sensor based on step vi.

6 ILLUSTRATIVE EXAMPLE

The proposed redesign modelling approach has been illustrated by considering an electric motor which is used as a prime mover of a centrifugal pump. As discussed earlier three types of sensors (current, temperature, and vibration) are used for monitoring the parameters of this motor. The data required for this illustration are presented in the tables 2, 3, & 4. The desired target reliability of sensor network system is 0.9892. It is proposed that the system (motor & sensor networks) be observed for 12 hours continuous operation and the information thus collected be used for further improved reliability design. Develop solutions to fulfil the following objectives:

a) Evaluate the expected mean life of each sensor network using the data collected on FAD, FID, & importance factors during 12 hours of operation.
b) Reliabilities of each sensor network at 12 hours.
c) Develop a new allocation that improves the reliabilities of each sensor network at 12 hours than the reliabilities obtained in (b).
d) Evaluate expected mean life and reliability of each new allocated sensor network and compare the same with the values obtained for (a) & (b).

The data required for this illustrative example are given in [1].

Table 2. Reliabilities & costs of sensors

Sensor type	**Reliability**	**Cost ($)**
Current	0.84393	15
Temperature	0.94690	22
Vibration	0.63580	30

Table 3. Threshold levels at normal load condition

S. No	**Sensor type**	**Threshold**		**Units**
		LL	***UL***	
1	Current	3	6	amps
2	Temperature	30	90	^{0}C
3	Vibration	2	5	g

Table 4. Threshold levels at full load condition

S. No	**Sensor type**	**Threshold**		**Units**
		LL	***UL***	
1	Current	6	9	amps
2	Temperature	90	150	^{0}C
3	Vibration	5	8	g

UL: upper limit; *LL*: Lower limit

Solution: - (a) The FAD for the system is defined as follows:

0.1: for any one or more of the following situations:

- Motor current consumption > 9 amperes but current sensor network readings are showing 3 to 6 amperes.
- Motor temperature >150^{0}C, but temperature sensor network readings are showing 30 to 90^{0}C.
- Motor vibration > 8 g, but vibration sensor network readings are showing 2 to 5 g.

Due to above reasons, significant system failure occurs that can result in injury, or major damage

0.5: Motor operating in full load region, but all sensor networks' readings are showing motor is in healthy region. Due to this reason, complete loss of system (motor) may occur; performance is unacceptable.

0.8: for any one or more of the following situations:

- Motor current consumption is 3 to 6 amperes, but current sensor network readings are showing > 9 amperes.
- Motor temperature is 30 to 90^0 C, but temperature sensor network readings are showing $>150^0$ C.
- Motor vibration is 2 to 5 g, but vibration sensor network readings are showing > 8 g.

Due to these reasons, user turns off the motor which causes no output from centrifugal pump. This leads to partial economical losses and impede progress of work.

1: Motor operating in healthy region, but all sensor network's readings are showing motor is in full load region. Due to this reason, minor economical losses occur, with no effect on acceptable system performance.

As discussed in section 4, FID for the system under consideration is defined as unity.
The following tables show the values defined for FAD & FID for the system under consideration operating for 12 hours. Readings are observed for every 3 hours of interval.

Table 5. FDD & FID for sensor networks operated for 12 hours

	Fault acceptance degree				**Fault influence degree**			
Sensor type	**3 hr**	**6 hr**	**9 hr**	**12 hr**	**3 hr**	**6 hr**	**9 hr**	**12 hr**
SI	0.1	0.1	0.1	0.1	1	1	1	1
SII	NF	0.8	0.5	0.1	NF	1	1	1
SIII	NF	NF	0.8	0.8	NF	NF	1	1

NF: No fault

According to the Wang et al. [2], the importance factor (α) for each sensor network is evaluated as follows:

$\alpha_{SI} = \frac{1}{4}[0.1, 0.1, 0.1, 0.1]\,[1, 1, 1, 1]^T = 0.100$

$\alpha_{SII} = \frac{1}{3}[0.1, 0.8, 0.5]\,[1, 1, 1]^T = 0.466$

$\alpha_{SIII} = \frac{1}{2}[0.8, 0.8]\,[1, 1]^T = 0.800$

Table 6. Initially allocation of importance factor & rankings

Sensor network type	**(α)**	**Ranking**
SI	0.100	3
SII	0.466	2
SIII	0.800	1

Then, using Eq. (2), we get the expected mean life of current sensor network as follows:

$$\theta^*_{SI} = \frac{12 \times 0.1 \times 12}{-4 \times \ln(0.989)} = 325.47 \text{ hours}$$

Similarly, for temperature and vibration sensor networks we get 2022.24, 2083.00 hours respectively.
The expected lives of sensor networks are tabulated in table 7.

Table 7. Expected mean life of each sensor network

Sensor network type	θ (in Hours)
Current	325.47
Temperature	2022.24
Vibration	2083.00

(b) Using Eq. (3), we get the reliabilities of each sensor network at 12 hours of operation. The reliabilities along with the initial allocation of each sensor network are presented in table 8.

Table 8. Reliability of each sensor network

Sensor network Type	Initial allocation	Initial network reliability	Network reliability at 12 hours
Current	4	0.99940	0.9638015
Temperature	3	0.99984	0.9940835
Vibration	5	0.99359	0.9900210

(c) Following the steps discussed in section 3, we get the following new allocation to each sensor network for achieving the target reliability of ≥ 0.9892.

New allocation: 7, 6, 8
New Cost: 477$

(d) The FAD & FID for new allocation of each sensor network are defined in table 9.

Table 9. FDD & FID for new allocated sensor networks for 12 hours

	Fault acceptance degree				Fault influence degree			
Sensor type	3 hr	6 hr	9 hr	12 hr	3 hr	6 hr	9 hr	12 hr
SI	NF	0.8	0.5	0.5	NF	1	1	1
SII	NF	NF	0.8	0.8	NF	NF	1	1
SIII	NF	1	1	1	NF	1	1	1

NF: No fault

For the data (given in table 9), the improved importance factor and expected mean life of each sensor network for new allocation are tabulated in table 10.

Table 10. Importance factors & expected mean life of sensor network

Sensor network Type	(α)	θ (in Hours)
Current	0.6	1952.81
Temperature	0.8	3037.71
Vibration	1	2847.85

Reliability of each sensor network (for new allocation) at 12 hours is as shown in table 11.

