STEP STRESS ACCELERATED LIFE TESTING PLAN FOR TWO PARAMETER PARETO DISTRIBUTION

Mustafa Kamal

●

Department of Statistics & Operations Research
(Aligarh Muslim University, Aligarh-202002, India)
E-mail: kamal19252003@gmail.com

Shazia Zarrin

●

Department of Statistics & Operations Research
(Aligarh Muslim University,Aligarh-202002, India)
E-mail: shaziazarrin@gmail.com

Arif-Ul-Islam

●

Department of Statistics & Operations Research
(Aligarh Muslim University, Aligarh-202002, India)
E-mail: arifislam2@yahoo.com

ABSTRACT

In Accelerated life testing if the accelerated test stress level is not high enough then many of the test items will not fail during the available time and one has to be prepared to handle a lot of censured data. To avoid such type of problems, a better way is step-stress ALT. In Step-stress ALT all test items are first tested at a specified constant stress for a specified period of time and then Items which are not failed will be tested at next higher level of stress for another specified time and so on until all items have failed or the test stops for other reasons. In this paper simple step stress pattern of ALT assuming that the lifetime of a product at any constant level of stress follow a two parameter Pareto distribution is considered. The maximum likelihood and asymptotic confidence interval estimate of the parameters are obtained. Optimal step stress ALT plan is proposed by minimizing the asymptotic variance of the MLE of the 100 P^{th} percentile of the lifetime distribution at normal stress condition. A simulation study is also performed to analyse the performance of parameter estimates.

KEYWORDS: Cumulative Exposure Model; Maximum Likelihood Estimation Method; Fisher Information Matrix; Asymptotic Confidence Intervals; Simulation Study.

1 INTRODUCTION

Accelerated life testing (ALT) is a quick way to obtain information about the life distribution of a material, component or product. In Accelerated life testing (ALT) items are subjected to conditions that are more severe than the normal ones, which yields shorter life but, hopefully, do not change the failure mechanisms. Some assumptions are needed in order to relate the life at high stress levels to life at normal stress levels in use. Based on these assumptions, the life distribution under normal stress levels can be estimated. Such way of testing reduces both time and cost.

Three types of stress loadings are usually applied in accelerated life tests: constant stress, step stress and Progressive-stress. Constant stress is the most common type of stress loading. Every item is tested under a constant level of the stress, which is higher than normal level. In this kind of testing, we may have several stress levels, which are applied for different groups of the tested items. This means that every item is subjected to only one stress level until the item fails or the test is stopped for other reasons. In Step-stress loading, the test items are subjected to successively higher levels of stress at pre-assigned test times. All items are first subjected to a specified constant stress for a specified period of time. Items that do not fail will be subjected to a higher level of stress for another specified time. The level of stress is increased step by step until all items have failed or the test stops for other reasons. Progressive-stress loading is quite like the step stress testing with the difference that the stress level increases continuously.

Failure data obtained from ALT can be divided into two categories: complete (all failure data are available) or censored (some of failure data are missing). Complete data consist of the exact failure time of test units, which means that the failure time of each sample unit is observed or known. In many cases when life data are analysed, all units in the sample may not fail. This type of data is called censored or incomplete data. Due to different types of censoring, censored data can be divided into time-censored (or type I censored) data and failure-censored (or type II censored) data. Time censored (or type I censored) data is usually obtained when censoring time is fixed, and then the number of failures in that fixed time is a random variable. Failure censored (or type II censored) data is obtained when the test is terminated after a specified number of failures, and then time to obtain that fixed number of failures is a random variable.

Simple step-stress ALT, where only one change of stress occurs, proposed by Nelson (1980) has been widely studied and referred as the Cumulative Exposure (CE) model. Many studies regarding SSALT planning based on the CE Model, have been performed. Miller and Nelson (1983) presented the optimum simple SSALT model. Bai *et al.* (1989) and Bai and Chun (1991) extended this model to the case where a prescribed censoring time is involved. Many authors also have provided the studies for statistical inference model for SSALT based on CEM; e.g., see Xiong (1998), Watkins (2001), Zhao and Elsayed (2005), Balakrishnan *el al.* (2009), Yeo and Tang (1999), Xiong and Ji (2004) and Xiong and Milliken (1999). Khamis and Higgins (1998) proposed a new model for SSALT as an alternative to the CEM, which is based on a time transformation of the exponential CEM. Most of works using the K-H model are concentrated on the optimal design plan for SSALT. Alhadeed and Yang (2002) provided the optimal plan for a simple SSALT using K-H model when the shape parameter is unknown.

