RELIABILITY MEASURES OF SYSTEMS WITH LOCATION-SCALE ACBVE COMPONENTS

B. Chandrasekar[1] and T.A. Sajesh[2]

•

[1]Loyola College, Chennai, India
[2]St. Thomas' College, Thrissur, India

e-mail: bchandrasekar2003@yahoo.co.in,
sajesh.abraham@yahoo.com

ABSTRACT

Block and Basu (1974) proposed an absolutely continuous bivariate exponential distribution whose marginals are weighted average of exponentials. Chandrasekar and Sajesh (2010) considered location, scale and location-scale families arising out of absolutely continuous bivariate exponential (ACBVE) distribution with equal marginals and derived the minimum risk equivariant estimators of location, scale and location-scale parameters. In this paper we consider a two-component system when failure times follow location-scale ACBVE distribution with equal marginals. We obtain the reliability performance measures of two-component parallel, series and standby systems. Also we provide the UMVUE, the MLE and the MREE of these reliability performance measures.

Keywords and phrases: Bivariate exponential, location – scale, reliability measures, two-component system.

1 Introduction

Evaluating performance measures associated with systems having dependent component failure times is rare in the literature. In this paper we consider a two-component system when failure times follow location – scale absolutely continuous bivariate exponential (ACBVE) distribution with equal marginals. Chandrasekar and Sajesh (2010) discussed about the equivariant estimation for parameters of location-scale exponential models. We obtain the reliability performance measures of the two component systems. Also we discuss the estimation of these reliability performance measures.

The plan of the paper is as follows: Section 2 provides some definitions and notations required in this paper. Some distributional results are discussed in Section 3. In Section 4 we obtain the reliability performance measures of two-component parallel, series and standby systems when the component failure times follow location – scale ACBVE with equal marginals. Section 5 provides the UMVUE, the MLE and the MREE of the reliability performance measures.

2 Preliminaries

Consider a two component system with failure times T_1 and T_2 respectively. Assume that (T_1, T_2) follows location – scale ACBVE $(\alpha, \beta, \xi, \tau)$ with pdf

$$f(t_1, t_2; \alpha, \beta, \xi, \tau) = \frac{(\alpha+\beta)(2\alpha+\beta)}{2\tau^2}\exp\left[-\frac{1}{\tau}\{\alpha(t_1+t_2)+\beta(t_1\vee t_2)-(2\alpha+\beta)\xi\}\right],$$

$$\alpha, \beta \text{ fixed}, \xi\in R, \tau>0; t_1\wedge t_2>\xi. \qquad (1)$$

It is assumed that (α, β) is known.

Suppose we observe n identical systems with observations (t_{1i}, t_{2i}), i = 1, 2, 3,…, n. Then the joint pdf of the sample is

$$p_{\xi,\tau}(t_1, t_2) = \left\{\frac{(\alpha+\beta)(2\alpha+\beta)}{2\tau^2}\right\}^n \exp\left[-\frac{1}{\tau}\sum_i\{\alpha(t_{1i}+t_{2i})+\beta(t_{1i}\vee t_{2i})-(2\alpha+\beta)\xi\}\right],$$

$$\operatorname*{Min}_i(t_{1i}\wedge t_{2i})>\xi;\ \xi\in R, \tau>0. \qquad (2)$$

It is easy to observe that the maximum likelihood estimator (MLE) of (ξ, τ) is given by $\left(\hat{\xi}, \hat{\tau}\right)$, where

$\hat{\xi} = \underset{i}{\mathrm{Min}}\,(t_{1i} \wedge t_{2i})$ and $\hat{\tau} = \frac{1}{n}\sum_{i}\left\{\alpha(t_{1i} + t_{2i}) + \beta(t_{1i} \vee t_{2i}) - (2\alpha+\beta)\hat{\xi}\right\}$.

3 Distributional results

Theorem 1: Let (T_1, T_2) follow location – scale ACBVE (α, β, ξ, τ) with pdf given in (1). Then

$(T_1 \wedge T_2) \sim E\left(\xi, \frac{\tau^2}{2\alpha+\beta}\right)$. Here E (a, b) refers to a random variable with pdf $\left(\frac{1}{b}\right)\exp\left\{-\frac{1}{b}(x-a)\right\}$, $x > a;\ a \in R,\ b > 0$.