Table 11. Reliability of sensor network at 12 hours

Sensor network Type	New allocation	Initial network reliability	Network reliability at 12 hours
Current	7	0.997103	0.993873
Temperature	6	0.999994	0.996057
Vibration	8	0.998421	0.995795

Therefore, new allocation of sensor networks provides the improved reliabilities at 12 hours compared to initial allocation of sensor networks. The mean life of current, temperature and vibration sensor networks is also increased considerably.

7 DISCUSSION

If ambiguity cases like the one mentioned in table 1 arises, we give first priority to SIII network for confident decision making based on rankings mentioned in table 6. It is proposed to replace the sensor networks periodically at times given in table 7 for initial allocation. The improved expected life of each sensor network is also presented in table 9. This procedure can be repeated for further improvement of sensor configuration.

8 CONCLUSIONS

A methodology is proposed in this paper to improve the initial optimally allocated sensor networks reliabilities for critical systems. Additional information such as fault acceptance degree (FAD), fault influence degree (FID), importance factor, and the actual operational data of the critical system for an initial operational period are used for this purpose. The methodology proposed for estimation of expected life of sensor networks, can be effectively utilized for planning preventive replacements of sensor networks used for critical systems such as: propulsion systems of a satellite, nuclear power plants, aircraft systems, and turbines of thermal power plants etc.

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DEVELOPMENT OF THE METHOD OF PREDICTION PARAMETER OF RELIABILITY CHEMICAL CURRENT SOURCES OPERATING IN A "SESSION" MODE

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Abstract

The paper describes the calculation method of reliability and conservability products class of chemical current sources (CCS) in the design to the specific conditions in the electronic means. The technical specification for the indicators of CCS reliability are for specific operation modes and their use in the calculation gives a large error. In the U.S. (MIL-HDBK-217F, Telcordia (Bellcore) SR 332), French (CNET RDF-2000), English (British Telecom HRD5) and Chinese (GJB/z 299B) references to the reliability of electronic devices there is no information for the calculation of reliability and conservability CCS, no calculation models.

The cumulative accounting model of physical factors affecting the calculated capacity of the chemical current sources, which is the main factor affecting the reliability is shown.

1. Introduction

Electronic devices (ED) for different purposes entered widely everyday life: ranging from cell-phones, laptops, medical devices, vehicles and ending spacecraft. Almost all of them can be used in stand-alone mode and uses as a power source chemical current sources (CCS). Most often CCS are divided by ability or inability to reuse: primary CCS (eg, nickel-cadmium batteries), secondary CCS (eg, lead-acid batteries) and electrochemical generators. Based on the harsh environment of the ED, such as a wide range of changes in ambient temperature (from -40 °C to +50 °C), to predict working capacity of ED, appears necessary to study the reliability of CCS for specific operating conditions (or a model of exploitation).

Therefore, there is the problem of assessing the reliability associated with the fact that the electronic part of the ED (resistors, capacitors, integrated circuits, etc.) that is responsible for the basic functions of the device can be determined by the methods described in the references [1, 5-9], and determine the reliability of the power supply CCS tied to specific conditions is not possible or is of great complexity. At present there is only one way - statistics on the reliability CCS in reference [1], but they do not allow us to estimate an accurate picture of the behavior of the element in accordance with the model of operation even in the later stages of design. No models for calculating CCS in references [5-9]. In this situation, the only solution is tests that cost developers in larger budgets.

Therefore, a fundamental solution to this problem is required: to create a method of forecasting performance of the reliability of CCS in the early stages of design.

In addition, according to tests of CCS various technological groups [4] were found that the greatest influence on the reliability is provided by:

- operating mode, i.e. during the discharge CCS, given to the load capacity at a given operating voltage is not less than the specified element to the load;
- The moment when a CCS on discharge;
- Storage mode (standby) mode, i.e. duration of storage.

For more details on the types of distributed denial of CCS class is shown in Fig. 1, where:

- Short circuit - 8%;
- Leaking electrolyte - 17%;
- Reduction of capacity – 43%;
- Swelling, depressurization - 17%;

- Assembly failures - 5%;
- Other - 10%.

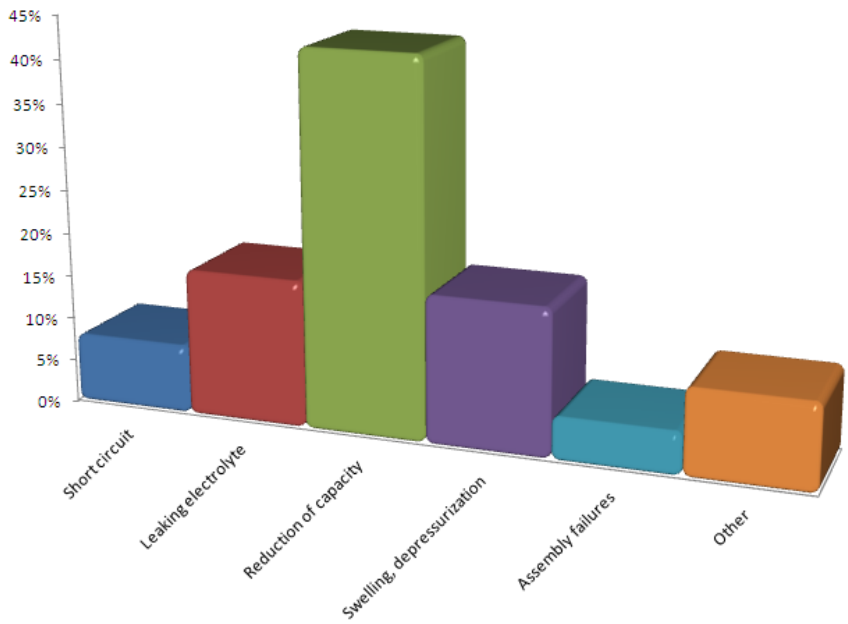


Figure 1. Percentage distribution by type of failure

2. The basic principles of construction of the calculation model of reliability and conservability for ED CCS

Indicators of reliability and conservability for the electronic part of ED is calculated by model, given in references [1, 5-9], depending on the country of origin of electrical product (EP).