More recently Lu and Rudy (2002) have dealt with the Weibull CE model under the inverse power law in the simple SSALT. McSorley, Lu and Li (2002) have shown the properties of the maximum likelihood (ML) estimators of parameters in the Weibull CE model with a log-linear function of stress on three-step SSALT data. Gounu, Sen and Balakrishnan (2004) tackled the optimal stress change points for multiple-step SSALT based on minimizing the asymptotic confidence interval of MLE of the mean life at design stress. Wu, Lin and Chen (2006) discussed the ALT with progressively Type-I group-censored exponential data. Balakrishnan and Han (2008) considered modification for censoring scheme in small sample sizes. Fan, Wang and Balakrishnan (2008) discussed the maximum likelihood (ML) estimation and Bayesian inference in group data ALT models under the relationship between the failure rate and the stress variables is linear under Box-Cox transformation. Al-Masri and Al-Haj Ebrahem (2009) derived the optimum times of changing stress level for simple step-stress plans under a cumulative exposure model assuming that the life time of a test unit follows a log-logistic distribution with known scale parameter by minimizing the asymptotic variance of the maximum likelihood estimator of the model parameters at the design stress with respect to the change time. Hassan and Al-Ghamdi (2009) obtained the optimal times of changing stress level for simple stress plans under a cumulative exposure model using the Lomax distribution for a wide range of values of the model parameters. Xu and Fei (2012) introduced and compared the four basic models for step-stress accelerated life testing: cumulative exposure model (CEM), linear cumulative exposure model (LCEM), tampered random variable model (TRVM), and tampered failure rate model (TFRM). Limitations of the four models are also introduced for better use of the models.

In this paper the two-parameter Pareto distribution as a lifetime model under simple-step-stress ALT is considered. Maximum likelihood estimates of parameters and their asymptotic confidence intervals are obtained. The performance of the estimates is evaluated by a simulation study with different pre-fixed values of parameters.

2 THE MODEL

2.1 The Pareto Distribution

The concept of this distribution was first introduced by Vilfredo Pareto (1897) in his well-known economics text "*Cours d'Economie Politique"*.

The two parameter forms of Pareto probability density function (pdf), cumulative distribution function (CDF), the reliability function (RF) and the hazard rate (HR) with shape parameter α and scale parameter θ given respectively by

$$f(t;\theta,\alpha)=\frac{\alpha\theta^{\alpha}}{(\theta+t)^{\alpha+1}}; \qquad t>0,\theta>0,\alpha>0 \tag{2.1}$$

$$F(t)=1-\frac{\theta^{\alpha}}{(\theta+t)^{\alpha}}; \qquad t>0,\theta>0,\alpha>0 \tag{2.2}$$

$$R(t)=\frac{\theta^{\alpha}}{(\theta+t)^{\alpha}} \tag{2.3}$$

$$h(t)=\frac{\alpha}{\theta+t} \tag{2.4}$$

The hazard rate (HR) is a decreasing function as $t>0$ and an increasing function as $t<0$.

2.2 Assumptions and Test Procedure

1. There two stress levels x_1 and x_2 $(x_1<x_2)$.
2. The failure time of a test unit follows a two-parameter Pareto distribution at every stress level.
3. A random sample of n identical products is placed on test under initial stress level x_1 and run until time τ, and then the stress is changed to x_2 and the test is continued until all products fail.
4. The lifetimes of the products at each stress level are i.i.d.
5. The scale parameter is a log-linear function of stress. That is, $\log\theta(x_i)=a+bx_i$, $i=1,2$ where a and b are unknown parameters depending on the nature of the product and the test method. Therefore, the lifetime of a test product at lower stress x_1 is longer than at higher stress x_2.
6. The Pareto shape parameter α is constant, i.e. independent of stress.
7. A cumulative exposure model holds, that is, the remaining life of test items depends only on the current cumulative fraction failed and current stress regardless of how the fraction accumulated. Moreover, if held at the current stress, items will fail according to the CDF of stress, but starting at the previously accumulated fraction failed, for more detail on CE Model see Nelson (1990). According to cumulative exposure model the CDF in step-stress ALT are given by