Proof: For $u > \xi$,

$$P_\xi[T_1 \wedge T_2 > u] = P_\xi[T_1 > u, T_2 > u]$$
$$= \frac{(\alpha+\beta)(2\alpha+\beta)}{2\tau^2}\int_u^\infty\int_u^\infty \exp\left[-\left(\frac{1}{\tau}\right)\{\alpha(x+y) + \beta(x \vee y) - (2\alpha+\beta)\xi\}\right] dt_1\, dt_2 \qquad (3)$$

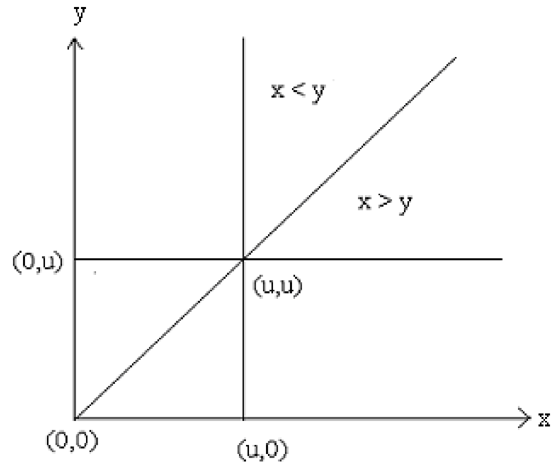

Figure 1

Using Figure 1, equation (3) can be written as,

$$P_\xi[T_1 > u, T_2 > u] = \frac{(\alpha+\beta)(2\alpha+\beta)}{2\tau^2}\int_u^\infty\left\{\int_u^{t_1}\exp\left[-\left(\frac{1}{\tau}\right)\{\alpha(t_1+t_2) + \beta t_1 - (2\alpha+\beta)\xi\}\right]dt_2\right\}dt_1$$
$$+ \frac{(\alpha+\beta)(2\alpha+\beta)}{2\tau^2}\int_u^\infty\left\{\int_{t_1}^\infty\exp\left[-\left(\frac{1}{\tau}\right)\{\alpha(t_1+t_2) + \beta t_2 - (2\alpha+\beta)\xi\}\right]dt_2\right\}dt_1$$
$$= \frac{(\alpha+\beta)(2\alpha+\beta)}{2\tau^2}\left\{\frac{\tau}{\alpha}\int_u^\infty \exp\left[-\left(\frac{1}{\tau}\right)\{(\alpha+\beta)t_1 + \alpha u - (2\alpha+\beta)\xi\}\right]dt_1\right\}$$
$$\left. - \frac{\tau}{\alpha}\int_u^\infty \exp\left\{-\left(\frac{1}{\tau}\right)(2\alpha+\beta)(t_1-\xi)\right\}dt_1 + \frac{\tau}{(\alpha+\beta)}\int_u^\infty \exp\left\{-\left(\frac{1}{\tau}\right)(2\alpha+\beta)(t_1-\xi)\right\}dt_1\right]$$
$$= \frac{(\alpha+\beta)(2\alpha+\beta)}{2\tau^2}\left[\frac{\tau^2}{\alpha(\alpha+\beta)}\exp\{-(2\alpha+\beta)(u-\xi)\} - \frac{\tau^2}{\alpha(2\alpha+\beta)}\exp\left\{-\left(\frac{1}{\tau}\right)(2\alpha+\beta)(u-\xi)\right\}\right.$$
$$\left. + \frac{\tau^2}{(\alpha+\beta)(2\alpha+\beta)}\exp\left\{-\left(\frac{1}{\tau}\right)(2\alpha+\beta)(u-\xi)\right\}\right]$$

$$= \frac{(\alpha+\beta)(2\alpha+\beta)}{2\tau^2}\left[\exp\left\{-\left(\frac{1}{\tau}\right)(2\alpha+\beta)(u-\xi)\right\}\right]\left[\frac{\tau^2\{(2\alpha+\beta)-(\alpha+\beta)+\alpha\}}{\alpha(\alpha+\beta)(2\alpha+\beta)}\right]$$

$$= \exp\left\{-\left(\frac{1}{\tau}\right)(2\alpha+\beta)(u-\xi)\right\}.$$

Therefore, $P_\xi\,[T_1 \wedge T_2 > u] = \exp\left\{-\left(\frac{1}{\tau}\right)(2\alpha+\beta)(u-\xi)\right\}$, $u > \xi$.

Hence $(T_1 \wedge T_2) \sim E\left(\xi, \frac{\tau}{2\alpha+\beta}\right)$.