Using as the basis mathematical models of EP [1, 5-9] for calculations reliability for CCS is not possible, because the elements are referred to as electro-chemical power products often combine both types of faults (sudden and gradual). And to a great extent, as the distribution by type of CCS failure (see Fig. 1), dominated by the gradual failure (degradation factors and aging). Actual sudden failure can happen, but in parallel is the "aging" of the element, which leads to phase out, if it has not had a sudden failure. Therefore, the mathematical model of reference [1, 5-9] can not be used as a basis for assessing CCS. Improvement the method of calculation of reliability and conservability is needed.

Based on the above analysis, these elements can be considered as consisting of two parts, one of which can only occur sudden failure, and in the other - only gradual. Elements work until the first of these failures. If $P_s(t)$ - the probability that sudden failure does not happen during time t, and $P_g(Z, t)$ - the probability that during time t the value of the safety factor for the centered container (key parameter CCS) will remain within the limits (set limits), i.e. does not happens gradual failure, the assumption that failures occur independently of each other, we find that the reliability function of CCS is:

$$P(Z,t) = P_s(t) \cdot P_g(Z,t) \qquad (1)$$

where:

$$P_s = \begin{cases} 0{,}999 \\ 0{,}9999 \end{cases}$$

- it is the probability of failure-free state at the time of CCS engagement from reference [1], which is determined by the results of tests;

$$P_g = \frac{1}{\sqrt{2\cdot\pi}} \int_{-\infty}^{Z} e^{\frac{-x^2}{2}} dx$$

- it is the probability of failure-free operation, defined by the normal distribution [3];
Z - normalized centered safety factor of capacity, a random variable.

To introduce the opportunities for a separate assessment of reliability and storage, put forward the following premise: "If the state probability at session mode λ_s spread out on two independent events - the probability of failure-free operation in the storage mode λ_{st} and the reliabilities in operation λ_{op}, and calculated using the following model with an exponential distribution [3]:

$$\begin{cases} \lambda_s = \dfrac{\lambda_{op}\cdot t_{op} + \lambda_{st}\cdot t_{st}}{t_{op}+t_{st}}; \\ \lambda_{tr} = \dfrac{\ln(P_s(t)\cdot P_g(Z,t_s))}{t_s}; \\ \lambda_s = \lambda_{tr}, \text{ provided that } t_{op}+t_{st}=t_s. \end{cases} \tag{2}$$

where: t_{op} – operating time, t_{st} – storing time; t_{op} + t_{st} = t_s – CCS operating time; λ_{tr} – traditional, calculated by reference [1].

When determining the indicators of reliability and conservability is necessary to divide the model into two operation mode to the power on the discharge, based on the extended postulate. Thus, we have the following general model of reliability:

$$P_g(Z,t) = \begin{cases} P_{op}(Z,t) = P_s \cdot P_{g.op}(Z,t); \\ P_{st}(Z,t) = P_s \cdot P_{g.st}(Z,t), \end{cases} \tag{3}$$

where: $P_{g.op}$ and $P_{g.st}$ – defined by the normal distribution, as in the model (1).

Thus, evaluating the reliability of a CCS, take into account the mode of operation, we are interested in a particular case: only storage, only operation; session mode, alternate storage and operation.

3. The calculation of the failure rate during operation and storage mode

CCS key parameter, as the statistical analysis shows in Fig. 1, is the average electrical capacity, determined by cumulative formula:

$$C = C_t - A_t\cdot\Delta_t + A_j\cdot\Delta J - A_{st}\cdot t_{st}^{'} - A_c\cdot N_c, \tag{4}$$

where: A_t, A_j, A_{st}, A_c – table gradients of changes in capacitance of temperature, duration of storage, discharge current and best practices in the charge-discharge cycles; C_t - the average capacity of CCS; $\Delta t = t_f$ - 20°C where t_f - functioning temperature; $\Delta J = J_{req}$ - J_f, where: J_{req} - the required discharge current, J_f - actual critical value of the discharge current; t_{st} – storage time.

Empirical model (4) is to be divided according to the mode of operation to a simple model provided the independence of the two events. A simplified calculation model:

- Average electrical capacitance for the storage is a function of the storage time and the discharge current and is given by:

$$C(J_f, t_{st}) = C_t + A_j\cdot\Delta J - A_{st}\cdot t_{st} \tag{4.1}$$

where: t_{st} - the actual storage time.

- Average electrical capacity for the mode of operation is defined by the formula:

$$C(T, \Delta J, N_c) = C_t - A_t \cdot \Delta T + A_j \cdot \Delta J - A_c \cdot N_c, \tag{4.2}$$

Normalized centered safety factor for the capacitance is determined by the formula:

$$Z = \frac{(C - C_{req})}{\sigma}, \tag{5}$$

where: C - result from the formula (4.1) or (4.2), the capacitance value of CCS; C_{req} – the required level of capacity in the application of CCS, a reference value; σ – standard deviation value of CCS capacity, defined by the formula:

$$\sigma = \sigma_t \cdot \frac{(B_t \cdot B_j)}{(B_{st} \cdot B_c)},$$

where:

$$B_j = \frac{C_j}{C_t}$$

- coefficients, representing a ratio of capacity C_j, calculated by the formula (3) taking into account only the *j*-th factor to the specified value average capacity C_t.

σ_t - table value of standard deviation, for the type, reference value.

For storage conditions formula for the standard deviation value of CCS capacity looks as follows:

$$\sigma = \sigma_t \frac{B_j}{B_{st}}$$

Respectively for the operation:

$$\sigma = \sigma_t \frac{B_t \cdot B_j}{B_c}$$

Reliabilities estimated by the formula (3).

The problem of direct calculation of the model (1) and (3) is that the improper integral can not be solved by simple methods to get the recurrence formula, required for automation, so polynomial regression methods used to obtain the recursive formula for calculating the probability of failure-free operation CCS, depending on the parameter.