$$F(t)=\begin{cases}F_1(t) & 0\le t<\tau\\ F_2(t-\tau+\tau') & \tau\le t<\infty\end{cases}$$

where the equivalent starting time, τ', is a solution of $F_1(\tau) = F_2(\tau')$ solving for τ', then $\tau' = \frac{\theta_2}{\theta_1}\tau$ and now the CDF is of the form

$$F(t) = \begin{cases} F_1(t), & 0 < t < \tau \\ F_2\left(\frac{\theta_2}{\theta_1}\tau + t - \tau\right), & \tau \le t < \infty \end{cases} \tag{2.5}$$

and corresponding pdf is obtained as as

$$f(t) = \begin{cases} f_1(t), & 0 < t < \tau \\ f_2\left(\frac{\theta_2}{\theta_1}\tau + t - \tau\right), & \tau \le t < \infty \end{cases} \tag{2.6}$$

From the assumptions of cumulative exposure model and the equation (2.2), the CDF of a test product failing according to Pareto distribution under simple step-stress test is given by

$$F(t) = \begin{cases} 1 - \dfrac{\theta_1^\alpha}{(\theta_1 + t)^\alpha}, & 0 < t < \tau \\ 1 - \dfrac{\theta_2^\alpha}{\left[\theta_2 + \tau\left(\dfrac{\theta_2}{\theta_1} - 1\right) + t\right]^\alpha} & \tau \le t < \infty \end{cases} \tag{2.7}$$

The PDF corresponding to (2.6) becomes

$$f(t) = \begin{cases} \dfrac{\alpha\theta_1^\alpha}{(\theta_1 + t)^{\alpha+1}}, & 0 < t < \tau \\ \dfrac{\alpha\theta_2^\alpha}{\left[\theta_2 + \tau\left(\dfrac{\theta_2}{\theta_1} - 1\right) + t\right]^{\alpha+1}} & \tau \le t < \infty \end{cases} \tag{2.8}$$

2.3 Objective of Study

For pre-fixed sample size n and the testing stress levels x_1 and x_2, the first objective is estimating the parameters a, b and α in a simple step-stress accelerated life test. The second objective is to obtain the optimal stress changing time τ which minimizes the asymptotic variance of the MLE of the P^{th} percentile of the lifetime distribution at normal stress condition $t_p(x_0)$.

3 ESTIMATION PROCEDURE

3.1 Point Estimates

Here the maximum likelihood method of estimation is used because ML method is very robust and gives the estimates of parameter with good statistical properties. In this method, the estimates of parameters are those values which maximize the sampling distribution of data. However, ML estimation method is very simple for one parameter distributions but its implementation in ALT is mathematically more intense and, generally, estimates of parameters do not exist in closed form, therefore, numerical techniques such as Newton Method, Some computer programs are used to compute them.

For obtaining the MLE of the model parameters, let t_{ij}, $j = 1,2,\cdots n_i$, $i = 1,2$ be the observed failure times of a test unit j under stress level i, where n_1 denotes the number of units failed at the low stress x_1 and n_2 denotes the number of units failed at higher stress level x_2. Therefore, the likelihood function for two-parameter Pareto distribution for simple step stress pattern can be written in the following form

$$L(\theta_1, \theta_2, \alpha) = \prod_{j=1}^{n_1} \frac{\alpha\theta_1^{\alpha}}{(\theta_1 + t_{1j})^{\alpha+1}} \prod_{j=1}^{n_2} \frac{\alpha\theta_2^{\alpha}}{\left[\theta_2 + \tau\left(\frac{\theta_2}{\theta_1} - 1\right) + t_{2j}\right]^{\alpha+1}} \tag{3.1}$$

The log-likelihood function $l\ (= \log L)$ corresponding to equation (3.1) can be rewritten as

$$\begin{aligned} l = \log L = n\log\alpha + n_1\alpha\log\theta_1 + n_2\alpha\log\theta_2 - (\alpha+1)\sum_{j=1}^{n_1}\log(\theta_1 + t_{1j}) \\ - (\alpha+1)\sum_{j=1}^{n_2}\log\left[\theta_2 + \tau\left(\frac{\theta_2}{\theta_1} - 1\right) + t_{2j}\right] \end{aligned} \tag{3.2}$$

where $n = n_1 + n_2$.