Theorem 2: Let (T_1, T_2) follow location – scale ACBVE $(\alpha, \beta, \xi, \tau)$ with pdf given in (1). Then

(i) $T_1 + T_2 - 2\xi \overset{d}{=} U_1 + U_2$,

(ii) $T_1 \vee T_2 - \xi \overset{d}{=} U_1 + U_3$ and

(iii) $T_1 \wedge T_2 - \xi \overset{d}{=} U_3$,

where $U_1 \sim E\left(0, \frac{\tau}{\alpha+\beta}\right)$, $U_2 \sim E\left(0, \frac{2\tau}{2\alpha+\beta}\right)$ and $U_3 \sim E\left(0, \frac{\tau}{2\alpha+\beta}\right)$.

Proof: MGF of $(T_1 + T_2, T_1 \vee T_2)$ at (u_1, u_2) is

$m(u_1, u_2) =$

$$\frac{(\alpha+\beta)(2\alpha+\beta)}{2\tau^2}\int_\xi^\infty\int_\xi^\infty \exp\left[-\frac{1}{\tau}\{(\alpha-u_1\tau)(t_1+t_2)+(\beta-u_2\tau)(t_1\vee t_2)-(2\alpha+\beta)\xi\}\right]dt_1\,dt_2$$

$$= \frac{(\alpha+\beta)(2\alpha+\beta)}{2\tau^2}\,\frac{2\tau^2\exp\left[\left\{\left(\frac{2\alpha+\beta}{\tau}\right)\xi - \frac{2(\alpha-u_1\tau)+(\beta-u_2\tau)}{\tau}\xi\right\}\Big/\tau\right]}{\{(\alpha-u_1\tau)+(\beta-u_2\tau)\}\,\{2(\alpha-u_1\tau)+(\beta-u_2\tau)\}}$$

$$= \left(1-\frac{u_1+u_2}{\alpha+\beta}\tau\right)^{-1}\left(1-\frac{2u_1+u_2}{2\alpha+\beta}\tau\right)^{-1}\exp\{(2u_1+u_2)\xi\}.$$

(i) $m(u_1, 0) = \left(1-\frac{u_1}{\alpha+\beta}\tau\right)^{-1}\left(1-\frac{2u_1}{2\alpha+\beta}\tau\right)^{-1}\exp(2u_1\xi)$.

This implies that $T_1 + T_2 - 2\xi \overset{d}{=} U_1 + U_2$, where $U_1 \sim E\left(0, \frac{\tau}{\alpha+\beta}\right)$,

$U_2 \sim E\left(0, \frac{2\tau}{2\alpha+\beta}\right)$ and $U_1 \amalg U_2$.

(ii) $m(0, u_2) = \left(1-\frac{u_2}{\alpha+\beta}\tau\right)^{-1}\left(1-\frac{u_2}{2\alpha+\beta}\tau\right)^{-1}\exp(u_2\xi)$.

This implies that $T_1 \vee T_2 - \xi \overset{d}{=} U_1 + U_3$, where $U_1 \sim E\left(0, \frac{\tau}{\alpha+\beta}\right)$, $U_3 \sim E\left(0, \frac{\tau}{2\alpha+\beta}\right)$

and $U_1 \amalg U_3$.

(iii) From Theorem 1, we have,

$T_1 \wedge T_2 - \xi \overset{d}{=} U_3$, where $U_3 \sim E\left(0, \frac{\tau}{2\alpha+\beta}\right)$.

4 Reliability performance measures

4.1 Parallel system

MTBF: Consider a two-unit parallel system with component failure times T_1 and T_2 respectively. Then the system failure time is

$$T = T_1 \vee T_2.$$

Assume that $(T_1, T_2) \sim$ location – scale ACBVE $(\alpha, \beta, \xi, \tau)$. Then from Theorem 2, we have

$$T - \xi \overset{d}{=} U_1 + U_3.$$

Therefore, MTBF = E (T)