The failure rate is defined as:

$$\lambda(t) = \frac{P_s \dfrac{\partial P_{g.st}(Z,t))}{\partial t}}{P_{g.st}(t)} \tag{6}$$

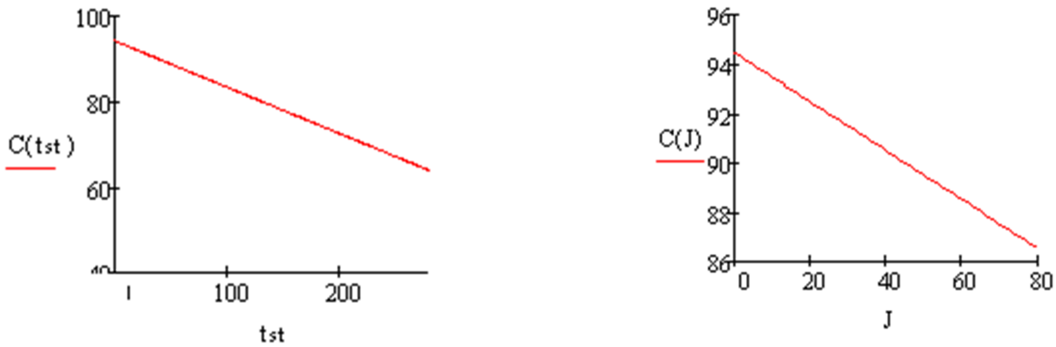
Or, by using the exponential distribution [3], we obtain the following relation to calculate the failure rate:

$$\lambda 1(t) = -\frac{\ln(P_s P_{g.st}(Z,t))}{t_{st}} \tag{7}$$

For example, we estimate the failure rate in the storage mode and operation of the battery related to the technological lead acid batteries group [10] - one of the groups most commonly used in vehicles [4].

As shown above, the reliability of the battery is affected by ambient temperature, discharge current, the number of charge-discharge cycles, the shelf life and storage conditions.

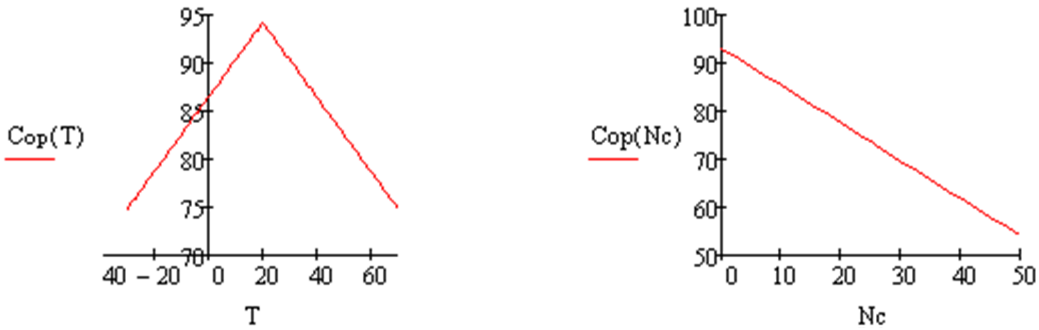
Plot the dependence $C(J_f, t_{st})$ of the model (4.1) (Fig. 2). Fig. 2a shows that with the increase of storage time, the average capacitance decreases linearly. With the increase of the discharge current, $C(J)$ also decreases linearly (Fig. 2b).



a) b)

Figure 2. Dependence of the average capacitance of storage time and discharge current: a) *C(tst)*; b) *C(J)*

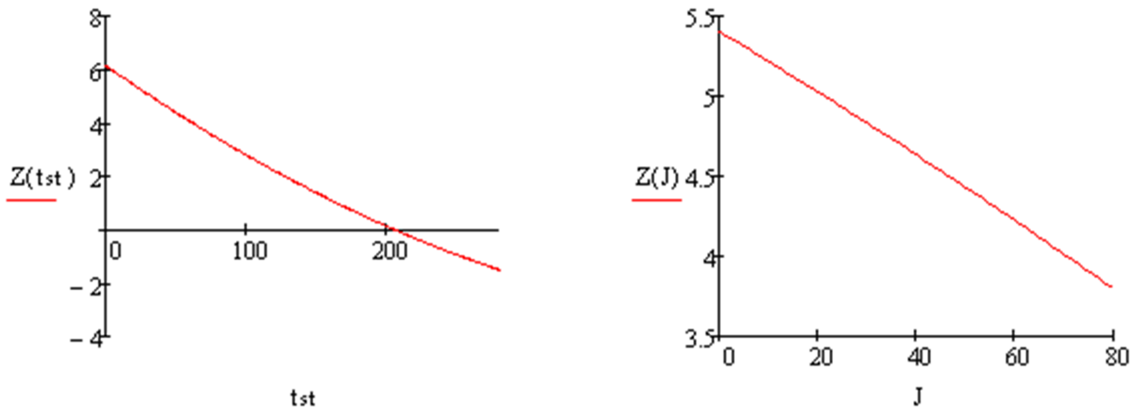
For operation (model (4.2)) plot the dependence of *Cop(T,Nc)*. Fig. 3a shows the dependence of *Cop(T)*. It is seen that up to *T*=+20°*C* schedule capacity increases linearly (*T*=+20°*C* corresponds to normal operating conditions), that is, at *T*=+20°*C* average electrical capacity reaches its maximum value, and then begins to decline linearly.



a) b)

Figure 3. Dependence of the average capacitance of the temperature and the number of charge-discharge cycles in operation: a) *Cop(T)*; b) *Cp(Nc)*

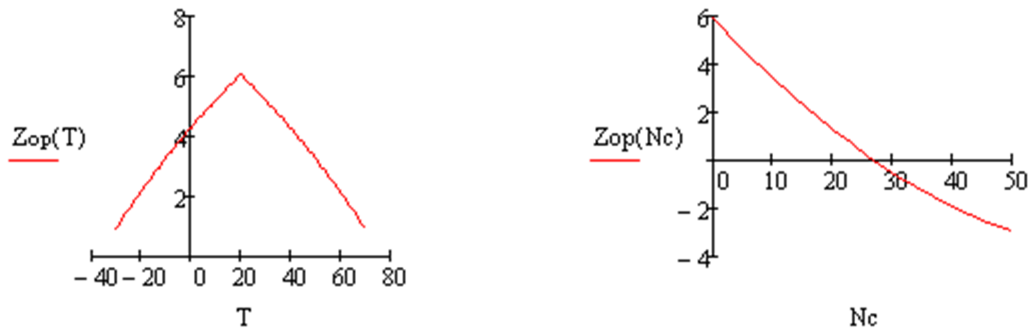
Using equation (5) plot the dependence of the normalized centered safety factor of the capacity for mode of storage and the mode of operation. Fig. 4 shows the dependence *Z(tst, J)* (storage mode). With the passage of time in storage mode normalized centered safety factor for the capacity drops almost linearly (Fig. 4a). The same can be said about the dependence of *Z(J)* (Fig. 4b).