Now by using the relation $\log\theta(x) = a + bx_i$, $i = 1,2$ for the scale parameter θ, in (3.2), the likelihood function becomes,

$$l = n\log\alpha + n_1\alpha(a + bx_1) + n_2\alpha(a + bx_2) - (\alpha+1)\sum_{j=1}^{n_1}\log(e^{a+bx_1} + t_{1j})$$

$$- (\alpha+1)\sum_{j=1}^{n_2}\log\left[e^{a+bx_2} + \tau\left(\frac{e^{a+bx_2}}{e^{a+bx_1}} - 1\right) + t_{2j}\right] \tag{3.3}$$

Differentiating (3.3) partially with respect to a, b and α, we get

$$\frac{\partial l}{\partial a} = n\alpha - (\alpha+1)\sum_{j=1}^{n_1}\frac{e^{a+bx_1}}{[e^{a+bx_1} + t_{1j}]} - (\alpha+1)\sum_{j=1}^{n_2}\frac{e^{a+bx_2}}{[e^{a+bx_2} + \tau(e^{b(x_2-x_1)} - 1) + t_{2j}]} \tag{3.4}$$

$$\frac{\partial l}{\partial b} = n_1x_1\alpha + n_2x_2\alpha - (\alpha+1)\sum_{j=1}^{n_1}\frac{x_1e^{a+bx_1}}{[e^{a+bx_1} + t_{1j}]} - (\alpha+1)\sum_{j=1}^{n_2}\frac{x_2e^{a+bx_2} + \tau(x_2 - x_1)e^{b(x_2-x_1)}}{[e^{a+bx_2} + \tau(e^{b(x_2-x_1)} - 1) + t_{2j}]} \tag{3.5}$$

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} + n_1(a + bx_1) + n_2(a + bx_2) - \sum_{j=1}^{n_1}\log[e^{a+bx_1} + t_{1j}] - \sum_{j=1}^{n_2}\log[e^{a+bx_2} + \tau(e^{b(x_2-x_1)} - 1) + t_{2j}] \tag{3.6}$$

From (3.6) the maximum likelihood estimates of α is given by the following equation:

$$\hat{\alpha} = \frac{n}{\psi_1 + \psi_2 - n_1(a + bx_1) - n_2(a + bx_2)} \tag{3.7}$$

where,

$$\psi_1 = \sum_{j=1}^{n_1} \log[e^{a+bx_1} + t_{1j}],$$

$$\psi_2 = \sum_{j=1}^{n_2} \log[e^{a+bx_2} + \tau(e^{b(x_2-x_1)} - 1) + t_{2j}].$$

By substituting for α into (3.4) and (3.5), the system equations are reduced into the following two non-linear equations:

$$\begin{aligned}\frac{\partial l}{\partial a} &= \frac{n^2}{\psi_1 + \psi_2 - n_1(a + bx_1) - n_2(a + bx_2)} \\ &- \left(\frac{n}{\psi_1 + \psi_2 - n_1(a + bx_1) - n_2(a + bx_2)} + 1\right)\sum_{j=1}^{n_1} \frac{e^{a+bx_1}}{[e^{a+bx_1} + t_{1j}]} \\ &- \left(\frac{n}{\psi_1 + \psi_2 - n_1(a + bx_1) - n_2(a + bx_2)} + 1\right)\sum_{j=1}^{n_2} \frac{e^{a+bx_1}}{[e^{a+bx_2} + \tau(e^{b(x_2-x_1)} - 1) + t_{2j}]}\end{aligned} \tag{3.8}$$

$$\begin{aligned}\frac{\partial l}{\partial b} &= \left(\frac{n}{\psi_1 + \psi_2 - n_1(a + bx_1) - n_2(a + bx_2)}\right)(n_1x_1 + n_2x_2) \\ &- \left(\frac{n}{\psi_1 + \psi_2 - n_1(a + bx_1) - n_2(a + bx_2)} + 1\right)\sum_{j=1}^{n_1} \frac{x_1e^{a+bx_1}}{[e^{a+bx_1} + t_{1j}]} \\ &- \left(\frac{n}{\psi_1 + \psi_2 - n_1(a + bx_1) - n_2(a + bx_2)} + 1\right)\sum_{j=1}^{n_2} \frac{x_2e^{a+bx_2} + \tau(x_2 - x_1)e^{b(x_2-x_1)}}{[e^{a+bx_2} + \tau(e^{b(x_2-x_1)} - 1) + t_{2j}]}\end{aligned} \tag{3.9}$$

Since (3.8) and (3.9) are non linear equations, their solutions are numerically obtained by using Newton Raphson method. They are solved simultaneously to obtain a and b. Then by substitution in (3.7) an estimate of α is easily obtained.