$$= E(U_1 + U_3) + \xi$$

$$= \frac{\tau}{\alpha+\beta} + \frac{\tau}{2\alpha+\beta} + \xi$$

$$= \frac{3\alpha+2\beta}{(\alpha+\beta)(2\alpha+\beta)}\tau + \xi. \tag{4}$$

Reliability function: Consider, $P(T - \xi > t),\ t > 0.$

$$= P(U_1 + U_3 > t),\ t > 0.$$

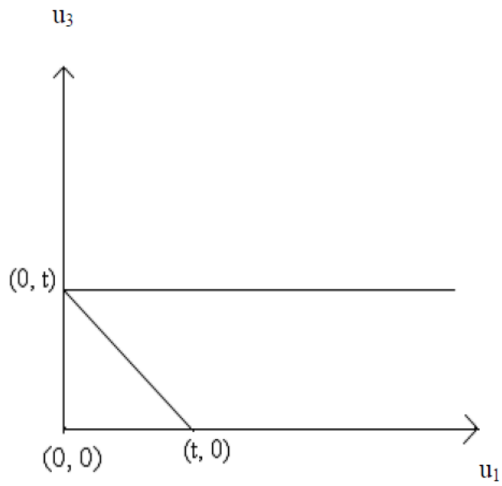

Figure 2

From figure 2 we have,

$$P(T - \xi > t) = \int_0^t \int_{t-u_3}^{\infty} \frac{(\alpha+\beta)(2\alpha+\beta)}{\tau^2} \exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)u_1 - \left(\frac{2\alpha+\beta}{\tau}\right)u_3\right\} du_1 du_3$$

$$+ \int_t^{\infty}\int_0^{\infty} \frac{(\alpha+\beta)(2\alpha+\beta)}{\tau^2} \exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)u_1 - \left(\frac{2\alpha+\beta}{\tau}\right)u_3\right\} du_1 du_3$$

$$= \int_0^t \frac{(2\alpha+\beta)}{\tau} \exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)(t-u_3) - \left(\frac{2\alpha+\beta}{\tau}\right)u_3\right\} du_3\ u_1$$

$$+ \int_t^{\infty} \frac{(2\alpha+\beta)}{\tau} \exp\left\{-\left(\frac{2\alpha+\beta}{\tau}\right)u_3\right\} du_3$$

$$= \left(\frac{2\alpha+\beta}{\alpha}\right)\exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)t\right\}\left\{1 - \exp\left(-\frac{\alpha}{\tau}t\right)\right\} + \exp\left\{-\left(\frac{2\alpha+\beta}{\tau}\right)t\right\}$$

$$= \left(\frac{2\alpha+\beta}{\alpha}\right)\left[\exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)t\right\} - \exp\left\{-\left(\frac{2\alpha+\beta}{\tau}\right)t\right\}\right] + \exp\left\{-\left(\frac{2\alpha+\beta}{\tau}\right)t\right\}$$

$$= \left(\frac{2\alpha+2\beta}{\alpha}\right)\exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)t\right\} - \left(\frac{\alpha+\beta}{\alpha}\right)\exp\left\{-\left(\frac{2\alpha+\beta}{\tau}\right)t\right\},\ t > 0.$$

Therefore,

$$P\ (T - \xi > t - \xi) = \left(\frac{2\alpha+\beta}{\alpha}\right)\exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)(t-\xi)\right\} - \left(\frac{\alpha+\beta}{\alpha}\right)\exp\left\{-\left(\frac{2\alpha+\beta}{\tau}\right)(t-\xi)\right\},\ t > \xi.$$

Then, the reliability function is given by

$$R\ (t) = P(T > t) = \left(\frac{2\alpha+\beta}{\alpha}\right)\exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)(t-\xi)\right\} - \left(\frac{\alpha+\beta}{\alpha}\right)\exp\left\{-\left(\frac{2\alpha+\beta}{\tau}\right)(t-\xi)\right\},\ t > \xi. \quad (5)$$

4.2 Series system

MTBF: Consider a two unit series system with component failure times T_1 and T_2 respectively. Then the system failure time is

$$T = T_1 \wedge T_2.$$

Assume that $(T_1, T_2) \sim$ location – scale ACBVE (α, β, ξ, τ). Then from Theorem 2, we have

$$T - \xi \stackrel{d}{=} U_3.$$

Therefore, MTBF = E (T)

$$= E\ (U_3) + \xi$$

$$= \frac{\tau}{2\alpha+\beta} + \xi. \quad (6)$$

Reliability function: Consider, $P\ (T - \xi > t),\ t > 0$

$$= P\ (U_3 > t),\ t > 0.$$

$$= \int_t^{\infty}\left(\frac{2\alpha+\beta}{\tau}\right)\exp\left\{-\left(\frac{2\alpha+\beta}{\tau}\right)u_3\right\}du_3$$

$$= \exp\left\{-\left(\frac{2\alpha+\beta}{\tau}\right)t\right\},\ t > 0.$$

Therefore,

$$P\ (T - \xi > t - \xi) = \exp\left\{-\left(\frac{2\alpha+\beta}{\tau}\right)(t-\xi)\right\},\ t > \xi.$$