а) б)

Figure 4. Dependence of the normalized centered safety factor in capacity Z(tst, J) in the storage mode: a) Z (tst); b) Z (J)

Fig. 7 shows the normalized centered safety factor for the capacity for the operation mode. Change in *Zop(T)* (Fig. 5a) corresponds to the variation *Cop(T)* (Fig. 3a). *Z(Nc)* (Fig. 5b) decreases with increasing number of cycles of charge discharge.



а) б)

Figure 5. Dependence of the normalized centered safety factor in capacity *Z(T, ΔJ, Nc)* in operation mode: a) *Zop(T)*; b) *Zop(Nc)*

Recursive formula for calculating the probability of failure-free operation of CCS during storage time t_{st}, obtained with the use of polynomial regression is:

$$P_{g.st}(t_{st}) = 0.999999 - 2.626\cdot 10^{-3}\cdot t_{st} + 6.786\cdot 10^{-5}\cdot t_{st}^{\ 2} - 4.76\cdot 10^{-7}\cdot t_{st}^{\ 3} + 5.724\cdot 10^{-10}\cdot t_{st}^{\ 4} + 7.959\cdot 10^{-13}\cdot t_{st}^{\ 5} \quad (8)$$

Fig. 6 shows the dependence of the probability of failure in the storage mode *P(tst)* (solid line) and dependence $P_{g.st}(t_{st})$ (dashed line), built on the model (1). As we can see diagrams of dependence of the models (1) and (8) virtually identical.

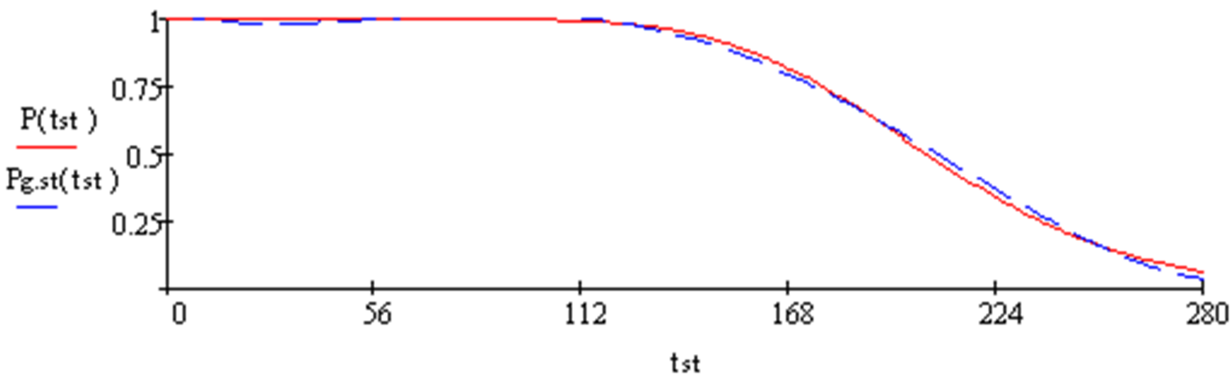


Figure 6. Dependence *P(tst)*, dependence *Pg.st(tst)*

Types of charts for the probability of failure-free operation of the discharge current, the number of charge-discharge cycles, and temperature are shown in Fig. 7-9.