3.2 Interval Estimates

According to large sample theory, the maximum likelihood estimators, under some appropriate regularity conditions, are consistent and normally distributed. Since ML estimates of parameters are not in closed form, therefore, it is impossible to obtain the exact confidence intervals, so asymptotic confidence intervals based on the asymptotic normal distribution of ML estimators instead of exact confidence intervals are obtained here.

The Fisher-information matrix composed of the negative second partial derivatives of log likelihood function can be written as

$$F = \begin{bmatrix} -\frac{\partial^2 l}{\partial a^2} & -\frac{\partial^2 l}{\partial a \partial b} & -\frac{\partial^2 l}{\partial a \partial \alpha} \\ -\frac{\partial^2 l}{\partial b \partial a} & -\frac{\partial^2 l}{\partial b^2} & -\frac{\partial^2 l}{\partial b \partial \alpha} \\ -\frac{\partial^2 l}{\partial \alpha \partial a} & -\frac{\partial^2 l}{\partial \alpha \partial b} & -\frac{\partial^2 l}{\partial \alpha^2} \end{bmatrix}$$

The elements of information matrix F are:

$$\frac{\partial^2 l}{\partial a^2} = -(\alpha+1)\sum_{j=1}^{n_1}\frac{[e^{a+bx_1}+t_{1j}]e^{a+bx_1}-e^{2(a+bx_1)}}{[e^{a+bx_1}+t_{1j}]^2}$$

$$-(\alpha+1)\sum_{j=1}^{n_2}\frac{[e^{a+bx_2}+\tau(e^{b(x_2-x_1)}-1)+t_{2j}]e^{a+bx_2}-e^{2(a+bx_2)}}{[e^{a+bx_2}+\tau(e^{b(x_2-x_1)}-1)+t_{2j}]^2}$$

$$\frac{\partial^2 l}{\partial b^2} = -(\alpha+1)\sum_{j=1}^{n_1}\frac{[(x_1^2 e^{a+bx_1})t_{1j}]}{[e^{a+bx_1}+t_{1j}]^2}$$

$$-(\alpha+1)\sum_{j=1}^{n_2}\frac{[e^{a+bx_2}+\tau(e^{b(x_2-x_1)}-1)+t_{2j}][x_2^2 e^{a+bx_2}+\tau(x_2-x_1)^2 e^{b(x_2-x_1)}]}{[e^{a+bx_2}+\tau(e^{b(x_2-x_1)}-1)+t_{2j}]^2}$$

$$-\frac{[x_2 e^{a+bx_2}+\tau(x_2-x_1)e^{b(x_2-x_1)}]^2}{[e^{a+bx_2}+\tau(e^{b(x_2-x_1)}-1)+t_{2j}]^2}$$

$$\frac{\partial^2 l}{\partial \alpha^2} = -\frac{n}{\alpha^2}$$

$$\frac{\partial^2 l}{\partial a \partial b} = \frac{\partial^2 l}{\partial b \partial a} = -(\alpha+1)\sum_{j=1}^{n_1}\frac{[(x_1 e^{a+bx_1})t_{1j}]}{[e^{a+bx_1}+t_{1j}]^2}$$

$$-(\alpha+1)\sum_{j=1}^{n_2}\frac{[e^{a+bx_2}+[\tau(x_1 e^{b(x_2-x_1)}-x_2)+x_2 t_{2j}]}{[e^{a+bx_2}+\tau(e^{b(x_2-x_1)}-1)+t_{2j}]^2}$$

$$\frac{\partial^2 l}{\partial a \partial \alpha} = \frac{\partial^2 l}{\partial \alpha \partial a} = n - \sum_{j=1}^{n_1}\frac{e^{a+bx_1}}{[e^{a+bx_1}+t_{1j}]} - \sum_{j=1}^{n_2}\frac{e^{a+bx_1}}{[e^{a+bx_2}+\tau(e^{b(x_2-x_1)}-1)+t_{2j}]}$$

$$\frac{\partial^2 l}{\partial \alpha \partial b} = \frac{\partial^2 l}{\partial b \partial \alpha} = n_1 x_1 + n_2 x_2 - \sum_{j=1}^{n_1}\frac{x_1 e^{a+bx_1}}{[e^{a+bx_1}+t_{1j}]} - \sum_{j=1}^{n_2}\frac{x_2 e^{a+bx_2}+\tau(x_2-x_1)e^{b(x_2-x_1)}}{[e^{a+bx_2}+\tau(e^{b(x_2-x_1)}-1)+t_{2j}]}$$