Then, the reliability function is given by

$$R\ (t) = P\ (T > t) = \exp\left\{-\left(\frac{2\alpha+\beta}{\tau}\right)(t-\xi)\right\},\ t > \xi. \quad (7)$$

4.3 Standby system

MTBF: Consider a two unit standby system with component failure times T_1 and T_2 respectively. Then the system failure time is

$$T = T_1 + T_2.$$

Assume that $(T_1, T_2) \sim$ location – scale ACBVE (α, β, ξ, τ). Then from Theorem 3.2, we have

$$T - 2\xi \stackrel{d}{=} U_1 + U_2$$

Therefore, MTBF = E (T)

$$= E(U_1 + U_2) + 2\xi$$
$$= \frac{\tau}{\alpha+\beta} + \frac{2\tau}{2\alpha+\beta} + 2\xi$$
$$= \frac{4\alpha+3\beta}{(\alpha+\beta)(2\alpha+\beta)}\tau + 2\xi. \quad (8)$$

Reliability function: Consider, P (T - 2ξ > t), t > 0.

$$= P(U_1 + U_2 > t),\ t > 0.$$

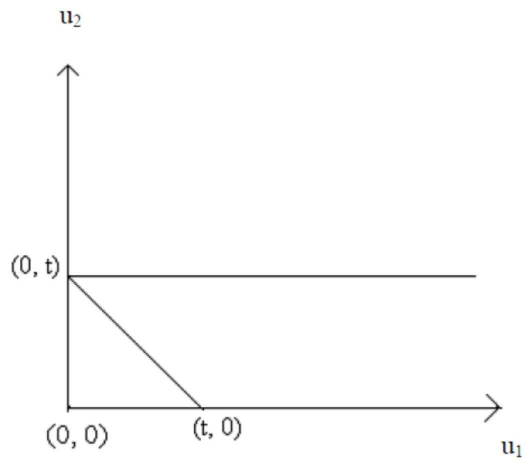

Figure 3

From figure 3 we have,

$$P(T - 2\xi > t) = \int_0^t \int_{t-u_2}^{\infty} \frac{(\alpha+\beta)(2\alpha+\beta)}{2\tau^2} \exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)u_1 - \left(\frac{2\alpha+\beta}{2\tau}\right)u_2\right\} du_1 du_2$$
$$+ \int_t^{\infty}\int_0^{\infty} \frac{(\alpha+\beta)(2\alpha+\beta)}{2\tau^2} \exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)u_1 - \left(\frac{2\alpha+\beta}{2\tau}\right)u_2\right\} du_1 du_2$$
$$= \int_0^t \frac{(2\alpha+\beta)}{2\tau} \exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)(t-u_2) - \left(\frac{2\alpha+\beta}{2\tau}\right)u_2\right\} du_2$$
$$+ \int_t^{\infty} \frac{(2\alpha+\beta)}{2\tau} \exp\left\{-\left(\frac{2\alpha+\beta}{2\tau}\right)u_2\right\} du_2$$
$$= \left(\frac{2\alpha+\beta}{\beta}\right)\exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)t\right\}\left\{\exp\left(\frac{\beta}{2\tau}t\right) - 1\right\} + \exp\left\{-\left(\frac{2\alpha+\beta}{2\tau}\right)t\right\}$$
$$= \left(\frac{2\alpha+\beta}{\beta}\right)\left[\exp\left\{-\left(\frac{2\alpha+\beta}{2\tau}\right)t\right\} - \exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)t\right\}\right] + \exp\left\{-\left(\frac{2\alpha+\beta}{2\tau}\right)t\right\}$$
$$= \left(\frac{2\alpha+2\beta}{\beta}\right)\exp\left\{-\left(\frac{2\alpha+\beta}{2\tau}\right)t\right\} - \left(\frac{2\alpha+\beta}{\beta}\right)\exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)t\right\},\ t > 0.$$