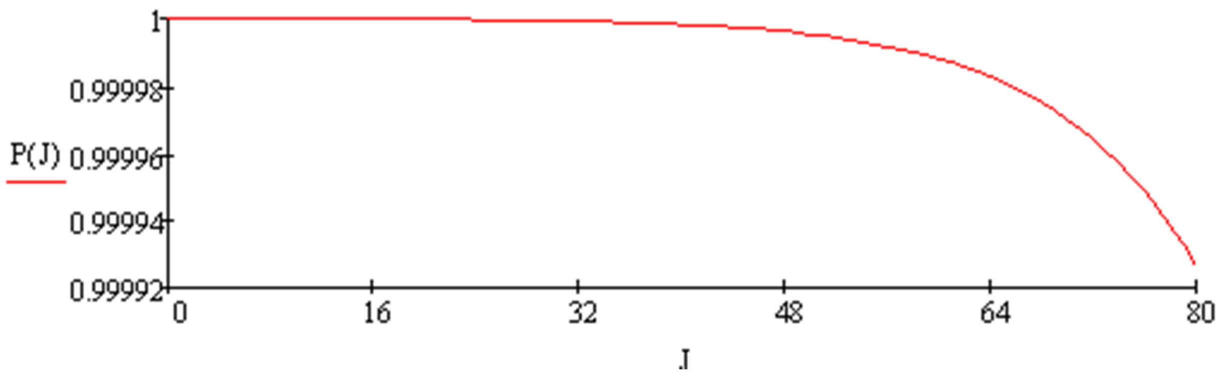


Figure 7. The dependence *P (J)*

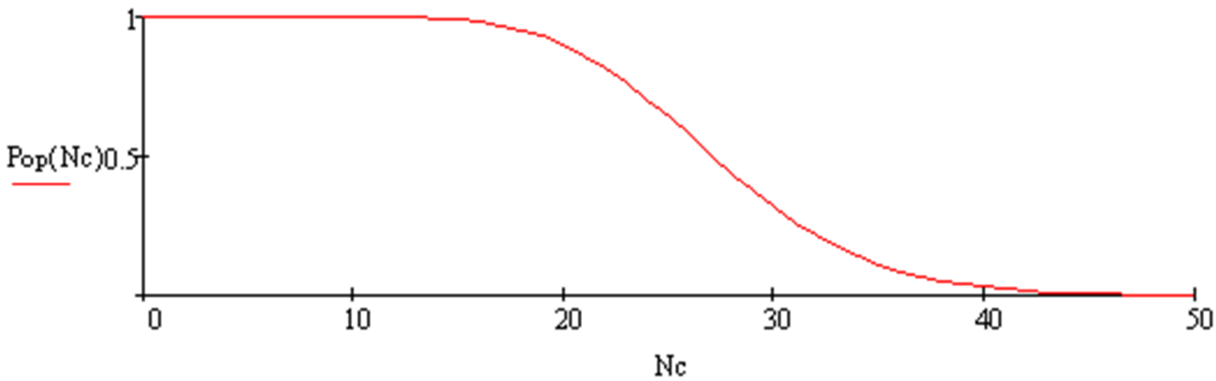


Figure 8. Dependence *Pop(Nc)*

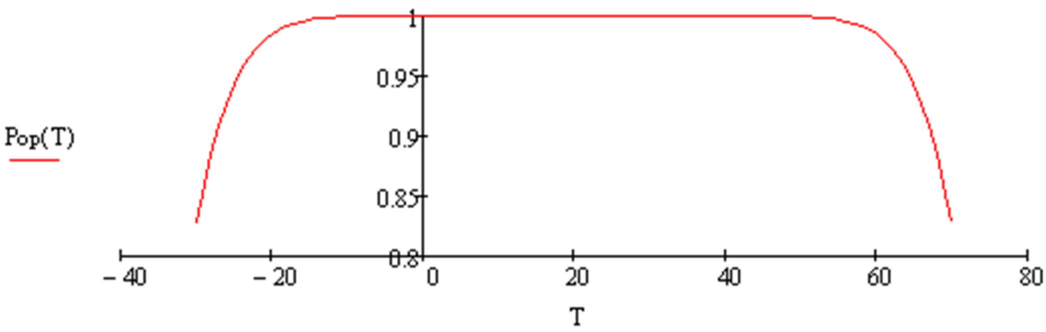


Figure 9. Dependence *Pop(T)*

Next, estimate the intensity to phase in the storage mode, depending on the storage time (see Fig. 10).

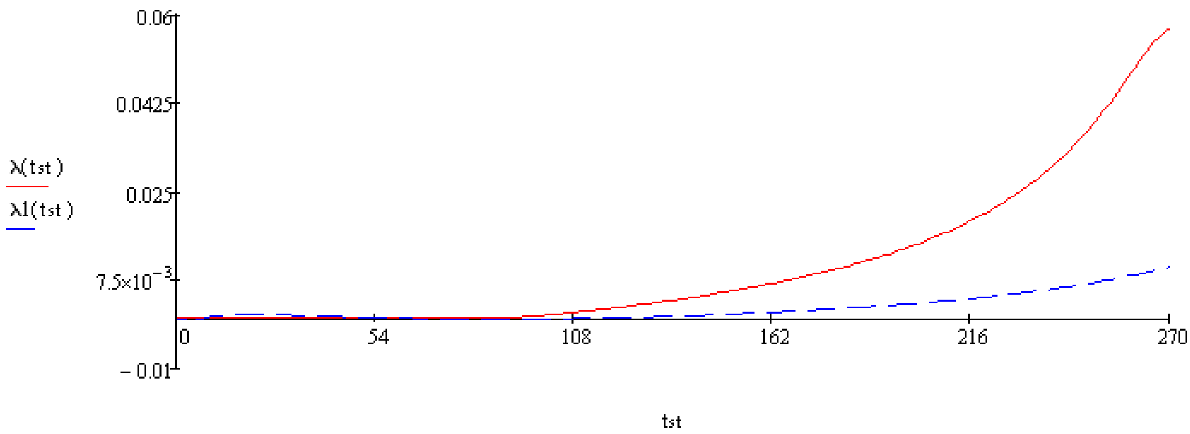


Figure 10. The dependence of the failure rate in the storage mode of storage time

Fig. 10 clearly shows that the warranty period for the estimated storage battery (≈ 100 months) plots of the failure rate of the storage time, constructed in two different models (models (6) and (7)) are practically the same, a sharp contrast to begin after the warranty period, when models already in use there is no need. This gives reason to believe that the assessment of the conservability of CCS can use the exponential distribution.

Types of dependence $\lambda(J)$, $\lambda op(T)$, $\lambda op(Nc)$ are shown in Fig. 11-13.