The asymptotic variance-covariance matrix of $\hat{a}, \hat{b}$ and $\hat{\alpha}$ is obtained by inverting the Fisher-information matrix that is

$$\Sigma = \begin{bmatrix} -\frac{\partial^2 l}{\partial a^2} & -\frac{\partial^2 l}{\partial a \partial b} & -\frac{\partial^2 l}{\partial a \partial \alpha} \\ -\frac{\partial^2 l}{\partial b \partial a} & -\frac{\partial^2 l}{\partial b^2} & -\frac{\partial^2 l}{\partial b \partial \alpha} \\ -\frac{\partial^2 l}{\partial \alpha \partial a} & -\frac{\partial^2 l}{\partial \alpha \partial b} & -\frac{\partial^2 l}{\partial \alpha^2} \end{bmatrix}^{-1} = F^{-1}$$

$$= \begin{bmatrix} AVar(\hat{a}) & ACov(\hat{a}\hat{b}) & ACov(\hat{a}\hat{\alpha}) \\ ACov(\hat{b}\hat{a}) & AVar(\hat{b}) & ACov(\hat{b}\hat{\alpha}) \\ ACov(\hat{\alpha}\hat{a}) & ACov(\hat{\alpha}\hat{b}) & AVar(\hat{\alpha}) \end{bmatrix}$$

Now, the two-sided approximate $100\lambda\%$ confidence limits for population parameters $\hat{a}, \hat{b}$ and $\hat{\alpha}$ can be constructed as

$$\left[\hat{a} \pm Z_{\lambda} \sqrt{AVar(\hat{a})}\right]$$
$$\left[\hat{b} \pm Z_{\lambda} \sqrt{AVar(\hat{b})}\right]$$
$$\left[\hat{\alpha} \pm Z_{\lambda} \sqrt{AVar(\hat{\alpha})}\right]$$

4 OPTIMAL TEST PLAN

The optimum criterion here is to find the optimum stress change time τ. Since the accuracy of ML method is measured by the asymptotic variance of the MLE of the $100\,P^{th}$ percentile of the lifetime distribution at normal stress condition $t_p(x_0)$, therefore the optimum value of the stress change time will the value which minimizes the asymptotic variance of the MLE of $t_p(x_0)$.

The $100\,P^{th}$ percentile of a distribution $F()$ is the age t_p by which a proportion of population fails Nelson (1990). It is a solution of the equation $P = F(t_p)$, therefore the $100\,P^{th}$ percentile for Pareto distribution is

$$t_p = \frac{\theta\left\{1-(1-P)^{1/\alpha}\right\}}{(1-P)^{1/\alpha}}$$

The $100\,P^{th}$ percentile for Pareto distribution at use condition is

$$t_p(x_0) = \frac{\exp(a+bx_0)\left\{1-(1-P)^{1/\alpha}\right\}}{(1-P)^{1/\alpha}}$$

Now the asymptotic variance of MLE of the $100\,P^{th}$ percentile at normal operating conditions is given by

$$AVar(t_p(\tilde{x}_0)) = \left[\frac{\partial t_p(\hat{x}_0)}{\partial \hat{a}}, \frac{\partial t_p(\hat{x}_0)}{\partial \hat{b}}, \frac{\partial t_p(\hat{x}_0)}{\partial \hat{\alpha}}\right] \Sigma \left[\frac{\partial t_p(\hat{x}_0)}{\partial \hat{a}}, \frac{\partial t_p(\hat{x}_0)}{\partial \hat{b}}, \frac{\partial t_p(\hat{x}_0)}{\partial \hat{\alpha}}\right]^{-1}$$

The optimum stress change time τ will be the value which minimizes $AVar(t_p(\hat{x}_0))$.