Therefore,

$$P(T - 2\xi > t - 2\xi) = \left(\frac{2\alpha+2\beta}{\beta}\right)\exp\left\{-\left(\frac{2\alpha+\beta}{2\tau}\right)(t-2\xi)\right\} - \left(\frac{2\alpha+\beta}{\beta}\right)\exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)(t-2\xi)\right\},\ t > 2\xi.$$

Then, the reliability function is given by

$$R(t) = P(T > t) = \left(\frac{2\alpha+2\beta}{\beta}\right)\exp\left\{-\left(\frac{2\alpha+\beta}{2\tau}\right)(t-2\xi)\right\}-\left(\frac{2\alpha+\beta}{\beta}\right)\exp\left\{-\left(\frac{\alpha+\beta}{\tau}\right)(t-2\xi)\right\},\ t > 2\xi. \quad (9)$$

5 Estimation of reliability measures

Sajesh (2007) considered location, scale and location-scale families arising out of ACBVE distribution proposed by Block and Basu (1974) with equal marginals and derived MREE, UMVUE and MLE of location, scale and location-scale parameters. In this section we discuss optimal estimation of reliability measures for parallel, series and standby systems.

5.1 Parallel system

UMVUE: The MTBF is $M = \frac{3\alpha+2\beta}{(\alpha+\beta)(2\alpha+\beta)}\tau+\xi$.

From Sajesh [3], the UMVUEs of ξ and τ are $\left\{T_1^* - \frac{T_2^*}{n(2n-1)(2\alpha+\beta)}\right\}$ and $\frac{T_2^*}{(2n-1)}$ respectively, where $T_1^* \sim E\left(\xi, \frac{\tau}{n(2\alpha+\beta)}\right)$ and $T_2^* \sim G\left(\frac{1}{\tau}, 2n-1\right)$.

Hence the UMVUE of M is

$$M^* = \left\{\frac{3\alpha+2\beta}{(\alpha+\beta)(2\alpha+\beta)}\right\}\left(\frac{T_2^*}{2n-1}\right)+\left\{T_1^* - \frac{T_2^*}{n(2n-1)(2\alpha+\beta)}\right\}.$$

$$= T_1^* + \left\{\frac{(3n-1)\alpha+(2n-1)\beta}{n(2n-1)(\alpha+\beta)(2\alpha+\beta)}\right\}T_2^*. \quad (10)$$

MREE: From Chandrasekar and Sajesh (2010), the MREE of $c\xi + d\tau$ is

$$M^{**} = cT_1^* + \frac{1}{2n}\left\{d - \frac{c}{n(2\alpha+\beta)}\right\}T_2^*.$$

Here $c = 1$, $d = \left\{\frac{3\alpha+2\beta}{(\alpha+\beta)(2\alpha+\beta)}\right\}$.

Then $$M^{**} = T_1^* + \left\{\frac{(3n-1)\alpha+(2n-1)\beta}{2n^2(\alpha+\beta)(2\alpha+\beta)}\right\}T_2^*. \quad (11)$$

MLE: The MLE of (ξ, τ) is $\hat{\xi} = T_1^*$ and $\hat{\tau} = \frac{1}{2n}T_2^*$.

Then the MLE of MTBF is $$\hat{M} = \frac{3\alpha+2\beta}{(\alpha+\beta)(2\alpha+\beta)}\hat{\tau}+\hat{\xi}. \quad (12)$$

The MLE of the reliability function is

$$\hat{R}(t) = \left(\frac{2\alpha+\beta}{\alpha}\right)\exp\left\{-\left(\frac{\alpha+\beta}{\hat{\tau}}\right)(t-\hat{\xi})\right\}-\left(\frac{\alpha+\beta}{\alpha}\right)\exp\left\{-\left(\frac{2\alpha+\beta}{\hat{\tau}}\right)(t-\hat{\xi})\right\},\ t > \hat{\xi}. \quad (13)$$

5.2 Series system

UMVUE: The MTBF is $M = \frac{\tau}{(2\alpha+\beta)}+\xi$.

From Sajesh (2007), the UMVUEs of ξ and τ are $\left\{T_1^* - \frac{T_2^*}{n(2n-1)(2\alpha+\beta)}\right\}$ and $\frac{T_2^*}{(2n-1)}$ respectively.