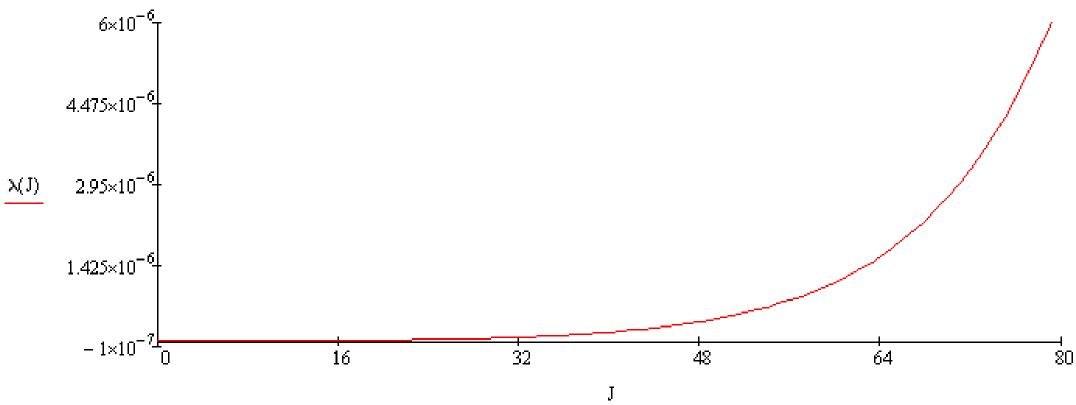


Figure 11. The dependence of the failure rate of the discharge current

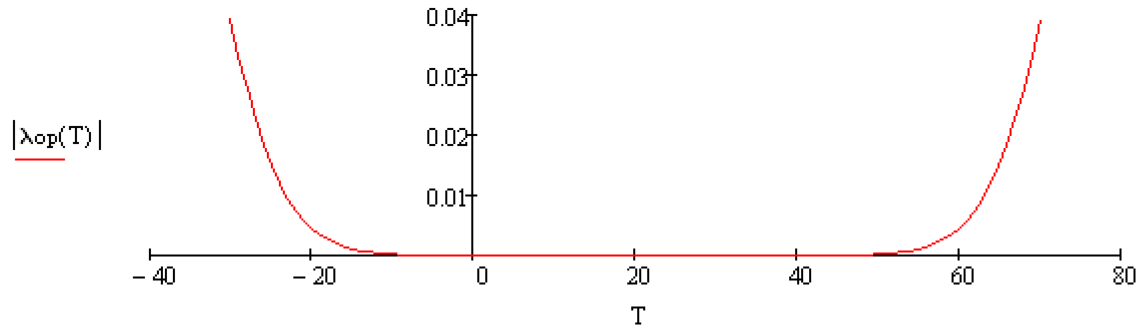


Figure 12. The dependence of the failure rate of the operating temperature

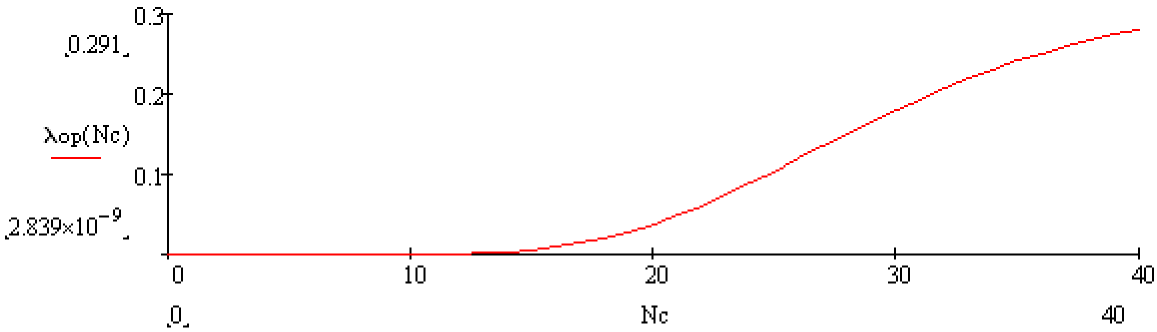


Figure 13. The dependence of the failure rate of the number of charge-discharge cycles

Now, to prove the postulate given in section 2, and the validity of the separation model (4), calculate the failure rate for the battery in the session mode in two ways and define the error.

1. Using the model (4) calculate λ. Storage time *tst* = 1 month, the number of cycles *Nc* = 1 .. 100. A dependence plot of λ of the number of cycles is shown in Fig. 14 (dashed line).
2. Calculate the *λs*, *λst* and *λop*, using the position of the postulate and of the models (4.1) and (4.2). Dependence *λs(Nc)* is shown in Fig.14 (solid line).

Taking *λ(Nc)* for the truth, we calculate the error:

$$\delta = \frac{\lambda(Nc) - \lambda_s(Nc)}{\lambda(Nc)} \cdot 100\%$$

The average error between the two methods in assessing the failure rate for the active lifetime is:

$$\delta_{ave} = \frac{\sum_{Nc=1}^{100} \delta}{100} \cdot 100\% = 9,3\%$$

The resulting value of the error is within acceptable engineering error (5-10%).

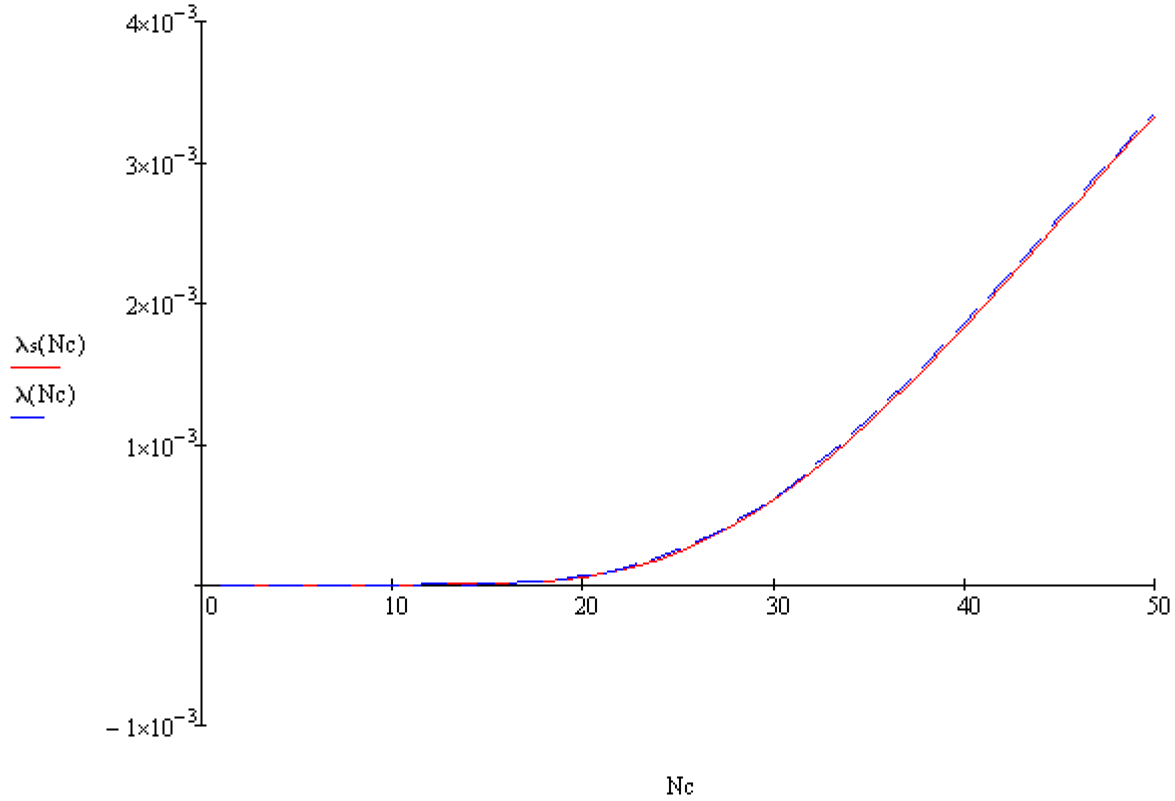


Figure 14. Dependence the failure rate *λtr(Nc)* - according to the traditional formula (1), *λs(Nc)* by (7)

4. The influence of parameters on the failure rate

The results of analysis of the influence of various parameters on the failure rate are presented in Fig. 15. As can be seen from the histogram, the strongest gradual failures depend on the number of charge-discharge cycles, which is consistent with the principle of battery life. Least contribution to gradual failure makes the discharge current.