5 SIMULATION STUDY

To evaluate the performance of the method of inference described in present study, several data sets with sample sizes n=100, 200,...,500 are generated for from two-parameter Pareto distribution. The values for true parameters and stress combinations are chosen to be $a = 0.5, b = 0.2$ $\alpha = 1.5$ and $(x_1, x_2) = (2,4), (3,5)$. The estimates and the corresponding summary statistics are obtained by the present Step Stress ALT model and the Newton iteration method. For different given samples and stresses combinations with $a = 0.5, b = 0.2$ and $\alpha = 1.5$, the ML estimates , asymptotic variance, the asymptotic standard error (SE), the mean squared error (MSE) and the coverage rate of the 95% confidence interval for a, b and α are obtained. Table-1 and 2 summarize the results of the estimates for a, b and α. The numerical results presented in Table-1 and 2 are based on 1000 simulation replications.

Table1: Simulations results based on Step stress with $a = 0.5, b = 0.2$ $\alpha = 1.5$ and $(x_1, x_2) = (2, 4)$

Sample Size *n*	Parameter	MLE	Variance	SE	MSE	95% Asymptotic CI Coverage
	a	0.51069	0.01349	0.01368	0.01360	0.94282
100	b	0.20353	0.00084	0.00083	0.00085	0.95030
	α	1.52507	0.04789	0.04737	0.04851	0.94984
	a	0.50734	0.00614	0.00653	0.00619	0.95866
200	b	0.20098	0.00038	0.00040	0.00038	0.95766
	α	1.50620	0.02194	0.02290	0.02198	0.95595
	a	0.50339	0.00425	0.00428	0.00426	0.94789
300	b	0.20132	0.00027	0.00027	0.00027	0.95075
	α	1.50997	0.01561	0.01527	0.01571	0.95090
	a	0.50258	0.00302	0.00318	0.00303	0.95290
400	b	0.20096	0.00019	0.00020	0.00019	0.96192
	α	1.50735	0.01095	0.01141	0.01100	0.96200
	a	0.50323	0.00239	0.00255	0.00241	0.95795
500	b	0.20059	0.00015	0.00016	0.00015	0.95595
	α	1.50444	0.00895	0.00908	0.00897	0.95595

Table2: Simulations results based on Step stress with $a = 0.5, b = 0.2$ $\alpha = 1.5$ and $(x_1, x_2) = (3, 5)$

Sample Size *n*	Parameter	MLE	Variance	SE	MSE	95% Asymptotic CI Coverage
	a	0.51740	0.01333	0.01404	0.01363	0.94964
100	b	0.20057	0.00040	0.00040	0.00040	0.95066
	α	1.50444	0.02299	0.02258	0.02301	0.95159
	a	0.50652	0.00793	0.00888	0.00797	0.94478
200	b	0.20171	0.00027	0.00027	0.00028	0.94979
	α	1.51273	0.01582	0.01521	0.01598	0.94887
	a	0.50692	0.00578	0.00633	0.00583	0.95030
300	b	0.20072	0.00020	0.00020	0.00020	0.94726
	α	1.50522	0.01152	0.01133	0.01155	0.94736
	a	0.50487	0.00443	0.00501	0.00446	0.96146
400	b	0.20067	0.00015	0.00015	0.00015	0.95740
	α	1.50506	0.00872	0.00900	0.00874	0.95740
	a	0.50257	0.00279	0.00298	0.00279	0.95595
500	b	0.20084	0.00011	0.00011	0.00011	0.95682
	α	1.50453	0.00555	0.00568	0.00557	0.95095

6 SUMMARY AND CONCLUDING REMARKS

This paper deals with parameter estimation of Pareto distribution under simple step stress ALT plan. The MLEs of the model parameters were obtained. The MLEs, the asymptotic variance and covariance of model parameters were obtained. Based on the asymptotic normality, the coverage rate of 95% confidence intervals of the model parameters are obtained. Optimal plan for step stress ALT is also determined by minimizing the asymptotic variance of the MLE of the $100P^{th}$ percentile of the lifetime distribution at normal stress condition.

From results in Table 1 and 2, it is observed that $\hat{a},\hat{b}$ and $\hat{\alpha}$ estimates the true parameters a, b and α quite well respectively with relatively small mean squared errors. The estimated standard error also approximates well the sample standard deviation. For a fixed a, b and α we find that as n increases, variance, standard error and the mean squared errors of $\hat{a},\hat{b}$ and $\hat{\alpha}$ get smaller. This is because that a larger sample size results in a better large sample approximation. It is also noticed that the coverage probabilities of the asymptotic confidence interval are close to the nominal level and do not change much across the five different sample sizes. In short, it is reasonable to say that the present step stress ALT plan works well and has a promising potential in the analysis of accelerated life testing.