Hence the UMVUE of M is

$$M^* = \left\{\frac{1}{(2\alpha+\beta)}\right\}\left(\frac{T_2^*}{2n-1}\right) + \left\{T_1^* - \frac{T_2^*}{n(2n-1)(2\alpha+\beta)}\right\}.$$

$$= T_1^* + \left\{\frac{(n-1)}{(2n-1)(2\alpha+\beta)}\right\}T_2^*. \qquad (14)$$

MREE: From Chandrasekar and Sajesh (2010), the MREE of $c\xi + d\tau$ is

$$M^{**} = cT_1^* + \frac{1}{2n}\left\{d - \frac{c}{n(2\alpha+\beta)}\right\}T_2^*.$$

Here c =1, d = $\frac{1}{(2\alpha+\beta)}$.

Then $M^{**} = T_1^* + \left\{\frac{(n-1)}{2n^2(2\alpha+\beta)}\right\}T_2^*. \qquad (15)$

MLE: The MLE of (ξ, τ) is $\hat{\xi} = T_1^*$ and $\hat{\tau} = \frac{1}{2n}T_2^*T_2$.

Then the MLE of MTBF is $\hat{M} = \frac{\hat{\tau}}{(2\alpha+\beta)} + \hat{\xi}. \qquad (16)$

The MLE of the reliability function is $\hat{R}(t) = \exp\left\{-\left(\frac{2\alpha+\beta}{\hat{\tau}}\right)(t-\hat{\xi})\right\},\ t > \hat{\xi}. \qquad (17)$

5.3 Standby system

UMVUE: The MTBF is $M = \frac{4\alpha+3\beta}{(\alpha+\beta)(2\alpha+\beta)}\tau + 2\xi$.

From Sajesh (2007), the UMVUEs of ξ and τ are $\left\{T_1^* - \frac{T_2^*}{n(2n-1)(2\alpha+\beta)}\right\}$ and $\frac{T_2^*}{(2n-1)}$ respectively.

Hence the UMVUE of M is

$$M^* = \left\{\frac{4\alpha+3\beta}{(\alpha+\beta)(2\alpha+\beta)}\right\}\left(\frac{T_2^*}{2n-1}\right) + 2\left\{T_1^* - \frac{T_2^*}{n(2n-1)(2\alpha+\beta)}\right\}.$$

$$= 2T_1^* + \left\{\frac{(4n-2)\alpha + (3n-2)\beta}{n(2n-1)(\alpha+\beta)(2\alpha+\beta)}\right\}T_2^*. \qquad (18)$$

MREE: From Chandrasekar and Sajesh(2010), the MREE of $c\xi + d\tau$ is

$$M^{**} = cT_1^* + \frac{1}{2n}\left\{d - \frac{c}{n(2\alpha+\beta)}\right\}T_2^*.$$

Here c =2, d = $\frac{4\alpha+3\beta}{(\alpha+\beta)(2\alpha+\beta)}$.

Then $M^{**} = 2T_1^* + \left\{ \frac{(4n-2)\alpha + (3n-2)\beta}{2n^2(\alpha+\beta)(2\alpha+\beta)} \right\} T_2^*.$ (19)

MLE: The MLE of (ξ, τ) is $\hat{\xi} = T_1^*$ and $\hat{\tau} = \frac{1}{2n} T_2^*$.

Then the MLE of MTBF is $\hat{M} = \frac{4\alpha + 3\beta}{(\alpha+\beta)(2\alpha+\beta)} \hat{\tau} + 2\hat{\xi}.$ (20)

The MLE of the reliability function is

$$\hat{R}(t) = \left(\frac{2\alpha + 2\beta}{\beta} \right) \exp\left\{ -\left(\frac{2\alpha+\beta}{2\hat{\tau}} \right)(t - 2\hat{\xi}) \right\} - \left(\frac{2\alpha+\beta}{\beta} \right) \exp\left\{ -\left(\frac{\alpha+\beta}{\hat{\tau}} \right)(t - 2\hat{\xi}) \right\},\ t > 2\hat{\xi}. \quad (21)$$

References

Block, H.W. and Basu, A.P. (1974). A continuous bivariate exponential extension. *Journal of American Statistical Association*, 69, 1031 - 1037.
Chandrasekar, B. and Sajesh, T.A. (2010), Equivariant estimation for parameters of location-scale bivariate exponential models. *Journal of Applied Statistical Science*, 17, 541-548.
Sajesh, T.A. (2007). Absolutely continuous bivariate exponential location-scale models. *M.Phil Dissertation submitted to Loyola College (Autonomous), Chennai-600 034, University of Madras, India.*