To evaluate the accuracy of the proposed method construct on the same graph (Fig. 17) the calculated (solid line) failure rate, and reference (dotted line) as a function of time. Supplemental failure rate is constant, independent of any parameters, while, as in real-life conditions, each of the main parameters - temperature, time, discharge current and the number of charge-discharge cycles - will make its contribution to the gradual failure. The point of intersection of the graphs shows the guaranteed life of the product.

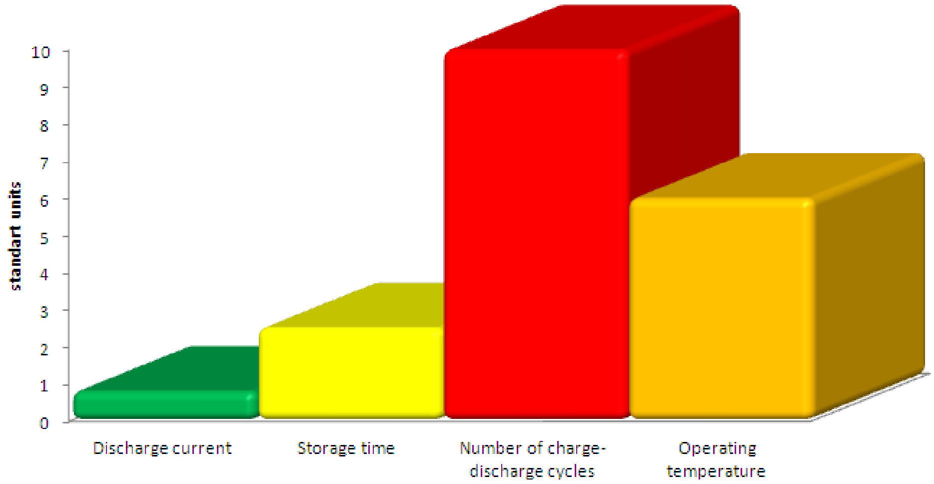


Figure 15. Histogram parameters influence on the value of the failure rate

5. Conclusions

Divide the graph into two parts: I - to the intersection graphs (short working time), II - after the intersection (life in excess of the guaranteed). On the section I the error, between the calculated and experimental values of the failure rate at time *t*1 = 24 will be:

$$\delta_1(under\ t_{st} = t_1 = 14) = \frac{\lambda 2 - \lambda(t = 14)}{\lambda 2} = 1\%$$

On the section II error between the calculated and experimental values of the failure rate at time *t*2 = 70 will be:

$$\delta_2(under\ t_{st} = t_2 = 70) = \frac{\lambda(t = 70) - \lambda 2}{\lambda 2} = 22\%$$

Thus, the error of this method ranges from 1% to 22% (this is the average score, with increasing time, the error will continue to grow). If the equipment is functioning a short time (or before the testing time before failure), the experimental failure rate gives even higher valuation. If it is required to assess reliability in a long period of time, perhaps even more than your initial period of service, the reference data, not taking into account the specific conditions of use, do not correspond to the real situation.

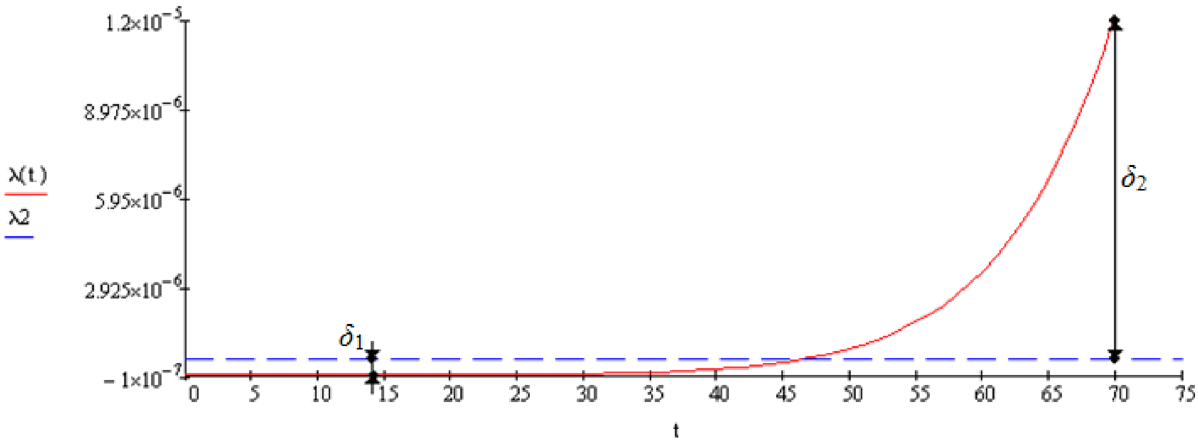


Figure 16. The failure rate obtained by the proposed model and the failure rate of reference

The error between the proposed method and the failure rate in terms of CCS technical conditions, an average of 1 - 22% for a given period of active existence. While the reference failure rate is constant, the value obtained by the proposed model in the article, is changing over time, which more accurately reflects the true picture of the reliability of any products, including CCS.

The substantiation of the possibility of separation failure rate in the session mode, the failure rate in the mode of operation and the failure rate in the storage mode is shown. The average error (at a hundred measurements) in this case is 9.3%, i.e. is within acceptable limits of engineering accuracy.

Proceeding from the premise that the main parameter is the average CCS capacitance model was developed predicting uptime hit, taking into account the main parameters that influence the capacity of: temperature exploitation, storage time, the number of charge-discharge cycles, the discharge current. Reduced reliability prediction CCS model allows more accurate assessment of the failure rate based on the model of operation (storage mode, operating mode, the session mode) in the specific electronic means. This model allows us to estimate the effect of various specifications of storage / work mode on the failure rate of CCS.

Given above mathematical model for predicting the reliability of CCS has been added to the base of mathematical models of «Automated system reliability and quality" [2].

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