REFERENCES

Alhadeed, A. A. and Yang, S. S. (2002): Optimal Simple Step-Stress Plan for Khamis-Higgins Model, IEEE Transactions on Reliability, vol. 51, 212-215.

Al-Masri, A.Q. and Al-Haj Ebrahem, M. (2009): Optimum Times for Step-Stress Cumulative Exposure Model Using Log-Logistic Distribution with Known Scale Parameter, Austrian Journal of Statistics, vol. 38, 59–66.

Bai, D.S., Chun, Y.R. (1991): Optimum simple step-stress accelerated life tests with competing causes of failure, IEEE Transactions on Reliability, vol.40, 622-627.

Bai, D.S., Kim, M.S. and Lee, S.H. (1989): Optimum simple step-stress accelerated life tests with censoring, IEEE Transactions on Reliability, vol. 38, 528-532.

Balakrishnan, N. and Han, D. (2008): Optimal step-stress testing for progressively Type-I censored data from exponential distribution, Journal of Statistical Planning and Inference, vol. 138, 4172-4186.

Balakrishnan, N., Beutner, E., and Kateri, M. (2009): Order Restricted Inference for Exponential Step-Stress Models, IEEE Transactions on Reliability, vol. **58**, 132-142.

Fan T.H., Wang W.L. and Balakrishnan, N. (2008): Exponential progressive step-stress life testing with link function based on Box-Cox transformation, Journal of Statistical Planning and Inference, vol. 138, 2340-2354.

Gouno, E., Sen, A. and Balakrishnan, N. (2004): Optimal step-stress test under progressive Type-I censoring, IEEE Transactions on Reliability, vol. 53, 383-393.

Hassan, A. S. and Al-Ghamdi, A. S. (2009): Optimum Step Stress Accelerated Life Testing for Lomax Distribution, Journal of Applied Sciences Research, vol. 5, 2153-2164.

Khamis, I.H. and Higgins, J.J. (1998): A new model for step-stress testing, IEEE Transactions on Reliability, vol.47, 131-134.

Lu, M.W. & Rudy, R.J. (2002): Step-stress accelerated test, International Journal of Materials & Product Technology, vol. 17, 425-434.

McSorley, E.O., Lu, J.C. and Li, C.S. (2002): Performance of parameter-estimates in step-stress accelerated life-tests with various sample-sizes, *IEEE Transactions on Reliability*, vol. 51, 271-277.

Miller, R. and Nelson, W. (1983): Optimum simple step stress plans for accelerated life testing, IEEE Transactions on Reliability, vol. 32, 59-65.

Nelson, W. (1980): Accelerated life testing step-stress models and data analysis, IEEE Transactions on Reliability, vol. 29, 103-108.

Nelson, W. (1990): Accelerated Testing: Statistical Models, Test Plans and Data Analyses, John Wiley & Sons, New York.

Pareto, V. (1897): *Cours d'Economie Politique*, 2. F. Rouge, Lausanne, Switzerland.

Watkins, A. J. (2001): Commentary: inference in simple step-stress models, IEEE Transactions on Reliability, vol. 50, 36-37.

Wu, S.J., Lin, Y.P. and Chen, Y.J. (2006): Planning step-stress life test with progressively type I group-censored exponential data, Statist. Neerlandica, vol. 60, 46-56.

Xiong, C. (1998): Step stress model with threshold parameter, *Journal of Statistical Computation and Simulation*, vol. 63, 349–360.

Xiong, C. and Ji, M. (2004): Analysis of grouped and censored data from step-stress life test, IEEE Transactions on Reliability, vol. 53, 22–28.

Xiong, C. and Milliken, G. A. (1999): Step-stress life-testing with random stress-change times for exponential data, IEEE Transactions on Reliability, vol. 48, 141-148.

Xu, H.Y. and Fei, H. (2012): Models Comparison for Step-Stress Accelerated Life Testing, Communications in Statistics - Theory and Methods, vol. 41, 3878-3887.

Yeo, K.P. and Tang, L.C. (1999): Planning step-stress life-test with a target accelerated factor, IEEE Transactions on Reliability, vol. 48, 61-67.

Zhao, W. and Elsayed, E. A. (2005): A General accelerated life model for step-stress testing. IIE Transactions, vol. **37**, 1059-1069.